The Effects of Outliers and Model Misspecification on Recursive Modelling of Volatility*

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Abstract

This paper is concerned with recursive estimation, testing and forecasting of the volatility of daily returns in Standard and Poor's 500 Composite Index in the presence of outliers, or significant spikes in the volatility of daily returns, and model misspecification. The empirical analysis increases the sample size up to 12000 observations recursively to examine the effects of outliers and misspecification on: (i) the parameter estimates of the ARCH(1) and GARCH(1,1) processes; (ii) their associated asymptotic and robust t-ratios; (iii) the second and fourth moment conditions for stationarity, consistency and asymptotic normality; and (iv) the forecast performance for periods with significant spikes in volatility and for periods of relative calm.

Keywords: Outliers and extreme observations, volatility models, model misspecification, recursive modelling, structural change.

1 Introduction

The modelling of volatility has been an active area of research in finance in recent years, and has been largely motivated by the importance of risk considerations in economic and financial markets. Estimates of volatility are used widely for a variety of reasons, including modelling the premium in forward and futures markets, portfolio selection, asset management, pricing primary and derivative assets, valuation of warrants and options, designing optimal hedging strategies for options and futures markets, evaluating risk spill-overs across markets, measuring announcement effects in event studies, and examining asymmetries and leverage effects.

Engle (1982) first captured the time-varying nature of volatility with the autoregressive conditional heteroscedasticity (ARCH(p)) model. Bollerslev (1986) generalized the ARCH model to GARCH (p, q), and this has proved to be the single most popular time-varying volatility model in practice. GARCH has several quite attractive features, namely the ability to accommodate two key stylised facts of volatility in financial data, the persistence of volatility and volatility clusters, leptokurtic data, and mathematical and computational simplicity. Many theoretical results, including the statistical properties of the models and the asymptotic properties of several estimation methods, are now available, and these provide a solid foundation for applications of the various models (see Li et al. (1999) for a survey, directed towards practitioners, of recent important theoretical results for GARCH models).

A common feature encountered in high frequency financial time series is the occurrence of extreme observations, or significant spikes in volatility, which can adversely affect the estimates and forecasts of volatility. Questions arise as to how practitioners should handle these observations. Franses and Ghijsels (1999) suggested steps to correct the data for outliers when using GARCH models to forecast volatility. In this paper, we investigate a separate issue as to how many observations should be used from a large data set that includes extreme observations. We also investigate the effects of model misspecification on the estimates, forecasts, and outliers.

This paper is concerned with recursive estimation, testing and forecasting of the asymmetric volatility of daily returns in Standard and Poor's Composite 500 Index (S&P 500) in the presence of extreme observations in the volatility of daily returns. The empirical analysis increases the sample size up to 12000 observations recursively to examine the effects of extreme observations on: (i) the parameter estimates of the ARCH(1) and GARCH(1,1) processes; (ii) their associated asymptotic and robust t-ratios; (iii) the second and fourth moment conditions for stationarity, consistency and asymptotic normality; and (iv) the forecast performance for periods with significant spikes in volatility and for periods of relative calm.

Several interesting results emerge from the analysis, namely: expanding the sample sizes recursively and including an extreme observation does not necessarily improve the accuracy of predicting future extreme observations; the parameter estimates of the ARCH(1) and GARCH(1,1) processes, their associated asymptotic and robust t-ratios, the second and fourth moment regularity conditions, and various forecast performance measures, are all highly volatile in small samples, but stabilise when an extreme observation is included in the estimation period at sample sizes in excess of 2000; increasing the sample size recursively beyond an extreme observation is unnecessary; the robust t-ratios are, in general, dramatically superior to the asymptotic t-ratios; the second moment condition is always satisfied; the fourth moment condition is generally satisfied if we assume that the conditional error is normal, but not if it follows a fatter-tailed distribution such as the $t(5)$ distribution; increasing the sample sizes recursively does not necessarily lead to these conditions being satisfied, or to improving the forecasting performance; and neither model is superior in forecasting volatility.

The plan of the paper is as follows. Section 2 presents the ARCH(1) and GARCH(1,1) models. Section 3 describes the data. The empirical results are analysed in Section 4. Some concluding remarks are given in Section 5.

2 The ARCH and GARCH Models

Both models to be estimated have an AR(1) conditional mean (or logarithmic returns of the S&P 500 Index) specification given by

$$y_t = \mu + \phi y_{t-1} + \varepsilon_t \,. \tag{1}$$

2.1 The ARCH(1) Model

For the ARCH(1) model, the conditional variance of the unconditional shock ε_t is given by

$$\varepsilon_t = \eta_t \sqrt{h_t} \tag{2}$$

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 \tag{3}$$

where η_t is a sequence of normally, independently and identically distributed random variables with zero mean and unit variance. Sufficient conditions for h_t to be positive and for the ARCH process to exist are that $\omega > 0$ and $\alpha > 0$. A sufficient condition for the existence of the fourth moment of ε_t is $k\alpha^2 < 1$, where k is the conditional fourth moment of η_t. Under the assumption of conditional normality, $k \equiv E_t(\eta_t^4) = 3$, so that the condition becomes $3\alpha^2 < 1$. An alternative assumption is that η_t is distributed according to the t distribution with $\nu > 4$ degrees of freedom, in which case $k = 3(\nu - 2)/(\nu - 4)$, with $3 \leq k \leq 9$. In the extreme case $\nu = 5$, the condition becomes $9\alpha^2 < 1$.

2.2 The GARCH(1,1) Model

The GARCH(1,1) model differs from the ARCH(1) model only in the addition of a lagged variable h_{t-1} in the conditional variance equation, which takes the form

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}. \tag{3}$$

Sufficient conditions for h_t to be positive and for the GARCH process to exist are that $\omega > 0$, $\alpha > 0$, and $\beta \geq 0$.

Several statistical properties have been established for the GARCH(1,1) process in order to define the unconditional moments of ε_t. First, the second moment of ε_t exists if $\alpha + \beta < 1$, which ensures that the GARCH(1,1) process is strictly stationary and ergodic, and $E\varepsilon_t^2 < \infty$ (see Bollerslev (1986) and Ling and Li (1997)). Second, a sufficient condition for the existence of the fourth moment of ε_t is $k\alpha^2 + 2\alpha\beta + \beta^2 < 1$ (see Bollerslev (1986)). Under the assumption of conditional normality, the condition becomes $3\alpha^2 + 2\alpha\beta + \beta^2 < 1$. An alternative assumption is that η_t is distributed according to the t distribution with $\nu > 4$

degrees of freedom, in which case $k = 3(\nu - 2)/(\nu - 4)$, with $3 \leq k \leq 9$. In the extreme case $\nu = 5$, the condition becomes $9\alpha^2 + 2\alpha\beta + \beta^2 < 1$.

For the GARCH(1,1) model, Nelson (1990) obtained the necessary and sufficient condition for strict stationarity and ergodicity as:

$$E(\ln(\alpha\eta_t^2 + \beta)) < 0. \tag{4}$$

A difficulty in applying the necessary and sufficient condition in (4) is that it is a function of a random variable and hence needs to be simulated. Condition (4) allows $\alpha + \beta$ to be greater than unity, in which case $E\varepsilon_t^2 = \infty$. The condition for a finite variance of the GARCH(1,1) process is $\alpha + \beta < 1$ and, as given above, the condition for a finite fourth moment is $3\alpha^2 + 2\alpha\beta + \beta^2 < 1$. The fourth moment condition is clearly the most stringent.

3 Data

The daily closing values of the S&P 500 Index for the period 3 January 1950 to 31 March 1999 were extracted from the Datastream database, and the daily return was calculated as the ratio of the close-to-close change in the index to the previous trading day's close.

The long sample period includes many significant spikes in the volatility of daily returns, the largest of which occurred on 19 October 1987, and also includes many episodes of relative calm. Consequently, this data set offers an invaluable opportunity to study the effects of extreme observations on the estimation, testing and forecasting of volatility over an extended period.

Various subsets of the data are used for estimation, testing and forecasting. To evaluate the effects of extreme observations on estimation, 12000 observations from the period 3 January 1950 to 7 May 1997 are used. For the evaluation of forecasting performance, two separate "out-of-sample" periods are used, each consisting of 250 observations; the first of these, from 8 May 1997 to 5 May 1998, includes some significant spikes in the volatility of daily returns, while the second, from 13 May 1996 to 7 May 1997, is a period of relative calm.

4 Empirical Results

4.1 Estimation Results

In order to evaluate the effects of increasing the sample size and including extreme observations, the ARCH(1) and GARCH(1,1) models are estimated recursively. In each set of estimates, the end observation of the sample is fixed at 7 May 1997. The sample begins with 200 observations from 23 July 1996 to 7 May 1997, and is then expanded backward recursively until it reaches 12000 observations at 3 January 1950.

The ARCH(1) model is obviously a misspecification of the true model because the values of the estimated t-ratios of β, both asymptotic and robust, exceed the critical value for all sample sizes. The results from the ARCH(1) model, however, serve as an indication of the extent of the effects such a misspecification has on estimation and forecasting.

Figure 1 gives the estimated values of the parameter of the ARCH(1) model as the sample size is increased recursively. The actual volatility of the daily returns is shown in the lower half of the figure to indicate where the volatility spikes occur. It is clear that the estimates of

α are highly volatile when the sample sizes are below 2400. Significant spikes in the actual volatility correspond to huge variations in the estimates of α. The most obvious feature is the huge shift in the α estimates with the 19 October 1987 spike in volatility, after which the variations in the α estimates are much smaller in magnitude. Another notable feature is the general U-shape in the middle of the figure, which indicates that α is not constant over time.

Figure 2 presents the asymptotic t-ratios, and the robust t-ratios of Bollerslev and Wooldridge (1992), for estimates of α in the ARCH(1) model. The robust t-ratios are designed to be insensitive to departures from normality, especially extreme observations. Both sets of t-ratios are somewhat erratic at small sample sizes and are more sensitive to extreme observations before the inclusion of the 19 October 1987 spike. The effects of significant spikes in volatility on the two sets of t-ratios are also dramatically different. Each spike in volatility increases the asymptotic t-ratios but decreases the robust t-ratios, with the magnitudes of the shifts being far greater for the asymptotic t-ratios. It is worth noting the huge increase in the asymptotic t-ratios when the 19 October 1987 spike is included. In contrast, the impact of this extreme observation on the robust t-ratios is barely visible.

Figure 3 shows the fourth moment condition for asymptotic normality of the ARCH(1) model. As discussed above, the fourth moment condition is $\mathrm{S}_{\mathrm{A(N)}} \equiv 3\alpha^2 < 1$ when η_t is distributed as $N(0,1)$, and $\mathrm{S}_{\mathrm{A(t)}} \equiv 9\alpha^2 < 1$ when η_t is distributed as $t(5)$. The values of both $S_{A(N)}$ and $S_{A(t)}$ in the recursions follow patterns that are identical to that of the parameter estimate, and are less than unity for all sample sizes.

Figure 4 shows the estimates of the ARCH parameter α of the GARCH(1,1) model. As in the ARCH(1) model, the values of α are highly volatile when the sample sizes are below 2400, and follow a general U-shaped pattern as the sample size is increased. Also present are the huge variations in the estimates of α that occur when there are significant spikes in the actual volatility. The only notable difference between the α estimates of the ARCH(1) and GARCH(1,1) models is that the values of the estimates are much smaller for the latter model, especially after the inclusion of the 19 October 1987 extreme observation.

The asymptotic and robust t-ratios for estimates of α in the GARCH(1,1) model are shown in Figure 5. Both sets of t-ratios have identical trends to those of the ARCH(1) model, but are slightly higher and show greater variations around the trends, especially when sample sizes are small. The effect of 19 October 1987 volatility spike on the t-ratios of the GARCH(1,1) model is also similar to that on the ARCH(1) model, shifting the asymptotic t-ratio up significantly while shifting the robust t-ratio down slightly.

