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Theory of supply chains: a working capital approach*

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Abstract

This paper presents a "time-to-build" theory of supply chains which implies a key role for the financing of working capital as a determinant of supply chain length. We apply our theory to offshoring and trade, where firms strike a balance between the productivity gain due to offshoring against the greater financial cost due to longer supply chains. In equilibrium, the ratio of trade to GDP, inventories and productivity are procyclical and closely track financial conditions.

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1 Introduction

Production takes time, especially when conducted through long supply chains. Working capital in the form of inventories and receivables bridges the timing mismatch between incurring costs and receiving cash from sales. To the extent that the financing cost of working capital matters, the length of supply chains is not only a matter of the economic fundamentals (such as the production technology or trade barriers) but is also shaped by financial conditions.

In this paper, we lay out a theory of supply chains where financial conditions play a pivotal role in the determination of the length of supply chains. Through this theory, we highlight a novel channel for macro fluctuations through *investment in working capital*, which bears a strong analogy with investment in physical capital, but which operates across groups of firms, rather than at the individual firm level. We highlight the associated repercussions of financing conditions on productivity and the volume of international trade.

By highlighting the analogy between physical capital and working capital on the firm's balance sheet, our theory suggests a reorientation in the way that economists think of inventories. Rather than being a buffer stock that smooths shocks, inventories in transit reflect the choice in working capital investment underpinning global supply chains. Tom Friedman's (2005) popular book on globalization ("The World is Flat") has an apt quote from the chief executive officer of UPS in this respect. The UPS CEO is quoted as saying:

> "When our grandfathers owned shops, inventory was what was in the back room. Now it is a box two hours away on a package car, or it might be hundreds more crossing the country by rail or jet, and you have thousands more crossing the ocean" [Friedman (2005, p. 174)]

Our theory gives more concrete form to the idea that inventories in transit reflect the investment necessary to set up and sustain global value chains.

The key feature of our theory is a non-linearity in the working capital necessary to sustain long supply chains. This feature is best explained through

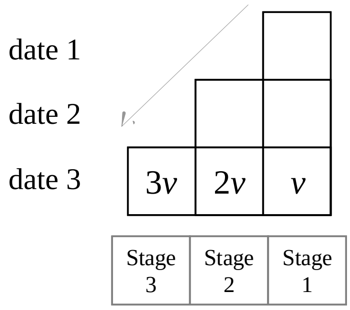

Figure 1. Inventories of a firm with a three-stage production process. At date 3, the firm has three vintages of inventories, and older vintages have higher value reflecting greater inputs in the past.

a diagram. Consider Figure 1 which depicts the inventories of a firm with a three-stage production process. The firm undertakes the first production stage at date 1, sends the intermediate good to stage 2 in date 2. At date 3, the firm has three vintages of inventories. The oldest inventory (3 periods old) has the highest value ($3v$) reflecting greater inputs in the past. The next oldest inventory (2 periods old) has the next highest value ($2v$), and so on.

In this setting, the value of the total *stock* of inventories carried by the firm (given by $v + 2v + 3v$) can be represented by the area of the triangle in Figure 1. Since the area of the triangle is increasing at the rate of the *square* of the length of the production chain, the value of the stock of inventories is also increasing at the same rate. In this way, the working capital need becomes highly sensitive to the length of the chain, necessitating much greater incremental financing needs as production chains become longer.

We make these ideas more precise in an equilibrium setting by starting with a benchmark 'Austrian' model of production where the output from one stage of production can be used as an input into production at a subsequent step. The 'Austrian' label makes reference to the idea of "roundabout production" in the terminology of Böhm-Bawerk (1884) and the Austrian school of capital theory, where the time dimension of production introduces an intertemporal tradeoff between the interest rate and productivity. Antras (2022, 2023) has breathed new life into the Austrian approach in studying global value chains (a more detailed comparison follows below).

In our context, the extent of roundabout production entails a choice in investment in total working capital employed, together with an associated credit demand. In our model, firms strike a balance between the productivity gain due to more roundabout production chain against the greater financing cost due to larger working capital. We show that an increase in the interest rate leads to a reduction in the the number of production stages and credit demand. Closing the model with a credit supply function leads to the joint determination of the interest rate and the total utilized working capital.

The focus of our analysis is on the ratio of gross sales along the chain to the value of the final good. Equivalently, our focus is on the ratio of gross sales to total value-added along the chain. The gross sales to value-added ratio serves as a summary measure of the extent of supply chain activity in the economy that utilizes intermediate inputs. We derive a tractable closed form solution for the gross sales to value-added ratio.

In the context of international trade, the ratio of gross sales to value-added has a natural counterpart in the ratio of global trade to global GDP. The trade-to-GDP ratio is an important indicator of globalization through trade activity, and our model identifies the determinants of this key ratio. We do so by applying our framework to the optimal offshoring decision of a multinational firm. Even when the sequential production process is largely determined by the technology, a multinational firm may nevertheless have considerable leeway to choose its production time profile through the extent of offshoring. Offshoring can lower costs and raise productivity, but the financial cost of holding larger inventories introduces a countervailing element. The firm must finance inventories in transit as assets on the balance sheet, and the cost of financing will affect the net benefits of offshoring. We derive closed form solutions and show that the ratio of trade to GDP is highly procyclical and fluctuates with financial conditions. Easier credit conditions are associated with higher trade relative to GDP, higher inventories in transit and higher productivity.

Our paper touches on several strands of the literature. Most closely related to our paper is the recent work of Antras (2022, 2023) who proposes a model of sequential production with a pre-determined number of stages, but in which the time length of each stage is a choice variable in the spirit of

Findlay (1978). In such a setting, Antras (2022) shows that a lower interest rate is associated with longer production times at each stage, higher wages and higher final goods output.

Our model explores the complementary notion of 'roundaboutness' in the *number of stages* in the supply chain, where each stage takes a fixed unit of time to complete. As with Antras (2022), a lower interest rate in our model is associated with higher output, higher productivity and wages, but one key difference is our focus on the gross sales to value-added ratio and the associated trade-to-GDP ratio. Another difference is how the model is closed to derive the equilibrium interest rate that determine the real economic outcomes. Antras (2022) introduces a capital market where the interest rate follows from the rate of time preference of agents. In our case, the model is closed by introducing a banking sector and the equilibrium interest rate is determined by credit market clearing. Given these differences, the model in Antras (2022) is a good match for longer-term economic questions, while our model is perhaps better suited for questions of how fluctuations in credit conditions at the business cycle frequency impact supply chain activity.

Our focus on financial conditions as a determinant of trade fluctuations places our work in the literature on trade and finance. It is well known that merchanise trade is dependent on bank finance for working capital (Amiti and Weinstein (2011), Niepmann and Schmidt-Eisenlohr (2017a)) and that global banks play the key intermediation role (BIS (2014), Niepmann and Schmidt-Eisenlohr (2017b), Caballero, Candelaria and Hale (2018), Claessens and Van Horen (2021)). In this vein, Minetti et al. (2019) find that credit conditions play a role in firms' decision to participate in supply chains. Our theory sheds light on the mechanisms involved.

Our approach holds promise in identifying the role of supply chains in the co-movement of macro fluctuations. Huo et al. (2019) find that the degree of co-movement in output across economies is larger than predicted by macro models and argue that two correlated shocks across countries (TFP and labor shocks) can go some way toward a reconciliation. In our model, fluctuations in supply chain activity inject shocks that resemble TFP shocks. In this respect, supply chains may be a useful ingredient in thinking about the international co-movement puzzle (Backus et al. (1993), Kose and Yi (2006)).

Our theory also sheds light on the role of exchange rates in trade and macro fluctuations. Cook and Patel (2022) show in a model with dollar invoicing and global value chains that a contractionary monetary shock reduces the ratio of gross to value added exports, a pattern confirmed in the data. Bruno, Kim and Shin (2018) and Bruno and Shin (2020) show that the broad dollar index has attributes of a barometer of financial conditions, whereby a stronger dollar is associated with tighter credit conditions and a slower growth in trade. In particular, in a detailed micro study, Bruno and Shin (2020) match export shipments with loans to show that exporting firms that are more reliant for credit from banks that have a greater reliance on wholesale dollar funding suffer a sharper slowdown in exports due to the greater fluctuations in credit availability from such banks.

Our paper has a point of contact with the large literature on global value chains (see Antras and Chor (2022) for a recent survey) and the formation of production networks (Acemoglu and Azar (2020)) and the propagation of shocks through interconnected sectors (Di Giovanni, Kalemli-Ozcan, Silva and Yildrim (2022)). Our focus on the role of financing conditions also provides a point of comparison with the literature on trade volumes at times of financial crises, especially during the Great Financial Crisis of 2008 (see Chor and Manova (2009) and Manova (2012)). Relatedly, our paper builds on Kashyap, Lamont and Stein (1994), who documented the sensitive nature of inventories to financial conditions, especially to shocks that reduced bank credit supply.

The remaining sections of this paper proceed as follows. Section 2 presents the benchmark model. Sections 3 explores optimal offshoring and its financial determinants. Section 4 discusses banks' choice of credit supply. Section 5 concludes the paper.

2 Benchmark 'Austrian' Model

We begin with an elementary model of supply chains that isolates the time dimension of production. There are no product or labor market distortions. The only friction is that production takes time. In this sense, our benchmark model has an Austrian theme that echoes the capital theory of Böhm-Bawerk (1884).

