Estimation of a Life-Cycle Model with Human Capital, Labor Supply and Retirement[*]

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Abstract

We develop and estimate a life-cycle model in which individuals make decisions about consumption, human capital investment, and labor supply. Retirement arises endogenously as part of the labor supply decision. The model allows for both an endogenous wage process through human capital investment (which is typically assumed exogenous in the retirement literature) and an endogenous retirement decision (which is typically assumed exogenous in the human capital literature). We estimate the model using Indirect Inference to match the life-cycle profiles of wages and hours from the SIPP data. The model replicates the main features of the data—in particular the large increase in wages and small increase in labor supply at the beginning of the life-cycle as well as the small decrease in wages but large decrease in labor supply at the end of the life cycle. We also estimate versions of the model in which human capital is completely exogenous and in which human capital is exogenous conditional on work (learning-by-doing). The endogenous human capital model fits the data the best; the learning-by-doing model is able to fit the overall life-cycle pattern; the exogenous model does not. We find that endogenous labor supply is essential for understanding life-cycle human capital investment and life-cycle human capital investment is essential for understanding life-cycle labor supply.

KEYWORDS: human capital, Ben-Porath, labor supply, retirement

JEL Classification: J22, J24, J26

1   Introduction

The Ben-Porath (1967) model of life-cycle human capital production and the life-cycle labor supply model are two of the most important models in labor economics. The former is the dominant framework used to rationalize wage growth over the life-cycle; the latter has been used to study hours worked over the life-cycle, including retirement. Quite surprisingly, aside from the seminal work in Heckman (1976, 1975), there has been little effort integrating these two important paradigms. This paper attempts to fill this void by estimating a life-cycle model in which workers choose human capital and labor supply jointly. Perhaps the most important aspect of our model is that we do not treat retirement as a separate decision. It occurs endogenously as part of the optimal life-cycle labor supply decision.

The retirement literature typically takes the wage process as given and estimates the incidence of retirement. Cross-section raw wages for people who work fall substantially before retirement. They decline by over 25% between ages 55 and 65. In much of the retirement literature, this trend is critical to understanding retirement behavior. By contrast, life-cycle human capital models take the retirement date as given, but model the formation of the wage process. While most work to date on the life-cycle human capital model aims to explain wage growth early in the life-cycle, there has been little work studying the interaction between human capital and labor supply at the end of the working life. We estimate a model wherein the wage, labor supply and retirement choices are rationalized in one unified setting. After endogenizing both labor supply and human capital, this model is rich enough to explain the life-cycle patterns of both wages and labor supply, with a focus on wage patterns and retirement at the end of working life.

Specifically, we develop and estimate a Ben-Porath type human capital model in which workers make consumption, human capital investment, and labor supply decisions. We estimate the model using Indirect Inference, matching the wage and hours profiles of male high school graduates from the Survey of Income and Program Participation (SIPP). With a parsimonious life-cycle model in which none of the parameters explicitly depend upon age or experience, we are able to replicate the main features of the data. In particular we match the large increase in wages and very small increase in labor supply at the beginning of the life-cycle as well as the small decrease in wages but very large decrease in labor supply at the end of the life-cycle.

The key to our ability to fit both ends of the life-cycle is human capital depreciation. In a simple model without human capital depreciation, there is no a priori reason for workers to concentrate their leisure towards the end of the life-cycle. However, this is no

longer the case with human capital depreciation which imposes a shadow cost on leisure. When workers take time off in the middle of their career, their human capital depreciates and they earn less when they return to the labor market. On the other hand, if this period of nonworking occurs at the end of the career, the shadow cost is much less a concern because the horizon is shorter. Older workers may choose not to re-enter at a lower wage so they continue to stay out of the labor market. We show that when we restrict our framework to exogenous human capital accumulation across the life-cycle, the model does not fit both the end and beginning of the life-cycle. When tastes for leisure do not vary across the life-cycle, the exogenous model cannot simultaneously reconcile the small increase in labor supply and large increase in wages at the beginning of the life-cycle and the small decrease in wages and large decrease in labor supply at the end. By contrast, the learning-by-doing model includes depreciation in much the same way and is able to reconcile the main features of the data. Of course if one exogenously allowed both wages and labor supply to depend upon age in a completely flexible way one could easily fit the joint pattern with an exogenous model. But, it is not clear that this model would have any testable implications. The goal of this paper is to try to fit the profiles without resorting to arbitrary age varying taste preferences and exogenous wage variation.

An interesting aspect of our model is that even though the preference for leisure does not vary systematically over the life-cycle, we do find that measured "labor supply elasticities" do vary over the life-cycle. In our dynamic model, the shadow cost of not working is much higher early in the life-cycle (as pointed out by e.g. Imai and Keane, 2004) and it is lower for older workers as opposed to peak earners. We find that early in the life-cycle the measured labor supply elasticity is low, around 0.2. However, workers around the standard retirement age are more sensitive to wage fluctuations with elasticities between 0.6 and 1.0.

While our baseline model does not incorporate health, we estimate a specification that allows the taste for leisure to depend on health and for this effect to increase with age. Surprisingly, such an "enhanced" model does not significantly improve the fit of the life-cycle patterns of wage and labor supply of the SIPP data. We also show that even within this model that allows a direct and flexible effect of health on labor supply, health plays a relatively minor role in the decline in labor supply late in life.

We use the estimated model to simulate the impacts of various Social Security policy changes. Much serious work has been developed to quantitatively estimate the economic consequences of an aging population and evaluate the remedy policies (Gustman and Steinmeier, 1986; Rust and Phelan, 1997; French, 2005; French and Jones, 2011; Haan and Prowse, 2014). They model retirement as a result of combinations of declining wages,

increasing actuarial unfairness of the Social Security and pension system, and increasing tastes for leisure. However, there is a major difference between our model and the previous retirement literature. Prior work typically takes the wage process as given and focuses on the retirement decision itself. For example, when conducting the counterfactual experiment of reducing the Social Security benefit by 20%, the previous literature takes the same age-wage profile as in the baseline model and re-estimates the retirement behavior under the new environment. As the wage has already been declining significantly and exogenously approaching the retirement age, under the new policy working is still less likely attractive for many workers. However, as we show in our model, less generous Social Security benefits result in higher labor supply later in the life-cycle, so workers adjust their investment over the life-cycle, which results in a higher human capital level as well as higher labor supply earlier. On average the observed wage levels are 5% higher between 65 and 80. Over the whole life-cycle, observed average yearly wages, total labor income, and total labor force participation rates increase by 1.5%, 2.17%, and 1.57%, respectively. By contrast, in the model with exogenous human capital, the percentage increases in yearly wages, total labor income and total labor supply are less significant, by 0.2%, 1.26%, and 1.31%, respectively. The differences are more dramatic in the experiments in which we remove the Social Security system, with the exogenous model underestimating most effects.

2   Relevant Literature

Human capital models have been widely accepted as a mechanism to explain life-cycle wage growth as well as the labor supply and income patterns. In his seminal paper, Ben-Porath (1967) develops the human capital model with the idea that individuals invest in their human capital "up front." In what follows we often use the term—"human capital model" to mean "Ben-Porath model." Heckman (1975, 1976) further extends the model and present more general human capital models in which each individual makes decisions on labor supply, investment and consumption. In both papers, each individual lives for finite periods and the retirement age is fixed. In their recent paper, Manuelli et al. (2012) calibrate a Ben-Porath model to include the endogenous retirement decision. All three models are deterministic.

Relative to the success in theory, there hasn't been as much work empirically estimating the Ben-Porath model. Mincer (1958) derives an approximation of the Ben-Porath model and greatly simplifies the estimation with a quadratic in experience, which is used in numerous empirical papers estimating the wage process (Heckman et al., 2006, survey

the literature). Early work on explicit estimation of the Ben-Porath model was done by Heckman (1975, 1976), Haley (1976), and Rosen (1976). Heckman et al. (1998a) is a more recent attempt to estimate the Ben-Porath model. They utilize the implication of the standard Ben-Porath model where at old ages the investment is almost zero. However, this implication does not hold any more when the retirement is uncertain, where each individual always has an incentive to invest a positive amount in human capital. Browning et al. (1999) survey much of this literature.[1]

Another type of human capital model, the learning-by-doing model, draws relatively more attention in empirical work. In the learning-by-doing model human capital accumulates exogenously, but only when an individual works. Thus workers can only impact their human capital accumulation through the work decision. In these models, the total cost of leisure is not only the direct lost earnings at the current time, but also includes the additional lost future earnings from the lower level of human capital. Shaw (1989) is among the first to empirically estimate the learning-by-doing model, using the PSID model and utilizing the Euler equations on consumption and labor supply with translog utility. Keane and Wolpin (1997) and Imai and Keane (2004) are two classic examples of research that directly estimate a dynamic life-cycle model with learning-by-doing. Blundell et al. (2015) is a more recent example. These papers assume an exogenously fixed retirement age. Wallenius (2009) points out that such a learning-by-doing model does not fit the pattern of wages and hours well at old ages.[2] Heckman et al. (2003) study the potential effects of wage subsidies on skill formulation by comparing on-the-job training models with learning-by-doing models. They simulate the effects of the 1994 EITC schedule for families with two children and find evidence that EITC lowers the long-term wages of people with low levels of education. They find that the learning-by-doing model predictions of the EITC policy effects fit the actual changes better than the Ben-Porath style model.

There is a large and growing literature on many aspects of retirement. In these models, typically retirement is induced either by increasing utility toward leisure (e.g. Gustman and Steinmeier, 1986) or increasing disutility toward labor supply (e.g. Blau, 2008). Haan and Prowse (2014) estimate the extent to which the increase in life expectancy affects retirement. Blau (2008) evaluates the role of uncertain retirement ages in the retirement-consumption puzzle.

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[1]Other more recent work includes Taber (2002), who incorporates progressive income taxes into the estimation, and Kuruscu (2006), who estimates the model nonparametrically.

[2]However, if one interprets the hourly wages as labor income and hours as labor force participation rates (since there is no participation decision in their model), the fit in Imai and Keane (2004) would be improved at older ages.

Retirement can also be induced by declining wages at old ages and/or fixed costs of working. Rust and Phelan (1997) estimate a dynamic life-cycle labor supply model with endogenous retirement decisions to study the effect of Social Security and Medicare in retirement behavior. French (2005) estimates a more comprehensive model including savings to study the effect of Social Security and pension as well as health in retirement decisions. French and Jones (2011) evaluate the role of health insurance in shaping retirement behavior. Casanova (2010) studies the joint retirement decision among married couples. Prescott et al. (2009) and Rogerson and Wallenius (2010) present models where retirement could be induced by a convex effective labor function or fixed costs.

In all the retirement literature listed above—theoretical or empirical—the wage process is assumed to be exogenous. That is, even when the environment changes while conducting counterfactual experiments, for example changing the Social Security policies, the wage process is kept the same and only the response in the retirement decision is studied.

3 Model

We present and estimate a Ben-Porath style human capital model with endogenous labor supply and retirement in which individuals choose consumption, human capital investment, and labor supply (including retirement as a special case). For simplicity we suppress the individual subscript i for all variables. We allow for heterogeneity in some of the parameters when estimating the model. We delay discussion of this to Section 4.1 for expositional convenience.

3.1 Environment

Demographics

Time is discrete and measured in years. Each individual lives from period $t = 0$ to $t = T$. At the beginning of the initial period, each individual is endowed with an initial asset $A_0 \in \mathbb{R}$ and an initial human capital level $H_0 \in \mathbb{R}^+$. Family status is an exogenous discrete state variable, including marital status and spouse's working status if married. A single or divorced individual is denoted by $M_t = 0$, while a married individual is indicated by either $M_t = 1$ (spouse not working) or $M_t = 2$ (spouse working). The family status evolves following an age-dependent Markov transition matrix.

Preference

In the baseline model we model the extensive margin of labor supply, so at each period the individual decides either to work or not. The flow utility at period t is

$$u_t \left(c_t, \ell_t, \gamma_t \right) = \psi_t \frac{c_t^{1-\eta_c}}{1 - \eta_c} + \gamma_t \ell_t \tag{1}$$

where c_t is family consumption and $\ell_t \in \{0, 1\}$ is leisure. The coefficient ψ_t shifts the marginal utility of consumption (e.g., Gourinchas and Parker, 2002) and is assumed a parametric form,

$$\psi_t = \exp \left(\varphi_1 t + \varphi_2 t^2 + \varphi_3 t^3 + \varphi_4 \mathbf{1} \{ M_t \neq 0 \} \right)$$

Note the shifter may differ across the single and the married couple. The coefficient γ_t represents taste for leisure and also depends on the family status. We allow for shocks in γ_t which is assumed to be an i.i.d. random variable for each individual and is specified in the next subsection.[3]

Human Capital

If an individual chooses to work, $\ell_t = 0$, he decides on how much time, I_t, to invest in human capital and spends the rest, $1 - I_t$, at effective (or productive) work from which the wage income is earned. Human capital is produced according to the production function

$$H_{t+1} = (1 - \delta) H_t + \xi_t \pi I_t^{\alpha_I} H_t^{\alpha_H} \tag{2}$$

where H_t is the human capital level at period t. The ξ_t is an idiosyncratic shock to the human capital innovation. If an individual chooses not to work, he does not invest in human capital (so $I_t = 0$) and human capital depreciates at rate δ.

The labor market is perfectly competitive. We normalize the rent of human capital to one so that the wage for the effective labor supply equals the human capital H_t. Thus pre-tax labor income at any point in time is

$$w_t = H_t \left(1 - \ell_t \right) \left(1 - I_t \right).$$

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[3]A key part of our exercise is that we do not explicitly allow γ_t to vary systematically across age. We describe the exact process in the next subsection. The two terms—"period" and "age"—are used interchangeably throughout the paper.

Social Security and Budget Constraint

While we have tried to keep the basic model as simple as possible, the social security system in the U.S. is such a crucial part of the retirement decision that we incorporate it into the model. We model the social security enrollment decision as a one time decision. Once a person turns 62 they can start claiming social security and once they have started claiming, they continue to collect benefits until their death. We will let ssa_t denote a binary decision variable indicating whether a person starts claiming at period t and let ss_t be a state variable indicating whether a person began claiming prior to period t. Since claiming is irreversible, once $ss_t = 1$ then ssa_t is no longer a relevant choice variable. Thus the law of motion can be written as

$$ss_0 = 0$$
$$ss_t = \max\{ss_{t-1}, ssa_{t-1}\}. \tag{3}$$

The claiming decision (ssa_t) is made separately from the labor force participation decision (ℓ_t) so that one can receive the social security benefit while working (subject to applicable rules such as the earnings test).

Once they have begun claiming, an individual collects benefits ssb_t which are a function of the claiming age and the Average Indexed Monthly Earnings ($AIME_t$). In practice we approximate the AIME and use the social security rules as of 2004. Details are in the Appendix. This is incorporated into the budget constraint

$$A_{t+1} = (1+r)A_t + Y_t\left(w_t, Y_t^s\left(M_t\right), ssb_t\right) - c_t + \tau_t, \tag{4}$$

where A_t stands for asset and r is the risk free interest rate. $Y_t\left(\cdot, \cdot\right)$ is the after-tax income which is a function of wage income, spousal income (if available), the social security benefit ssb_t (if available), and the tax code. $Y_t^s\left(M_t\right)$ is the spousal income,

$$Y_t^s\left(M_t\right) = y_t^s \cdot \mathbf{1}\left\{M_t = 2\right\}, \ \log\left(y_t^s\right) \sim \mathcal{N}\left(\bar{y}_t, \sigma_{y_t}^2\right) \tag{5}$$

where y_t^s is an age-dependent log-normal random variable. Government transfers, τ_t, provide a consumption floor $\underline{c}$ as in Hubbard et al. (1995) so

$$\tau_t = \max\left\{0, \ \underline{c} - \left((1+r)A_t + Y_t - \underline{A}_{t+1}\right)\right\}, \tag{6}$$

where $\underline{A}_{t+1}$ is the asset lower bound at period $t+1$.[4]

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[4]We define the asset lower bound as the amount that each individual can pay back for sure before

Life ends at the end of period T and each individual values the bequest he will leave. It takes the form

$$b(A_{T+1}) = b_1 \frac{(b_2 + A_{T+1})^{1-\eta_c}}{1 - \eta_c} \qquad (7)$$

where b_1 captures the relative weight of the bequest motive and b_2 determines its curvature as in DeNardi (2004).

3.2 Solving the model

Four shocks affect individuals: the evolving marital status, M_t, the spousal income, $Y_t^s(M_t)$, the shock in leisure taste, γ_t, and the human capital innovation shock, $\tilde{\varsigma}_t$. The timing of the model works as follows: at the beginning of each period, M_t is realized, followed by γ_t. He then simultaneously chooses consumption, labor supply, human capital investment, and social security application when relevant. After these decisions are made, the spousal income and the human capital innovation shock are realized, which determine the asset and the human capital level in the following period, respectively. All M_t, $Y_t^s(M_t)$, γ_t and $\tilde{\varsigma}_t$ are i.i.d. shocks from the perspective of the agents—so agents have no private information about their value prior to their realizations.[5]

The recursive value function can be written as

$$V_t(X_t, \gamma_t) = \max_{c_t, \ell_t, I_t, ssa_t} \{ u_t(c_t, \ell_t, \gamma_t) + \beta E\left[V_{t+1}(X_{t+1}, \gamma_{t+1}) | X_t, c_t, \ell_t, I_t, ssa_t \right] \} \qquad (8)$$

where $X_t = \{M_t, A_t, H_t, AIME_t, ss_t\}$ is the vector of state variables. The expectation is over the leisure shock in γ_{t+1} and the human capital innovation $\tilde{\varsigma}_t$.

The solution to the agent's problem each period is done in two stages. We first solve for the optimal choices conditional on the labor supply decision and then we determine the labor supply decision.

The optimal consumption $\mathcal{C}_{t,0}(X_t)$, investment $\mathcal{I}_{t,0}(X_t)$, and social security claiming $\mathcal{SSA}_{t,0}(X_t)$ decisions conditional on participating in the labor market ($\ell_t = 0$) depend only on X_t and can be obtained from

$$\{\mathcal{C}_{t,0}(X_t), \mathcal{I}_{t,0}(X_t), \mathcal{SSA}_{t,0}(X_t)\} \equiv \underset{c_t, I_t, ssa_t}{\operatorname{argmax}} \left\{ \frac{c_t^{1-\eta_c}}{1-\eta_c} + \beta E\left[V_{t+1}(X_{t+1}, \gamma_{t+1}) | X_t, c_t, \ell_t = 0, I_t, ssa_t \right] \right\} \qquad (9)$$

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death, as in Aiyagari (1994). Since the probability of not working at each period is positive, the lower bound is characterized by the nonnegative consumption and the bequest function specified below, which is $\underline{A}_t = -b_2 / (1+r)^{T-t+1}$.

[5]We assume the stochastic spousal income, when available, is realized after all the decisions (consumption, labor supply, ...) are made, to reduce the state space size and save computation time. Assuming its realization before the decisions will not affect the estimation of the model significantly.

and the conditional value function is

$$\widetilde{V}_{t,0}\left(X_t\right) \equiv \frac{\left(\mathcal{C}_{t,0}\left(X_t\right)\right)^{1-\eta_c}}{1-\eta_c} + \beta E\left[V_{t+1}\left(X_{t+1},\gamma_{t+1}\right)|\,X_t,\mathcal{C}_{t,0}\left(X_t\right),\ell_t=0,\mathcal{I}_{t,0}\left(X_t\right),\mathcal{SSA}_{t,0}(X_t)\right] \quad (10)$$

Similarly, conditional on not working ($\ell_t = 1$), we can calculate the optimal consumption and claiming decision from

$$\{\mathcal{C}_{t,1}\left(X_t\right),\mathcal{SSA}_{t,1}(X_t)\} \equiv \underset{c_t,ssa_t}{\mathrm{argmax}}\left\{\frac{c_t^{1-\eta_c}}{1-\eta_c} + \beta E\left[V_{t+1}\left(X_{t+1},\gamma_{t+1}\right)|\,X_t,c_t,\ell_t=1,I_t=0,ssa_t\right]\right\} \quad (11)$$

and define the conditional value function apart from γ_t to be

$$\widetilde{V}_{t,1}\left(X_t\right) \equiv \frac{\left(\mathcal{C}_{t,1}\left(X_t\right)\right)^{1-\eta_c}}{1-\eta_c} + \beta E\left[V_{t+1}\left(X_{t+1},\gamma_{t+1}\right)|\,X_t,\mathcal{C}_{t,1}\left(X_t\right),\ell_t=1,I_t=0,\mathcal{SSA}_{t,1}(X_t)\right]. \quad (12)$$

We use the parametric form for γ_t,

$$\gamma_t = exp\left(\tilde{a}_0 + a_\varepsilon\varepsilon_t\right) \quad (13)$$

where $\tilde{a}_0 = a_0 + a_1\mathbf{1}\left\{M_t = 1\right\} + a_2\mathbf{1}\left\{M_t = 2\right\}$ and ε_t follows an independent and identically-distributed standard normal distribution. Therefore γ_t follows a log-normal distribution, $\ln\gamma_t \sim \mathcal{N}\left(\tilde{a}_0, a_\varepsilon^2\right)$. Notice that since there is no serial correlation in the stochastic shocks of leisure, ε_t, the conditional policy and value functions defined in equations (9)-(12) do not depend on γ_t.

The optimal labor supply solution is

$$\ell_t = \underset{\ell_t\in\{0,1\}}{\mathrm{arg\ max}}\ \widetilde{V}_{t,\ell_t}\left(X_t\right) + \gamma_t\ell_t \quad (14)$$

Define

$$\varepsilon_t^* \equiv \frac{1}{a_\varepsilon}\left\{\log\left(\gamma_t^*\left(X_t\right)\right) - \tilde{a}_0\right\}. \quad (15)$$

where

$$\gamma_t^* = \widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right)$$

and we have the following proposition.

