

Volatility Models of Currency Futures in Developed and Emerging Markets

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Abstract

This paper examines volatility models of currency futures contracts for three developed markets and two emerging markets. For each contract, standard models of the Unbiased Expectations Hypothesis (UEH) and Cost-of-Carry hypothesis (COC) are extended to derive volatility models corresponding to each of the two standard approaches. Each volatility model is formulated as a system of individual equations for the conditional variances of futures returns, spot returns and the domestic risk-free interest rate. The empirical results suggest that the conditional volatility of futures return for emerging markets is significant in explaining the conditional volatility of returns in the underlying spot market. For developed markets, however, the conditional volatility of the spot returns is significant in explaining the conditional volatility of futures returns. Moreover, it is found that the domestic risk-free interest rate has little impact on the conditional variances of the futures, spot and domestic risk-free interest rates.

Keywords: Cost-of-Carry Volatility Systems; Unbiased Expectations Hypothesis
Volatility System.

1. Introduction

Over the last 25 years, financial innovation and competitive pressures have forced massive changes on the structure and institutions of the foreign exchange market. The Bank for International Settlements surveys [1] estimate that the total daily worldwide foreign exchange trading volume in 1998 alone was \$1.5 trillion per day, or nearly \$400 trillion per year. Trading volume on the foreign exchange markets is clearly massive. By comparison, only \$58.8 billion in equities was traded on the busiest day in the history of the New York Stock Exchange [9], on 19 April 1999. The volatility in these markets became apparent after the devaluation of the pound sterling in November 1967, when a series of international financial crisis ensued until the Smithsonian agreement in 1971. Consequently, this led to the introduction of trading in foreign currency futures on the International Monetary Market (IMM) of the Chicago Mercantile Exchange in May 1972.

Increasingly, studies in futures markets tend to focus on some common issues, namely, the determination of optimal hedge ratios [6], international transmission of information across different international futures markets trading identical futures contracts ([11], [8]) , and price volatility and trading volume [2], among others. There are few known studies that examine the interactions between the volatilities of the spot and futures markets. In a recent study [10], spot and futures market volatilities of Australian dollar futures contracts traded on the IMM are examined using a univariate approach. They provided evidence that the volatility in futures returns was strongly affected by the volatility in the underlying spot market and the volatility in the foreign risk-free rate, but not by volatility in the domestic risk-free interest rate.

In this paper we formulate volatility models of currency futures contracts. The approach differs from [7] in that a system of equations is formulated to represent a volatility model. Two standard models in [10] are extended to estimate a volatility model for each of these two well-known approaches in modelling futures prices, namely the Cost-of-Carry (COC) model and the Unbiased Expectations Hypothesis (UEH). This estimation method is preferred because a systems approach is more efficient than the univariate methods in [7].

The COC and UEH volatility models are estimated for currency futures contracts from three developing and two emerging countries traded on the International Monetary Market of the Chicago Mercantile Exchange (CME). A primary objective in analyzing the models according to their separate grouping is examine the relative impacts of spot or futures volatilities in each of the markets and to identify patterns that are common within each group. Interestingly, the results indicate that there is systematic behaviour in the conditional variances in both developed and emerging markets.

The remainder of the paper is organized as follows. In Section 2, we discuss the formulation of volatility models of futures contracts based on the two main hypotheses for futures pricing. Then we examine the data in Section 3. In Section 4, we present some results from unit root and cointegration tests. Section 5 highlights the main results obtained in the paper. Section 6 provides some concluding remarks.

2. *Volatility Models of futures contracts*

In [10], systems equations of Australian dollar futures contracts based on the two main hypotheses for pricing futures contracts, namely the Cost-of-Carry (COC) model and Unbiased Expectations hypotheses (UEH) are developed. The error correction representation

of the COC model, with one cointegrating vector among the futures price, spot price, domestic interest rate and foreign interest rate, assuming that all four variables are all I(1) and the domestic interest rate is determined exogenously (the foreign risk-free rate is assumed to have a negligible influence on the domestic rate) is given as:

$$\begin{aligned}\Delta s_t = & a_0 + a_1 \Delta s_{t-1} + a_2 \Delta f_{t-1} + a_3 \Delta r_{t-1}^d + a_4 \Delta r_{t-1}^f + \dots \\ & \dots + a_5 (f_{t-1} - b_1 s_{t-1} - b_2 r_{t-1}^f - b_3 r_{t-1}^d) + \varepsilon_t^s\end{aligned}\quad (1a)$$

$$\begin{aligned}\Delta f_t = & a_{10} + a_{11} \Delta s_{t-1} + a_{12} \Delta f_{t-1} + a_{13} \Delta r_{t-1}^d + a_{14} \Delta r_{t-1}^f + \dots \\ & \dots + a_{15} (f_{t-1} - b_1 s_{t-1} - b_2 r_{t-1}^f - b_3 r_{t-1}^d) + \varepsilon_t^f\end{aligned}\quad (1b)$$

$$\begin{aligned}\Delta r_t^f = & a_{20} + a_{21} \Delta s_{t-1} + a_{22} \Delta f_{t-1} + a_{23} \Delta r_{t-1}^d + a_{24} \Delta r_{t-1}^f + \dots \\ & \dots + a_{25} (f_{t-1} - b_1 s_{t-1} - b_2 r_{t-1}^f - b_3 r_{t-1}^d) + \varepsilon_t^r\end{aligned}\quad (1c)$$

with the single error correction term given by $(f_{t-1} - b_1 s_{t-1} - b_2 r_{t-1}^f - b_3 r_{t-1}^d)$ in the COC model.