Estimates of the GARCH parameter β of the GARCH(1,1) model are given in Figure 6. This is virtually a mirror image of the estimates of α, with the β estimates moving in the opposite direction to those of α. There is also much variability in the β estimates at sample sizes below 2400, and an inverted U-shape as the sample size is increased beyond 2400. The spikes in volatility also have larger impacts on the β estimates when the sample size is small.

Figure 7 shows the t-ratios for the β estimates in the GARCH(1,1) model. Both the asymptotic and robust t-ratios show great variability for sample sizes below 2500, prior to the inclusion of the 19 October 1987 spike in volatility. The inclusion of this extreme observation shifts the asymptotic t-ratio up and the robust t-ratio down. Beyond that, both t-ratios become much smoother, especially the robust t-ratios.

The second moment condition for stationarity and consistency of the GARCH(1,1) model as discussed above is $\alpha + \beta < 1$. Figure 8 shows the value of the estimated $\alpha + \beta$. Spikes in the volatility of returns have large impacts on this value when the sample size is below 2400. For large sample sizes, and with the inclusion of the 19 October 1987 spike, this value is less volatile, but it is also not constant. It is significant to note that the second moment condition is satisfied for all sample sizes in the backward recursions.

As discussed previously, the condition for the existence of the fourth moment of ε_t in the GARCH(1,1) model is $S_{G(N)} \equiv 3\alpha^2 + 2\alpha\beta + \beta^2 < 1$ when η_t is distributed as $N(0,1)$, and $S_{G(t)} \equiv 9\alpha^2 + 2\alpha\beta + \beta^2 < 1$ when η_t is distributed as $t(5)$. Figure 9 shows the values of $S_{G(N)}$ and $S_{G(t)}$. The value of $S_{G(N)}$ in the recursions follows a pattern that is identical to that of the second moment condition, and is less than unity for most of the sample. Surprisingly, some of the violations of the fourth moment condition occur for relatively large samples (at around 8000 observations). While following the same pattern in fluctuations, $S_{G(t)}$ is greater in value and exceeds unity for all sample ranges that include the 19 October 1987 volatility spike.

4.2 Forecasting Results

To evaluate the effects of increasing sample sizes and including extreme observations on the forecast performance of the ARCH(1) and GARCH(1,1) models, similar backward recursions are used. For each model, two sets of forecasts are performed. In the first set of forecasts, the forecast period is from 8 May 1997 to 5 May 1998, which includes an extreme observation at 27 October 1997. Estimation of the parameters to obtain these forecasts is in the same manner as the backward recursions explained above, with sample sizes ranging from 200 observations to 5000 observations. For each sample size, 250 one-day ahead forecasts are made for the period 8 May 1997 to 5 May 1998. The prediction errors from these 250 forecasts are then combined in the three measures of forecast performance, namely mean absolute prediction error (MAPE), mean absolute percentage prediction error (MAPPE), and root mean square prediction error (RMSPE).

Figures 10 to 12 show the forecast performance measures of the ARCH(1) model for the 8 May 1997 to 5 May 1998 period. In Figure 10, MAPE decreases initially as the sample size is increased recursively and reaches a minimum at a sample size of about 1500. It then increases slightly before evening out at a sample size of about 1900. The inclusion of the 19 October 1987 extreme observation in the estimation period, however, causes MAPE to increase steeply and then stabilise at a higher level. This implies that when the forecast period includes extreme observations, the inclusion of an extreme observation in the estimation period does not lead to improved forecasting accuracy using the MAPE measure.

MAPPE in Figure 11 does not show any initial trend. Instead, it appears very sensitive to the inclusion of extreme observations in the estimation period. The inclusion of the 19 October

1987 extreme observation, for example, causes MAPPE to experience a huge increase. Hence, with the MAPPE measure as well, the inclusion of an extreme observation does not help in the prediction of volatility in periods with extreme observations.

Figure 12 shows that the value of RMSPE is very volatile and follows a declining trend when the sample size is increased from 200 to about 1100, but shows no trend beyond that. It is also very sensitive to the inclusion of extreme observations in the estimation period. Unlike the two previous measures, however, the inclusion of an extreme observation like that of 19 October 1987, decreases both the value and volatility of RMSPE. Hence, with the RMSPE measure, the inclusion of an extreme observation helps in the prediction of volatility in periods with extreme observations.

The second set of forecasts is for the period 13 May 1996 to 7 May 1997, which does not contain any large spikes in the volatility of returns. As before, the same backward recursion procedure and averaging of one-day ahead forecasts is used to obtain the forecast performance measures. As one would expect, Figure 13 shows that MAPE is much lower for this relatively calmer forecast period than for the forecast period that includes 19 October 1987. The pattern that MAPE traces, as the sample size is increased recursively, follows the same U-shaped pattern for sample sizes below 1800. The inclusion of the 19 October 1987 extreme observation in the estimation period again causes MAPE to increase steeply. This implies that, for a relatively calm forecast period, the inclusion of an extreme observation in the estimation period also does not lead to improved forecasting accuracy using the MAPE measure.

Figure 14 shows that MAPPE, the average of the prediction error expressed as a percentage of the actual error, is higher for the relatively calmer forecast period. The pattern that MAPPE shows for this forecast period is quite different from that for the volatile forecast period. For the calm forecast period, MAPPE traces a U-shape as the sample size is increased from 200 to1700. The introduction of the 19 October 1987 extreme observation into the estimation sample has a much smaller impact on MAPPE in this forecast period, increasing it only slightly.

Figure 15 shows that RMSPE is smaller for the calmer forecast period regardless of sample size. This is consistent with the finding of lower values for MAPE in the calmer forecast period and confirms the intuitive expectation that forecasts are more accurate when forecasting for less volatile periods. The pattern traced by RMSPE when the sample size is increased recursively is different from that for the more volatile forecast period. In the calmer forecast period, RMSPE shows an inverse U-shape initially. When the extreme observation of 19 October 1987 is introduced into the estimation sample, RMSPE shows a distinct increase. Thus, all the three measures of forecast accuracy indicate that the introduction of an extreme observation into the estimation sample does not help to improve the accuracy of the forecasts when forecasting for a relatively calm period.

The forecast performance measures of the GARCH(1,1) model for the 8 May 1997 to 5 May 1998 period are graphed in Figures 16 to 18. The patterns these graphs show are generally similar to those for the ARCH(1) model. There are differences, however, in the magnitudes and sensitivities of the measures. Surprsingly, based on the MAPE and RMSPE measures, the ARCH(1) model performs better at forecasting than the GARCH(1,1) model. With the

MAPPE measure, however, the results are ambiguous, with ARCH(1) performing better when the estimation samples are small, but worse when the sample is expanded beyond 1800.

While all three measures for the GARCH(1,1) model are more sensitive to changes in the estimation sample, they are more robust to extreme observations in the sample. Figures 16 and 17 show that the shifts in MAPE and MAPPE due to the inclusion of the 19 October 1987 extreme observation are relatively smaller in the GARCH(1,1) model, while Figure 18 shows that the RMSPE measure for the GARCH(1,1) model is hardly affected by the inclusion of extreme observations.

The forecast performances of the GARCH(1,1) model for the relatively calm forecast period of 3 May 1996 to 7 May 1997, as shown in Figures 18 to 21, also follow patterns that are generally similar to those in Figures 13 to 15 for the ARCH(1) model. Again, the performances of the GARCH(1,1) model are more sensitive to changes in the estimation sample, but more robust to the inclusion of extreme observations.

Comparing the magnitudes of each forecasting performance graph for the ARCH(1) model against the corresponding graph for the GARCH(1,1) model, we find neither model dominating the other. While the GARCH(1,1) model clearly outperforms the ARCH(1) model when we use the RMSPE measure, the same cannot be said when we use the other two measures. The ARCH(1) model performs better when when the estimation samples are small, but much worse when extreme observations are included.

5 Concluding Remarks

This paper has investigated the effects of increasing sample sizes recursively, both with and without the inclusion of extreme observations, on the parameter estimates, t-tests, moment conditions and forecasts of the ARCH(1,) and GARCH(1,1) models. The results indicate that the ARCH and GARCH parameter estimates, their asymptotic and robust t-ratios, the second and fourth moment regularity conditions, and various forecast performance measures for both models, are all highly volatile for small sample sizes. However, when an extreme observation is included in the estimation period for sample sizes above 2000, all the sample estimates and their associated statistics stabilise. An important implication of these results is that increasing the sample sizes recursively beyond the extreme observation is unnecessary.

Another important result is that the robust t-ratios are dramatically superior to the asymptotic t-ratios, especially in the presence of high volatility in the returns. The second moment condition for stationarity is always satisfied for the GARCH(1,1) model as is the fourth moent condition for the ARCH(1) model. Under normality of the conditional errors, the fourth moment condition for asymptotic normality is generally satisfied for the GARCH(1,1) model. An interesting finding is that this condition is not necessarily satisfied with larger sample sizes. If it is assumed that the conditional error follows a fatter-tailed distribution such as $t(5)$, then the fourth moment condition is generally not satisfied for the GARCH(1,1) model when an extreme observation like 19 October 1987 is included, regardless of the sample sizes used.

For some measures of forecasting performance like MAPE and MAPPE, the inclusion of an extreme observation in the estimation sample leads to a marked deterioration in forecasting performance of both models, especially the ARCH(1) model. Increasing the sample sizes recursively does not necessarily improve the forecasting performance of either model. Neither model shows a clear superiority in forecasting volatility.

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Figure 1: Estimates of ARCH(1)

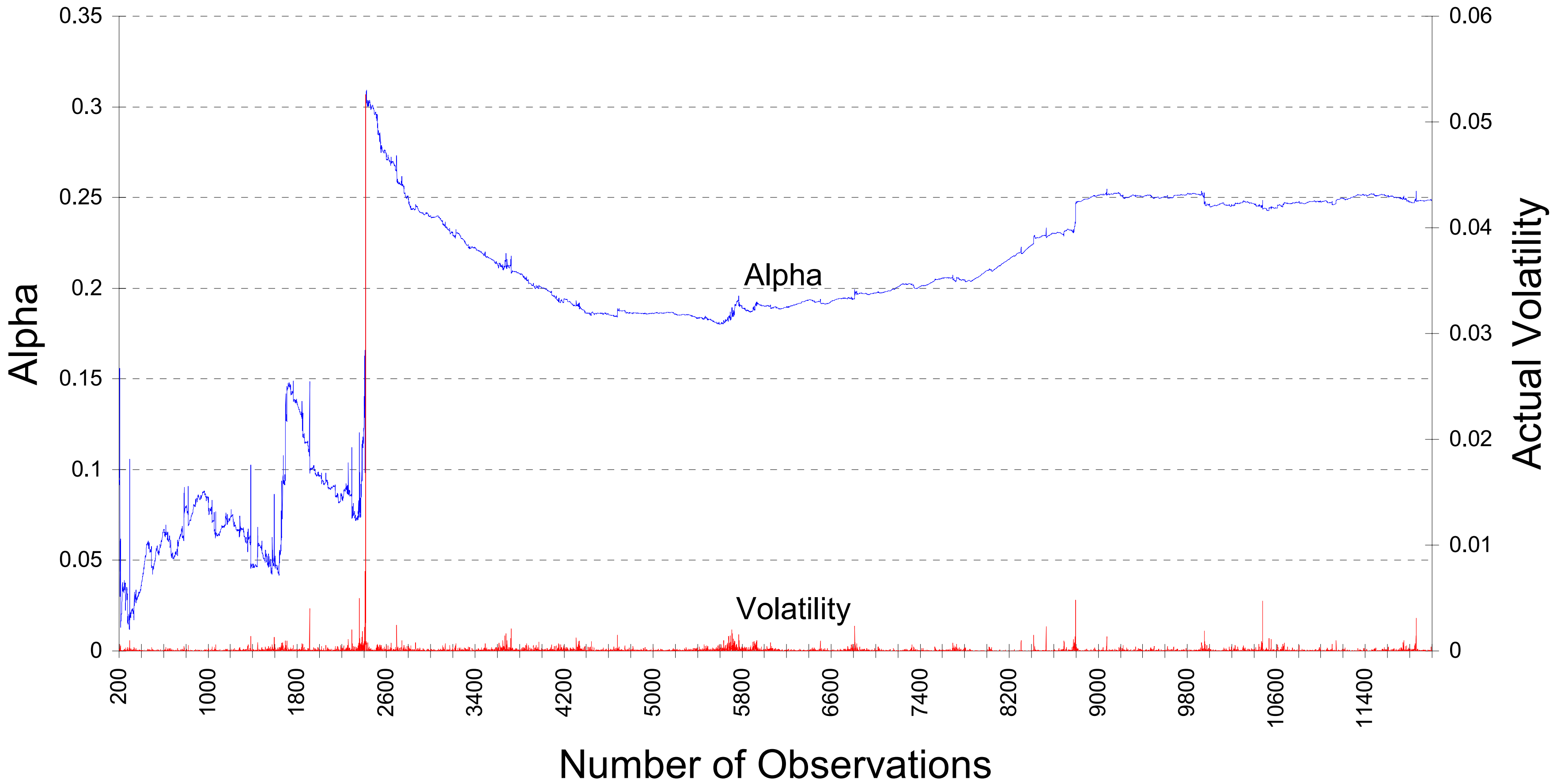

Figure 2: t-ratios of ARCH(1)