PROPOSITION 1: The optimal labor supply decision is

$$\ell_t = \begin{cases} 1, & \text{if } \varepsilon_t \geq \varepsilon_t^* \\ 0, & \text{if } \varepsilon_t < \varepsilon_t^* \end{cases}. \quad (16)$$

and the expected value function is

$$E\left[V_t\left(X_t, \gamma_t\right) | X_t\right] = \Phi\left(\varepsilon_t^*\right) \widetilde{V}_{t,0}\left(X_t\right) + \left(1 - \Phi\left(\varepsilon_t^*\right)\right)\left[\widetilde{V}_{t,1}\left(X_t\right) + E\left(\gamma_t | \varepsilon_t \geq \varepsilon_t^*\right)\right]$$

where

$$E\left(\gamma_t | \varepsilon_t \geq \varepsilon_t^*\right) = exp\left(\tilde{a}_0 + \frac{a_\varepsilon^2}{2}\right)\frac{\Phi\left(a_\varepsilon - \varepsilon_t^*\right)}{\Phi\left(-\varepsilon_t^*\right)}$$

PROOF: Appendix A.

Finally note that X_{t+1} is a known function of $X_t, c_t, \ell_t, I_t, ssa_t, \xi_t, Y_t^s$, and M_{t+1}, so to solve for

$$E\left[V_{t+1}\left(X_{t+1}, \gamma_{t+1}\right) | X_t, c_t, \ell_t, I_t, ssa_t\right] = E\left[E\left(V_{t+1}\left(X_{t+1}, \gamma_{t+1}\right) | X_{t+1}\right) | X_t, c_t, \ell_t, I_t, ssa_t\right]$$

we just need to integrate over the distributions of Y_t^s, M_{t+1}, and ξ_t. We assume ξ_t is i.i.d and follows a log-normal distribution,

$$\log\left(\xi_t\right) \sim \mathcal{N}\left(-\frac{\log\left(\sigma_\xi^2 + 1\right)}{2}, \log\left(\sigma_\xi^2 + 1\right)\right) \tag{17}$$

so that ξ_t has mean of one and variance of σ_ξ^2.

4   Estimation

The estimation of the model is carried out using a two-step strategy. First, we pre-set parameters that either can be cleanly identified without explicitly using our model or are not the focus of this paper. In the second step, we estimate the remaining preference and production parameters of the model using Indirect Inference. The model is described by equations (1)-(8) and we summarize the parameters here. The parameters related to preferences are the discount rate, β, the intertemporal elasticity of consumption, η_c, the consumption shifter, φ_{1-4}, the taste for leisure, a_{0-2}, a_ε, and the bequest parameters, b_1 and b_2. Human capital production is determined by δ, π, α_I, α_H and σ_ξ. Parameters related to the budget constraint are the interest rate r and the consumption floor $\underline{c}$. Finally there are initial values for the state variables, assets, A_0, human capital, H_0, and Averaged Indexed Monthly Earnings, $AIME_0$.

Table 1: Normalized or pre-set parameters

Parameters		Normalized/Pre-set Values
Interest rate	r	0.03
Discount	β	0.97
Initial wealth[a]	A_0	0.0
Initial AIME[a]	$AIME_0$	0.0
Consumption floor[b]	$\underline{c}$	2.19
Bequest shifter[c]	b_2	222.0

[a]The initial age is 18.
[b]The consumption floor is equivalent to $4380 in 2004$, since we normalize the total time endowment for labor supply at one period—which is 2000 hours—as one.
[c]The bequest shifter is equivalent to $444,000.

4.0.1 Pre-set Parameters

The set of parameters pre-set in the first stage includes the interest rate, initial wealth and initial AIME, the time discount rate, consumption floor, and bequest shifter. In Section 8.2 we look at the sensitivity of some of our results to these values.

One period is defined as one year.[6] The initial period in our model corresponds to age 18 and ends at age 80.[7] The early retirement age is 62 and the normal retirement age is 65. The risk free real interest rate is set as $r = 0.03$ and the time discount rate is set as $\beta = 0.97$. The consumption floor is set as $\underline{c} = 2.19$, as estimated in French and Jones (2011).[8]

The parameter which determines the curvature of the bequest function is set as $b_2 = 222$, as in French and Jones (2011).[9] We assume all individuals start off their adult life with no wealth and zero level of AIME at age 18. These normalized or pre-set parameters are summarized in Table 1.

4.1 Heterogeneity

This leaves the following parameters: η_c, a_{0-2}, a_ε, b_1, δ, π, α_I, α_H, σ_ξ, and H_0. We allow for heterogeneity in three of these: ability to learn (π), ability to earn (H_0), and tastes for leisure (a_0). For computational reasons we only have nine types determining the joint distribution of (a_0, π). Specifically, we model it as a nine-point Gauss-Hermite

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[6]Mid-year retirement might be an issue. However, more than half of workers are never observed working half-time approaching retirement, so it would not be a big issue.

[7]The life expectancy for white males is 74.1 in 2000 and 76.5 in 2010.

[8]$\underline{c} = 4380/2000 = 2.19$ since we normalize the total time endowment for labor supply at one period as one.

[9]It is equivalent to $444,000 in 2004 U.S. dollar.

approximation of a joint normal distribution, which depends on five parameters: the mean and variance of a_0, the mean and variance of π, and the correlation between the two. Respectively we write this as $(\mu_{a_0}, \sigma_{a_0}, \mu_\pi, \sigma_\pi, \rho)$. We emphasize that since we are only using nine points we are not assuming that the Gauss-Hermite is a good approximation of a normal, but rather view this as the parametrization itself. That is, we assume that the joint distribution of (a_0, π) is a parametric discrete distribution with 9 points determined by the parameter vector $(\mu_{a_0}, \sigma_{a_0}, \mu_\pi, \sigma_\pi, \rho)$.

Since human capital is already a state variable in our model, we can be more flexible in modeling initial human capital. We allow it to be correlated with (a_0, π) through the functional form

$$H_0 = \exp\left(\gamma_0 + \gamma_{a_0} a_0 + \gamma_\pi \pi + \sigma_{H_0} \nu\right) \tag{18}$$

where $\nu \sim \mathcal{N}(0,1)$ is an i.i.d standard normal random variable.

4.2 Estimation Procedure

We apply Indirect Inference to estimate the parameters of interest, Θ,

$$\Theta = \left\{ \underbrace{\eta_c, \varphi_{1-4}}_{c}, \underbrace{\mu_{a_0}, \sigma_{a_0}, a_\varepsilon, a_1, a_2}_{\text{leisure}}, \underbrace{b_1}_{\text{bequest}}, \underbrace{\delta, \alpha_I, \alpha_H, \sigma_\xi, \mu_\pi, \sigma_\pi, \rho}_{\text{human capital production}}, \underbrace{\gamma_0, \gamma_{a_0}, \gamma_\pi, \sigma_{H_0}}_{\text{initial human capital}} \right\}$$

according to the following procedure.

i) Calculate the auxiliary model from the data.

ii) Iterate on the following procedure for different values of Θ until the minimum distance has been found.

   (a) Given a set of parameters, solve value functions and policy functions for the entire state space grid.

   (b) Generate the life-cycle profile for each simulated individual.

   (c) Calculate the auxiliary model from the simulation.

   (d) Calculate the distance between the simulated auxiliary model and the data auxiliary model.

4.3 Data and the Auxiliary Parameters

Our primary data set is the Survey of Income and Program Participation (SIPP). The SIPP is comprised of a number of short panels of respondents and we use all of the panels

starting with the 1984 panel and ending with the 2008 panel. To focus on as homogeneous a group as possible, the sample only includes white male high school graduates.[10]

Our measure of labor force participation is a dummy variable for whether the individual worked during the survey month.[11] Clearly the aggregation is imperfect. We construct the hourly wage as the earnings in the survey month divided by the total number of hours worked in the survey month.

We begin estimation of the model from age 22 rather than 18 for two reasons. First, we have a short panel meaning that many 19 year old high school graduates may return to college after they leave the panel. Second, our model does not include any search or matching behavior, which might be important for the labor force patterns among very recent labor force entrance as they transition from school to work as suggested by literature (Topel and Ward, 1992; Neal, 1999). Our model does over-predict the labor supply for those individuals.

Six sets of moment conditions at each age from 22 to 65 (except the last two) are chosen to assemble the auxiliary model. We use a total of 230,657 panel observations from 80,519 different respondents.

  i) The labor force participation rates (LFPR);
 ii) The first moments of the logarithm of observed wages;
iii) The first moments of the logarithm of observed wages after controlling for individual fixed effects.[12]
 iv) The second moments (standard deviation) of the logarithm of observed wages.
  v) The first moment of consumption from 27 to 65[13]
 vi) The overall transition probabilities between age 35 and 50
      (a) from working to not working
      (b) from not working to working

As is standard in the literature on estimation of Ben-Porath style human capital we as-

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[10]Estimation results for college graduates are presented in Appendix F.

[11]In SIPP an individual is observed in at most three months each year. If an individual is observed working more than 50% of the time then he is categorized as participating in the labor force, otherwise not. If one is sampled twice for the year and is observed working in one month only, the participation status is determined randomly (50% for each possibility).

[12]To construct these moments we first regress log wage on the age dummies and survey year dummies and obtain the predicted log wage, denoted as z. We pick a base age (age 30) and calculate the average predicted log wage at the base age for each year, denoted as $\bar{z}_{a,j}$, where a is the base age and j is for survey year. We then pick a base year y and calculate the difference of $\bar{z}_{a,j}$ between each year j and the base year y, denoted as $\Delta\bar{z}_{a,j}$. Finally we calculate the difference between the original log wage and $\Delta\bar{z}_{a,j}$ and define the result as $\ln \tilde{W}_t$, which is the log wage after filtering out the time fixed effects.

[13]The adult equivalent consumption profile is constructed from the Consumer Expenditure Survey as in Fernández-Villaverde and Krueger (2007).

sume that wages in the data correspond to

$$W_t = H_t \left(1 - I_t\right) \tag{19}$$

in the model. We match both age-wage profiles, with and without controlling for individual fixed effect as the two have quite different patterns.

Figures 1a-1c present these four profiles. Figure 1a plots the labor force participation rates between age 22 and 65. Figure 1b plots two log wage profiles. The first one is the log wage profile from the pooled sample, while the second one is the log wage profile after controlling for individual fixed effects. The original log wage profile has a hump shape, but the one filtering out individual fixed effects does not decline within the examined period which is between age 22 and 65. Figure 1c shows the extent to which the variance of log wages increases with age.

The most interesting result in Figures 1a-1c is the discrepancy between the age-wage profiles with or without controlling for individual fixed effects. This has been documented in various data sets, including the National Longitudinal Survey of Older Men (NLSOM) data (Johnson and Neumark, 1996), the Panel Study of Income Dynamics (PSID) data (Rupert and Zanella, 2012), and the Health and Retirement Survey (HRS) data (Casanova, 2013). These papers find that after controlling for individual fixed effects the age-wage profile is flatter than the hump-shaped age-wage profile estimated using pooling observations, and it does not decline until 60s or late 60s. All of these papers argue that this evidence is not consistent with the traditional human capital model since the traditional human capital model would predict a hump-shaped wage. The intuition is that when the human capital depreciation outweighs the investment, wages start to decline which generates a hump-shaped profile. Fitting the wage profile after controlling for fixed effects makes our problem more challenging because we need to explain the decrease in labor supply later in life when there is little evidence that wages decline.

To further verify this result we compare our SIPP results with the Current Population Survey (CPS) data. From the CPS Merged Outgoing Rotation Groups (MORG) data, we match the same respondent in two consecutive surveys using the method proposed in Madrian and Lefgren (2000), and we have a short panel with each individual interviewed twice, one year apart.[14] We construct a similar short panel from the CPS March Annual Social and Economic Supplement files (March). The difference is that the wage information is collected from the reference week in the CPS MORG data and from the previous year in the CPS March data.

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[14]For MORG data, they are the fourth and eighth interview.

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Figure 2 presents the age-wage profiles with or without controlling for individual fixed effects for male high school graduates from the 1979-2012 CPS MORG data and the 1979-2007 CPS March data. We find a somewhat even larger discrepancy in the age-wage profiles as in the SIPP data presented in Figure 1b.[15]

5 Estimation Results

The estimates of the parameters are listed in Table 2. Of particular importance are the depreciation rate, δ, curvature in the human capital production function, α_I, and a_ε which determines the elasticity of labor supply. Before discussing these parameter values we examine the fit of the model in Figures 3a-3d. The fit of the model in the two overall transition probabilities is presented in the first two rows in Table E.[16]

The first and central point is that our parsimonious model can reconcile the main facts in the data: a small increase in labor supply/large increase in wages at the beginning of the life-cycle along with the large decrease in labor supply/small decrease in wages at the end of the life-cycle.[17]

The simulated labor force participation rate increases slightly between age 22 and 30 as shown in Figure 3a. Our main result is that this simple model is able to generate a massive decline in labor supply between age 55 and 65, which fits the sharp decline of labor force participation rates within that age period in the data and simultaneously the flat wage profile in the fixed effect model.

Our model generates similar discrepancy between the log wages with and without controlling for individual fixed effects, as shown in Figures 3b and 3c, and both profiles fit the data well. Log wages after filtering out individual fixed effects increase at a decreasing pace from age 22 to age 58 and then decreases slightly (Figure 3b). On the other hand, Figure 3c shows that the original log wage profile presents a hump shape which resembles the data profile.

The model also replicates the log wage variation as in the data (Figure 3d). This increasing variation mainly comes from the heterogeneity in the parameters. Without het-

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[15]Time fixed effects are filtered out, as described in footnote 12. We use the same starting year for the CPS MORG data and the CPS March data. Using the 1979-2007 CPS MORG data generates essentially same profiles.

[16]The overidentification test statistic is reported in the bottom of Table 2. The model is rejected at the 0.1% level. The fact that we reject is not surprising given the simplicity of our model and the size of our sample. One could easily add some extra parameters to pass the statistical criterion, but this is not our goal. Our goal is to use a simple model that does a very good job of capturing the life-cycle patterns.

[17]One should keep in mind that our parsimonious specification might be a limitation on our policy counterfactuals as other features that we have not explicitly modeled might impact those simulations.

Table 2: Estimates in the baseline model[a]

Parameters		Estimates	Standard Errors
HC depreciation[b]	δ	0.109	(0.009)
HC production function: I factor	α_I	0.067	(0.024)
HC production function: H factor	α_H	0.123	(0.015
Standard deviation of HC innovation	σ_ξ	0.002	(0.003)
Consumption: CRRA	η_c	4.040	(0.042)
Consumption shifter: coef on t ($\times 10$)	φ_1	0.259	(0.073)
Consumption shifter: coef on t^2 ($\times 10^2$)	φ_2	0.123	(0.020)
Consumption shifter: coef on t^3 ($\times 10^3$)	φ_3	-0.032	(0.003)
Consumption shifter: coef on married	φ_4	0.569	(0.160)
Leisure: Standard Deviation of Shock	a_ε	0.265	(0.018)
Leisure: spouse not working	a_1	0.597	(0.088)
Leisure: spouse working	a_2	-0.566	(0.081)
Bequest weight	b_1	18,069,750	(4,611,752)
Parameter heterogeneity[c]			
Leisure: mean of intercept	μ_{a_0}	-5.582	(0.118)
Leisure: standard deviation of intercept	σ_{a_0}	0.907	(0.045)
HC productivity, mean	μ_π	1.805	(0.110)
HC productivity, standard deviation	σ_π	0.675	(0.048)
Correlation between a_0 and π	ρ	-0.989	(0.088)
Initial human capital level at age 18			
Intercept	γ_0	1.660	(0.086)
Coefficient on a_0	γ_{a_0}	0.064	(0.017)
Coefficient on π	γ_π	0.609	(0.067)
Standard deviation of error term	σ_{H_0}	0.007	(0.015)
χ^2 Statistic = 529[d]		Degrees of freedom = 200	

[a]Indirect Inference estimates. Estimates use a diagonal weighting matrix. Standard errors are given in parentheses.
[b]HC: Human Capital.
[c]The joint distribution of (a_0, π) is a parametric discrete distribution with nine points determined by these five parameters, using a nine-point Gauss-Hermite approximation.
[d]This is the J-statistic. The critical values of the χ^2 distribution are $\chi^2_{(200,0.01)} = 249$, $\chi^2_{(200,0.005)} = 255$, $\chi^2_{(200,0.001)} = 268$.

erogeneity in parameters, the wage variation would decrease with age as human capital would converge due to concavity of the production function. With heterogeneity, the human capital level might diverge, depending on parameter values.

Our model fits the shape and the level of the adult equivalent consumption profile reasonably well, except for the young ages. The model generates the similar overall transition probabilities between working and not working as shown in Table E.

We obtain our fit of the life-cycle profiles of labor supply and log wages despite the lack of any explicit time-dependent preference, production or constraints in our model. Two key features of our model make them possible: the human capital depreciation and the separation between the effective labor and observed labor. We discuss each of these in turn.

We argued above that human capital depreciation is essential for matching the labor force participation profile. This discussion implies that our estimate of a depreciation value $\delta = 0.109$ plays a major role explaining the pattern of wages and life cycle labor supply. Given this, it is important to place this value into the range of estimates in the literature. This is not easily done as there is a very large range of estimates—some larger than our 10.9% estimate and some smaller. There are broadly three different literatures that estimate related parameters. The first of these is motivated by family leave for women and tries to estimate the effect of career interruption on wages. It finds estimates ranging from 1.5% per year to 25%.[18] A second literature looks at displacement from the Displaced Worker Survey and also finds a wide range of estimates—many of which are not directly comparable to ours.[19] A third literature examines the effect of the

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[18]A classic early paper on this topic is Mincer and Polachek (1974) which estimates a net depreciation rate of around 1.5 percent per year. Mincer and Ofek (1982) go beyond this to discuss the difference between short term and long term losses from interruption. In the long run individuals invest in human capital to offset the initial loss, so Mincer and Ofek (1982)'s definition of short term losses is more closely related to our concept of depreciation. Using panel data methods for the National Longitudinal Survey of Mature Women they find estimates ranging from 5.6% to 8.9%. Light and Ureta (1995) use National Longitudinal Survey of Youth 1979 data and estimate that the immediate effect of a year of non-participation in the labor market leads to a decline in earnings of 25%. Kunze (2002) and Gorlich and de Grip (2009) both use German data (IAB employment sample and German Socio-economic panel respectively). Kunze (2002) finds estimates of about 2-5% wages losses for women from unemployment spells but about 13-18% from parental leave. Gorlich and de Grip (2009) find a variety of results ranging from around 1.5% to 5% depending on the type of spell.

[19]While much of this literature is more focused on earnings than wages, some papers look at weekly earnings. Both Farber (1993) and Ruhm (1991) estimate the effect of a displacement on re-employment wages and obtain a range of estimates with most being around declines of 10% but varying from 6.5% to 16.9%. These numbers are not annualized but are just from the incidence of displacement. Li (2013) uses the same data but produces annualized versions so that the effects can be more easily compared to our estimate of δ. She estimates the effects for many different occupations with a huge range of estimates across occupations. Focusing on the three largest occupations she finds a deprecation of 9.4% for Installation and Repair workers, 7.7% for Production workers, and 17.4% for workers in Transportation.

length of an unemployment spell on the wage at rehire. Schmieder et al. (2014) is a recent and convincingly identified paper of this type. They estimate the effect using a regression discontinuity with German data. In Germany the length of eligibility for unemployment insurance depends on age with jumps at ages 42 and at 44. They see an increase in unemployment duration at these two discontinuity points, so they use the kink points as instruments in order to estimate the effect of the length of unemployment duration on re-employment wages. They find that one extra month of unemployment leads to a decrease in wages of 0.8% which gives an annual rate remarkably close to our estimate of 10.9%. While it looks at women in England, Blundell et al. (2015) is of similar style to our paper in the sense that it is a structural life-cycle model of labor supply and human capital formation. Interestingly, their analysis reveals a substantial depreciation of human capital ranging from 6% to 11%.

A second important feature for explaining the life-cycle profiles comes from a point emphasized by Heckman et al. (1998a): observed wages are different than observed human capital. We see in figure 3b that in both the model and the data, once fixed effects are accounted for, wages are close to flat for ages 50-65 despite the fact that there is a large decrease in labor supply. This distinction between human capital and wages can help explain this effect. As shown in Figure 4a, at older ages (around 60) the actual human capital level has already depreciated to a relatively low level, even though the observed wage level is still quite high. This is due to the quick decline in investment that happens around that time. This means that measured wages, $H_t (1 - I_t)$, can be flat while H_t is decreasing as long as I_t is decreasing as well. The time investment profile in Figure 4b matches this implication. The solid line is the unconditional investment profile while the dashed line is the average investment profile conditional on working. These two profiles are very close to each other at prime ages, and both decrease over time.

The relatively high value of investment late in the working career is also related to why we find a much smaller level of the human capital curvature parameter, α_I, compared to the literature summarized in Browning et al. (1999). The larger is α_I the steeper is the decline in human capital investment with age. At the extreme when $\alpha_I = 1$ one gets a "bang-bang" solution with full investment to a point and then zero investment thereafter. Because depreciation is large, in order to fit the relatively flat wage profile that we see at older ages one needs a lot of investment at this age which requires a small value of α_I. Heckman et al. (1998a) fit the wage data with a much larger value of α_I but our models are quite different in a number of ways including the fact that this model includes leisure and in their model they set deprecation to zero.

At the early stage of the life-cycle, workers invest a considerable amount of time in

human capital production which drives up both the human capital level and the wage. Once the worker reaches his mid-career (around age 45), he reduces the time investment at an increasing rate and human capital starts to decrease. As the worker spends less of his working time investing, wages continue to increase. One can see in Figure 4a that the observed wage keeps increasing after age 45 and peaks around 52, after which the observed wage starts declining slowly. After age 62, however, since the worker has already allocated most of his time in effective working, there is no further room for such adjustment. As a result, the observed wage declines at almost the same rate at which human capital depreciates. This leads to large falls in labor supply at older ages.

Such separation also helps generate the pattern that the working hours profile peaks earlier than the wage profile (Weiss, 1986). Working hours increase slightly with age when the worker is young, with a large portion devoted to human capital investment. The working hours profile peaks around age 40 and starts declining. However, with proportionally less time devoted to human capital investment and more time to effective labor supply (Figure 4b), the observed wage increases from labor market entry to about age 52.

5.1 Elasticity of Labor Supply

In this subsection, we investigate the model's implications for elasticities of labor supply. Since labor supply is discrete, we examine the elasticity along the extensive margin. At the individual level, the labor supply elasticity is zero unless the worker is exactly indifferent between working or not, in which case it is infinite. Therefore, we can not construct the standard Marshallian and Hicksian labor supply elasticities. Instead we construct counterparts to these by increasing the human capital rental rate at different ages by 10% (from 1 to 1.1), and then simulating the percentage change in the labor force participation rate using the baseline model.[20]

Let h_t^b be the labor force participation rate at age t in the baseline model and h_t^t be the labor force participation rate at age t (denoted by the subscript) in the simulation in which we increase the rental rate at age t (denoted by the superscript) by 10%. Then our version of the Marshallian elasticity is calculated as

$$me = \frac{\log\left(h_t^t\right) - \log\left(h_t^b\right)}{\log(1.1)}. \tag{20}$$

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[20]In both simulations we assume that the increase in rental rates is anticipated.