For the UEH, assuming that the futures and spot prices are I(1) and that a cointegrating relationship exists between the two prices, is given as:

$$\Delta s_t = c_0 + c_1 \Delta s_{t-1} + c_2 \Delta f_{t-1} + c_3 (f_{t-1} - d_1 s_{t-1}) + \varepsilon_t^s \quad (2a)$$

$$\Delta f_t = c_{10} + c_{11} \Delta s_{t-1} + c_{12} \Delta f_{t-1} + c_{13} (f_{t-1} - d_1 s_{t-1}) + \varepsilon_t^f \quad (2b)$$

$$\Delta r_t^f = c_{20} + c_{21} \Delta r_{t-1}^f + c_{22} \Delta r_{t-1}^d + \varepsilon_t^r \quad (2c)$$

where $(f_{t-1} - d_1 s_{t-1})$ represents the error correction term between the futures and spot prices.

There is no cointegrating relationship between the domestic and foreign risk-free rate, so that interest rate parity is not necessary for equation (2c), which is optional for the system. Equation (2c) is included in the system to enable a comparison between the UEH and COC models.

In this paper, we extend the formulations in [10] to incorporate the second moments of futures returns, for which the variance of the COC model is given corresponding to the particular equations (1a)-(1c) as follows:

$$\begin{aligned} \text{var } \Delta s_t = & a_0 + a_1 \text{var } \Delta s_{t-1} + a_2 \text{var } \Delta f_{t-1} + a_3 \text{var } \Delta r_{t-1}^d + a_4 \text{var } \Delta r_{t-1}^f + \dots \\ & \dots + a_5 \text{var}(f_{t-1} - b_1 s_{t-1} - b_2 r_{t-1}^f - b_3 r_{t-1}^d) + \varepsilon_t^s \end{aligned} \quad (3a)$$

$$\begin{aligned} \text{var } \Delta f_t = & a_{10} + a_{11} \text{var } \Delta s_{t-1} + a_{12} \text{var } \Delta f_{t-1} + a_{13} \text{var } \Delta r_{t-1}^d + a_{14} \text{var } \Delta r_{t-1}^f + \dots \\ & \dots + a_{15} \text{var}(f_{t-1} - b_1 s_{t-1} - b_2 r_{t-1}^f - b_3 r_{t-1}^d) + \varepsilon_t^f \end{aligned} \quad (3b)$$

$$\begin{aligned} \text{var } \Delta r_t^f = & a_{20} + a_{21} \text{var } \Delta s_{t-1} + a_{22} \text{var } \Delta f_{t-1} + a_{23} \text{var } \Delta r_{t-1}^d + a_{24} \text{var } \Delta r_{t-1}^f + \\ & \dots + a_{25} \text{var}(f_{t-1} - b_1 s_{t-1} - b_2 r_{t-1}^f - b_3 r_{t-1}^d) + \varepsilon_t^r \end{aligned} \quad (3c)$$

Equations (3a)-(3c) are denoted as the COC Volatility Systems model (COCVS) model with one cointegrating vector. Covariances between the variables in each equation are subsumed into the error terms.

A similar procedure is applied to the UEH model given by equations (2a)-(2c) to obtain the UEH Volatility Systems (UEHVS) model, as follows:

$$\text{var } \Delta s_t = c_0 + c_1 \text{var } \Delta s_{t-1} + c_2 \text{var } \Delta f_{t-1} + c_3 \text{var}(s_{t-1} - d_1 f_{t-1}) + \varepsilon_t^s \quad (4a)$$

$$\text{var } \Delta f_t = c_{10} + c_{11} \text{var } \Delta s_{t-1} + c_{12} \text{var } \Delta f_{t-1} + c_{13} \text{var}(s_{t-1} - d_1 f_{t-1}) + \varepsilon_t^f \quad (4b)$$

$$\text{var } \Delta r_t^f = c_{20} + c_{21} \text{var } \Delta r_{t-1}^f + c_{22} \text{var } \Delta r_{t-1}^d + \varepsilon_t^r \quad (4c)$$

The covariances are, as before, subsumed into the error term.

For the COC model with two cointegrating vectors, we assume a cointegrating relationship between the spot and futures returns, and also between the domestic and foreign risk-free rate, as follows:

$$\begin{aligned} \Delta s_t = & a_0 + a_1 \Delta s_{t-1} + a_2 \Delta f_{t-1} + a_3 \Delta r_{t-1}^d + a_4 \Delta r_{t-1}^f + a_5 (f_{t-1} - b_1 s_{t-1}) \\ & + a_6 (r_{t-1}^f - b_2 r_{t-1}^d) + \varepsilon_t^s \end{aligned} \quad (5a)$$

$$\Delta f_t = a_{10} + a_{11}\Delta s_{t-1} + a_{12}\Delta f_{t-1} + a_{13}\Delta r_{t-1}^d + a_{14}\Delta r_{t-1}^f + a_{15}(f_{t-1} - b_1 s_{t-1}) + a_{16}(r_{t-1}^f - b_2 r_{t-1}^d) + \varepsilon_t^f \quad (5b)$$

$$\Delta r_t^f = a_{20} + a_{21}\Delta s_{t-1} + a_{22}\Delta f_{t-1} + a_{23}\Delta r_{t-1}^d + a_{24}\Delta r_{t-1}^f + a_{25}(f_{t-1} - b_1 s_{t-1}) + a_{26}(r_{t-1}^f - b_2 r_{t-1}^d) + \varepsilon_t^r \quad (5c)$$

where $(f_{t-1} - b_1 s_{t-1})$ is the error correction term between the futures and spot prices and $(r_{t-1}^f - b_2 r_{t-1}^d)$ is the error-correction term between the domestic and foreign risk-free interest rate.