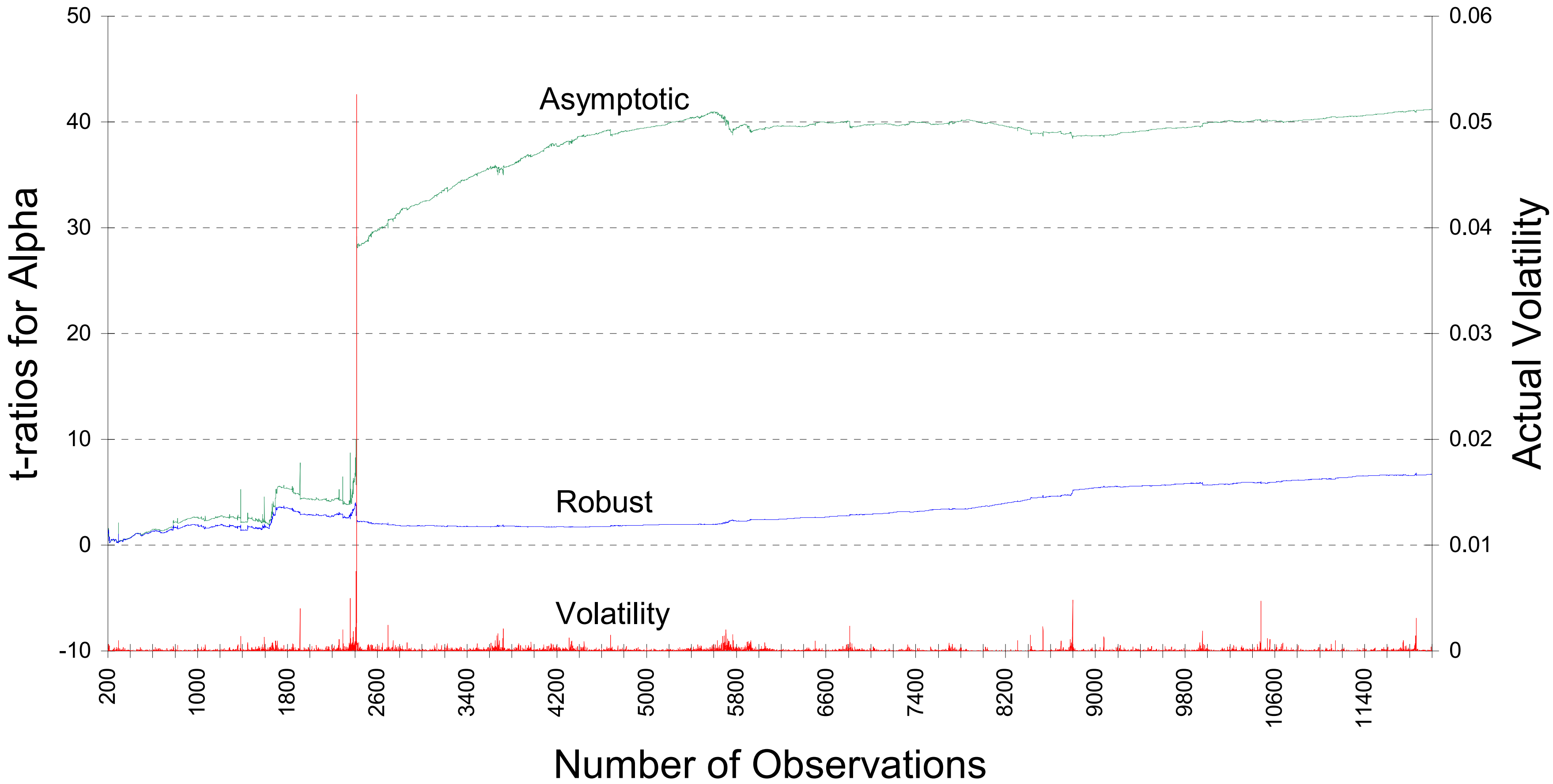

Figure 3: Fourth Moments of ARCH(1)

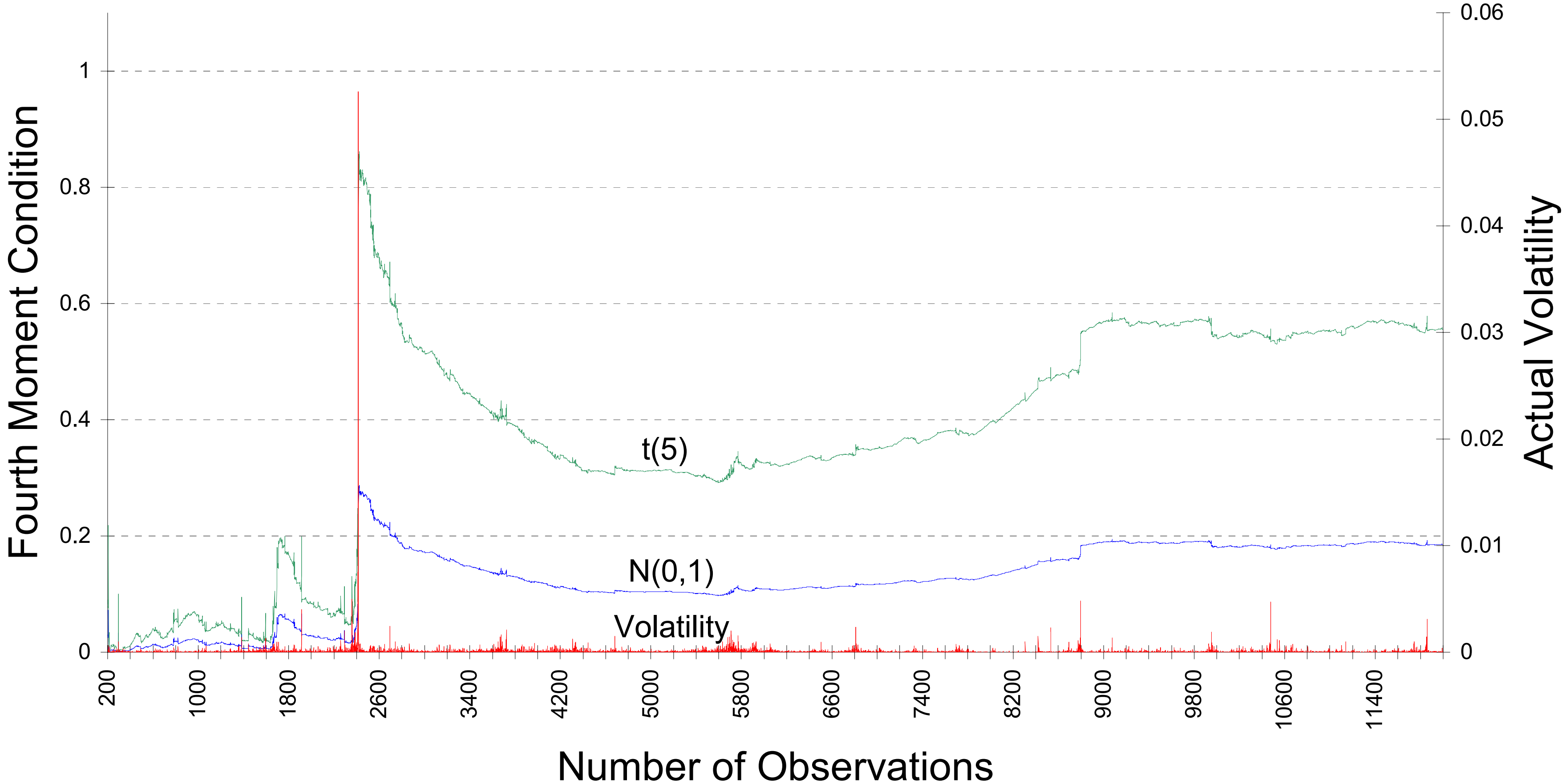

Figure 4: α Estimates of GARCH(1,1)

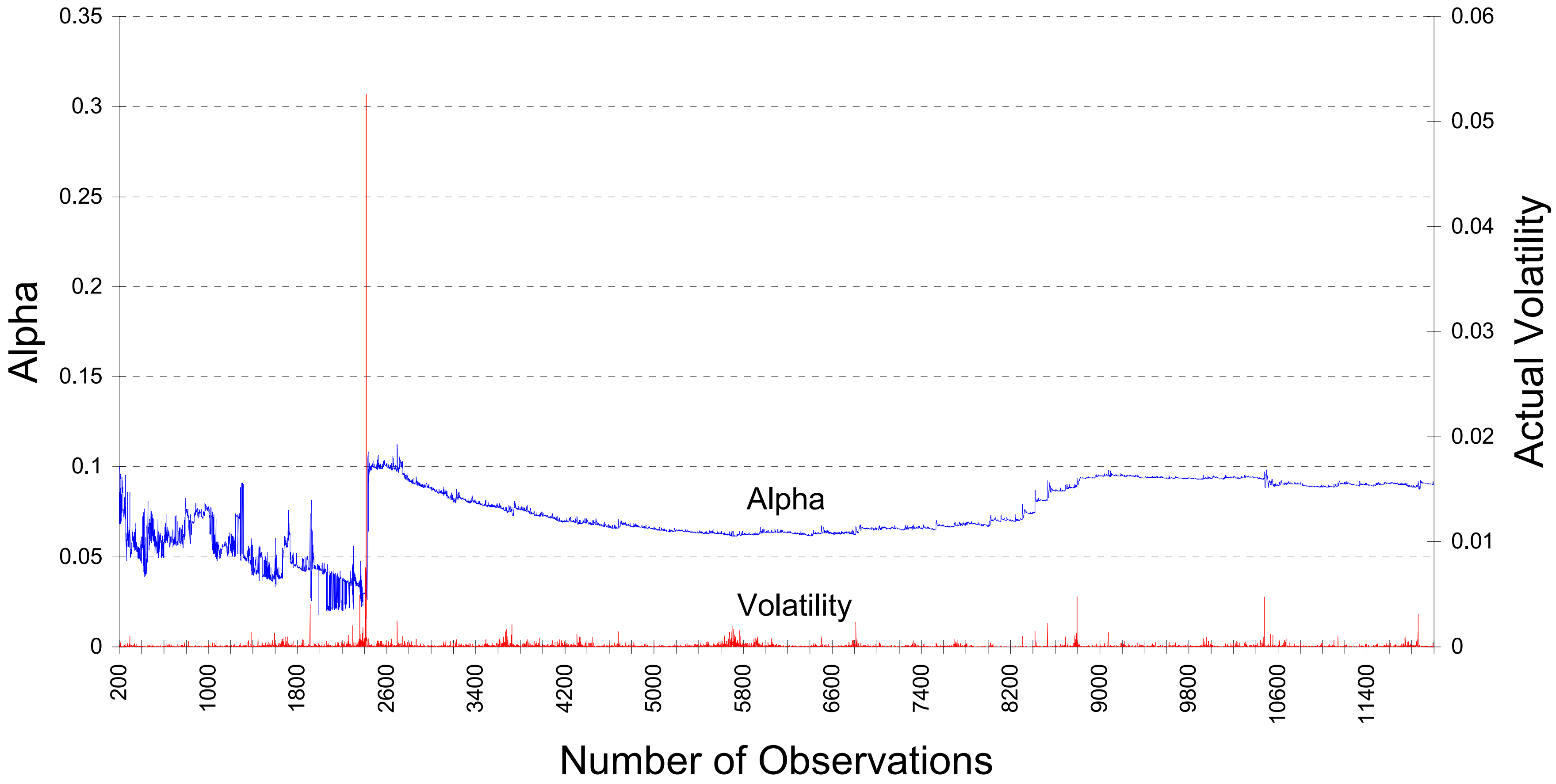

Figure 5: α t-ratios of GARCH(1,1)