2.1 Working capital and productivity

Production takes place through chains of length n. There is a population of L workers. There are firms each owned by a penniless entrepreneur. Each firm is matched with one worker, so that there are L/n production chains in the economy. We assume L is large relative to n, so that the economy consists of a large number of production chains.

Within each production chain, there is a consumer-facing firm, labeled as firm 1, which sells the final output. The other firms produce intermediate inputs in the production of the final good. Firm n supplies its output to firm $n-1$, who in turn supplies output to $n-2$, and so on.

There is a "time to build" element. Each step of the production process takes one unit of time, where time is indexed by $t \in \{1, 2, \cdots\}$. A production chain of length n takes n units of time to produce the final output.

Although each step of the production process is identified with a firm, this is for narrative purposes only. In applied settings, our model may be better interpreted as a single multi-plant firm with each stage corresponding to a plant. The model is silent on where the boundary of the firm lies along the chain. What matters for us is the *aggregate* financing need of the supply chain as a whole. For simplicity of exposition, we will say "firm" with the understanding that they can be units of a single multi-plant firm.

Wage costs cannot be deferred and must be paid immediately in the period when the production is carried out. Labor is provided inelastically, and total labor supply is fixed at L. There is no physical capital. The wage rate is w per period. A production chain hires one worker for each stage. The cashflow to the chain is given in the table below.

		Firms					cumulative
		1	2	$\cdots$	$n-1$	n	cashflow
	1					$-w$	$-w$
	2				$-w$	$-w$	$-3w$
date	$\vdots$			$\cdots$	$-w$	$-w$	$\vdots$
t	$n-1$		$-w$	$\cdots$	$-w$	$-w$	$-\frac{1}{2}n^2w$
	n	$-w$	$-w$	$\cdots$	$-w$	$-w$	$-\frac{1}{2}n(n+1)w$
	$n+1$	$y(n)-w$	$-w$	$\cdots$	$-w$	$-w$	
	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	

At date 1, firm n begins production by hiring a worker and paying wages. It sends its output to firm $n-1$ at date 2. Firm $n-1$ takes delivery and begins production at date 2, and sends the intermediate good to firm $n-2$ at date 3, and so on. Meanwhile, at date 2, firm n starts another sequence of production by producing its output, which is sent to firm $n-1$ at date 3.

The first positive cashflow to the chain comes at date $n+1$ when firm 1 sells the final output $y(n)$. The cash transfer upstream is instantaneous, so that all upstream firms are paid for their contribution to the output.

Firms face a borrowing rate of $r > 0$. For the analysis in this section, we take r as given. We will later endogenize r by introducing a banking sector and solve for r as the equilibrium interest rate that clears the credit market.

We assume for simplicity that firms face a financing cost of zero in the initial set-up phase until the first positive cashflow materializes from the sale of the final good. In Appendix A, we provide the solution for the general case where firms face positive interest cost from the outset, and show that the assumption of zero interest cost in the initial set-up phase is without loss of generality for our main results.

The working capital needed in the initial set-up phase is given by:

$$\tfrac{1}{2}n(n+1)w \tag{1}$$

reflecting the sum of all wages paid until the first cashflow from the sale of the final good. Firms start with no equity and all financing is done by borrowing. Note that the total borrowing is of the order of the *square* of the

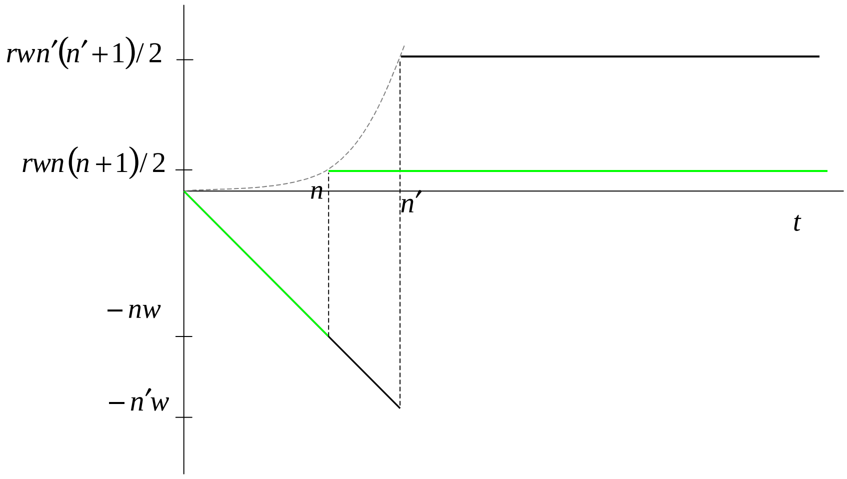

Figure 2. Profile of lenders' cash flow from lending to a production chain of length n (light line) and to a chain of length n' (dark line)

length of the production chain, corresponding to the area of the "triangle" in the cashflow diagram above.

There are L/n production chains, so that the aggregate working capital demand in the economy, denoted by K, is

$$\begin{aligned} K &= \tfrac{1}{2} n\,(n+1)\,w \times \frac{L}{n} \\ &= \frac{n+1}{2} wL \end{aligned} \tag{2}$$

For firms, the choice of the length of the production chain takes account of the marginal increase in productivity from lengthening the chain against the increased cost of financing for working capital. From the lenders' perspective, the cash flow is negative until date n, but then they start receiving interest payment on the outstanding stock of loans. Figure 2 compares the profile of lenders' cash flows depending on the length of the supply chain. The light line gives the cash flow profile by lending to a supply chain of length n, while the dark line gives the profile from lending to a chain of length $n' > n$.

The production chain consisting of n stages has final output $y\,(n)$, where

$$y\,(n) = A(n)l \tag{3}$$

and l is total labor employed by the chain ($l = n$ when each stage employs one worker), and

$$A(n) = n^{\alpha}, \qquad (0 < \alpha < 1) \tag{4}$$

so that productivity is an increasing and concave function of the length of production chain. Our assumption that $0 < \alpha < 1$ harks back to Böhm-Bawerk's (1884) discussion of "roundabout production" in which:

> "[t]he indirect method entails a sacrifice of time but gains the advantage of an increase in the quantity of the product. Successive prolongations of the roundabout method of production yield further quantitative increases though in diminishing proportion."[1]

The parameter α is the only "deep" technological parameter in our model, as the interest rate on working capital will be obtained by closing the model with credit supply.

2.2 Optimal length of supply chain

Supply chain length n maximizes the surplus of the chain as a whole, reflecting the interpretation of our model as the decision of a multi-plant firm. Since the borrowing rate is zero until date n and is r from date $n+1$, the choice of n at date 0 maximizes the discounted surplus:

$$\begin{aligned} V &= \sum_{t=n+1}^{\infty} \frac{(n^{\alpha} zL - wzL - rzK)}{(1+r)^{t-n}} \\ &= (n^{\alpha} zL - wzL - rzK)\frac{1}{r} \end{aligned} \tag{5}$$

where z is the proportion of the labor force employed by the production chain. The above maximization problem boils down to the problem of maximizing the per period surplus:

$$\begin{aligned} \pi &= n^{\alpha} zL - wzL - rzK \\ &= \left[n^{\alpha} - w\left(1 + \tfrac{r(n+1)}{2}\right)\right] zL \end{aligned} \tag{6}$$

The first-order condition for n gives

$$n = \left(\frac{2\alpha}{wr}\right)^{\frac{1}{1-\alpha}} \tag{7}$$

[1] Böhm-Bawerk (1884), p. 88 of 1959 English translation by G. Huncke, Libertarian Press.

In competitive markets firms bid away their surplus by competing for workers. The wage rate is determined by the zero profit condition:

$$n^{\alpha} = w\left(1 + \tfrac{r(n+1)}{2}\right) \tag{8}$$

The left-hand side of (8) is the marginal product of labor, while the the right-hand side is its marginal cost taking account of working capital costs. The horizontal labor demand curve meets the vertical labor supply curve when the wage rate is given by (8).

Given this simple set-up, we can solve the model in closed form. The optimal chain length is

$$n = \frac{\alpha}{1-\alpha}\left(1 + \frac{2}{r}\right) \tag{9}$$

so that production chains are longer when the interest rate r is lower.

Productivity, or output per worker is

$$A(n) = \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\left(1 + \frac{2}{r}\right)^{\alpha}$$

and total output of the economy Y is

$$Y = n^{\alpha}L = \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\left(1 + \frac{2}{r}\right)^{\alpha} L \tag{10}$$

so that productivity and output are decreasing in the interest rate r. The equilibrium wage w is also decreasing in r, since we have:

$$w = 2\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\left(1 + \frac{2}{r}\right)^{\alpha}\left(\frac{1-\alpha}{2+r}\right) \tag{11}$$

Total working capital of all production chains in the economy is given by:

$$\begin{aligned} K &= \frac{n+1}{2}wL \\ &= \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\left(1 + \frac{2}{r}\right)^{\alpha}\left(\frac{\alpha}{r} + \frac{1-\alpha}{2+r}\right) L \end{aligned} \tag{12}$$

In our model, investment in working capital raises productivity and increases output. However, the increase in working capital comes at the cost of greater financing cost. Within the credit market, K is the *aggregate credit* *demand* in the economy. Equation (12) shows that credit demand is decreasing in the interest rate r. The credit to GDP ratio has the simple form as below, which also declines with the interest rate.

$$\frac{K}{Y}=\frac{\alpha}{r}+\frac{1-\alpha}{2+r} \tag{13}$$

2.3 Total factor productivity

Our model draws out the analogy between working capital and fixed capital. Indeed, Ramey's (1989) investigation of modeling inventories as a factor of production suggests that the analogy can be explored further. From (2) and (3), total output can be written as

$$\begin{aligned} Y(K,L) &= n^{\alpha}L \\ &= \left(\frac{2K}{wL}-1\right)^{\alpha}L \\ &= \left(\frac{2}{w}-\frac{L}{K}\right)^{\alpha}K^{\alpha}L^{1-\alpha} \end{aligned} \tag{14}$$

where K here represents working capital.