We calculate the Intertemporal Elasticity of Substitution (IES) as

$$ies = \frac{\log \left(h_t^t / h_{t-1}^t \right) - \log \left(h_t^b / h_{t-1}^b \right)}{\log(1.1)}. \tag{21}$$

The whole life-cycle age-wage profile will be different in this model even when the only change is in the rental rate at age t. An alternative way of calculating these elasticities is to compute the percentage changes in the labor supply responding to the percentage changes in the observed wages,

$$me' = \frac{\log \left(h_t^t \right) - \log \left(h_t^b \right)}{\log(w_t^t) - \log(w_t^b)} \tag{22}$$

$$ies' = \frac{\log \left(h_t^t / h_{t-1}^t \right) - \log \left(h_t^b / h_{t-1}^b \right)}{\log \left(w_t^t / w_{t-1}^t \right) - \log \left(w_t^b / w_{t-1}^b \right)}. \tag{23}$$

The calculated Marshallian elasticity and IES at each age from both methods are plotted in Figure 5a. Table 3 also documents both elasticities at selected ages. One can see that labor supply is much more elastic at older ages than at younger ages in both calculations. This is due in large part to the shadow cost of leisure. The shadow cost is substantially larger for young workers than for older workers since the older workers have a shorter time horizon. As a result, the labor supply of young workers is less responsive to temporary wage shocks than is the labor supply of older workers. It is also due to the density of the tastes for leisure γ_t. When the probability of working is closer to 50% the density of people close to indifferent will be larger which results in a larger elasticity. Note that the second measure of the Marshallian elasticity or IES is almost universally smaller than the first.[21] The reason is that at age t the percentage change in the wage is larger than that in the human capital rental rate. As a result of workers' responses to the anticipated rental rate increase, they adjust their investment strategy to take advantage of the higher rental rate at age t.

Figure 5b provides some sense of how these temporary effects impact lifetime labor supply. Panel (i) presents the effect of LFPR profiles for cases where the 10% increase in the human capital rental rate occurs at different ages, specifically at ages 25, 35, 60, and 65. This shows the response in LFPR relative to the baseline model at different ages for the positive shock at one specific age. The LFPR rises closer to the shock age and rises sharply at the shock, due to dominating substitution effect. When it is distant from the shock, the LFPR is lower than the baseline model, due to dominating income effect. The

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[21]Except at very old ages.

Table 3: Elasticities at selected ages

	Responding to %Δ in H rental rate[a]		Responding to %Δ in wages[b]	
Age	Marshallian (*me*)	IES (*ies*)	Marshallian (*me'*)	IES (*ies'*)
20	0.578	0.548	0.520	0.486
25	0.370	0.352	0.340	0.321
30	0.224	0.198	0.205	0.181
35	0.161	0.138	0.152	0.130
40	0.142	0.132	0.138	0.127
45	0.155	0.138	0.150	0.133
50	0.218	0.195	0.215	0.191
55	0.272	0.239	0.267	0.235
60	0.596	0.518	0.572	0.503
65	1.483	1.221	1.207	1.019
70	2.457	1.921	2.235	1.980

[a]The Marshallian is $me = \frac{\log\left(h_t^t\right) - \log\left(h_t^b\right)}{\log(1.1)}$; the IES is $ies = \frac{\log\left(h_t^t/h_{t-1}^t\right) - \log\left(h_t^b/h_{t-1}^b\right)}{\log(1.1)}$.

[b]The Marshallian is $me' = \frac{\log\left(h_t^t\right) - \log\left(h_t^b\right)}{\log(w_t^t) - \log(w_t^b)}$; the IES is $ies' = \frac{\log\left(h_t^t/h_{t-1}^t\right) - \log\left(h_t^b/h_{t-1}^b\right)}{\log\left(w_t^t/w_{t-1}^t\right) - \log\left(w_t^b/w_{t-1}^b\right)}$

shock also affects the time allocation at work. When approaching the shock age, workers gradually increase time investment to achieve a higher human capital when the positive shock arrives (Panel (ii) and (iii)). While at the shock, workers dramatically decrease time investment and increase effective working hours to take advantage of the higher human capital and its higher rental rate. Panel (iv) of Figure 5b summarizes the total change in LFPR for such positive shocks at different ages. Assume that the human capital rental rate only increases at age t and the timing of this shock is represented by the X-axis of this figure. For this case, the "Overall" represents the overall change in LFPR over the entire life-cycle (from age 18 to 80); the "Before t" represents the total change in LFPR before age t; the "After t" is the total change after age t and the "At t" is the spot change at age t. Notice that if the positive shock occurs during the early career, the wealth effect causes a decline in the LFPR at later ages and the overall effect is negative in the LFPR. However, a positive shock at older ages would encourage higher LFPR afterwards and the overall LFPR increases. This is because one individual allocates more time in effective working at old ages than at young ages. Thus the substitution effect is more prominent at older ages, when the wage is around the peak.

For individuals under age 50 these estimates are very close to the estimates of labor

supply elasticities found in the literature. For example, the early literature estimates the Frisch elasticity being 0.09 (Browning et al., 1999), 0.15 (MaCurdy, 1981), and 0.31 (Altonji, 1986). Chetty (2012) reports extensive (Hicksian) labor supply elasticities around 0.25 combining estimates from many different studies and approaches. Focusing on the extensive margin, Rogerson and Wallenius (2013) suggest that the IES is 0.75 or greater given empirically reasonable level of nonconvexities or fixed costs. The average of our estimates between ages 55 and 65 is remarkably close to theirs.

6 Roles of Health or Part Time

We have intentionally kept our model simple to show that human capital can explain the dramatic fall in labor supply at the end of the life-cycle. However, there are many alternative reasons why labor supply might decline. Aside from Social Security rules, which we have already incorporated, the most important is health (e.g. Currie and Madrian 1999, French and Jones 2011). If the primary reason for retirement is health, its omission might seriously distort our results. In this section we incorporate health into our model in a very flexible way. We show that while it is an important factor, it is not the primary driver of retirement. We also investigate the case where individuals can choose to work part time and show that the option of partial labor supply is not the main contributing factor of retirement either.

6.1 The Role of Health

We allow for an additional state variable—health status, $S_t \in \{0,1\}$, with 0 being in good health and 1 in bad health. Each individual is assumed to have good health from the beginning of the first period up to age 49, $S_t = 0$, $t \leq 49$. Since age 50, the health status evolves exogenously according to a time-dependent probability transition matrix, and is realized at the beginning of each period before any choice is made.[22]

We allow the taste for leisure in the utility function (1) to depend on the health status and change with age,

$$\gamma_t = exp\left(\tilde{a}_0 + S_t\left(a_{s0} + a_{st}t\right) + a_\varepsilon \varepsilon_t\right). \tag{24}$$

That is, individuals with bad health have a different taste for leisure than those with good

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[22]The health transition matrix is estimated from the Panel Study of Income Dynamics (PSID) data. We include the health status from age 50 for two reasons. First, most individuals have good health before age 50. Second, this simplification saves computation time.

health and this difference changes as they age.[23] We refer to this model as the baseline health model.

To estimate these two new parameters, a_{s0} and a_{st}, we include the difference in labor force participation rates between workers with good health and workers with bad health, from age 50 to 65 into our moment conditions. The data moments are derived from the 1963-2007 CPS March data.

We then re-estimate the whole model. The estimates of the parameters are listed in the first two columns of Table 4. The fit of the model is presented in Figure 6a. Including health (and the additional moments shown in panel (ii) of Figure 6a) into the model does not improve its performance on the original moments in any significant way.

However, just because the fit does not improve much does not imply that health does not play an important role. It may just be that either health or human capital could explain retirement.[24] To explore the implications of health we use the model estimated with health, but then simulate a counterfactual in which there was no health change. Specifically, we eliminate the importance of health for individuals over 50 in two different ways—we do not allow their health to worsen and we eliminate the interaction between health and preferences for work. Specifically, we simulate an experiment in which the health status that an individual had at age 50 remains for the rest of their life. Secondly, in addition to fixing the health status at age 50, for individuals with bad health status on and after age 50, we assume their taste for leisure does not increase with age. That is, letting t^* be the time period in which the individual turns 50, we assume that the taste for leisure is now

$$\gamma_t = exp\left(\tilde{a}_0 + S_t\left(a_{s0} + a_{st} \cdot \min\{t, t^*\}\right) + a_\varepsilon \varepsilon_t\right) \tag{25}$$

and $S_t = S_{50}$ for $t > t*$. We then re-solve the modified model and simulate the life-cycle profile for each individual using the same estimates from the aforementioned baseline health model.[25] The profiles of labor supply and human capital from the second experiment are plotted in Figure 6b. The difference between the counterfactual and the baseline health model is very small in both the labor force participation rate and the human capital

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[23]A key aspect of the thought experiment behind this paper is to not allow preferences to vary systematically with age in our baseline model. In practice we can only fit the interaction of health and labor supply in the data by allowing for an interaction between health and tastes for leisure. The main point of this subsection is that health is not essential to explain the profiles, so even though we are favoring the model with health by allowing this extra flexibility, health has a relatively minor role.

[24]Note that this is not to say they are not separately identified. The extra moments we use for the health model identify the importance of health.

[25]We are assuming that agents have rational expectations and are aware that their health status will not change. We have also simulated models in which they are not aware that their health status will remain fixed—it does not change the basic message.

level. This implies that at least in our model health is not a major factor driving retirement. This result confirms findings in the previous literature. French (2005) estimates that the changes in health attribute to roughly 10% of the drop in the labor force participation rates between ages 55 and 70, and the contribution to hours worked by workers near retirement is much smaller. Blau and Shvydko (2011) also report that health deterioration is an important but not major cause of retirement.

6.2

6.3 The Role of Part Time

We now include the choice of part time working in the model. At each period, individual decides to work full time ($\ell_t = 0$), or to work part time ($\ell_t = p_t \in (0,1)$), or not work ($\ell_t = 1$). In general, the utility from working part time, p_t, can vary over time and is to be estimated. For simplicity of notation, we surpress the subscript for p_t in the following discussion and bring it back in estimation.

If an individual chooses to work part time, the investment in human capital is $I_t \in \left[0, \frac{1}{2}\right]$ and the effective work time is $\frac{1}{2} - I_t$, with wage earning

$$w_t = H_t \cdot \left(\frac{1}{2} - I_t\right).$$

Note that working part time means spending half time in the labor market and the other half time at leisure, which yields utility of $\gamma_t p$.

The solution is similar to the baseline model with binary labor supply choices. The optimal labor supply solution is

$$\ell_t = \arg \max_{\ell_t \in \{0, p, 1\}} \widetilde{V}_{t, \ell_t}(X_t) + \gamma_t \ell_t \tag{26}$$

where

$$\{\mathcal{C}_{t,p}(X_t), \mathcal{I}_{t,p}(X_t), \mathcal{SSA}_{t,p}(X_t)\} \equiv \arg\max_{c_t, I_t, ssa_t} \left\{ \psi_t \frac{c_t^{1-\eta_c}}{1-\eta_c} + \beta E\left[V_{t+1}(X_{t+1}, \gamma_{t+1}) \mid X_t, c_t, p, I_t, ssa_t\right] \right\} \tag{27}$$

$$\widetilde{V}_{t,p}(X_t) \equiv \psi_t \frac{(\mathcal{C}_{t,p}(X_t))^{1-\eta_c}}{1-\eta_c} + \beta E\left[V_{t+1}(X_{t+1}, \gamma_{t+1}) \mid X_t, \mathcal{C}_{t,p}(X_t), p, \mathcal{I}_{t,p}(X_t), \mathcal{SSA}_{t,p}(X_t)\right] \tag{28}$$

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Table 4: Estimates in the models with health and with part time option[a]

Parameters		with Health		with Part Time	
		Estimates	S.E.	Estimates	S.E.
HC depreciation[b]	δ	0.121	0.011	0.152	0.001
HC production function: I factor	α_I	0.038	0.013	0.064	0.001
HC production function: H factor	α_H	0.128	0.026	0.232	0.002
Standard deviation of HC innovation	σ_ξ	0.118	0.062	0.006	0.0001
Consumption: CRRA	η_c	4.043	0.026	4.004	0.019
Consumption shifter: coef on t $(\times 10)$	φ_1	0.245	0.062	0.740	0.006
Consumption shifter: coef on t^2 $(\times 10^2)$	φ_2	0.213	0.042	-0.012	0.0002
Consumption shifter: coef on t^3 $(\times 10^3)$	φ_3	-0.054	0.007	-0.019	0.0003
Consumption shifter: coef on married	φ_4	1.108	0.389	1.728	0.024
Leisure: Standard Deviation of Shock	a_ε	0.240	0.030	0.551	0.005
Leisure: spouse not working	a_1	0.417	0.126	1.936	0.045
Leisure: spouse working	a_2	-1.235	0.185	-1.047	0.039
Leisure: unhealthy	a_{s0}	-0.088	0.029	-	-
Leisure: unhealthy time trend	a_{st}	0.019	0.001	-	-
Part time utility: constant	a_{p0}	-	-	1.359	0.015
Part time utility: coef on t $(\times 10)$	a_{p1}	-	-	-1.147	0.008
Part time utility: coef on t^2 $(\times 10^2)$	a_{p2}	-	-	0.238	0.001
Bequest weight	b_1	27,839,860	5,664,820	8,830,654	53,919
Parameter heterogeneity[c]					
Leisure: mean of intercept	μ_{a_0}	-5.667	0.116	-4.912	0.024
Leisure: standard deviation of intercept	σ_{a_0}	1.462	0.090	2.544	0.013
HC productivity, mean	μ_π	1.856	0.169	1.748	0.018
HC productivity, standard deviation	σ_π	0.529	0.045	0.863	0.003
Correlation between a_0 and π	ρ	-0.155	0.052	-0.993	0.003
Initial human capital level at age 18					
Intercept	γ_0	1.427	0.303	2.236	0.019
Coefficient on a_0	γ_{a_0}	0.162	0.019	-0.415	0.009
Coefficient on π	γ_π	0.928	0.140	-1.410	0.025
Standard deviation of error term	σ_{H_0}	0.280	0.096	0.001	0.0004
χ^2 Statistic[d]		1489		2905	
Degrees of freedom		214		241	

[a]Indirect Inference estimates. Estimates use a diagonal weighting matrix. Standard errors are given in parentheses.
[b]HC: Human Capital.
[c]The joint distribution of (a_0, π) is a parametric discrete distribution with nine points determined by these five parameters, using a nine-point Gauss-Hermite approximation.
[d]This is the J-statistic. The critical values of the χ^2 distribution are $\chi^2_{(214,0.01)} = 265$, $\chi^2_{(214,0.005)} = 271$, $\chi^2_{(214,0.001)} = 283$; $\chi^2_{(241,0.01)} = 295$, $\chi^2_{(241,0.005)} = 301$, $\chi^2_{(241,0.001)} = 315$.

Define

$$\gamma^*_{t\,\min} \equiv \min \left\{ \frac{\widetilde{V}_{t,0}(X_t) - \widetilde{V}_{t,p}(X_t)}{p}, \ \widetilde{V}_{t,0}(X_t) - \widetilde{V}_{t,1}(X_t) \right\}$$

$$\gamma^*_{t\,\max} \equiv \max \left\{ \frac{\widetilde{V}_{t,p}(X_t) - \widetilde{V}_{t,1}(X_t)}{1-p}, \ \widetilde{V}_{t,0}(X_t) - \widetilde{V}_{t,1}(X_t) \right\}$$

and we have the following proposition.

PROPOSITION 2: The optimal labor supply decision is

$$\ell_t = \begin{cases} 0, & \text{if } \varepsilon_t \leq \varepsilon^*_{t\,\min}, \\ p, & \text{if } \varepsilon^*_{t\,\min} < \gamma_t < \varepsilon^*_{t\,\max}, \\ 1, & \text{if } \varepsilon_t \geq \varepsilon^*_{t\,\max}. \end{cases}$$

where

$$\varepsilon^*_{tj} \equiv \frac{1}{a_\varepsilon} \left\{ \log \left(\gamma^*_{tj} \right) - \tilde{a}_0 \right\}, \ j \in \{\min, \max\},$$

and

$$\begin{aligned} E\left[V_t(X_t, \gamma_t)\,|\,X_t\right] &= \Phi\left(\varepsilon^*_{t\,\min}\right) \widetilde{V}_{t,0}(X_t) + \left(1 - \Phi\left(\varepsilon^*_{t\,\max}\right)\right) \cdot \left[\widetilde{V}_{t,1}(X_t) + E\left(\gamma_t\,|\,\varepsilon_t \geq \varepsilon^*_{t\,\max}\right)\right] \\ &\quad + \left(\Phi\left(\varepsilon^*_{t\,\max}\right) - \Phi\left(\varepsilon^*_{t\,\min}\right)\right) \cdot \left[\widetilde{V}_{t,p}(X_t) + E\left(\gamma_t\,|\,\varepsilon^*_{t\,\min} < \varepsilon_t < \varepsilon^*_{t\,\max}\right)\right] \end{aligned}$$

where

$$E\left(\gamma_t\,|\,\varepsilon_t \geq \varepsilon^*_{t\,\max}\right) = exp\left(\tilde{a}_0 + \frac{a_\varepsilon^2}{2}\right) \frac{\Phi\left(a_\varepsilon - \varepsilon^*_{t\,\max}\right)}{\Phi\left(-\varepsilon^*_{t\,\max}\right)}$$

$$E\left(\gamma_t\,|\,\varepsilon^*_{t\,\min} < \varepsilon_t < \varepsilon^*_{t\,\max}\right) = exp\left(\tilde{a}_0 + \frac{a_\varepsilon^2}{2}\right) \frac{\Phi\left(a_\varepsilon - \varepsilon^*_{t\,\min}\right) - \Phi\left(a_\varepsilon - \varepsilon^*_{t\,\max}\right)}{\Phi\left(\varepsilon^*_{t\,\max}\right) - \Phi\left(\varepsilon^*_{t\,\min}\right)}.$$

PROOF: Appendix B.

Following the same strategy as in Section 4 and assuming

$$p_t = \frac{1}{1 + \exp\left(-a_{p0} - a_{p1} \cdot t - a_{p2} \cdot t^2\right)},$$

we re-estimate the model with part time option.[26] Table 4 presents parameter estimates. The fit of the model, as shown in Figure 7a, is similar as the baseline model.

To investigate the effect of having a part time option, we use the estimates in Table

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[26]We include the part time rate at each age from 22 to 65 as additional moments.

4 and simulate a counterfactual without such an option. Figure 7b presents the profiles of labor force participation and the human capital. It appears that removing the part time option does not change the retirement pattern significantly, suggesting that the more flexible labor supply arrangement is not the major source of retirement.

7   Alternative Human Capital Models

We compare our baseline human capital accumulation model with two variants. All other aspects of the model remain the same. The first variation assumes the innovation part in the human capital production function is completely exogenous. The second variation assumes the innovation only occurs if individuals work, but is exogenous conditional on work. This is essentially a learning-by-doing model as in, for example, Imai and Keane (2004). To keep this comparable, we alter our baseline model as little as possible. We also restrict the number of total parameters to remain the same so that we are comparing models with similar levels of flexibility.

First we consider the model with exogenous human capital. In this case human capital evolves according to the function

$$H_{t+1} = (1 - \delta) H_t + \xi_t \pi \left(1 + \alpha_1 t + \alpha_2 t^2 \right)$$

where t is potential experience. Notice that this is very close to our standard model from equation (2). We have exactly the same parameter names, except that (α_I, α_H) are replaced with (α_1, α_2) since their roles have changed considerably. In this case human capital evolves completely exogenously in the sense that individuals can do nothing to change their human capital.

The parametrization of the second model is analogous. Here we alter the exogenous model so that human capital only grows for workers:

$$H_{t+1} = (1 - \delta) H_t + (1 - \ell_t) \xi_t \pi \left(1 + \alpha_1 t + \alpha_2 t^2 \right).$$

We refer to this as the "learning-by-doing" model. Even though it looks quite similar to the exogenous model, as a practical matter it is very different as workers can control their human capital through their labor force participation decision. When individuals do not work, their human capital depreciates at rate δ.

In section 5 above we discuss two different reasons why our model can fit the life-cycle profiles of wages and labor supply and in particular the large increase in wages but

small increase in labor supply at the beginning of the life-cycle and the large decrease in labor supply but small decrease in wages at the end. The first is human capital depreciation: when workers stop working their human capital falls. The second is the distinction between observed wages and human capital. These two models allow for us to see the relative importance for these two different explanations because the exogenous human capital model lacks both of these features while the learning-by-doing allows for the former but not the latter.

The estimates of these models are presented in Table 5 and the fits of the two models are presented in Figure 8a. We first discuss the completely exogenous model. The within sample fit of the model is reasonably well on all moments between Age 22 and 65. However, the exogenous model has very poor out of sample fit at the worker's early career. Between age 20 and 22, there is a sharp increase in the labor force participation rate (Panel (i)), accomponies by a spike in the wage (Panel (iii)). The reason is that to fit the decrease in labor supply at the end requires a very large labor supply elasticity (as well as a lot of sample selection bias to give an estimated flat wage). However, the large elasticity to explain labor supply at the end leads to a huge increase in labor supply and the sample selection bias leads to a spike in the wage at the beginning, neither of which we see in the data. To see the size of the elasticity, we estimate our version of the Intertemporal Elasticity of Substitution as above and present it in figure 8b as well as in table 6 at selected ages. The exogenous model requires a substantially larger elasticity.

By contrast the learning-by-doing model fits the data well—though not quite as well as our baseline model, especially in the labor force participation at early career. The elasticity of labor supply is much closer to the baseline model than it is to the exogenous model—as one can see from figure 8b or from the fact that a_ε takes on a similar value 0.180 as opposed to 0.002 in the exogenous model. In comparing the fit, all three models explain the fixed effect wage profile, the standard deviation profile and the consumption profile fairly well. The exogenous model does not fit the early career aspect of the labor force participation rate well. The learning-by-doing fits labor supply better than the exogenous model, but not quite as well as the baseline model. The exogenous model performs considerably worse in the log wage profile at early career. It is important to note here that we did not try a wide range of exogenous or learning-by-doing models; we just did a comparison between our baseline model and an exogenous or learning-by-doing model chosen to be close to our baseline model. Presumably alternative and more flexible models could fit the data better—though this is true of our baseline model as well.