Based on equations (5a)-(5c), we formulate the COC Volatility Systems (COCVS) model with two cointegrating vectors, as follows:

$$\text{var } \Delta s_t = a_0 + a_1 \text{var } \Delta s_{t-1} + a_2 \text{var } \Delta f_{t-1} + a_3 \text{var } \Delta r_{t-1}^d + a_4 \text{var } \Delta r_{t-1}^f + a_5 \text{var}(f_{t-1} - b_1 s_{t-1}) + a_6 \text{var}(r_{t-1}^f - b_2 r_{t-1}^d) + \varepsilon_t^s \quad (6a)$$

$$\text{var } \Delta f_t = a_{10} + a_{11} \text{var } \Delta s_{t-1} + a_{12} \text{var } \Delta f_{t-1} + a_{13} \text{var } \Delta r_{t-1}^d + a_{14} \text{var } \Delta r_{t-1}^f + a_{15} \text{var}(f_{t-1} - b_1 s_{t-1}) + a_{16} \text{var}(r_{t-1}^f - b_2 r_{t-1}^d) + \varepsilon_t^f \quad (6b)$$

$$\text{var } \Delta r_t^f = a_{20} + a_{21} \text{var } \Delta s_{t-1} + a_{22} \text{var } \Delta f_{t-1} + a_{23} \text{var } \Delta r_{t-1}^d + a_{24} \text{var } \Delta r_{t-1}^f + a_{25} \text{var}(f_{t-1} - b_1 s_{t-1}) + a_{26} \text{var}(r_{t-1}^f - b_2 r_{t-1}^d) + \varepsilon_t^r \quad (6c)$$

The UEHVS model is a special case of the COCVS model with two cointegrating vectors as equations (6a)-(6c) reduce through parametric restrictions to the system of equations given by (4a)-(4c). Equation (6) reduces to (4) by eliminating: (i) the conditional variances of the two interest rates; (ii) the conditional variances of the error correction term between the two interest rates from the spot and futures equations (6a)-(6b); (iii) the conditional covariances of both spot and futures prices; (iv) the conditional variances of the error correction term between the spot and futures prices and between the two interest rates from the foreign interest rate equation.

As the UEHVS model is nested within the COCVS model with two cointegrating vectors, it can be tested by applying the following parametric restrictions on the COCVS model (with two cointegrating vectors) as follows:

$$H_0 : a_3 = a_4 = a_6 = a_{13} = a_{14} = a_{16} = a_{21} = a_{22} = a_{25} = a_{26} = 0 \quad (7)$$

The Wald statistics on the parametric restrictions can be used to test the validity of these restrictions. Under the null hypothesis, the error correction term between the interest rates is deleted from equations (6a)-(6c) and the error correction term between the futures and spot prices is deleted from equation (7).

3. *Data*

Daily spot and futures settlement prices for the Brazilian Real (BRR), French Franc (FRF), German Deutsche Mark (DEM), Japanese Yen (JPY) and the Mexican Peso (MXN) traded on the International Monetary Market (IMM) of the Chicago Mercantile Exchange (CME) are analyzed in this paper. These futures contracts and their corresponding spot rates represent a sample of currencies from both developed and emerging markets. In this paper, currency futures contracts on developed markets are the French Franc, German Deutsche Mark and Japanese Yen, while those on emerging markets are the Brazilian Real and Mexican Peso. The sample for the DEM and JPY covers the period October 1989 to October 2000, for a total of 2878 observations. Sample observations for the other currencies have different starting dates due to the unavailability of data prior to October 1989. The FRF contract commences in September 1993 and ends in October 2000; the BRR covers the period November 1995 until October 2000; and the MXN is available from June 1996 until October 2000. We use the risk-free interest rate of the domiciled currency as the foreign risk-free rate, and the US Treasury Bill rate as the domestic risk-free interest rate. Daily observations on the futures and spot prices, and domestic and foreign risk-free rates, are obtained from the DATASTREAM

International database. For futures contracts, the nearest to delivery contract is rolled over to the next contract to avoid maturity and thin trading effects. Returns of the futures and spot are taken as the first logarithmic differences of their respective prices.

Table 1 presents summary statistics on the futures and spot prices, and domestic and foreign interest rates pertaining to each currency analyzed. Notable observations from these statistics are the values for the Mexican Peso. Specifically, the interest rates associated with the Mexican Peso tend to have the largest magnitude and range. Japan, on the other hand, has the lowest interest rate among the currencies used in our paper. As expected, the futures and spot prices follow similar time paths, clearly tracking the long-run relationship between these two variables.

4. Unit root and cointegration tests

The Augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) tests are applied to the futures and spot price, domestic and foreign risk-free interest rate of all five currencies to determine their order of integration. Results of these tests are given in Tables 2 to 4. The ADF and the PP statistics for all the variables are not more negative than their respective critical values, suggesting that all variables used in the study are nonstationary in levels.¹ Significant ADF statistics are obtained for first differences of these variables, implying that they are integrated of order one, or $I(1)$.

For the Cost-of-Carry systems model, a long run relationship is assumed to exist between four variables, namely the futures price, spot price, domestic risk-free rate and foreign risk-free rate. Cointegration tests among the four variables are conducted using the Johansen

¹ A trend and intercept are included as the ADF statistics with and without trend are significantly different.

procedures [5] to identify the number of cointegrating relationships. Two cointegrating vectors are obtained among the four variables for the Brazilian Real and Deutsche Mark, comprising one long-run relationship between the futures and spot prices, and another between the domestic and foreign risk-free rates. One cointegrating vector is obtained among the four variables for the French Franc, Japanese Yen and Mexican Peso, describing one long run relationship among the four variables. Johansen's procedures [5] confirm the existence of a cointegrating relationship between the spot and future prices in the Unbiased Expectations Hypothesis. A summary of the results of Johansen's procedures is given in Table 5.