Imposing a Cobb-Douglas functional form for output will result in a production function where the total factor productivity term $(2/w-L/K)^{\alpha}$ depends on endogenous variables, where financial factors are at play. The TFP term is not well defined when $r=0$ as the denominators of both expressions inside the brackets in (14) go to infinity. However, Figure 3 which plots TFP as a function of the borrowing rate r when $\alpha=0.033$ suggests that for most parameter values, the TFP term is decreasing in the borrowing rate. To outside observers who impose a Cobb-Douglas production function, they would observe that TFP undergoes shocks as financial conditions change. When financial conditions are tight and the risk premium in the borrowing rate increases, they will also observe that total factor productivity falls.

Our approach holds promise in shedding light on the role of supply chains in the co-movement of macro fluctuations and in addressing the so-called international co-movement puzzle - namely, that the co-movement in output across economics is larger than suggested by macro models (Backus et al. (1993), Kose and Yi (2006)).

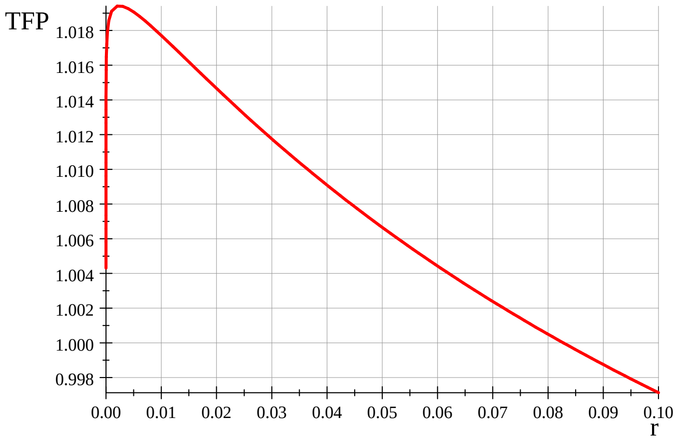

Figure 3. Total factor producitivity as a function of the market interest rate ($\alpha = 0.033$)

Regarding TFP more directly, Huo et al. (2019) find that two correlated shocks across countries (TFP and labor shocks) can go some way toward a reconciliation of the data with the macro model predictions. Our finding above that supply chain activity can inject shocks that resemble TFP shocks may be a useful basis for revisiting international co-movement of output.

2.4 Sales and value-added

Our model is well-suited to distinguishing total sales (gross output) from value-added (final good sale), which will be useful in our discussion of international trade (measured in gross terms) relative to GDP (measured in value-added terms).

Denote by p_i the price of the intermediate good produced by firm i. Given the zero profit condition, the intermediate good price is just sufficient to cover wages and the cost of intermediate inputs including the cost of working capital. Thus, in steady state, the prices of intermediate goods are given by:

$$\begin{aligned}
p_n &= w + rwn \\
p_{n-1} &= w + rw\,(n-1) + p_n \qquad (15) \\
p_{n-2} &= w + rw\,(n-2) + p_{n-1} \\
&\vdots \\
p_1 &= w + rw + p_2 \qquad (16)
\end{aligned}$$

By recursive substitution, we have

$$\begin{aligned}
p_n &= w\,(1+rn) \qquad (17) \\
p_{n-1} &= 2w\,(1+rn) - wr \\
p_{n-2} &= 3w\,(1+rn) - wr\,(1+2) \qquad (18) \\
&\vdots \\
p_1 &= nw\,(1+rn) - wr\,(1+2+\cdots+(n-1))
\end{aligned}$$

The economy has L/n of such production chains. Therefore, gross sales in steady state, denoted by S, can be written as:

$$S = \sum_{i=1}^{n} p_i \left(\frac{L}{n}\right) = \left[w\,(1+rn)\left(\sum_{i=1}^{n} i\right) - wr\sum_{i=1}^{n-1} i\,(n-i)\right]\left(\frac{L}{n}\right) \qquad (19)$$

Using the algebraic identity:

$$\textstyle\sum_{i=1}^{n-1} i\,(n-i) = \frac{1}{6}n\,(n-1)\,(n+1)\,,$$

gross sales can be solved in closed form as:

$$\begin{aligned}
S &= \left[\tfrac{1}{2}\,(1+rn)\,(n+1) - \tfrac{1}{6}r\,(n-1)\,(n+1)\right] wL \\
&= \tfrac{1}{6}\,(n+1)\,(r+2rn+3)wL \qquad (20)
\end{aligned}$$

Meanwhile, total value-added is the value of the final good (denoted by Y), which amounts to

$$Y = p_1\left(\frac{L}{n}\right) = \left[wn\,(1+rn) - wr\textstyle\sum_{i=1}^{n-1} i\right]\left(\frac{L}{n}\right) \qquad (21)$$

Therefore

$$Y = \left(\tfrac{1}{2}r\,(n+1) + 1\right) wL \tag{22}$$

and the ratio of sales to value-added is:

$$\frac{S}{Y} = \frac{(n+1)\,(\frac{1}{3}r + \frac{2}{3}rn + 1)}{r(n+1)+2} \tag{23}$$

Substituting in the solution for n, we have

$$\begin{aligned} \frac{S}{Y} &= \frac{(r+2\alpha)\,(r+\alpha\,(1+r)+3)}{3r\,(1-\alpha)\,(r+2)} \\ &= \frac{1}{3\,(1-\alpha)}\left(1+\frac{2\alpha}{r}\right)\left(1+\alpha+\frac{1-\alpha}{2+r}\right) \end{aligned} \tag{24}$$

The sales to value-added ratio is decreasing in the interest rate r, reflecting the shorter production chains when financial conditions are tighter. Total inventories of intermediate goods in steady state is

$$I = S - Y = \frac{(n-1)\,(1+\frac{2}{3}r(n+1))}{2}wL$$

which gives the inventory-GDP ratio:

$$\frac{I}{Y} = \frac{S}{Y} - 1 = \frac{(n-1)\,(1+\frac{2}{3}r(n+1))}{r(n+1)+2} \tag{25}$$

which also declines with the interest rate.

Gathering the findings in (9), (10), (24) and (25), we can summarize the main features of our model in terms of the following proposition.

Proposition 1 *A higher borrowing rate r is associated with (1) shorter production chains, (2) lower productivity per worker, (3) lower GDP, (4) lower sales-to-GDP ratio and (5) a lower inventory-to-GDP ratio.*

3 Application to international trade

We now turn to an application of our theory to offshoring and trade, and begin with a motivating example of offshoring for a multinational firm.

		Stages	
		2nd	1st
Date t	1		w
	2	w	w
	3	w	w
	⋮	⋮	⋮

		Stages		
		3rd	2nd	1st
Date t	1			c
	2		0	c
	3	w	0	c
	4	w	0	c
	⋮	⋮	⋮	⋮

Figure 4. **Costs of two-step production with and without offshoring.** A good is produced with two rounds of value-added. The left-hand diagram depicts production without offshoring. The right-hand diagram depicts the case when there is offshoring of the first stage of production. Without offshoring, each production stage takes one period and incurs cost of w. By offshoring the first stage, the firm reduces the first-stage cost to c but lengthens the time to produce the final good to three periods due to the transport stage.

3.1 Motivating example

Consider a two-stage production process without offshoring ($n = 2$), where a final good can be produced with two rounds of value-added. This case is depicted by the left-hand diagram in Figure 4. Each step in the production of the good takes one time period, and incurs a cost of w. At date 1, the firm that produces both stages completes the first production step at cost w and sends the intermediate good to the second step. At date 2, the firm goes through the second step of production incurring cost w. Meanwhile, the firm begins the first stage of the production of the next unit at cost w.

The firm begins to receive revenue of p from date 3 onwards, when it sells the good at price p. Before then, the firm finances the costs incurred during the initial phase (dates 1 and 2) by borrowing at interest rate r'.

In steady state (from date 3 onwards), the firm's cashflow is

$$p - 2w - r\left(2w\left(1+r\right) + w\left(1+r\right)^2\right) \tag{26}$$

consisting of sales revenue p, per-period production cost $2w$ and the interest expense on the debt incurred during the initial phase of production (at the steady state interest rate $r > 0$).

Now, suppose that the firm can offshore the first stage of production abroad. The right-hand panel of Figure 4 depicts production with offshoring. By offshoring the first step of production, the firm enjoys a productivity gain and also save on the cost of the first step of production. But it has to lengthen the total production time to three periods to take account of the time taken to transport the intermediate good. The cost of the first step of production with offshoring (including transport cost) is c. At date 2, the intermediate good is transported, and the second step of production takes place at date 3. The firm receives revenue from the sale of the good from date 4 onwards.

