On Flexibility of Soft-Regulation with the 'Comply or Explain' Approach*

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Abstract

This paper theoretically examines whether or under what circumstances it is socially optimal that soft-regulation with the 'comply or explain' approach (SR) is inflexible in the sense that 'explain' is virtually unavailable for all firms. Moreover, we compare social welfare under SR and under conventional regulation of a uniform standard (CR) and investigate what is advantage (or disadvantage) of SR relative to the CR. We have the following main results. First, the optimal SR is not inflexible only when relatively high standard-compliance cost firms are a minority. Second, the optimal SR generates weakly higher social welfare than the optimal CR regardless of the type of the source of private information. However, what guarantees the superiority of the SR to the CR depends on the type of source of private information.

1 Introduction

In the 1960s and 1970s, many 'command-and-control' regulations in form of legislation have been adopted. Many of these regulations have been successful to quickly respond to problems related to environment, health, safety, etc. However, problems of conventional 'command-and-control' regulation were emphasized in the 1980s. One of major problems is inflexibility (or no discretion) that causes inefficiency in case of regulating heterogeneous firms (in particular, with private information). As alternatives of the inflexible conventional regulation, soft-regulation, legally non-binding forms of regulation such as guidelines and recommendations, resolutions, declarations, codes of practice and conduct and agreements between regulator and regulatees, has attracted interest of the regulators (Coglianese and Mendelson, 2010; Koutalakis et al., 2010).

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Soft-regulation delegates discretion to firms that know their compliance costs of regulation better than governments do. On one hand, such delegation enables exploiting information advantages of the regulated firms. The aim of exploiting the information advantages is to mitigate the lack of information need to flexibly regulate heterogenous firms. On other hand, too much delegation might lead hallowing out of the regulation(Koutalakis et al., 2010). Many soft-regulations before the mid-1990s did not include monitoring and reporting requirements (European Environment Agency, 1997). Under such regulations, the reported compliance status is unlikely credible. Therefore, firms under most soft-regulations have become to be required to report their performance or compliance status. In addition to reporting compliance status, some soft-regulations require firms to explain reasons for non-compliance if they do not comply. This type of soft-regulation is known as 'comply or explain' approach. 'comply or explain' approach has been adopted around the world, especially EU, to address issues in corporate governance and corporate social responsibility (Leyens, 2018).

Under the 'comply or explain' approach, as argued by Seidl et al. (2013), firms have an option to comply with (soft-)regulations other than complying with the standard of the regulations; to 'explain' or to verify their legitimacy of non-compliance with the standard by explanation. Moreover, the firms can choose how much they deviate from full-compliance though efforts to explain the reasons for non-compliance generally increase with the extent of deviation from full-compliance. Judging from the existence of this option or 'explain', it is supposed not to be socially optimal that all firms comply with the standard due to the firms' heterogeneity. However, in some cases, it might be socially optimal that 'explain' is virtually unavailable for all firms even though they would verify their legitimacy by an explanation if necessary.

This paper examines whether or under what circumstances it is socially optimal that soft-regulation with the 'comply or explain' approach is inflexible in the sense that 'explain' is virtually unavailable for all firms. In doing so, we take into account another feature of soft-regulation that is legally non-binding; legislative approval is not necessary to implement soft-regulations though regulated firms' acceptance is necessary to do so. Unnecessity of legislative approval, as well as flexibility, contributes to the wide use of soft-regulation, especially, for environmental issues because legally binding forms of environmental policy have been politically difficult to be implemented. Therefore, we incorporate the unnecessity of legislative approval into our theoretical model that is built to analyze whether or under what circumstances inflexible soft-regulation with the 'comply or explain' approach is socially optimal.

In our model, there are two types of players; a regulator and heterogeneous firms with private information on their standard compliance cost. The regulator has two regulatory options, a conventional regulation (CR) and a soft-regulation (SR). The CR is a uniform standard that all firms have to comply with. To enforce the CR, legislative approval to a bill on it is necessary but might not be given. On the other hand, the SR is implemented as long as all firms accept the regulator's proposal on the SR. If they reject it, then the regulator submits a bill on the CR. What we do is to examine whether and under what circumstances the optimal SR is inflexible in this setting. If the optimal SR is not inflexible and generates higher social welfare than the CR, SR's flexibility contributes to making regulation better. We examine this in different types of sources of private information and different structure of marginal external benefit from standard compliance.

Our main results are summarized as follows. First, the optimal SR is not inflexible only when relatively high standard-compliance cost firms are a minority regardless of the type of sources of private information and structure of the marginal external benefit from standard compliance. Of course, the results are a bit different between the different structures of the marginal external benefit and the different types of the source of private information. If the marginal external benefit of some firm's standard compliance depends on other firms' standard compliance levels, then the optimal SR is more likely to be inflexible than it is in the case where the marginal external benefit does not so. As for the source of private information, relative to the case when the slope of marginal standard compliance cost is unobservable (unobservable MSCC slope case), the optimal SR is likely to allow the deviation from the standard in the unobservable pre-regulation performance case.

Second, the optimal SR generates weakly higher social welfare than the CR regardless of the type of the source of private information and structure of marginal external benefit from standard compliance. However, what guarantees the superiority of the SR to the CR depends on the type of source of private information. The flexibility of the SR guarantees the superiority in unobservable pre-regulation performance case. In the unobservable MSCC slope case, the superiority is generated by what the SR is implemented for sure as long as all firms accept the proposal on the SR.

Soft-regulation has been studied in the context of environmental issues. Soft-regulation for these issues is defined as voluntary approach. Many researchers have studied voluntary approach to environmental protection due to an increase in the popularity of voluntary approaches. Because main focus is different from ours, Most researchers have developed models of a single regulated firm (e.g., Segerson and Miceli, 1998; Hansen, 1999; Segerson and Miceli, 1999; Glachant, 2007; and Fleckinger and Glachant, 2011) or of multiple regulated firms without informational asymmetry (Lutz et al., 2000; Maxwell et al., 2000; Manzini and Mariotti, 2003; Dawson and Segerson, 2008; and Brau and Carraro, 2011). Although Lyon and Maxwell (2003) examined a soft-regulation for multiple polluters with private information, the soft-regulation gives firms discretion to decide whether to adopt green technology with a uniform amount of adoption subsidy. Thus, the literature has not examined how soft-regulation can flexibly control the performance of a large number of firms under an asymmetric information case.

Technically, this paper analyzes some sort of delegation problem that was first defined and analyzed by Holmström (1977). In particular, because costly explanation can be interpreted as money burning, our problem is similar to the delegation problem with money burning that has been studied by Amador and Bagwell (2019), Amador et al. (2018) and Ambrus and Egorov (2017). In contrast with their single agent models, we study a model of multiple agents with different reservation utility. We examine relationship between money burning (costly explanation) and structure of reservation utility by considering two types of source of private information.

The remainder of this paper is organized as follows. In the next section, we describe the model environment. Section 3 presents the main results. Section 4 examines the voluntary policy in different settings. Section 5 concludes this paper.

2 Setup

We consider a case where a regulator wants to control some action or performance of firms with private information about their cost due to change in their action or performance such as pollution emitting, resource usage, etc by soft-regulation (SR) or conventional regulation (CR). To examine the potential of soft-regulation with the "comply-or-explain" approach in such a case, we develop a two-stage policy game played by a regulator and heterogeneous firms in an industry. In addition, we focus on a case where the benevolent regulator has strong negotiation power under the soft-regulation making process and makes a take-it-or-leave-it offer to the industry.

In the first stage, the regulator makes the take-it-or-leave-it offer of SR to the industry and then, the industry decides whether to accept the offer. The industry accepts the offer when all firms prefer the SR to the CR. If the industry accepts the offer, all firms follow the soft-regulation. If the industry rejects the offer, then, in the second stage, the regulator tries to introduce a CR for the industry with a probability of failure of $1-p$ ($0 < p \leq 1$). By introducing a possibility that the regulator fails to introduce the CR, we incorporate political difficulties in the conventional regulation-making process[1] as simple as possible to focus on firm heterogeneity. Thus, due to the risk of having no CR, the regulator has an incentive to introduce the soft-regulation or to make the offer of the SR to the industry. Once the CR is introduced, all firms must follow it. The timing of the game is summarized as follows:

Stage 1 (Soft-regulation (SR)) The regulator makes the take-it-or-leave-it offer a SR to the industry. If the industry accepts the offer, then firms follow the SR.

Stage 2 (Conventional regulation (CR)) If the industry rejects the offer, then the regulator tries to introduce a CR for the industry. The CR is adopted with probability p. Once the CR is adopted, all firms comply with it. Otherwise, firms do not change their performance.

Both soft-regulation (SR) and conventional regulation (CR) generate standard compliance costs of firms and external benefits from standard compliance. Let $\frac{1}{2}c_i a_i^2$ be firm i's standard compliance cost, where a_i and c_i are the standard for firm i and the slope of marginal standard compliance cost (MSCC) of firm i. We assume that c, the MSCC slope, is privately known by each firm and is distributed over $[\underline{c}, \bar{c}]$ ($\bar{c} > \underline{c} > 0$) with continuously differentiable probability density $f(c)$[2]. Then, the aggregate standard compliance cost is given by $\int_{\underline{c}}^{\bar{c}} \frac{1}{2}c[a(c)]^2 f(c)dc$. On the other hand, the external benefit from standard compliance is assumed to be $b\int_{\underline{c}}^{\bar{c}} a(c)f(c)dc$.

[1]Fleckinger and Glachant (2011) pointed out that "the assumption that the adoption of legislation is subject to uncertainty is both realistic and common in papers dealing with voluntary abatement" (p.43). The simplest way to incorporate the uncertainty of adopting the legislation into a model is to assume that p is purely exogenous like Segerson and Miceli (1998). We follow Segerson and Miceli's method. See p.43 of Fleckinger and Glachant (2011) for other ways to incorporate the uncertainty of adopting the legislation in the literature.

[2]We normalize the number of firms to unity.

First, we consider stage 2 or the conventional regulation (CR). We assume that the CR forces any firm to comply with the same standard, a_{CR}, irrespective of the MSCC slope. This assumption seems reasonable because the conventional "command-and-control" approach has been criticized as being inflexible and cost-ineffective. Because the objective of the benevolent regulator is the maximization of social welfare or social benefits from the CR, a_{CR}, is set as follows;

$$\begin{aligned} a_{CR} &= \underset{0\le a_{CR}\le e}{\arg\max}\ \ p\left\{b\int_{\underline{c}}^{\bar{c}} a_{CR}f(c)dc - \int_{\underline{c}}^{\bar{c}}\frac{1}{2}ca_{CR}^2 f(c)dc\right\} \\ &= \underset{0\le a_{CR}\le e}{\arg\max}\ \ ba_{CR} - \frac{1}{2}E[c]a_{CR}^2 \\ &= \frac{b}{E[c]} \end{aligned}$$

where $E[c] = \int_{\underline{c}}^{\bar{c}} cf(c)dc$.

