

# Auctions of Real Options\*

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## Abstract

Corporations and governments frequently sell assets with embedded real options to competing buyers using both cash and contingent bids. Examples include natural resource leases, real estate, patents and licenses, and start-up firms with growth options. This paper models these auctions of real options, incorporating both endogenous auction initiation and post-auction dynamics. I find that common security bids create moral hazard and distort investment. Strategic auction timing affects auction initiation, security ranking, equilibrium bidding, and investment, and should be considered jointly with security design and seller's commitment level. Optimal auction design aligns investment incentives using a combination of down payment and royalty payment, but inefficiently delays sale and investment. I also provide suggestive evidence for model predictions using data from the leasing and exploration of oil and gas tracts. Altogether, these results reconcile theory with several empirical puzzles and imply novel predictions when compared with those from the existing literature.

**JEL Classification:** D44; D81; D82; G13; G31; G32; L24

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On September 8, 2003, Regeneron, a New York based biotech company, granted the pharmaceutical giant Aventis a license to develop and commercialize Regeneron’s Vascular Endothelial Growth Factor (VEGF) Trap, a potential cancer therapeutic.<sup>1</sup> In return, Aventis would pay an \$80 million upfront fee—the highest to date for a product in phase I clinical trials—and \$430 million in stock purchase and milestone awards, and would cover all development costs and split future product revenue evenly with Regeneron.<sup>2</sup> Ten years later in an oil lease auction that netted the U.S. Government \$1.2 billion, Exxon-Mobil emerged as the highest bidder and was entitled, but not obliged, to explore and drill on seven of the 320 auctioned tracts in the Central Gulf of Mexico for 5–10 years, and was to pay 18.75% royalty of future revenues from oil production to the U.S. Department of Interior.<sup>3</sup>

In these two deals, both the license and the lease constitute classic examples of a broader class of projects and assets with embedded real options whose sale and exercise underlie some of the most crucial decisions for entrepreneurs, firm executives, and government officials. Moreover, these transactions routinely involve competing bids in combinations of cash and payment contingent on the asset’s future cash flows, and can be effectively viewed through the lens of security-bid auctions.<sup>4</sup> Why did Regeneron negotiate for both cash and royalty payments? Why are nearly 72% of oil and gas tracts offshore and 56% of those on federal lands neither producing nor under active exploration despite the imbalance in supply and demand?<sup>5</sup> More fundamentally, how should a seller trade off benefits of security bids

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<sup>1</sup>In fact, Aventis acquired a portfolio of options: the timing option to observe the progress of Genentech’s drug Avastin before developing VEGF, the exit option if VEGF fails clinical tests, and the operations option to use research knowledge of VEGF to anchor into the anti-angiogenesis therapeutic market.

<sup>2</sup>Fax watch Inc., Pharma Biotech: News, Regeneron, Aventis Enter Into Agreement to Develop Cancer Therapy (Sept. 5, 2003), [www.sec.gov/Archives/edgar/data/872589/000095012303012534y91481exv10w26.htm](http://www.sec.gov/Archives/edgar/data/872589/000095012303012534y91481exv10w26.htm).

<sup>3</sup>ENERCOM Consulting, 360 articles: Central Gulf of Mexico Lease Sale - Come on in, the Water if Fine! (March 21, 2013), <http://oilandgas360.com/central-gulf-of-mexico-lease-sale-come-on-in-the-water-is-fine>.

<sup>4</sup>Oil leases have been auctioned using cash contracts, bonus-bid contracts, royalty contracts and pure profit share contracts. In transfer and licensing of technologies, as in the example of pharmaceutical industry, rival bidders offer various contingent contracts (Vishwasrao (2007) and Bessy and Brousseau (1998)). In sales of large assets such as the wireless spectrum auction for FCC bandwidth, aggressive bidders can declare bankruptcy and the bids are essentially debts (Board (2007a) and Zheng (2001)). Equities, preferred convertibles, call options, and debts are frequently used in M&A and venture capital financing (Martin (1996), Kaplan and Stromberg (2003), and Hellmann (2006)). Other examples include advance and royalty payments in publishing contracts (Dessauer (1981) and Caves (2003)), motion picture deals (Chisholm (1997)), business licenses such as electronic gambling machines with pre-specified profit tax, and military procurement contracts (McAfee and McMillan (1987b)).

<sup>5</sup>“Oil and Gas Lease Utilization - Onshore and Offshore. Report to the President” by U.S. Department of the Interior dated May 2012. This revelation has triggered a huge public outcry and heated debate in congress on the reason for the purported sluggish development of natural resources, and has policy implications in

with incentive distortions in option exercise? How to jointly decide security choice and auction timing? What is the role of the seller's commitment and who initiates the auction in equilibrium?

To simultaneously address these important questions, this paper builds a model of auctions of real options, endogenizing auction initiation and tying together option exercises with selling mechanisms. I derive the following main results under the unifying intuition that economic agents enjoy different optionalities in the sale and operation of an asset: First, common security bids cause inefficient and often suboptimal investments. Second, strategic auction timing affects auction initiation, security choice, bidding equilibrium, and post-auction investment, and should be considered jointly with security design and seller's commitment. Third, optimal auction design entails delayed auction initiation and investment, but aligns investment incentives using cash and royalties, as is prevalent in real-life business practice. Fourth, when a seller lacks commitment to auction design, bidders always initiate the auction and bidding equilibria are equivalent to those in cash auctions. These findings imply that many conclusions from traditional auction and real options models need to be modified in dynamic settings with learning and strategic interactions. I argue that the model offers explanations for several outstanding empirical puzzles, and further show that oil exploration data and business anecdotes corroborate model implications.

Auctions of real options are prevalent in licensing and patent acquisitions, leasing of natural resources, real estate development, M&A deals, venture capital and private equity markets, and privatization of large national enterprises.<sup>6</sup> They entail tremendous financial resources and mainly come in two categories in practice.<sup>7</sup> While in formal settings such as oil lease auctions or wireless spectrum auctions, the seller specifies explicitly and commits to the time and rules of the auction and an ordered set of security bids, many other cases

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the backdrop of Obama's proposal to increase onshore royalty by 50%.

<sup>6</sup>Bolton, Roland, Vickers, and Burda (1992) describe the privatization policies in Central and Eastern Europe. Pakes (1986) and Schwartz (2004) discuss patents as real options. One prominent M&A case involves Microsoft's \$8.5 billion cash acquisition of the voice-over-IP service Skype—its largest acquisition to date—for the portfolio of options such as Windows phone integration, and technology merger with outlook which it exercised in 2013. Google's largest acquisition with \$12.5 billion for Motorola in 2012 also gave it the option to develop the portfolio of patents Motorola held, and other options such as the one to sell the set-top box business which it did in 2013 to Arris Group. Both deals are against the backdrop of potential rival bids and can be effectively viewed as auctions.

<sup>7</sup>In the Gulf of Mexico alone, the oil and gas leases auctioned by the U.S. federal government in 1954-2007 have exceeded 300 billion, and annual licensing deals by pharmaceutical giants exceed \$20 billion even in the aftermath of the financial crisis. M&A volume world-wide is also in the trillions of dollars annually.

can be thought of as informal auctions where the seller lacks such commitment. Prominent examples of informal auctions include corporate takeovers and project finance, where bidders decide what to offer and often can initiate the contact or negotiation. Still others such as licensing agreements and contracts in the entertainment industry appear in both categories. This paper addresses both formal and informal auctions.

While prior studies offer insights into auctions of real options, existing applications typically take auction initiation as exogenous and do not consider post-auction dynamics, leaving out the stochastically evolving market environment and the dependence of an asset's payoff on post-auction actions. They also analyze the sales and exercises of real options in isolation, and exogenously specify the agency conflicts in the latter. This paper differs by taking into consideration that auction initiation affects the security choice, and the security design shapes the investment decision.

This study therefore attempts to bridge the gap between auction theory and corporate finance, and adds to the emerging literature on agency conflicts in real options and on auction initiation and security bids. The first key contribution is recognizing the optionality and dynamic nature of auctions. This paper shows equilibrium behaviors can differ significantly from static settings and underscores that auction timing, security design, and real option exercise are interdependent and should not be studied in isolation. The second key contribution is in linking agency issues in real option exercises to selling mechanisms. Auction design endogenously induce agency conflicts. In addition, this paper casts auction initiation as an optimal stopping problem to capture a new dimension of strategic interaction amongst sellers and bidders, and empirically examines its implications on corporate investment.

Specifically, this paper models the sale and exercise of a typical investment option with endogenous participation in both formal or informal settings. The model involves a seller and multiple potential bidders who are risk-neutral and maximize their expected payoffs. They interact in continuous time in three sequential stages. In the first stage, the seller (or potentially a bidder in informal auctions) strategically initiates the auction. In the second stage, participating bidders bid cash and contingent securities and the seller allocates. In the final stage, the winning bidder rationally times the exercise of the investment option and delivers the contingent payment to the seller.<sup>8</sup>

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<sup>8</sup>Although the paper focuses on the post-auction action of investment timing, which is also fitting for oil lease auctions and licensing agreements, the key intuitions certainly manifest themselves in many other types

There are two key frictions in the model. The first is the non-contractibility of the bidders' private information. Contingent payment does not account for the bidders' private costs and thus misaligns investment incentives in the third stage. This leads to a tradeoff for the seller between the post-auction moral hazard in investments and the benefits of contingent bids such as enhanced rent extraction.<sup>9</sup> As contingent bids become increasingly prevalent, this tradeoff could have a first-order impact on projects with high option values, such as the development of real estate and natural resources, as well as the transfer and licensing of technologies. Moreover, there is no "one-size-fits-all" in security ranking, as any comparison has to be made in conjunction with considerations of auction timing and market environment.

The second friction is the cost associated with the ownership transfer, such as legal fees for underwriting contingent securities, initial opportunity cost to the winning bidder, the seller's discontinued benefit from the asset's alternative use, or irreversible loss of the option for more efficient allocation of the asset in future. Delaying the auction saves the time value of money on these costs and encourages greater participation, but risks missing the opportune exercise of the investment option. These tradeoffs endogenize auction timing in the first stage. The seller times the auction to maximize the option value less the information rent, and a bidder times the auction to maximize the information rent, neither of which maximizes social welfare. Thus the seller inefficiently delays the auction and the bidder prefers earlier initiation than the seller. I find that strategic auction timing is a salient feature in real-life business practice and is integral to auction design.

By combining cash and royalty payments, the optimal security design eliminates post-auction moral hazard, but inefficiently delays auction and investment. The intuition is that the seller faces a real option with an added exercise cost that is the information rent, thus needs a contingent payment to pass this cost to the winning bidder, who then invests at the optimal investment threshold that is higher than the efficient one. Cash payments complements the design by ensuring individual rationality and incentive-compatibility. This

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of post-auction actions. While moral hazard associated with capital allocation has been well-studied (see Stein (2003)), the one associated with investment timing is equally important and deserves more attention.

<sup>9</sup>Prima facie, the type of bids should not matter as there is always a cash equivalent. One advantage to contingent bids is that they enhance the seller's revenue by effectively linking payoff to a variable affiliated with bidders' private information—the "linkage" principle in Milgrom (1985). Contingent bids also mitigate liquidity or legal constraints and reduce valuations gaps amongst various parties.

result is consistent with the popular use of negotiated royalty payment and down payment in sales of marketing rights, licensing agreements, publishing and movie contracts, and many other franchise business practices.

It further turns out that the seller's commitment level to auction timing and security design significantly impacts the bidding and investment outcomes. Absent such commitment, bidding equilibria are equivalent to those in cash auctions. The intuition is that cash-like bids allow a bidder to generate the maximum social surplus, and at the same time outbid competitors in the cheapest way. For example, a bidder with higher valuation finds it easier to outbid others using cash than equity shares because it costs him more than it does someone with lower valuation to offer the same shares. The auction timing game is also complicated by Bayesian updates of beliefs on the types present absent initiation. In equilibrium, bidders always initiate and invest efficiently.

These results have the following implications. First, post-auction investments can be both inefficiently delayed or accelerated depending on the security design. In particular, the security design in oil and gas lease auctions causes the winning bidder to delay exploration beyond efficient rational waiting due to optionality. A greater royalty rate in highly uncertain environment exacerbates the delay, which suggests that the large number of idle tracts reported can be the consequence of inefficient security design. Anecdotal evidence and empirical studies corroborate these predictions.<sup>10</sup>

Second, auction timing is an integral part of auction design and depends crucially on the seller's commitment. While many studies have focused on security design or security ranking, their conclusions are sensitive to auction timing, whose impact could be larger. Moreover, when the seller cannot commit to auction timing, bidders always initiate when they expect to exercise the investment option. This is consistent with Fidrmuc, Roosenboom, Paap, and Teunissen (2012) which finds that strategic acquisitions are more often bidder-initiated. I also use the data on leasing and exploration of oil and gas tracts in the Gulf of Mexico to show that when bidders could initiate lease auctions before the implementation of Area-Wide Leasing in May 1983, they did so, and carried out first exploratory drills at least 10% faster.

Finally, and perhaps most surprisingly, my findings stand in contrast to conventional predictions derived from several classic studies. For example, it is well-established that

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<sup>10</sup>See Humphries (2009), Opaluch, Grigalunas, Anderson, Trandafir, and Jin (2010), and Cong (2013b).

having more bidders is beneficial to a seller (Bulow and Klemperer (1996)), which makes it all the more puzzling why the sellers in some corporate auctions restrict the number of bidders (Hansen (2001) and French and McCormick (1984)). I show that depending on the security design, more bidders could decrease revenue and social welfare due to aggressive bidding and increased moral hazard. Another widely held belief is that security bids generate higher revenue than cash, but this paper argues cash dominates common securities as the bidders' market becomes very competitive. In addition, most auctions traditionally deemed efficient (including cash auctions) are actually not once we consider endogenous initiation.