In steady state (from date 4 onwards), the firm's cashflow is

$$\widetilde{p} - (c + w) - r\left((c + w)(1 + r) + c(1 + r)^2 + c(1 + r)^3\right) \quad (27)$$

consisting of sales revenue $\widetilde{p}$ net of production cost $c + w$ and the cost of building up and carrying working capital. By offshoring the first step of production, the firm raises revenue to $\widetilde{p}$ and lowers the first stage cost to c, but incurs a higher working capital cost.

Denote by $\tilde{k}$ and k the working capital with and without offshoring, respectively. The firm chooses to offshore when the firm's steady-state cashflow with offshoring (27) is larger than without offshoring (26), or equivalently, when

$$(\widetilde{p} - p) + (w - c) > r(\tilde{k} - k) \quad (28)$$

where $\tilde{k} = (c + w)(1 + r) + c(1 + r)^2 + c(1 + r)^3$ and $k = 2w(1 + r) + w(1 + r)^2$. The firm can increase steady state profit through offshoring when the financing cost of offshoring is sufficiently small. However, higher r entails a higher hurdle for offshoring.

In what follows, we develop our model of offshoring by extending the benchmark model of general n-stage supply chains into a multi-country setting.

3.2 Model of offshoring and productivity

Consider a multinational firm with a presence in multiple locations. Each location has an absolute advantage in precisely one stage of the production process. The absolute advantage derives from the location, not the worker.

Specifically, there is a constant $b > 0$ such that the location with the absolute advantage in production stage i has productivity of $1+b$ compared to productivity of 1 in any other location for that task.

The output of the multinational firm ($y(n) = A(n)l$) then depends on the extent of offshoring and is given by:

$$A(n,s) = \left(\sum_{i=1}^{n} x_i\right)^{\alpha} \qquad (0 < \alpha < 1) \tag{29}$$

where $x_i = 1+b$ if the production of the ith stage takes place in the location with the absolute advantage in stage i while $x_i = 1$ if the production takes place elsewhere. Thus, if there are s stages where production takes place in the location with the absolute advantage, productivity is given by

$$A(n,s) = (n+bs)^{\alpha} \tag{30}$$

To highlight the choice of offshoring, we fix the length of the production chain at $n = \bar{n}$, and there are $\bar{n}$ locations. Then the productivity of a chain is

$$A(s) = (\bar{n}+bs)^{\alpha} \tag{31}$$

The firm's key decision is to choose s, the extent of offshoring.[2]

3.3 Inventories in transit

We assume that transport requires labor services just as production does. Offshoring incurs additional financing costs due to time needed to transport intermediate goods. We assume that if an intermediate good is transported to another location, transport takes one unit of time, which is the same as the time needed for production of an intermediate good. Within the same country, we assume that transport happens instantaneously and without labor cost.

If s stages are offshored, the time to production of the final good in this offshoring model rises from $\bar{n}$ to $\bar{n}+s$. With offshoring, a new type of

[2]It is possible to allow for endogenous n by first deriving s and w as a function of n and then solve for n. The qualitative features of the model remain.

inventory emerges - *inventory in transit*. Without offshoring, the multinational firm has $\bar{n}$ vintages of inventories at the steady state. With offshoring, multinational firms hold $\bar{n}+s$ vintages of inventories including s vintages of inventories in transit.

As in the benchmark model, wages cannot be deferred and firms that engage in intermediate good production or overseas transport need working capital to pay wages. Wages at each stage of production or transportation is paid in the period when the activity of the stage takes place. We assume that w is equal across locations and activities.[3] We maintain the assumption that firms finance working capital with debt from banks at zero interest rates during the initial periods ($t \leq \bar{n}+s$). The relaxation of the assumption of zero interest rates during the transition period does not alter the main results of the model (see Appendix A).

The multinational firm starts the production of the first stage intermediate good ($i=n$) at date 1. At date $\bar{n}+s$, it begins producing the final goods by inputing intermediate goods for which it has paid $\frac{1}{2}(\bar{n}+s)(\bar{n}+s-1)w$, and labor, for which it currently pays $(\bar{n}+s)w$. The steady state working capital is

$$\tfrac{1}{2}(\bar{n}+s)(\bar{n}+s+1)w \tag{32}$$

which is equal to the sum of all the wages that have been paid during the initial set-up period. Note that offshoring raises the working capital from $\frac{1}{2}\bar{n}(\bar{n}+1)w$ to $\frac{1}{2}(\bar{n}+s)(\bar{n}+s+1)w$ due to inventories in transit.

Total working capital for the global economy as a whole, denoted by K, is then

$$\begin{aligned} K &= \tfrac{1}{2}(\bar{n}+s)(\bar{n}+s+1)w \cdot \frac{L}{(\bar{n}+s)} \\ &= \frac{\bar{n}+s+1}{2}wL \end{aligned} \tag{33}$$

[3] We may introduce a model which allows for wage difference across countries, for example due to restrictions on international labor mobility. In this variant of the model, multi-national firms have incentive to offshore each stage of production chain to the country with the lowest wage in the stage. The solution of the variant model with wage difference for the optimal offshoring is similar to that of this section with productivity difference.

where L is the population of workers. K also has the interpretation as the *total demand for credit* to finance working capital investment. Taking the borrowing rate as given for now, the per period borrowing cost is

$$r \cdot \frac{\bar{n}+s+1}{2} wL \tag{34}$$

The cost of financing for working capital increases with the number of offshored stages.

3.4 Optimal offshoring

The firm chooses s to maximize its value, given by the discounted sum of surpluses:

$$V = \sum_{t=\bar{n}+s+1}^{\infty} \frac{(\bar{n}+bs)^{\alpha} zL - wzL - rzK}{(1+r)^{t-\bar{n}-s}} \tag{35}$$

where z is the proportion of the workforce employed by the firm. The maximization problem reduces to maximizing the per-period profit

$$\begin{aligned} \pi &= (\bar{n}+bs)^{\alpha} zL - wzL - rzK \\ &= \left[(\bar{n}+bs)^{\alpha} - w\left(1 + \frac{r\left(\bar{n}+s+1\right)}{2}\right)\right] zL \end{aligned} \tag{36}$$

The first-order condition for s yields

$$\bar{n} + bs = \left(\frac{2b\alpha}{wr}\right)^{\frac{1}{1-\alpha}} \tag{37}$$

and the zero profit condition is

$$(\bar{n}+bs)^{\alpha} = w\left(1 + \frac{r}{2}(1+\bar{n}) + \frac{r}{2}s\right) \tag{38}$$

Assume that $b > \frac{1}{\alpha}$. From eqs. (37) and (38) we can solve the model in closed form. Optimal extent of offshoring is

$$s = \frac{\alpha}{1-\alpha}\left(1 + \bar{n} + \frac{2}{r}\right) - \frac{\bar{n}}{b(1-\alpha)} \tag{39}$$

which is positive since $b > \frac{1}{\alpha}$ and is decreasing in r. The offshoring ratio $s/\bar{n}$ captures what fraction of production is offshored, and is given by

$$\frac{s}{\bar{n}} = \frac{\alpha}{1-\alpha}\left(\left(1+\frac{2}{r}\right)\frac{1}{\bar{n}} + 1\right) - \frac{1}{b(1-\alpha)} \tag{40}$$

Appendix B describes an accounting framework which can be used to approximate the offshoring ratio $s/\bar{n}$ using available data.

The productivity of any location is given by

$$A(\bar{n}, r) = \left[\frac{(b-1)\alpha}{1-\alpha}\bar{n} + \frac{b\alpha}{1-\alpha}\left(1+\frac{2}{r}\right)\right]^{\alpha} \tag{41}$$

and the world output Y^{World} is

$$Y^{\text{World}} = (\bar{n}+bs)^{\alpha} L = \left[\frac{(b-1)\alpha}{1-\alpha}\bar{n} + \frac{b\alpha}{1-\alpha}\left(1+\frac{2}{r}\right)\right]^{\alpha} L \tag{42}$$

so that productivity and world output are declining in r.

By plugging (39) into (38), we derive the equilibrium wage as

$$w = \frac{2b\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\left[(b(1+\bar{n})-\bar{n})r^{\frac{1}{1-\alpha}} + 2br^{\frac{\alpha}{1-\alpha}}\right]^{1-\alpha}} \tag{43}$$

which is also declining in r. The tightening of financial condition reduces offshoring and has a negative impact on the world productivity, wages and output.

Using eqs. (39) and (43), we can then solve for working capital K as $(\bar{n}+s+1)\, wL/2$, which is decreasing in r. Global demand for credit is therefore decreasing in r.