This comparison between the fit of the three models suggests that the human capital depreciation rate seems to be relatively more important for fitting the data than the

Parameters		Exogenous[b]		Learning-by-Doing[c]	
		Estimates	S.E.	Estimates	S.E.
HC depreciation[d]	δ	0.098	0.003	0.090	0.004
HC production function: on t	α_1	0.014	0.003	0.001	0.001
HC production function: on t^2 $(\times 10^4)$	α_2	-4.116	(0.559)	-0.018	(0.008)
Standard deviation of HC innovation	σ_ξ	1.495	0.092	0.003	0.005
Consumption: CRRA	η_c	4.258	0.031	3.636	0.021
Consumption shifter: on t $(\times 10)$	φ_1	0.202	0.035	0.365	0.056
Consumption shifter: on t^2 $(\times 10^2)$	φ_2	0.100	0.015	0.043	0.013
Consumption shifter: on t^3 $(\times 10^3)$	φ_3	-0.040	0.002	-0.021	0.002
Consumption shifter: coef on married	φ_4	2.249	0.168	0.004	0.002
Leisure: Standard Deviation of Shock	a_ε	0.002	0.001	0.180	0.026
Leisure: spouse not working	a_1	1.129	0.132	5.214	0.533
Leisure: spouse working	a_2	-1.199	0.257	-1.887	0.201
Bequest weight	b_1	25,157,726	4,132,866	6,564,966	898,777
Parameter heterogeneity					
Leisure: mean of intercept	μ_{a_0}	-6.513	0.041	-4.965	0.109
Leisure: standard deviation of intercept	σ_{a_0}	1.351	0.058	1.699	0.122
HC productivity, mean	μ_π	1.567	0.042	1.766	0.065
HC productivity, standard deviation	σ_π	0.459	0.035	0.622	0.043
Correlation between a_0 and π	ρ	-1.000	0.064	0.031	0.026
Initial human capital level at age 18					
Intercept	γ_0	2.292	0.392	1.024	0.288
Coefficient on a_0	γ_{a_0}	0.253	0.039	0.065	0.016
Coefficient on π	γ_π	0.802	0.140	0.777	0.181
Standard deviation of error term	σ_{H_0}	0.626	0.055	0.124	0.090
χ^2 Statistic[e]		1185		1839	
Degrees of freedom		200		200	

[a]Indirect Inference estimates. Estimates use a diagonal weighting matrix. Standard errors are given in parentheses.

[b]In the exogenous model, the human capital production function is $H_{t+1} = (1-\delta)H_t + \xi_t \pi (1 + \alpha_1 t + \alpha_2 t^2)$.

[c]In the learning-by-doing model, the human capital production function is $H_{t+1} = (1-\delta)H_t + (1-\ell_t)\xi_t \pi (1 + \alpha_1 t + \alpha_2 t^2)$.

[d]HC: Human Capital.

[e]This is the J-statistic. The critical values of the χ^2 distribution are $\chi^2_{(200,0.01)} = 249$, $\chi^2_{(200,0.005)} = 255$, $\chi^2_{(200,0.001)} = 268$.

Table 6: Elasticities at selected ages, responding to % changes in wages[a]

	Baseline Model		Exogenous Model		Learning-by-doing Model	
Age	Marshallian	IES	Marshallian	IES	Marshallian	IES
20	0.520	0.486	5.054	5.499	0.228	0.224
25	0.340	0.321	2.801	3.065	0.233	0.209
30	0.205	0.181	1.190	1.492	0.189	0.176
35	0.152	0.130	0.926	1.105	0.111	0.098
40	0.138	0.127	0.798	0.954	0.100	0.090
45	0.150	0.133	0.895	1.025	0.117	0.093
50	0.215	0.191	1.097	1.273	0.226	0.202
55	0.267	0.235	1.812	2.010	0.333	0.305
60	0.572	0.503	3.096	3.285	0.586	0.587
65	1.207	1.019	3.627	4.383	0.324	0.258
70	2.235	1.980	9.087	9.801	0.200	0.135

[a]The Marshallian is $me' = \frac{\log(h_t^t) - \log(h_t^b)}{\log(w_t^t) - \log(w_t^b)}$; the IES is $ies' = \frac{\log(h_t^t/h_{t-1}^t) - \log(h_t^b/h_{t-1}^b)}{\log(w_t^t/w_{t-1}^t) - \log(w_t^b/w_{t-1}^b)}$.

difference between human capital and observed wages.

8 Changes in Tax and Social Security

The preceding sections show that the model fits the life-cycle profiles of labor supply and log wages in the data well. In this section, we use the model to predict how changes in the taxes or Social Security rules would affect behavior in labor supply, human capital investment and the resulting log wage profile. We conduct seven counterfactual policy experiments which reflect various changes in the tax codes and Social Security rules. The results of these experiments are summarized in columns 2-8 in Table 7, where the first column is the baseline model. All numbers are summations or averages throughout the life-cycle (from age 18 to 80).

8.1 The Baseline Model

The first experiment increases the income tax proportionally by 50%. Column 2 shows that after the tax increase, an average individual works additional 1.061 years over the life-cycle, equivalent to 2.7% of the total labor supply. Most of the increase in the labor supply is allocated to the effective labor, which increases by almost one year or 2.69%. The investment increases by 3.5%, which leads to 2.8% increase in the human capital level and

1.06% increase in the observed log wages.[27] Annual consumption reduces by 2.37%. A tax hike has both substitution and income effects. The substitution effect discourages labor supply while the income effect encourages labor supply. Our first experiment indicates that in our model the income effect dominates the substitution effect and this is the case with most of our experiments. We also see that human capital investment increases in this experiment. The direct effect of taxes discourages human capital investment, but the increase in labor supply (and in particular delayed retirement) increases human capital investment.

The manner in which Social Security rules affect labor supply and wages is of central interest to policy makers. The six experiments in columns 3-8 are devoted to answering these questions. In the first three we manipulate the current Social Security rules (columns 3-5) while in the last three we decompose the distortionary effects of the current Social Security system (columns 6-8).

First we remove the Social Security earnings test, which is effective between age 62 and 70 in the baseline model. In the second one, we delay Normal Retirement Age (NRA) by two years: the new NRA is age 67 in this counterfactual experiment while it is age 65 in the baseline model. In the third one, we reduce the Social Security benefit proportionally by 20%. The results are presented in columns 3-5 in Table 7. Removing the Social Security earnings test between ages 62 and 70 has a smaller effect on all variables except the log wages; delaying the normal retirement age by two years, has a slightly larger impact; reducing the generosity of the social security benefit has the largest effect among these three. For instance, they increase the labor force participation by four-and-a-half, five, or seven-and-a-half months, respectively. One important feature is that the change in the labor supply does not only happen later in the life-cycle when the policy change is directly effective, it takes place over the whole life-cycle, as indicated in Figure 9a. When the NRA is delayed two years or the Social Security benefit is reduced, workers also invest more and therefore have higher human capital levels, which leads to higher wages at old ages (Figure 9a). The wage difference is negligible before age 60 but increases substantially after that, reaching 5% or 8% around age 67. On the other hand, removing the Social Security earnings test induces substantially higher labor force participation as well as more investment, resulting up to 15% higher wages within the relevant age window.[28] Ignoring such a wage response in experiments involving retirement policy will

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[27]Other papers have looked at the effects of taxes and human capital with this type of model. Examples are Heckman et al. (1998b), Heckman et al. (1999), and Taber (2002). These experiments are quite different as labor supply makes a large difference here so the results are not directly comparable.

[28]Removing the Social Security earnings test is equivalent to a positive shock to the human capital rental rate for age 62-70, where substitution effect is more prominent.

most likely introduce bias.

In the last three experiments, we decompose the effect of the current U.S. Social Security system into the individual effects of the Social Security taxes and the Social Security benefit. In Column 6 we keep the Social Security benefit but eliminate the Social Security taxes (the payroll taxes);[29] in Column 7 we remove the Social Security benefit completely but keep the Social Security taxes; in Column 8 we remove the entire Social Security system, that is, both the Social Security taxes and the benefit. Removing the Social Security taxes induces an average individual to supply 2.7 years less labor. This is not surprising because removing the Social Security taxes is essentially a universal cut in the tax rate. In our tax hike counterfactual, the income effect dominates the substitution effect as is true for the cut in social security taxes as well. Analogously, removing the Social Security benefit induces more labor supply. However, the increase in the labor supply is 5.2 years, which is much higher than 2.7 years reduction of labor supply in the case of removing Social Security Taxes. The combination of these two effects leads to the results in the last experiment where both the Social Security taxes and benefit are removed. Column 8 indicates that eliminating the current Social Security system increases average labor supply by 1.2 years over the life-cycle. Such observation is also mentioned qualitatively in Gustman and Steinmeier (1986) and Rust and Phelan (1997). Figure 9b shows that the changes in the labor supply and log wages are most phenomenal at old ages in the experiment without Social Security system.

Another point worth emphasizing is that in almost every policy counterfactual, the increase in the endogenously determined wage levels are substantial. This is especially true at old ages: 15% when removing the earnings test, 5% when delaying NRA by two years, 8% when reducing Social Security benefit, over 15% when removing Social Security taxes, benefit or the entire Social Security system. These are caused by changes in the human capital levels as a result of higher or lower investment. For this reason, it is likely that ignoring the human capital investment channel would generate bias in terms of predicting LFPR at old ages in similar experiments.

8.2 Sensitivity to Alternative Models

Table 8 presents the results of experiments from the alternative models, specifically, Panel A from the exogenous model, Panel B from the learning-by-doing model, Panel C from the model with health, and Panel D from the model with part time option. Across four different models the responses to the policy changes are qualitatively similar to our

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[29]The income taxes are still effective.

baseline model in all experiments, although they differ quantitatively in some experiments.

First we focus on the three models of exogenous or endogenous human capital accumulation. In the first experiment (column 2), the labor supply response to tax hike is largest in the exogenous model and smaller in the learning-by-doing model. This is consistent with the pattern of labor supply elasticities as discussed in subsection 5.1. In the experiments of altering Social Security benefits (columns 3-5), the labor supply response to the policy changes is smallest in the learning-by-doing model, larger when the human capital is exogenous, and largest in the baseline model. This result comes from several different features of these three models. Consider the experiment that reduces the Social Security benefit by 20% (Column 5). The change in labor supply is essentially purely due to the income effect. We see that the largest impact on labor supply comes from the baseline model, the second highest from the exogenous model, and the lowest in the learning-by-doing model. The key to understanding this difference is human capital. When the Social Security benefit is reduced, the reduction in the expected wealth induces higher labor force participation particularly for older workers. In the two human capital models this "delayed retirement" increases the expected return to the human capital investment, which in turn induces higher participation at earlier ages. This "adjacent complementarity" channel makes the labor supply response more efficient. Such channel does not operate for the exogenous model, so the change in labor supply leads to larger response in the exogenous model than in the learning-by-doing model. On the other hand, the baseline model gives a worker an extra channel for adjustment—the allocation of time between investment and working. This channel serves as an augmentation for the labor supply response, especially during early career.[30] This results a larger labor supply response in the baseline model than in the learning-by-doing model.

In Panel C, we present the experiment results for the model with health as described in subsection 6.1 in Panel D the results for the model with part time option in subsection 6.3. In most cases, the results are qualitatively similar to our baseline model but quantitatively differ in some experiments.

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[30]This is one of the reasons why the labor supply elasticity is higher in the baseline model than in the learning-by-doing model before age 50, as shown in Figure 8b. However, one should not confuse the income effect discussed in this subsection with the substitution effect discussed in subsection 5.1.

Table 7: Effects of changing taxes or Social Security rules, baseline model

	1 Baseline	2 Tax Increase 50%		3 No Earnings Test		4 NRA = 67		5 Reduce SSB 20%		6 No SS Taxes		7 No SS Benefit		8 No SS System	
	Level[a]	ΔLevel[b]	%Δ[c]	ΔLevel	%Δ	ΔLevel	%Δ	ΔLevel	%Δ	ΔLevel	%Δ	ΔLevel	%Δ	ΔLevel	%Δ
LFPR	38.861	1.061	2.731	0.379	0.976	0.446	1.147	0.639	1.645	-2.694	-6.932	5.202	13.385	1.216	3.128
Effective Labor	0.585	0.016	2.690	0.006	0.972	0.007	1.134	0.010	1.648	-0.040	-6.843	0.079	13.416	0.019	3.235
Pre-tax Income	9.815	0.433	4.412	0.102	1.041	0.134	1.370	0.206	2.096	-0.914	-9.307	1.520	15.484	0.207	2.104
Average lnw	2.618	0.028	1.059	0.015	0.570	0.007	0.285	0.010	0.388	-0.031	-1.196	0.036	1.387	-0.012	-0.445
Human Capital	13.371	0.374	2.798	0.108	0.810	0.125	0.937	0.184	1.373	-0.836	-6.250	1.254	9.379	0.193	1.443
Investment	0.032	0.0011	3.489	0.0003	1.053	0.0004	1.384	0.0005	1.599	-0.0027	-8.569	0.0041	12.822	0.0004	1.155
Consumption	9.075	-0.215	-2.368	0.010	0.107	-0.033	-0.361	-0.049	-0.540	0.241	2.651	-0.164	-1.811	0.030	0.333

[a]The "Level" column refers to the annual value averaged over the whole life-cycle, except the "LFPR" which is the total number of years worked over the whole life-cycle. For example, in the baseline model, the total LFPR is 38.861 years from 18 to 80.

[b]The "ΔLevel" column refers to the difference of the total value between the current experiment and the baseline model. For example, in the "No Earnings Test" case, the LFPR is 0.379 years higher than that in the baseline model across the whole life-cycle from 18 to 80.

[c]The "%Δ" column refers to the percentage of the difference in the "ΔLevel" column relative to the level in the baseline model. For example, in the "No Earnings Test" case, the LFPR increases by 0.379 years which is equivalent to 0.976% of the LFPR in the baseline model.

Table 8: Effects of changing taxes or Social Security rules, alternative models

	1 Baseline	2 Tax Increase 50%		3 No Earnings Test		4 NRA = 67		5 Reduce SSB 20%		6 No SS Taxes		7 No SS Benefit		8 No SS System	
	Level[a]	ΔLevel[b]	%Δ[c]	ΔLevel	%Δ	ΔLevel	%Δ	ΔLevel	%Δ	ΔLevel	%Δ	ΔLevel	%Δ	ΔLevel	%Δ
Panel A: Exogenous Model															
LFPR	38.675	1.061	2.744	0.072	0.186	0.304	0.787	0.487	1.260	-2.187	-5.655	2.802	7.245	0.127	0.328
Pre-tax Income	9.845	0.278	2.823	0.021	0.211	0.071	0.725	0.113	1.144	-0.529	-5.375	0.645	6.549	0.009	0.094
Average lnw	2.665	0.001	0.055	0.0001	0.003	-0.001	-0.042	-0.001	-0.050	0.0004	0.015	-0.008	-0.300	-0.005	-0.173
Consumption	8.593	-0.243	-2.831	0.009	0.106	-0.040	-0.468	-0.064	-0.739	0.343	3.992	-0.339	-3.946	0.027	0.313
Panel B: Learning-by-doing Model															
LFPR	41.702	0.667	1.599	0.074	0.177	0.166	0.397	0.242	0.580	-1.662	-3.985	1.902	4.561	0.175	0.420
Pre-tax Income	10.307	0.252	2.449	0.022	0.209	0.053	0.515	0.083	0.804	-0.596	-5.781	0.796	7.721	0.187	1.815
Average lnw	2.634	0.008	0.305	0.001	0.031	0.001	0.050	0.003	0.105	-0.018	-0.691	0.018	0.690	-0.004	-0.144
Human Capital	13.425	0.235	1.748	0.020	0.146	0.042	0.314	0.065	0.481	-0.532	-3.964	0.674	5.023	0.102	0.761
Consumption	8.529	-0.303	-3.548	0.011	0.133	-0.050	-0.589	-0.088	-1.035	0.416	4.880	-0.403	-4.724	0.007	0.080
Panel C: Model with Health															
LFPR	41.696	0.893	2.143	0.146	0.349	0.188	0.451	0.305	0.731	-2.167	-5.198	2.961	7.102	0.496	1.189
Effective Labor	0.640	0.014	2.144	0.002	0.347	0.003	0.453	0.005	0.745	-0.033	-5.145	0.042	6.563	0.007	1.117
Pre-tax Income	10.259	0.291	2.839	0.030	0.290	0.063	0.618	0.104	1.016	-0.673	-6.565	0.724	7.056	0.050	0.487
Average lnw	2.654	0.007	0.269	-0.001	-0.024	0.002	0.079	0.004	0.153	-0.016	-0.604	0.015	0.565	-0.003	-0.095
Human Capital	13.138	0.292	2.222	0.036	0.274	0.051	0.388	0.082	0.623	-0.651	-4.957	0.719	5.475	0.059	0.449
Investment	0.022	0.0005	2.113	0.0001	0.425	0.0001	0.399	0.0001	0.333	-0.0015	-6.730	0.002	9.157	0.0002	0.915
Consumption	8.711	-0.249	-2.860	0.007	0.080	-0.040	-0.454	-0.059	-0.679	0.347	3.986	-0.303	-3.484	0.046	0.530
Panel D: Model with Part Time Option															
LFPR	41.587	0.735	1.767	0.100	0.241	1.283	3.085	1.327	3.190	-0.742	-1.783	1.612	3.875	0.652	1.568
Effective Labor	0.599	0.010	1.749	0.002	0.256	0.015	2.468	0.017	2.811	-0.016	-2.719	0.027	4.444	0.009	1.518
Pre-tax Income	11.405	0.226	1.985	0.028	0.247	0.107	0.937	0.147	1.290	-0.514	-4.503	0.707	6.196	0.079	0.696
Average lnw	2.631	0.001	0.031	0.001	0.036	-0.065	-2.467	-0.061	-2.321	-0.088	-3.331	0.165	6.280	-0.017	-0.628
Human Capital	14.083	0.205	1.455	0.025	0.175	0.108	0.765	0.138	0.977	-0.438	-3.110	0.615	4.365	0.051	0.365
Investment	0.044	0.0009	2.023	0.00001	0.018	0.0022	4.977	0.0022	5.096	-0.0006	-1.267	0.0026	5.854	0.0020	4.470
Consumption	10.097	-0.425	-4.213	0.000	0.005	-0.074	-0.737	-0.063	-0.626	0.424	4.195	-0.303	-3.003	0.186	1.847

[a]The "Level" column refers to the annual value averaged over the whole life-cycle, except the "LFPR" which is the total number of years worked over the whole life-cycle. For example, in the exogenous model, the total LFPR is 38.675 years from 18 to 80.

[b]The "ΔLevel" column refers to the difference of the total value between the current experiment and the baseline model. For example, in the "No Earnings Test" case, the LFPR is 0.072 years higher than that in the baseline exogenous model across the whole life-cycle from 18 to 80.

[c]The "%Δ" column refers to the percentage of the difference in the "ΔLevel" column relative to the level in the baseline model. For example, in the "No Earnings Test" case, the LFPR increases by 0.072 years which is equivalent to 0.186% of the LFPR in the baseline exogenous model.

[d]In the exogenous model, the effective labor is same as LFPR.

9 Robustness Check

Recall that some of parameters are set to certain values taken from the previous literature. In this section we vary those pre-set parameters to see how they affect our estimation results. In particular, we check following variants: (1) increase the consumption floor $\underline{c}$ from 2.19 to 2.5; (2) decrease the consumption floor $\underline{c}$ from 2.19 to 1.8; (3) decrease the time discount rate β from 0.97 to 0.96 but increase the interest r from 0.03 to 0.04; (4) increase the initial asset A_0 from 0.0 to 50,000. In each case, all other pre-set parameters are kept the same as the baseline model, and then we re-estimate all of the parameters of the model. The estimation results are listed in Table 9, and the moments are plotted in Figure 10.

In all cases the simulated moments fit the data moments quite well. Varying pre-set parameters does change the estimated values of some parameters, but in all variants our model generates simulated auxiliary model which match data auxiliary model quite well.

10 Conclusion

This paper develops and estimates a rich life-cycle model that merges a Ben-Porath style human capital framework with a neoclassical style framework with endogenous labor supply and retirement framework. In the model, each individual makes decisions on consumption, human capital investment, labor supply and retirement. Investment in human capital generates wage growth over the life-cycle, while depreciation of human capital is the main force generating retirement. We show that the parsimonious model is able to fit the main features of life-cycle labor supply, wages (with and without fixed effects) as well as retirement. In particular we can fit both the large increase in wages and small changes in labor supply at the beginning of the life-cycle along with the small changes in wages but large changes in labor supply at the end. We incorporate health and part time option into the model individually and show that while both are important factors, human capital remains the main explanation for the decline in labor supply for older workers.