Let's move to stage 1 or soft-regulation (SR). As we mentioned above, we focus on SR with the "comply-or-explain" approach that is a set of a "not legally required" standard and explanation for non-compliance with the standard. The explanation for non-compliance is supposed to be credible or high-quality. The high-quality explanation is very likely to be costly and therefore, the explanation can be interpreted as costly verification. In addition, firms complying with a lower compliance level of the standard are very likely to be required for a more detailed explanation (more costly verification). In a nutshell, we assume that when firms do not fully comply, they have to verify and verification cost depends on their compliance level of the standard.

Let a_{SR} and r be a standard under the SR and compliance level of the standard ($0 \le r \le 1$), respectively. Because the verification cost under the SR (or explanation cost under the SR) is assumed to depend on the compliance level of the standard, it can be defined as $v(r)$. Then, social welfare under the SR is given by

$$\int_{\underline{c}}^{\bar{c}}\left[br(c)a_{SR} - \frac{1}{2}c(r(c)a_{SR})^2 - v(r(c))\right]f(c)dc.$$

Because the difference in compliance level of the standard can be interpreted as a difference in the level of standard, we can let $a(c) = r(c)a_{SR}$ and call $a(c)$ virtual standard under the SR. In addition, by the Revelation Principle, we can focus on a direct mechanism to analyze what happens in equilibrium. Therefore, we can write the regulator's problem under the SR as

$$\max_{a(c),\ v(c)} \int_{\underline{c}}^{\bar{c}}\left[ba(c) - \frac{1}{2}c[a(c)]^2 - v(c)\right]f(c)dc \tag{1}$$

subject to

$$\text{(IC)}\quad c = \arg\min_{c'}[\frac{1}{2}c(a(c'))^2 + v(c')]\quad \forall c \tag{2}$$

$$\text{(PC)}\quad \frac{1}{2}c(a(c))^2 + v(c) \le p\frac{1}{2}c(a_{CR})^2\quad \forall c \tag{3}$$

where IC is the incentive compatibility condition and PC is the participation condition. The participation condition is that firms' cost under the SR is smaller than their expected cost under the CR (taking into account the possibility that the regulator fails to introduce the CR). Otherwise, the industry rejects the SR proposed by the regulator.

Because $\frac{1}{2}c[a(c)]^2$ satisfies the single-crossing property, the IC condition can be rewritten as $a'(c) \leq 0$ (monotonicity) and $\frac{1}{2}c(a(c))^2 + v(c) = \frac{1}{2}[\underline{c}a(\underline{c})^2 + \int_{\underline{c}}^{c}[a(\tilde{c})]^2 d\tilde{c}]$ (LIC)[3] like the standard screening model. In addition, from monotonicity and LIC, we have

$$\begin{aligned}\frac{1}{2}c(a(c))^2 + v(c) &= \frac{1}{2}[\underline{c}[a(\underline{c})]^2 + \int_{\underline{c}}^{c}[a(\tilde{c})]^2 d\tilde{c}] \\ &\leq \frac{1}{2}[\underline{c}[a(\underline{c})]^2 + [a(\underline{c})]^2 \int_{\underline{c}}^{c} 1 d\tilde{c}] = \frac{1}{2}c[a(\underline{c})]^2.\end{aligned}$$

As a result, we hereinafter use a condition $a(\underline{c}) \leq \sqrt{p}a_{CR}$ (PC$'$) instead of using PC condition. Finally, using LIC, we can rewrite the regulator's objective function as

$$\int_{\underline{c}}^{\bar{c}} \left\{ ba(c)f(c) - \frac{1}{2}[a(c)]^2[1-F(c)] \right\} dc - \frac{1}{2}\underline{c}[a(\underline{c})]^2.$$

Therefore, the regulator's problem under the SR rewrite as follows;

$$\max_{a(c)} \int_{\underline{c}}^{\bar{c}} \left\{ ba(c)f(c) - \frac{1}{2}[a(c)]^2[1-F(c)] \right\} dc - \frac{1}{2}[\underline{c}a(\underline{c})^2] \tag{4}$$

subject to

$$\text{(Monotonicity)} \quad a'(c) \leq 0 \quad \forall c \in [\underline{c}, \bar{c}] \tag{5}$$

$$\text{(PC}'\text{)} \quad a(\underline{c}) \leq \sqrt{p}a_{CR} \tag{6}$$

Because of the Monotonicity condition, we can interpret the virtual standard for firms with $\underline{c}$, $a(\underline{c})$, as the standard under the SR, a_{SR}. From PC$'$, the standard under the SR, a_{SR}, must be lower than $\sqrt{p}a_{CR}$. $\sqrt{p}a_{CR}$ is the best among uniform virtual standards satisfying (5) and (6) because a_{CR} is the best uniform standard from the property of CR and $\sqrt{p}a_{CR}$ is the nearest to a_{CR}. Therefore, if the regulator implements an "inflexible" SR or a SR with a uniform virtual standard, then the standard and the virtual standard must be $\sqrt{p}a_{CR}$. We define such a SR as inflexible SR.

Definition 1. Let a SR that $a(c) = \sqrt{p}a_{CR}$ and $v(c) = 0$ for all c be inflexible SR.

We use this inflexible SR to discuss the flexibility of SR. As long as the inflexible SR is optimal, the flexibility of SR does not make SR better. However, if the optimal SR is not the inflexible SR, the flexibility makes SR better. We define such a case as "SR is flexible" and its special case as "SR is perfectly flexible".

Definition 2. SR is flexible if the inflexible SR is not optimal. Particularly, SR is perfectly flexible if there exists $a(c)$ that is strictly monotonically decreasing or $a(c) = b\frac{f(c)}{1-F(c)}$ for all c.

[3]Local incentive compatibility.

From the F.O.Cs for (4), the virtual standard under the perfectly flexible SR is the solution for the regulator's problem (4) without constraints (5) and (6). In the standard screening model, there exists a distribution of MSCC slope, $f(c)$, such that the solution for a problem WITHOUT constraints is also the solution for the problem WITH the constraints. However, the following proposition states that this is not true of our problem.

Proposition 1. *$\frac{f(c)}{1-F(c)}$ must be strictly increasing for $\exists c \in [k, \bar{c}]$ where $\underline{c} < k < \bar{c}$ or the SR cannot be perfectly flexible.*

Proof. Suppose $d[f(c)/(1-F(c))]/dc \leq 0$ for $\forall c \in [k, \bar{c}]$ where $\underline{c} < k < \bar{c}$. Define $h(c) = -\log(1-F(c)) + \log(1-F(k))$ and $g(c) = h(c) - (c-k)h'(k)$. Then, $h''(c) \leq 0$ because $h'(c) = f(c)/(1-F(c))$. In addition, $g(k) = 0$, $g'(k) = 0$ and $g''(c) \leq 0$. Therefore, $g(c) \leq 0$. However, $g(\bar{c}) = h(\bar{c}) - (\bar{c} - k)h'(k) = +\infty$ because $F(\bar{c}) = 1$. This is contradiction. Therefore, $d[f(c)/(1-F(c)]/dc > 0$ for $\exists c \in [k, \bar{c}]$ and the optimal SR cannot be perfectly flexible. □

Note that we do not take into account the PC condition to show that the SR cannot be perfectly flexible. We just show monotonicity does not hold for some c or for some MSCC slope if $a(c) = b\frac{f(c)}{1-F(c)}$ for all c. Thus, the optimal virtual standard for some firms with different slopes of MSCC must be the same even though we ignore the PC condition. Even for some interval where $\frac{f(c)}{1-F(c)}$ is strictly decreasing (let it be $[c_1, c_2]$), the optimal virtual standard might be the same due to the PC condition (for example, $a(c) = \sqrt{p}a_{CR}$ for $c \in [c_1, c_2]$ if $b\frac{f(c)}{1-F(c)} > \sqrt{p}a_{CR}$). Unlike the standard screening model, c for which $a(c) = b\frac{f(c)}{1-F(c)}$ is the optimal virtual standard might be few. Therefore, we focus on whether the SR is flexible (flexibility makes SR better) and generates higher social welfare than the CR does.

When we compare social welfare under SR and CR, we check whether not only the optimal SR but also the inflexible SR generates higher social welfare than the CR does for the following reasons. First, if the inflexible SR generates higher social welfare than CR, then the optimal SR also generates higher social welfare than CR. Second, implementation certainty, as well as flexibility, is the difference between SR and CR in our model. Therefore, the superiority of the optimal SR over CR does not immediately imply that flexibility is the merit of SR . The flexibility is clearly merit of SR if the inflexible SR does not generate higher social welfare than the CR does but the optimal SR does so. However, if the inflexible SR always generates higher social welfare than the CR does and there are few cases where SR is flexible, then implementation certainty is clearly the merit of SR but flexibility is not so very much. Thus, by comparing the inflexible SR with the CR, we check the effects of implementation certainty and investigate which, flexibility or implementation certainty, is the main merit of SR.

3 Main results

The following proposition and corollary are our main results.

Proposition 2. *SR is flexible if and only if*

$$E[c|c \geq k] > c_{CR} + k \quad \exists k \in [\underline{c}, \bar{c}]. \tag{7}$$

where $E[c|c \geq k] = \int_k^{\bar{c}} cf(c)dc/[1 - F(k)]$ *and* $c_{CR} = E[c]/\sqrt{p} = b/(\sqrt{p}a_{CR})$. *The inflexible SR always generates at least the same social welfare as the CR does. If* $p < 1$, *the inflexible SR generates strictly higher social welfare than the CR does.*

Proof. See Appendix A.1. □

Corollary 1. *SR is not flexible if* $f(\cdot)$ *is weakly monotonically increasing.*

Proof. See Appendix A.2. □

Proposition 2 claims that SR is flexible if and only if social welfare increases by deviation from the inflexible SR and inequality (7) characterizes when the deviation is socially beneficial. Actually, by multiplying (7) by $\sqrt{p}a_{CR}[1 - F(k)]$, the LHS of (7) becomes marginal social benefit of the deviation and the RHS becomes marginal social cost;

$$\underbrace{\sqrt{p}a_{CR}\int_k^{\bar{c}} cf(c)dc}_{Marginal\ Standard\ Compliance\ Cost} > \underbrace{b[1 - F(k)]}_{Marginal\ benefit} + \underbrace{k[1 - F(k)]\sqrt{p}a_{CR}}_{Marginal\ Verification\ Cost} \quad \exists k \in [\underline{c}, \bar{c}] \tag{8}$$

To consider why the RHS and LHS of this inequality are marginal social benefit and marginal social cost of the deviation from the inflexible SR, first remember that the highest (virtual) standard under the SR accepted by firms is the one under the inflexible SR. Therefore, to deviate the inflexible SR, the regulator must lower the virtual standard and lowering the virtual standard induces decrease in standard compliance cost, decrease in external benefit from standard compliance and increase in verification cost. Decrease in standard compliance costs increases social welfare whereas decrease in external benefits from standard compliance and increase in verification costs decreases social welfare.