3.5 Ratio of trade to output

Our model's distinction between total sales and value-added allows us to track the ratio of trade to total output, or equivalently, the ratio of trade to value-added. From the zero profit condition, we can express the intermediate prices p_i as in eq. (16)

$$p_i = w + rwi + p_{i+1}$$

Total sales in steady state can be obtained as:

$$\sum_{i=1}^{\bar{n}+s} p_i \left(\frac{L}{\bar{n}+s}\right) = \frac{1}{6}\left(\bar{n}+s+1\right)\left(r+2r(\bar{n}+s)+3\right) wL \tag{44}$$

We can obtain total trade per period (denoted by T^{World}) by multiplying total sales by the proportion of production stages that are offshored. Therefore,

$$\begin{aligned} T^{\text{World}} &= \frac{s}{\bar{n}+s}\sum_{i=1}^{\bar{n}+s} p_i \left(\frac{L}{\bar{n}+s}\right) \\ &= \frac{s}{\bar{n}+s}\left(\frac{r+2r(\bar{n}+s)+3}{6}\right)(\bar{n}+s+1)\, wL \end{aligned} \tag{45}$$

which is increasing in s.[4]

Total output (Y^{World}) is given by the value of the final good, or equivalently, the total value-added, and is given by

$$Y^{\text{World}} = p_1\left(\frac{L}{\bar{n}+s}\right) = \left[\frac{1}{2}r(\bar{n}+s+1)+1\right] wL \tag{46}$$

Therefore, the ratio of total trade to output is

$$\begin{aligned} \frac{T^{\text{World}}}{Y^{\text{World}}} &= \frac{\frac{s}{\bar{n}+s}\sum_{i=1}^{\bar{n}+s} p_i(\frac{L}{\bar{n}+s})}{p_1(\frac{L}{\bar{n}+s})} \\ &= \frac{s}{\bar{n}+s}\left[\frac{(\bar{n}+s+1)\left(\frac{1}{3}r+\frac{2}{3}r(\bar{n}+s)+1\right)}{r(\bar{n}+s+1)+2}\right] \end{aligned} \tag{47}$$

Note that $\frac{s}{\bar{n}+s}$ increases with s, and the fraction inside the square bracket also tends to increase with s since the numerator is a convex function of s while the denominator is linear in s. Thus, a higher s is associated with a higher ratio of trade to output.

Recall that the optimal offshoring s is decreasing in r (eq. (39)). Therefore, eq. (47) implies that the ratio of trade to output decreases with the interest rate r.[5]

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[4] In reality, overseas transport companies receive fees for shipment from exporters or importers rather than buy and sell goods with them. Considering this, total sales among the firms (S^{World}) can be approximated by

$$S^{World} = \sum_{i=1}^{\bar{n}+s} p_i(\frac{L}{\bar{n}+s}) - T^{World} = \frac{\bar{n}}{\bar{n}+s}\sum_{i=1}^{\bar{n}+s} p_i(\frac{L}{\bar{n}+s}).$$

[5] To formally prove this, we can rewrite $\frac{T^{World}}{Y^{World}}$ as the product of three functions of the interest rate r as

Lastly, total inventories of intermediate goods is given by

$$\begin{aligned} I &= \sum_{i=1}^{\bar{n}+s} p_i \left(\frac{L}{\bar{n}+s}\right) - p_1 \left(\frac{L}{\bar{n}+s}\right) \\ &= \frac{(\bar{n}+s-1)\left(1+\frac{2}{3}r(\bar{n}+s+1)\right)}{2} wL \end{aligned} \tag{48}$$

We multiply eq. (48) by $\frac{s}{\bar{n}+s}$ to get total inventories in transit (I^{tr}):

$$\begin{aligned} I^{\text{tr}} &= \left(\frac{s}{\bar{n}+s}\right) I \\ &= \frac{s}{\bar{n}+s}\left[\frac{(\bar{n}+s-1)\left(1+\frac{2}{3}r(\bar{n}+s+1)\right)}{2}\right] wL \end{aligned} \tag{49}$$

The ratio of inventory-in-transit to output is

$$\frac{I^{\text{tr}}}{Y^{\text{World}}} = \frac{s}{\bar{n}+s}\left[\frac{(\bar{n}+s-1)\left(1+\frac{2}{3}r(\bar{n}+s+1)\right)}{r(\bar{n}+s+1)+2}\right] \tag{50}$$

so that inventories-in-transit relative to output is increasing in s and hence decreasing in r.

We summarize our findings in terms of the following proposition.

Proposition 2 *A higher interest rate r is associated with (1) lower off-shoring, (2) lower productivity per worker, (3) lower output, (4) lower trade-output ratio, and (6) lower inventories in transit as fraction of output.*

$$\frac{T^{World}}{Y^{World}} = B(r)C(r)D(R)$$

where

$$\begin{aligned} B(r) &= (\bar{n}+s+1) \\ C(r) &= \frac{s}{r(\bar{n}+s+1)+2} r \\ D(r) &= \frac{\frac{1}{3}r + \frac{2}{3}r(\bar{n}+s)+1}{\bar{n}+s}\left(\frac{1}{r}\right) \end{aligned}$$

Given the assumption $b > \frac{1}{\alpha}(> 1)$, we can show that $\frac{dB(r)}{dr} < 0$, $\frac{dC(r)}{dr} < 0$ and $\frac{dD(r)}{dr} < 0$, and hence $\frac{d\left(\frac{T^{World}}{Y^{World}}\right)}{dr} < 0$.

4 Closing the model with credit supply

So far we have treated the rate of interest r as given. We now close the model by introducing credit supply through a banking sector.

An advantage of closing the model with a banking sector is that we can address how short-term fluctuations in credit conditions that affect lending condition (such as through fluctuations in the leverage of the banking sector, or the erosion of bank equity) can affect macro fluctuations through productivity and trade. In this way, our analysis opens up additional avenues for exploration in trade and finance. We begin our analysis by introducing the risk of failure of supply chains and a banking sector whose total lending is determined through a contracting problem to overcome a moral hazard problem among banks, following Bruno and Shin (2015).

4.1 Supply chains with credit risk

We introduce risk of failure of supply chains. Starting at date $\bar{n}+s+1$ (when each supply chain starts receiving positive cash from sales), the supply chain associated with a multinational firm is subject to a hazard rate $\varepsilon > 0$ of failure with zero liquidation value. We assume that if firm fails, the constituent units can re-group costlessly under a new multinational firm who can borrow afresh.

The multinational firm's optimisation problem at date 0 is to choose s to maximize the expected firm value, V:

$$V = \sum_{t=\bar{n}+s+1}^{\infty} \frac{(1-\varepsilon)^{t-\bar{n}-s}\left((\bar{n}+bs)^{\alpha} zL - wzL - rzK\right)}{(1+r)^{t-\bar{n}-s}} \tag{51}$$

Note that V the discount rate now also incorporates the hazard rate ε, as well as the interest rate r. The firm value (51) can be simplified to:

$$V = \left((\bar{n}+bs)^{\alpha} zL - wzL - rzK\right) \frac{1-\varepsilon}{r+\varepsilon} \tag{52}$$

Despite the inclusion of risk factor ε, therefore, the maximization problem is reduced to maximizing certain profits as in the previous sections:

$$\pi = (\bar{n}+bs)^{\alpha} zL - wzL\left(1 + \frac{r(\bar{n}+s+1)}{2}\right)$$

while the borrowing rate r here is an endogenous variable, reflecting risk premium, which is determined when the demand and supply of credit markets are equalized.

We have similar first-order condition for s and zero profit condition as in eqs. (37) and (38), which yields the optimal extent of offshoring as eq. (39)

$$s = \frac{\alpha}{1-\alpha}\left(1+\bar{n}+\frac{2}{r}\right) - \frac{\bar{n}}{b(1-\alpha)} \tag{53}$$

Then the global demand for credit (for working capital), denoted by K, is derived as a function of r:

$$K(r) = \frac{\bar{n}+s(r)+1}{2} w(r) L \tag{54}$$

where $s(r)$ and $w(r)$ satisfy eqs. (41) and (53).

4.2 Credit supply by banks

Credit is supplied through banks which are subject to a moral hazard problem. The bank's equity e is fixed, with equity ownership evenly distributed among the investor population in the world. Bank credit is short-term, and rolled over every period.

Along the steady state, the bank lends k^S (for "credit") at date t at the lending rate of interest r, so that the bank is owed $(1+r)\,k^S$ at date $t+1$. The lending is financed from the combination of equity e and deposit funding d, which is raised from investors. The cost of debt financing (deposit) is f so that the bank owes $(1+f)\,d$ at date $t+1$ (its notional liabilities). We will show shortly that f is determined to be equal to the risk-free rate r^f, which is set at zero.

Each production chain is subject to a hazard rate $\varepsilon > 0$ of failure from date$\bar{n}+s+1$ onwards, while the correlation in failure across chains follows the Vasicek (2002) model. More specifically, production chain j survives into the next period (so that the loan is repaid) when $z_j > 0$ along the steady state (from date $\bar{n}+s+1$ on), where z_j is the random variable:

$$z_j = -\Phi^{-1}(\varepsilon) + \sqrt{\rho}H + \sqrt{1-\rho}h_j \tag{55}$$

Here $\Phi(.)$ is the cumulative distribution function of the standard normal, H and $\{h_j\}$ are independent standard normals, and ρ is a constant between zero and one. H has the interpretation of the economy-wide fundamental factor that affects all chains, while h_j is the idiosyncratic factor for chain j. The parameter ρ is the weight on the common factor. Note that the unconditional probability of default of each production chain is given by $\Pr(z_j < 0) = \Pr\left(\sqrt{\rho}H + \sqrt{1-\rho}h_j < \Phi^{-1}(\varepsilon)\right) = \Phi\left(\Phi^{-1}(\varepsilon)\right) = \varepsilon$, consistent with our assumption that each chain has a constant hazard rate of failure of ε. Given the economy-wide factor H, defaults of different chains may have positive correlation.

Banks are able to diversify their loan by lending to a large number of separate production chains. In this situation, banks' leverage is determined through the following contracting problem, which follows Bruno and Shin (2015).