This paper builds on the literature on security-bid auctions and their applications in corporate finance.<sup>11</sup> DeMarzo, Kremer, and Skrzypacz (2005) give an extensive exposition of security-bid auctions, showing that “steeper” securities lead to higher expected value to the seller. Samuelson (1987) suggests that adverse selection and moral hazard complicate the effect. Che and Kim (2010) and Rhodes-Kropf and Viswanathan (2000) demonstrate, respectively, that adverse selection could reverse the ranking of securities and lead to inefficiencies in bankruptcy reorganizations and privatizations. This paper examines post-auction moral hazard—the second issue Samuelson (1987) emphasized. Kogan and Morgan (2010) compare equity and debt auctions under moral hazard in an experimental study. McAfee and McMillan (1987a) is another related study which derives optimal linear incentive contract under competition, information asymmetry, and moral hazard. This paper is unique in that it considers post-auction moral hazard in a dynamic setting and jointly with endogenous auction timing.

This paper also complements the emerging literature on agency issues and auction initiation in a real options framework. Maeland (2002), Grenadier and Wang (2005), and Cong (2012) study distortion of investment incentives due to adverse selection and moral hazard. Board (2007b) derives optimal selling mechanisms of options. This paper differs primarily in considering a broader class of standard securities and auction timing. It links agency conflicts directly to the security design. Gorbenko and Malenko (2013b) examine

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<sup>11</sup>Hansen (1985), Crémer (1987) and Riley (1988) are among the early contributions. Also related are the theories of incentive contracting, typically applied to defense procurement (Engelbrecht-Wiggans (1987), McAfee and McMillan (1986), and Laffont and Tirole (1987)). Hansen (2001) reviews the corporate auction process. Dasgupta and Hansen (2007) and Boone and Mulherin (2007) justify viewing corporate takeovers as auctions. Eckbo, Giammarino, and Heinkel (1990) consider the role of non-cash bids. Skrzypacz (2013) gives an overview.

bidder-initiated takeover attempts in cash and stocks with heterogenous cash constraints. Gorbenko and Malenko (2013a) is another detailed study on auction initiation, focusing on time-varying types and signaling without security bids. This paper examines initiations driven by aggregate market conditions and Bayesian learning. In addition, I highlight the role of seller’s commitment, and link auction initiation to post-auction investment.

The remainder of the paper proceeds as follows. Section 1 introduces the model and illustrates its main features. Section 2 analyzes optimal investment strategies. Section 3 derives bidding equilibria and optimal auction design. Section 4 characterizes informal auctions as signaling and timing games. Section 5 discusses implications and provides empirical evidence. Section 6 considers extensions and limitations. Section 7 concludes. Appendix A contains all the proofs. Appendix B lists technical conditions for “well-behaved” distributions. Appendix C describes institutional details, data construction, and empirical tests based on sales and explorations of oil and gas tracts in the Gulf of Mexico.

## 1 A Model of Auctions of Real Options

This section describes the economic environment and sets up the model, before illustrating the main features of the model using examples of cash and bonus-bid auctions.

### 1.1 Setup

A risk-neutral seller with discount rate  $r > 0$  owns a project with an embedded option. Once developed, the project generates a verifiable lump sum cash flow whose value  $P_t$  is publicly observed and evolves stochastically according to a geometric Brownian motion (GBM)

$$dP_t = \mu P_t dt + \sigma P_t dB_t, \tag{1}$$

where  $B_t$  is a standard Brownian Motion under the equivalent martingale measure,  $\mu < r$  is the instantaneous conditional expected percentage change per unit time in  $P_t$ , and  $\sigma$  is the instantaneous conditional standard deviation per unit time.<sup>12</sup>

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<sup>12</sup> $r > \mu$  ensures finite value of the option. See McDonald and Siegel (1986) or Dixit and Pindyck (1994). The lump sum could represent the present value of a stream of future cashflows.



The seller does not have the expertise to exploit the option but can sell the project to  $N$  risk-neutral potential bidders with the same discount rate  $r$  who have the expertise to exploit the option.<sup>13</sup> Bidder  $i$  can develop the project by paying a private investment cost  $\theta_i$ .<sup>14</sup> The distribution of the  $\theta_i$ s are i.i.d. with positive support  $[\underline{\theta}, \bar{\theta}]$ , and follows either Uniform Distribution or Generalized Pareto Distribution.<sup>15</sup> Denote the cumulative distribution and density function by  $F(\theta)$  and  $f(\theta)$  respectively. Similar to DeMarzo, Kremer, and Skrzypacz (2005), the winning bidder has to pay an up-front cost  $X \geq 0$ , which we can interpret as the initial resources required by the project.<sup>16</sup> The project is worthless to him if it is never developed. The seller loses a reservation value  $Y$  when the asset is sold.<sup>17</sup>

When the auction is held at time  $t_a$ , bidders compete by offering security bids that are combinations of contingent payments from the cash flow of the project and non-contingent payments which, for simplicity, can be viewed as upfront cash at the time of the auction. Unless stated otherwise, the discussion focuses on standard security bids as defined next.

**DEFINITION.** A *standard security bid* is an upfront cash payment  $C \in \mathbb{R}$  and a contingent payment  $S(P_\tau) \in [0, P_\tau]$  paid at the time of investment  $\tau$  such that  $S(P)$  and  $P - S(P)$  are weakly increasing.<sup>18</sup>

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<sup>13</sup>For clarity, I refer to the seller as female and the bidders as male.

<sup>14</sup>The analysis also applies when bidders differ in other quantities, such as production capacities.

<sup>15</sup>This specification includes common distributions used in the literature, with both bounded and unbounded supports. It renders the auction design problem “regular”, standard in auction literature, and rules out unrealistic mathematical constructions in discussing auction timing. Results in the paper hold more generally for “well-behaved” distributions. Appendix B details the technical requirements.

<sup>16</sup>Examples include illiquid human capital, the cost of underwriting in the case of security issuance, or simply an opportunity cost for the winning bidder.

<sup>17</sup>For example, the seller may derive a continuous stream of cash flow  $rY \geq 0$  through alternative uses of the asset till it is sold. In pharmaceutical licensing, the alternative use could be sales without marketing. In leasing natural resources, it could be environmental benefits or income from utilizing the land as a national park. Though not modeled in this paper, the value from alternative use can also depend on  $P$ , for example,  $Y(P) = y_0 + y_1P$  with  $y_0 \geq 0$  and  $y_1 \in [0, 1]$ . The main results carry through as long as  $Y(P)$  and  $P$  evolve differently.  $Y$  can also represent the option value of more efficient allocation and use of the asset when technology improves.  $X$  and  $Y$  represent in reduced-form auction frictions that are commonplace. Startups, project finance, etc, all have high reservation values and the legal fees are high. Potential bidders may incur cost of due diligence, which can be modeled as an entry cost. Holding the auction may incur heavy costs such as that of revealing proprietary information to rivals (Hansen (2001)). The losers in an M&A could have diminished value (Gorbenko and Malenko (2013b)). Other pre-contract costs are common too (French and McCormick (1984)).

<sup>18</sup>This definition rules out directly contracting on private cost  $\theta$ , which is familiar in real-life practices. This is frequently due to important practical problems in validating profits reported. Consequently, payments are usually contingent on top-line revenue in the development of natural resources, contracts on marketing and licensing rights, as well as franchise chain operations.

Limited liability for the seller and the winning bidder require  $0 \leq S(P_\tau) \leq P_\tau$ ; double monotonicity is standard in the security design literature.<sup>19</sup> Standard security bids thus admit most securities and contracts used in practice. For example, with equity bids, the seller receives a fraction  $\alpha$  of the payoff:  $S(P) = \alpha P$ ; with call option bids, the seller can pay a strike price  $k$  for the project cash flow:  $S(P) = [P - k]^+$ ; with bonus bids on fixed royalty rate  $\phi$ , the seller receives bonus  $C$  and royalty payment  $S(P) = \phi P$ .

The agents interact in continuous time as shown in Figure 1. To analyze the dynamics, I work backward to first solve for the optimal investment strategy for the winning bidder, then derive the bidding equilibrium given the bidders' valuations based on their investment strategies, and then study the impact of strategically timing the auction.

$t_0 = 0$ : Interaction starts.

$\hookrightarrow t_a$ : Auction held at  $P_a$ , project allocated, cash part  $C$  of bid paid, auction frictions  $X$  &  $Y$  incurred.

$\hookrightarrow \tau$ : Project invested at  $P_\tau$ ,  $\theta$  incurred, contingent part  $S(P_\tau)$  of bid paid.

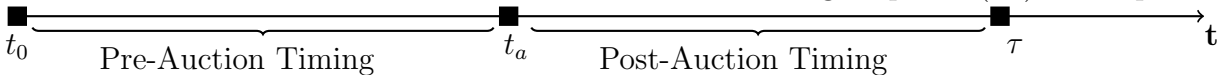


Figure 1: Timeline

The seller's commitment to auction design plays a fundamental role in equilibrium. In formal auctions, the seller decides the timing of the auction and pre-specifies a set of permissible bids ordered by an index (the bid with the highest index wins). In other settings, the seller cannot commit to pre-specified bids or auction timing, thus considers all types of offers and potentially allows the bidders to initiate the auction. I follow DeMarzo, Kremer, and Skrzypacz (2005)'s convention to call such cases informal auctions and discuss them in Section 4. Throughout the paper, I focus on First-Price Auctions (FPAs) and Second-Price Auctions (SPAs) where the bidder with the highest bid wins and pays the highest bid or the second-highest bid respectively.<sup>20</sup>

<sup>19</sup>See Hart and Moore (1995), DeMarzo and Duffie (1999), and DeMarzo, Kremer, and Skrzypacz (2005). An alternative weaker condition to get many of the main results is  $S(P)$ 's being continuous.

<sup>20</sup>Following Skrzypacz (2013) and Ding and Wolfstetter (2011), I assume the seller commits to no renegotiation, and to no contracting or resale to losing or non-participating bidders. Section 6 discusses relaxations of these assumptions.

## 1.2 Two Simple Examples

### Cash Auctions

The bidding strategies and post-auction investments in cash auctions serve as a useful benchmark for later sections. Upon winning, a bidder of type  $\theta$  owns the project entirely, and optimally develops the project at time  $t \geq t_a$  to maximize  $\mathbb{E}[e^{-rt}(P_t - \theta)]$ . This is a standard problem in the real options literature.<sup>21</sup> The optimal strategy involves immediate investment upon reaching an upper threshold  $P^*(\theta)$ . Let  $P_a$  denote the cash flow level when the auction is held. The value of the investment option  $W$  and  $P^*(\theta)$  are independent of  $X$  and  $t$ , and are given by

$$P^*(\theta) = \max \left\{ P_a, \frac{\beta}{\beta - 1} \theta \right\}, \quad \text{where } \beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left( \frac{\mu}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma^2}} > 1, \quad \text{and} \quad (2)$$

$$W(P_a; \theta) = D(P_a; P^*(\theta))(P^*(\theta) - \theta), \quad \text{where } D(P; P') = \left( \frac{P}{P'} \right)^\beta \quad \text{for } P \leq P'. \quad (3)$$

Appendix A.1 shows that  $D(P_t; P')$  corresponds to the time- $t$  price of an Arrow-Debreu security that pays one dollar the first moment threshold  $P' \geq P_t$  is reached. The option value of the project is simply the total value of Arrow-Debreu securities that replicate the payoff of the investment option at exercise. It can also be viewed as the value of the exercise payoff  $P^*(\theta) - \theta$  discounted by the “expected discount factor”  $D(P_a; P^*(\theta))$ .

Bidder  $i$ 's private valuation in cash auctions is then  $W(P_a; \theta_i) - X$ , which is decreasing in  $\theta_i$ . There exists a cutoff type for participation  $\theta_c = \min\{\bar{\theta}, \theta_{BE}\}$ , where the break-even type  $\theta_{BE}$  solves  $W(P_a; \theta) - X = 0$  and is given explicitly as

$$\theta_{BE} = (\beta - 1)(P_a^\beta \beta^{-\beta} X^{-1})^{\frac{1}{\beta-1}} \mathbb{1}_{\{P_a > \beta X\}} + (P_a - X) \mathbb{1}_{\{P_a \leq \beta X\}} \quad (4)$$

Types with costs higher than  $\theta_c$  do not participate. Since there is no misalignment of incentives in post-auction investments, FPAs and SPAs generate equivalent revenues to the seller, and efficiently allocate the project to type  $\theta_{(1)}$  if  $\theta_{(1)} \leq \theta_c$ , where  $\theta_{(j)}$  is the  $j$ th lowest realized  $\theta$ . The cases with reserve price or entry fee are similar.

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<sup>21</sup>For example, in McDonald and Siegel (1986) and Dixit and Pindyck (1994).

## Bonus-bid Auctions

In many countries, the predominant design for leasing natural resources involves fixing a royalty rate  $\phi$  and having the contractors bid up-front payment  $C$ —the so-called “bonus”.<sup>22</sup>

The winning bidder owns a fraction  $1 - \phi$  of the project and has a real option value  $L(\theta) = \max_{\tau} \mathbb{E}[e^{-r\tau}((1 - \phi)P_{\tau} - \theta)] - X$ . Scaling the cash flow in Eq.(2) gives the optimal investment threshold  $P^{bonus}(\theta) = \max\{P_a, \frac{\beta}{\beta-1} \frac{\theta}{1-\phi}\}$ . In SPAs, every participant bids up to the expected value  $L(\theta)$ , which is decreasing in  $\phi$ .  $L(\theta_{BE}) = 0$  gives the breakeven type. The seller has a real option value  $\max_{\tau} \mathbb{E}[e^{-r\tau} \phi P_{\tau}]$  and prefers immediate investment. But the winning bidder only invests when  $P^{bonus}$  is first hit. The expected total revenue is simply  $R_{bonus} = \mathbb{E} \left[ L(\theta_{(2)}) 1_{\{\theta_{(2)} < \theta_c\}} + [D(P_a; P^{bonus}) \phi P^{bonus}(\theta_{(1)}) - Y] 1_{\{\theta_{(1)} < \theta_c\}} \right]$ . The cut-off type, revenue and social welfare in FPA bonus-bid auctions are the same, because this is essentially a cash auction for  $1 - \phi$  fraction of the project and revenue equivalence applies.