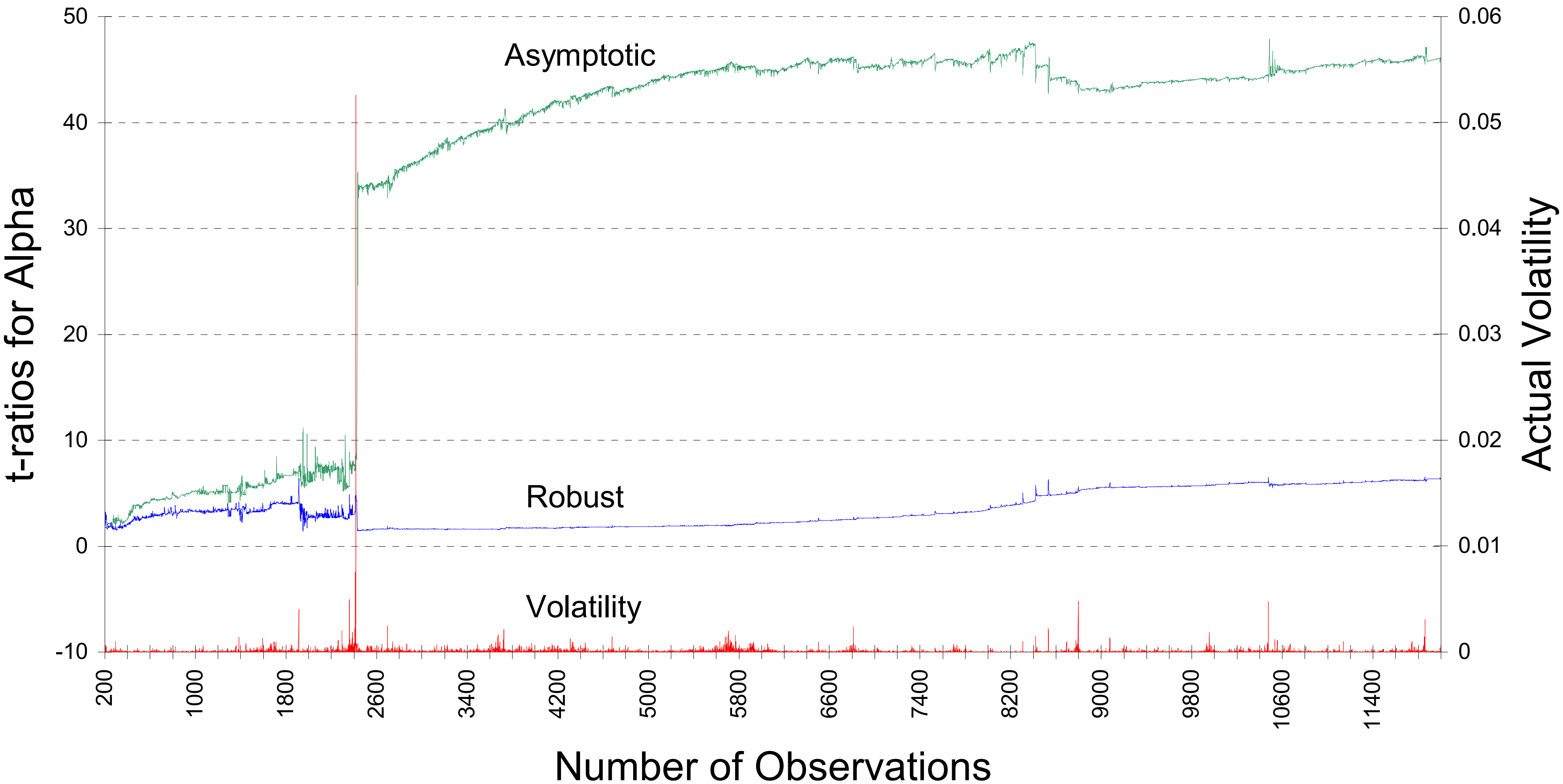

Figure 6: β Estimates of GARCH(1,1)

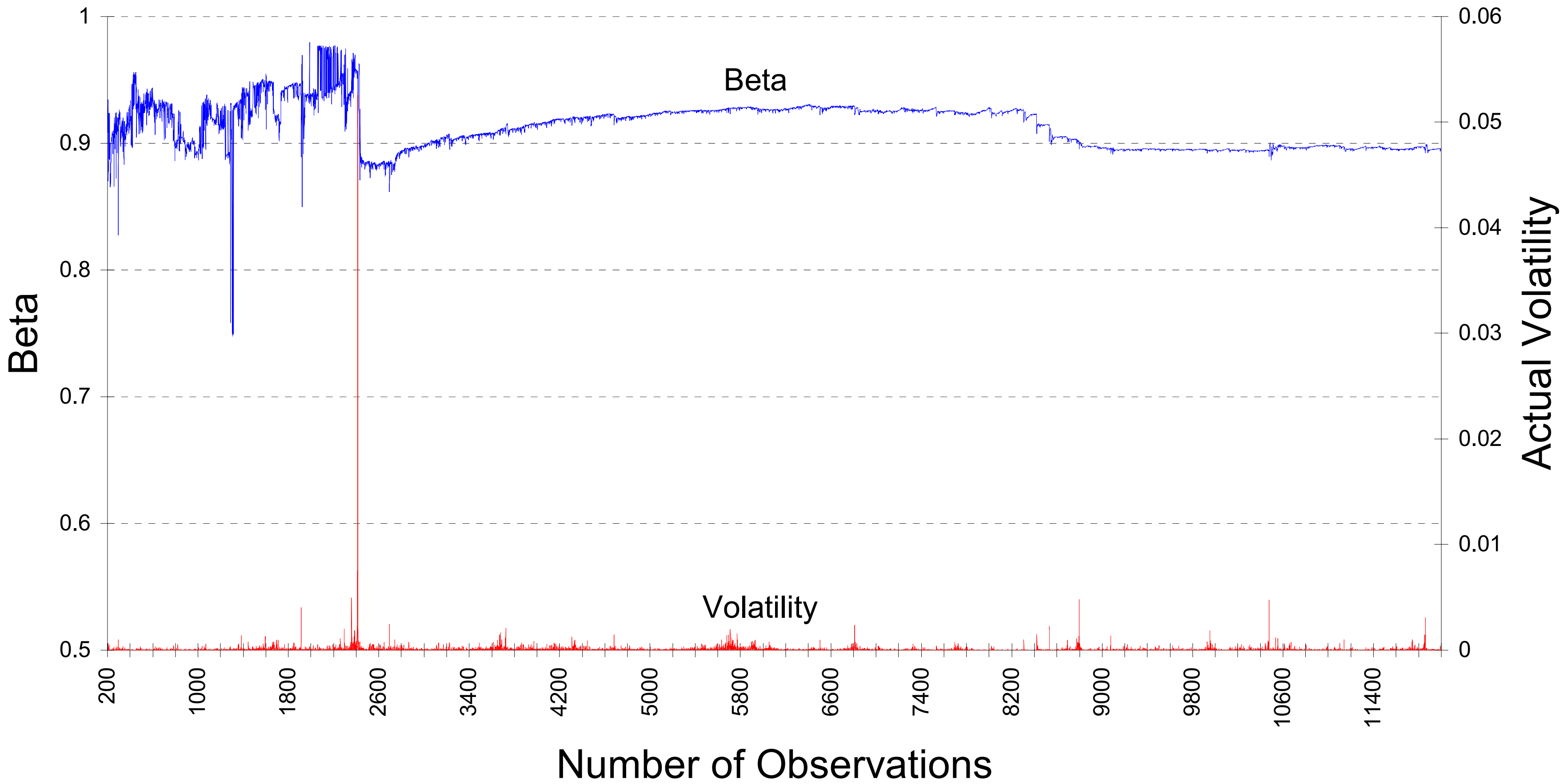

Figure 7: β t-ratios of GARCH(1,1)

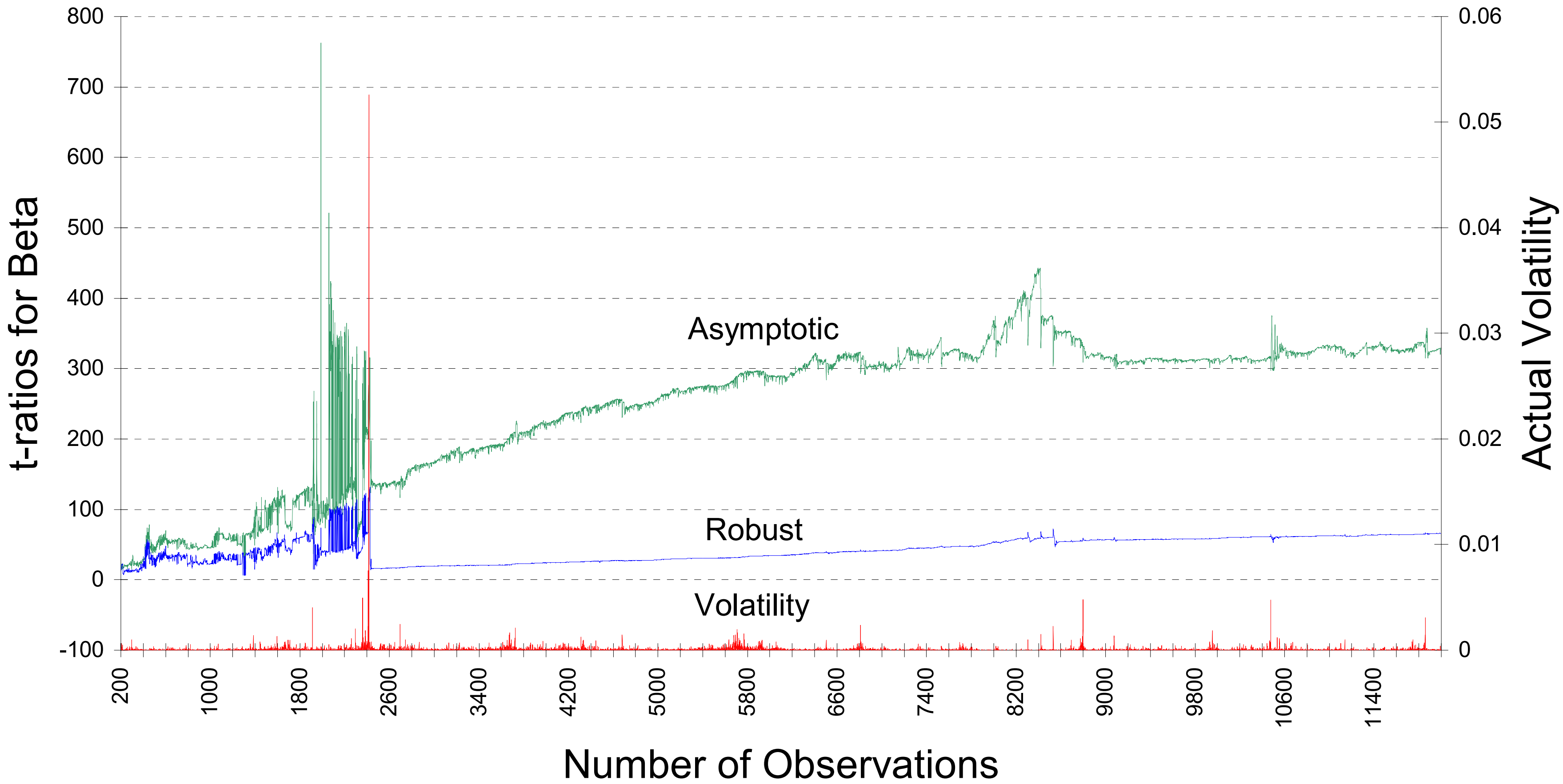

Figure 8: Second Moments of GARCH(1,1)

