Having the cards: Aggressive and passive third-party intervention in a rent-seeking conflict

Ryota Tsuchiya
Graduate School of Economics, The University of Tokyo

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1. Motivation

- Changes in the size of support from the U.S. to Ukraine

U.S. provides more aid to Ukraine, threatens to step up sanctions on Russia

By Alessandra Prentice

January 29, 2015 7:34 AM GMT+9 · Updated 10 years ago

U.S. Treasury Secretary Jack Lew departs after a Financial Stability Oversight Council open meeting at the Treasury Department in Washington January 21, 2015. REUTERS/Jonathan Ernst Purchase Licensing Rights

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Aid to Ukraine: How much have Kyiv's Western allies provided?

By Reuters

March 4, 2025 8:16 PM GMT+9 · Updated 3 months ago

Ukrainian President Volodymyr Zelenskiy gestures during a meeting with members of the media on the outskirts of London, Britain, March 2, 2025. REUTERS/Carlos Jasso/File Photo Purchase Licensing Rights

Before 2022

- $2 billion in financial aid

Source: Reuters, January 29, 2015

After 2022

- $66.9 billion in military assistance

Source: Reuters, March 4, 2025

- Outside intervention is often full-fledged after the outbreak of the conflict.

1. Key assumption in previous research

- **Implicit assumption**: Ex-ante intervention
- They construct a two-stage model in which the third parties act as Stackelberg leaders and the combatants are Stackelberg followers.

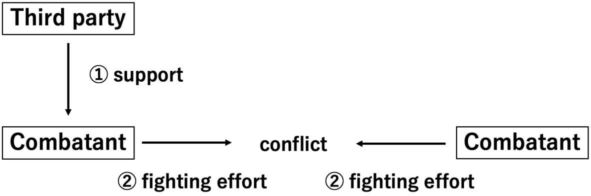

Figure: Ex-ante intervention

1. Research Question

- **Research Question**: How does the *timing* of third-party intervention shape the equilibrium conflict outcome?

1. Main findings of this study

- **Answer**: Passive intervention (ex-post) can effectively reduce fighting efforts compared to aggressive intervention (ex-ante).

$$\boxed{X^P < X^T < X^A}$$

2. Model

- There are two contestants and a third party.
  - Players 1 (e.g., Ukraine) and 2 (e.g., Russia) engage in a conflict by making a costly fighting effort, x_1, x_2.
  - Player 3 (e.g., the United States or some European countries) externally provides military or financial supports, M, for player 1 (i.e., biased intervention).

- Following Epstein and Hefeker (2003) and Hentschel (2022), the contest success function (CSF) is assumed as

$$p_1 = \frac{(1+M)x_1}{(1+M)x_1 + x_2} \quad \text{and} \quad p_2 = \frac{x_2}{(1+M)x_1 + x_2}, \tag{1}$$

where $p_1 + p_2 = 1$.

  - $M = 0$ reduces to a simple Tullock (1980) form of the CSFs.

2. Model

- Using the standard Tullock contest framework, the expected payoff for player $i = 1, 2$ is written as

$$U_i = p_i V_i - x_i$$

where $V_i > 0$ is the valuation to the rents of player i, and p_i is given by (1).

- Following Chang et al. (2007) and Chang and Sanders (2009), the expected payoff for player 3 is given by

$$\begin{aligned} U_3 &= p_1 S_1 + p_2 S_2 - M \\ &= S_2 + p_1 \underbrace{(S_1 - S_2)}_{\equiv S} - M, \end{aligned}$$

where S_i represents the value that the third party will obtain when faction $i = 1, 2$ wins the conflict, and we assume $S_1 > S_2 \geq 0$.

3.1 Passive intervention

- Two-stage contest model
  - Stage 1: Players 1 and 2 simultaneously choose non-negative fighting effort, x_1 and x_2.
  - Stage 2: Player 3 chooses non-negative support, M.