### 1.3 Post-auction Investment and Pre-auction Timing

There are a few observations from these two examples. When using contingent payment in bonus-bid auctions, the winning bidder’s real option differs from the seller’s, leading to post-auction misalignment of incentives. This distortion in investment timing is costly to social welfare and the seller’s revenue. Moreover, since the real option value, break-even type, and cash bids all depend on  $P_a$ , the timing of the auction clearly matters.

#### Cost of Inefficient Investment and Post-auction Moral Hazard

As  $P^{bonus}(\theta) \geq P^*(\theta)$ , investment is inefficiently delayed in bonus-bid auctions, which could potentially explain the higher idle rate of oil tracts observed—a topic I revisit in Section 5 on model implications. This delay is costly to the seller ex post the auction. Whether it is costly ex ante depends on the tradeoff between the additional rent extracted with contingent payments and the cost of post-auction moral hazard. For example, bonus-bid auctions yield the seller higher revenues than cash auctions only when  $X$  is relatively

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<sup>22</sup>In the US, the Minerals Lands Leasing Act prescribes the base share of royalty rate at 1/8 the value of production for onshore leases, and the Outer Continental Shelf Lands Act used 1/6 for offshore leases. The offshore rate for leasing beginning in 2008 is set at 18.75%. This form of payments are also common for technology license and marketing rights. See Rothkopf and Engelbrecht-Wiggans (1992), Reece (1979), Hendricks, Porter, and Tan (1993), and Haile, Hendricks, and Porter (2010) for more details.

small and either  $\beta$  or  $\phi$  is also small.

Let us examine a situation absent moral hazard to understand how contingent securities enhance the seller’s revenue. Suppose  $\theta$  were contractible, the seller can use a profit share SPA to completely align investment incentives, i.e.,  $S(\alpha, P) = \alpha(P - \theta)$ . The winning bidder simply faces a scaled optimization problem, and because the individual rationality constraints for participation is the same as in a cash-bid auction, the cut-off type is identical. If  $\theta_{(2)} < \theta_c$ , she bids up to its valuation, i.e.,  $(1 - \alpha(\theta_{(2)}))W(P_a; \theta_{(2)}) = X$ . The seller’s expected revenue  $\mathbb{E}[(\alpha(\theta_{(2)})W(P_a; \theta_{(1)}) - Y)\mathbb{1}_{\{\theta_{(2)} < \theta_c\}}]$  can be expressed as

$$\mathbb{E} \left[ (W(P_a; \theta_{(2)}) - X - Y)\mathbb{1}_{\{\theta_{(2)} < \theta_c\}} \right] + \mathbb{E} \left[ \alpha(\theta_{(2)})(W(P_a; \theta_{(1)}) - W(P_a; \theta_{(2)}))\mathbb{1}_{\{\theta_{(2)} < \theta_c\}} \right].$$

The first term represents the seller’s expected revenue in cash auctions and the second term is the linkage benefit: for every realization that  $\theta_{(2)}$  participates, the seller recovers a portion  $\alpha(\theta_{(2)})$  of the winning bidder’s information rent. This leads to higher revenues than in cash auctions.

In reality,  $\theta$  is not observable or difficult to contract upon.<sup>23</sup> Either revenue extraction or efficiency loss can dominate. Table 1 gives an illustration. The profit share auction yields higher revenue and the same welfare compared to the cash auction, as anticipated. A bonus-bid auction with  $\phi = 1/8$  also results in higher revenue, but welfare is reduced due to inefficient investment. But both equity-bid auction and bonus-bid auction with  $\phi = 1/4$  yield significantly lower revenues and welfares compared to the cash auction. Depending on the exact real option involved, one could lose more than 90% of revenue and welfare using a security-bid auction compared to a cash auction.

$P_a = 15, N = 18, \beta = 1.8, \theta \sim Unif[10, 50], X = 3, Y = 0$

Security Design	Cash-bid	Profit Share	Bonus-bid $\phi = 1/8$	Equity-bid	Bonus-bid $\phi = 1/4$
Revenue	1.6578	1.8384	1.7403	1.6416	1.4429
Welfare	2.2565	2.2565	2.1862	2.0597	1.6027

Table 1: Expected Welfare and Seller’s Revenue.

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<sup>23</sup>Profit share auctions are rare due to high monitoring costs, limited comparability, and landowner’s risk aversion (Robinson (1984)). Past experience in oil lease auctions has shown considerable difficulties in reaching agreement on the proper profits (Opaluch, Grigalunas, Anderson, Trandafir, and Jin (2010)).

## Impacts of Auction Timing

Auction timing affects the post-auction investments, bidding behaviors, and agents' revenues. First, auctions held at a higher  $P_a$  results in less delays in post-auction investment. In particular, when  $P_a$  is higher than the investment thresholds in both cash and bonus-bid auctions, investment follows the auction immediately. Second, conditional on the security design, the bidders's valuations are higher for a higher  $P_a$ ; thus bidders participate more and bid more cash or bonus. Third, while the seller receives more at an auction held at a higher  $P_a$ , she has to discount the payoff, since she needs to wait to hold the auction.

Auction timing has both direct effects independent of security design, and interaction effects with security design. Figure 2 plots time zero present values of the expected revenues from cash and bonus-bid auctions held when  $P_t$  first reaches  $P_a$ . Surprisingly, auction timing affects seller's revenue even in cash auctions.  $P_a$  determines which security design dominates and changing it leads to substantial variations in the seller's revenue. For example, Cash dominates only for  $P_a \leq 44$ , and optimal timing of bonus-bid auctions can increase revenue by more than 60%.

## 2 Post-auction Investments under Uncertainty

This section takes the standard security bid in equilibrium as given to formally derive the optimal investment strategy, and shows how it is shaped by both auction timing and security design.

Suppose the winning bidder of type  $\theta$  pays cash  $C$  at the time of auction  $t_a$  and pays contingent security  $S(P_t)$  when project is invested at time  $t \geq t_a$ , his private valuation at  $t_a$  is

$$\tilde{V}(C, S(\cdot), \theta) = \max_{\tau \geq t_a} \mathbb{E}_P[e^{-r(\tau-t_a)}(P_\tau - S(P_\tau) - \theta)] - X - C, \quad (5)$$

where  $\tau$  is any stopping time. The key innovation from traditional real options models is the term  $S(P)$  which is of general form. It is not clear if  $V(\theta)$  is well-defined a priori. The following proposition shows that an optimal investment strategy exists and characterizes it.

**Proposition 1.** *For any standard security bid, there exists a threshold investment strategy that is optimal among all stopping times. Moreover, the valuation  $\tilde{V}(C, S(\cdot), \theta)$  is non-*

decreasing in  $P_a$ , and continuously decreasing in  $\theta$ .

As detailed in Appendix A.2, the optimal strategy is independent of  $X$  and  $C$ , which are sunk costs when deciding on the investment. The strategy generally involves both upper and lower thresholds that are dependent on  $P_a$ —auction timing clearly matters. The remainder of this section turns to the following special case.

**Condition (Conv):** *The function  $P^{-\beta}[P - S(P) - \theta]$  of  $P$  is quasi-concave with a maximum achieved at  $\tilde{P}(\theta)$ . For  $P \geq \tilde{P}(\theta)$ ,  $S(P)$  is piecewise twice-differentiable with only positive jumps in  $S'(P)$  and  $S''(P)$  (when exists) satisfying  $PS''(P) \geq (1 - \beta)[1 - S'(P)]$ .*

Condition (Conv) only requires that the security is not too “concave” at large  $P$ , and is non-restrictive as it holds for most common securities used in real life and in equilibria in this paper, such as equities or call options. The following Lemma simplifies discussions in later sections:

**Lemma 1.** *With (Conv), the optimal investment follows an upper threshold strategy with threshold  $\tilde{P}(\theta)$ .*

The tradeoff is similar to that in traditional real options models. At  $P_t$ , the gain of waiting to reach  $P_t + dP$  is the increase in payoff  $d[P_t - S(P_t)] = [1 - S'(P_t)]dP + o(dP)$ , but the payoff is also discounted by  $D(P_t; P_t + dP)$ , resulting in a fractional loss  $1 - D(P_t; P_t + dP) = \beta P_t^{-1} + o(dP)$  of the original payoff. The net benefit decreases in  $P_t$  and becomes negative beyond  $\tilde{P}(\theta)$ . Thus, waiting is beneficial if and only if  $P_t < \tilde{P}(\theta)$ .

Security bidding generally causes inefficient investments and the distortion is closely tied to the shape of the security. This is best seen with  $S(P)$  differentiable at  $\tilde{P}$ , in which case

$$\tilde{P}(\theta) = \frac{\beta}{\beta - 1 + S'(\tilde{P})}[\theta + S(\tilde{P})] \quad (6)$$

Compared to the traditional real options investment threshold  $\frac{\beta}{\beta-1}\theta$ , the security payment has two effects. Intuitively, the bidder faces an additional cost  $S(P)$ , and thus requires a higher threshold. But at the same time, the sensitivity of the security payment to cash flow

implies a smaller option premium and prompts an earlier investment.<sup>24</sup> Depending which effect dominates, the threshold could be either higher or lower than the efficient threshold. The following proposition describes the direction of distortion:

**Proposition 2.** *With condition (Conv), relative to what is socially efficient,*

(a) *A project invested at  $P$  is weakly delayed if  $\beta S(P) - PS'(P) > 0$ , and weakly accelerated if  $\beta S(P) - PS'(P) < 0$ , regardless of the winning bidder's type;*

(b) *A winning bidder of type  $\theta$  will invest weakly late if  $(\beta - 1)S\left(\frac{\beta}{\beta-1}\theta\right) > \theta S'\left(\frac{\beta}{\beta-1}\theta\right)$ , and weakly early if  $(\beta - 1)S\left(\frac{\beta}{\beta-1}\theta\right) < \theta S'\left(\frac{\beta}{\beta-1}\theta\right)$ .*

These results follow directly from Lemma 1 and (Conv). While prior literature mostly points to investment delays due to agency conflicts, the timing distortion really depends on the security design. The cash flow elasticity of security (CES)  $E_S = \frac{PS'(P)}{S(P)}$  is thus informative:

**Corollary 1.** *Investment is inefficiently delayed (accelerated) if  $E_S < (>)\beta$ .*

For example,  $E_S = 1$  in bonus-bid auctions, resulting in inefficient delays in investments.

### 3 Formal Auctions

The knowledge of optimal investment strategies allows bidders to value the real option. This section continues to analyze bidding equilibria in formal auctions. There are three key findings. First, bidding and investment equilibria exist under mild regularity conditions. Second, bidding outcome, investment, and security ranking are all sensitive to auction timing and the market condition. Third, optimal auction designs entail combinations of cash and royalty payments, and lead to inefficiently delayed investments auction timing.

In formal auctions, the seller commits to a pre-specified auction timing and a well-ordered set of allowed bids. Thus the bidders compete by offering allowed security bids, which in real life are ranked by simple, easily implementable rules. A variant of the definition in DeMarzo, Kremer, and Skrzypacz (2005) formalizes this notion of well-orderedness:

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<sup>24</sup>As in Grenadier (2002) and Grenadier and Malenko (2011), option premium is the NPV of investment at the moment of exercise divided by the total cost:  $OP(\theta) = \frac{\tilde{P}(\theta) - S(\tilde{P}(\theta)) - \theta}{S(\tilde{P}(\theta)) + \theta}$ .



**DEFINITION** An *ordered set of securities* ranked by index  $s$  is defined by a left-continuous map  $\Pi(s) = \{C(s), S(s, \cdot)\}$  from  $[s_L, s_H] \subset \mathbb{R}$  to the set of standard security bids such that for each voluntary participant of type  $\theta$ ,  $V(s, \theta) \equiv \tilde{V}(C(s), S(s, \cdot), \theta)$  is non-negative on  $[s_L, \tilde{s}]$  and negative on  $(\tilde{s}, s_H]$  for some  $\tilde{s} \in [s_L, s_H]$ .

In addition to being standard, an ordered set of securities admits ranking with index  $s$  for any payoff from the project and permissible bids cover a range wide enough such that each participant earns non-negative profit by bidding low enough but earns no profit bidding too high. The seller allocates the project to the bidder with the highest index. The winning bidder pays a security using the highest-bid index in FPAs or the next-highest-bid index in SPAs. This notion of formal auctions is consistent with real-life practice:  $s$  could be the fraction of shares  $\alpha$  in a pure equity auction  $\{C(\alpha) = 0, S(\alpha, P) = \alpha P\}$ , the (negative) strike price  $k$  in a call option auction  $\{C(-k) = 0, S(-k, P) = \max\{P - k, 0\}\}$ , or the bonus  $b$  in a bonus-bid auction with royalty rate  $\phi$  fixed  $\{C(b) = b, S(b, P) = \phi P\}$ . Such securities are routinely used in M&As, VC contracts, and lease auctions where the winning bidder is indeed the one offering the highest  $s$ .<sup>25</sup>

### 3.1 Equilibrium Bidding Strategies

Using the fact that  $V(s, \theta)$  is well-defined (Proposition 1), I characterize equilibrium bidding strategies, assuming any indifference in bidding is resolved by bidding a higher index.

**Proposition 3.** *In first-price auctions, when  $\ln V(s, \theta)$  is absolutely continuous in  $s$  with the derivative (when exists) decreasing in  $\theta$ , there exists a unique symmetric Bayesian Nash equilibrium that is decreasing, differentiable, and is characterized by:*

$$s'(\theta) = \frac{(N-1)f(\theta)}{1-F(\theta)} \frac{V(s(\theta), \theta)}{V_1(s(\theta), \theta)} \quad (7)$$

for  $\theta \leq \hat{\theta}$  with the boundary condition  $s(\hat{\theta}) = \sup\{s \in [s_L, s_H] \mid V(s, \hat{\theta}) = 0\}$ . The cut-off type for participation is  $\hat{\theta} = \sup\{\theta \leq \bar{\theta} \mid \max_s V(s, \theta) \geq 0\}$ .