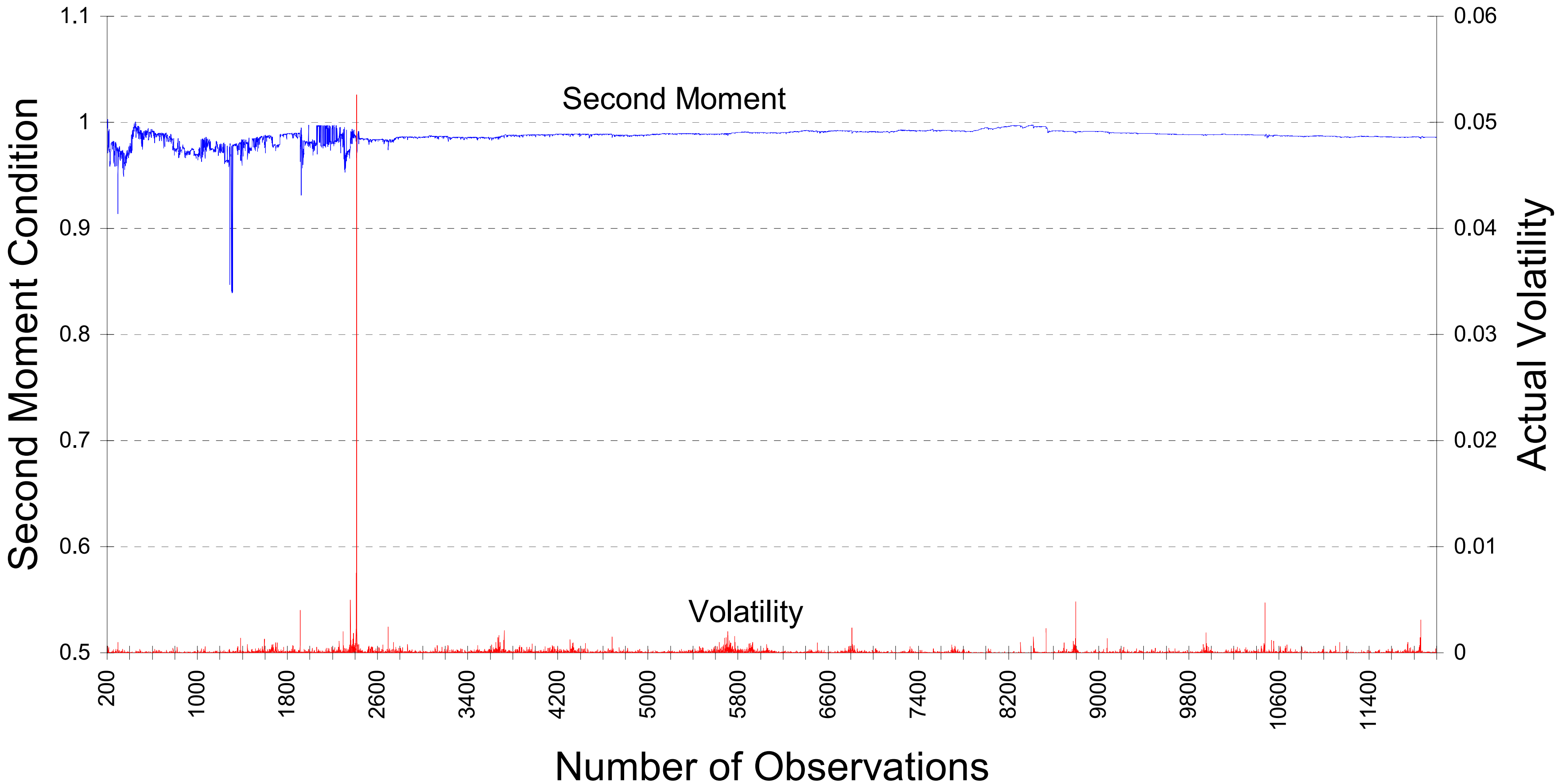

Figure 9: Fourth Moments of GARCH(1,1)

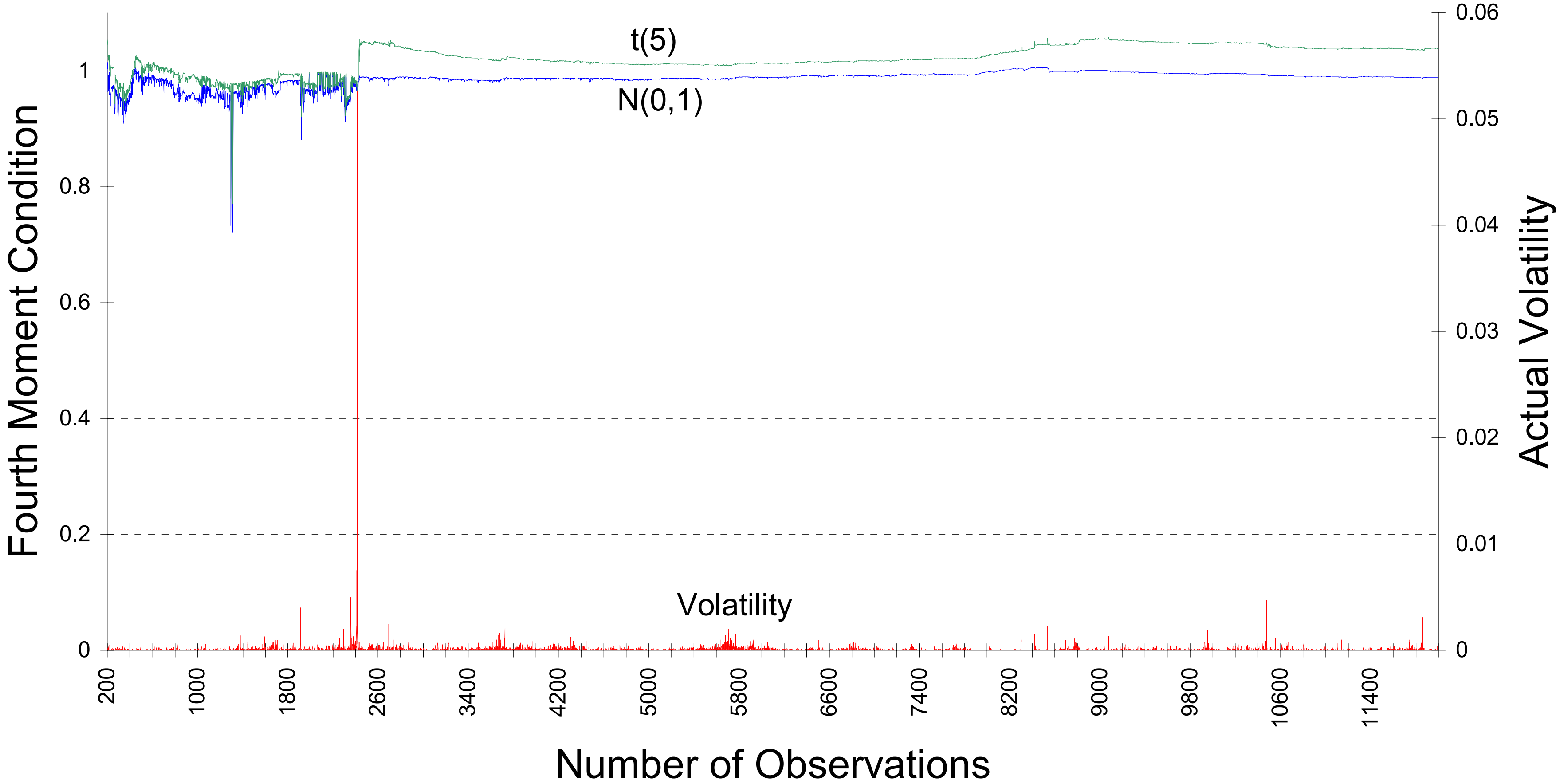

Figure 10: ARCH(1) Forecast Performance for 8/5/97 to 5/5/98 (including October 1997)

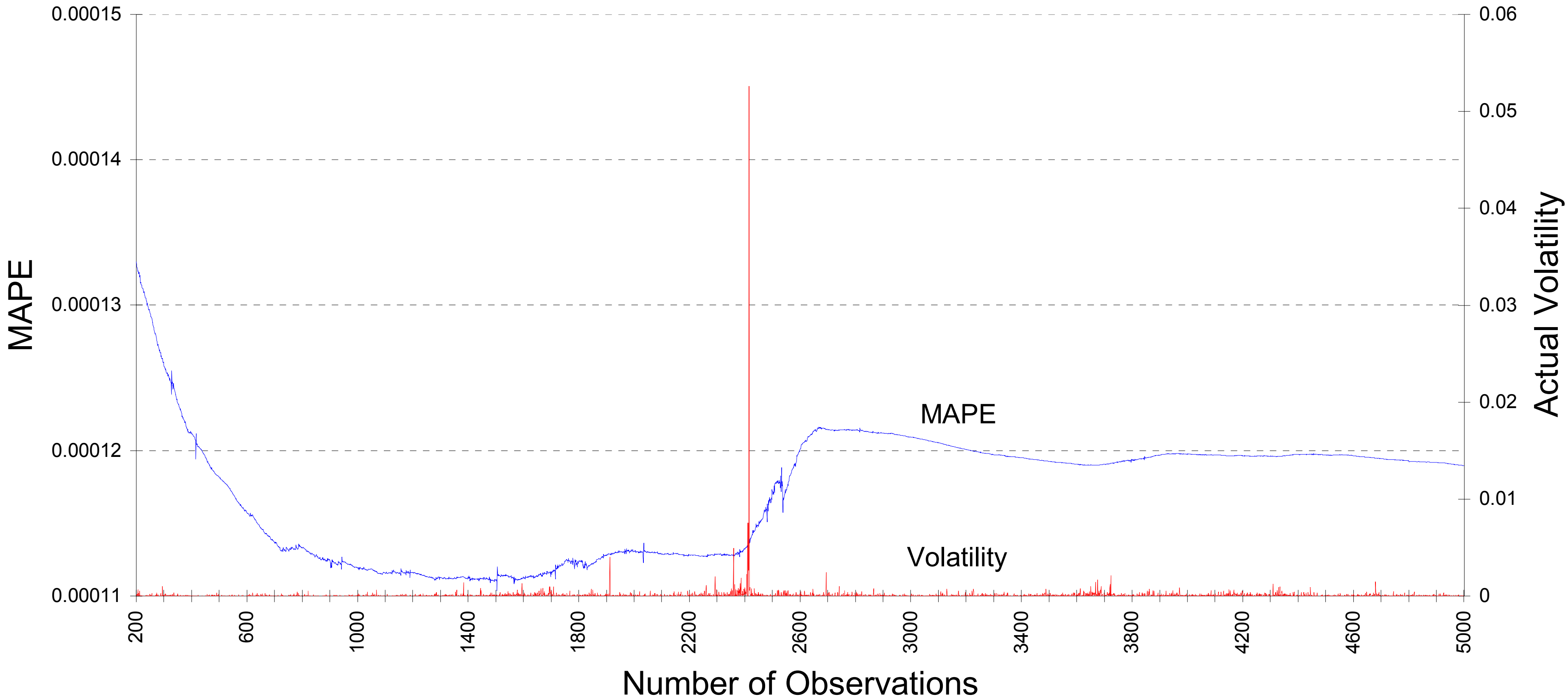

Figure 11: ARCH(1) Forecast Performance for 8/5/97 to 5/5/98 (including October 1997)