5. *Estimation results*

The symmetric GARCH (1,1) model (see [3]) is estimated for the conditional variances of the futures and spot returns, and the domestic and foreign interest rates. Conditional variances for the error correction terms for both the Cost-of-Carry Volatility System and Unbiased Expectation Hypothesis Volatility System for the five currencies are also estimated. Using these estimates of the conditional variances, both the Cost-of-Carry and the Unbiased Expectation Hypothesis volatility systems are estimated using the Seemingly Unrelated Regression Equations method. Estimates for the three equation systems corresponding to the Cost-of-Carry volatility system with one cointegrating vector, the Cost-of-Carry volatility system model with two cointegrating vectors, and the Unbiased Expectation Hypothesis volatility system, are given in Tables 6, 7 and 8, respectively.

5.1 *Cost-of-carry volatility system with one cointegrating vector*

Table 6 presents the estimates of the COC volatility system with one cointegrating vector. The conditional variance of the futures returns for the Mexican Peso is significant in explaining the conditional variance of the respective spot returns. However, the conditional

variance of the futures returns is not significant in explaining the conditional variance of the spot returns for either the French Franc or Japanese Yen. It is found that the conditional variance of the spot returns is significant in explaining the conditional variance of futures returns for both the French Franc and Japanese Yen. The reverse does not, however, hold for both these contracts. It is also found that the conditional variance of the domestic interest rates does not have a significant effect on the conditional variance of the futures returns for these three currencies. The conditional variance of the domestic risk-free interest rate does not have a significant influence on the conditional variance of the foreign risk-free interest rate in France, Japan and Mexico. However, the conditional variance of the foreign interest rate in Japan and Mexico is significant in explaining the conditional variance of their respective futures returns. For Japan, the conditional variance of the foreign interest rate is significant in explaining the conditional variance of the spot returns.

5.2 *Cost-of-carry volatility systems with two cointegrating vectors*

Table 7 presents the estimates for the Cost-of-Carry Volatility System with two cointegrating vectors, which applies only to models for the Brazilian Real and Deutsche Mark. The conditional variance of futures returns for the Brazilian Real is significant in explaining the conditional variance of spot returns, but the reverse does not hold. This result is consistent for the emerging markets analyzed in this paper. The conditional variance of spot returns for the Deutsche Mark is significant in explaining the conditional variance of futures return. Again, the reverse does not hold, a common pattern observed for developed markets. The conditional variance of the domestic risk-free rate is not significant in explaining the conditional variance of the foreign risk-free rate, but the conditional variance of the foreign risk-free rate is significant in explaining the conditional variance of the spot rate. It is also found that the

conditional variance of the foreign risk-free rate is not significant in explaining the conditional variance of both the futures and spot returns.

5.3 Unbiased expectations hypothesis volatility system

Table 8 presents the estimates of the Unbiased Expectations Hypothesis Volatility System. As for the Cost-of-Carry Volatility Systems Model, the conditional variance of futures returns is significant in explaining the conditional variance of spot returns for the currencies of emerging markets, but not vice-versa. The conditional variance of the spot returns is significant in explaining the conditional variance of the futures returns, as observed for currencies in developed markets. An exception is the Mexican Peso, where the conditional variance of the futures returns is significant in explaining the conditional variance of the futures returns. The domestic risk-free rate is not significant in explaining the conditional variance of the futures, spot and foreign risk-free rate for currencies using the UEHVS model. This suggests that the conditional variance of the domestic risk-free interest rate does not affect the conditional variance of the currency spot and futures markets in either the developed or emerging markets.

5.4 Comparisons between the two models

Estimates of the error-correction terms for the two Cost-of-Carry models, and the Unbiased Expectations Hypothesis Volatility models, are presented in Tables 9 and 10. The coefficient of lagged spot prices in the Cost-of-Carry Volatility System with one cointegrating vector is close to minus unity, with values in the range (-0.99819, -1.00720), while the values are in the range (-0.99942, -1.0064) for the Cost-of-Carry Volatility System with two cointegrating vectors. Coefficients of lagged spot prices in the Unbiased Expectations Hypothesis Volatility Systems are also very close to minus unity, with values in the range (-0.99150, -1.01294).

These results are consistent with recent empirical results² in [4]. Moreover, the magnitudes of the interest rate variables are very close to zero, and insignificant, in most of the currency contracts.

As the Cost-of-Carry Volatility System with two cointegrating vectors nests the Unbiased Expectations Hypothesis Volatility System, it is possible to determine the appropriate model on the basis of testing parametric restrictions. If the restrictions are valid, the Cost-of -Carry Volatility System reduces to the Unbiased Expectations Hypothesis Volatility System. The Wald test procedure is used to test the null hypothesis that the restrictions are valid. Of the five currencies, only the models for the Brazilian Real and the Deutsche Mark with two cointegrating vectors are tested. The Wald test statistics, which are highly significant at 30.5 and 39.1 (see Table 11) for the Brazilian Real and the Deutsche Mark, respectively, suggest that the appropriate model for the two currencies is the Cost-of-Carry Volatility System with two cointegrating vectors.

6. *Conclusion*

Multinational firms are subject to the changing patterns of currency volatility that have a tremendous impact on their performance. The introduction of derivative products has increased in recent years, indicating the emphasis that financial institutions place on these products to counter both exchange rate and interest rate movements. Although corporations are aware of such innovations, there is still a heavy reliance on the traditional hedging tools afforded by forward and futures contracts. There is a need for a deeper understanding of the nature and behaviour of currency futures contracts and their impact on spot markets.

² In [4], cointegrating vectors were found to be in the range of (-1.03, -0.95).