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<sup>25</sup>In M&As with acquirer's stocks as bids,  $C$  simply corresponds to the value of acquirer's cash flows that are independent of the acquisition,  $X$  corresponds to the opportunity cost of incorporating the target firm, and  $P$  is the payoff from the acquired assets and projects, and the synergy created.

Notice I have assumed “Single-Crossing” here, which is standard in the literature.

Next for SPAs, the equilibrium bidding strategy is characterized by

**Proposition 4.** *In second-price auctions, the unique Bayesian Nash equilibrium in weakly undominated strategies is for type  $\theta$  to bid  $s(\theta) = \sup\{s \in [s_L, s_H] \mid V(s, \theta) \geq 0\}$ , which is decreasing in  $\theta$ . The cut-off type for participation is  $\hat{\theta} = \sup\{\theta \leq \bar{\theta} \mid \max_s V(s, \theta) \geq 0\}$ .*

The next corollary follows directly from the fact that the bidding strategies are monotone.

**Corollary 2.** *In security-bid FPAs and SPAs as described above, the investment option is allocated, if at all, to a bidder with the least investment cost. Moreover, the level of participation is the same for FPAs and SPAs, and is weakly smaller than that in cash auctions.*

In addition, the amount of competition as indicated by  $N$ , the initial commitment cost  $X$ , and the timing of the auction  $P_a$  all have fundamental impacts on bidding behavior.

**Proposition 5.** *Bidders bid more aggressively (weakly greater  $s$  for all types, and strictly greater  $s$  for a positive measure of types) in FPAs with security bids as  $N$  increases or  $X$  decreases, or if  $V/V_1$  is increasing in  $P_a$ , as  $P_a$  increases. They bid more aggressively in SPAs with security bids as  $X$  decreases or  $P_a$  increases .*

Consequently, the winner bids a greater index with more competition, smaller initial cost, or higher threshold for auction. Intuitively, a smaller  $X$  or higher  $P_a$  correspond to higher valuations of the project by the bidders, which allows them to promise more to the seller to increase their chances of winning. When  $N$  is bigger in FPAs, one has to increase the bid to outbid more competitors. However, this does not apply in SPAs because one bidder’s bidding strategy is independent of others’ bids.

These results allow the characterization of many common auctions with standard securities in addition to cash and bonus-bid auctions. Below are two examples.

## Equity Auctions and Investment Delays

Suppose  $\alpha$  is the fraction of shares the winning bidder has to pay and  $Y$  is the reserve price, the  $S(\alpha, P) = \alpha P$  and  $C(\alpha) = Y$ . By Lemma 1, a participant’s present value conditional on winning and exercising at  $P_\tau$  is  $\mathbb{E}_P[D(P_a; P_\tau)][(1 - \alpha)P_\tau - \theta] - X - Y$ . First and second order conditions give the following corollary,

**Corollary 3.** *In auctions with equity bids, the winning bidder invests when cash flow first reaches  $P^{equity}(\theta) = \max\left\{P_a, \frac{\beta\theta}{(\beta-1)(1-\alpha)}\right\}$ .*

Comparing the threshold to  $P^*(\theta)$ , the investment is inefficiently delayed—undesirable to the seller because her revenue  $D(P_a; P)S(\alpha, P)$  is decreasing in  $P$ . Ex post the auction, investing some time earlier could improve both seller’s revenue and welfare.

Bidding equilibria exist for both FPAs and SPAs, and the cut-off type  $\hat{\theta}$  is identical to that in a cash auction with the same reserve price. In SPAs, the bidder  $\theta$  increases  $\alpha$  until  $V(\alpha, \theta) = 0$ . In FPAs, since  $\frac{\partial^2 \ln V(\alpha, \theta)}{\partial \alpha \partial \theta} < 0$  is well-defined except on the boundary  $P_a = \frac{\beta\theta}{(\beta-1)(1-\alpha)}$ , Proposition 3 applies and  $\alpha(\theta)$  is continuous and decreasing. For example, when  $X = 0$ ,

$$\alpha(\theta) = \int_{\theta}^{\hat{\theta}} \frac{(N-1)f(\theta'')}{\beta(1-F(\theta''))} \exp\left[\int_{\theta''}^{\theta} \frac{(N-1)f(\theta')}{\beta(1-F(\theta'))} d\theta'\right] d\theta'', \quad \text{for } \theta \leq \hat{\theta}. \quad (8)$$

With uniform distribution, this translates to  $\alpha(\theta) = 1 - \left(\frac{\hat{\theta}-\theta}{\hat{\theta}-\theta}\right)^{\frac{N-1}{\beta}}$ . Clearly bidders bid more shares when  $N$  or  $P_a$  increases or  $X$  decreases (cutoff  $\hat{\theta}$  is increasing in  $P_a$  and decreasing in  $X$ ). Note inefficient delays of investments also follow directly from Corollary 1 since  $E_S = 1 < \beta$ .

### Call Option Auctions and Investment Accelerations

Consider call option bids with no reserve price. Let  $k$  be the strike price the winning bidder of type  $\theta$  contracts, then  $S(-k, P) = \max\{P - k, 0\}$  and  $C(-k) = 0$ . Bidder  $\theta$ ’s present value conditional on winning and exercising at  $\tau$  is  $\mathbb{E}_P[D(P_a; P_\tau)(P_\tau - \max\{P_\tau - k, 0\} - \theta)] - X$ . If a bidder of type  $\theta$  bids a strike less than  $X + \theta$ , with non-trivial probability he wins with a required strike  $k < X + \theta$  in both FPA and SPA, and fails to break even. So he is better off bidding  $k \geq X + \theta$ . If he bids a strike greater than  $P^*(\theta)$ , the required strike conditional on winning in either FPA or SPA satisfies  $k > P^*(\theta)$ . He always invests with the threshold  $P^*(\theta)$  and the call is never exercised. But he could bid lower  $k$  to increase the chance of winning. Hence  $k \in [X + \theta, P^*(\theta)]$ . The investment threshold maximizes the winning bidder’s value,

**Corollary 4.** *In auctions with call option bids, a bidder of type  $\theta$  always bids  $k \in [X + \theta, P^*(\theta)]$ , and upon winning, invests when the cash flow first reaches  $P^{call}(\theta) = \max\{P_a, k\}$ .*

Notice  $P^{call}(\theta) \leq P^*(\theta)$  and the equality holds when  $P_a > \frac{\beta}{\beta-1}\theta$ . Inefficiency therefore lies in the potential *acceleration* of investments.<sup>26</sup> Basically, if the call option is going to be exercised, there is no incentive for the bidder to keep timing the market because that delays his payment  $k$ . When  $P_a \leq k$ ,  $V = \frac{P_a^\beta(k-\theta)}{k^\beta} - X$ , otherwise  $V = k - \theta - X$ .

Proposition 3 applies for the bidding equilibrium.<sup>27</sup> Next for SPAs, for those who participate, they bid up to their valuations, in other words,  $k = \theta + X$  if  $\theta < P_a - X$  or  $k$  solves  $\frac{P_a^\beta(k-\theta)}{k^\beta} = X$  otherwise. In either case,  $k < \frac{\beta}{\beta-1}\theta$  and the investment is strictly accelerated. In fact, the seller makes a profit only when  $\theta < P_a - X$ , otherwise the strike price is simply the value of the project, netting her no profit. The cutoff type is the same as that in FPAs.

The numerical illustrations to follow sometimes include another common form of security: a fixed promise of payment  $B$  from the project's payoff—essentially debts without interests,  $S(B, P) = \min(P, B)$ , also known as friendly debt, or in Islamic finance, Qard/Qardul hassan.<sup>28</sup> Since  $E_S < \beta$  for friendly debt, investments are delayed.

### 3.2 Optimal Auction Design

DeMarzo, Kremer, and Skrzypacz (2005) show that “steeper” securities yield higher revenues for the seller. This ranking breaks down due to post-auction moral hazard: a “steeper” security extracts more from the winning bidder's information rent, but it also reduces his incentive investment efficiently post-auction. As seen in the examples of cash and bonus-bid auctions, a “steeper” security (bonus with royalty) does not always dominate. This subsection approaches security ranking from a mechanism design perspective and allow general structures of security payments.

By direct revelation principle, it suffices to examine a truth-telling mechanism corresponding to the auction. Let  $Q(\tilde{\theta}_i, \theta_{-i})$  be the probability of allocating the project to bidder  $i$ , where  $\tilde{\theta}_i$  is the reported type by  $i$  and  $\theta_{-i}$  are other participants' reported types. A general

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<sup>26</sup>Existing dynamic agency models of investment often predict decreased or delayed investments (e.g., Grenadier and Wang (2005) and DeMarzo and Fishman (2007)). It really depends on the security design.

<sup>27</sup>Note  $\frac{\partial \ln V}{\partial(-k)} < 0$ . For  $k \leq P_a$ ,  $\frac{\partial^2 \ln V}{\partial(-k)\partial\theta} = -\frac{1}{(k-\theta-X)^2} < 0$ ; for  $k > P_a$ , it is  $-\frac{(\beta-1)}{V^2 k^\beta} \left[ \left(\frac{P_a}{k}\right)^\beta \frac{\theta}{\beta-1} - X \right] \leq -\frac{(\beta-1)}{V^2 k^\beta} \left[ \left(\frac{P_a}{k}\right)^\beta (k-\theta) - X \right] = -\frac{(\beta-1)}{V k^\beta} < 0$  using the fact  $k \leq \frac{\beta}{\beta-1}\theta$ .

<sup>28</sup>Interestless debts are used frequently in contractual agreements in Islamic banking and microfinance, and are equivalent to granting the winning bidder instead of the seller call options - the exact opposite situation to that for call option bids.

security payment then has the form  $S(\tilde{\theta}_i, \theta_{-i}, \mathcal{I}_t)$  at time  $t$ , where  $\mathcal{I}_\tau$  is the set of contractible information up to time  $t$ .<sup>29</sup> The expected utility at time zero to type  $\theta_i$  upon participating and optimally investing is

$$U(\theta_i, \tilde{\theta}_i) = \mathbb{E}_{\theta_{-i}} \left[ Q(\tilde{\theta}_i, \theta_{-i}) \max_{\tau \geq t_a} \mathbb{E}_P \left[ e^{-r\tau} (P_\tau - \theta_i) - \int_{t_a}^{\infty} e^{-rt} S(\tilde{\theta}_i, \theta_{-i}, \mathcal{I}_t) dt - e^{-rt_a} X \right] \right].$$

Let  $\tau^*(\theta_i, \tilde{\theta}_i, \theta_{-i})$  denote the optimal stopping time that is almost surely bigger than  $t_a$ , and  $\tau_i^* = \tau^*(\theta_i, \theta_i, \theta_{-i})$ . Incentive compatibility requires  $U(\theta_i) \equiv U(\theta_i, \theta_i) \geq U(\theta_i, \tilde{\theta}_i)$  and the individual rationality requires  $U(\theta_i) \geq 0$ .<sup>30</sup> Equivalently,

**Lemma 2.** *Any incentive compatible and individually rational mechanism satisfies*

$$U(\theta_i) = \mathbb{E}_{\theta_{-i}} \left[ \int_{\theta_i}^{\bar{\theta}} Q(\theta_j, \theta_{-i}) \mathbb{E}_P [e^{-r\tau_j^*}] d\theta_j \right] + U(\bar{\theta}) \quad (9)$$

where  $U(\bar{\theta}) \geq 0$ . Moreover  $\tau_i \geq t_a \forall i$  for time consistency.

The design of the auction naturally decomposes into two parts: auction timing and security design. I now examine the ranking of security design given  $t_a$ , and derive the optimal security. Auction timing in the optimal auction design is the subject of next subsection. For notational convenience, let  $z(\theta) = \theta + F(\theta)/f(\theta)$ .<sup>31</sup>

## Ranking Security Design

**Proposition 6.** *The seller's revenue in FPAs and SPAs with standard security bids is given by*

$$\mathbb{E} \left[ \mathbb{1}_{\{\theta_{(1)} \leq \hat{\theta}\}} \left[ e^{-r(\tau_1^* - t_a)} (P_{\tau_1^*} - z(\theta_{(1)})) - X - Y \right] \right] \quad (10)$$

where  $\theta_{(1)}$  is the smallest realized cost, and  $\tau_1^*$  is the bidder's corresponding optimal stopping time for investment. Cutoff type  $\hat{\theta}$  is as given in Propositions 3 and 4.

<sup>29</sup>For standard security bids,  $\mathcal{I}_\tau$  contains cash flow from the project  $P_\tau$  when invested at  $\tau$ , but in general  $\mathcal{I}_t$  could include the history of  $P$  up to  $t$ , and  $t$  itself if they are contractible.

<sup>30</sup>An extension with positive reserve utility or entry cost/fee is straightforward.

<sup>31</sup> $z(\theta)$  is increasing - a standard assumption in the auctions literature, for example, see Krishna (2009). One sufficient condition is the "inverse hazard function"  $F(\theta)/f(\theta)$ 's being non-decreasing.

The seller's payoff thus depends on the "virtual valuation" of the best type rather than the actual valuation.<sup>32</sup> The seller essentially owns the best type's real option with an additional cost, which is stochastic. In general, the winning bidder's optimal investment timing differs from the seller's. This proposition, together with the bidding equilibria for formal auctions derived earlier, allow the ranking of various security designs.

Security ranking depends on parameters such as  $N$  and  $\sigma$ . In particular, different auction timings lead to different security ranking. This phenomenon is best seen in Figure 7(a): Among several pure contingent securities, equity gives the highest expected revenue and call option the lowest at  $P_a = 280$ , whereas call option is the highest and debt is the lowest at  $P_a = 360$ .

### Optimal Security Design

Despite the complexity in security ranking, one can find the optimal security design. To maximize revenue, the seller wants  $\tau_i^*$  to be the first-hitting time of threshold  $P^*(z(\theta_i))$ , in which case she allocates the project to the bidder with the lowest  $\theta$ .