In addition, due to IC condition or $a'(c) \leq 0$ for all c, the regulator must also lowers the virtual standard for all firms with a higher steepness of MSCC slope than k (for all $c > k$) if it lowers the virtual standard for firms whose MSCC slope is k. Therefore, if the regulator changes a at k from $\sqrt{p}a_{CR}$ to $\sqrt{p}a_{CR} - \epsilon$, then decrease in standard compliance costs is equal to $\epsilon \int_k^{\bar{c}} c\sqrt{p}a_{CR}f(c)dc$, decrease in external benefit from standard compliance is $b[1 - F(k)]\epsilon$, and increase in verification costs $k\sqrt{p}a_{CR}[1 - F(k)]\epsilon$ from $v'(k) = -ka(k)a'(k)$. Thus, the LHS of (8) is marginal benefit of the deviation from the inflexible SR and the RHS is the marginal cost. Therefore, the deviation is socially beneficial if (7) holds and Proposition 2 claims that SR is flexible if and only if social welfare increases by deviation from the inflexible SR.

If c_{CR} is small and $E[c|c \geq k]$ is not close to k (much larger than k), (7) is likely to hold or SR is likely to flexible. $c_{CR}(= E[c]/\sqrt{p})$ is small if the distribution of firms is biased toward low steepness of MSCC slope (low c or of low standard compliance cost) and the CR is very likely to be adopted (high p). On the other hand, if high stabdard compliance cost firms are a minority of minority, $E[c|c \geq k]$ is likely close to k. Therefore, $E[c|c \geq k]$ is likely large if high standard compliance cost firms are not a minority of minority. Summarizing the above, SR is likely to flexible when both of the following conditions hold; (1) majority of firms are of low standard compliance cost but high standard compliance cost are not a minority of minority and (2) CR is very likely to be adopted. The following firms' distribution is a typical example satisfying (1);

$$f(c) = \begin{cases} \frac{7}{4} & (1 \leq c < 1.5) \\ \frac{1}{28} & (1.5 \leq c \leq 5) \end{cases}.$$

Figure 1 describes virtual standards ($a_{SR}(c)$), virtual standards without constraints of IC and PC ($a(c) = b\frac{f}{1-F}$), inflexible SR ($\sqrt{p}a_{CR}$) and standards under the complete information case ($a(c) = b/c$) where the distribution of firms' MSCC slope is the above. The firms' distribution (PDF) of figure 1 is biased toward low steepness of MSCC slope and if the CR is adopted for sure ($p = 1$). Therefore, c_{CR} is not large ($c_{CR} = 1.5$). In addition, high standard compliance cost firms are not a minority of a minority and $E[c|c > k]$ is not close to k ($E[c \mid c \geq 1.5] = 3.25$ when $k = 1.5$). Thus, SR is flexible because $c_{CR} + k < E[c|c > k]$ when $k = 1.5$, for example.

Generally, virtual standards under flexible SR are likely shaped like a_{SR} in figure 2. $\tilde{k}$ is c (MSCC slope) such that

$$\tilde{k} = \underset{\underline{c} \leq k \leq \bar{c}}{\arg\min} a_{SR}(k) \ where \ a_{SR}(k) = b\frac{1 - F(k)}{\int_k^{\bar{c}} cf(c)dc - k[1 - F(k)]}$$

$a_{SR}(\tilde{k})$ must be smaller than $\sqrt{p}a_{CR}$ because SR is flexible or (8) holds. Due to the monotonicity constraint (nondecreasingness of $a_{SR}(c)$), $a_{SR}(c) = a_{SR}(\tilde{k})$ for $c \geq \tilde{k}$. When c is small (in figure 2, c is smaller than $\tilde{c}$), $a_{SR}(c) = \sqrt{p}a_{CR}$ because $f(c)/(1 - F(c)) > \sqrt{p}a_{CR}$(since $f(c)$ is large if SR is flexibile as argued before). Thus, for small c ($c \leq \tilde{c}$ in figure 2), $a_{SR}(c) = \sqrt{p}a_{CR}$ whereas $a_{SR}(c) = a_{SR}(\tilde{k})$ for large c ($c \geq \tilde{k}$ in figure 2). How about $a_{SR}(c)$ for $\tilde{c} < c < \tilde{k}$? If $f(c)/(1-F(c))$ is monotonically nonincreasing, $a_{SR}(c) = bf(c)/(1 - F(c))$. However, If there exisits an interval between $\tilde{c}$ and $\tilde{k}$ such that $f(c)/(1 - F(c))$ is not monotonically nonincreasing like figure 2, there might exists a bunching interval like interval between $\underline{k}$ and $\overline{k}$. Please see Appendix A.3 for a formal discussion about the shape of virtual standards in the case where SR is flexible.

Finally, we have to mention that the inflexible SR always generates higher welfare than the CR does. PC condition induces this. More specifically, it is crucial that the virtual standard under the inflexible SR satisfies PC condition with equality for all firms or expected standard compliance costs of all firms under the inflexible SR are the same as those under the CR. Therefore, SR's implementation certainty is its merit relative to CR in the sense that implementation certainty guarantees the superiority of the SR to the CR. In the next section, we characterize the property of SR under different settings to examine under what circumstance SR's flexibility makes SR better and SR's implementation certainty is not the merit of SR.

4 Different settings

4.1 Influential team performance case

In the last section, external benefit from some firm's standard compliance was considered to be independent of other firms' standard compliance or "team" performance. However, team performance influences effects of individual firm's standard compliance on social welfare in some cases when the regulator addresses problems such as damage due to pollutant emissions. In this subsection, we consider a such case (and define it as influential team performance case). Concretely, marginal external benefit from standard compliance change from b in the last section to $b\int [e-a(c)]\, f(c)dc$ where e is pre-regulation performance[4]. In this case, standard under the CR is given by $a'_{CR} = \frac{be}{b+E[c]}$ and viratual standard under the inflexible SR is given by $\sqrt{p}a'_{CR}$ for all c (because both IC and PC do not change at all). Then, we have the following proposition.

Proposition 3. *SR is flexible iff*

$$E[c|c \geq k] > c'_{CR} + k \ \exists k \in [\underline{c}, \bar{c}]$$

where $c'_{CR} = [E[c] + b(1-\sqrt{p})]/\sqrt{p} = be/(\sqrt{p}a'_{CR}) - b$. *The inflexible SR always generates at least the same social welfare as the CR does. If* $p < 1$*, the inflexible SR generates strictly higher social welfare than the CR does.*

Proof. See Appendix A.4. □

Note that if $p = 1$, this necessary and sufficient condition coincides with that in the non-influential team performance case because $c_{CR} = c'_{CR}$ when $p = 1$ (in the linear case, $E[c|c \geq k] \leq c_{CR} + k \ \forall k \in [\underline{c}, \bar{c}]$ where $c_{CR} = E[c]/\sqrt{p}$). Otherwise, c_{CR} and c'_{CR} are different by $b(1-\sqrt{p})/\sqrt{p}$ if $p < 1$. This difference reflects the impact of some firm's standard compliance on marginal external benefit from other firms' standard compliance and makes the optimal SR more likely inflexible than the one in non-influential team performance case. Thus, flexibility makes SR better less often than it does in the non-influential team performance case.

The proposition also states that the inflexible SR always generates higher social welfare than the CR does. This result is also the same as in the non-influential team performance case. Thus, whether team performance affects marginal external benefit from individual firm's standard compliance does not change what implementation certainty guarantees the superiority of the SR to the CR. However, results are a bit different if information asymmetry of standard compliance costs results from pre-regulation performance rather than MSCC slope. We show this in the next subsection.

[4]Taking pollutant emissions as an example, we can interpret e as the amount of pollutant emissions before regulation and a as the amount of emissions reduction. As for the form of marginal external benefit, we have this marginal external benefit from standard compliance if the external benefit from standard compliance is given by $e^2 - (\int [e-a(c)]\, f(c)dc)^2$, for instance.

4.2 Unobservable pre-regulation performance case

In this subsection, we consider the case where the regulator cannot observe the pre-regulation performance of bad action,e but it can observe the MSCC slope, c, that is the same for all firms. Let e be distributed in $[\underline{e}, \bar{e}]$ with probability distribution $f(\cdot)$ that satisfies $f(\bar{e}) > 0$. In this case, we define standard under CR and SR by the performance (e_{CR} and $e_{\bar{SR}}$, respectively) rather than the performance change (a_{CR} and a_{SR}) because the regulator cannot observe the pre-regulation performance, e. As a result, the regulator cannot specify the performance change of individual firms. We define the inflexible SR in the same manner as an unobservable MSCC slope case and let virtual performance of the SR be e_{SR} (for details of $e_{\bar{SR}}$ as well as IC and PC conditions, see Appendix A.5). Then, we have the following proposition that shows when SR is flexible or when the inflexible SR is optimal.

Proposition 4. *The optimal SR always generates higher social welfare than the CR but the inflexible SR does not always do so. The condition for the flexibility of SR depends on whether the CR can be adopted for sure ($p = 1$).*

(i) Suppose that $p = 1$. Social welfare under the optimal SR (SWSR) is strictly greater than social welfare under CR(SWCR) if the SR is flexible. The SR is flexible if and only if $c[1-F(e)]-bf(e) > 0$ for some $e \in (\underline{e}, \bar{e})$.

(ii) Suppose that $p < 1$. SWSR is strictly greater than SWCR. The inflexible SR is optimal if $cf(e) + bf'(e) \geq 0$ $\forall e$.

Proof. See Appendix A.6. □

This proposition shows sufficient conditions that the inflexible SR is optimal. The inflexible SR is optimal unless the distribution of firms' pre-regulation performance, $f(e)$, is strongly biased toward low or medium level. If $f(e)$ is strongly biased toward low or medium level, $f(e)$ is very likely to sharply decrease in some range or $cf(e) + bf'(e) \geq 0$ $\forall e$ does not hold in the range. Therefore, if the distribution of firms' pre-regulation performance is not strongly biased toward a high or medium level, SR is very likely to be inflexible. In addition, if $cf(e) + bf'(e) \geq 0$ $\forall e$, $c[1 - F(e)] - bf(e)$ increases with e and $-bf(\bar{e}) < 0$ at $\bar{e}$. Thus, $cf(e) + bf'(e) \geq 0$ $\forall e$ is a sufficient condition for the optimality of inflexible SR or $c[1 - F(e)] - bf(e) \leq 0$ $\forall e$ regardless of whether the CR can be adopted for sure or not.

However, SR is likely flexible if $f(e)$ sharply decreases ($cf(e) + bf'(e) < 0$) for some range (but not short-range). For example, the distribution of firms' pre-regulation performance is strongly biased toward low or medium level like a normal distribution with low variance and high or medium mean. Actually, figure 3 gives an example when SR is flexible. In this example, the distribution of firms' pre-regulation performance is strongly biased toward the medium level. SR(e_{SR}) has inflexible part ($3.5 \leq e \leq 5.8$; $e_{SR} = 3.62$ and $e_{CR} = 3.5$) and flexible part ($5.8 < e \leq 10$). SR is also likely flexible if medium or high pre-regulation performance firms are few because if $f(e)$ is very small for medium or high e, then $f(e)$ is very likely to sharply decrease around this e.