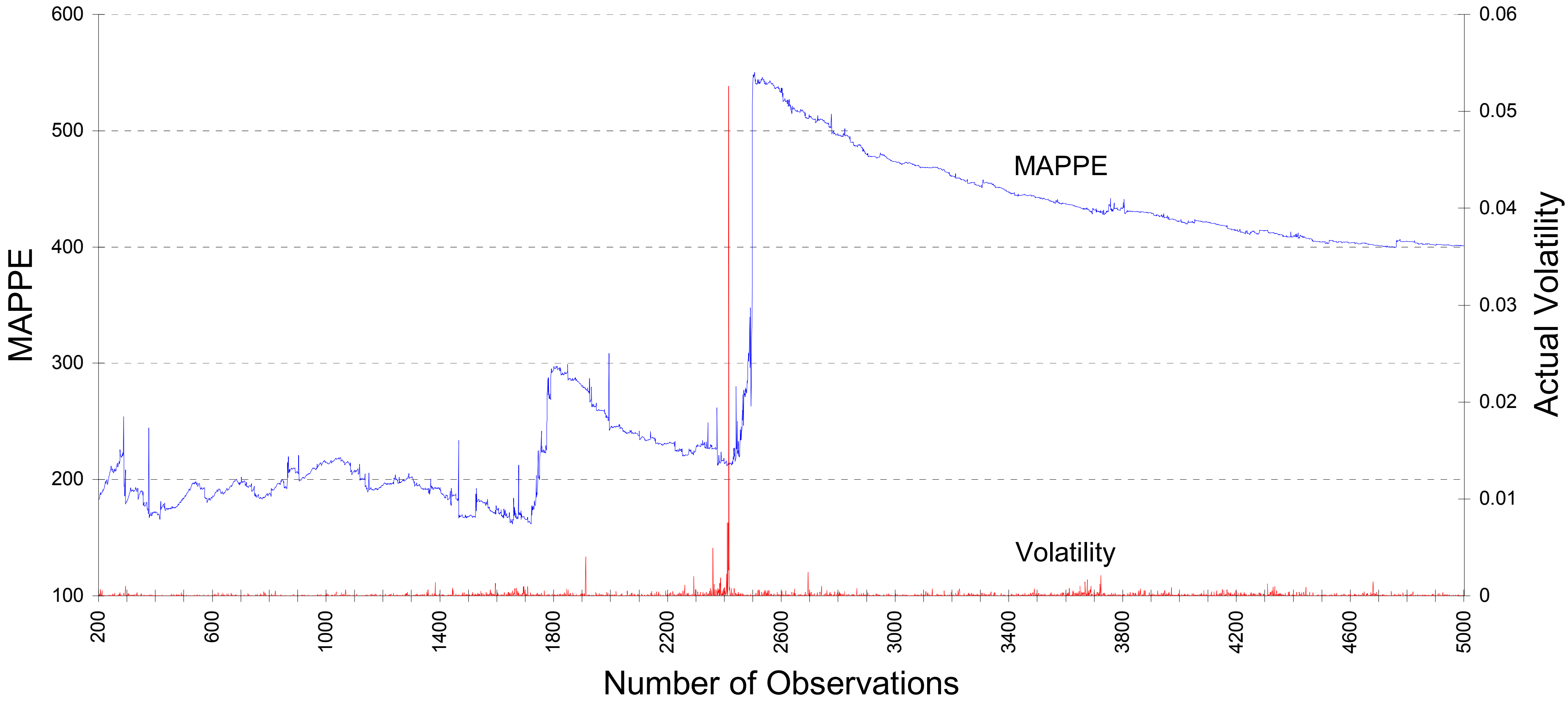

Figure 12: ARCH(1) Forecast Performance for 8/5/97 to 5/5/98 (including October 1997)

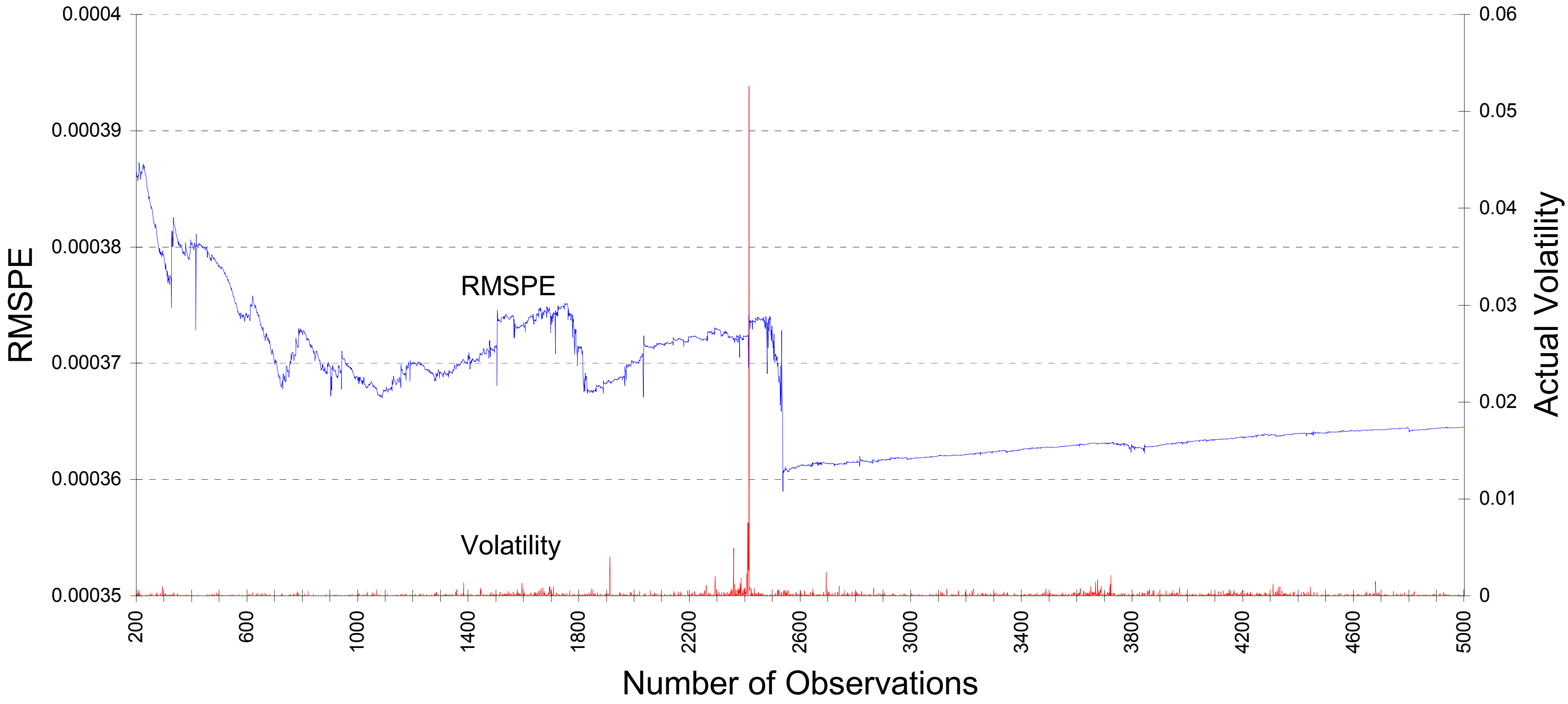

Figure 13: ARCH(1) Forecast Performance for 3/5/96 to 7/5/97 (excluding October 1997)

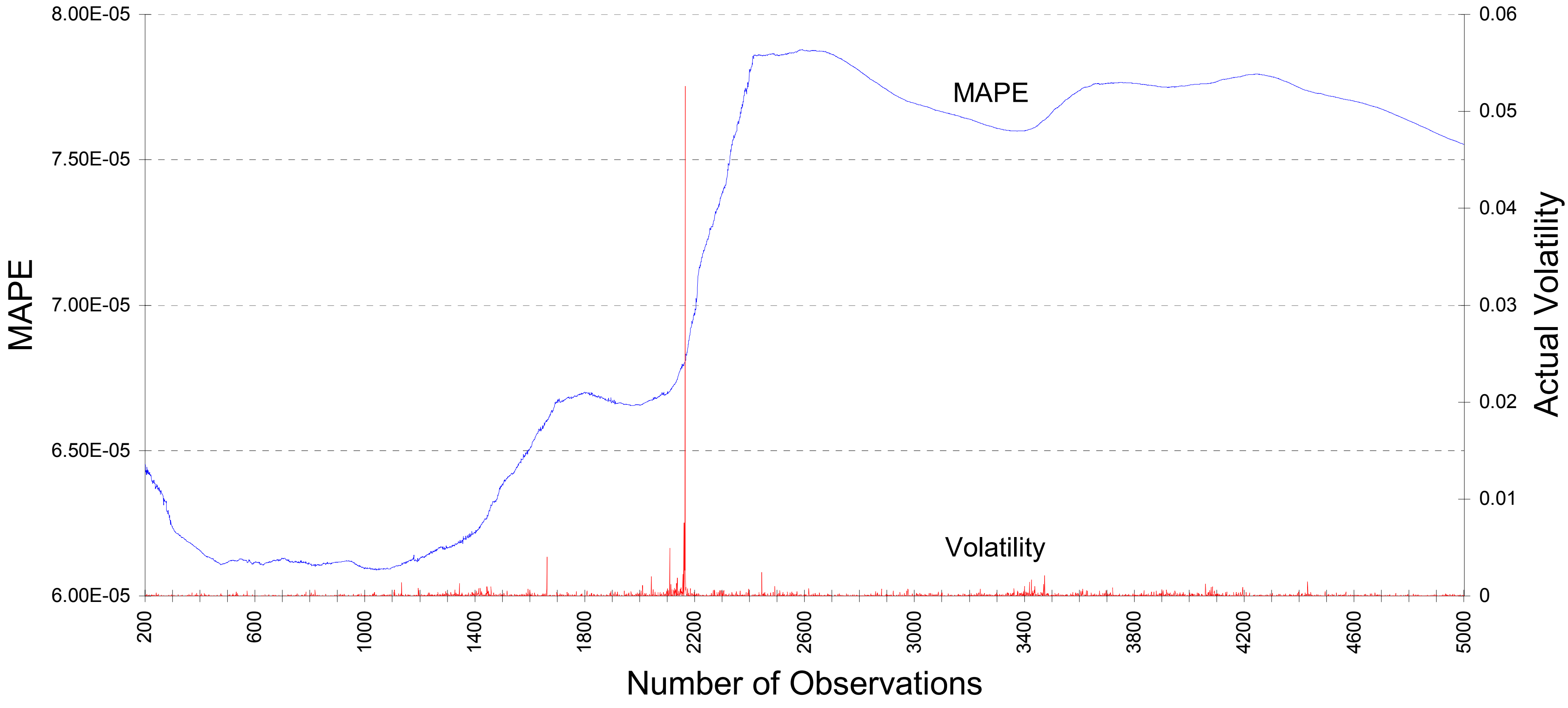

Figure 14: ARCH(1) Forecast Performance for 3/5/96 to 7/5/97 (excluding October 1997)