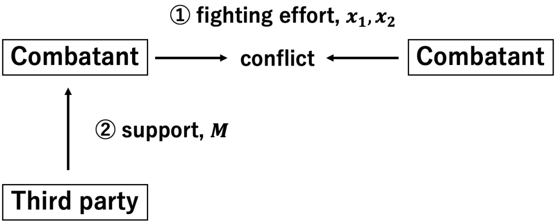

Figure: Passive intervention

3.1 Passive intervention (2nd stage)

- Given any $(x_1, x_2) \in \mathbb{R}^2$, the maximization problem for a third party is written as

$$\max_{M \geq 0} \quad U_3 = S_2 + p_1 S - M \tag{2}$$

subject to (1).

- The Karush-Kuhn-Tucker (KKT) conditions imply

$$\frac{\partial U_3}{\partial M} = \underbrace{\frac{x_1 x_2}{\left[(1+M)x_1 + x_2\right]^2}}_{= \partial p_1 / \partial M} S - 1 \leq 0.$$

  - There exist both interior and corner solutions.

3.1 Passive intervention (2nd stage)

- Solving the KKT conditions yields

$$M(x_1, x_2) = \sqrt{\frac{x_2 S}{x_1}} - \frac{x_2}{x_1} - 1, \tag{3}$$

if $S > \frac{(x_1+x_2)^2}{x_1 x_2}$, and $M(x_1, x_2) = 0$ otherwise.[1]

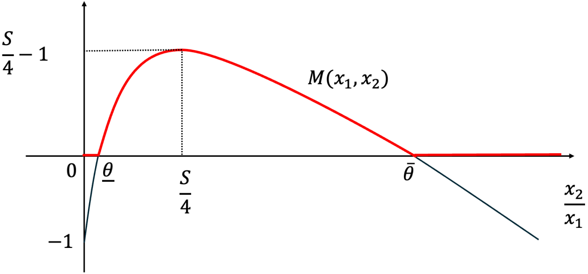

Figure: Best response function of the third party

[1]Note that $M(x_1, x_2)$ is homogeneous with degree zero.

3.1 Passive intervention (2nd stage)

- When $0 < S \leq \frac{(x_1+x_2)^2}{x_1 x_2}$, it is optimal for the third player not to intervene in the conflict, i.e., $M(x_1, x_2) = 0$, leading to the standard results of the Tullock contest (see p21).

- To focus on the analysis where there exists third-party intervention, suppose now that $S > \frac{(x_1+x_2)^2}{x_1 x_2}$.

3.1 Passive intervention (1st stage)

- Substituting (3) into (1), we obtain[2]

$$p_1 = 1 - \sqrt{\frac{x_2}{x_1 S}} \quad \text{and} \quad p_2 = \sqrt{\frac{x_2}{x_1 S}}. \tag{4}$$

  - These satisfy the standard properties of the CSFs: (i) $\partial p_i / \partial x_i > 0$, (ii) $\partial p_i / \partial x_j < 0$, (iii) $\partial^2 p_i / \partial x_i^2 < 0$, (iv) $\sum_i p_i = 1$.

[2] Note that $0 \leq p_i \leq 1$ holds from the assumption of $S > \frac{(x_1+x_2)^2}{x_1 x_2}$.

3.1 Passive intervention (1st stage)

- The maximization problem for player $i = 1, 2$ is written as

$$\max_{x_i \geq 0} \quad U_i = p_i V_i - x_i, \tag{5}$$

where p_i is given by (4).

- The FOCs for players 1 and 2 are[3]

$$\frac{\partial U_1}{\partial x_1} = \underbrace{\frac{1}{2x_1}\sqrt{\frac{x_2}{x_1 S}}}_{= \partial p_1 / \partial x_1} V_1 - 1 = 0, \tag{6}$$

$$\frac{\partial U_2}{\partial x_2} = \underbrace{\frac{1}{2}\sqrt{\frac{1}{x_1 x_2 S}}}_{= \partial p_2 / \partial x_2} V_2 - 1 = 0, \tag{7}$$

respectively.