In contrast with the unobservable MSCC slope case, the inflexible SR does not always generate higher social welfare than the CR does. The PC condition plays a key role. Firms' performance level under the SR that satisfies the PC condition with equality when verification cost is zero is increasing with their pre-regulation performance if $p < 1$ or the regulator might fail to introduce the CR (such performance level is independent of pre-regulation performance if $p = 1$). This implies that the inflexible SR can be too lenient for low pre-regulation performance firms and that they accept a more stringent performance target than the inflexible SR. Actually, the inflexible SR is too lenient from the viewpoint of social welfare if the distribution of firms' pre-regulation performance is strongly biased toward a low level or the number of firms sharply decreases as pre-performance level increases[5]. In addition, the inflexible SR is more lenient if the regulator is more likely to fail to introduce the CR (p is small) and as a result, SR is more likely to be flexible (if $f(e)$ sharply decreases for some range).

Although the inflexible SR might generate lower social welfare than the CR does, the SR with $e_{SR} = e - p(e - e_{CR})$ guarantees the same social welfare as the CR does and if $p < 1$[6]. As a result, the optimal SR always generates higher social welfare than the SR does. Thus, flexibility guarantees the superiority of the SR to the CR in the unobservable pre-regulation performance case, whereas implementation certainty guarantees the superiority of the SR to the CR in the unobservable MSCC slope case. In addition, SR is flexible with many forms of firms' distribution relative to the unobservable MSCC slope case. SR is flexible as long as firms with medium or high standard compliance costs are the minority in the unobservable pre-regulation performance case (so, SR can be flexible even if firms with low standard compliance costs are the minority). On the other hand, SR is flexible (only) if firms with low standard compliance costs are majority and firms with high standard compliance costs are not a minority of a minority in unobservable MSCC slope case. These facts indicate that the observability of individual firms' pre-regulation performance affects the merit of the SR's flexibility. The flexibility is beneficial in the unobservable case relative to the observable case.

5 Conclusion

This paper examines when the optimal soft-regulation (SR) allows firms with private information on their standard compliance cost to deviate from the standard by developing a game-theoretic model of regulation and compare social welfare under the optimal SR and conventional regulation (CR). In our model, CR is inflexible and might have implementation uncertainty, whereas SR has implementation certainty and is flexible in the sense that the SR can allow firms to deviate from

[5]In the Appendix A.6, we give an numerical example where the inflexible SR is worse than the CR ($p = 0.9$ $\delta = c = 1$. $\underline{e} = 1$, $\bar{e} = 10$ and $f(x) = ke^{-x^2}$ where $k = 1/(\int_1^{10} e^{-x^2} dx)$.).

[6]This standard is equal to the expected performance change (of type e) under the CR and the sum of standard compliance and verification costs under this standard is weakly smaller than the expected standard compliance cost under the CR. Note that this standard is inflexible or equal to e_{CR} if $p = 1$.

the standard if they submit costly evidence on their compliance cost to a regulator. By using this model, we argue that how types of sources of private information and shape of external benefit from standard compliance affect the shape of the optimal SR and the superiority of the SR to the CR. We can summarize our findings into the following two.

First, the optimal SR allows relatively high standard compliance cost firms to deviate from the standard only when they are a minority regardless of the shape of marginal external benefit from standard compliance and type of source of private information. Of course, the results are a bit different between the different functional forms and the different types of the source. If the marginal external benefit depends on other firms' standard compliance, then the optimal SR is less likely to allow firms to deviate relative to the case when If the marginal external benefit does not so. As for the source of private information, in the case of unobservable marginal standard compliance cost slope, the optimal SR allows high standard compliance cost firms to deviate from the standard when they are a minority and low standard compliance cost firms are a majority. On the other hand, in the unobservable pre-regulation performance case, the optimal SR does so when high standard compliance cost firms are a minority and low standard compliance cost firms are a majority or when high compliance cost firms are a minority and medium compliance cost firms are a majority. Thus, relative to the unobservable marginal standard compliance cost slope case, the optimal SR is likely to allow the deviation from the standard in the unobservable pre-regulation performance case.

Second, the optimal SR generates weakly higher social welfare than the optimal CR regardless of the type of the source of private information and functional form of the marginal external benefit. However, what guarantees the superiority of the SR to the CR depends on type of source of private information. On the one hand, flexibility guarantees the superiority in unobservable pre-regulation performance case where the most stringent standard accepted by "different" firms is different. On the other hand, implementation certainty does so in unobservable marginal standard compliance cost slope case where the most stringent standard by "different" firms is the same. From these two points, we have the following implication. When the regulator does not know about firms' performance that it wants to control, flexibility of SR is beneficial if explanation works well as costly verification. However, if the regulator knows about the performance, then there exist many cases when the inflexible SR is better than any flexible SR, relative to the unobservable pre-regulation performance case. Judging from this, it might be better that the regulator revises the SR to make it inflexible once the regulator understands firms' performance by policy implementation

In our model, we assumed that explanation works well as costly verification. However, it is an important issue to improve quality of explanation in the field of corporate governance code where the 'comply or explain' approach is very popular. In addition, many soft-regulations in environmental issues cannot enforce a firm's commitment but soft-regulation in our model is assumed to be enforceable. Therefore, it would be very interesting to consider the case in which soft-regulation is not enforceable. Exploring this type of extensions remains an endeavor for future research.

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A Appendix

A.1 Proof of Proposition 2

Proof. Let us assume $\bar{c} > \frac{b}{\sqrt{p}a_{CR}}$ hereinafter (otherwise, the Proposition is already solved). From (4), we just have to maximize $\int_{\underline{c}}^{\bar{c}} \left\{ ba(c)f(c) - \frac{1}{2}[a(c)]^2[1-F(c)] \right\} dc$.

($\Rightarrow$) Suppose that $a(c) = \sqrt{p}a_{CR}$ for all $c \in [\underline{c}, \bar{c}]$ is not optimal when $E[c|c \geq k] \leq c_{CR}+k \quad \forall k \in [\underline{c}, \bar{c}]$. Then, because $\dot{a}(c) \leq 0$ for all c must hold, optimal $a(c)$ must be strictly smaller than $\sqrt{p}a_{CR}$ for $c \geq \tilde{c}$ or for $c > \tilde{c}$. Let us define the following infinitedesimal function for sufficiently small positive value η and $0 < \lambda < 1$:

$$\ell(c) = \begin{cases} 0 & \text{when } c \leq \tilde{c} \\ \lambda(\sqrt{p}a_{CR} - a(c)) & \text{when } \tilde{c} \leq c \leq \tilde{c} + \eta \\ \lambda(\sqrt{p}a_{CR} - a(\tilde{c}+\eta)) & \text{when } \tilde{c} + \eta < c. \end{cases}$$

In addition, let us define $\epsilon = \lambda(\sqrt{p}a_{CR} - a(\tilde{c}+\eta))$ and $\hat{c} = \tilde{c} + \eta$. By adding $\ell(c)$ to $a(c)$, the increment of $\int_{\underline{c}}^{\bar{c}} \left\{ba(c)f(c) - \frac{1}{2}[a(c)]^2[1-F(c)]\right\} dc$ is

$$\begin{aligned}
b\epsilon \int_{\hat{c}}^{\bar{c}} f(c)dc - \int_{\underline{c}}^{\bar{c}} a(c)\ell(c)[1-F(c)]dc &= b\epsilon \int_{\hat{c}}^{\bar{c}} f(c)dc - \int_{\hat{c}}^{\bar{c}} a(c)\epsilon[1-F(c)]dc \\
&= b\epsilon \int_{\hat{c}}^{\bar{c}} f(c)dc - \epsilon \int_{\hat{c}}^{\bar{c}} a(c)[1-F(c)]dc \\
&> b\epsilon \int_{\hat{c}}^{\bar{c}} f(c)dc - \epsilon \int_{\hat{c}}^{\bar{c}} \sqrt{p}a_{CR}[1-F(c)]dc \\
&= b\epsilon[1-F(\hat{c})] - \epsilon\sqrt{p}a_{CR}\left\{[c(1-F(c))]_{\hat{c}}^{\bar{c}} + \int_{\hat{c}}^{\bar{c}} cf(c)dc\right\} \\
&= \epsilon\sqrt{p}a_{CR}[1-F(\hat{c})]\left\{(\hat{c} + b/\sqrt{p}a_{CR}) - \frac{\int_{\hat{c}}^{\bar{c}} cf(c)dc}{[1-F(\hat{c})]}\right\} \\
&= \epsilon\sqrt{p}a_{CR}[1-F(\hat{c})]\left\{(\hat{c} + b/\sqrt{p}a_{CR}) - E[c|c \geq \hat{c}]\right\} \geq 0.
\end{aligned}$$

This contradicts with optimality of $a(c)$. Therefore, if $E[c|c \geq k] \leq c_{CR} + k \quad \forall k \in [\underline{c}, \bar{c}]$, the inflexible SR is optimal. Or if SR is flexible, $E[c|c \geq k] > c_{CR} + k \quad \exists k \in [\underline{c}, \bar{c}]$.

($\Leftarrow$) Suppose $a(c) = \sqrt{p}a_{CR}$ for all c. Let us define the following infinitedesimal function for sufficiently small positive value ϵ where $k \in [\underline{c}, \bar{c}]$:

$$\ell_k(c) = \begin{cases} 0 & \text{when } c \leq k \\ k - c & \text{when } k < c \leq k + \epsilon \\ -\epsilon & \text{when } k + \epsilon < c. \end{cases}$$

By adding $\ell_k(c)$ to $a(c)$, the increment of $\int_{\underline{c}}^{\bar{c}} \left\{ba(c)f(c) - \frac{1}{2}[a(c)]^2[1-F(c)]\right\} dc$ is

$$\begin{aligned}
-b\epsilon \int_{k}^{\bar{c}} f(c)dc - \int_{\underline{c}}^{\bar{c}} \sqrt{p}a_{CR}\ell_k(c)[1-F(c)]dc &= -\int_{k}^{\bar{c}} \sqrt{p}a_{CR}\epsilon[1-F(c)]dc - b\epsilon \int_{k}^{\bar{c}} f(c)dc \\
&= -\epsilon\sqrt{p}a_{CR}\left\{[c(1-F(c))]_{k}^{\bar{c}} + \int_{k}^{\bar{c}} cf(c)dc\right\} - b\epsilon[1-F(k)] \\
&= \epsilon\sqrt{p}a_{CR}[1-F(k)]\left\{\frac{\int_{k}^{\bar{c}} cf(c)dc}{[1-F(k)]} - \frac{b}{\sqrt{p}a_{CR}} - k\right\} \\
&= \epsilon\sqrt{p}a_{CR}[1-F(k)]\left\{E[c|c \geq k] - c_{CR} - k\right\}
\end{aligned}$$

This increment must be smaller than zero if $a(c)$ is optimal. Therefore, if the inflexible SR is optimal, $E[c|c \geq k] \leq c_{CR} + k \quad \forall k \in [\underline{c}, \bar{c}]$. Or if $E[c|c \geq k] > c_{CR} + k \quad \exists k \in [\underline{c}, \bar{c}]$, SR is flexible.