**Proposition 7.** *An optimal security design exists and is implemented by FPAs using well-ordered securities indexed by  $s$ : Denote  $\hat{\theta}$  the solution to  $P_a^\beta(\beta - 1)^{\beta-1} = (X + Y)\beta^\beta z(\theta)^{\beta-1}$ ,*

$$C(s) = \begin{cases} \frac{P_a^\beta}{\beta P^*(z(-s))^{\beta-1}} - X - \int_{s_L}^s \left[ \frac{1-F(-s')}{1-F(-s)} \right]^{N-1} \left[ \frac{P_a}{P^*(z(-s'))} \right]^\beta ds', & \text{if } s \in [s_L, -\underline{\theta}] \\ C(-\underline{\theta}) + \underline{\theta} + s, & \text{if } s > -\underline{\theta} \end{cases} \quad (11)$$

$$S(s, P) = \phi(s)P, \quad \text{where } \phi(s) = \frac{(\beta - 1)F(-s)\mathbb{1}_{\{s \in [s_L, -\underline{\theta}]\}}}{(\beta - 1)F(-s) - \beta s f(-s)}, \quad \text{and } s_H = \infty, s_L = \max\{-\bar{\theta}, -\hat{\theta}\}.$$

The optimal security can be interpreted as a cash down payment plus a royalty payment, which is frequently used in the sales of licensing or marketing rights and contracts in publishing or movie production. In equilibrium, type  $\theta$  bids  $s = -\theta$ . Recall  $P^{bonus} = \frac{\beta}{\beta-1} \frac{\theta}{1-\phi}$ , which implies  $S(s, P^{bonus}) = \frac{F(\theta)}{f(\theta)}$ , i.e., the information rent in Equation (10). This equates the bidder's marginal benefit of waiting to his marginal cost of waiting. The winner and the seller then face the same optimization problem for investment. The contingent security thus aligns the winner's post-auction incentives with the seller's, and the cash payment ensures

<sup>32</sup>This payoff is equivalent to the expected marginal revenue (MR), see Bulow and Roberts (1989).

incentive compatibility.<sup>33</sup>

The project is allocated, if at all, to the best type. However, the investment threshold for type  $\theta$  satisfies  $P^*(z(\theta)) \geq P^*(\theta)$ , and it can be verified that some projects of positive social value are not allocated and invested. Thus despite the elimination of moral hazard, investments are weakly delayed or missed entirely relative to the socially efficient outcome.

Moreover, to ensure incentive compatibility, the royalty payment is increasing in  $\theta$  while the cash payment is decreasing in  $\theta$ . A high-cost type has less incentive to mimick a low-cost type because he has to pay more cash, and has a contingent residual that is more sensitive to his investment timing which is more distorted in equilibrium. This extends McAfee and McMillan (1987a)'s work on optimal linear incentive contracts in the following ways: First, my setup include realistic cases where the contractible output are generated post-auction. Second, the result shows linear incentive contracts are rather robust to time discounting, especially that the discounted project payoff is actually decreasing in contractible output  $P$ . Moreover, linear incentive contracts are also robust when we include all dynamic payments based on available contractible information. Finally, optimal security involves negatively correlated cash down payment and contingent royalty payment, a novel and testable prediction that is of interests for empirical studies.

### Optimal Auction Timing

In addition to security design, the seller has the liberty to decide when to hold the auction. Figure 7(a) illustrates how auction timing affects the seller's revenue for several common securities. This subsection derives the auction timing in an optimal auction.

The seller's present expected utility holding the auction with optimal security design at time  $t_a$  is  $\mathbb{E}[e^{-rt_a}[W(P_a; z(\theta_{(1)})) - X - Y]^+]$ , where  $P_a$  is the cash flow level at  $t_a$ . The seller essentially owns a timing option with irreversible cost  $X + Y$ .

**Proposition 8.** *There is a unique threshold strategy for timing the auction that is optimal*

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<sup>33</sup>Another interpretation of the optimal security is a cash payment to acquire the real option, plus a strike payment  $\frac{F(-s)}{f(-s)}$  to the seller for exercising the option. This corresponds to the insight in Board (2007b) to use a revenue-independent contingent payment to align incentives.

among all stopping times. The threshold is the largest root  $P_a$  to

$$\int_{\underline{\theta}}^{\hat{\theta}_z} f(\theta)[1 - F(\theta)]^{N-1} [\beta(X + Y + z(\theta)) - (\beta - 1)P^*(z(\theta))] d\theta = 0, \quad (12)$$

where  $P^*(\theta) = \max\{P_a, \frac{\beta}{\beta-1}\theta\}$ , and cutoff type  $\hat{\theta}_z$  solves  $W(P_a; z(\hat{\theta}_z)) = X + Y$ . In particular, the auctioneer never sells the project when she expects no chance of immediate investment.

Intuitively, option values erode as  $P_a$  increases, thus it would not be optimal to postpone the auction indefinitely. If the seller sells at a level where she expects no bidder to invest right away, she can profitably deviate by delaying the incidence of cost  $X + Y$  and waiting for greater participation. The bigger  $X + Y$  is, the more the seller endogenously delay the auction. When  $X + Y \rightarrow \infty$ , the project is never sold.<sup>34</sup>

Since the seller owns an investment option with a greater exercise cost  $z(\theta_{(1)}) \geq \theta_{(1)}$  due to information rent, the threshold to incur  $X + Y$  to sell the option is higher:

**Proposition 9.** *Optimal formal auctions happen weakly later than efficient formal auctions.*

I show this in Appendix A.11 by first proving that both optimal and efficient security designs have unique threshold for holding the auction, and then arguing that at the efficient threshold, it is better to wait further using the optimal security.

As in Myerson (1981)'s analysis on optimal auction design in a static setting, there is a wedge between the seller's revenue and welfare, but with post-auction actions and auction timing, the optimal design here has several distinct features. Although the seller still excludes bidders, the auction is held under better market condition (higher  $P_a$ ), which encourages participation and mitigates the exclusion. This implies that in real life one may not see sellers excluding bidders as much using entry fee or reserve price, because she has the alternative tool of choosing a more propitious time to hold the auction. Moreover, the auction and investment are inefficiently delayed - inefficiencies in dynamic settings are multi-dimensional.

While in formal auctions the seller commits to the auction timing and security design, in reality bidders often play a more active role as discussed next.

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<sup>34</sup>Though not considered in the current setup,  $X + Y < 0$  simply implies that the seller holds the auction as soon as it is feasible to avoid missing the investment threshold.



## 4 Informal Auctions

Many economic interactions such as corporate takeovers, competition for supply contracts, and talent recruitment can have characteristics of auctions. The discussion in formal auctions restricts bids to a pre-specified ordered set. Yet the seller often cannot ignore offers from outside the set, and thus considers all bids and chooses the most desirable one ex post, especially in informal auctions. This essentially leaves the security design of the auction to the bidders as they can bid any contingent payment. Moreover, the seller may have to consider offers before she puts the asset up for sale, in which case both the auctioneer and the bidders can strategically initiate the auction.<sup>35</sup>

It turns out investments are always efficient when the seller cannot commit to auction timing and pre-specified security design. As such, the conclusion in DeMarzo, Kremer, and Skrzypacz (2005) still applies: cash is the cheapest way for a better type to separate from worse types and, in equilibrium, every bid is equivalent to cash. This section further shows that when only the seller can initiate, the auction is inefficiently delayed (ex ante). When the bidders can initiate the auction as well, the auctioneer always waits for an offer in equilibrium, and the auction is inefficiently accelerated (ex post).

### 4.1 The Signaling and Timing Game

If the seller commits to neither a pre-specified timing of the auction nor a bidding and allocation rule, she chooses the bid that gives her the highest expected payoff based on her beliefs regarding the type of each bidder at the time the auction is held. The auction therefore exhibits features of a signaling and timing game of the following form:

1. Either the seller or a bidder initiates the auction at some time  $t_a \geq 0$ .
2. Participating bidders submit informal bids simultaneously.<sup>36</sup> An informal bid  $\Pi^i$  by bidder  $i$  is a cash payment  $C^i$  and a security payment  $S^i(\cdot)$  contingent on the project cash flow subject to limited liability  $S^i(P) \in [0, P]$ .

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<sup>35</sup>When an asset of a Delaware corporation is for sale after being approached by a buyer, the Revlon rule imposes upon directors a duty to solicit bids and conduct an auction.

<sup>36</sup>Any indifference between bidding or not when knowing that a better type has initiated can be resolved by having a liquidity shock arriving at Poisson rate  $\lambda \rightarrow 0$  precipitates the auction.

3. The seller chooses the winning bidder rationally according to the valuation function  $R(\Pi^i) = C^i + \mathbb{E}[R_\theta(S^i)|\Theta(\Pi^i)]$  provided she values the bid more than the reservation value  $Y$ .  $\Theta(\Pi^i)$  is her belief of bidder  $i$ 's type upon seeing the bid, and  $R_\theta(S^i) = \mathbb{E}[e^{-r\tau_\theta^i} S^i(P_{\tau_\theta^i})]$  where  $\tau_\theta^i$  is the optimal stopping rule for type  $\theta$  when bidding  $\Pi^i$ , i.e.  $\tau_\theta^i = \operatorname{argmax}_{\tau \geq t_a} \mathbb{E}[e^{-r\tau}(P_\tau - S^i(P_\tau) - \theta)] - X - C^i$ .
4. The winning bidder  $i$  pays the upfront cash  $C^i$  and the initial cost  $X$  at  $t_a$ , then invests rationally at  $\tau_{\theta_i}^i$  and makes the contingent payment.

Note that the seller's valuation  $R(\Pi^i)$  is not necessarily the same as the value of the security to bidder  $i$ ,  $C^i + R_{\theta_i}(S^i)$ . One may question if the setup of the game misses out any informal offers, such as contracting on the timing of investment when feasible. The results are robust to additional side contracts because one can enlarge the security space from  $S(P_t)$  to  $S(\mathcal{I}_t)$  where  $\mathcal{I}_t$  is the entire contractible information set, as long as limited liabilities hold. The proofs apply with minor changes in notations.<sup>37</sup>

For informal auctions, FPAs simply mean allocating the project to the bidder with the highest bid according to the valuation function, and SPAs are understood as a variant of English auctions where the seller announces the valuation of bids as bidders continuously adjust their offers until all have stopped.

## 4.2 Bidding Equilibrium in Informal Auctions

Taking the cash flow level at  $t_a$  as given, there is an *essentially unique* bidding equilibrium. Moreover, equilibrium investments are efficient. The proofs for the following results are robust to the belief on the distribution of types, thus are independent of how learning takes place in the timing game. This allows a nice separation of auction timing and signaling by security bids, rendering the problem tractable.

**Lemma 3.** *In a bidding equilibrium, a participating bidder  $i$  has  $\tau_{\theta_i}^i = \tau_i^*$  where  $\tau_i^*$  is the stopping time corresponding to the threshold strategy with investment trigger  $P^*(\theta_i)$ .*

The intuition is that if a bidder does not invest efficiently upon winning, he can always deviate to a bid that results in efficient investment, and offer more cash to the seller to

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<sup>37</sup>The results also hold for non-standard security bids as long as optimal investment strategy exists. The extension discusses this further.

increase his marginal probability of winning without reducing the payoff upon winning.

**Lemma 4.** *Informal auctions only admit fully-separating equilibria.*

As every bidder upon winning invests efficiently, a better type has greater valuation than worse types and can separate. This also implies that no two bidders place the same bid.

**Proposition 10.** *There is an essentially unique bidding equilibrium for the informal auction, which is equivalent, in terms of allocation outcome and expected payoffs, to a first-price cash auction with reserve price  $Y$ . In particular, post-auction investment is efficient.*

Basically, the bids are all cash-like in equilibrium. A better type finds it cheaper to use a security that is less sensitive to the true type and creates more social surplus. Take equity bids for example. Not only do they inefficiently delays investment, but a better type finds them costly to use because his  $\alpha$  shares are worth more than a worse type's. Cash-like securities that ensure efficient investment and cheap separation are the most attractive. Because a better type is indifferent from mimicking a marginally worse type in equilibrium, all bidders must be using cash-like securities.

### 4.3 Endogenous Timing of Informal Auctions

I assume that when forming initiation and bidding strategies, any indifference in timing is resolved by initiating later, and in participation by participating.<sup>38</sup> First, consider the simpler game where only the seller can initiate the auction. Since in equilibrium, the payoffs are equivalent to a first-price cash auction and FPAs and SPAs with cash generate the same revenue, the seller's timing problem is equivalent to the strategic timing of a second-price cash auction. Her expected utility for holding the auction at time  $t_a$  with cash flow  $P_a$  is  $\mathbb{E}[e^{-rt_a}[W(P_a; \theta_{(2)}) - X - Y]^+]$ . Similar to timing an formal auction design,

**Proposition 11.** *When only the seller can initiate an informal auction, there is an optimal timing with auction threshold  $P_a$  inefficiently high.*

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<sup>38</sup>This makes the equilibrium more robust and can be formally justified by assuming a small probability of initiation failure and a small Poisson arrival rate of exogenous auction initiation, then taking their limits to zero.

Cash auctions and informal auctions timed by the seller are thus inefficiently delayed. The intuition is that a welfare maximizer initiates only when the auction payoff exceeds the cost of ownership transfer by a certain threshold. But the seller faces the additional cost in the form of information rent paid to the winning bidder, thus times her option value starts to erode only with higher  $P_a$ , commanding a higher option premium for holding the auction.

In real life, especially in M&As and patent sales, we often see bidders initiating the auction.<sup>39</sup> To fully endogenize the auction timing in an informal auction, one has to consider the Bayesian equilibrium in the optimal stopping game where both seller and bidders can initiate the auction. This timing game is complicated by two facts: first, all parties dynamically update their beliefs about the distribution of types; second, this learning process potentially renders FPAs different from SPAs because the bidding strategy in FPAs is dependent on the belief of other bidders' types whereas in SPAs the undominated strategy is to bid one's own value.

