Consumption and labor supply decisions in a neoclassical growth model with quasi-hyperbolic discounting

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Outline

1. Theoretical and experimental literature on non-constant discounting
2. Motivation
3. Model
    - two-period
    - infinite horizons.
4. Results on tax policy

Criticism on geometric discounting

- Constant discounting function δ^t is popular in macroeconomics.
- Some experimental evidence (Benzion et al. (1989)) suggests that discounting of future rewards is not constant.
- **Quasi-hyperbolic** discounting (Laibson (1997))

$$U_0 = u(c_0) + \beta \sum_{t=1}^{\infty} \delta^t u(c_t). \tag{1}$$

- **Time-inconsistency :** the trade-off between date 1 and date 2 at date 0, is different from the one at date 1.

Ramsey model with quasi-hyperbolic discounting

- Krusell, Kuruşçu and Smith (2002, henceforth KKS) incorporate quasi-hyperbolic discounting into the neoclassical growth model.
- Findings of KKS
  1. Competitive economy (CE) performs better than the planning economy (PE).
  2. The time-consistent tax policy including positive capital tax replicates the PE and then reduces welfare.
- Their message: The market mechanism is good! Leave it as it is.
- **My questions**: Is the mechanism always good? Is the optimal capital tax always positive?

Literature

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- Schwarz, M., Sheshinski, E., 2007. Quasi-hyperbolic discounting and social security systems. EER.
- Ubfal, D., 2016. How general are time preferences? Eliciting good-specific discount rates, Journal of Development Economics.

Literature

- Bisin, A., Lizzeri, A., Yariv, L., 2014. Government policy with time inconsistent voters. AER.
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- Hori, T., Futagami, K. 2017. A non-unitary discount model. Economica.
- Krusell, P., Kuruşçu, B., Smith, A., 2002. Equilibrium welfare and government policy with quasi-geometric discounting. JET.

Model of KKS

- Resource constraint and budget constraint:

$$k_{t+1} = f(k_t) - c_t, \quad (2)$$
$$k_{t+1} = r_t k_t + w_t - c_t. \quad (3)$$

- First best. Consume a lot today, and save a lot after tomorrow. The allocation is time-inconsistent.
- KKS assume that planner and consumer adopt a time consistent consumption-saving strategy. They have to save today more than the First best.
  - Consumer: The marginal gain from saving, r is constant.
  - Planner: The marginal gain from saving, $f'(k)$ is diminishing.
  - Planner suffers more from the self-control problem.

Literature on hyperbolic discounting

- Partial equilibrium:
  1. Laibson (2001) studies undersavings.
  2. Diamond and Kőszegi (2003) investigate early retirement.
- Government policy:
  1. Schwarz and Sheshinski (2007) study **social security** in OG model.
  2. Paserman (2008) shows that **labor market policies** that encourage workers with hyperbolic discounting to search job improve welfare.
  3. Bisin et al. (2014) study a model with time inconsistent **voters** .
  4. Pavoni and Yaziki (2017) assume that agents' ability to self-control increases with **age** and show that savings should be subsidized.
  5. Graham and Snower (2008) study **inflation**.

Literature

- Abdellaoui.M., E. Attema., H. Bleichrodt, 2010. Intertemporal tradeoffs for gains and losses: an experimental measurement of discounted utility. EJ.
- Attema, A.E., Bleichrodt, H., L'Haridon, O. 2018. Discounting health and money: New evidence using a more robust method. J Risk Uncertain.
- Augenblick, N., Niederle, M., Sprenger, C., 2015. Working over time: dynamic inconsistency in real effort tasks. QJE.
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- Benzion, U., Rapaport, A., Yagil, J., 1989. Discount rates inferred from decisions: an experimental study. Management Science.

Conclusion

- Investigate a neoclassical growth model with quasi-hyperbolic discounting.
- Obtain the competitive equilibrium allocation and the social planner's allocation explicitly .
- Find that the results of KKS do hold if the time inconsistency in labor supply is differs from one in consumption.