Social welfare under the inflexible SR ($a(c) = \sqrt{p}a_{CR}$ for all c) is always weakly greater than the one under the CR because expected standard compliance cost under the inflexible SR and the CR are the same where expected external benefit under the inflexible SR is weakly greater than that under the CR because $\sqrt{p} \geq p$ holds. Since the inequality of $\sqrt{p} \geq p$ is the strict inequality if and only if $p < 1$, social cost under the inflexible SR is always strictly smaller than the one under the CR if and only if $p < 1$. If the inflexible SR is not optimal, social welfare under the optimal SR is strictly greater than the one under the CR regardless of the value of p. $\square$

A.2 Proof of Corollary 1

To prove $c_{CR} + k \geq E[c|c \geq k]$, it is sufficient to prove $E[c] + k \geq E[c|c \geq k]$. Because $\underline{c} \geq 0$, $E[c] + k \geq E[c|c \geq k]$ apparently holds when $k = \underline{c}$. Thus, it is sufficient to prove $\frac{\partial E[c|c \geq k]}{\partial k} \leq 1 \ \forall k$ because $\frac{\partial E[c]+k}{\partial k} = 1 \ \forall k$ clearly holds.

Note that $\frac{\partial E[c|c \geq k]}{\partial k} = \frac{\partial}{\partial k}\frac{\int_k^{\bar{c}} cf(c)dc}{1-F(k)} = (E[c|c \geq k] - k)\frac{f(k)}{1-F(k)} = (E[c|c \geq k] - k)\frac{f(k)}{\int_k^{\bar{c}} f(c)dc}$. Here, $0 \leq E[c|c \geq k] - k \leq \bar{c} - k$. In addition, because $f(c) \leq f(k)$ for all $c \in [k, \bar{c}]$, $\int_k^{\bar{c}} f(c)dc \geq \int_k^{\bar{c}} f(k)dc = f(k)(\bar{c} - k)$ and $\frac{f(k)}{\int_k^{\bar{c}} f(c)dc} \leq \frac{1}{\bar{c}-k}$. Therefore, $(E[c|c \geq k] - k)\frac{f(k)}{\int_k^{\bar{c}} f(c)dc} \leq (\bar{c} - k)\frac{1}{\bar{c}-k} = 1$ and consequently $\frac{\partial E[c|c \geq k]}{\partial k} \leq 1$ is satisfied.

A.3 Virtual standard in the basic setting

Proof. Suppose that there exists $k \in [\underline{c}, \bar{c}]$ such that $E[c|c \geq k] > c_{CR}+k$. Let $a_{SR}(c)$ be the solution for the problem. Then, $a_{SR}(c) < \sqrt{p}a_{CR}$ for some c. In addition, we know that $a_{FOC}(c) = b\frac{f(c)}{1-F(c)}$ obtained from F.O.Cs without constraints cannot be the solution because $\frac{f(c)}{1-F(c)}$ is increasing around $\bar{c}$ and the monotonicity constraint is not satisfied. Thus, there exists k such that the monotonicity constraint forces $a_{SR}(c)$ to be equal to $a_{SR}(k)$ for $c \geq k$. Impact of infinitesimal change in $a_{SR}(k)$ on social welfare is as follows;

$$\int_k^{\bar{c}} bf(c)dc - a_{SR}(k)\int_k^{\bar{c}} [1 - F(c)]dc \tag{9}$$

Setting $a_{SR}(k)$ is optimal such that (9) is equal to 0. Therefore,

$$a_{SR}(k) = b\frac{\int_k^{\bar{c}} f(c)dc}{\int_k^{\bar{c}}[1 - F(c)]dc}.$$

Let

$$\tilde{a}(c) = \min[\sqrt{p}a_{CR},\ b\frac{\int_c^{\bar{c}} f(\tilde{c})d\tilde{c}}{\int_c^{\bar{c}}[1 - F(\tilde{c})]d\tilde{c}}]$$

and

$$k' = \min[c|c \in \arg\min \tilde{a}(c)].$$

Then, k should be equal to k'. Suppose that $k < k'$. Because $\tilde{a}(k) > \tilde{a}(k')$, we have

$$\int_{k'}^{\bar{c}} bf(c)dc - \tilde{a}(k)\int_{k'}^{\bar{c}}[1 - F(c)]dc < bf(c)dc - \tilde{a}(k')\int_{k'}^{\bar{c}}[1 - F(c)]dc = 0.$$

Therefore, by chaging from $a_{SR}(c) = \tilde{a}(k)$ for $c \geq k'$ to $a_{SR}(c) = \tilde{a}(k) - \epsilon$ for $c \geq k'$, the social welfare increases and this change in $a_{SR}(c)$ satisfies the monotonicity constraint. This contradicts with optimality of $a_{SR}(c)$. Thus, k must not be smaller than k'.

Next, suppose that $k > k'$and $k \notin \arg\min \tilde{a}(c)$. Then, because $\tilde{a}(k) > \tilde{a}(k')$ for $k \notin \arg\min \tilde{a}(c)$, we have

$$\int_{k'}^{\bar{c}} bf(c)dc - \tilde{a}(k) \int_{k'}^{\bar{c}} [1 - F(c)]dc < \int_{k'}^{\bar{c}} bf(c)dc - \tilde{a}(k') \int_{k'}^{\bar{c}} [1 - F(c)]dc = 0.$$

Therefore, by chaging from $a_{SR}(c) = \begin{cases} a_{SR}(c) > \tilde{a}(k) & \text{for } k' \leq c \leq k \\ \tilde{a}(k) & \text{for } c \geq k \end{cases}$ to $a_{SR}(c) = \tilde{a}(k) - \epsilon$ for $c \geq k'$, the social welfare increases. Thus, k must not be smaller than k' and $k \notin \arg\min \tilde{a}(c)$.

Finally, suppose that $k > k'$and $k \in \arg\min \tilde{a}(c)$. Then, because $\tilde{a}(k) = \tilde{a}(k')$, $a_{SR}(c) > \tilde{a}(k)$ for all $k' \leq c < k$, and

$$\begin{aligned} \int_{k'}^{k} bf(c)dc - \tilde{a}(k') \int_{k'}^{k} [1 - F(c)]dc \qquad\qquad & \\ + \int_{k}^{\bar{c}} bf(c)dc - \tilde{a}(k') \int_{k}^{\bar{c}} [1 - F(c)]dc & = \int_{k'}^{k} bf(c)dc - \tilde{a}(k') \int_{k'}^{k} [1 - F(c)]dc \\ & \qquad + \int_{k}^{\bar{c}} bf(c)dc - \tilde{a}(k) \int_{k}^{\bar{c}} [1 - F(c)]dc \\ & = \int_{k'}^{k} bf(c)dc - \tilde{a}(k') \int_{k'}^{k} [1 - F(c)]dc = 0, \end{aligned}$$

we have

$$\int_{k'}^{k} bf(c)dc - \int_{k'}^{k} a_{SR}(c)[1 - F(c)]dc < \int_{k'}^{k} bf(c)dc - \tilde{a}(k') \int_{k'}^{k} [1 - F(c)]dc = 0.$$

Therefore, by chaging from $a_{SR}(c)$ to $a_{SR}(c) = a_{SR}(c) - \epsilon$ for $k' \leq c \leq k$, the social welfare increases. Thus, k must not be smaller than k' and $k \in \arg\min \tilde{a}(c)$. From the above argument, $k = k'$ and $a_{SR}(c) = \tilde{a}(k')$ for all $c \geq k'$.

How about $c \leq k'$? Let

$$\hat{a}(c) = \min[\sqrt{p}a_{CR},\ b\frac{f(c)}{1 - F(c)}].$$

Then, if $\hat{a}(c)$ is monotinic decreasing, then

$$a_{SR}(c) = \begin{cases} \hat{a}(c) & \text{if } c < k \\ \tilde{a}(k') & \text{if } c \geq k'. \end{cases}$$

If there exist intervals where $\hat{a}(c)$ is strictly increasing, let $\underline{q_i}$ and $\overline{q_i}$ starting point and end point of the ith interval, respectively. Then, there exists $\underline{k_i}$ and $\overline{k_i}$ such that $\underline{k_i} < \underline{q_i} < \overline{q_i} < \overline{k_i}$ and

$$\frac{f(\overline{k_i})}{1-F(\overline{k_i})} = \frac{f(\underline{k_i})}{1-F(\underline{k_i})} = \frac{\int_{\underline{k_i}}^{\overline{k_i}} f(c)dc}{\int_{\underline{k_i}}^{\overline{k_i}}[1-F(c)]dc}.$$

In this case,

$$a_{MR}(c) = \begin{cases} \hat{a}(c) & \text{if } c < \underline{k_1} \\ \min[a_{FOC}(\overline{k_1}),\ \sqrt{p}a_{CR}] & \text{if } \underline{k_1} \leq c \leq \overline{k_1} \\ \hat{a}(c) & \text{if } \overline{k_1} < c < \underline{k_2} \\ \vdots & \vdots \\ \min[a_{FOC}(\overline{k_n}),\ \sqrt{p}a_{CR}] & \text{if } \underline{k_n} \leq c \leq \overline{k_n} \\ \hat{a}(c) & \text{if } \overline{k_n} < c < k' \\ \tilde{a}(k') & \text{if } c \geq k'. \end{cases}$$

□

A.4 Proof of Proposition 3

Let us assume $\bar{c} \geq \frac{b(e-\sqrt{p}a'_{CR})}{\sqrt{p}a'_{CR}}$ here (otherwise, the Proposition is already solved.). In addition, $\dot{a}(c) \leq 0$ for all c must hold because firm's cost function is the same as non-influential team performance case. Verification cost is calculated as$v(c) = \frac{1}{2}[\int_{\underline{c}}^{c} a(c')^2dc' + p\underline{c}a'^2_{CR} - ca(c)^2]$. Firm's cost is $\frac{1}{2}[\int_{\underline{c}}^{c} a(c')^2dc' + p\underline{c}a'^2_{CR}]$ Taking also into account the external benefit from controlling firms' performance, the social welfare is $\frac{1}{2}b\int_{\underline{c}}^{\bar{c}} a(c)f(c)dc[e - \frac{1}{2}\int_{\underline{c}}^{\bar{c}} a(c)f(c)dc] - \frac{1}{2}\int_{\underline{c}}^{\bar{c}}\int_{\underline{c}}^{c} a(c')^2dc'f(c)dc - \frac{1}{2}p\underline{c}a'^2_{CR}\int_{\underline{c}}^{\bar{c}} f(c)dc$. Our interest is whether $a(c) = \sqrt{p}a'_{CR}\forall c$ maximizes $\frac{1}{2}b\int_{\underline{c}}^{\bar{c}} a(c)f(c)dc[e - \frac{1}{2}\int_{\underline{c}}^{\bar{c}} a(c)f(c)dc] - \frac{1}{2}\int_{\underline{c}}^{\bar{c}}\int_{\underline{c}}^{c}[a(c')]^2dc'f(c)dc$.