Despite the fact that our framework does not rely on age or time varying preference or production function parameters, our model implies a rather small and empirically plausible Marshallian elasticity rises with age. We also estimate the same basic framework using two different approaches to human capital accumulation—exogenous human capital as well as learning-by-doing. While we find that the baseline model is better at replicating the main features of the data, the learning-by-doing captures the main features. The

Table 9: Estimates in the baseline model and variants[a]

MODEL SPECIFICATIONS		Baseline	1 Larger $\underline{c}$	2 Lower $\underline{c}$	3 Change δ,r	4 Larger A_0
Interest rate	r	0.03			0.04	
Discount	β	0.97			0.96	
Initial wealth	A_0	0.0				50,000
Consumption floor	$\underline{c}$	2.19	2.5	1.8		
HC depreciation[b]	δ	0.109	0.109	0.109	0.108	0.123
HC prod: I factor	α_I	0.067	0.067	0.064	0.010	0.065
HC prod: H factor	α_H	0.123	0.123	0.123	0.056	0.133
St. Dev. of HC innovation	σ_ξ	0.002	0.001	0.001	0.001	0.002
Consump: CRRA	η_c	4.040	4.040	4.040	4.055	4.041
CS: coef on t $(\times 10)$[c]	φ_1	0.259	0.259	0.281	0.454	0.325
CS: coef on t^2 $(\times 10^2)$	φ_2	0.123	0.123	0.131	0.079	0.111
CS: coef on t^3 $(\times 10^3)$	φ_3	-0.032	-0.032	-0.035	-0.031	-0.032
CS: coef on married	φ_4	0.569	0.569	0.525	0.662	0.633
Leisure: St. Dev. of shock	a_ε	0.265	0.265	0.265	0.194	0.269
Leisure: spouse not working	a_1	0.597	0.597	0.597	0.625	0.490
Leisure: spouse working	a_2	-0.566	-0.566	-0.566	-0.794	-0.363
Bequest weight	b_1	18,069,750	18,070,066	18,018,962	17,443,348	31,409,836
Parameter heterogeneity						
Leisure: mean of intercept	μ_{a_0}	-5.582	-5.582	-5.581	-5.569	-5.758
Leisure: St. Dev. of intercept	σ_{a_0}	0.907	0.907	0.890	0.754	0.710
HC prod., Mean	μ_π	1.805	1.805	1.805	1.802	1.948
HC prod., Stand. Dev.	σ_π	0.675	0.674	0.673	0.646	0.718
Correlation between $a_0 \& \pi$	ρ	-0.989	-0.993	-0.987	-1.000	-0.999
Initial HC level at age 18						
Intercept	γ_0	1.660	1.659	1.654	1.513	1.582
Coefficient on a_0	γ_{a_0}	0.064	0.064	0.063	0.049	0.025
Coefficient on π	γ_π	0.609	0.609	0.605	0.578	0.515
St. Dev. of error term	σ_{H_0}	0.007	0.005	0.002	0.169	0.017

[a]Indirect Inference estimates. Estimates use a diagonal weighting matrix.
[b]"HC" stands for "Human Capital."
[c]"CS" stands for "Consumption Shifter."

exogenous model does not. In our baseline model, the level of human capital falls for people working with their wages being flat due to investment on the job. This mechanism which is intrinsic to the Ben-Porath framework is not in play in either the learning-by-doing framework or the exogenous human capital model and this plays a central role in generating a better fit. The model is robust to several robustness checks.

We use the estimated model to simulate the impacts of various policy changes. While prior work typically takes the wage process as given and focuses on the retirement decision, we are able to model the effect of the policy change on the wage process and the labor supply decisions. As we show in our model, less generous Social Security benefits result in higher labor supply later in the life-cycle, so workers adjust their investment over the life-cycle. This results in a higher human capital level as well as higher labor supply earlier in the life-cycle. Compared with the baseline model, the labor supply response to policy changes are much smaller in most experiments in the learning-by-doing model, but slightly larger in most experiments when human capital is completely exogenous. The bottom line is that modeling labor supply and human capital decisions jointly is critical in an analysis of the effects of policy changes. While presumably other factors would be important for explaining other features of labor markets, endogenous labor supply is critical for understanding life-cycle human capital investment and life-cycle human capital investment is critical for understanding life-cycle labor supply.

Figure 1a: Labor force participation rate—SIPP data
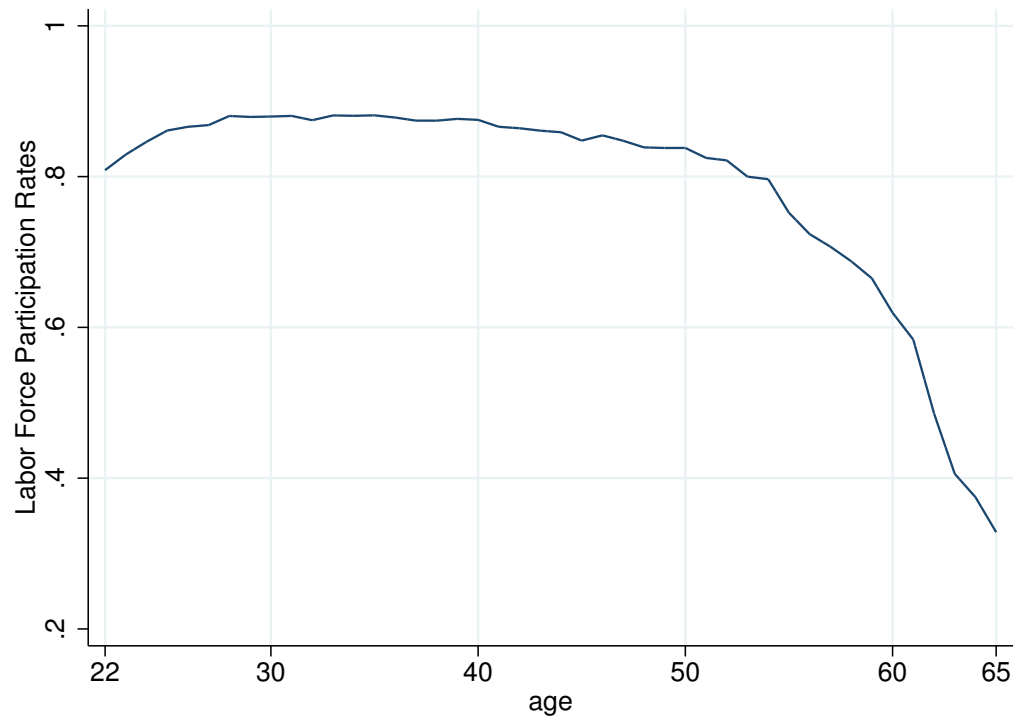
Figure 1b: Log wages with and without controlling for individual fixed effects—SIPP data
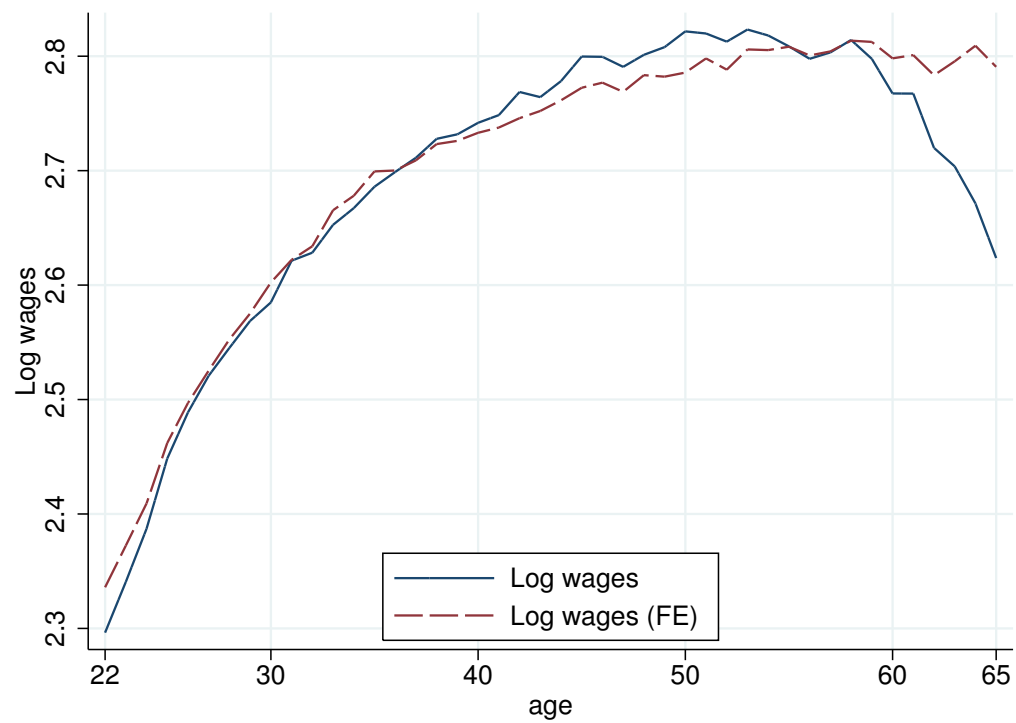
Figure 1c: Standard deviation of log wages—SIPP data
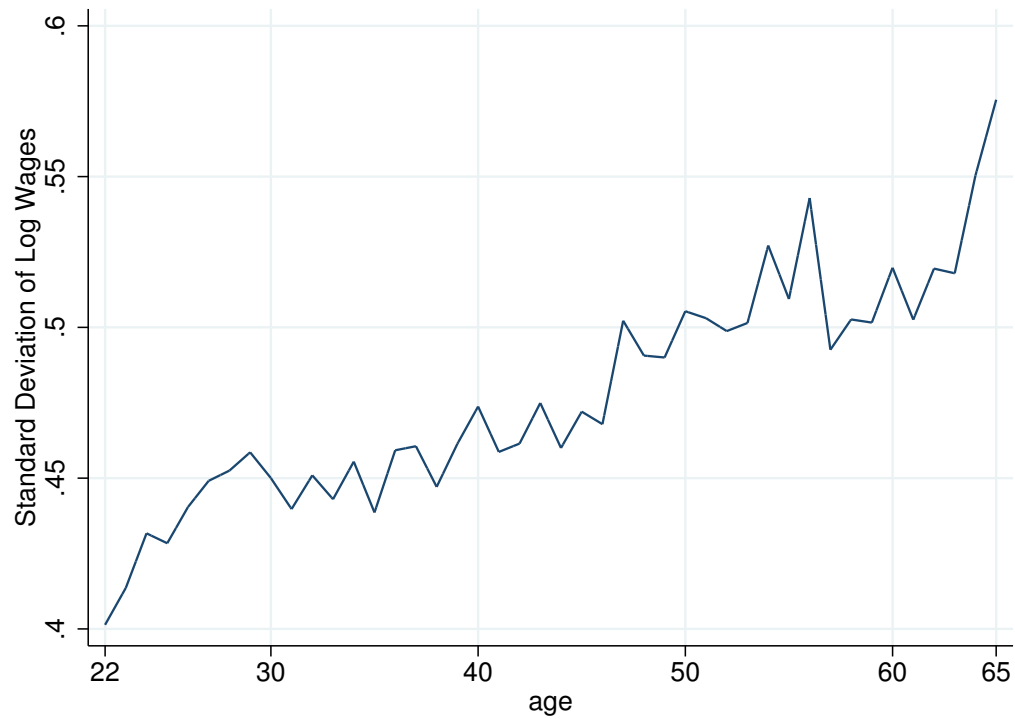
Figure 2: Log wage profiles of male high school graduates with and without controlling for individual fixed effects, CPS MORG and March data.
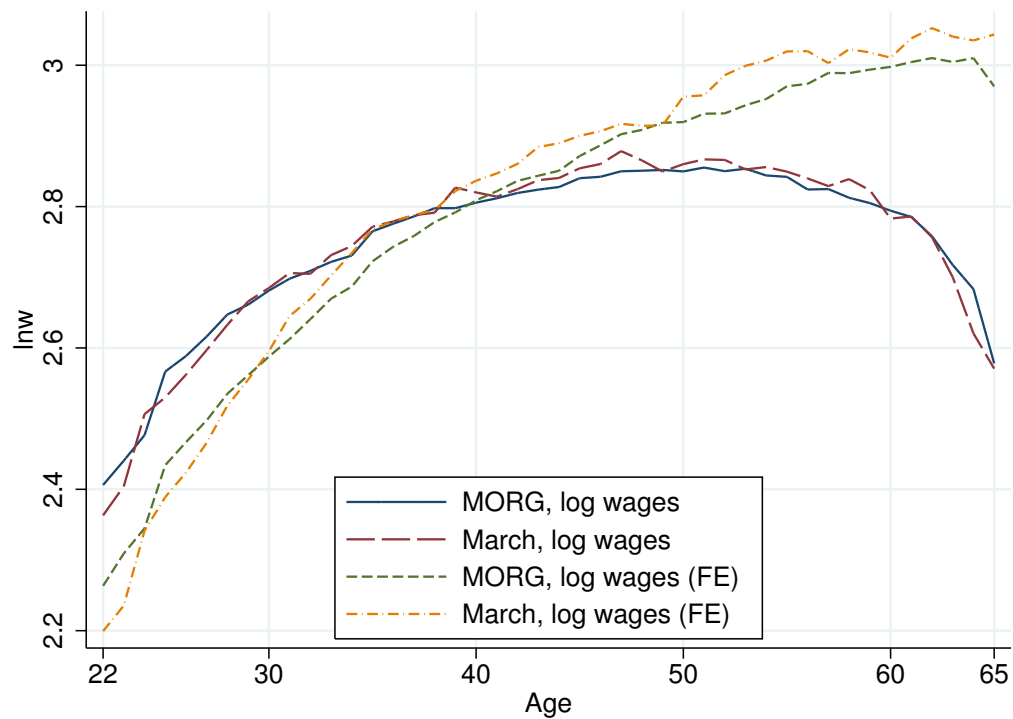
Figure 3a: Fit of model: labor force participation rate
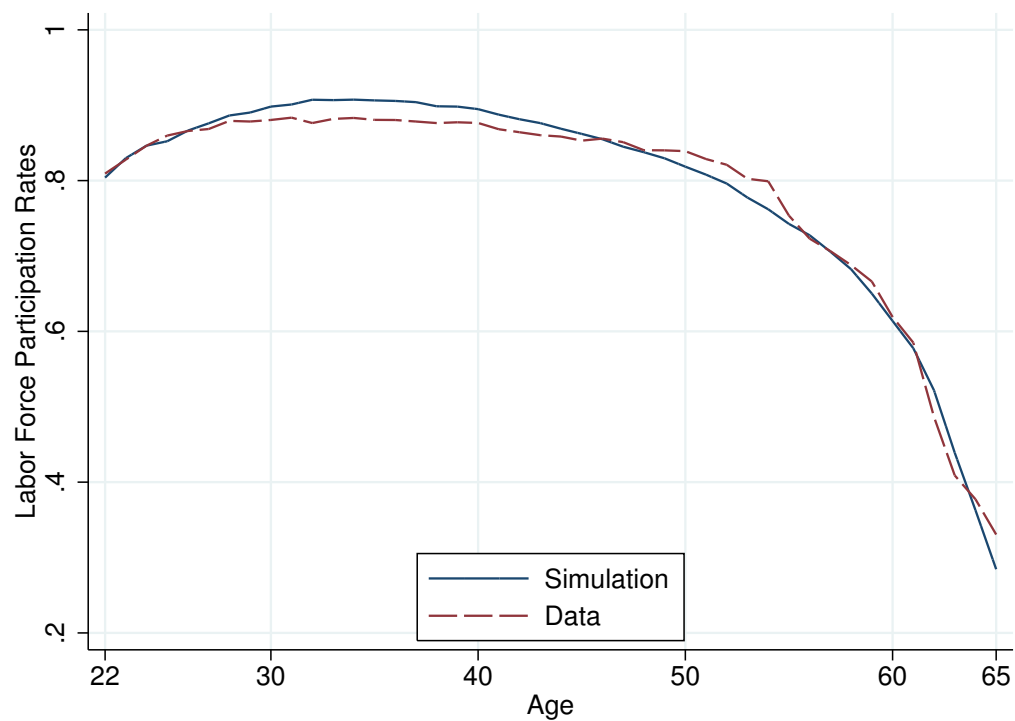
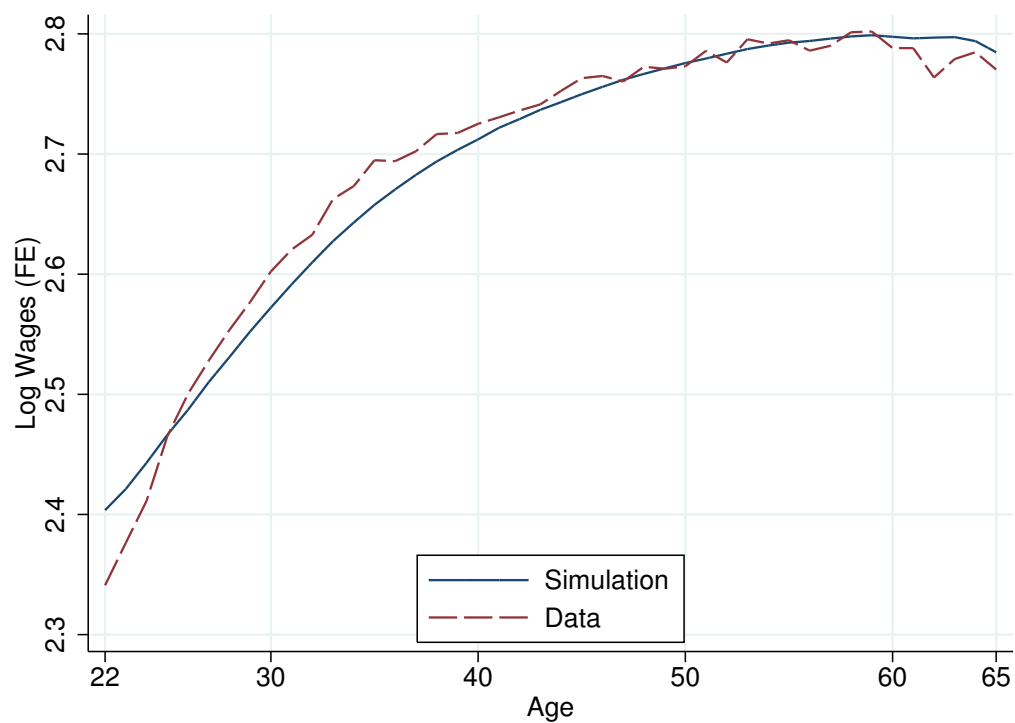Figure 3b: Fit of model: log wages after controlling for individual fixed effects