In this paper, the conditional variances between spot and futures markets was modelled for both developed and emerging markets. It was found that the conditional variance of the futures market is significant in explaining the conditional variance of the spot market returns in emerging markets. For developed markets, the conditional variance of spot returns is significant in explaining the conditional variance of futures returns. These results are interesting because they suggest that exchange rate volatility in emerging markets is driven by volatility in their respective futures contracts. The currencies of emerging markets are subject to international influences, which provides some support for governmental intervention to maintain exchange rate stability. For developed markets, the influence of foreign agents tend to be more controlled as the results suggest that the conditional volatility in spot returns drives the conditional volatility in futures returns. The case for reduced intervention in foreign exchange markets becomes apparent for developed markets.

The empirical results also show that the conditional variance of the domestic risk-free rate does not have a significant influence on the conditional variance of the spot, futures or foreign risk free rate. Moreover, the conditional variances of the foreign risk-free rates in the Japanese and Mexican markets are significant in explaining the conditional volatility of futures returns. In the same way, the conditional variances of the foreign risk-free rate in the Japanese and Brazilian markets are significant in explaining the conditional variance of spot returns.

The Cost-of-Carry Volatility System with two cointegrating vectors and the Unbiased Expectation Hypothesis Volatility System were compared on the basis of nested tests. It was found that the Cost-of-Carry Volatility System outperforms the Unbiased Expectations Hypothesis Volatility System for both the Brazilian Real and Deutsche Mark.

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References

- [1] Bank for International Settlements, 1999, *Central Bank Survey of Foreign Exchange and Derivative Market Activity*, Basle, Switzerland.
- [2] Bessembinder H. and Seguin P. J., 1993, Price Volatility, Trading Volume and Market Depth: Evidence from Futures Markets, *Journal of Financial and Quantitative Analysis*, 28, 1, 21-38.
- [3] Bollerslev, T., 1986, A Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics*, 31, 307-327.
- [4] Brenner, R. J. and Kroner, K.F., 1995, Arbitrage, cointegration and testing the Unbiasedness Hypothesis in financial markets, *Journal of Financial and Quantitative Analysis*, 30, 23-42.
- [5] Johansen, S., 1991, Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models, *Econometrica*, 59, 6, 1551-1580.
- [6] Kroner, K. F., and Sultan, J., 1993, Time Varying Distributions and Dynamic Hedging with Foreign Currency Futures, *Journal of Financial and Quantitative Analysis*, 28, 4, 535-551.
- [7] McAleer, M., and Sequeira, J. M., 1999, Testing Alternative Models of Volatility in Currency Futures Markets, in L. Oxley, F. Scrimgeour and M. McAleer (eds.), *Proceedings of the International Congress on Modelling and Simulation, Volume 2*, Hamilton, New Zealand, 411-416.
- [8] Najand, M., Rahman, H., and Yung, K., 1992, Inter-Currency Transmission of Volatility in Foreign Exchange Futures, *Journal of Futures Markets*, 12, 6, 609-620.
- [9] *New York Stock Exchange Factbook*, 1999, New York Stock Exchange, New York.

- [10] Sequeira, J. M., McAleer, M., and Chow, Y.F., 2001, Efficient Estimation and Testing of Alternative Models of Currency Futures Contracts, *Economic Record*, 77, 238, 270-282.
- [11] Tse, Y., Lee T.H., and Booth, G.G., 1996, The International Transmission of Information in Eurodollar Futures Markets: A Continuously Trading Market Hypothesis, *Journal of International Money and Finance*, 15, 3, 447-465.

Table 1
Descriptive Statistics

Data	Sample Size	Mean	Maximum	Minimum	Std. Dev	Skewness	Kurtosis
<i>Brazilian Real</i>	1296						
Futures price		0.791	1.036	0.455	0.190	-0.394	1.433
Spot exchange rate		0.794	1.040	0.463	0.190	-0.404	1.441
Domestic Interest rates (US)		5.167	2.420	0.214	0.456	0.385	3.624
Foreign Interest Rate (Bra)		1.102	6.360	3.640	0.358	0.855	3.756
<i>German Deutsche Mark</i>	2878						
Futures price		0.603	0.741	0.424	0.060	-0.191	2.928
Spot exchange rate		0.603	0.739	0.424	0.061	-0.249	2.904
Domestic Interest rates (US)		5.109	8.260	2.650	1.304	0.376	3.059
Foreign Interest Rate (Ger)		5.691	9.933	2.570	2.467	0.433	1.560
<i>French Franc</i>	1853						
Futures price		0.176	0.210	0.126	0.018	-0.235	2.452
Spot exchange rate		0.175	0.210	0.126	0.019	-0.227	2.442
Domestic Interest rates (US)		5.036	6.360	2.920	0.721	-1.038	4.227
Foreign Interest Rate (Fre)		4.471	9.000	2.570	1.392	0.803	2.621
<i>Japanese Yen</i>	2878						
Futures price		0.009	0.012	0.006	0.001	0.373	2.963
Spot exchange rate		0.009	0.012	0.006	0.001	0.382	2.997
Domestic Interest rates (US)		5.109	8.260	2.650	1.304	0.376	3.059
Foreign Interest Rate (Jap)		2.636	8.469	0.016	2.731	0.863	2.275
<i>Mexican Peso</i>	1133						
Futures price		0.111	0.132	0.084	0.011	0.092	1.773
Spot exchange rate		0.114	0.134	0.094	0.011	0.264	1.557
Domestic Interest rates (US)		5.166	6.360	3.640	0.484	0.368	3.272
Foreign Interest Rate (Mex)		23.462	52.500	14.050	6.065	1.018	4.055

Notes:

- (1) All futures prices and spot prices are expressed in terms of US dollars per unit of foreign currency.
- (2) *Data* refers to the sample sets used in the study. For each sample, four sets of variables are collected. The domestic interest rate for all sample sets refers to the US risk-free rate.
- (3) *Sample size* refers to the number of observations collected for each of the variables in the sample sets.