Proof. ($\Rightarrow$) Suppose that $a(c) = \sqrt{p}a'_{CR}$ for all $c \in [\underline{c}, \bar{c}]$ is not optimal when $E[c|c \geq k] \leq c'_{CR} + k \quad \forall k \in [\underline{c}, \bar{c}]$. Then, because $\dot{a}(c) \leq 0$ for all c must hold, optimal $a(c)$ must be strictly smaller than $\sqrt{p}a'_{CR}$ for $c \geq \tilde{c}$ or for $c > \tilde{c}$. Let us define the following infinitedesimal function for sufficiently small positive value η and $0 < \lambda < 1$:

$$\ell(c) = \begin{cases} 0 & \text{when } c \leq \tilde{c} \\ \lambda(\sqrt{p}a'_{CR} - a(c)) & \text{when } \tilde{c} \leq c \leq \tilde{c} + \eta \\ \lambda(\sqrt{p}a'_{CR} - a(\tilde{c} + \eta)) & \text{when } \tilde{c} + \eta < c. \end{cases}$$

In addition, let us define $\epsilon = \lambda(\sqrt{p}a'_{CR} - a(\tilde{c} + \eta))$ and $\hat{c} = \tilde{c} + \eta$. By adding $\ell(c)$ to $a(c)$, the

[6]If $\frac{\int_{\underline{k_i}}^{\overline{k_i}} f(c)dc}{\int_{\underline{k_i}}^{\overline{k_i}}[1-F(c)]dc} = \frac{f(\underline{k_i})}{1-F(\underline{k_i})} < \frac{f(\overline{k_i})}{1-F(\overline{k_i})}$ or $\frac{f(\underline{k_i})}{1-F(\underline{k_i})} < \frac{f(c)}{1-F(c)}$ for $\forall c > \overline{q_i}$, then q_i or some $c \in (\underline{k_i}, q_i)$ must be k'.

increment of $\frac{1}{2}b\int_{\underline{c}}^{\bar{c}} a(c)f(c)dc[e-\frac{1}{2}\int_{\underline{c}}^{\bar{c}} a(c)f(c)dc] - \frac{1}{2}\int_{\underline{c}}^{\bar{c}}\int_{\underline{c}}^{c}[a(c')]^2dc'f(c)dc$ is

$$
\begin{aligned}
& be\int_{\underline{c}}^{\bar{c}} \ell(c)f(c)dc - b\int_{\underline{c}}^{\bar{c}} a(c)f(c)dc\int_{\underline{c}}^{\bar{c}} \ell(c)f(c)dc - \int_{\underline{c}}^{\bar{c}}[\int_{\underline{c}}^{c} a(c')\ell(c')dc']f(c)dc \\
=& be\epsilon\int_{\hat{c}}^{\bar{c}} f(c)dc - b\int_{\underline{c}}^{\bar{c}} a(c)f(c)dc\int_{\hat{c}}^{\bar{c}} \epsilon f(c)dc - \int_{\hat{c}}^{\bar{c}} \epsilon\int_{\hat{c}}^{c} a(c')dc'f(c)dc \\
>& be\epsilon\int_{\hat{c}}^{\bar{c}} f(c)dc - b\int_{\underline{c}}^{\bar{c}} \sqrt{p}a'_{CR}f(c)dc\int_{\hat{c}}^{\bar{c}} \epsilon f(c)dc - \int_{\hat{c}}^{\bar{c}} \epsilon\int_{\hat{c}}^{c} \sqrt{p}a'_{CR}dc'f(c)dc \\
=& \epsilon b[e-\sqrt{p}a'_{CR}]\int_{\hat{c}}^{\bar{c}} f(c)dc - \epsilon\sqrt{p}a'_{CR}\int_{\hat{c}}^{\bar{c}}\int_{\hat{c}}^{c} dc'f(c)dc \\
=& \epsilon b[e-\sqrt{p}a'_{CR}]\int_{\hat{c}}^{\bar{c}} f(c)dc - \epsilon\sqrt{p}a'_{CR}\int_{\hat{c}}^{\bar{c}}(c-\hat{c})f(c)dc \\
=& \epsilon\sqrt{p}a'_{CR}[(\hat{c}+be/(\sqrt{p}a'_{CR})-b)\int_{\hat{c}}^{\bar{c}} f(c)dc - \int_{\hat{c}}^{\bar{c}} cf(c)dc] \\
=& \epsilon\sqrt{p}a'_{CR}[1-F(\hat{c})]\left(\hat{c}+c'_{CR}-E[c|c\geq\hat{c}]\right)\geq 0
\end{aligned}
$$

where inequality holds because $a(c) < \sqrt{p}a'_{CR}$ for $c \geq \tilde{c}$ or for $c > \tilde{c}$. This contradicts with optimality of $a(c)$. Therefore, if $E[c|c\geq k] \leq c'_{CR}+k \quad \forall k \in [\underline{c},\bar{c}]$, the inflexible SR is optimal. Or if SR is flexible, $E[c|c\geq k] > c'_{CR}+k \quad \exists k \in [\underline{c},\bar{c}]$.

($\Leftarrow$) Suppose $a(c) = \sqrt{p}a'_{CR}$ for all c. Let us define the following infinitedesimal function for sufficiently small positive value ϵ where $k \in [\underline{c},\bar{c}]$:

$$
\ell_k(c) = \begin{cases} 0 & \text{when } c \leq k \\ k-c & \text{when } k < c \leq k+\epsilon \\ -\epsilon & \text{when } k+\epsilon < c. \end{cases}
$$

For any virtual standard function $\bar{a}(\cdot)$ which is infinitesimally different from $a(c) = \sqrt{p}a'_{CR}\forall c$, the set of the above infinitesimal functions constitutes the basis for $\bar{a}-a$, i.e. $\bar{a}-a$ can be expressed as the superposition of the above infinitesimal functions. Therefore, the necessary and sufficient condition that $a(c) = \sqrt{p}a'_{CR}\forall c$ is the (locally) best SR is that $\frac{1}{2}b\int_{\underline{c}}^{\bar{c}} a(c)f(c)dc[e-\frac{1}{2}\int_{\underline{c}}^{\bar{c}} a(c)f(c)dc] - \frac{1}{2}\int_{\underline{c}}^{\bar{c}}\int_{\underline{c}}^{c}[a(c')]^2dc'f(c)dc$ under virtual standard function $a+\ell_k$ is always (weakly) smaller than that under virtual standard function a for all $k \in [\underline{c},\bar{c}]$. Taking the difference between them, this condition can be calculated as

$$
\begin{aligned}
& b[e\int_{\underline{c}}^{\bar{c}}\ell_k(c)f(c)dc-\int_{\underline{c}}^{\bar{c}}\ell_k(c)f(c)dc\int_{\underline{c}}^{\bar{c}}\sqrt{p}a'_{CR}f(c)dc]-\int_{\underline{c}}^{\bar{c}}\{\int_{\underline{c}}^{c}\sqrt{p}a'_{CR}\ell(c')dc'\}f(c)dc \\
=& -b[e-\sqrt{p}a'_{CR}]\epsilon\int_{k}^{\bar{c}}f(c)dc+\sqrt{p}a'_{CR}\epsilon\int_{k}^{\bar{c}}(c-k)f(c)dc \\
=& -b\epsilon[e-\sqrt{p}a'_{CR}](1-F(k))+\sqrt{p}a'_{CR}\epsilon\int_{k}^{\bar{c}}(c-k)f(c)dc \\
=& -\epsilon\sqrt{p}a'_{CR}[1-F(k)]\left((k+be/(\sqrt{p}a'_{CR})-b)-\int_{k}^{\bar{c}}cf(c)dc/[1-F(k)]\right) \\
=& -\epsilon\sqrt{p}a'_{CR}[1-F(k)]\left(k+c'_{CR}-[E[c|c\geq k]\right)
\end{aligned}
$$

This increment must be smaller than zero if $a(c)$ is optimal. Therefore, if the inflexible VP is optimal, $E[c|c\geq k]\leq c'_{CR}+k \quad \forall k\in[\underline{c},\bar{c}]$. Or if $E[c|c\geq k]>c'_{CR}+k \quad \exists k\in[\underline{c},\bar{c}]$, SR is flexible.

Social welfare under the inflexible SR ($a(c)=\sqrt{p}a'_{CR}$ for all c) is always weakly greater than the one under the CR because expected standard compliance cost under both regulations are the same where expected external benefit from standard compliance under the CR is weakly greater than that under optimal SR because $\sqrt{p}\geq p$ holds. Since the inequality of $\sqrt{p}\geq p$ is the strict inequality if and only if $p<1$, social welfare under the inflexible SR is always strictly greater than the one under the CR if and only if $p<1$. Therefore, if SR is flexible, social welfare under the optimal SR is strictly greater than the one under the CR regardless of the value of p. □

A.5 CR, inflexible SR and IC and PC conditions under unobservable pre-regulation performance case

Under unobservable pre-regulation performance case, the CR is a (local) optimum of the following problem;

$$\max_{e_R}\int_{e_R}^{\bar{e}}\left\{b(e-e_R)-\frac{1}{2}c(e-e_R)^2\right\}f(e)de.$$

From the F.O.C for this problem, its local optimum, e_{CR}, is given by $e_{CR}=\int_{\underline{e}}^{\bar{e}}ef(e)de-b/c$ if $\underline{e}\geq\int_{\underline{e}}^{\bar{e}}ef(e)de-b/c$ or $\int_{\underline{e}}^{\bar{e}}\{bf(e)-c(1-F(e))\}de\leq 0$ and characterized by

$$\int_{e_{CR}}^{\bar{e}}[b-c(e-e_{CR})]f(e)de=\int_{e_{CR}}^{\bar{e}}\left[bf(e)-c(1-F(e))\right]de=0 \tag{10}$$

if $\underline{e}<\int_{\underline{e}}^{\bar{e}}ef(e)de-b/c$. Thus, if $e_{CR}\geq\underline{e}$, then $e_{CR}=\int_{\underline{e}}^{\bar{e}}ef(e)de-b/c$ and if $e_{CR}<\underline{e}$, e_{CR} satisfies $\int_{e_{CR}}^{\bar{e}}\{bf(e)-c(1-F(e))\}de=0$.

Let $e_{MR}(e)$ be performance target under SR for firms whose pre-regulation performance is e.

Then, like unobservable MSCC slope case, we can write the regulator's problem under the SR as

$$\max_{a(e),\ s(e)} \int_{\underline{e}}^{\bar{e}} \left[ba(e) - \frac{1}{2}c[a(e)]^2 - v(e)\right] f(e)de \tag{11}$$

subject to

$$\text{(IC)} \quad e = \arg\min_{e'}[\frac{1}{2}c(e - e' + a(e'))^2 I_{[e>e'-a(e')]} + v(e')] \quad \forall e \tag{12}$$

$$\text{(PC)} \quad \frac{1}{2}c(a(e))^2 + v(e) \leq p\frac{1}{2}c(e - e_{CR})^2 I_{[e>e_{CR}]} \quad \forall e. \tag{13}$$

where $a(e) = e - e_{MR}(e)$. We derive the inflexible SR, $e_{\bar{M}R}$, by using (13). It is given by $e_{\bar{M}R} = \sqrt{p}e_{CR} + (1-\sqrt{p})\bar{e}$. Note that SR is inflexible (uniform performance standard) if $a'(e) = 1$ for all $e > e_{CR}$ and that SR is perfectly flexible if $a'(e) \neq 1$ for all $e > e_{CR}$.