[3] Note that $x_i = 0$ does not satisfy the KKT conditions.

3.1 Passive intervention (1st stage)

- The FOCs give the following best reply functions.

$$x_1(x_2) = \left[\frac{V_1^2 x_2}{4S}\right]^{\frac{1}{3}} \quad \text{and} \quad x_2(x_1) = \frac{V_2^2}{4Sx_1}. \tag{8}$$

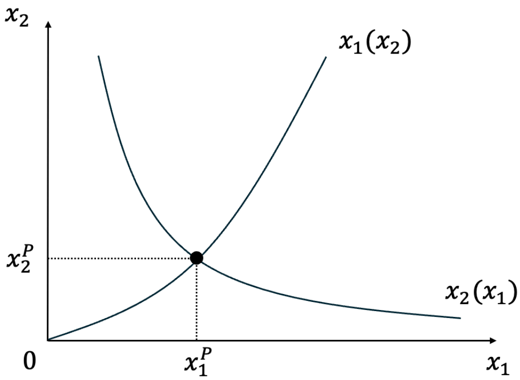

Figure: Best reply functions of the two combatants

3.1 Passive intervention (1st stage)

- Observing (8), we can see that the game has both strategic substitutes and complements.
- **Intuition**: Asymmetry in payoff structures
  - Player 1 has an incentive to increase x_1 against an increase in x_2 in order to defend his/her own share, p_1 (strategic complements).
  - Player 2, on the other hand, has an incentive to decrease x_2 against an increase in x_1 in order to make his/her own effort efficient (strategic substitutes).

3.1 Passive intervention (Equilibrium)

- Solving (8), the equilibrium conflict level under passive third-party intervention is determined as

$$x_1^P = \frac{V_1}{2}\sqrt{\frac{V_2}{V_1 S}} \quad \text{and} \quad x_2^P = \frac{V_2}{2}\sqrt{\frac{V_2}{V_1 S}}. \tag{9}$$

Assumption 1

In order for the stability condition, $|x_i'(x_j)| < 1$, to be satisfied, we assume $V_2 < V_1 < 3V_2$.

- Comparing (9), we have

$$x_1^P - x_2^P = \frac{V_1 - V_2}{2}\sqrt{\frac{V_2}{V_1 S}},$$

  which implies $x_1^P > x_2^P$ from Assumption 1.
  - This is a common feature that the stronger player (i.e, player with higher valuation) expends more resources into the conflict.

3.1 Passive intervention (Equilibrium)

- Substituting (9) into (3), the equilibrium support level can be expressed as

$$M^P = \sqrt{\frac{V_2 S}{V_1}} - \frac{V_2}{V_1} - 1, \qquad (10)$$

if $S > \frac{(V_1+V_2)^2}{V_1 V_2} \equiv \Theta(V_1, V_2)$, and $M^P = 0$ otherwise.

Assumption 2

In order for passive intervention to emerge as an equilibrium, we assume $S > \Theta(V_1, V_2)$.

Proposition 1

Passive third-party intervention emerges in equilibrium when there are strong ties between alliances, i.e., a large S, or the strength of the disputants is balanced, i.e., V_2/V_1 lies between the interval of $[\underline{\theta}, \overline{\theta}]$.

- **Implication**: The basic Tullock formulation without outside intervention is only valid when the two contestants are of heterogeneous natures.

3.1 Passive intervention (Equilibrium)

- From (9) and (10), the equilibrium conflict intensity under passive third-party intervention is defined as

$$X^P \equiv (1+\lambda M^P)x_1^P + x_2^P = \lambda \frac{V_2}{2} + (1-\lambda)\frac{V_1+V_2}{2}\sqrt{\frac{V_2}{V_1 S}}, \tag{11}$$

where $\lambda \in \{0,1\}$ is a binary parameter.