Table 2
Augmented Dickey-Fuller Test for Variables in Levels

Currency		Futures	Spot	Domestic Interests Rate	Foreign Interests Rates
<i>Brazilian Real</i> 1-1296	Number of Lags	11	11	8	11
	ADF statistic	-2.571	-2.318	-1.989	-2.934
	Critical Value (5%)	-3.416	-3.416	-3.416	-3.416
<i>Deutsche Mark</i> 1-2878	Number of Lags	6	10	8	8
	ADF statistic	-1.400	-1.698	-1.575	0.570
	Critical Value (5%)	-3.414	-3.414	-3.414	-3.414
<i>French Franc</i> 1-1853	Number of Lags	0	0	10.00	7.00
	ADF statistic	-1.354	-1.350	-2.152	-1.379
	Critical Value (5%)	-3.415	-3.415	-3.415	-3.415
<i>Japanese Yen</i> 1-2878	Number of Lags	10	10	8	10
	ADF statistic	-1.803	-1.789	-1.575	-0.236
	Critical Value (5%)	-3.414	-3.414	-3.414	-3.414
<i>Mexican Peso</i> 1-1133	Number of Lags	0	10	8	4
	ADF statistic	-2.395	-1.852	-0.999	-2.284
	Critical Value (5%)	-3.416	-3.416	-3.416	-3.416

Notes:

- (1) All ADF statistics are found to be significant at the 5% level.
- (2) A time trend is included in all ADF regressions as the results with and without trend are significantly different.

Table 3
Philips-Perron Test for Variables in Levels

Currency		Futures	Spot	Domestic Interest Rates	Foreign Interest Rates
<i>Brazilian Real</i> 1-1296	Number of Lags	11	11	8	11
	PP test statistic	-2.053	-2.143	-1.384	-2.064
	Critical Value (5%)	-3.416	-3.416	-3.416	-3.416
<i>Deutsche Mark</i> 1-2878	Number of Lags	6	10	8	8
	PP test statistic	-1.549	-1.617	-1.391	0.944
	Critical Value (5%)	-3.414	-3.414	-3.414	-3.414
<i>French Franc</i> 1-1853	Number of Lags	0	0	10	7.00
	PP test statistic	-1.355	-1.350	-2.154	-3.415
	Critical Value (5%)	-3.415	-3.415	-3.415	-3.415
<i>Japanese Yen</i> 1-2878	Number of Lags	10	10	8	10
	PP test statistic	-1.697	-1.682	-1.391	-0.236
	Critical Value (5%)	-3.414	-3.414	-3.414	-3.414
<i>Mexican Peso</i> 1-1133	Number of Lags	0	10	8	4
	PP test statistic	-2.395	-1.833	-1.149	-2.478
	Critical Value (5%)	-3.416	-3.416	-3.416	-3.416

Notes:

(1) All PP statistics are found to be significant at the 5% level.

(2) A time trend is included in all PP-regressions as the results with and without trend are significantly different.

Table 4
Augmented Dickey-Fuller Test for Variables in First Differences

Currency		Futures	Spot	Domestic Interest Rates	Foreign Interest Rates
<i>Brazilian Real</i> 1-1296	Number of Lags	12	10	7	9
	ADF statistic	-8.440	-9.849	-13.754	-12.099
	Critical Value (5%)	-3.416	-3.416	-3.416	-3.416
<i>Deutsche Mark</i> 1-2878	Number of Lags	5	5	9	7
	ADF statistic	-23.339	-23.008	-16.767	-15.772
	Critical Value (5%)	-3.414	-3.414	-3.414	-3.4140
<i>French Franc</i> 1-1853	Number of Lags	0	0	9	6
	ADF statistic	-43.555	-44.447	-13.159	-15.442
	Critical Value (5%)	-3.415	-3.415	-3.415	-3.415
<i>Japanese Yen</i> 1-2878	Number of Lags	9	9	9	12
	ADF statistic	-16.111	-15.984	-16.767	-15.624
	Critical Value (5%)	-3.414	-3.414	-3.414	-3.414
<i>Mexican Peso</i> 1-1133	Number of Lags	0	9	7	3
	ADF statistic	-35.121	-10.232	-12.989	-18.205
	Critical Value (5%)	-3.416	-3.416	-3.416	-3.416

Notes:

- (1) All ADF statistics are found to be significant at the 5% level.
- (2) A time trend is included in all ADF regressions as the results with and without trend are significantly different.

Table 5
Results from the Cointegration Test

Currency	Number of Cointegrating Vectors	
	COC	UEH
Brazilian Real	2	1
French Franc	1	1
Deutsche Mark	2	1
Japanese Yen	1	1
Mexican Peso	1	1

Notes:

- (1) The Johansen test is used to determine the number of cointegrating vectors among the four variables according to the assumption of COC. The number of cointegrating vectors will then determine the COC volatility systems model to employ for each set of data.
- (2) For the UEH volatility system model, it is assumed that only one error correction term exists between the futures and spot returns, so that all the samples are treated uniformly.