We can rewrite the IC and PC conditions. By the following lemma (Lemma 1), the IC condition can be written as $a'(e) \leq 1\ \forall e$ and

$$v(e) = c\{A(e) - \frac{(a(e))^2}{2} + \frac{(a(\underline{e}))^2}{2}\}\ \forall e \tag{14}$$

where $A'(\cdot) = a(\cdot)$ and $A(\underline{e}) = 0$. (14) can be written in differentiable form as $c(1-a'(e))a(e) = v'(e)$ if $a(\cdot)$ and $v(\cdot)$ are differentiable. Note that the IC condition implies that $v(\cdot)$ is weakly monotonically increasing derived from $c(1-a'(e))a(e) = v'(e)$ and $a'(e) \leq 1$. Substituting (14) for the PC condition, the PC condition is simplified to:

$$A(e) \leq \frac{p(e-e_{CR})^2}{2} - \frac{(a(\underline{e}))^2}{2}\ \forall e. \tag{15}$$

Lemma 1. *IC condition $\Leftrightarrow$ (14) and $a'(e) \leq 1$ for all e.*

Proof of $\Rightarrow$

It is obvious that (14) holds for all $e \in [e_{CR}, \bar{e}]$ if the IC condition holds. Therefore, what we have to show is that $a'(e) \leq 1$ for all $e \in [e_{CR}, \bar{e}]$ if the IC condition holds. From the IC condition, $\forall e,\ e' \in [e_{CR}, \bar{e}]$,

$$\frac{1}{2}c(a(e))^2 + v(e) \leq \frac{1}{2}c(e - (e' - a(e')))^2 + v(e') \tag{16}$$

$$\frac{1}{2}c(a(e'))^2 + v(e') \leq \frac{1}{2}c(e' - (e - a(e)))^2 + v(e). \tag{17}$$

Without loss of generality, we can assume $e > e'$. Therefore, combining 16 and 17,

$$\begin{aligned}\frac{1}{2}c(e-(e'-a(e')))^2 - \frac{1}{2}c(a(e'))^2 &\geq \frac{1}{2}c(a(e))^2 - \frac{1}{2}c(e'-(e-a(e)))^2 \\ (e-e')(e-e'+2a(e')) &\geq (e-e')(e'-e+2a(e)) \\ e-e' &\geq a(e)-a(e') \\ \int_{e'}^{e} 1dt &\geq \int_{e'}^{e} a'(t)dt\end{aligned} \tag{18}$$

Therefore, $a'(e) \leq 1$ for all $e \in [\underline{e}, \bar{e}]$.

Proof of $\Leftarrow$

Suppose that the IC condition does not hold. Then, there exist e' and e'' (we can assume $e' > e''$without loss of generality) such that

$$\frac{1}{2}c(e'-(e''-a(e'')))^2 + v(e'') < \frac{1}{2}c(a(e'))^2 + v(e').$$

Therefore,

$$\begin{aligned}\frac{1}{2}c(e'-(e''-a(e'')))^2 + v(e'') - [\frac{1}{2}c(a(e''))^2 + v(e'')] &< \frac{1}{2}c(a(e'))^2 + v(e') - [\frac{1}{2}c(a(e''))^2 + v(e'')]. \\ \frac{1}{2}(e'-e''+a(e''))^2 - \frac{1}{2}(e''-e''+a(e''))^2 &< A(e') - A(e'') \\ \int_{e''}^{e'} [e-e''+a(e'')]de &< \int_{e''}^{e'} a(e)de. \\ \int_{e''}^{e'} [e-e'']de &< \int_{e''}^{e'} [a(e)-a(e'')]de.\end{aligned}$$

However, $\int_{e''}^{e'}[e-e'']de \geq \int_{e''}^{e'}[a(e)-a(e'')]de$ must hold because $a'(e) \leq 1$ implies that $e - e'' \geq a(e) - a(e'')$ for all $e \in [e'', e']$.

A.6 Proof of Proposition 4

Social welfare under the CR, written as SWCR, is written as follows:

$$SWCR = p\int_{\underline{e}}^{\bar{e}} \{b(e-e_{CR}) - \frac{c(e-e_{CR})^2}{2}\} I_{[e \geq e_{CR}]} f(e)de.$$

Then, let us define $\xi(e)$ as follows:

$$\xi(e) = \begin{cases} \frac{1}{2}p(e-e_{CR})^2 - A(e) - \frac{1}{2}(a(\underline{e}))^2 & \text{when } e \geq e_{CR} \\ 0 & \text{otherwise.}\end{cases}$$

Because of PC, $\xi(e) \geq 0$ always holds.

Social welfare under a SR, written as SWSR, is calculated as follows:

$$\begin{aligned}
SWSR &= \int_{\underline{e}}^{\bar{e}} \{ba(e) - \frac{1}{2}ca(e)^2 - v(e)\} I_{[e \geq e_{CR}]} f(e) de \\
&= \int_{\underline{e}}^{\bar{e}} \{ba(e) - cA(e) - \frac{1}{2}c(a(\underline{e}))^2\} I_{[e \geq e_{CR}]} f(e) de \\
&= \int_{\underline{e}}^{\bar{e}} \{pb(e - e_{CR}) - c\frac{1}{2}p(e - e_{CR})^2 + c\xi(e) - b\xi'(e)\} I_{[e \geq e_{CR}]} f(e) de \\
&= SWCR + \int_{\underline{e}}^{\bar{e}} \{c\xi(e) - b\xi'(e)\} f(e) de \\
&= SWCR + \int_{\underline{e}}^{\bar{e}} c\xi(e) f(e) de - [b\xi(e) f(e)]_{\underline{e}}^{\bar{e}} + \int_{\underline{e}}^{\bar{e}} b\xi(e) f'(e) de \\
&= SWCR + \int_{\underline{e}}^{\bar{e}} \{cf(e) + bf'(e)\} \xi(e) de + b\xi(\underline{e}) f(\underline{e}) - b\xi(\bar{e}) f(\bar{e})
\end{aligned}$$

where the second equality holds due to IC condition (2). Because the regulator can always choose $\xi(e) = 0$ for all e and $SWSR = SWCR$ if $\xi(e) = 0$ for all e, $SWSR \geq SWCR$ and in particular, $SWSR > SWCR$ if $\xi(e) = 0$ for all e is not optimal. However, inflexible SR, $e_{\bar{M}R} = \sqrt{p} e_{CR} + (1 - \sqrt{p})\bar{e}$, might underperform relative to the CR. For example, $\int_{\underline{e}}^{\bar{e}} \{cf(e) + bf'(e)\} \tilde{\xi}(e) de + b\tilde{\xi}(\underline{e}) f(\underline{e}) - b\tilde{\xi}(\bar{e}) f(\bar{e}) < 0$ where $\xi(e) = \frac{1}{2}p(e - e_{CR})^2 - \int_{\underline{e}}^{e} \max(\hat{e} - e_{\bar{M}R}, 0) d\hat{e} = \frac{1}{2}p(e - e_{CR})^2 - \frac{1}{2}(e - e_{\bar{M}R})^2 I_{e \geq e_{MR}}$ when $p = 0.9$ $b = c = 1$. $\underline{e} = 1$, $\bar{e} = 10$ and $f(x) = ke^{-x^2}$ where $k = 1/(\int_1^{10} e^{-x^2} dx)$. In this case, $SWSR < SWCR$ if the regulator adopts the inflexible SR.

Next, we consider when $p = 1$. We show that (i) SR is never perfectly flexible and (ii) SR is flexible and $SWSR > SWCR$ if $\frac{c}{b} > \frac{f(e)}{1 - F(e)}$ for some $e \in (\underline{e}, \bar{e})$. To show (i), we use the following lemma.

Lemma 2. *$\xi(e) = 0$ for all e must hold under the optimal $a(e)$ if SR is perfectly flexible.*

Proof. Suppose that $\forall e$; $a'(e) < 1$ (SR is perfectly flexible) and $\exists e$ s.t. $\xi(e) > 0$. Due to the continuity of $\xi(\cdot)$, there exists open interval (e_1, e_2) s.t. $\forall e \in (e_1, e_2)$; $\xi(e) > 0$. In (e_1, e_2), PC is not binding. Then, consider the infinitesimal function as follows.

$$\lambda(e) = \begin{cases} -\epsilon(e - e_1) & \text{when } e_1 \leq e \leq e_1 + \eta \\ -\epsilon\eta + \epsilon(e - e_1 - \eta) & \text{when } e_1 + \eta < e \leq e_1 + 3\eta \\ \epsilon\eta - \epsilon(e - e_1 - 3\eta) & \text{when } e_1 + 3\eta < e \leq e_1 + 4\eta. \\ 0 & \text{otherwise} \end{cases}$$

where $e_1 + 4\eta \leq e_2$. If positive variables ϵ and η are sufficiently small, neither $a(e) + \lambda(e)$ nor $a(e) - \lambda(e)$ violates IC and PC. Such addition or subtraction of $\lambda(\cdot)$ only changes the value of $A(e)$ in $[e_1, e_1 + 4\eta]$. If addition or subtraction of $\lambda(\cdot)$ changes SWSR, then the optimality of $a(e)$ is violated. Therefore, $cf(e) + bf'(e) = 0$ holds for all $e \in [e_1, e_2]$. By adding and subtracting countably many $\lambda(\cdot)$, $\xi(e) = 0$ can be achieved for $e \in [e_1, e_2]$. Thus, the optimal $a(e)$ satisfies$\xi(e) = 0$ for all

e if SR is perfectly flexible. $\square$

Lemma 2 implies that if SR is perfectly flexible, then both of the following must hold; under the optimal $a(e)$, $\xi(e) = 0$ for all e and $a'(e) < 1$ for all $e > e_{CR}$. However, $a(e)$ does not satisfy both$\xi(e) = 0$ for all e and $a'(e) < 1$ for all $e > e_{CR}$. When $p = 1$, $\xi(e) = 0$ for all e requires that $a(e) = \max(e - e_{CR}, 0)$ for all e (as a result, $a'(e) = 1$ for all $e > e_{CR}$) because

$$\xi(e) = \begin{cases} \int_{\underline{e}}^{e} [e - a(e) - e_{CR}]de - \frac{1}{2}[(\underline{e} - e_{CR})^2 - (a(\underline{e}))^2] & \text{if } \underline{e} \geq e_{CR} \\ \int_{\underline{e}}^{e} [e - a(e) - e_{CR}] I_{[e \geq e_{CR}]} de & \text{otherwise} \end{cases}$$

when $p = 1$. Thus, SR cannot be perfectly flexible.

Now, we show (ii). Suppose that the inflexible SR ($\xi(e) = 0$ for all e) is optimal and consider the following deviation from the inflexible SR; $\xi(\tilde{e}) = \epsilon$ holds for some $\tilde{e} \in (\underline{e}, \bar{e})$. Because $\dot{a}(e) \leq 1$, $\xi(e) = \epsilon$ for all $e \geq \tilde{e}$ must also hold. The difference in SWSR between the deviated SR and the inflexible SR is

$$\epsilon \int_{\tilde{e}}^{\bar{e}} \{cf(e) + bf'(e)\}de - b\epsilon f(\bar{e}) = \epsilon[c(1 - F(\tilde{e})) - bf(\tilde{e})].$$

This must be negative for all $\tilde{e} \in (\underline{e}, \bar{e})$ if the inflexible SR is optimal. Therefore, SR is flexible and $SWSR > SWCR$ if $\frac{c}{b} > \frac{f(e)}{1-F(e)}$ for some $e \in (\underline{e}, \bar{e})$.