Recent experimental results

- Abdellaoui et al. (2010)
  - Find that the discounting on monetary rewards is **constant**.
  - Their experiments focus on money, not consumption, but they argue that the distinction is not relevant for the experiments.
- Augenbrick et al. (2015) estimate

$$\begin{aligned} \text{Money} &: \max[c_t^\alpha + \beta_m \delta^k c_{t+k}^\alpha] \text{ s.t. } c_t + Rc_{t+k} = \bar{I}, \\ \text{Effort} &: \min[e_t^\gamma + \beta_e \delta^k e_{t+k}^\gamma] \text{ s.t. } e_t + Re_{t+k} = \bar{E}. \end{aligned}$$

  and find that $\beta_m \cong 1$, $\beta_e \cong 0.90$. Choices over effort is more present biased than the one over monetary rewards.
- My guess: Time inconsistency on consumption may be different from the one on effort.

Motivation

- We construct Ramsey model with quasi-hyperbolic discounting and endogenous labor supply.
- Assume that the degree of time-inconsistency over work may be different from the one over consumption.
- Check whether the result of KKS continue to hold.
- Study two period model and the infinite horizons model.

Time consistent tax policy

Theorem

If $\beta_l < 1$ and β_c is close to one, the government policy is welfare-improving and the optimal capital tax rate is negative. If $\beta_c = \beta_l < 1$, then the policy reduces welfare.

- A time-consistent government policy $\tau = (\tau^k, \tau^l, \tau^c)$ which consists of capital income tax, labor income tax and consumption tax exists and is given by $(\tilde{\alpha} = 1 - \alpha)$

$$\tau^k = \frac{s^e - s^*}{s^e}, \tau^l = 1 - \frac{(1-\tau^k)\alpha - s^*}{1 - (1+\theta)l^*}\frac{l^*\theta}{\tilde{\alpha}}, \tau^c = -\frac{\tau^k\alpha + \tau^l\tilde{\alpha}}{1 - s^*}. \quad (30)$$

- Equilibrium allocation under the policy τ coincides with the PE.

Welfare comparison

Theorem

If $\beta_l < 1$ and β_c is close to one, the PE performs better than the CE. If $\beta_c = \beta_l < 1$, the PE performs worse than the CE.

- Let $\hat{s}$ and $\hat{l}$ denote the level of constant savings rate and labor supply that maximizes U_0
- If $\beta_l < 1$ and β_c is sufficiently close to one,

$$s^e < s^* < \hat{s} \text{ and } l^e < l^* < \hat{l}. \tag{28}$$

- If $\beta_l = \beta_c < 1$,

$$s^* < s^e < \hat{s} \text{ and } l^* < l^e < \hat{l}. \tag{29}$$

Good-specific discounting

- Experimental result on good-specific discounting.
  1. Ubfal (2016): Higher discount rates for sugar or meat than for money.
  2. Attema et al. (2018): Higher discount rates for health than for money.
- Model with good-specific discounting
  1. Hori and Futagami (2018): Assume that discounting on consumption differs from the one on labor supply. Saving subsidy improves welfare.
  2. Ohdoi et al. (2015): Ramsey model based on Hori and Futagami (2018). Find that PE is better than the CE.
  3. Cheng and Chu (2018): Two period model in which the discounting on health differs from the one on sin good, and is also different from planner's discount rate. Study optimal saving and consumption tax.
- Our model focus on the difference in the time inconsistency.

Two-period example

- There is a continuum of individuals with a unit measure.
- The utility functions are $(u' > 0, u'' < 0, g' > 0, g'' > 0.)$

$$\begin{aligned} \text{date } 0 &: U_0(c_0, l_0, c_1, l_1) = u(c_0) - g(l_0) + u(c_1) - \beta g(l_1), \quad (4) \\ \text{date } 1 &: U_1(c_1, l_1) = u(c_1) - g(l_1). \quad (5) \end{aligned}$$

where c is consumption, l is labor, and $\beta < 1$ is the discount rate on labor. The discount rate on consumption is equal to one.