Table 6
Estimation Results for the Cost-of-Carry Volatility System with One Cointegrating Vector

Variables	French Franc			Japanese Yen			Mexican Peso		
	$\text{var}(\Delta s_t)$	$\text{var}(\Delta f_t)$	$\text{var}(\Delta r_t^f)$	$\text{var}(\Delta s_t)$	$\text{var}(\Delta f_t)$	$\text{var}(\Delta r_t^f)$	$\text{var}(\Delta s_t)$	$\text{var}(\Delta f_t)$	$\text{var}(\Delta r_t^f)$
Constant	0.000** (0.002)	0.000** (0.000)	0.002 (0.119)	0.000 (0.086)	0.000 (0.132)	0.000 (0.765)	0.000 (0.195)	0.000** (0.001)	0.017 (0.602)
$\text{Var}(\Delta s_{t-1})$	0.972** (0.000)	0.043** (0.002)	370.255** (0.000)	0.983** (0.000)	0.037* (0.016)	- 16.137 (0.333)	0.776** (0.000)	0.064 (0.132)	-1.090 (0.995)
$\text{Var}(\Delta f_{t-1})$	0.008 (0.609)	0.915** (0.000)	-428.463** (0.000)	-0.014 (0.402)	0.940** (0.000)	26.634 (0.101)	- 0.051** (0.000)	0.455** (0.000)	- 85.804 (0.454)
$\text{Var}(\Delta r_{t-1}^d)$	0.000 (0.955)	0.000 (0.671)	-0.078 (0.547)	0.000 (0.544)	0.000 (0.502)	0.000 (0.996)	-0.001 (0.465)	- 0.001 (0.697)	-0.339 (0.955)
$\text{Var}(\Delta r_{t-1}^f)$	0.000 (0.673)	0.000 (0.901)	0.783** (0.000)	0.000** (0.000)	0.000** (0.000)	0.934** (0.000)	0.000 (0.589)	0.000* (0.023)	0.884** (0.000)
Ect	0.003* (0.032)	0.001 (0.742)	48.714** (0.000)	0.124** (0.000)	0.103** (0.000)	-5.775 (0.790)	0.074** (0.000)	0.131** (0.000)	507.179** (0.000)
R ²	0.968	0.933	0.746	0.959	0.970	0.891	0.605	0.329	0.824
DW	1.908	1.929	1.910	1.899	1.929	1.559	1.935	1.997	2.110

Notes:

- (1) Ect represents coefficients of the error correction term between the futures, spot, domestic interest rates and the foreign interest rates in the COCVS from equation (3a)-(3c).
- (2) *p*-values are given in parentheses.
- (3) ** Denotes significance at the 1% level; * denotes significance at the 5% level.

Table 7
Estimation Results of the Cost-of-Carry Volatility System with Two
Cointegrating Vectors

Variables	Brazilian Real			Deutsche Mark		
	$\text{var}(\Delta s_t)$	$\text{var}(\Delta f_t)$	$\text{var}(\Delta r_t^f)$	$\text{var}(\Delta s_t)$	$\text{var}(\Delta f_t)$	$\text{var}(\Delta r_t^f)$
Constant	0.000 (0.133)	0.000 (0.311)	0.001** (0.002)	0.000* (0.015)	0.000** (0.000)	0.000** (0.000)
$\text{Var}(\Delta s_{t-1})$	0.616** (0.000)	0.606 (0.247)	0.991 (0.576)	0.981** (0.000)	0.102** (0.000)	-0.292 (0.828)
$\text{Var}(\Delta f_{t-1})$	0.118** (0.000)	0.881** (0.000)	0.631 (0.457)	0.004 (0.178)	0.630** (0.000)	1.824 (0.095)
$\text{Var}(\Delta r_{t-1}^d)$	-0.001 (0.254)	0.000 (0.999)	0.013 (0.799)	0.000 (0.986)	0.000 (0.526)	0.000 (0.958)
$\text{Var}(\Delta r_{t-1}^f)$	-0.000* (0.027)	0.000 (0.377)	0.912** (0.000)	0.000 (0.845)	0.000 (0.392)	0.448** (0.000)
Ect1	0.132** (0.000)	0.095** (0.000)	-0.118 (0.852)	0.002* (0.022)	0.019** (0.001)	1.545** (0.000)
Ect2	0.000 (0.728)	0.000 (0.682)	0.000** (0.000)	0.000 (0.381)	0.000 (0.587)	0.000 (0.402)
R^2	0.909	0.882	0.875	0.946	0.478	0.217
DW	2.267	2.275	1.63	1.984	1.996	2.014

Notes:

- (1) Ect1 represents the coefficients of the error correction term between the futures and spot price, and Ect2 represents the coefficient of the error correction term between the foreign domestic risk free interest rate in the and error correction term between domestic interest rates and the foreign interest rates in the COC system with two cointegrating vectors given by equations (6a)-(6c).
- (2) p -values are given in parentheses.
- (3) ** Denotes significance at the 1% level; * denotes significance at the 5% level.

Table 8
Estimation Results of the Unbiased Expectations Hypothesis Volatility System

Variables	Brazilian Real			Deutsche Mark			French Franc		
	$\text{var}(\Delta s_t)$	$\text{var}(\Delta f_t)$	$\text{var}(\Delta r_t^f)$	$\text{var}(\Delta s_t)$	$\text{var}(\Delta f_t)$	$\text{var}(\Delta r_t^f)$	$\text{var}(\Delta s_t)$	$\text{var}(\Delta f_t)$	$\text{var}(\Delta r_t^f)$
Constant	0.000 (0.463)	0.000 (0.437)	0.001** (0.000)	0.000** (0.000)	0.000** (0.000)	0.001** (0.000)	0.000* (0.015)	0.000** (0.000)	0.001** (0.003)
$\text{Var}(\Delta s_{t-1})$	0.612** (0.000)	0.053 (0.310)	—	0.982** (0.000)	0.100** (0.000)	—	0.978** (0.000)	0.040** (0.001)	—
$\text{Var}(\Delta f_{t-1})$	0.112** (0.000)	0.874** (0.000)	—	0.003 (0.270)	0.627** (0.000)	—	0.003 (0.801)	0.917** (0.000)	—
$\text{Var}(\Delta r_{t-1}^d)$	—	—	0.002 (0.973)	—	—	-0.002 (0.828)	—	—	-0.128 (0.324)
$\text{Var}(\Delta r_{t-1}^f)$	—	—	0.935** (0.000)	—	—	0.459** (0.000)	—	—	0.856** (0.000)
Ect	0.138** (0.000)	0.107** (0.000)	—	0.002** (0.012)	0.016** (0.002)	—	0.010** (0.001)	0.002 (0.611)	—
R ²	0.909	0.882	0.873	0.976	0.478	0.211	0.968	0.933	0.735
DW	2.235	2.267	1.640	1.978	1.991	2.018	1.917	1.931	2.027