  - When $\lambda = 0$, X^P is a simple sum of the fighting effort made by the two contestants.
  - When $\lambda = 1$, X^P is the sum of the "effective" fighting effort, which includes the effects of military assistance provided by the third party.

3.1 Passive intervention (Equilibrium)

- Substituting (9) into (4), the equilibrium winning probability under passive intervention is calculated as[4]

$$p_1^P = 1 - \sqrt{\frac{V_2}{V_1 S}} \quad \text{and} \quad p_2^P = \sqrt{\frac{V_2}{V_1 S}}. \tag{12}$$

  - Under Assumptions 1 and 2, $p_1^P > p_2^P$ holds.

- The equilibrium payoffs of players 1 and 2 are

$$U_1^P = V_1 - \frac{3V_1}{2}\sqrt{\frac{V_2}{V_1 S}} \quad \text{and} \quad U_2^P = \frac{V_2}{2}\sqrt{\frac{V_2}{V_1 S}}. \tag{13}$$

  - Under Assumption 2, $U_1^P > U_2^P$ holds.

- The equilibrium payoff of the third party is

$$U_3^P = S_1 + \frac{V_2}{V_1} - 2\sqrt{\frac{V_2 S}{V_1}} + 1. \tag{14}$$

[4]Note that $0 \leq p_i^P \leq 1$ holds under Assumption 2.

3.2 Passive intervention vs. No intervention

- To evaluate the effect of passive intervention, recall that the equilibrium of the Tullock contest is described as[5]

$$x_1^T = \frac{V_1^2 V_2}{(V_1+V_2)^2} \quad \text{and} \quad x_2^T = \frac{V_1 V_2^2}{(V_1+V_2)^2}, \tag{15}$$

$$X^T \equiv x_1^T + x_2^T = \frac{V_1 V_2}{V_1+V_2}, \tag{16}$$

$$p_1^T = \frac{V_1}{V_1+V_2} \quad \text{and} \quad p_2^T = \frac{V_2}{V_1+V_2}, \tag{17}$$

$$U_1^T = \frac{V_1^3}{(V_1+V_2)^2} \quad \text{and} \quad U_2^T = \frac{V_2^3}{(V_1+V_2)^2}. \tag{18}$$

[5] See Konrad (2009).

3.2 Passive intervention vs. No intervention

Lemma 1

Suppose that Assumptions 1 and 2 hold. Then, passive third-party intervention mitigates the individual fighting effort of each combatant, $x_i^P < x_i^T$*. Furthermore, passive intervention can effectively reduce the aggregate conflict intensity, i.e.,* $X^P < X^T$*, for both* $\lambda = 0$ *and* $\lambda = 1$*.*

Proof.

See Appendix A. □

3.2 Passive intervention vs. No intervention

Lemma 2

Suppose that Assumptions 1 and 2 hold. Then, passive third-party intervention increases the winning probability of its ally and decreases that of its enemy, i.e., $p_1^P > p_1^T$ *and* $p_2^P < p_2^T$. *Moreover, passive intervention increases the expected payoff of its ally and decreases that of its enemy, i.e.,* $U_1^P > U_1^T$ *and* $U_2^P < U_2^T$.

Proof.

See Appendix B. □

4.1 Aggressive intervention

- Two-stage contest model
  - Stage 1: Player 3 chooses non-negative support, M.
  - Stage 2: Players 1 and 2 simultaneously choose non-negative fighting effort, x_1 and x_2.