Figure 3c: Fit of model: log wages
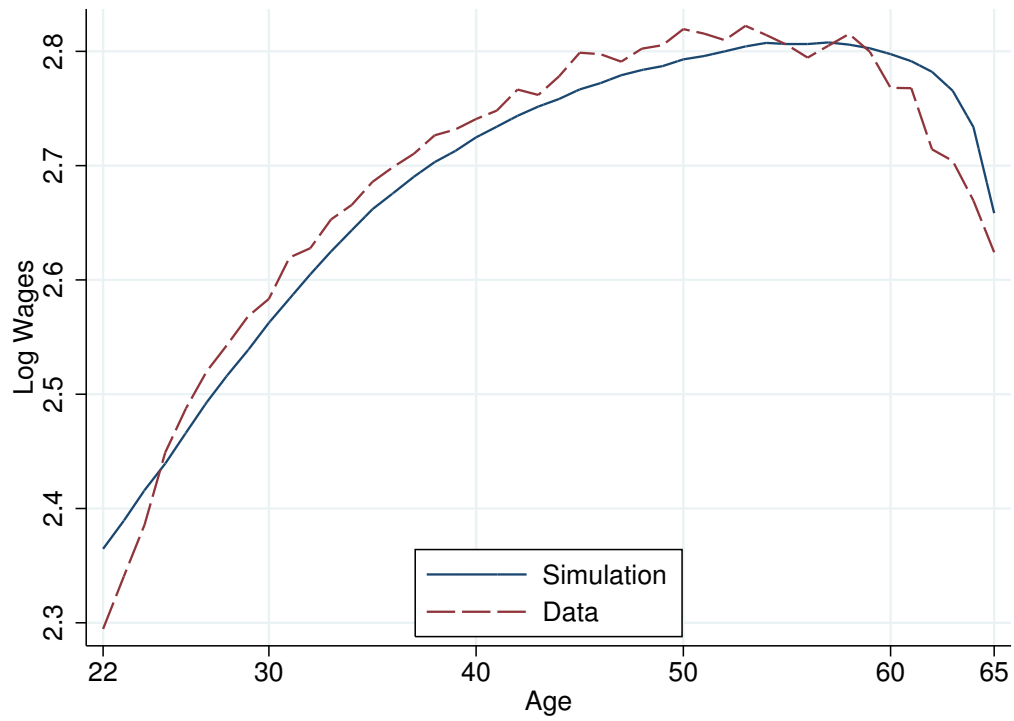
Figure 3d: Fit of model: standard deviations of log wages
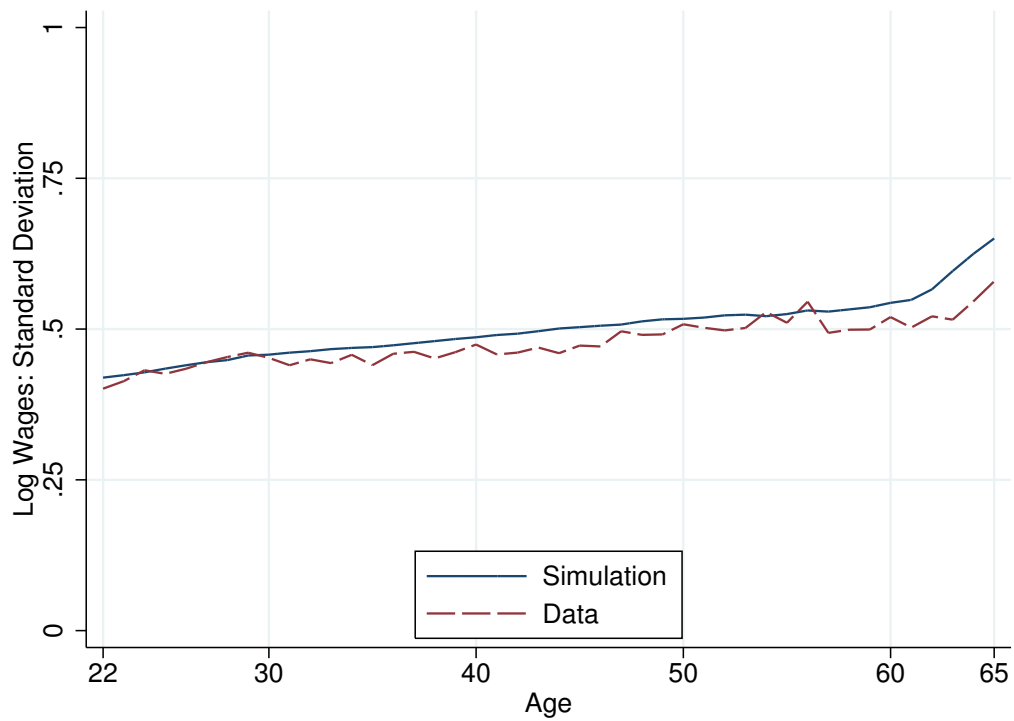
Figure 3e: Fit of model: adult equivalent consumption
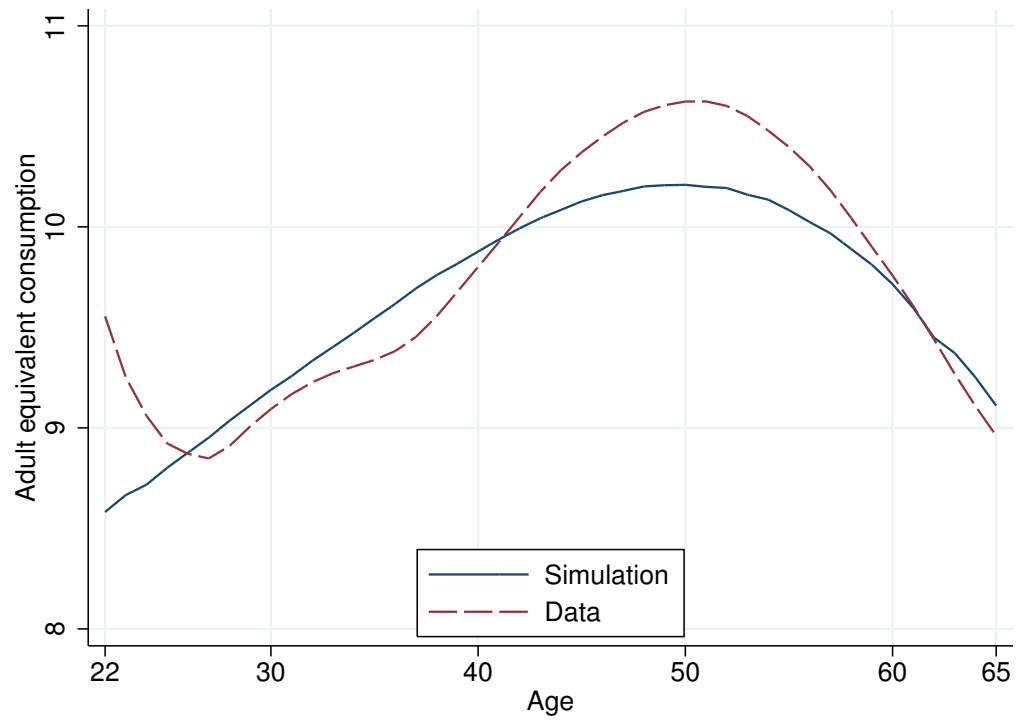
Figure 4a: Log wages and human capital
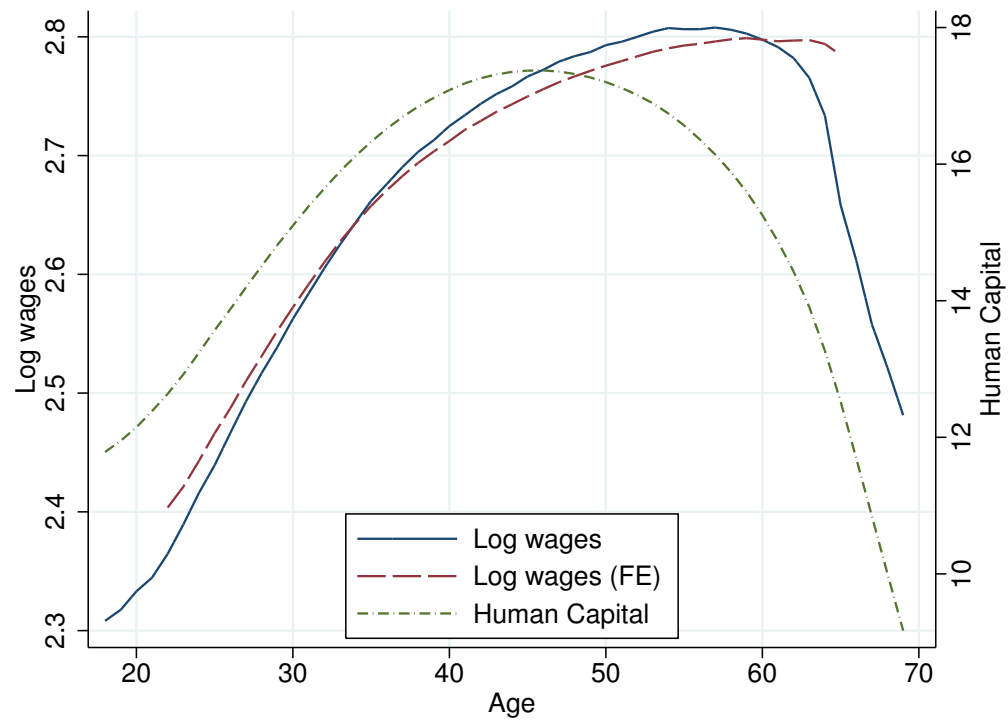
Figure 4b: Investment, and human capital
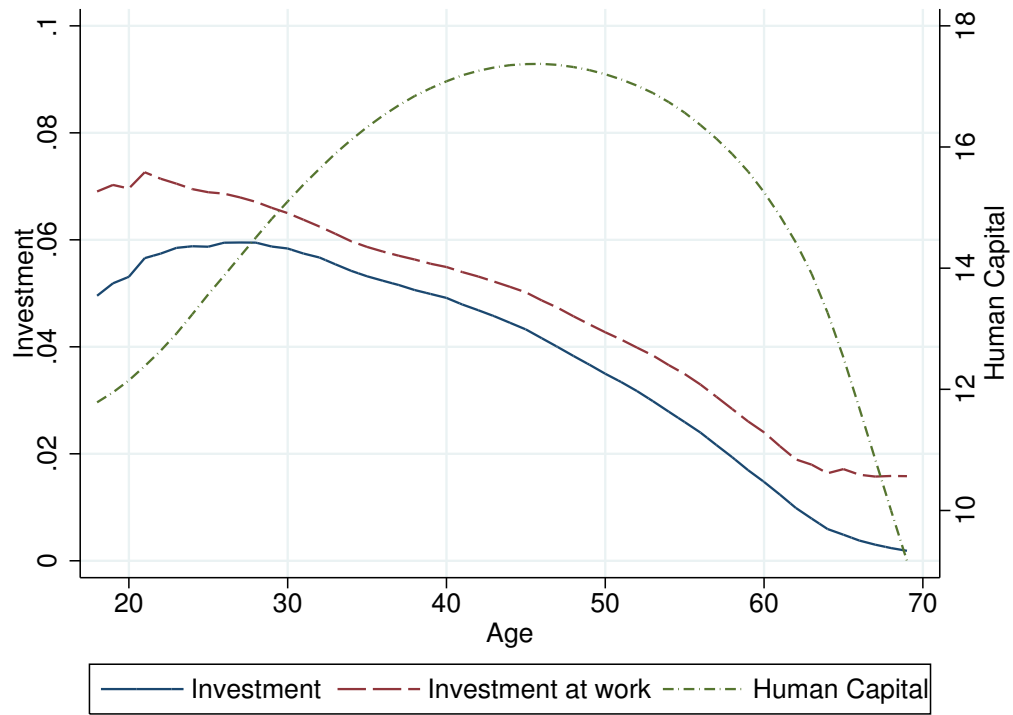
Figure 5a: Calculated elasticities
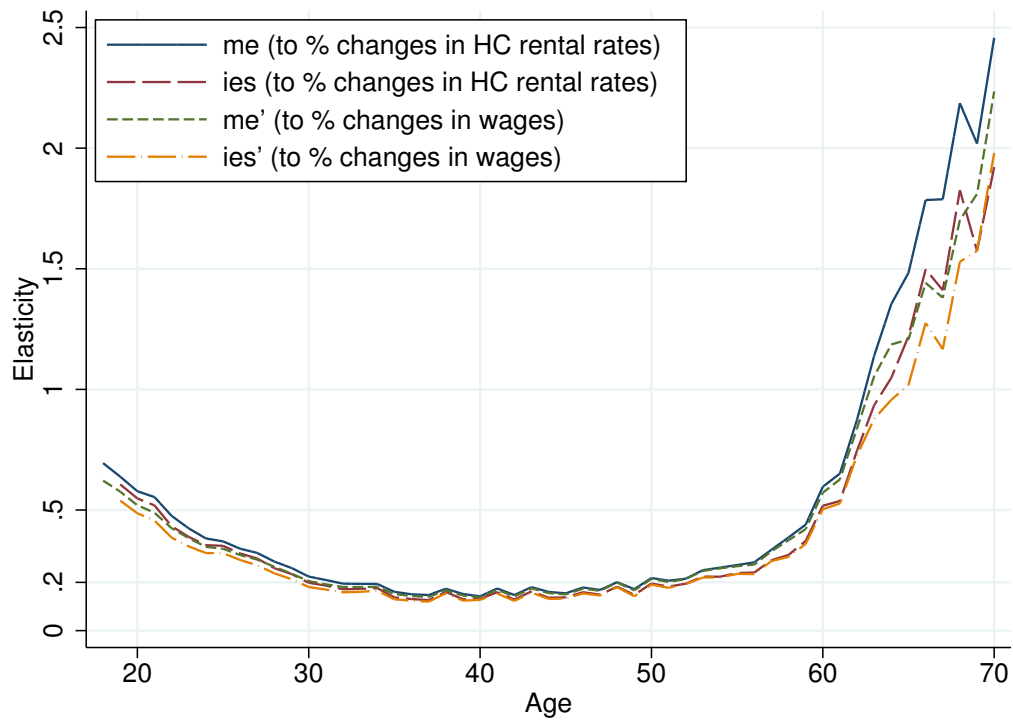
Figure 5b: Labor force participation rates (LFPR) for positive shocks

(i) Responses in LFPR.

(ii) Responses in investment.

(iii) Responses in human capital.

(iv) Decomposition of LFPR Profiles.
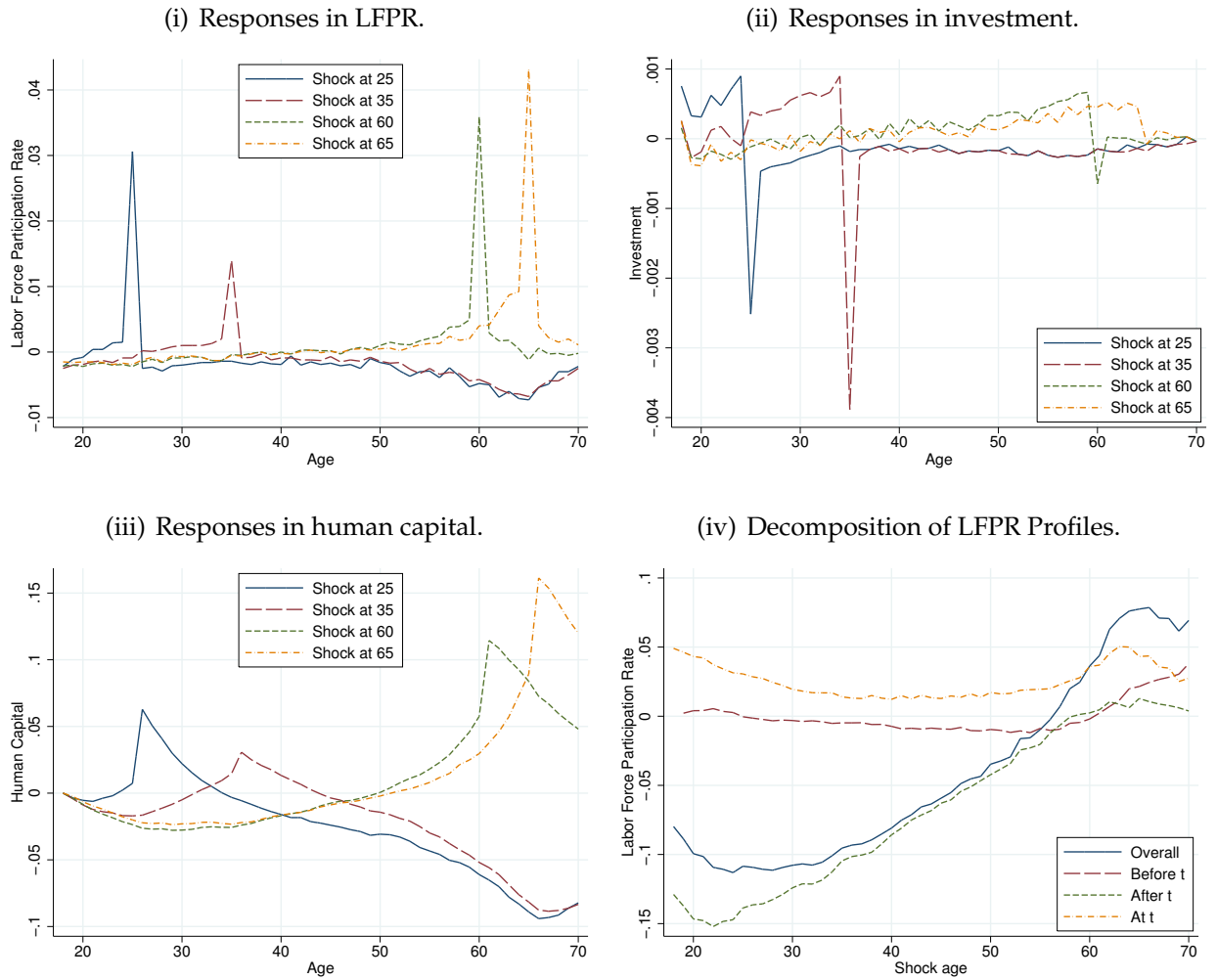
Figure 6a: Fit of model with health

(i) Labor Force Participation Rates

(ii) LFPR difference, good vs bad health

(iii) Log Wages & Log Wages (FE)

(iv) Standard Deviation of Log Wages

(v) Consumption
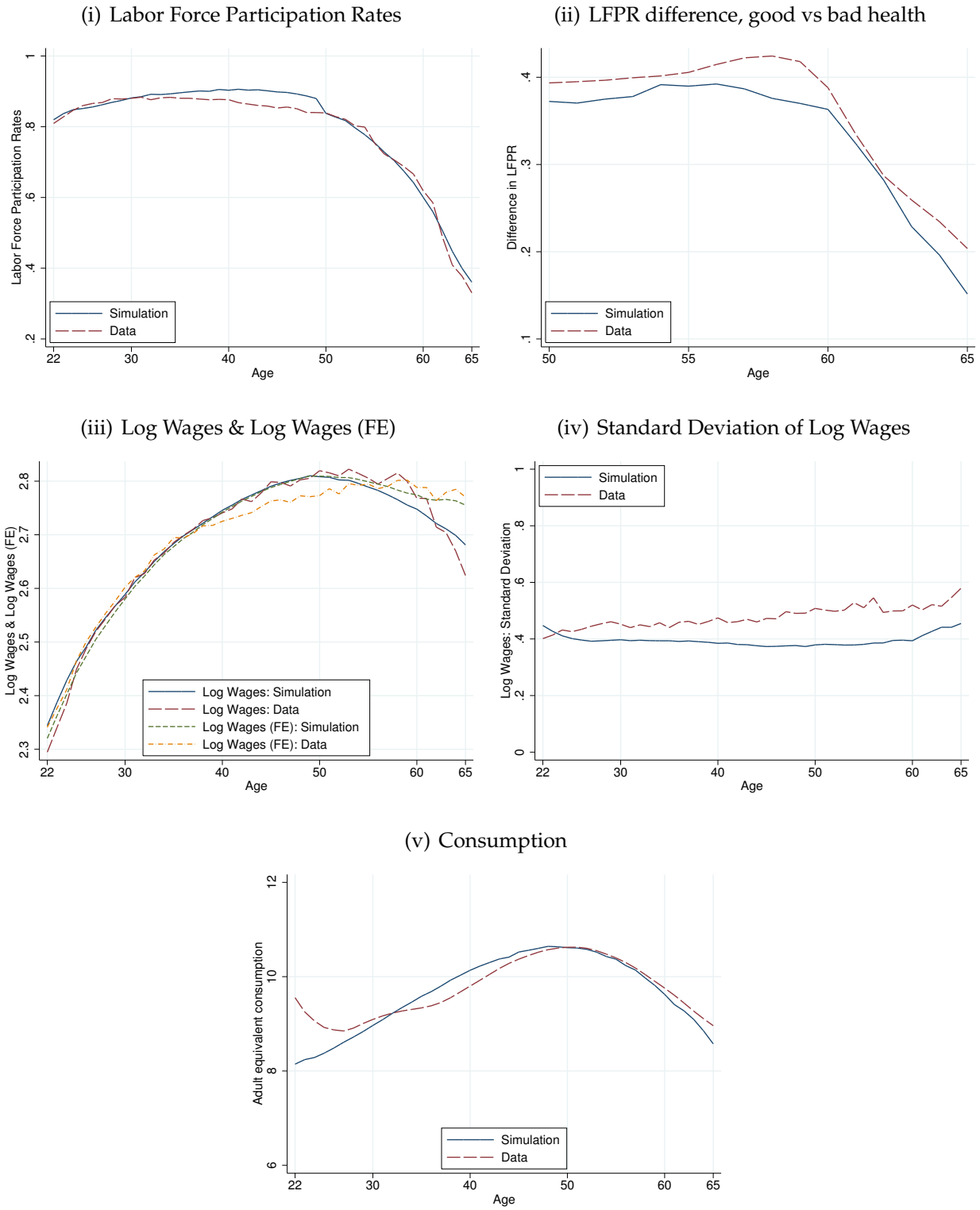
Figure 6b: Sensitivity to heath preferences: health status fixed and taste for leisure unchanged after age 50
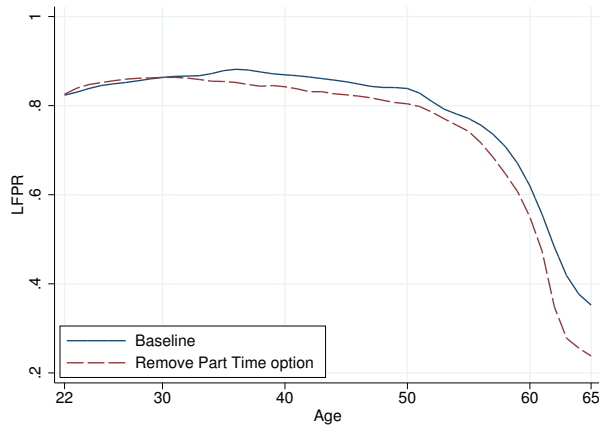
(i) Labor Force Participation Rates

(ii) Human Capital Levels
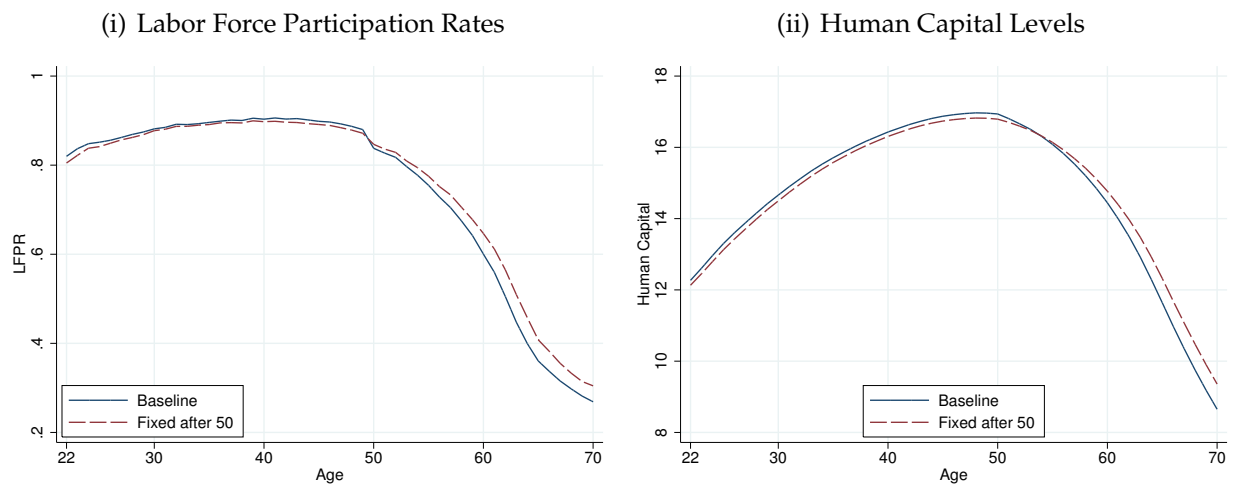
Figure 7a: Fit of model with part time

(i) Labor Force Participation Rates

(ii) LFPR, Part Time

(iii) Log Wages & Log Wages (FE)

(iv) Standard Deviation of Log Wages

(v) Consumption

48
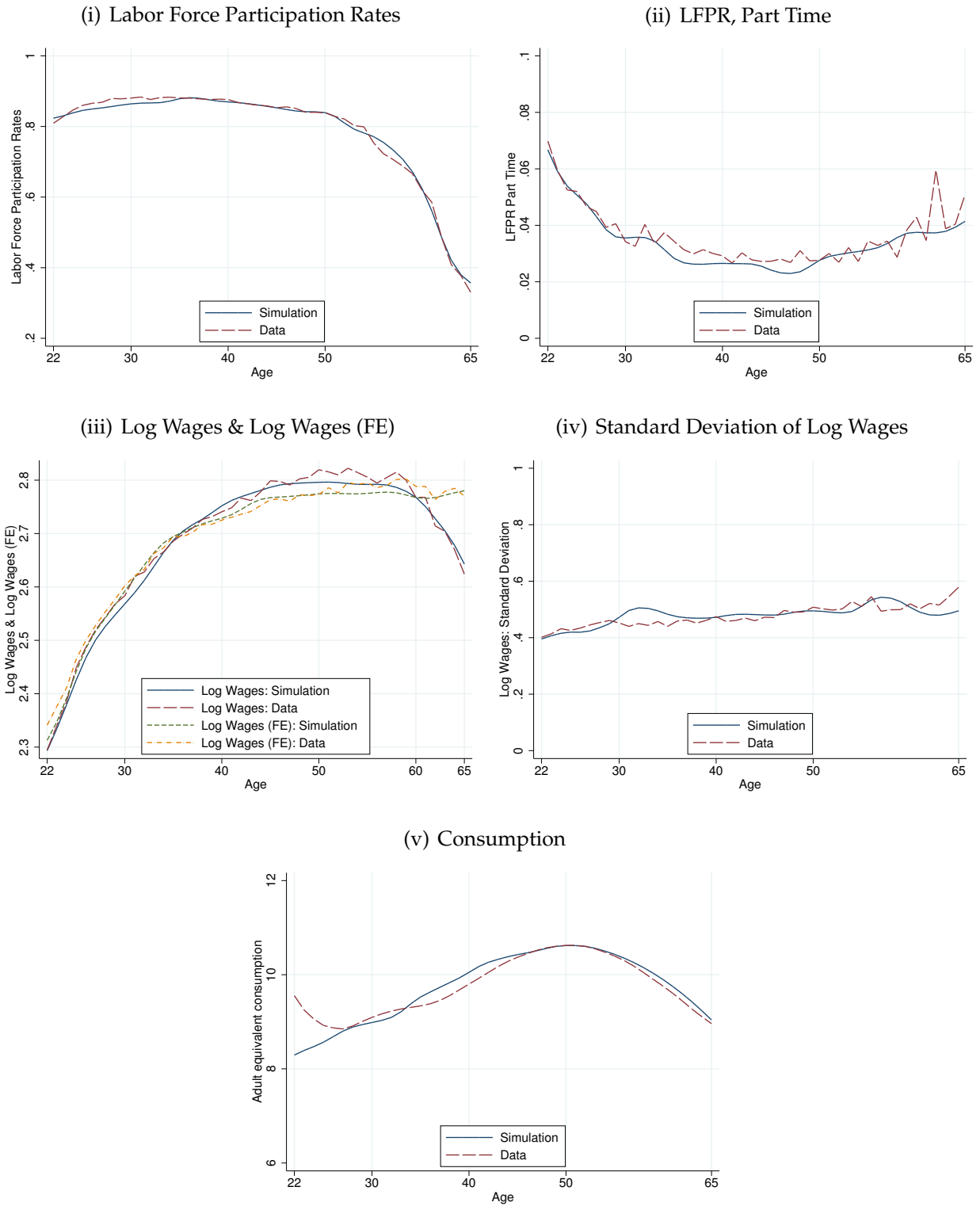
Figure 7b: Sensitivity to part time option: turn off the part time option