Table 8 (contd)

Variables	Japanese Yen			Mexican Peso		
	$\text{var}(\Delta s_t)$	$\text{var}(\Delta f_t)$	$\text{Var}(\Delta r_t^f)$	$\text{var}(\Delta s_t)$	$\text{var}(\Delta f_t)$	$\text{var}(\Delta r_t^f)$
Constant	0.000** (0.003)	0.000* (0.031)	0.001** (0.004)	0.000** (0.001)	0.000** (0.000)	0.094** (0.001)
$\text{Var}(\Delta s_{t-1})$	0.981** (0.000)	0.035* (0.022)	—	0.788** (0.000)	0.102* (0.015)	—
$\text{Var}(\Delta f_{t-1})$	-0.003 (0.875)	0.950** (0.000)	—	-0.049** (0.001)	0.448** (0.000)	—
$\text{Var}(\Delta r_{t-1}^d)$	—	—	0.048 (0.361)	—	—	-3.216 (0.591)
$\text{Var}(\Delta r_{t-1}^f)$	—	—	0.940** (0.000)	—	—	0.906** (0.000)
Ect	0.013** (0.001)	0.014** (0.000)	-	0.018** (0.000)	0.058** (0.000)	-
R ²	0.959	0.969	0.890	0.596	0.332	0.818
DW	1.881	1.915	1.564	1.926	1.968	2.080

Notes:

- (1) Ect represents the coefficient of the error correction term the UEH volatility system, given by equations (4a)-(4c).
- (2) p -values are given in parentheses.
- (3) ** Denotes significance at the 1% level and * denotes significance at the 5% level..

Table 9
Estimates of the Error-Correction Terms in the Cost-of- Carry
Volatility System

Panel A: Cost-of-Carry Volatility System with one
Cointegrating Vector

Estimates	Currency Contract		
	French Franc	Japanese Yen	Mexican Peso
b_1	-0.99819** (0.000)	-1.00050** (0.002)	-1.00720** (0.016)
b_2	-0.00164** (0.000)	-0.00179** (0.000)	-0.00160** (0.005)
b_3	-0.00174** (0.000)	0.00171** (0.000)	0.00142** (0.000)

Panel B: Cost-of-Carry Volatility System with Two
Cointegrating Vectors

Estimates	Currency Contract	
	Brazilian Real	Deutsche Mark
b_1	-1.0023** (0.002)	-0.99942** (0.000)
b_2	3.6991 (0.051)	-0.96269* (0.047)

Notes:

- (1) b_1, b_2, b_3 in Panel A are the estimates of the coefficients of the error correction term $f_{t-1} - b_1 s_{t-1} - b_2 r_{t-1}^f - b_3 r_{t-1}^d$ for Cost-of-Carry volatility System, given by equations (3a)-(3c).
- (2) b_1 in Panel B is the estimate of the coefficients of the error correction term to the term $f_{t-1} - b_1 s_{t-1}$ and b_2 is the estimate of the error correction term $r_{t-1}^f - b_2 r_{t-1}^d$ for Cost-of-Carry Volatility System with two cointegrating vectors given by equations (6a)-(6c).
- (3) p -values are given in parentheses
- (4) ** Denotes significance at the 1% level and * denotes significance at the 5% level

Table 10
Estimates for the Error Correction Terms for the Unbiased Expectations Hypothesis System

Estimates	Currency Contract				
	Brazilian Real	French Franc	Deutsche Mark	Japanese yen	Mexican Peso
d_1	-1.009329** (0.002)	-0.999066** (0.000)	-0.99991** (0.001)	-0.99898** (0.000)	-1.01294** (0.001)

Notes:

- (1) d_1 is the estimate of the coefficients of the error correction term given in $f_{t-1} - d_1 s_{t-1}$ for the Unbiased Expectations hypotheses given by equations (4a)-(4c).
- (2) p -values are given in parentheses
- (3) ** Denotes significance at the 1% level and * denotes significance at the 5% level

Table 11
Wald Tests Results of the Null Hypothesis

	Currency Contract	
	Brazilian Real	Deutsche Mark
Test Statistics	30.501** (0.000)	39.061** (0.000)

Notes:

- (1) The Wald test is calculated for the Cost-of-Carry volatility model with two cointegrating vectors for the following null hypothesis:

$$H_0 : a_3 = a_4 = a_6 = a_{13} = a_{14} = a_{16} = a_{21} = a_{22} = a_{25} = a_{26} = 0$$

The equation system of (6a) - (6b) will collapse to (4a)-(4b), which is the Unbiased Expectations Hypothesis under the restrictions above. Under these conditions, the error correction terms between the interest rates are removed from (6a)-(6c), and the error correction term between the futures and spot returns is removed from (6c). Finally, the conditional variances of the domestic and foreign interest rates are removed from (6a)-(6b). The Wald test determines whether the null hypothesis is rejected, indicating that the Cost-of-Carry model dominates when there are two cointegrating vectors.

- (2) p -values are given in parentheses.
(3) **Denotes significance at the 1% level.