Let's move the case when $p < 1$. If under the optimal $a(e)$,$\xi(e) > 0$ for some e, then, from lemma 2, SR is never perfectly flexible. Therefore, we show that under the optimal $a(e)$, $\xi(e) > 0$ for some e. First, consider a case where $e_{CR} \leq \underline{e}$. Because difference in SWSR between $\xi(e) = \epsilon > 0$ for all e and $\xi(e) = 0$ for all e is

$$\begin{aligned} \epsilon \int_{\underline{e}}^{\bar{e}} \{cf(e) + bf'(e)\}de + b\epsilon f(\underline{e}) - b\epsilon f(\bar{e}) =& \epsilon \int_{\underline{e}}^{\bar{e}} cf(e)de + \epsilon \int_{\underline{e}}^{\bar{e}} bf'(e)de - b\epsilon f(\bar{e}) + b\epsilon f(\underline{e}) \\ =& c\epsilon + b\epsilon[f(\bar{e}) - f(\underline{e})] - b\epsilon f(\bar{e}) + b\epsilon f(\underline{e}) \\ =& c\epsilon > 0, \end{aligned}$$

$a(e)$ with $\xi(e) = \epsilon > 0$ for all e is better than the one with $\xi(e) = 0$ for all e. Therefore, under the optimal $a(e)$, $\xi(e) > 0$ for some e if $e_{CR} \leq \underline{e}$ and $p < 1$.

Second, consider a case where $e_{CR} > \underline{e}$. In this case, SWSR is equal to $SWCR + \int_{e_{CR}}^{\bar{e}} \{cf(e) + bf'(e)\}\xi(e)de - b\xi(\bar{e})f(\bar{e})$. Suppose that under the optimal $a(e)$,$\xi(e) = 0$ for all e. Then, consider $\xi(e) = \epsilon$ for all $e \geq \tilde{e} \geq e_{CR}$ instead where ϵ is sufficiently small positive value. It increases SWSR by the amount of $\epsilon\{\int_{e_{CR}}^{\bar{e}} \{cf(e) + bf'(e)\}de - bf(\bar{e})\}$. If under the optimal $a(e)$, $\xi(e) = 0$ for all e, the amount must be negative. Therefore, $\int_{\tilde{e}}^{\bar{e}} \{cf(e) + bf'(e)\}(e)de - bf(\bar{e}) < 0$ $\forall \tilde{e} \in [e_{CR}, \bar{e})$ holds. The LHS of this inequality can be rewritten as

$$c(1 - F(\tilde{e})) + b(f(\bar{e}) - f(\tilde{e})) - bf(\bar{e}) = c(1 - F(\tilde{e})) - bf(\tilde{e}).$$

Therefore, $c(1-F(\tilde{e}))-bf(\tilde{e})<0\ \forall\tilde{e}\in[e_{CR},\bar{e})$ holds if under the optimal $a(e)$, $\xi(e)=0$ for all e. If $c(1-F(\tilde{e}))-bf(\tilde{e})<0\ \forall\tilde{e}\in[e_{CR},\bar{e})$, then $\int_{e_{CR}}^{\bar{e}}[c(1-F(e))-bf(e)]<0$. This contradicts that e_{CR} must satisfy $\int_{e_{CR}}^{\bar{e}}\{c(1-F(e))-bf(e)\}de=0$ if $e_{CR}>\underline{e}$. Therefore, under the optimal $a(e)$,$\xi(e)>0$ for some e if $e_{CR}>\underline{e}$ and $p<1$.

Next, we show that a sufficient condition for the optimality of the inflexible SR is $cf(e)+bf'(e)\geq 0\ \forall e$ (therefore, a necessary condition that SR is flexible is $cf(e)+bf'(e)<0\ \exists e$) when $p<1$. Suppose that $cf(e)+bf'(e)\geq 0\ \forall e$. Then, $A(\bar{e})>0$ must hold because our maximand is rewrited as follows;

$$\int_{\underline{e}}^{\bar{e}}\{ba(e)-cA(e)-\frac{1}{2}c(a(\underline{e}))^2\}f(e)de=bf(\bar{e})A(\bar{e})-\frac{1}{2}c(a(\underline{e}))^2$$
$$-\int_{\underline{e}}^{\bar{e}}\{cf(e)+bf'(e)\}A(e)de.$$

Otherwise, by letting $a(e)=\max\{0,e-\bar{e}+\epsilon\}$, the maximand is

$$bf(\bar{e})\frac{\epsilon^2}{2}-\{cf(\bar{e})+bf'(\bar{e})\}\frac{\epsilon^3}{6}>0$$

if ϵ is sufficiently small. When $A(\bar{e})=0$, the maximand is 0. This contradicts with optimality of $a(e)$. Therefore, $A(\bar{e})>0$.

$\int_{\underline{e}}^{\bar{e}}\{cf(e)+bf'(e)\}A(e)de$ is minimized by minimizing $A(e)$ for all e. However, $A(\bar{e})$ must be some positive value (letting it be $k<\frac{p}{2}(\bar{e}-e_{CR})^2$). Therefore, when $A(\bar{e})$ is fixed at k, $a(e)=0$ should hold for as long an interval as possible and $a(e)$ increases around $\bar{e}$ as sharply as possible. Due to the IC condition, $\dot{a}(e)\leq 1$ must hold. Therefore, $a(e)=\min\{0,e-\bar{e}+a(\bar{e})\}$. In addition, $A(\bar{e})=k>0$ must hold. This $a(e)$ is the same as setting uniform performance standard e_U for all firms. Because $e_{\bar{M}R}$ for all firms is the best uniform performance standards that satisfies the PC condition, $a(e)=\min\{0,e-e_{\bar{M}R}\}$ is optimal.

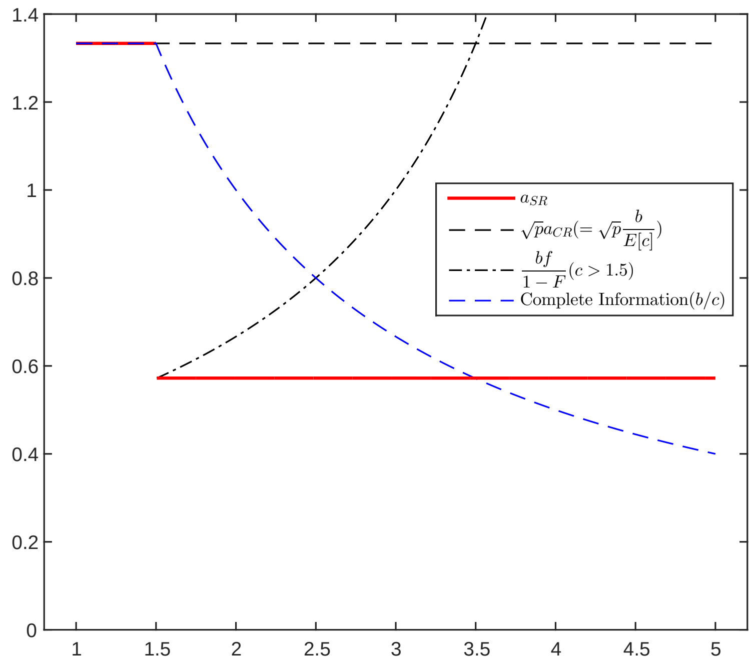

Figure 1: An example where SR is flexible ($p = 1$, $b = 2$, $c \in [1, 5]$, and $f(c) = \begin{cases} \frac{7}{4} & (1 \leq c < 1.5) \\ \frac{1}{28} & (1.5 \leq c \leq 5) \end{cases}$)

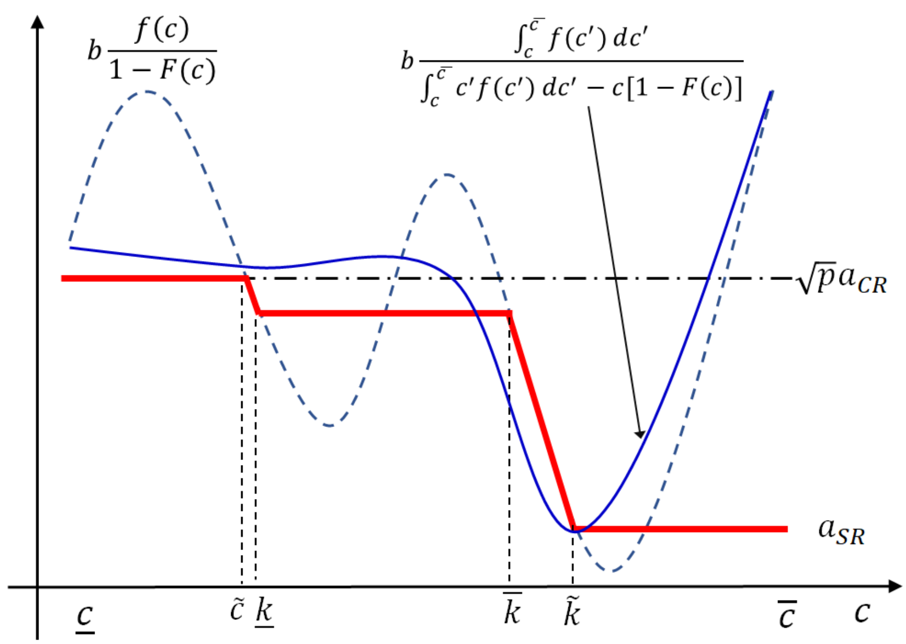

Figure 2: Virtual standards when SR is flexible

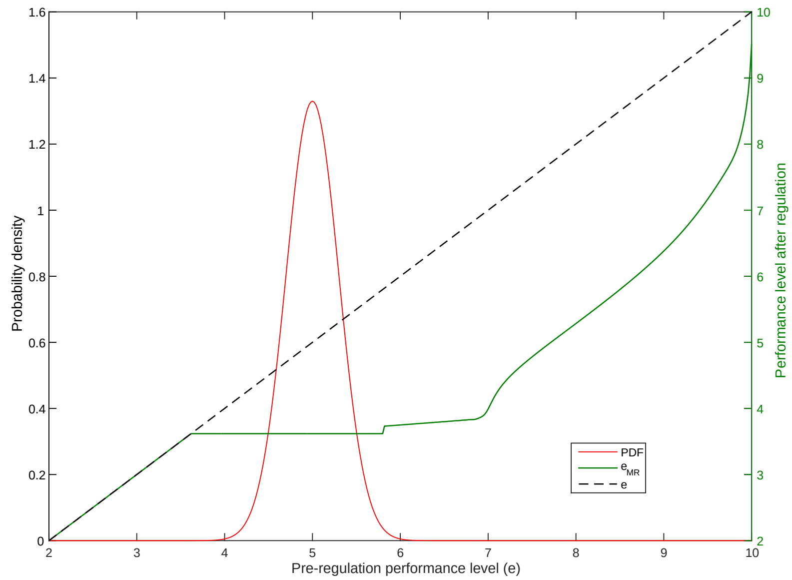

Figure 3: An example where SR is flexible (but not perfectly flexible) ($p = 0.9$, $b = 1.5$, $c = 1$, $e \in [2, 10]$ and $f(e) = \phi(e; 5, 0.3)$)