(i) Labor Force Participation Rates

(ii) Human Capital Levels
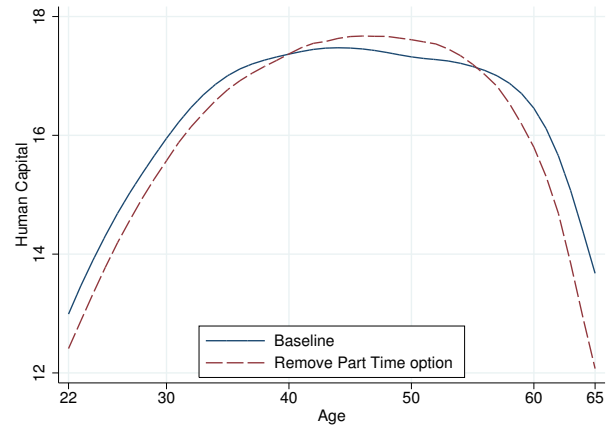
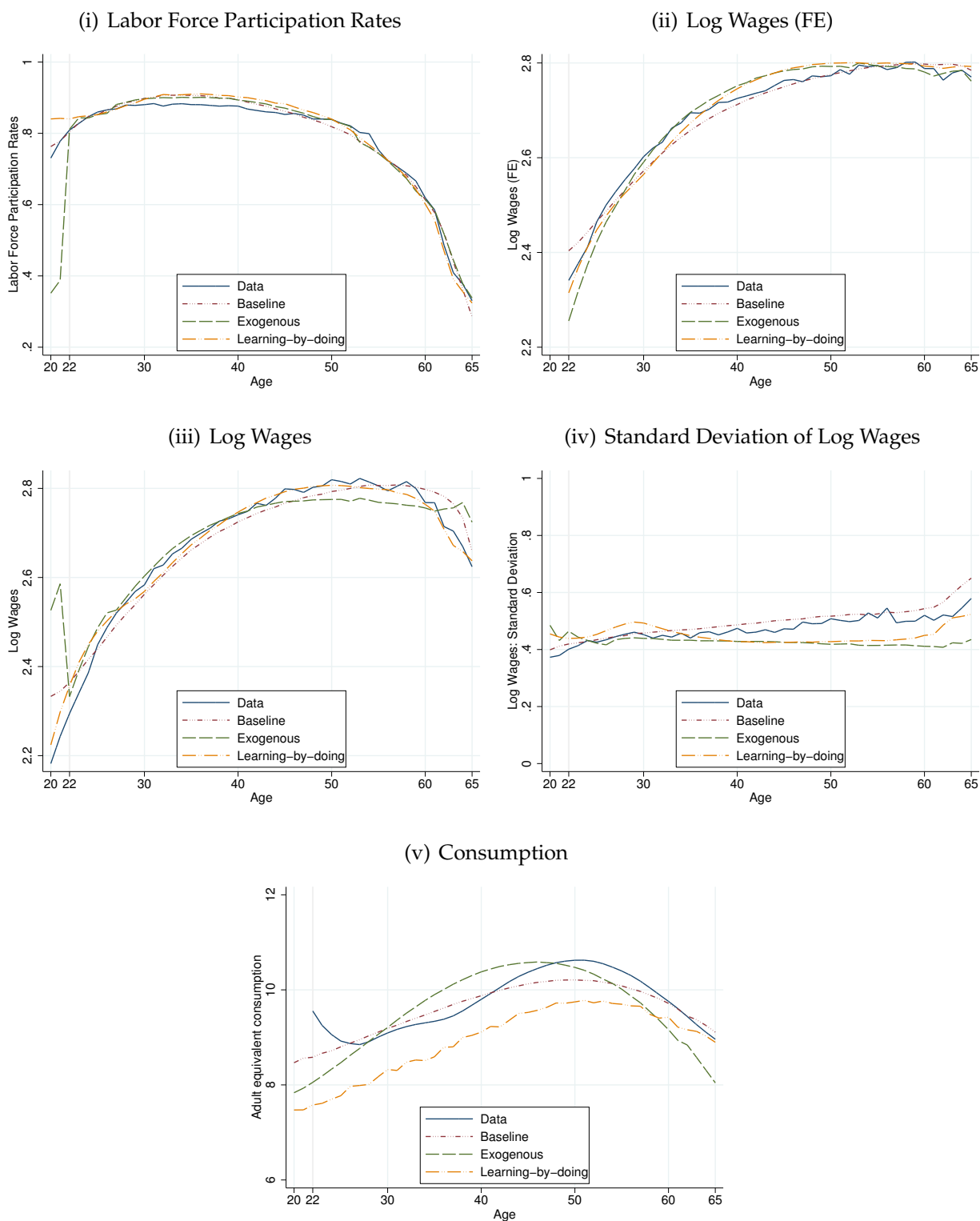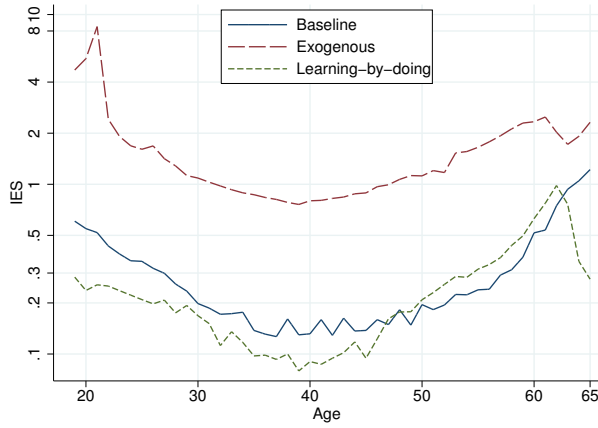# Figure 8a: Exogenous and learning-by-doing models moments

(i) Labor Force Participation Rates

(ii) Log Wages (FE)

(iii) Log Wages

(iv) Standard Deviation of Log Wages

(v) Consumption

Figure 8b: Comparison of the Intertemporal Elasticities of Substitution (IES).

(i) LFPR Responses to % Changes in H Rental Rate
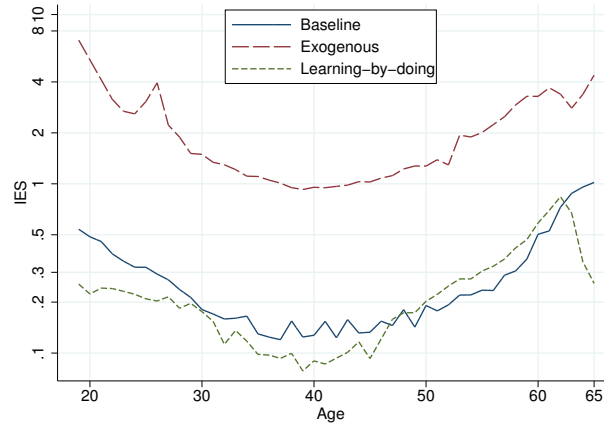

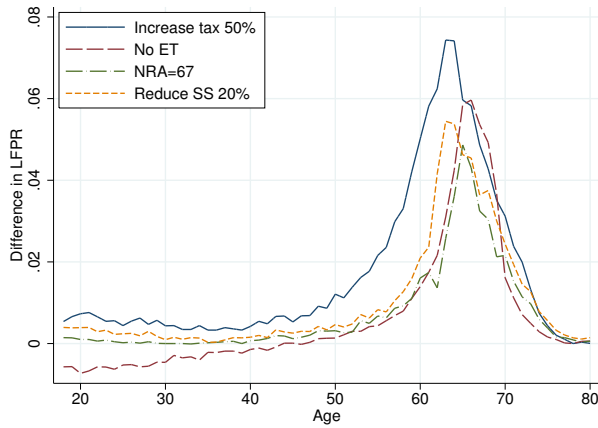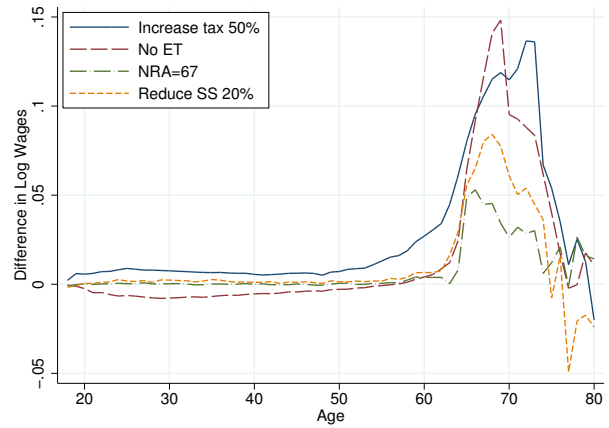### (ii) LFPR Responses to % Changes in Wages

Figure 9a: [Baseline model] Policy experiments: reduce Social Security benefits

(i) Difference in Labor Force Participation Rates

(ii) Difference in Log Wages

(iii) Difference in Investment

(iv) Difference in Human Capital

Figure 9b: [Baseline model] Policy experiments: remove Social Security taxes or benefits
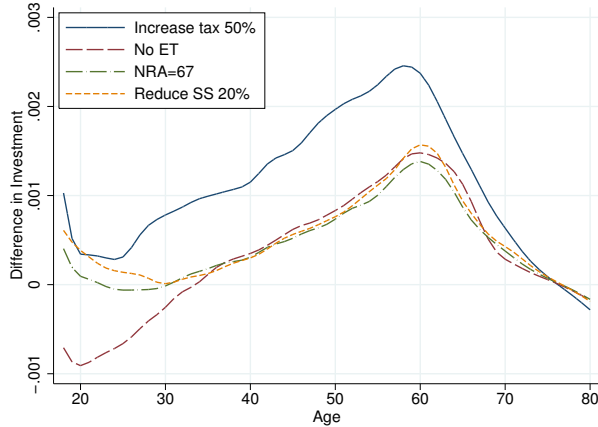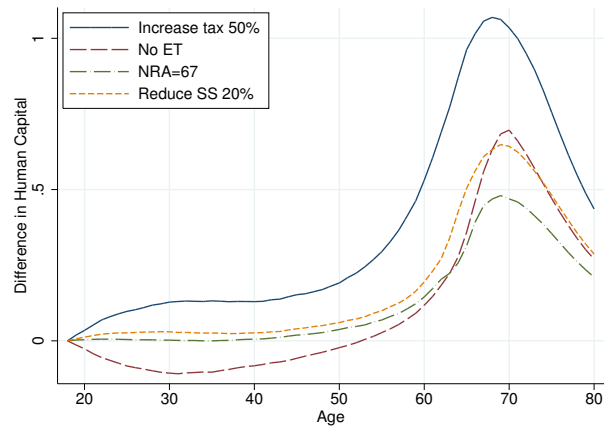
(i) Difference in Labor Force Participation Rates

(ii) Difference in Log Wages

(iii) Difference in Investment
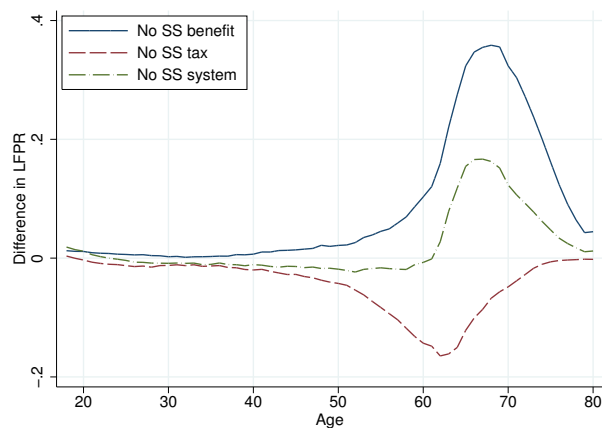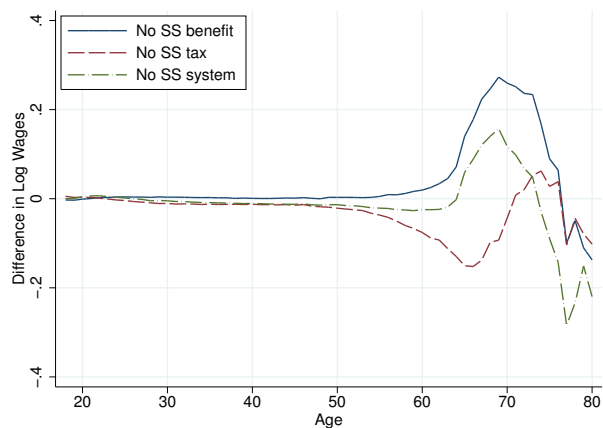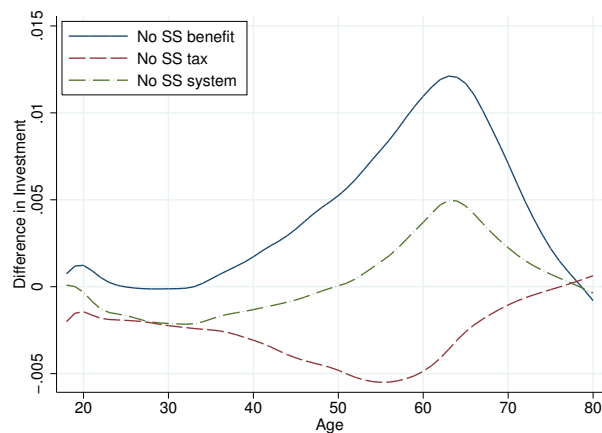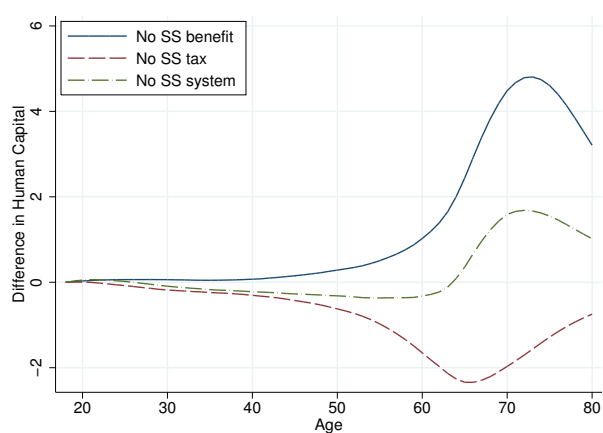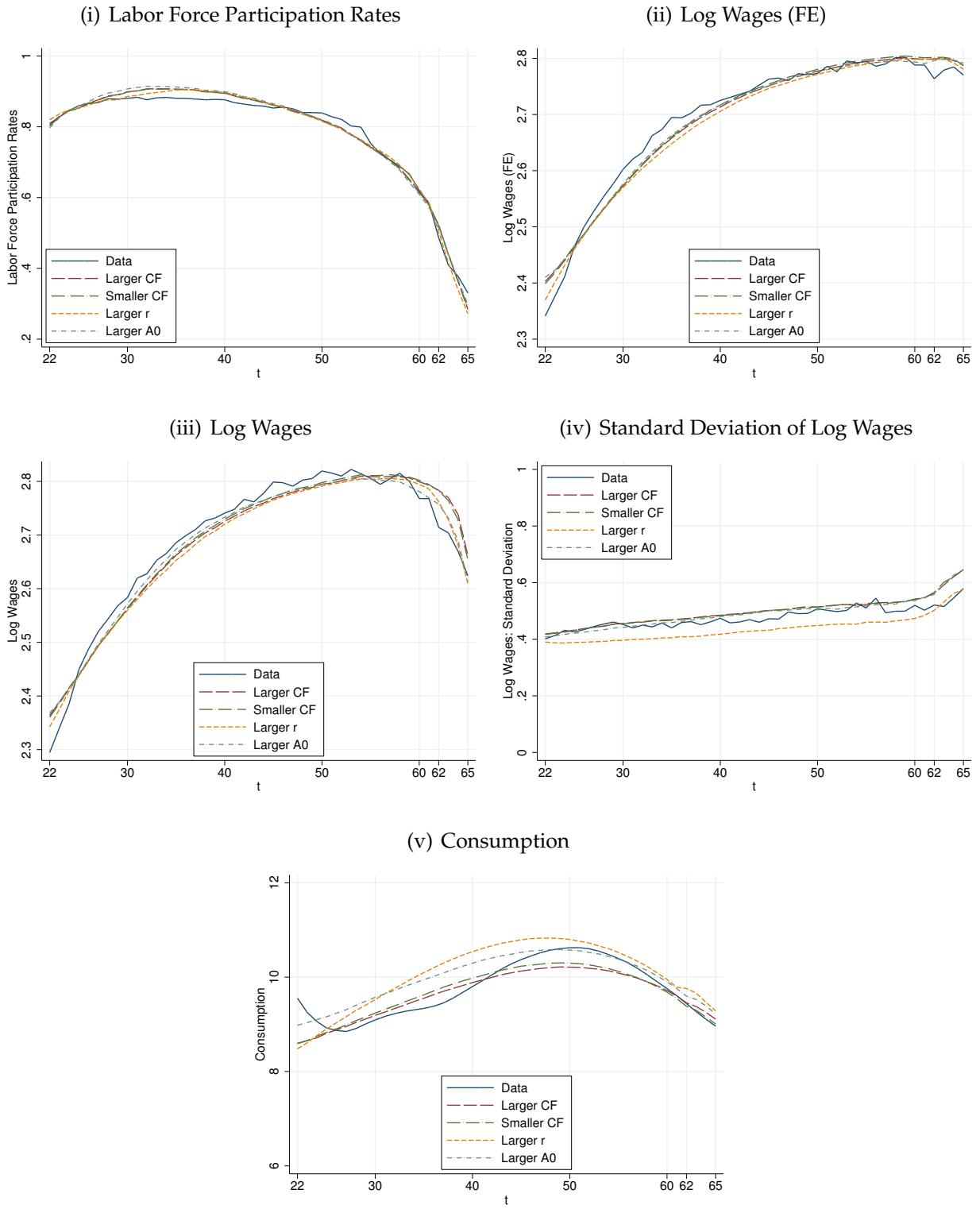

(iv) Difference in Human Capital

Figure 10: Fit of alternative models

(i) Labor Force Participation Rates

(ii) Log Wages (FE)

(iii) Log Wages

(iv) Standard Deviation of Log Wages

(v) Consumption

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Appendix

A Proof of Proposition 1

The solution to 14 is that the individual works if and only if

$$\widetilde{V}_{t,0}\left(X_t\right) \geq \widetilde{V}_{t,1}\left(X_t\right) + \gamma_t.$$

This means that there exists a threshold value

$$\gamma_t^* = \widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right)$$

such that

$$\ell_t = \begin{cases} 1, & \text{if } \gamma_t \geq \gamma_t^* \\ 0, & \text{if } \gamma_t < \gamma_t^* \end{cases}.$$

From (13) we have

$$\ell_t = \begin{cases} 1, & \text{if } \varepsilon_t \geq \varepsilon_t^* \\ 0, & \text{if } \varepsilon_t < \varepsilon_t^* \end{cases}.$$

where ε_t^* is defined as (15).

We know that

$$\widetilde{V}_{t,0}\left(X_t\right) > \widetilde{V}_{t,1}\left(X_t\right)$$

thus

$$\gamma_t^* = \widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right) > 0$$

and there exists

$$\varepsilon_t^* \equiv \frac{1}{a_\varepsilon}\left\{\log\left(\gamma_t^*\left(X_t\right)\right) - a_0\right\}$$

Conditional on X_t, the expected value is straightfoward,

$$E\left[V_t\left(X_t, \gamma_t\right)\mid X_t\right] = \Phi\left(\varepsilon_t^*\right)\widetilde{V}_{t,0}\left(X_t\right) + \left(1 - \Phi\left(\varepsilon_t^*\right)\right)\left[\widetilde{V}_{t+1,1}\left(X_{t+1}\right) + E\left(\gamma_t\mid \varepsilon_t \geq \varepsilon_t^*\right)\right]$$

Since γ_t is log-normal, we have

$$E\left(\gamma_t\mid \varepsilon_t \geq \varepsilon_t^*\right) = E\left(\gamma_t\mid \gamma_t \geq \gamma_t^*\right) = exp\left(\tilde{a}_0 + \frac{a_\varepsilon^2}{2}\right)\frac{\Phi\left(a_\varepsilon - \varepsilon_t^*\right)}{\Phi\left(-\varepsilon_t^*\right)}$$

Q.E.D.