**Proposition 12.** *The Bayesian auction game with informal bids admits an auction timing equilibrium where bidders always initiate with threshold  $P_I(\theta)$  increasing in  $\theta$  that uniquely solves*

$$\int_{\theta}^{\bar{\theta}} \frac{d}{dP} \frac{W(P; \theta) - W(P; \theta')}{P^{\beta}} \Big|_{P=P_I} f(\theta') [1 - F(\theta')]^{N-2} d\theta' = 0. \quad (13)$$

*This is an essentially unique equilibrium in SPAs, and the unique monotone equilibrium in FPAs.*

The intuition is simpler in SPAs. If by cash flow level  $P$  the auction has not been initiated, everyone updates their beliefs about types that are present. The seller times the auction to maximize the second-highest valuation, whereas type  $P_I^{-1}(P)$  times the auction to maximize the present value of informational rent (difference between the his valuation and the second highest valuation). The latter starts to erode earlier than the former does as the initiation threshold  $P_a$  increases. Therefore the seller always waits in such an equilibrium. In FPAs this may not hold because bidding strategies depend on the dynamically updated beliefs about the types present. Fortunately, in a monotone timing equilibrium, the bidders truncate the support of  $\theta$  from lower values corresponding to better types, but form bidding strategies using beliefs on types that are worse. This ensures that the bidding strategies are not affected

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<sup>39</sup>See Fidrmuc, Roosenboom, Paap, and Teunissen (2012), and Gorbenko and Malenko (2013b).

by dynamic learning.

The prediction that bidders initiate is broadly consistent with empirical evidence. For example, patent holders rarely organize an auction and instead are often approached by acquirers. Also, acquisitions by strategic bidders in informal negotiations are primarily bidder-initiated.<sup>40</sup>

**Corollary 5.** *Bidder-initiated informal auctions are inefficiently accelerated ex post.*

Conditional on knowing the least-cost type, the auction should be held later to maximize social welfare. To see this, a bidder gets the difference between his valuation and the second highest valuation, and does not bear the cost of ownership transfer  $X + Y$  unless he is the only participant, because this cost impacts the highest valuation and the second highest valuation in the same way. It in turn means that when  $X + Y$  increases, both the seller and the bidders would prefer a higher threshold for initiation, but the seller is differentially affected more. Numerical simulations show that in general the bidder-initiated informal auctions does better than the seller-initiated informal auctions. The intuition is that when bidders initiate, there is dynamic learning and their private information is utilized for the timing decision, which improves welfare, and often the revenue to the seller. When the seller initiates, she only uses the prior belief on the distribution of types, which differs from the realized types.

Finally, since a bidder would not initiate until the investment trigger for his real option is reached, the following corollary ensues.

**Corollary 6.** *The real option is exercised more quickly in informal auctions that are bidder-initiated than in those that are not bidder-initiated.*

In practice, the seller's level of commitment may lie in a continuous spectrum. For examples, the seller may commit to the security design but not the auction timing.<sup>41</sup> Though not explicitly discussed here, the tradeoffs regarding auction timing are similar and many results generalize. In particular, Corollary 6 holds for bonus-bid auctions.

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<sup>40</sup>For example, Fidrmuc, Roosenboom, Paap, and Teunissen (2012) document almost 80% are bidder-initiated. Note the current model is more applicable to strategic acquisitions where bidders are more likely to have private information regarding valuation than in financial acquisitions.

<sup>41</sup>The setup in Gorbenko and Malenko (2013b) resembles such a case.

## 5 Model Implications and Empirical Evidence

The model has three main implications. First, security bids cause misaligned incentives in post-auction investment; the distortion depends on auction timing and security design. Second, strategic timing of auctions affects auction initiation, security choice, investment and equilibrium payoffs, and is fundamental to auction design. Third, in settings rich in dynamics and information asymmetry, conventional wisdom should be applied with caution.

### 5.1 Inefficiencies in Investment Timing

Earlier sections have demonstrated that investments are delayed in equity auctions, bonus auctions, and friendly debt auctions, and are accelerated in call option auctions. Even the revenue-maximizing design delays investment. Figure 3 gives an illustration of investments at very different times under various security designs. Using the properties of the Wald distribution, the time delay  $t_D$  when the investment threshold is increased from  $P_1$  to  $P_2$  has PDF,<sup>42</sup>

$$f_{GBM}(t_D; P_1, P_2) = \frac{\ln \frac{P_2}{P_1}}{\sqrt{2\pi\sigma^2 t_D^3}} \exp \left[ -\frac{\left[ \ln \frac{P_2}{P_1} - \left( \mu - \frac{\sigma^2}{2} \right) t_D \right]^2}{2\sigma^2 t_D} \right] \quad (14)$$

with the mean and shape parameters  $m = \ln \left( \frac{P_2}{P_1} \right) / [\mu - \frac{\sigma^2}{2}]$  and  $y = \left[ \ln \left( \frac{P_2}{P_1} \right) / \sigma \right]^2$ . In the case of royalty auctions or bonus-bid auctions, the expected investment lag is  $\Gamma = -\ln(1 - \phi) / [\mu - \frac{\sigma^2}{2}]$ , where we have assumed  $\mu - \frac{\sigma^2}{2} > 0$  for the expectation to exist.<sup>43</sup>

Next, I examine the effects of changing the underlying parameters on the expected investment lags in bonus-bid auctions. The expected development delays are independent of  $N$ ,  $X$ ,  $P_a$ , and  $r$ , but  $\frac{\partial \Gamma}{\partial \phi} > 0$ ,  $\frac{\partial \Gamma}{\partial \mu} < 0$  and  $\frac{\partial \Gamma}{\partial \sigma} > 0$ . In addition,  $\frac{\partial^2 \Gamma}{\partial \phi^2} > 0$ ,  $\frac{\partial^2 \Gamma}{\partial \sigma^2} > 0$ ,  $\frac{\partial^2 \Gamma}{\partial \sigma \partial \phi} > 0$ . Not only do more volatile market or high royalty rates result in longer delays, but they are mutually reinforcing, with increasing marginal effects. What is the social cost of the investment lag? It can be shown that the option value is a fraction  $(1 - \phi + \phi\beta)(1 - \phi)^\beta$  of the socially efficient value, and the fractional loss  $L$  satisfies  $\frac{\partial L}{\partial \phi} > 0$ ,  $\frac{\partial^2 L}{\partial \phi^2} > 0$ . Again, royalty

<sup>42</sup>See Karlin and Taylor (1975) pg.363, and Chhikara and Folks (1989).

<sup>43</sup>If  $\mu < \sigma^2/2$ , the median lag  $M$  can be considered instead. It satisfies  $\Phi \left[ \frac{\ln(1-\phi) + (\mu - \sigma^2/2)M}{\sigma\sqrt{M}} \right] + (1 - \phi)^{1 - \frac{2\mu}{\sigma^2}} \Phi \left[ \frac{\ln(1-\phi) - (\mu - \sigma^2/2)M}{\sigma\sqrt{M}} \right] = \frac{1}{2}$ , and numerical simulations yield the same qualitative results under a wide range of parameters.

rate has a compounding effect on social cost.

These predictions are consistent with available empirical evidence. The US Department of the Interior experimented with royalty auctions in 1978–1983, where the government fixed a small up-front “bonus” payment and allowed the bidders to compete on royalty rates. Many bidders bid extremely high royalty rates and the tracts were never drilled.<sup>44</sup> Oil price and volatility were indeed extremely high during that period. Moreover, Humphries (2009) reports that the royalty relief programs in the 1990s significantly increased interest in deepwater leases, and oil production increased sharply. Opaluch, Grigalunas, Anderson, Trandafir, and Jin (2010) also conclude that increased royalty rates would have a net negative effect on the social value of offshore development. Appendix C.3 contains a Cox Hazard Rate estimation for a diff-in-diff test with the treatment being the Deep Water Royalty Relief Act (DWRRA) in 1996–1999 to eligible leases. There is some evidence that a reduced royalty rate increases propensity to explore tracts, and the mean estimate indicates a differential increase of 11% in the likelihood to drill between eligible and uneligible tracts. Cong (2013b) contains a more detailed discourse.

By highlighting the destructive powers of moral hazard, this paper suggests a potential explanation for why large tracts of land remain idle.<sup>45</sup> Without prescribing detailed policy changes, this paper suggests that there is always the trade-off between social efficiency and revenue extraction. Any useful policy recommendations should first focus on reducing the post-auction moral hazard that is inimical to both the revenue and social welfare. Moreover, instead of uniformly raising the royalty rate, allowing bidders to self select into differential rates as described in Proposition 7 could be a more effective way in increasing revenue to the government.

## 5.2 Strategic Timing of Auctions

As seen in propositions 5, 8-9,11, and 12, auction timing affects bidding strategy, auction outcome, and investment decision. More importantly, this shows many aspects of auction

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<sup>44</sup>See Dougherty and Lohrenz (1980) and Binmore and Klemperer (2002).

<sup>45</sup>In reality, many other strategic interactions among the bidders complicate the issue. For example, Beshears (2011) shows alliances in oil and gas drilling perform better than solo bidders; Hendricks and Porter (1996) attributes the delays in exploratory drilling to free-rider problem and war of attrition. The above analysis complements these studies.

design should be analyzed in conjunction with endogenous auction timing. In Figure 7(a), the worst security design at  $P_a = 300$  outperforms the best security design at  $P_a = 220$  by at least 1.5. Similar phenomena are observed in FPAs (Figure 2) and for welfare (Figure 7(b)) too. In this regard, strategic timing is equally important as security design.

There could be factors exogenous to the model that affect the strategic timing of auctions. For one, delaying the auction risks creating market uncertainty and delaying the introduction of new technology or development, and potentially losing the “first-mover” advantage.<sup>46</sup> Another motivation for timing the auction is the coordination of market players.<sup>47</sup> This paper shows that strategic timing due to cost of auction is an important complement to other factors in explaining the timing of auctions observed in real life.

### **Empirical Test of Auction Initiation and Investment**

In addition to auction timing, who initiates an auction also matters. When the seller lacks commitment to auction timing, the bidders always initiate and the real option is exercised more quickly (Proposition 12 and Corollary 6). Since bidders time the auction to maximize information rent, they only initiate when their option value starts to erode, implying that the investment option is, on average, exercised faster when bidders initiate. I test this using data on leasing and exploration of oil and gas tracts in the Gulf of Mexico. Prior to the introduction of Area Wide Leasing (AWL) in May 1983, energy firms could nominate oil and gas tracts to be auctioned. AWL eliminated the nomination process and made most of the offshore lands available in every sale. The auction environment thus underwent an important break from one where bidders can initiate to one where leases are always on sale in a region decided by the seller. The model predicts that, bidders explore and drill faster when they initiate sales than when seller initiates. I use the Cox proportional hazard model to study the time to first exploratory drill for over 20,000 leases sold in the Gulf of Mexico. The Cox model does not impose restrictions on the baseline hazard rate and allows time-varying covariates and censoring of observations, and has become standard for duration analysis.

Table 2 reports the estimations of the model  $\kappa(t) = \psi(t)[\exp(\vec{X}(t)^T \vec{\gamma})]$ , where  $\kappa(t)$  is the

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<sup>46</sup>The sale of the British 3G telecom licences is an illustrating case (Klemperer (2002), Binmore and Klemperer (2002)).

<sup>47</sup>Auctions of futures contracts on electricity provision is a good example where an early auction allows the winning bidder ample time to construct facilities for generating and delivering electricity.



hazard rate of exploratory drill at time  $t$  conditional on lack of drill until time  $t$ , and  $\psi(t)$  is the baseline hazard rate that is unrestricted.  $\vec{X}(t)$  is a vector of independent variables, including the variable of interest  $AWL$ , which is 0 before May 1983 and 1 after, and other controls for firm, lease, and market effects (Appendix C.2 details their constructions).  $\vec{\gamma}$  are their coefficients to be estimated. The hazard ratio associated with  $AWL$  is consistently negative and significant, indicating that *ceteris paribus*, the rate to explore decreased from that in the era with bidder initiations. I consider the years 1978-1989, though the results are robust to the size of the window.  $AWL$  consistently reduces the likelihood of exploratory drilling by at least 10%. I also estimate the model with year dummies instead of  $AWL$ , and Figure 4 clearly shows the break the estimated coefficients, which corresponds to a reduction of 40% in the hazard ratio to explore and drill for leases auctioned after the implementation of  $AWL$ .

### 5.3 Nonconventional Wisdom

Many standard results in auction theory may not hold in the presence of pre-auction timing and post-auction dynamics, prompting re-examination of conventional beliefs.

#### **Welfare Creation: Are Auctions as Efficient as We Think?**

Are auctions as socially efficient as traditionally believed once we consider endogenous timing of auctions? Take private-value cash auctions for example: They are considered efficient in the standard literature but are, in fact, inefficiently delayed. Many other auctions are also less efficient than one believes once strategic timing is taken into consideration. The intuition is that the seller times the auction to maximize her revenue, not the social welfare. For regulators concerned with social efficiency, it is critical to consider auction timing in addition to the seller's market power.

#### **Auctions versus Negotiations: The More, The Merrier?**

It is well-established that in private-value auctions increasing the number of bidders enhances the seller's revenue. The importance of competition in corporate takeovers is perhaps best articulated in Bulow and Klemperer (1996):

*“With independent signals and risk-neutral bidders, an absolute English auction with  $N + 1$  bidders is more profitable in expectation than any negotiation with  $N$  bidders.”*

But in reality, sellers restrict the number of bidders even absent entry fees, for example, in sales of private companies and divisions of public companies (see Hansen (2001) and French and McCormick (1984)). Close to half of all corporate takeovers in the 1990s avoided public auctions with more bidders and opted for private negotiations (Boone and Mulherin (2007)). The moral hazard associated with security bids provides a potential explanation.