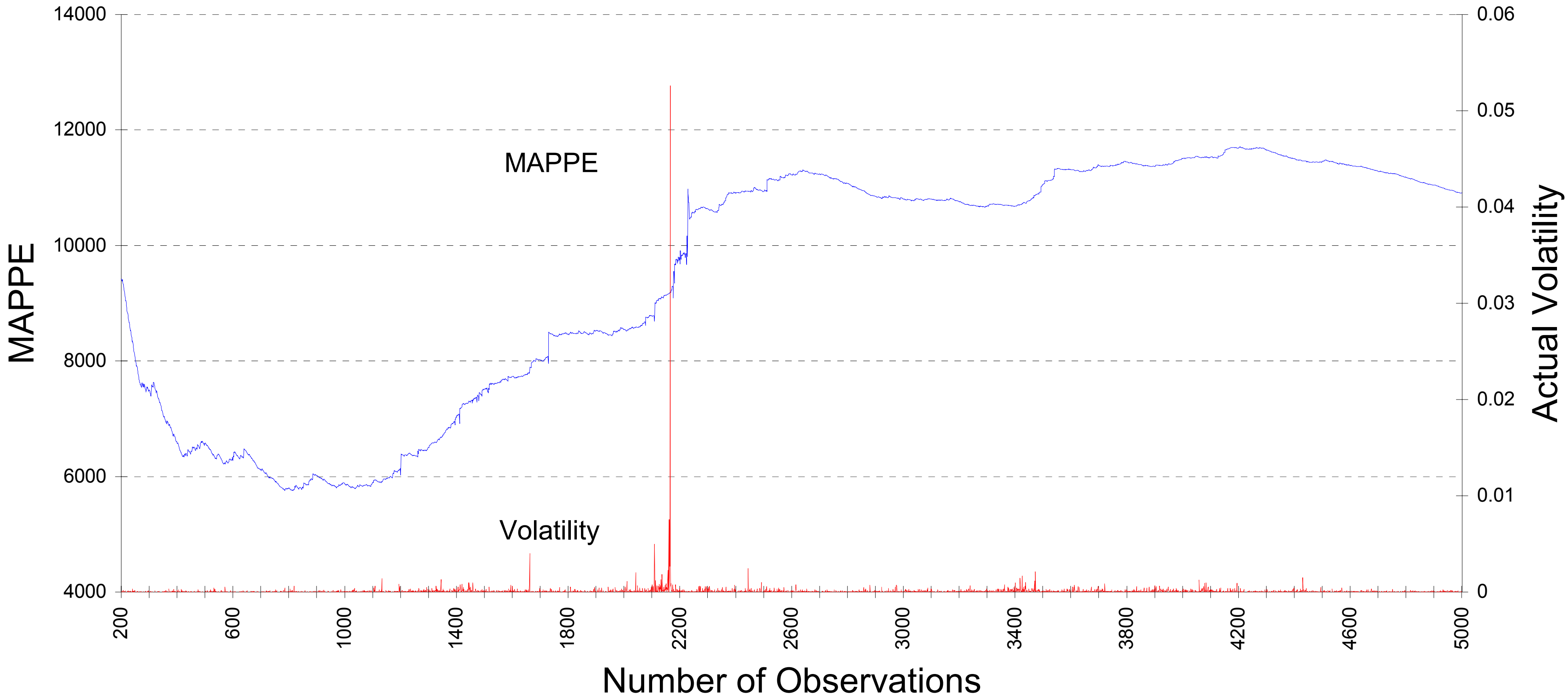

Figure 15: ARCH(1) Forecast Performance for 3/5/96 to 7/5/97 (excluding October 1997)

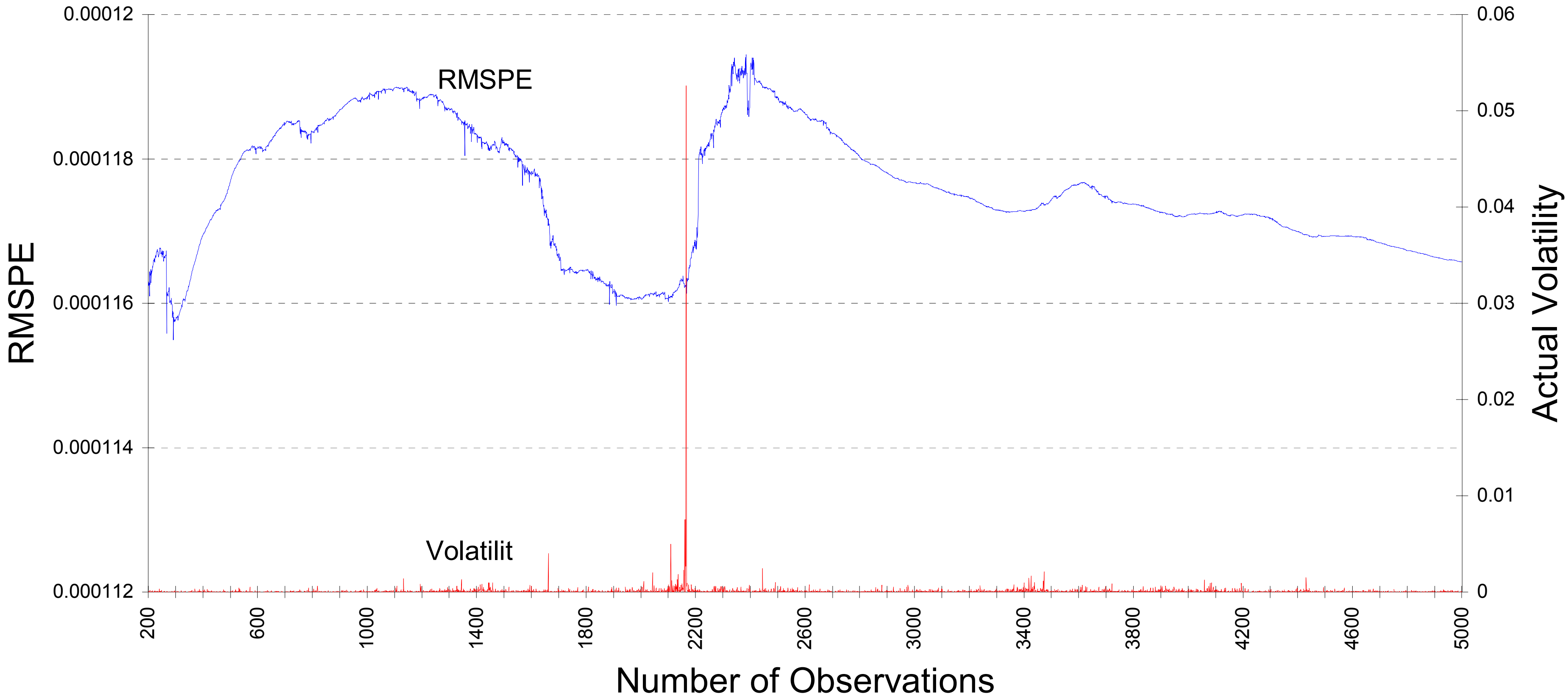

Figure 16: GARCH(1,1) Forecast Performance for 8/5/97 to 5/5/98 (including October 1997)

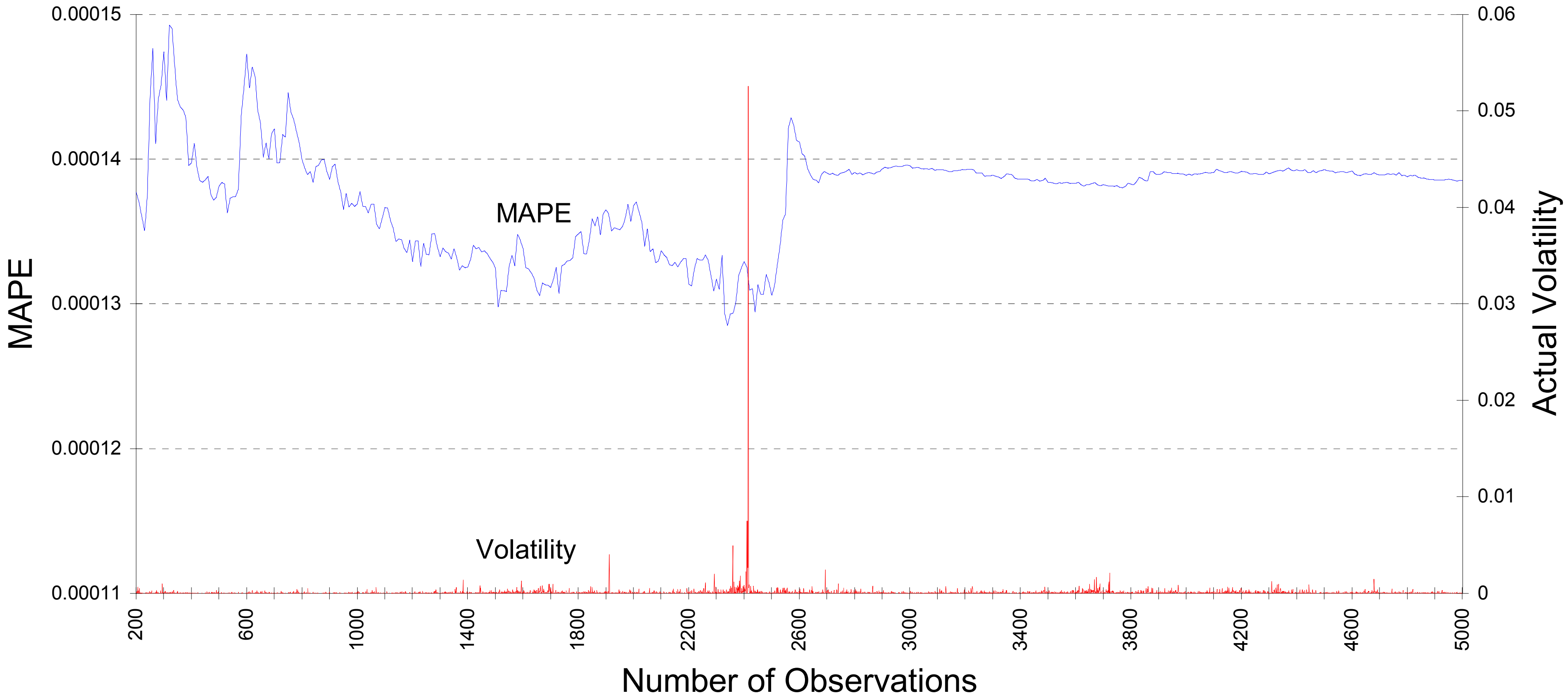

Figure 17: GARCH(1,1) Forecast Performance
for 8/5/97 to 5/5/98 (including October 1997)

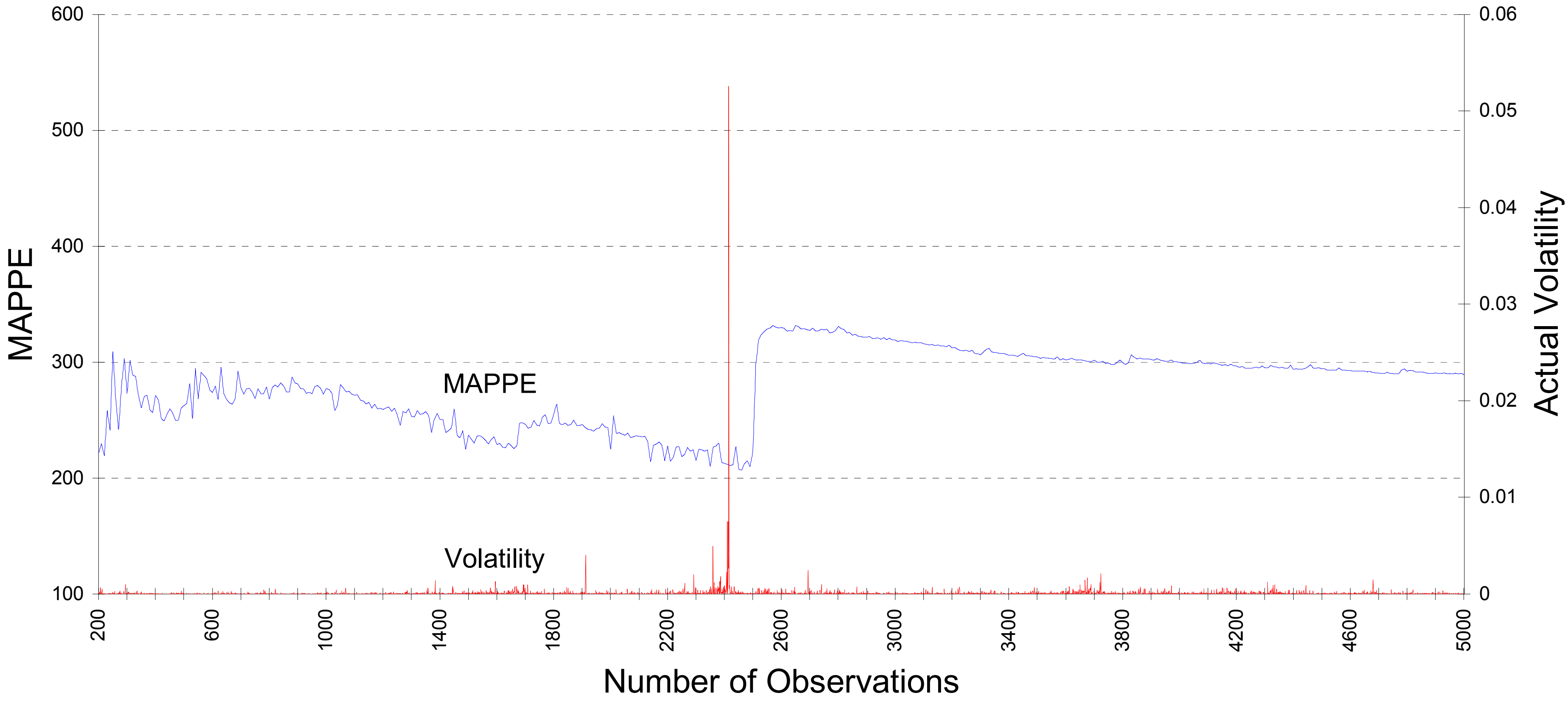

Figure 18: GARCH(1,1) Forecast Performance for 8/5/97 to 5/5/98 (including October 1997)