Suppose that the banks face the choice between two alternative portfolios. The good portfolio consists of loans to production chains which have a probability of default ε and zero correlation of defaults across loans $\rho = 0$ (so that $z_j = -\Phi^{-1}(\varepsilon) + h_j$). The bad portfolio consists of loans to chains with a higher probability of default $\hat{\varepsilon} = \varepsilon + v$, for $v > 0$ and positive correlation of defaults across loans $\hat{\rho} > 0$ (hence $z_j = -\Phi^{-1}(\varepsilon + v) + \sqrt{\hat{\rho}}H + \sqrt{1-\hat{\rho}}h_j$). The bad portfolio generates greater dispersion in the outcome density for the loan portfolio. Since banks have limited liability, a greater probability of bank failure is associated with a higher option value of limited liability.

The notional value of the bank's total loan is $(1+r)\,k^S$. Conditional on H, defaults of individual loans are independent. By taking the limit where the bank diversifies its lending across a large number of firms, the realized value of the bank's loan portfolio can be written as a function of H by the law of large numbers.

Suppose that the bank chooses the bad portfolio of loans to production chains with $v > 0$ and $\hat{\rho} > 0$. Then the realized value of the bank's loan portfolio conditional on H, $a_B(H)$, is

$$\begin{aligned} a_B(H) &= (1+r)\,k^S \cdot \Pr\left(\sqrt{\hat{\rho}}H + \sqrt{1-\hat{\rho}}h_j \geq \Phi^{-1}(\varepsilon + v)\,|H\right) \\ &= (1+r)\,k^S \cdot \Phi\left(\tfrac{\sqrt{\hat{\rho}}H - \Phi^{-1}(\varepsilon+v)}{\sqrt{1-\hat{\rho}}}\right) \end{aligned} \tag{56}$$

If we normalize a_B by the face value of assets, the c.d.f. of normalized $\hat{a}_B$ is given by

$$\begin{aligned} F_B(u) &= \Pr(\hat{a}_B \leq u) \\ &= \Pr\left(H \leq \hat{a}_B^{-1}(u)\right) \\ &= \Phi\left(\hat{a}_B^{-1}(u)\right) \\ &= \Phi\left(\tfrac{\Phi^{-1}(\varepsilon+v) + \sqrt{1-\rho}\,\Phi^{-1}(u)}{\sqrt{\hat{\rho}}}\right) \end{aligned} \tag{57}$$

where $\hat{a}_B(H) \equiv a_B(H) / (1+r)\,k^S$.

Now suppose that the bank chooses the good portfolio consisting of loans to production chains with $v = 0$ and $\rho = 0$. Setting $v = 0$ and let $\rho \to 0$ in eq. (57), the good portfolio has the outcome distribution:

$$F_G(u) = \begin{cases} 0 & \text{if } u < 1-\varepsilon \\ 1 & \text{if } u \geq 1-\varepsilon \end{cases} \tag{58}$$

The good portfolio consists of i.i.d. loans, each of which has the default probability of ε, and the bank can fully diversify away credit risk. With a fully diversified loans, banks face the default of exactly ε fraction of borrowers. The realized value of the bank's portfolio is certain at $(1-\varepsilon)(1+r)\,k^S$.

The notional debt of the bank to depositors is $(1+f)\,d$. The debt of the bank normalized by the face value of assets, φ, is

$$\varphi \equiv (1+f)\,d / (1+r)\,k^S \tag{59}$$

which captures normalized leverage.

Here, φ is the strike price of the embedded option for the bank from limited liability. Let $\pi(\varphi)$ denote the value of the put option when the strike price is φ. Following Merton (1974), the bank's expected repayment to depositors is the repayment made in full in all states of the world (φ) minus the option value to default $(\pi(\varphi))$.

Then the expected payoffs of the bank is

$$E(\hat{a}) - [\varphi - \pi(\varphi)] \tag{60}$$

where $E(\hat{a})$ is the expected realization of the (normalized) loan portfolio.

The bank chooses d, k^S (equivalently, φ) and f so as to maximize its expected payoff (60) subject to the incentive compatibility constraint for the bank to choose the good portfolio, and the break-even constraint for depositors. If the expected payoff increases with leverage φ, the bank will increase leverage, but only until it hits the level that binds the incentive compatibility constraint.

The bank's incentive compatibility constraint to choose the good portfolio is

$$E_G(\hat{a}) - [\varphi - \pi_G(\varphi)] \geq E_B(\hat{a}) - [\varphi - \pi_B(\varphi)] \tag{61}$$

where the left-hand side is the expected payoff of the good portfolio and the right-hand side is that of the bad portfolio.

Denote the difference in option value to default by $\Delta\pi(\varphi) = \pi_B(\varphi) - \pi_G(\varphi)$, and note that $E_G(\hat{a}) - E_B(\hat{a}) = v$. Then eq. (61) can be written more simply as

$$\Delta\pi(\varphi) \leq v \tag{62}$$

The bank needs to keep leverage φ low enough that the higher option value to default of the bad portfolio does not exceed the greater expected payoff of the good portfolio.

From Breeden and Litzenberger (1978), the state price density is the second derivative of the option price with respect to its strike price. Using this, the difference in option value to default $\Delta\pi(\varphi)$ is given by

$$\Delta\pi(\varphi) = \begin{cases} \int_0^{\varphi} F_B(u)\, du & \text{if } \varphi < 1-\varepsilon \\ \int_0^{1-\varepsilon} F_B(u)\, du - \int_{1-\varepsilon}^{\varphi} [1 - F_B(u)]\, du & \text{if } \varphi \geq 1-\varepsilon \end{cases} \tag{63}$$

Thus $\Delta\pi(\varphi)$ is single-peaked, reaching its maximum at $\varphi = 1-\varepsilon$. In addition, $\Delta\pi(\varphi)$ is increasing in leverage for $\varphi < 1-\varepsilon$, and $\Delta\pi(0) = 0$.

Note that

$$\begin{aligned}\Delta\pi(1) &= \int_0^1 [F_B(u) - F_G(u)]\,du \\ &= \int_0^1 [1 - F_G(u)]\,du - \int_0^1 [1 - F_B(u)]\,du \\ &= E_G(\hat{a}) - E_B(\hat{a}) = v \end{aligned} \tag{64}$$

that is, $\Delta\pi(\varphi)$ approaches v from above as $\varphi \to 1$. As $\varphi < 1$ for any bank with positive notional equity, there is a unique solution to $\Delta\pi(\varphi) = v$ in the range where $\Delta\pi(\varphi)$ is increasing. As $\Delta\pi(\varphi)$ is increasing in leverage for $\varphi < 1 - \varepsilon$, the solution for φ^* satisfies $\varphi^* < 1 - \varepsilon$. In sum, there is a unique level of (normalized) leverage φ^* that solves $\Delta\pi(\varphi) = v$, where $\varphi^* < 1 - \varepsilon$. As such, the bank chooses the good portfolio and the leverage φ^* which is less than $1 - \varepsilon$.

As a result of the bank's choice of good portfolio, the bank's probability of default becomes zero. Then the break-even constraint for depositors implies that the deposit rate of interest is equal to the risk-free rate, which is assumed to be zero: $f = r^f = 0$.

Using eq. (59) and the balance sheet identity $e + d = k^S$, we can solve for the bank's supply of credit, k^S, as

$$k^S = \frac{e}{1 - (1 + r)\varphi^*} \tag{65}$$

The total credit supply K^S across all banks is then given by:

$$K^S = \frac{me}{1 - (1 + r)\varphi^*} \tag{66}$$

where m is the number of banks in the world. This suggests that the global credit supply is increasing in r, e and φ^*. Especially, the credit supply increases with the bank lending rate r.

By combining the credit supply function given above (eq. (66)) with the credit demand function for financing working capital (eq. (54)), we can solve for the equilibrium borrowing rate r as the rate that clears the credit market. Any shock that reduces banking sector credit, such as credit losses that reduce bank equity e or an increase in hazard rate ε (which reduces leverage φ^*), will result in an upward shift of the credit supply curve, leading to an increase in the borrowing rate r. The increased borrowing rate will then kick in motion the combination of reduced productivity, reduced wages and lower offshoring activity described in Sections 2 and 3. We summarize our main result as follows.

Proposition 3 *A negative shock in the banking sector credit supply is associated with (1) an increase in the borrowing rate r (2) fall in productivity per worker, (3) fall in output (4) fall in the inventories in transit, and (5) fall in the trade to output ratio.*

5 Concluding remarks

Financial shocks that raise the cost of financing can have a substantial impact on macro and trade variables through their impact on the cost of working capital. Our results derive from the feature that production takes time and the operation of a production chain across national borders entails heavy demands on financing. One consequence of this feature is that long production chains and offshoring are sustainable only when credit is cheap, and chains that have become over-extended are vulnerable to financial shocks that raise the cost of borrowing. Our model has been deliberately stark so as to highlight the role of working capital. We have abstracted away from many of the standard ingredients that have been used to model financial frictions in the macro or trade literature. We have no fixed capital, no savings decisions, nor labor supply decisions. Having turned off these intertemporal and labor supply choices, we can isolate the effect of working capital better.

References

Acemoglu, Daron and Pablo Azar (2020) “Endogenous production networks” Econometrica, 88, 33-82.