As competition intensifies, bidders bid more (Proposition 5), resulting in greater moral hazard. And it can be shown that the seller does better charging an entry fee or reserve price. Thus negotiations with  $N$  bidders can yield higher revenue than an absolute English auction with  $N + 1$  bidders. Figure 5 gives numerical simulations in the same spirit as in Samuelson (1985) to indicate that revenue and welfare could vary in almost any way with  $N$ . There is no contradiction, however, because the monotonicity of revenue in competition is restored with the optimal security. The key lesson is that the impact of competition depends on the security design.

The result generalizes to auctions with standard securities such as friendly debts and call options (see Figure 6), and is robust to distributional assumptions and endogenous entries with entry costs. Since the expected social welfare and revenue to the seller need not increase with the number of potential bidders, limiting participation may improve revenue or welfare. Given that many public auctions of companies involve the use of standard securities, this is consistent with the aforementioned empirical observations.

### **Security Ranking: One Size Fits All?**

While DeMarzo, Kremer, and Skrzypacz (2005) derive elegant results on “steepness” ranking security designs, the ranking has to be considered in conjunction with potential mistalignment of incentives, timing of auctions, and number of bidders, etc. In Figure 6, ranking is sensitive to the number of potential bidders while in Figure 7(a), ranking is sensitive to auction timing. In both figures, though equity generally dominates friendly debt, it could either dominate or be dominated by call options despite being less “steep”. These results are robust to the introduction of entry fees or entry costs and are novel in linking security design to level of competition and endogenous auction initiation.

## Cash versus Contingent Securities: Who is the Winner?

Prior studies indicate that security bids usually perform better than cash bids. Rhodes-Kropf and Viswanathan (2000) show that any securities auction generates higher expected revenue to the seller than a cash auction. But since the linkage advantage of security bids lies in the extraction of the winning bidder's rent, it decreases in expectation when  $N$  increases. Yet moral hazard persists with many standard securities. In particular, cash could generate higher revenue than many standard securities such as equities. To illustrate, I focus on pure contingent securities.

First note that any contingent security  $S(s, P)$  can be approximated by  $\sum_{i \in I} a_i(s)[P - b_i(s)]^+$ , where  $I$  is a countable set and  $\sum_i a_i(s) \leq 1 \forall s$  to ensure  $P - S(s, P)$  is weakly increasing in  $P$ . Suppose  $\underline{s}$  is the security the type  $\underline{\theta}$  bids, without loss of generality  $b_i(\underline{s}) \leq b_j(\underline{s})$  if  $i \leq j$ . I define the following class of securities:

**DEFINITION.** An  *$M$ -regular security* is a contingent security for which the above approximation is exact such that for  $M > 0$ , and  $\frac{\beta}{\beta-1}\underline{\theta} \in [b_m, b_{m+1})$ ,

$$\min \left( \left| \frac{\beta}{\beta-1}\underline{\theta} - b_m \right|, \left| \frac{\beta}{\beta-1}\underline{\theta} - b_{m+1} \right|, \left| \sum_{i \leq m} a_i b_i - \underline{\theta} \sum_{i \leq m} a_i \right| \right) > M. \quad (15)$$

In fact, most common securities are  $M$ -regular securities. For example, equity corresponds to  $a_1 = \alpha(\underline{\theta})$ ,  $a_2 = b_1 = 0$ ,  $b_2 = \infty$ .

**Proposition 13.** *For any  $M > 0$ , cash bids dominate  $M$ -regular securities in FPAs and SPAs in terms of expected revenue and social welfare, as the number of bidders gets large.*

In particular, cash bids dominate equity bids, call option bids, and friendly debt bids. Given that most common securities can be approximated by an  $M$ -regular securities, the size of the bidders market is an important consideration in security choice. The result also predicts that security bids are seldomly used when the number of bidders is large.

Other considerations also influence security design. For example, the medium of exchange acts as a signal to the market (Eckbo, Giammarino, and Heinkel (1990), Betton, Eckbo, and Thorburn (2009) and Malmendier, Opp, and Saidi (2012)). Gorbenko and Malenko (2011) suggest another scenario where cash dominates equity bids when the number of sellers and

the corresponding bidders' markets are large, because sellers find cash more effective in attracting more bidders. The battle between cash and contingent securities goes on and there is unlikely to be a clear winner.

## 6 Extensions and Discussions

### 6.1 Regret and Renegotiation

A regret-proof mechanism is easy to implement. Some standard securities allow allocations to be sub-optimal based on the seller's inference of the bidders' types, which can cause disqualification of the bid ex post.<sup>48</sup> But people still use them for simplicity of bidding rules, in agreement with the spirit of formal auctions where allocation rules are fait accompli. Moreover, regret is rare in auctions with standard securities.<sup>1</sup>

Renegotiation is another form of ex post regret. In special cases, renegotiation can make everyone better off ex post. I use bonus-bid auctions to illustrate. Winning bidder of type  $\theta$  can renegotiate to invest efficiently and split the additional social surplus in proportion (this includes the Nash bargaining solution); this process is equivalent to contracting to invest at  $P^*(\theta)$  using a new royalty rate  $\hat{\phi} \in \left[ \phi(1 - \phi)^{\beta-1}, \frac{1-(1-\phi)^\beta}{\beta} \right]$ , where the bounds ensure that both parties are weakly better off. But this leads to  $1 - \hat{\phi}\beta \geq (1 - \phi)^\beta > 1 - \phi\beta$  by Bernoulli's inequality. Thus  $\hat{\phi} < \phi$ . Given that the bidders pay a smaller royalty rate with renegotiation, they bid more cash bonus upfront. This new equilibrium obviously improves welfare. Since the seller receives higher bonus bids, and weakly higher present value of royalty payment, her expected revenue is greater too. What about the winning bidder? By revenue equivalence, in both FPAs and SPAs with bonus bids, the information rent to the winner is proportional to  $(1 - \phi)^\beta [\theta^{1-\beta} - \mathbb{E}[\tilde{\theta}^{1-\beta} | \tilde{\theta} \leq \theta]]$ , where  $\tilde{\theta}$  refers to the second-highest bidder. A reduced royalty rate improves bidders' payoffs as they are decreasing in  $\phi$ . Hence all parties benefit from renegotiation.

The above example hinges on the contractibility of optimal investment. In general, committing to no renegotiation helps the seller to attain the highest payoff and maintain reputation in repeated interactions. Absent a commitment to no renegotiation, the original bidding equilibrium breaks down. There is also the question whether the seller can commit

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<sup>48</sup>See, for example, Che and Kim (2010).

to the initial bidding rule, the absence of which makes the process essentially an informal auction, as discussed in Section 4.

## 6.2 Options with Expirations

So far we have assumed that the investment option is perpetual. This is a reasonable simplification, because investment decisions are often made in relatively short time frames compared to the time scale of investment opportunities. For example, the duration for land leases are typically 30-50 years and constructions are often planned and implemented in a few years. However, it is useful to understand how the results are modified as a real option approaches expiration. Moreover, many investment options lose value due to unforeseen circumstances such as natural disasters and regulatory reforms—circumstances best modeled with stochastic expirations. Neither case would affect the implications of this paper.

**Stochastic Expiration:** Suppose with arrival intensity  $\delta$ , the investment option is rendered worthless. This is equivalent to augmenting the discount rate by  $\delta$ , which makes delaying less desirable. Stochastic expirations can work to the seller’s advantage, for example, when she decides expiration terms in combination with royalty rates.

**Deterministic Expiration:** With deterministic expiration time  $T$ , the project is equivalent to an American call option on a stock whose price process follows  $P_t$ , and pays a stream of dividend  $(r - \mu)P_t$ . Numerical solutions to optimal exercise and option valuations are well-known in mathematical finance. Figure 8 gives an illustration of bonus-bid auctions with a lease duration of 5 years. Although the investment thresholds converge close to the expiration, higher royalty rates still lead to greater investment delays due to post-auction moral hazard. Moreover, auction timing still affects bidding and payoffs in that having deterministic expirations are equivalent to having a different distribution of valuations.

## 6.3 Interdependent Values and Affiliated Signals

With interdependent values and affiliated signals, bidding strategies become more complicated. Moreover, optimal investments post-auction are sensitive to the auction format in addition to the security bid, because the winning bidder obtains different information set for auctions of different formats. Though I leave the detailed analysis for future research, it

is reasonable to expect that key results generalize. First, standard security bids still cause inefficiency in investments because the shape of the security is independent of the winning bidder’s bid. For example in equity auctions, the security bid inevitably causes investment delay because  $E_S < \beta$ . Second, even with interdependent values and affiliated signals, the seller’s payoff differs from welfare, and auction timings are generally inefficient. In the specific case of oil and gas leases, even with “mineral rights” models that involve common values, the investment threshold is bigger than the efficient one by a factor of  $1/(1 - \phi)$ , and  $P_a$  affects the valuations and bidding strategies.

## 6.4 Liquidity Constraints and External Financing

Anecdotal evidence suggests that liquidity constraints may often be a concern. To mitigate such constraints many bidders turn to contingent bids. The results in this paper are robust to cash-constraints with the use of non-standard securities, which correspond to side contracts, regulatory agreements, etc., in real practice.<sup>49</sup> Moreover, a bidder can raise financing from a third party.

External financing is an interesting extension in itself.<sup>50</sup> Post-auction investment is unaffected because we can view the financier and the seller as a coalition; from the bidders’ perspective, they face the same bidding and investment problem. For example, suppose in bonus-bid auctions for oil leases, the bidders have to externally finance the bonus bids. Further assume the financier only takes shares of the future revenue from the oil production, and the amount of shares increases with the cash demanded by the bidder. To the bidders, this is equivalent to an equity-bid auction on the revenue where the reserve equity share is the royalty rate to the seller. The development of oil will be further delayed, resulting in both loss of welfare and payoff to the bidders. The strategic timing of auctions changes because even though the total payoff to the seller and the financier is the same as in a security-bid auction without external financing, the split depends on the financing terms and form of the auction.<sup>51</sup>

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<sup>49</sup>A previous version of the paper solves the case with cash constraints, allowing  $\delta$ -function securities of the form  $S(P) = H(P') \mathbb{1}_{\{P=P'\}} + P \mathbb{1}_{\{P \neq P'\}}$ .

<sup>50</sup>For example, see Rhodes-Kropf and Viswanathan (2005), Povel and Singh (2010), and Liu (2012).

<sup>51</sup>Vladimirov (2013) discusses the payoffs to the seller and financier in a static setting.

## 6.5 Optimal Dynamic Mechanism

So far we have restricted the mechanism design to formal auctions where  $t_i = t_A \quad \forall i$ , which is realistic. In the hypothetical situation where pre-auction communication is costless, we can augment the design space to include dynamic mechanisms where the transaction timing  $t_i$  differs across  $i$ .<sup>52</sup> In a direct revelation mechanism, Lemma 2 still holds, and since  $t_i \leq \tau_i^*$ , the seller optimally sets  $t_i = \tau_i^*$  to postpone incurring the cost  $X + Y$  as much as possible. Therefore, optimal selling mechanisms of real options allocate the project to the type with the lowest  $\theta$  at cash flow  $P^*(z(\theta) + X + Y)$  and entails immediate investment upon transfer of ownership. The bidders pay cash from the project such that the IC condition in Lemma 2 holds.

In contrast to the optimal auctions in Myerson (1981) and in the previous discussion on formal auctions, this dynamic mechanism ensures full participation, because the marginal revenue from each type other than the worst type can become positive if  $P$  is high enough. This is a consequence of the flexibility of having a portfolio of options to time the allocation to each bidder, over having an option to time the auction for a portfolio of investment options. Nevertheless the investment and allocation timing are still inefficiently delayed ( $\max\{P_0, \frac{\beta}{\beta-1}[z(\theta) + X]\}$ ). The seller's costless communication of the screening contracts to the bidders is crucial here. Otherwise, the seller strategically times that communication and the analysis is identical to that of formal auction design.

## 7 Conclusion

Auctions of real options are ubiquitous, involve tremendous financial resources, and have policy implications. Prior studies have mostly analyzed the sales and exercises in isolation, treating the former as one-shot games while exogenously specifying the agency conflicts in the latter. To better understand these corporate transactions and reconcile theory with empirical observations, this paper introduces endogenous timing and post-sale dynamics into an auction model with investment. I show that common security bids lead to inefficient and often sub-optimal investment; and endogenous auction initiation and seller's commitment

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<sup>52</sup>With costless communication, revenue can also be maximized using a dynamic variant of the Vickery-Clarke-Groves mechanism even for  $\theta$ s that evolve over time, as Board (2007b) originally points out.

significantly impact bidding equilibrium, auction payoffs, and post-auction investment, and thus are integral to auction design. I further show that optimal auction design corresponds to the popular combination of cash and royalty payments in real life, and entails inefficient sales and investments. Taken together, the results of the paper challenge earlier approaches that analyze auction initiation, security design, and corporate investments separately: the interactions of these factors in dynamic settings give rich interplay that is not accessible otherwise, and as a consequence, many conventional beliefs should be revised.

As a first attempt to capture the salient features of auctions of real options and the underlying economics, this paper adds insights to security bids, endogenous initiation, and agency issues in the real options framework, and gives predictions more in accord with real life observations. Admittedly, the model is not the most general possible and more work is clearly needed for many of the aforementioned applications and extensions: In particular, selling real options with renegotiations and resales is worth exploring further. Incorporating seller's private information is also important in many applications, especially M&As. Moreover, while some of the novel predictions are consistent with stylized facts and available data, others require further empirical studies. The real challenge lies in developing these research options in a timely and responsible manner.