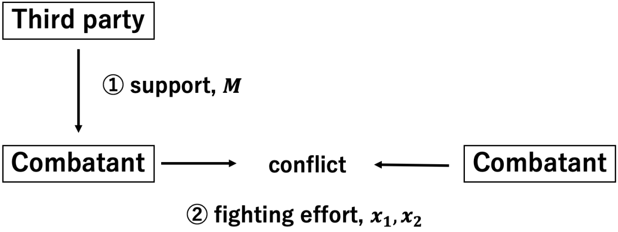

Figure: Aggressive intervention

4.1 Aggressive intervention (2nd stage)

- Given any $(M, x_j) \in \mathbb{R}^2$, the maximization problem for player $i = 1, 2$ is written as

$$\max_{x_i \geq 0} \quad U_i = p_i V_i - x_i,$$

subject to (1).

- The KKT conditions for players 1 and 2 are

$$\frac{\partial U_1}{\partial x_1} = \underbrace{\frac{(1+M)x_2}{[(1+M)x_1 + x_2]^2}}_{\partial p_1/\partial x_1} V_1 - 1 \leq 0, \tag{19}$$

$$\frac{\partial U_2}{\partial x_2} = \underbrace{\frac{(1+M)x_1}{[(1+M)x_1 + x_2]^2}}_{\partial p_2/\partial x_2} V_2 - 1 \leq 0, \tag{20}$$

respectively.

4.1 Aggressive intervention (2nd stage)

- The KKT conditions imply the following best response functions:

$$x_1(x_2, M) = \sqrt{\frac{x_2}{1+M}V_1} - \frac{x_2}{1+M} \quad (21)$$

if $x_2 \leq (1+M)V_1$ and $x_1(M, x_2) = 0$ otherwise, and

$$x_2(x_1, M) = \sqrt{(1+M)x_1V_2} - (1+M)x_1 \quad (22)$$

if $x_1 \leq \frac{V_2}{1+M}$ and $x_2(M, x_1) = 0$ otherwise.

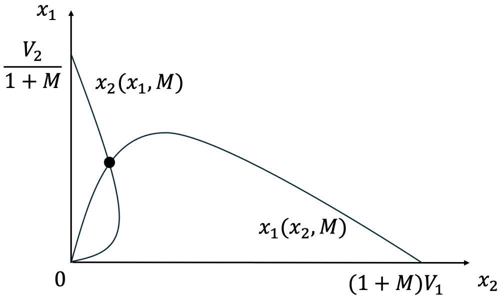

Figure: Best response functions of the two combatants.

4.1 Aggressive intervention (2nd stage)

- Solving the FOCs, we obtain

$$x_1(M) = \frac{(1+M)V_1^2 V_2}{\{(1+M)V_1 + V_2\}^2}, \tag{23}$$

$$x_2(M) = \frac{(1+M)V_1 V_2^2}{\{(1+M)V_1 + V_2\}^2}. \tag{24}$$

- From (23) and (24), the winning probability of player $i = 1, 2$ is given by[6]

$$p_1 = \frac{(1+M)V_1}{(1+M)V_1 + V_2} \quad \text{and} \quad p_2 = \frac{V_2}{(1+M)V_1 + V_2}. \tag{25}$$

[6] Note that for all $M \geq 0$, $0 \leq p_i \leq 1$ holds.

4.1 Aggressive intervention (1st stage)

- The maximization problem for a third party is formulated as

$$\max_{M \geq 0} \quad U_3 = S_2 + p_1 S - M,$$

where p_1 is given by (25).

- The KKT conditions imply

$$\frac{\partial U_3}{\partial M} = \underbrace{\frac{V_1 V_2}{[(1+M)V_1 + V_2]^2}}_{\partial p_1 / \partial M} S - 1 \leq 0. \tag{26}$$

4.1 Aggressive intervention (1st stage)

- Solving KKT conditions, the equilibrium aggressive support level can be derived as

$$M^A = \sqrt{\frac{V_2 S}{V_1}} - \frac{V_2}{V_1} - 1, \tag{27}$$

if $S > \Theta(V_1, V_2)$, and $M^A = 0$ otherwise.
  - This is equivalent to (10).