- The production function $F(k, l)$ (k : capital, l : labor) has constant returns to scale. Capital is fully depreciated.
- The resource constraint (RC) is

$$\begin{aligned} \text{RC0} &: k_1 = F(k_0, l_0) - c_0, \quad (6) \\ \text{RC1} &: c_1 = F(k_1, l_1). \quad (7) \end{aligned}$$

A solution to the planner's problem

- Planning economy (PE): $g^*(k) = s^* A k^\alpha (l^*)^{1-\alpha}$ and $l^*(k) = l^*$

$$s^* = \frac{\alpha\delta\beta_c}{1 - \alpha\delta(1-\beta_c)}, l^* = \frac{1-\alpha}{\frac{\theta(1-\alpha\delta)}{1-\alpha\delta(1-\beta_c)} + 1 - \alpha}. \quad (26)$$

- Competitive economy (CE):
  1. $G(\bar{k}) = s^e A \bar{k}^\alpha (\bar{l}^e)^{1-\alpha}$ and $L(\bar{k}) = \bar{l}^e$;
  2. $g(k, \bar{k}) = \alpha^{-1} s^e \bar{r} k$ and $l(k, \bar{k}) = \frac{1}{1+\theta}\{1 - \theta \frac{(\alpha - s^e)\bar{r}k}{\alpha \bar{w}}\}$.
  3. $\bar{r} = \alpha s^e A \bar{k}^{\alpha-1} (\bar{l}^e)^{1-\alpha}$, and $\bar{w} = (1-\alpha) s^e A \bar{k}^\alpha (\bar{l}^e)^{-\alpha}$,

$$s^e = \frac{\alpha\delta\frac{\beta_c + \theta\beta_l}{1+\theta}}{1 - (1 - \frac{\beta_c + \theta\beta_l}{1+\theta})\delta}, \bar{l}^e = \frac{1-\alpha}{\theta(1-s^e) + 1 - \alpha} \quad (27)$$

Competitive economy

- In the CE, the consumer chooses l and k', given
  1. the process of the aggregate capital and labor, $\bar{k}' = G(\bar{k})$ and $\bar{l} = L(\bar{k})$
  2. the factor prices $\bar{r} = r(\bar{k})$ and $\bar{w} = w(\bar{k})$,
  3. his future decision on capital $k' = g(k, \bar{k})$ and on labor $l = l(k, \bar{k})$
- The rules g and l solve

$$\begin{aligned} V_0^e(k, \bar{k}) &= \max_{c,l} [\ln c + \theta \ln(1-l) + \delta V^e(k', \bar{k}')], \quad (23) \\ \text{s.t. } k' &= \bar{r}k + \bar{w}l - c. \quad (24) \end{aligned}$$

- The function V^e satisfies $(c = \bar{r}k + \bar{w}l(k,\bar{k}) - g(k,\bar{k}))$

$$V^e(k,\bar{k}) = \beta_c \ln c + \beta_l \theta \ln[1 - l(k,\bar{k})] + \delta V^e(g(k,\bar{k}), G(\bar{k})). \quad (25)$$

Competitive economy at date 1

- The budget constraint in the CE is

$$k' = rk + wl - c. \quad (8)$$

- The factor market is competitive: $r = F_k(k, l)$ and $w = F_l(k, l)$.
- At date 1, given the initial state k_1, the individual solves

$$\begin{aligned} V_1(k_1) &= \max_{c_1, l_1} [u(c_1) - g(l_1)], \\ \text{s.t. } c_1 &= r_1 k_1 + w_1 l_1. \quad (9) \end{aligned}$$

- The FOCs are

$$w_1 u'(c_1) = g'(l_1). \quad (10)$$

- The decision rules are given by $c_1(k_1)$ and $l_1(k_1)$.