Adrian, Tobias, Paolo Colla and Hyun Song Shin (2013) “Which Financial Frictions? Parsing the Evidence from the Financial Crisis of 2007-9” 2012 NBER Macro Annual, volume 27, Daron Acemoglu, Jonathan Parker, and Michael Woodford, editors, 159 - 214, 2013.

Adrian, Tobias and Hyun Song Shin (2014) “Procyclical Leverage and Value-at-Risk” *Review of Financial Studies*, 27 (2), 373-403.

Alessandria, George, Joseph Kaboski and Virgiliu Midrigan (2011) “US trade and inventory dynamics” *American Economic Review*, 101, 303-07

Amiti, Mary, and David E. Weinstein (2011) “Exports and Financial Shocks.” Quarterly Journal of Economics (2011) 126, 1841–1877

Antras, Pol (2022) “An ‘Austrian’ model of global value chains”, working paper, Harvard University

Antras, Pol (2023) “Interest rates and world trade: an ‘Austrian’ perspective” forthcoming in AEA Papers and Proceedings

Antras, Pol and Davin Chor (2022) “Global Value Chain”, Handbook of international economics, Vol. 5, 297-376

Backus, D., P. J. Kehoe, and F. Kydland (1993) “International business cycles: theory and evidence.” Journal of Political Economy, 100 (4): 745–775

BIS (2014) “Trade finance: development and issues” CGFS papers 50, Bank for International Settlements, January 2014

Basel Committee on Banking Supervision (2005) “International Convergence of Capital Measurement and Capital Standards: A Revised Framework”, Bank for International Settlements, http://www.bis.org/publ/bcbs118.pdf

Bems, Rudolf, Robert C. Johnson and Kei-Mu Yi (2011) "Vertical Linkages and the Collapse of Global Trade" American Economic Review Papers & Proceedings, 101, 308–312

Blinder, Alan (1986) “Can the Production Smoothing Model of Inventory Behavior be Saved?” *Quarterly Journal of Economics*, 101, 431-53

Blinder, Alan and Louis Maccini (1991) “Taking Stock: A Critical Assessment of Recent Research on Inventories” Journal of Economic Perspectives, 5, 73-96

Böhm-Bawerk, Eugen von (1895) "The Positive Theory of Capital and Its Critics," *Quarterly Journal of Economics*, vol. 9, (January), 113-131.

Böhm-Bawerk, Eugen von (1895) *Capital and Interest* (3 vols. in one). South Holland, IL: Libertarian Press. Trans. By George D. Huncke and Hans F. Sennholz. 1959.

Boissay, F., N. Patel, and H. S. Shin. 2020. Trade Credit, Trade Finance, and the COVID-19 Crisis. BIS Bulletin 24.

Breeden, D. and R. Litzenberger (1978) “Prices of State-Contingent Claims Implicit in Option Prices,” *Journal of Business* 51(4), pp. 621-651.

Bruno, Valentina, Se-Jik Kim, and Hyun Song Shin (2018) “Exchange Rates and the Working Capital Channel of Trade Fluctuations” *American Economic Association Papers and Proceedings*, 108, 531-536

Bruno, Valentina and Hyun Song Shin (2015) “Cross-border banking and global liquidity” Review of Economic Studies, 82 (2), 535-564.

Chor, Davin, and Kalina Manova (2012) “Off the cliff and back? Credit conditions and international trade during the global financial crisis”. *Journal of International Economics*, 87(1): 117-133.

Claessens, Stijn, and Neltje Van Horen (2021) “Foreign Banks and Trade” *Journal of Financial Intermediation*, 45: 1008-56.

Cook, D., and N. Patel (2022): “Dollar Invoicing, Global Value Chains, and the Business Cycle Dynamics of International Trade,” IMF Working Papers, 2022.

Findlay, Ronald (1978) “An ‘Austrian’ model of international trade and interest rate equalization”, Journal of Political Econonomy, 86, 989-1007.

Friedman, Thomas (2005) *The World is Flat*, Farrar, Straus and Giroux, New York

Di Giovanni, Julian, Sebnem Kalemli-Ozcan, Alvaro Silva, and Muhammed A. Yıldırım (2022) “Global Supply Chain Pressures, International Trade, and Inflation” forthcoming in the ECB Sintra symposium volume

Huo, Z., A. A. Levchenko, and N. Pandalai-Nayar (2019): “International comove-ment in the global production network,”Technical report, National Bureau of Economic Research.

Kashyap, Anil, Owen Lamont and Jeremy Stein (1994) "Credit Conditions and the Cyclical Behavior of Inventories" Quarterly Journal of Economics, 109, 565-592.

Kim, Se-Jik and Hyun Song Shin (2013) "Working capital, trade and macro fluctuations", Griswold Center working paper 235, Princeton University, www.princeton.edu/~ce

Kim, Se-Jik (2017) "Working Capital Theory of Economic Growth" work in progress, SNU.

Kose, M. A., and K.-M. Yi (2006): "Can the standard international business cycle model explain the relation between trade and comovement?" Journal of international Economics, 68, 267–295.

Kydland, F. E. and E. C. Prescott (1982) "Time to Build and Aggregate Fluctuations" Econometrica, 50, 1345-70.

Manova, Kalina (2012) "Credit Constraints, Heterogeneous Firms, and International Trade" forthcoming in the *Review of Economic Studies*

Minetti, R, P Murro, Z Rotondi and S C Zhu (2019) "Financial constraints, firms' supply chains and internationalization" Journal of the European Economic Association, 17(2), 327-375.

Niepmann, F., and T. Schmidt-Eisenlohr (2017a) "International Trade, Risk and the Role of Banks" *Journal of International Economics*, 107: 111-126.

————. 2017b "No guarantees, no trade: How banks affect export patterns" *Journal of International Economics*, 108: 338-350.

Raddatz, Claudio (2006) "Liquidity Needs and Vulnerability to Financial Underdevelopment", *Journal of Financial Economics*, 80 (3), 677-722.

Ramey, Valerie (1989) "Inventories as a Factor of Production" *American Economic Review*, 79 (3), 338-354

Vasicek, Oldrich (1987) "Probability of Loss on Loan Portfolio" unpublished paper, available at:
http://www.moodyskmv.com/research/files/wp/Probability_of_Loss_on_Loan_Portfolio.pdf

Yi, Kei-Mu (2003) "Can Vertical Specialization Explain the Growth of World Trade?" Journal of Political Economy, 111 (1), 52-102.

Appendix

A General financing cost in benchmark model

In the body of the text, we assumed for simplicity that the interest rate in the initial set-up phase is zero. In this appendix, we solve for the case with positive interest rates to demonstrate that our main results in the benchmark Austian model (Section 2) are unchanged in the general case.

Consider firm n which produces the most upstream stage good within a production chain that hires one worker for each stage. It borrows w to pay wages from a bank at date 1. The firm pays back $w(1+r_1)$ and borrows the same amount at date 2, where r_j is the interest rate at date j. It continues to roll over the principal and interest of the loan until date $n+1$. The value of the original loan of w becomes $w[\Pi_{j=1}^{n}(1+r_j)]$ at date $n+1$.

The firm engages in a second-round production and borrows w at date 2, and so on. As a result, total working capital financed by firm n at date $n+1$ is

$$\sum_{t=1}^{n}[\Pi_{j=t}^{n}(1+r_j)]w \tag{67}$$

From date $n+1$ when the first final good of the production chain is sold, the firm receives the proceeds from its sales, with which it pays wages for the worker hired at the date and the interest for the working capital accumulated. Along the steady state, the firm rolls over the principal of the loan (67) but pays interest.

Firm i starts from date $n-i+1$. The working capital held by the firm at date $n+1$ is

$$\sum_{t=n-i+1}^{n}[\Pi_{j=t}^{n}(1+r_j)]w \tag{68}$$

Then total working capital of the production chain that hires one worker for each stage at date $n+1$ is

$$\sum_{i=1}^{n}\sum_{t=n-i+1}^{n}[\Pi_{j=t}^{n}(1+r_j)]w \tag{69}$$

In the steady state (that is, from date $n+1$ on), the production chain keeps rolling over the working capital.

There are L/n production chains of hiring one worker per stage, so that the aggregate demand for working capital along the steady state is given by

$$K=\left[\sum_{i=1}^{n}\sum_{t=n-i+1}^{n}(\Pi_{j=t}^{n}(1+r_j))\right]w\cdot\frac{L}{n} \tag{70}$$

Suppose that the interest rate in the initial set-up phase is the same at $r_j = \hat{r}$ for all $j \leq n$. Then the expression inside the square bracket of eq. (70) can be rewritten as

$$\begin{aligned} \sum_{i=1}^{n} \sum_{t=n-i+1}^{n} [\Pi_{j=t}^{n}(1+r_j)] &= \sum_{i=1}^{n} (1+\hat{r}) \frac{(1+\hat{r})^i - 1}{\hat{r}} \\ &= \frac{(1+\hat{r})}{\hat{r}} \left(\frac{(1+\hat{r})^n - 1}{\hat{r}} (1+\hat{r}) - n \right) \end{aligned} \quad (71)$$

Using $(1+\hat{r})^n = 1 + \frac{n!}{1!(n-1)!}\hat{r} + \frac{n!}{2!(n-2)!}\hat{r}^2 + \frac{n!}{3!(n-3)!}\hat{r}^3 + ...\hat{r}^n$, we then have

$$\begin{aligned} K(n) &= \frac{(1+\hat{r})}{\hat{r}} \left(\frac{(1+\hat{r})^n - 1}{\hat{r}} (1+\hat{r}) - n \right) w \cdot \frac{L}{n} \\ &= \frac{(1+\hat{r})}{\hat{r}^2} Q(n) \cdot wL \end{aligned} \quad (72)$$

where $Q(n) = \left(\hat{r} + \frac{(n-1)}{2}\hat{r}^2 + \frac{(n-1)(n-2)}{6}\hat{r}^3 + ... + \hat{r}^{n-1} + \frac{1}{n}\hat{r}^n \right)(1+\hat{r}) - \hat{r}$.