- To consider the case where there exists aggressive intervention (i.e., $M^A > 0$), suppose that Assumption 2 holds in the following.

4.1 Aggressive intervention (Equilibrium)

- Substituting (27) into (23) and (24), the equilibrium conflict level under aggressive intervention is determined as[7]

$$x_1^A = V_1\left(\sqrt{\frac{V_2}{V_1 S}} - \frac{V_2}{V_1 S}\right) \text{ and } x_2^A = V_2\left(\sqrt{\frac{V_2}{V_1 S}} - \frac{V_2}{V_1 S}\right). \quad (28)$$

Assumption 3

In order for the stability condition, $|x_i'(x_j)| < 1$, to be satisfied, we assume $1/2 < V_2 < V_1$ and $\Theta < S < 4\Theta$.

- Comparing (28),

$$x_1^A - x_2^A = (V_1 - V_2)\left(\sqrt{\frac{V_2}{V_1 S}} - \frac{V_2}{V_1 S}\right),$$

which implies that $x_1^A > x_2^A$ from Assumption 3.

[7] Note that $x_i^A > 0$ holds under Assumption 2.

4.1 Aggressive intervention (Equilibrium)

- From (27) and (28), the equilibrium conflict intensity under aggressive intervention is defined as

$$
\begin{aligned}
X^A &\equiv (1+\lambda M^A)x_1^A + x_2^A \\
&= \lambda \left[V_2 \left(1 - \sqrt{\frac{V_2}{V_1 S}} \right) \right] + (1-\lambda) \left[(V_1 + V_2) \left(\sqrt{\frac{V_2}{V_1 S}} - \frac{V_2}{V_1 S} \right) \right]. \quad (29)
\end{aligned}
$$

4.1 Aggressive intervention (Equilibrium)

- Substituting (27) into (25), the equilibrium winning probability is given by[8]

$$p_1^A = 1 - \sqrt{\frac{V_2}{V_1 S}} \quad \text{and} \quad p_2^A = \sqrt{\frac{V_2}{V_1 S}}, \tag{30}$$

which are equivalent to (12). From Assumptions 2 and 3, $p_1^A > p_2^A$.

- The equilibrium payoffs of players 1 and 2 are

$$U_1^A = V_1 \left(1 - \sqrt{\frac{V_2}{V_1 S}}\right)^2 \quad \text{and} \quad U_2^A = \frac{V_2^2}{V_1 S}. \tag{31}$$

  - Since $p_1^A > p_2^A$ and $V_1 > V_2$ from Assumptions 2 and 3, $U_1^A > U_2^A$.

- The equilibrium payoff of the third party is

$$U_3^A = S_1 + \frac{V_2}{V_1} - 2\sqrt{\frac{V_2 S}{V_1}} + 1, \tag{32}$$

which is equivalent to (14).

[8] Note that $0 \leq p_i^A \leq 1$ holds under Assumption 2.

4.2 Aggressive intervention vs. No intervention

Lemma 3

Suppose that Assumption 2 holds. Then, aggressive third-party intervention reduces the individual fighting effort, $x_i^A < x_i^T$. *Furthermore, while aggressive intervention can weaken the total fighting effort, it may intensify the effective total fighting effort, i.e.,* $X^A < X^T$ *if* $\lambda = 0$ *and* $X^A > X^T$ *if* $\lambda = 1$.

Proof.

See Appendix C. □

4.2 Aggressive intervention vs. No intervention

Lemma 4

Suppose that Assumption 2 holds. Then, aggressive third-party intervention increases the probability of winning for its ally and decreases that for its enemy, $p_1^A > p_1^T$ *and* $p_2^A < p_2^T$*. Moreover, aggressive intervention increases the expected payoff of its ally and decreases that of its enemy,* $U_1^A > U_1^T$ *and* $U_2^A < U_2^T$*.*

Proof.