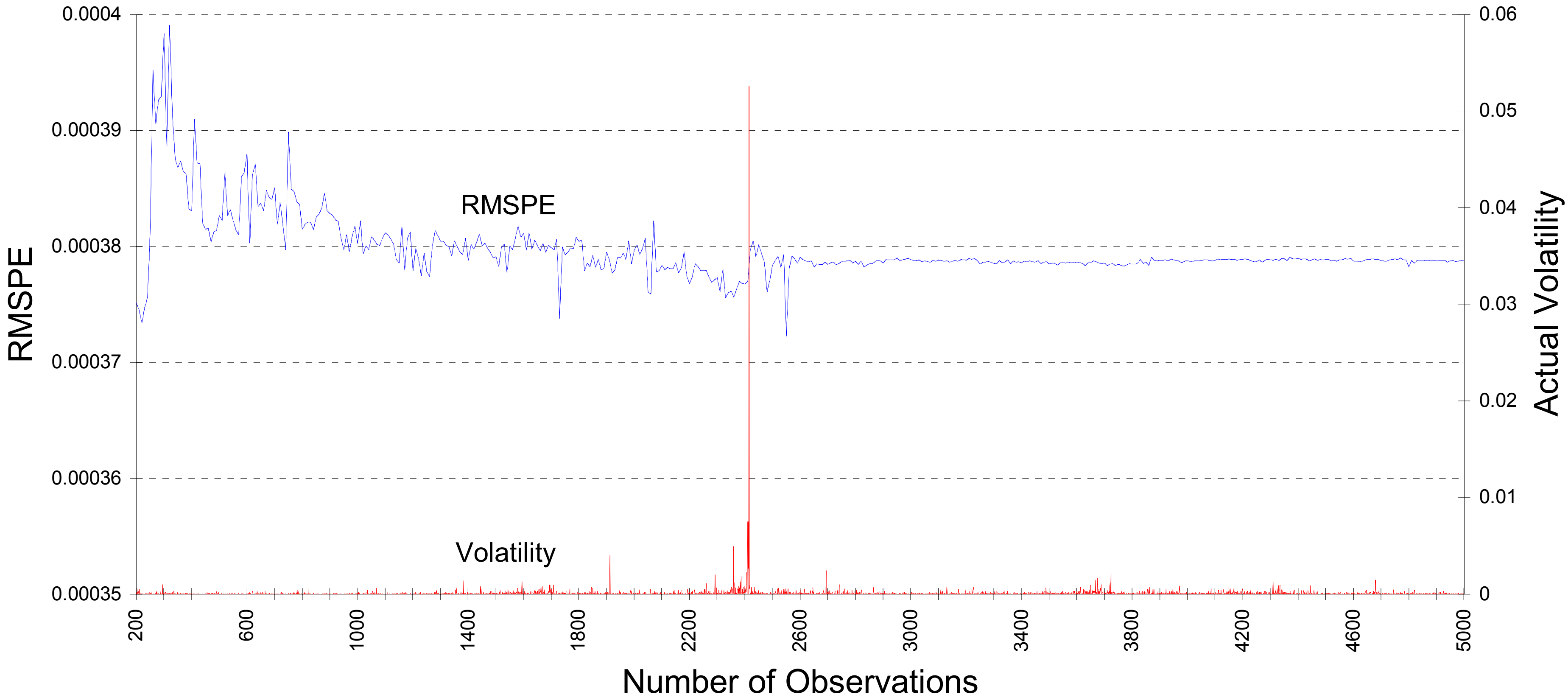

Figure 19: GARCH(1,1) Forecast Performance for 3/5/96 to 7/5/97 (excluding October 1997)

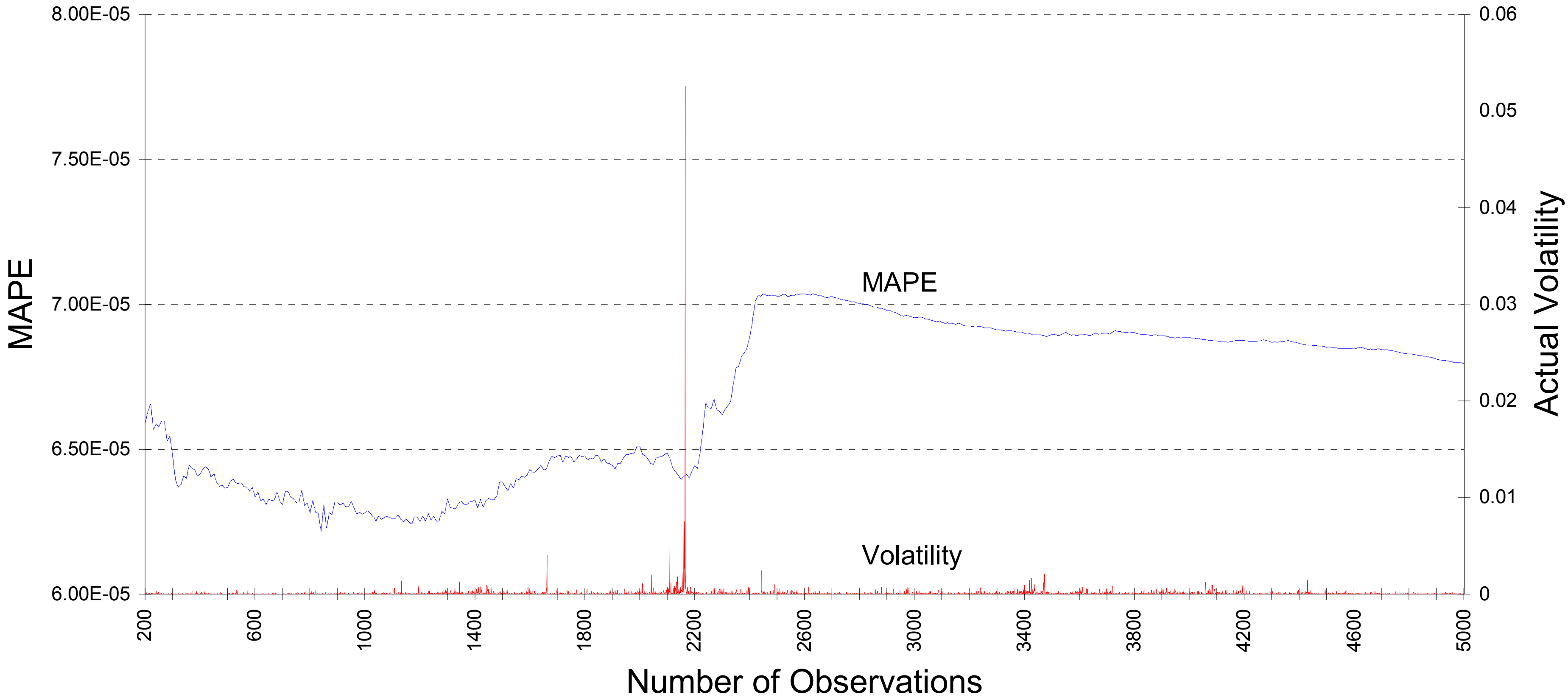

Figure 20: GARCH(1,1) Forecast Performance for 3/5/96 to 7/5/97 (excluding October 1997)

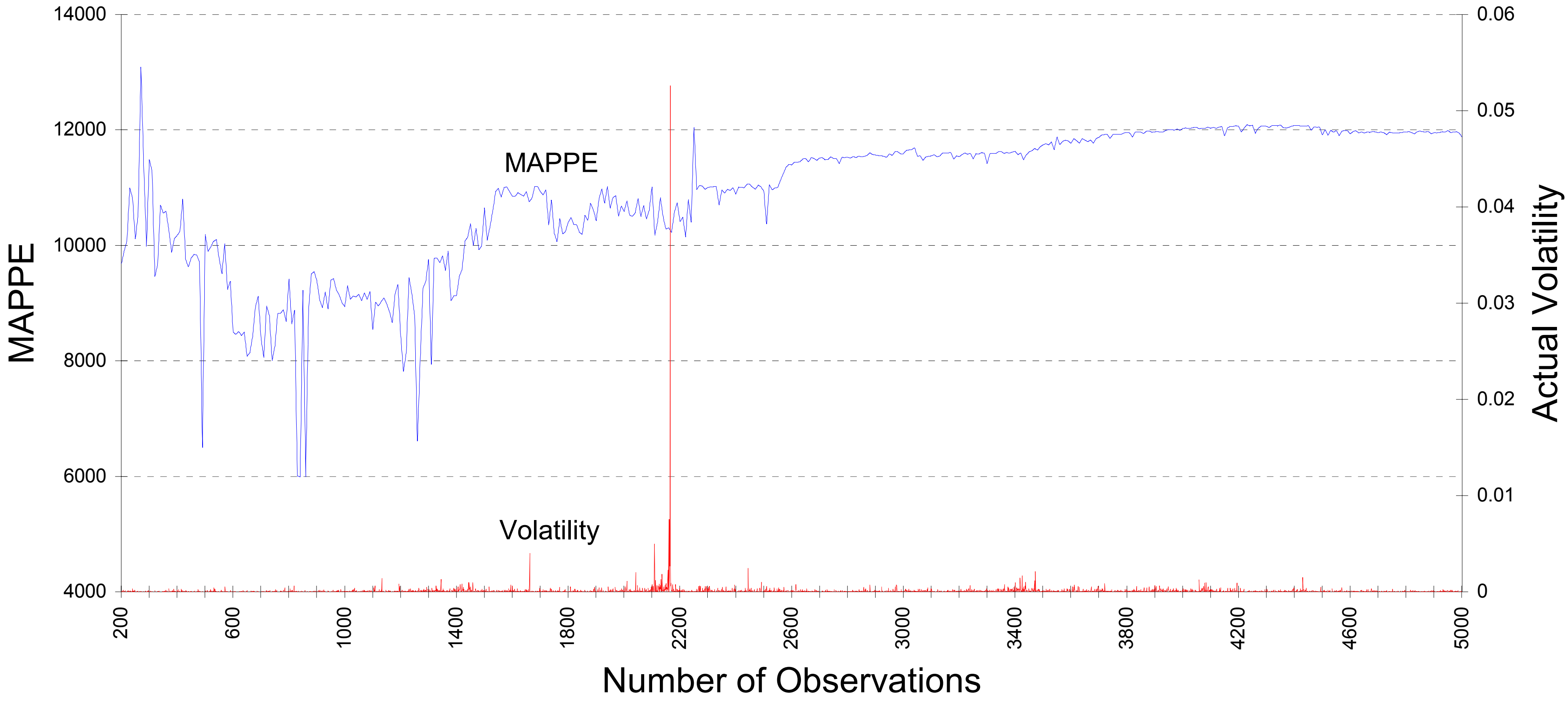

Figure 21: GARCH(1,1) Forecast Performance for 3/5/96 to 7/5/97 (excluding October 1997)