B Proof of Proposition 2

The solution to [26] is

$$
\ell_t = \begin{cases}
0 & \widetilde{V}_{t,0}\left(X_t\right) > \max\left\{\widetilde{V}_{t,p}\left(X_t\right) + \gamma_t p,\ \widetilde{V}_{t,1}\left(X_t\right) + \gamma_t\right\} \\
p & \widetilde{V}_{t,p}\left(X_t\right) + \gamma_t p > \max\left\{\widetilde{V}_{t,0}\left(X_t\right),\ \widetilde{V}_{t,1}\left(X_t\right) + \gamma_t\right\} \\
1 & \widetilde{V}_{t,1}\left(X_t\right) + \gamma_t > \max\left\{\widetilde{V}_{t,0}\left(X_t\right),\ \widetilde{V}_{t,p}\left(X_t\right) + \gamma_t p\right\}
\end{cases}
$$

or

$$
\ell_t = \begin{cases}
0, & \gamma_t < \min\left\{\dfrac{\widetilde{V}_{t,0}(X_t) - \widetilde{V}_{t,p}(X_t)}{p},\ \widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right)\right\} \\
p, & \dfrac{\widetilde{V}_{t,0}(X_t) - \widetilde{V}_{t,p}(X_t)}{p} < \gamma_t < \dfrac{\widetilde{V}_{t,p}(X_t) - \widetilde{V}_{t,1}(X_t)}{1-p}, \\
1, & \gamma_t > \max\left\{\dfrac{\widetilde{V}_{t,p}(X_t) - \widetilde{V}_{t,1}(X_t)}{1-p},\ \widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right)\right\}
\end{cases}
$$

First we know that

$$
\widetilde{V}_{t,0}\left(X_t\right) > \widetilde{V}_{t,p}\left(X_t\right) > \widetilde{V}_{t,1}\left(X_t\right)
$$

thus given $p \in (0,1)$, we have

$$
\begin{aligned}
\widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right) &> 0 \\
\frac{\widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,p}\left(X_t\right)}{p} &> 0 \\
\frac{\widetilde{V}_{t,p}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right)}{1-p} &> 0
\end{aligned}
$$

Since $\widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right)$ is a weighted average of $\frac{\widetilde{V}_{t,0}(X_t) - \widetilde{V}_{t,p}(X_t)}{p}$ and $\frac{\widetilde{V}_{t,p}(X_t) - \widetilde{V}_{t,1}(X_t)}{1-p}$, we have

$$
\min\left\{\frac{\widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,p}\left(X_t\right)}{p},\ \frac{\widetilde{V}_{t,p}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right)}{1-p}\right\} \le \widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right) \le \max\left\{\frac{\widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,p}\left(X_t\right)}{p},\ \frac{\widetilde{V}_{t,p}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right)}{1-p}\right\}
$$

Define

$$
\gamma^*_{t\,\min}\left(X_t\right) \equiv \min\left\{\frac{\widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,p}\left(X_t\right)}{p},\ \widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right)\right\}
$$

$$
\gamma^*_{t\,\max}\left(X_t\right) \equiv \max\left\{\frac{\widetilde{V}_{t,p}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right)}{1-p},\ \widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right)\right\}
$$

There are two cases.

Case 1.

$$\frac{\widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,p}\left(X_t\right)}{p} \leq \widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right) \leq \frac{\widetilde{V}_{t,p}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right)}{1-p}$$

where two equalities do not hold simultaneously. In this case, option $\ell_t = p$ will be chosen by any individual with a positive probability. We also have

$$\gamma_{t\min}^* = \frac{\widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,p}\left(X_t\right)}{p}$$

$$\gamma_{t\max}^* = \frac{\widetilde{V}_{t,p}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right)}{1-p}$$

Solving ε_t from (13) yields

$$\varepsilon_{tj}^* = \frac{1}{a_\varepsilon}\left\{\log\left(\gamma_{tj}^*\right) - \tilde{a}_0\right\}, \, j \in \{\min, \max\}$$

Since γ_t is log-normal, we have

$$E\left(\gamma_t | \varepsilon_t \geq \varepsilon_{tj}^*\right) = exp\left(\tilde{a}_0 + \frac{a_\varepsilon^2}{2}\right)\frac{\Phi\left(a_\varepsilon - \varepsilon_{tj}^*\right)}{\Phi\left(-\varepsilon_{tj}^*\right)}, \, j \in \{\min, \max\}$$

Note that

$$E\left(\gamma_t | \varepsilon_t \geq \varepsilon_{t\min}^*\right) = E\left(\gamma_t | \varepsilon_{t\min}^* < \varepsilon_t \leq \varepsilon_{t\max}^*\right)Pr\left(\varepsilon_t \leq \varepsilon_{t\max}^* \mid \varepsilon_t \geq \varepsilon_{t\min}^*\right)$$
$$+ E\left(\gamma_t | \varepsilon_t \geq \varepsilon_{t\max}^*\right)Pr\left(\varepsilon_t > \varepsilon_{t\max}^* \mid \varepsilon_t \geq \varepsilon_{t\min}^*\right)$$

so

$$E\left(\gamma_t \middle| \varepsilon^*_{t\min} < \varepsilon_t \le \varepsilon^*_{t\max}\right)$$

$$= \frac{E\left(\gamma_t \middle| \varepsilon_t \ge \varepsilon^*_{t\min}\right) - E\left(\gamma_t \middle| \varepsilon_t \ge \varepsilon^*_{t\max}\right) Pr\left(\varepsilon_t > \varepsilon^*_{t\max} \middle| \varepsilon_t \ge \varepsilon^*_{t\min}\right)}{Pr\left(\varepsilon^*_{t\min} < \varepsilon_t \le \varepsilon^*_{t\max} \middle| \varepsilon_t \ge \varepsilon^*_{t\min}\right)}$$

$$= \frac{exp\left(\tilde{a}_0 + \frac{a^2_\varepsilon}{2}\right) \frac{\Phi\left(a_\varepsilon - \varepsilon^*_{t\min}\right)}{\Phi\left(-\varepsilon^*_{t\min}\right)} - exp\left(\tilde{a}_0 + \frac{a^2_\varepsilon}{2}\right) \frac{\Phi\left(a_\varepsilon - \varepsilon^*_{t\max}\right)}{\Phi\left(-\varepsilon^*_{t\max}\right)} \frac{\Phi\left(-\varepsilon^*_{t\max}\right)}{\Phi\left(-\varepsilon^*_{t\min}\right)}}{\frac{\Phi\left(\varepsilon^*_{t\max}\right) - \Phi\left(\varepsilon^*_{t\min}\right)}{\Phi\left(-\varepsilon^*_{t\min}\right)}}$$

$$= \frac{exp\left(\tilde{a}_0 + \frac{a^2_\varepsilon}{2}\right) \frac{\Phi\left(a_\varepsilon - \varepsilon^*_{t\min}\right)}{\Phi\left(-\varepsilon^*_{t\min}\right)} - exp\left(\tilde{a}_0 + \frac{a^2_\varepsilon}{2}\right) \frac{\Phi\left(a_\varepsilon - \varepsilon^*_{t\max}\right)}{\Phi\left(-\varepsilon^*_{t\min}\right)}}{\frac{\Phi\left(\varepsilon^*_{t\max}\right) - \Phi\left(\varepsilon^*_{t\min}\right)}{\Phi\left(-\varepsilon^*_{t\min}\right)}}$$

$$= exp\left(\tilde{a}_0 + \frac{a^2_\varepsilon}{2}\right) \frac{\Phi\left(a_\varepsilon - \varepsilon^*_{t\min}\right) - \Phi\left(a_\varepsilon - \varepsilon^*_{t\max}\right)}{\Phi\left(\varepsilon^*_{t\max}\right) - \Phi\left(\varepsilon^*_{t\min}\right)}$$

Case 2.

$$\frac{\widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,p}\left(X_t\right)}{p} \ge \widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right) \ge \frac{\widetilde{V}_{t,p}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right)}{1 - p}.$$

In this case, option $\ell_t = p$ is a dominated strategy and will be chosen by any individual with zero probability. We also have

$$\gamma^*_{t\min} = \gamma^*_{t\max} = \widetilde{V}_{t,0}\left(X_t\right) - \widetilde{V}_{t,1}\left(X_t\right).$$

and

$$\Phi\left(\varepsilon^*_{t\max}\right) - \Phi\left(\varepsilon^*_{t\min}\right) = 0.$$

In either case, it is straightforward to calculate $E\left[V_t\left(X_t, \gamma_t\right) \middle| X_t\right]$. *Q.E.D.*

C   Taxes

We use tax codes in the year of 2004. There are two different kinds of taxes that the worker's wage income is subject to, namely the payroll taxes and the federal income taxes. We ignore the state income taxes. The payroll taxes include the Social Security portion, 6.2% capped at $87,900, and the Medicare tax, 1.45% uncapped. The federal income taxes are progressive and we use the tax rules under head of household. The personal exemption for each person is $3,100 and the standard deduction for head of

Table C1: Wage income tax codes (in 2004$).

Marginal Tax Rate	Pre-tax (Y)	Post-tax Income
0.0765	$\leq 10,250$	$0.9235Y$
0.1765	$10,251 - 20,450$	$9,465.88 + 0.8235\,(Y - 10,250)$
0.2265	$20,451 - 49,150$	$17,865.58 + 0.7735\,(Y - 20,450)$
0.3265	$49,151 - 87,900$	$40,065.03 + 0.6735\,(Y - 49,150)$
0.2645	$87,901 - 110,750$	$66,163.15 + 0.7355\,(Y - 87,900)$
0.2945	$110,751 - 172,950$	$82,969.33 + 0.7055\,(Y - 110,750)$
0.3445	$172,951 - 329,350$	$126,851.43 + 0.6555\,(Y - 172,950)$
0.3645	$\geq 329,351$	$229,371.63 + 0.6355\,(Y - 329,350)$

household is $7,150. These all together generate the tax codes used in the paper in Table C1.

D   Social Security

We use most Social Security rules in the year of 2004.[31]

D.1   The Social Security Benefits

The normal retirement age (NRA) is 65. The worker receives full Social Security benefits if he applies for the benefits at the NRA. The full retirement benefits are equal to the Primary Insurance Amount (PIA), which is a function of Average Indexed Monthly Earnings (AIME),

$$
\begin{aligned}
PIA \;=\; & 0.9 * min\,\{bp_1, AIME\} + 0.32 * min\,\{bp_2 - bp_1, max\,\{0, AIME - bp_1\}\} \\
& + 0.15 * max\,\{0, AIME - bp_2\}, \tag{D.1}
\end{aligned}
$$

where $(bp_1,\ bp_2) = (612,\ 3689)$.

The AIME is computed as the monthly average earning of the 35 years with highest inflation-adjusted earnings. Only earnings subject to the Social Security tax are used in the calculation and therefore AIME is capped. The included earning in a specific year is adjusted for wage inflation by multiplying the wage growth rate relative to the base year, which is at age 60. The wage growth rate is calculated by dividing the average wage in the base year by the average wage in that specific year. Earnings after the base year are not adjusted. Interestingly, the wage growth rate of the national average wage index is very similar to the growth rate of CPI-U after Year 1969, as shown in Figure D1, so we

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[31]Most of information about Social Security benefits in this section is extracted from http://www.ssa.gov.

ignore the small difference between these two and use the real wages to update AIME without adjustment.

Computing exact AIME requires keeping tracking of the worker's earning history, which is computationally infeasible. Instead we apply an approximating method, taking into account the wage growth pattern over the life-cycle,

$$AIME_{t+1} = AIME_t + max \left\{ 0, \frac{sse_t}{35 \times 12} - share_{min}(t) \cdot AIME_t \right\} \tag{D.2}$$

where $sse_t = min \{H_t(1 - \ell_t)(1 - I_t), s\bar{s}e\}$ is included earning, capped at $s\bar{s}e = \$87,900$. The $share_{min}$ is the share of minimum wage in AIME. Figure D2 lists the estimated $share_{min}(t)$ from CPS data for age 52 to 76, assuming the starting working age of 16, and $share_{min}(t < 52)=0$.

The early retirement age (ERA) is 62. Starting from ERA, the worker is eligible to receiving the Social Security benefits at a reduced level. In this case, the benefit is reduced 5/9 of one percent for each month before NRA, or 6.67% per year, up to three years. Beyond three years, the benefit is reduced 5/12 of one percent per month or 5% per year.

On the other hand, delayed receiving Social Security benefits after the NRA increases benefits. The delayed retirement credit (DRC) of 6% is given to the applicant for each delayed year up to age 69.[32] No DRC is given for applicants at age 70 or older.

D.2   The Social Security Earnings Test

We use the Social Security earnings test rules in 1999.[33] The Social Security benefits could be withheld partly or totally if the worker is earning income while taking the Social Security benefits at ages before 70.

For beneficiary under age 65, $1 of benefits for every $2 of earnings in excess of the exempt amount ($10,885 in 2004 dollars) is withheld. The benefit withholding rate for those aged 65-69 is $1 of benefits for every $3 of earnings in excess of the exempt amount ($17,575 in 2004 dollars).

If a whole year's worth of benefits is withheld between ages 62 to 64, benefits in the future will be raised by 6.7% each year. If the benefit is withheld between age 65 to 69, the future benefits will be raised by 6.0%. Given our terminal age at 80, it is favorable for individuals aged 62 to 64 but not actuarially fair for individuals aged 65 or older.

---

[32]The 6% DRC is for cohorts born between 1935 and 1936 (inclusive). The DRC varies from 3% for cohorts born in 1924 or earlier to 8% for cohorts born in 1943 or later. In between, it increases by 0.5% every two years.

[33]Before 2000, the earnings test applies to ages before 70. Since 2000, the earnings test is eliminated after reaching NRA.

D.3 Taxable Social Security Benefits

The Social Security benefits are not taxable if it is the only income. If there is other income, compute "total income" as the sum of half of the benefits and all other income. If total income is no more than the base amount ($25,000$ for head of household) then no benefits are taxable. If total income is higher than $\$34,000$ then up to 85% of the benefits could be taxable. Assume the Social Security benefits are y_{ss} and all the other income is y_o, the taxable part of Social Security benefits is calculated as

$$y_{ss,taxable} = \begin{cases} 0, & \text{if } y_o = 0 \text{ or } y_o + \frac{y_{ss}}{2} \leq 25000 \\ \min\left\{0.85 y_{ss}, \frac{1}{2}\min\left\{y_{ss}, y_o + \frac{y_{ss}}{2} - 25000, 9000\right\},\right. & \\ \left. + 0.85 \max\left\{0, y_o + \frac{y_{ss}}{2} - 34000\right\}\right\} & \text{otherwise.} \end{cases} \tag{D.3}$$

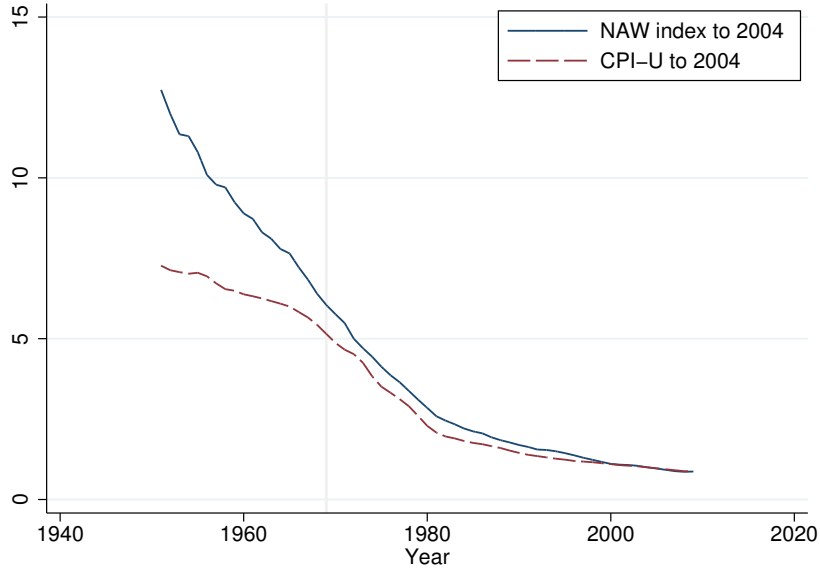Figure D1: Relative (to Year 2004) indices of National Average Wage Index and CPI-U.

Figure D2: Share of minimum wage on AIME, assuming starting working from age 16. CPS data.
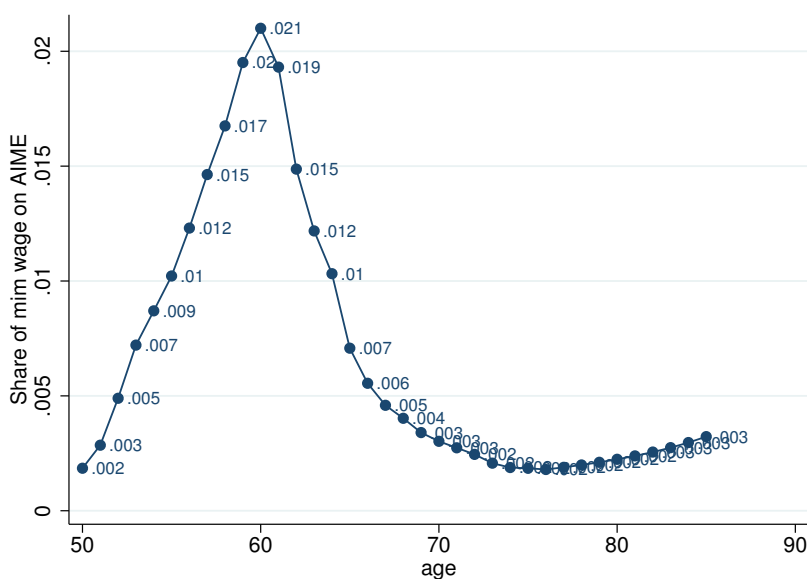

Table E: Transitions from various models[a]

	Models	Working to Not Working	Not Working to Working
	High school graduates:		
1	Data	0.041	0.230
2	Baseline model	0.044	0.253
Alternative human capital models:			
3	Exogenous	0.039	0.238
4	Learning-by-doing	0.036	0.245
Robustness check:			
5	Larger $\underline{c}$	0.046	0.271
6	Lower $\underline{c}$	0.047	0.272
7	Change δ, r	0.044	0.256
8	Smaller δ	0.039	0.224
9	Larger A_0	0.043	0.256
Other models:			
10	With health	0.032	0.228
11	With Part Time option	0.046	0.241
	College graduates:		
12	Data	0.022	0.334
13	Model	0.020	0.321

[a]The transition rate is the overal transition probability between age 35 and 50. Columns 1-11 are for high school graduates while columns 12-13 are for college graduates.

Table F: Estimates in the baseline model for college graduates[a]

Parameters		Estimates	Standard Errors
Human capital depreciation	δ	0.089	(0.007)
Human capital production function: I factor	α_I	0.015	(0.007)
Human capital production function: H factor	α_H	0.100	(0.008)
Standard deviation of human capital innovation	σ_ξ	0.006	(0.011)
Consumption: CRRA	η_c	3.784	(0.024)
Consumption shifter: coefficient on t $(\times 10)$	φ_1	0.256	(0.067)
Consumption shifter: coefficient on t^2 $(\times 10^2)$	φ_2	0.103	(0.027)
Consumption shifter: coefficient on t^3 $(\times 10^3)$	φ_3	-0.033	(0.004)
Consumption shifter: coefficient on married	φ_4	0.816	(0.231)
Leisure: Standard Deviation of Shock	a_ε	0.136	(0.028)
Leisure: spouse not working	a_1	0.779	(0.137)
Leisure: spouse working	a_2	-0.787	(0.146)
Bequest weight	b_1	16,594,247	(2,158,990)
Parameter heterogeneity[b]			
Leisure: mean of intercept	μ_{a_0}	-6.220	(0.058)
Leisure: standard deviation of intercept	σ_{a_0}	0.190	(0.076)
Human capital productivity, mean	μ_π	2.221	(0.119)
Human capital productivity, standard deviation	σ_π	0.907	(0.010)
Correlation between a_0 and π	ρ	-0.741	(0.208)
Initial human capital level at age 18			
Intercept	γ_0	1.759	(0.174)
Coefficient on a_0	γ_{a_0}	0.034	(0.015)
Coefficient on π	γ_π	0.482	(0.054)
Standard deviation of error term	σ_{H_0}	0.316	(0.121)
χ^2 Statistic = 874[c]		Degrees of freedom = 200	

[a]Indirect Inference estimates. Estimates use a diagonal weighting matrix. Standard errors are given in parentheses.
[b]The joint distribution of (a_0, π) is a parametric discrete distribution with nine points determined by these five parameters, using a nine-point Gauss-Hermite approximation.
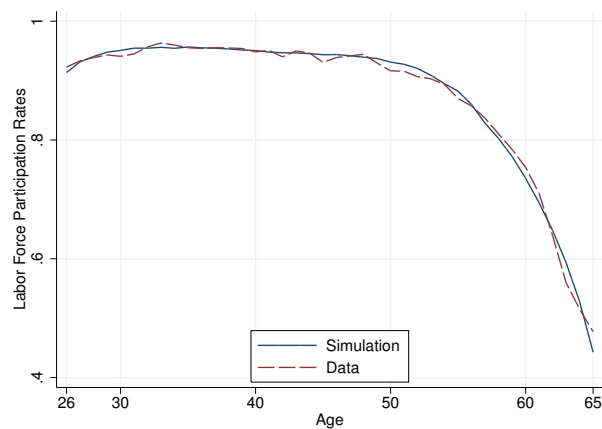[c]This is the J-statistic. The critical values of the χ^2 distribution are $\chi^2_{(200,0.01)} = 249$, $\chi^2_{(200,0.005)} = 255$, $\chi^2_{(200,0.001)} = 268$.

E   Transitions

F   College Graduates

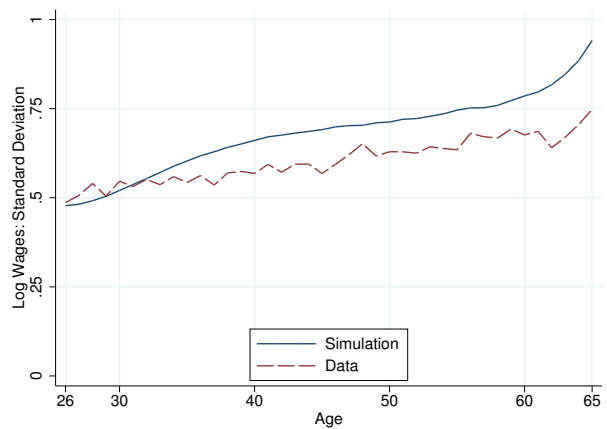Figure F: Fit of model with college graduates

(i) Labor Force Participation Rates

(ii) Log Wages & Log Wages (FE)

(iii) Standard Deviation of Log Wages

(iv) Consumption