See Appendix D. □

5. Aggressive intervention vs. Passive intervention

Proposition 2

Suppose that Assumptions 1 and 2 hold. Then, passive intervention can more effectively mitigate the individual fighting effort than aggressive intervention, $x_i^P < x_i^A \, (< x_i^T)$*. Furthermore, passive intervention can achieve a lower aggregate conflict intensity than aggressive intervention, i.e.,* $X^P < X^A \, (< X^T)$ *if* $\lambda = 0$ *and* $X^P \, (< X^T) < X^A$ *if* $\lambda = 1$*.*

Proof.

See Appendix E. □

5. Aggressive intervention vs. Passive intervention

Proposition 3

Suppose that Assumptions 1 and 2 hold. Then, the two combatants obtain higher expected payoffs from passive intervention than aggressive intervention, i.e., $(U_1^T <)\, U_1^A < U_1^P$ *and* $U_2^A < U_2^P (< U_2^T)$*. Furthermore, the third party is indifferent with respect to either aggressive or passive intervention, i.e.,* $U_3^P = U_3^A$*.*

Proof.

Noting that $p_i^P = p_i^A$ and $x_i^P < x_i^A$ for $i = 1, 2$, we have $U_i^P = p_i^P V_i - x_i^P > p_i^A V_i - x_i^A = U_i^A$ for $i = 1, 2$. In addition,(14) and (32) directly show that $U_3^P = U_3^A$. □

6. Summary

- **What I did**:
  To present a comprehensive model of both ex-ante and ex-post third-party intervention in a rent-seeking conflict model
- **What I found**:
  Compared to aggressive intervention, passive intervention can more effectively mitigate the intensity of conflict
- **Contribution**:
  To provide another perspective on “passive” third-party intervention and examines its potential effects on conflict outcomes

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Appendix A

- Comparing (9) and (15), we have

$$x_i^P \gtreqless x_i^T \iff S \lesseqgtr \frac{(V_1 + V_2)^4}{4V_1^3 V_2} \equiv \Omega(V_1, V_2)$$

Then, we have

$$\Theta(V_1, V_2) - \Omega(V_1, V_2) = \frac{(V_1 + V_2)^2(3V_1 + V_2)(V_1 - V_2)}{4V_1^3 V_2}.$$

Since $V_1 > V_2$ from Assumption 1, $\Theta(V_1, V_2) > \Omega(V_1, V_2)$ holds.
Thus, we have $S > \Omega(V_1, V_2)$, which shows $x_i^P < x_i^T$.

Appendix A

- Comparing (11) with $\lambda = 0$ and (16), we obtain

$$X^P\big|_{\lambda=0} - X^T = \frac{V_1 + V_2}{2}\sqrt{\frac{V_2}{V_1 S}} - \frac{V_1 V_2}{V_1 + V_2},$$

which implies that $X^P \gtreqless X^T \iff S \lesseqgtr \Omega(V_1, V_2)$. Since $S > \Omega(V_1, V_2)$, we have $X^P < X^T$.

- Comparing (11) with $\lambda = 1$ and (16), we obtain

$$X^P\big|_{\lambda=1} - X^T = \frac{V_2}{2} - \frac{V_1 V_2}{V_1 + V_2} = \frac{V_2(V_2 - V_1)}{2(V_1 + V_2)}.$$

Since $V_1 > V_2$ from Assumption 1, we have $X^P < X^T$.

Appendix B

- Comparing (12) and (17), we have

$$p_1^P \gtreqless p_1^T \text{ and } p_2^P \lesseqgtr p_2^T \iff S \gtreqless \Theta(V_1, V_2).$$

Since $S > \Theta(V_1, V_2)$ from Assumption 2, we obtain $p_1^P > p_1^T$ and $p_2^P < p_2^T$.