Note that the interest rate $\hat{r}$ is so small that we may assume $\hat{r}^k$ for $k \geq 3$ goes to zero. Then we can approximate $Q(n)$ by $\frac{(n+1)}{2}\hat{r}^2$, which gives

$$K(n, \hat{r}) = (1+\hat{r}) \left(\frac{n+1}{2} \right) wL \quad (73)$$

This suggests that the aggregate working capital is a function of the length of production chain and the interest rate in the initial set-up period.

The firm seeks to maximize the profit along the steady state

$$\begin{aligned} \pi &= n^\alpha zL - wzL - rzK(n, \hat{r}) \\ &= \left[n^\alpha - w - r(1+\hat{r}) \left(\frac{n+1}{2} \right) w \right] zL \end{aligned} \quad (74)$$

where r is the interest rate in the steady state, which may differ or be equal to the interest rate in the initial set-up period $\hat{r}$.

The first-order condition for n gives

$$\alpha n^{\alpha-1} = r(\frac{1+\hat{r}}{2})w \quad (75)$$

The zero profit condition is

$$n^\alpha = \left[1 + r(1+\hat{r}) \left(\frac{n+1}{2} \right) \right] w \quad (76)$$

From eqs. (75) and (76), the equilibrium chain length is

$$n = \frac{\alpha}{(1-\alpha)}\left(1 + \frac{2}{r(1+\hat{r})}\right) \tag{77}$$

which is reduced to $\frac{\alpha}{(1-\alpha)}\left(1 + \frac{2}{r(1+r)}\right)$ in case with $\hat{r} = r$, and eq. (9) in the benchmark case with $\hat{r} = 0$.

This suggests that an increase in the interest rate, regardless of whether in the initial set-up period or the steady state, results in fall in the length of production chain. An increase in the interest rate in the set-up period leads to a shortening of production chain by increaing the steady state working capital $K(n, \hat{r})$, while that of the steady state interest rate does the same by raising the interest charged on the working capital r.

B Generalization for offshoring model

In this appendix, we solve the model of offshoring in the general case where the interest rate in the initial set-up phase can be positive.

Consider firm i, which operates in the i-th from the most downstream among $\bar{n}+s+1$ stages of production/transportation of the global production chain. It begins production from date $\bar{n}+s-i+1$. The working capital that the firm holds at date $\bar{n}+s+1$ is

$$\sum_{t=\bar{n}+s-i+1}^{\bar{n}+s} [\Pi_{j=t}^{\bar{n}+s}(1+r_j)]w \tag{78}$$

Total working capital of a global production chain that hires one worker for each stage at date $\bar{n}+s+1$ is given by

$$\sum_{i=1}^{\bar{n}+s} \sum_{t=\bar{n}+s-i+1}^{\bar{n}+s} [\Pi_{j=t}^{\bar{n}+s}(1+r_j)]w \tag{79}$$

which the global production chain continues to roll over in the steady state.

The world's demand for working capital along the steady state is

$$K = \left[\sum_{i=1}^{\bar{n}+s} \sum_{t=\bar{n}+s-i+1}^{\bar{n}+s} (\Pi_{j=t}^{\bar{n}+s}(1+r_j))\right] w \cdot \frac{L}{(\bar{n}+s)} \tag{80}$$

In an analogous way to the benchmark Austrian model case in Appendix A, we can show that with the assumption $r_j = \hat{r}$ for all $j \leq n$, the expression inside the square bracket of eq. (80) is simplified to

$$\frac{(1+\hat{r})}{\hat{r}}\left(\frac{(1+\hat{r})^{\bar{n}+s}-1}{\hat{r}}(1+\hat{r}) - (\bar{n}+s)\right) \tag{81}$$

and we have

$$\begin{aligned} K(s) &= \frac{(1+\hat{r})}{\hat{r}}\left(\frac{(1+\hat{r})^{\bar{n}+s}-1}{\hat{r}}(1+\hat{r})-(\bar{n}+s)\right)w\cdot\frac{L}{(\bar{n}+s)} \\ &= \frac{(1+\hat{r})}{\hat{r}^2}Q(s)\cdot wL \end{aligned} \tag{82}$$

where $Q(s)=\left(\hat{r}+\frac{(\bar{n}+s-1)}{2}\hat{r}^2+\frac{(\bar{n}+s-1)(\bar{n}+s-2)}{6}\hat{r}^3+...+\frac{1}{\bar{n}+s}\hat{r}^{\bar{n}+s}\right)(1+\hat{r})-\hat{r}$.

Since $\hat{r}^k$ for $k\geq 3$ goes to zero, we can approximate $Q(s)$ by $\frac{(\bar{n}+s+1)}{2}\hat{r}^2$. Using this, we have

$$K(s,\hat{r})=(1+\hat{r})\left(\frac{\bar{n}+s+1}{2}\right)wL \tag{83}$$

The global production chain chooses s to maximize the profit along the steady state

$$\begin{aligned} \pi &= (\bar{n}+bs)^{\alpha}zL-wzL-rzK(s,\hat{r}) \\ &= \left[(\bar{n}+bs)^{\alpha}-w-r(1+\hat{r})\left(\frac{\bar{n}+s+1}{2}\right)w\right]zL \end{aligned} \tag{84}$$

which yields the first-order condition for s

$$\alpha b(\bar{n}+bs)^{\alpha-1}=r(\frac{1+\hat{r}}{2})w \tag{85}$$

The zero profit condition gives

$$(\bar{n}+bs)^{\alpha}=\left[1+r(1+\hat{r})\left(\frac{\bar{n}+s+1}{2}\right)\right]w \tag{86}$$

From eqs. (85) and (86), we derive the equilibrium extent of offshoring

$$s=\frac{\alpha}{(1-\alpha)}\left(1+\bar{n}+\frac{2}{r(1+\hat{r})}\right)-\frac{\bar{n}}{b(1-\alpha)} \tag{87}$$

which is expressed as $s=\frac{\alpha}{(1-\alpha)}\left(1+\bar{n}+\frac{2}{r(1+r)}\right)-\frac{\bar{n}}{b(1-\alpha)}$ in case where $\hat{r}=r$, and eq. (39) in the base case where $\hat{r}=0$.

This tells us that an increase in the interest rate, regardless of whether it is in the set-up period or the steady state, results in fall in the extent of offshoring.

C Trade growth accounting

In this appendix, we present a method of indirectly calculating the growth in offshoring. The growth in offshoring of intermediate good production is not easily observed directly. We describe an accounting framework which can be used to approximate it based on available data.

We define our measure of offshoring q as the ratio of imported intermediate goods to the total intermediate goods - both imported and domestically produced. Thus, we have:

$$q \equiv \frac{\text{imported intermediate goods}}{\begin{array}{c}\text{imported}\\ \text{intermediate goods}\end{array} + \begin{array}{c}\text{domestically produced}\\ \text{intermediate goods}\end{array}} \tag{88}$$

which approximates the offshoring ratio $(\frac{s}{\bar{n}})$ in the model. Data needed to directly calculate q is not readily available in most countries.

To present a method to approximate q, we use the following notation. Y is GDP, Y_m is manufacturing value-added, S_m is manufacturing sales (gross output) and M is total imports. Then, define β so that

$$\begin{aligned} S_m &= \beta \times \text{intermediate goods} \\ &= \beta \times \left(\begin{array}{c}\text{imported}\\ \text{intermediate goods}\end{array} + \begin{array}{c}\text{domestically produced}\\ \text{intermediate goods}\end{array} \right) \end{aligned} \tag{89}$$

Eq. (89) represents a production relationship between intermediate inputs and its gross output, where β is the coefficient for technology of the production function. So we take it as a constant parameter.

We also assume that imported intermediate goods is a constant fraction γ of total imports:

$$\text{imported intermediate goods} = \gamma \times M \tag{90}$$

Then by using our definitions of β and γ, we can write

$$\begin{aligned} M &= q \times S_m \times \frac{1}{\gamma\beta} \\ &= q \times \frac{S_m}{Y_m} \times \frac{Y_m}{Y} \times Y \times \frac{1}{\gamma\beta} \end{aligned} \tag{91}$$

So, import/GDP ratio is

$$\frac{M}{Y} = q \times \frac{S_m}{Y_m} \times \frac{Y_m}{Y} \times \frac{1}{\gamma\beta} \tag{92}$$

Given the assumption that β and γ are constants, then the growth of offshoring can be obtained as

$$g(q) = g\left(\frac{M}{Y}\right) - g\left(\frac{S_m}{Y_m}\right) - g\left(\frac{Y_m}{Y}\right) \tag{93}$$

In long hand we have: growth of Offshoring = growth of Imports/GDP − growth of Manufacturing Sales/Valued-added − growth of Manufacturing Value-added/ GDP.

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