Competitive economy at date 0

- At date 0, the individual takes his initial state $k_0 = k$ and the future rules $(c_1(k), l_1(k))$ as given, and maximizes his utility U_0:

$$
\begin{aligned}
V_0(k_0) &= \max_{c_0, l_0} \left[u(c_0) - g(l_0) + u(c_1(k_1)) - \beta g(l_1(k_1)) \right], && (11) \\
\text{s.t. } k_1 &= r_0 k_0 + w_0 l_0 - c_0. && (12)
\end{aligned}
$$

- CE satisfies RC0, RC1, and the FOC at date 1:

$$
w_1 u'(c_1) = F_l(k_1, l_1) u'(c_1) = g'(l_1). \qquad (13)
$$

- Eq (13) binds because unconditional optimization implies $w_1 u'(c_1) = \beta g'(l_1) < g'(l_1)$.

Planner's problem

- The planner chooses his current labor supply l and future state k', given his future decisions on capital $g^*(k)$ and on labor $l^*(k)$:

$$
V_0^*(k) = \max_{c,l,k'} [\ln c + \theta \ln(1 - l) + \delta V^*(k')], \text{ s.t. RC.} \qquad (21)
$$

- The value function V^* satisfies the functional equation

$$
V^*(k) = \beta_c \ln[c^*(k)] + \beta_l \theta \ln(1 - l^*(k)) + \delta V^*(g^*(k)), \qquad (22)
$$

where $c^*(k) = F(k, l^*(k)) - g^*(k)$.

- The solution to the planner's problem (g^*, l^*, V^*)
  1. given V^*, the rules g^* and l^* solves Eq. (21), and
  2. given g^* and l^*, V^* satisfies Eq. (22).

Infinite horizon model

- There is a continuum of agent with unit measure who lives forever.
- Preference :

$$U_0 = \ln c_0 + \theta \ln(1 - l_0) + \sum_{t=1}^{\infty} \delta^t \{\beta_c \ln c_t + \beta_l \theta \ln(1 - l_t)\}, \quad (19)$$

where $\theta > 0$, δ is the discount factor, β_c (β_l) is the time inconsistency parameter on the consumption (labor supply).

- The resource constraint is

$$\text{RC} : k' = F(k, l) - c. \quad (20)$$

- Cobb-Douglas production function: $F(k, l) = Ak^{\alpha} l^{1-\alpha}$.

The planning economy

- At date 1, given the state k_1, the planner solves

$$V_1^*(k_1) = \max_{c_1, l_1} [u(c_1) - g(l_1)], \text{s.t. RC1.} \quad (14)$$

- The planner's rule $(c_1^*(k_1), l_1^*(k_1))$ satisfy RC1 and

$$F_l(k_1, l_1) u'(c_1) = g'(l_1). \quad (15)$$

- At date 0, the planner solves

$$\begin{aligned} V_0^*(k) &= \max_{c_0, l_0, k_1} [u(c_0) - g(l_0) + u(c_1^*(k_1)) - \beta g(l_1^*(k_1))] \\ &= \max_{c_0, l_0, k_1} U_0 \text{ s.t. RC0, RC1 and } (15). \end{aligned}$$

Welfare comparison

Theorem

The PE performs (weakly) better than the CE in terms of welfare.

Proof.

Both the PE and the CE satisfy

$$
\begin{aligned}
\text{RC0} &: \ k_1 = F(k_0, l_0) - c_0, && (16)\\
\text{RC1} &: \ c_1 = F(k_1, l_1), && (17)\\
\text{FOC1} &: \ F_L(k_1, l_1)u'(c_1) = g'(l_1). && (18)
\end{aligned}
$$

Although the planner is maximizing the date-0 utility subject to the three constraints, the individual in the CE is not. □

Log utility function

- Case with $u(c) = \ln c$, $g(l) = -\theta \ln(1-l)$ and $F(k,l) = Ak^{\alpha}l^{1-\alpha}$
  - PE is **strictly** better than CE.
  - At date 0, the savings rate is lower in CE than in PE.
- The constraint: $F_L(k_1, l_1)u'(c_1) \geq g'(l_1)$.
  - The planner knows that if he accumulates capital k_1, the marginal product of labor (MPL) increases, and the constraint is relaxed.
  - For the consumer, MPL is equal to the fixed wage rate. The equilibrium capital level is insufficiently low.
- **Time consistent** government policy
  - At date 1, the government does not have an incentive to use tax.
  - At date 0, the government has an incentive to use capital subsidy.