Appendix B

- From (13) and (18), for player 1, we have

$$U_1^P \gtreqless U_1^T \iff S \gtreqless \frac{9(V_1+V_2)^4}{4V_1V_2(2V_1+V_2)^2} \equiv \Upsilon(V_1,V_2).$$

  Then, we have

$$\Theta(V_1,V_2) - \Upsilon(V_1,V_2) = \frac{(V_1+V_2)^2(7V_1+5V_2)(V_1-V_2)}{4V_1V_2(2V_1+V_2)^2}.$$

  Since $V_1 > V_2$ from Assumption 1, we obtain $U_1^P > U_1^T$.

- Similarly for player 2, we have

$$U_2^P \gtreqless U_2^T \iff S \lesseqgtr \Omega(V_1,V_2).$$

  Since $S > \Omega(V_1,V_2)$, we have $U_2^P < U_2^T$.

Appendix C

- Comparing (28) and (15), we have

$$x_i^A - x_i^T = V_i \underbrace{\left(\sqrt{\frac{V_2}{V_1 S}} - \frac{V_2}{V_1 S} - \frac{V_1 V_2}{(V_1 + V_2)^2} \right)}_{\equiv g(S; V_1, V_2)}$$

Since $g(S)$ is decreasing in the range of $S > \Theta(V_1, V_2)$ with $g|_{S=\Theta} = 0$, $g(S) < 0$ holds under Assumption 2. Hence, $x_i^A < x_i^T$.

Appendix C

- Noting that $x_i^A < x_i^T$ holds for $i = 1, 2$, it suggests that $X^A < X^T$ holds for $\lambda = 0$.

- Comparing (29) with $\lambda = 1$ and (16), we have

$$X^A\big|_{\lambda=1} - X^T = V_2\left(\frac{V_2}{V_1 + V_2} - \sqrt{\frac{V_2}{V_1 S}}\right),$$

  which implies that $X^A \gtreqless X^T \iff S \gtreqless \Theta(V_1, V_2)$. Since $S > \Theta(V_1, V_2)$ from Assumption 2, we have $X^A > X^T$.

Appendix D

- Noting that $p_i^P = p_i^A$ for $i = 1, 2$, we obtain $p_1^A > p_1^T$ and $p_2^A < p_2^T$.
- From (31) and (18), for player 1 we have

$$U_1^A - U_1^T = V_1 \left[\left(1 - \sqrt{\frac{V_2}{V_1 S}} \right)^2 - \left(\frac{V_1}{V_1 + V_2} \right)^2 \right],$$

  which implies that $U_1^A \gtreqless U_1^T \iff S \gtreqless \Theta(V_1, V_2)$. Since $S > \Theta(V_1, V_2)$ from Assumption 2, we have $U_1^A > U_1^T$.
- Similarly for player 2, we have

$$U_2^A - U_2^T = V_2^2 \cdot \frac{(V_1 + V_2)^2 - V_1 V_2 S}{V_1 S (V_1 + V_2)^2},$$

  which implies that $U_2^A \gtreqless U_2^T \iff S \lesseqgtr \Theta(V_1, V_2)$. Since $S > \Theta(V_1, V_2)$ from Assumption 2, we have $U_2^A < U_2^T$.

Appendix E

- Comparing (9) and (28), we can see that

$$x_i^P - x_i^A = V_i \cdot \frac{\sqrt{V_2}(2\sqrt{V_2} - \sqrt{V_1 S})}{2V_1 S},$$

which implies that $x_i^P \gtreqless x_i^A \iff S \lesseqgtr \Lambda(V_1, V_2)$. Since $\Theta(V_1, V_2) > \Lambda(V_1, V_2)$, $S > \Lambda(V_1, V_2)$ holds, which shows $x_i^P < x_i^A$.

- Since $x_i^P < x_i^A$ holds for $i = 1, 2$, we immediately reach the result that $X^P < X^A$ for $\lambda = 0$.
- In addition, when $\lambda = 1$, we have already proven that $X^P < X^T$ and $X^T < X^A$, which implies $X^P < X^A$.