## References

- Beshears, John, 2011, The performance of corporate alliances: Evidence from oil and gas drilling in the Gulf of Mexico, *Working Paper*.
- Bessy, Christian, and Eric Brousseau, 1998, Technology licensing contracts features and diversity, *International Review of Law and Economics* 18, 451–489.
- Betton, Sandra, B Espen Eckbo, and Karin S Thorburn, 2009, Merger negotiations and the toehold puzzle, *Journal of Financial Economics* 91, 158–178.
- Binmore, Ken, and Paul Klemperer, 2002, The biggest auction ever: The sale of the British 3G telecom licences, *The Economic Journal* 112, C74–96.
- Board, Simon, 2007a, Bidding into the red: A model of post-auction bankruptcy, *Journal of Finance* 112, 2695–2723.
- , 2007b, Selling options, *Journal of Economic Theory* 136, 324–340.
- Bolton, Patrick, Gérard Roland, John Vickers, and Michael Burda, 1992, Privatization policies in central and eastern europe, *Economic Policy* pp. 275–309.
- Boone, Audra L, and J Harold Mulherin, 2007, How are firms sold?, *The Journal of Finance* 62, 847–875.



- Bulow, Jeremy, and Paul Klemperer, 1996, Auctions versus negotiations, *The American Economic Review* 86, 180–194.
- Bulow, Jeremy, and John Roberts, 1989, The simple economics of optimal auctions, *The Journal of Political Economy* pp. 1060–1090.
- Caves, Richard E., 2003, Contracts between art and commerce, *Journal of economic Perspectives* 17(2), 73–83.
- Che, Yeon-Koo, and Jinwoo Kim, 2010, Bidding with securities: comment, *The American Economic Review* 100, 1929–1935.
- Chhikara, Raj S, and J Leroy Folks, 1989, *The inverse Gaussian distribution: theory, methodology, and applications* . vol. 95 (CRC Press).
- Chisholm, Darlene C, 1997, Profit-sharing versus fixed-payment contracts: Evidence from the motion pictures industry, *Journal of Law, Economics, and Organization* 13, 169–201.
- Choquet, Gustave, 1966, *Topology* (Academic Press New York and London).
- Cohn, Donald L, 1980, *Measure theory* . vol. 1993 (Birkhäuser Boston).
- Cong, Lin W., 2012, Costly learning and agency conflicts in investments under uncertainty, (*Working Paper*).
- , 2013a, Optimal stopping strategies of brownian motions, .
- , 2013b, Security design and investment: How royalties delay exploration of oil and gas tracts?, .
- Crémer, Jacques, 1987, Auctions with contingent payments: Comment, *American Economic Review* 77, 746.
- Dasgupta, Sudipto, and Robert G Hansen, 2007, Auctions in corporate finance, *Handbook of corporate finance: empirical corporate finance* 1, 87–143.
- DeMarzo, Peter, and Darrell Duffie, 1999, A liquidity-based model of security design, *Econometrica* 67, 65–99.
- DeMarzo, Peter, Ilan Kremer, and Andrzej Skrzypacz, 2005, Bidding with securities: Auctions and security design, *American Economic Review* 95(4), 936–959.
- DeMarzo, Peter M, and Michael J Fishman, 2007, Agency and optimal investment dynamics, *Review of Financial Studies* 20, 151–188.
- Dessauer, John P., 1981, *Book Publishing* (NY: Bowker).
- Ding, Wei, and Elmar G Wolfstetter, 2011, Prizes and lemons: procurement of innovation under imperfect commitment, *The RAND Journal of Economics* 42, 664–680.
- Dixit, Avinash K, and Robert S Pindyck, 1994, *Investment under uncertainty* (Princeton university press).
- Dougherty, EL, and John Lohrenz, 1980, Us offshore oil and gas lease royalty bidding, *Journal of Petroleum Technology* 32, 1445–1451.
- Duffie, Darrell, 2009, *Dynamic Asset Pricing Theory* (Author’s Manuscript, Princeton university Press).

- Eckbo, B Espen, Ronald M Giammarino, and Robert L Heinkel, 1990, Asymmetric information and the medium of exchange in takeovers: Theory and tests, *Review of Financial Studies* 3, 651–675.
- Engelbrecht-Wiggans, Richard, 1987, On optimal competitive contracting, *Management science* 33, 1481–1488.
- Fidrmuc, Jana P, Peter Roosenboom, Richard Paap, and Tim Teunissen, 2012, One size does not fit all: Selling firms to private equity versus strategic acquirers, *Journal of Corporate Finance* 18, 828–848.
- French, Kenneth R, and Robert E McCormick, 1984, Sealed bids, sunk costs, and the process of competition, *Journal of Business* pp. 417–441.
- Gorbenko, Alexander, and Andrey Malenko, 2011, Competition among sellers in securities auctions, *The American Economic Review* 101, 1806–1841.
- , 2013a, Auctions with endogenous initiation, *Work in Progress*.
- , 2013b, Means of payment and timing of mergers and acquisitions in a dynamic economy, *Working Paper*.
- Grenadier, Steven R, 2002, Option exercise games: An application to the equilibrium investment strategies of firms, *Review of Financial Studies* 15(3), 691–721.
- , and Andrey Malenko, 2011, Real options signaling games with applications to corporate finance, *Review of Financial Studies* 24, 3993–4036.
- Grenadier, Steven R, and Neng Wang, 2005, Investment timing, agency, and information, *Journal of Financial Economics* 75, 493–533.
- Haile, Philip, Kenneth Hendricks, and Robert Porter, 2010, Recent us offshore oil and gas lease bidding: A progress report, *International Journal of Industrial Organization* 28, 390–396.
- Hansen, Robert G., 1985, Auctions with contingent payments, *American Economic Review* 75(4), 862–865.
- Hansen, Robert G, 2001, Auctions of companies, *Economic Inquiry* 39, 30–43.
- Harrison, J. Michael, 2013, *Brownian Models of Performance and Control* (Cambridge University Press).
- Hart, Oliver, and John Moore, 1995, Debt and seniority: An analysis of the role of hard claims in constraining management, *American Economic Review* 85.
- Hellmann, Thomas, 2006, IPOs, acquisitions, and the use of convertible securities in venture capital, *Journal of Financial Economics* 81, 649–679.
- Hendricks, Kenneth, and Robert H Porter, 1996, The timing and incidence of exploratory drilling on offshore wildcat tracts, *The American Economic Review* pp. 388–407.
- , and Guofu Tan, 1993, Optimal selling strategies for oil and gas leases with an informed buyer, *The American Economic Review* pp. 234–239.
- Humphries, Marc, 2009, Royalty relief for U.S. deepwater oil and gas leases, .
- Kaplan, Steve, and Per Stromberg, 2003, Financial contracting theory meets the real world: an empirical analysis of venture capital contracts, *Review of Economic Studies* 70, 281–315.

- Karatzas, Ioannis A., and Steven Eugene Shreve, 1988, *Brownian Motion and Stochastic Calculus* (New York: Springer-Verlag).
- Karlin, Samuel, and Howard E Taylor, 1975, *A first course in stochastic processes* (Academic Press, New York).
- Klemperer, Paul, 2002, How (not) to run auctions: the European 3G telecom auctions, *European Economic Review* 46, 829–845.
- Kogan, Shimon, and John Morgan, 2010, Securities auctions under moral hazard: An experimental study, *Review of Finance* 14, 477–520.
- Krishna, Vijay, 2009, *Auction theory* (Academic press).
- Laffont, Jean-Jacques, and Jean Tirole, 1987, Auctioning incentive contracts, *The Journal of Political Economy* 95, 921–937.
- Lin, C-Y Cynthia, 2009, Estimating strategic interactions in petroleum exploration, *Energy Economics* 31, 586–594.
- , 2012, Strategic decision-making with information and extraction externalities: A structural model of the multi-stage investment timing game in offshore petroleum production, *Review of Economics and Statistics*.
- Liu, Tingjun, 2012, Takeover bidding with signaling incentives, *Review of Financial Studies* 25, 522–556.
- Maeland, Joril, 2002, Valuation of irreversible investments and agency problems, *Innovation, Organization and Strategy (Trigeorgis, L.)*, Cambridge University Press, forthcoming.
- Malmendier, Ulrike, Marcus Matthias Opp, and Farzad Saidi, 2012, Cash is king-revaluation of targets after merger bids, .
- Martin, Kenneth J, 1996, The method of payment in corporate acquisitions, investment opportunities, and management ownership, *The Journal of Finance* 51, 1227–1246.
- McAfee, R. Preston, and John McMillan, 1986, Bidding for contracts: a principal-agent analysis, *The RAND Journal of Economics* pp. 326–338.
- , 1987a, Competition for agency contracts, *The Rand Journal of Economics* pp. 296–307.
- , 1987b, *Incentives in government contracting* (Toronto: U. of Toronto Press).
- McDonald, Robert, and David Siegel, 1986, The value of waiting to invest, *Quarterly Journal of Economics* 101, 707–7288.
- Milgrom, Paul, 1985, Auction theory, *Lecture for the Fifth World Congress of the Econometric Society, Mimeo*.
- Myerson, R.B., 1981, Optimal auction design, *Mathematics of operations research* pp. 58–73.
- Opaluch, James, Thomas Grigalunas, Christopher Anderson, Simona Trandafir, and Di Jin, 2010, Policies to affect the pace of leasing and revenues in the Gulf of Mexico, .
- Pakes, Ariel, 1986, Patents as options: Some estimates of the value of holding European patent stocks, *Econometrica: Journal of the Econometric Society* pp. 755–784.
- Povel, Paul, and Rajdeep Singh, 2010, Stapled finance, *The Journal of Finance* 65, 927–953.

- Protter, Philip, 2004, *Stochastic Integration and Differential Equations: Version 2.1* . vol. 21 (Springer).
- Reece, Douglas K, 1979, An analysis of alternative bidding systems for leasing offshore oil, *The Bell Journal of Economics* pp. 659–669.
- Revuz, Daniel, and Marc Yor, 1999, *Continuous martingales and Brownian motion* . vol. 293 (Springer).
- Rhodes-Kropf, Matthew, and S. Viswanathan, 2000, Corporate reorganizations and non-cash auctions, *Journal of Finance* 55(4), 1807–1849.
- , 2005, Financing auction bids, *RAND Journal of Economics* 36(4), 789–815.
- Riley, John G., 1988, Ex post information in auctions, *Review of Economic Studies* 55(3), 409–429.
- Robinson, Mark S, 1984, Oil lease auctions: Reconciling economic theory with practice, *University of California at Los Angeles, mimeo*.
- Rothkopf, Michael H, and Richard Engelbrecht-Wiggans, 1992, Innovative approaches to competitive mineral leasing, *Resources and Energy* 14, 233–248.
- Samuelson, William F, 1985, Competitive bidding with entry costs, *Economics Letters* 17, 53–57.
- , 1987, Auctions with contingent payments: Comment, *The American Economic Review* 77, 740–745.
- Schwartz, Eduardo S, 2004, Patents and R&D as real options, *Economic Notes* 33, 23–54.
- Skrzypacz, Andrzej, 2013, Auctions with contingent payments-an overview, *International Journal of Industrial Organization*.
- Stein, Jeremy C, 2003, Agency, information and corporate investment, *Handbook of the Economics of Finance* 1, 111–165.
- Stokey, N.L., 2009, *The economics of inaction* (Princeton University Press).
- Topkis, Donald M., 1978, Minimizing a submodular function on a lattice, *Operations Research* 26(2), 305–21.
- Vishwasrao, Sharmila, 2007, Royalties vs. fees: How do firms pay for foreign technology?, *International Journal of Industrial Organization* 25, 741–759.
- Vladimirov, Vladimir, 2013, Financing bidders in takeover auctions, *Working Paper*.
- Zheng, Charles Z., 2001, High bids and broke winners, *Journal of Economic Theory* 100(1), 129–171.

# Appendix

## A Derivations and Proofs.

### A.1 The Arrow-Debreu Security

For sufficiently small  $dt$ ,  $D(P_t; P') = e^{-rdt}\mathbb{E}[D(P_t + dt; P')]$ , an application of Itô's formula shows  $D(P_t; P')$  satisfies  $\frac{1}{2}\sigma^2 P^2 D_{PP} + \mu P D_P - rD = 0$ , subject to the boundary conditions  $D(P'; P') = 1$  and  $D(0; P_a) = 0$ . This yields  $D(P_t; P') = \mathbb{E}[e^{-r(\tau-t)}] = (\frac{P_t}{P'} )^\beta$  where  $\tau = \inf\{s \geq t : P_s \geq P'\}$ .

### A.2 Proof of Proposition 1

*Proof.* First note  $\tilde{V} \in [-X - C, W(P_a; \theta) - X - C]$ , thus the valuation is finite. The value function of the optimal stopping is thus the infimum of a class of  $\mathcal{C}^2$  functions with non-positive drift that majorize  $P - S(P) - \theta$ , and the stopping time is first-hitting. (Proposition 5.8, and 5.10 in Harrison (2013)). Therefore,

$$\tilde{V}(C, S(\cdot), \theta) = D(P_a; P_L, P_U) [P_L - S(P_L) - \theta] + D(P_a; P_U, P_L) [P_U - S(P_U) - \theta] - C - X,$$

where  $D(P_a; P_L, P_U)$  is the Arrow-Debreu security that pays one dollar when  $P$  first hits before  $P_L$  before hitting  $P_U$ , and  $D(P_a; P_U, P_L)$  is similarly defined. And  $P_L \in [0, P_a]$  and  $P_U \in [P_a, \infty]$  are the optimal lower and upper thresholds for investment.  $D(P_a; P_L, P_U)$  satisfies  $\frac{1}{2}\sigma^2 P^2 D_{PP} + \mu P D_P - rD = 0$  with the boundary conditions  $D(P_L; P_L, P_U) = 1$  and  $D(P_U; P_L, P_U) = 0$ . The solution is  $D(P_a; P_L, P_U) = \left( P_a^\beta - P_a^\gamma P_U^{\beta-\gamma} \right) \left( P_L^\beta - P_L^\gamma P_U^{\beta-\gamma} \right)^{-1}$ . Similarly,  $D(P_a; P_U, P_L) = \left( P_a^\beta - P_a^\gamma P_L^{\beta-\gamma} \right) \left( P_U^\beta - P_U^\gamma P_L^{\beta-\gamma} \right)^{-1}$ . The optimal  $P_L$  and  $P_U$  are obviously independent of  $X$  and  $C$  and are functions of  $\theta$  and  $P_a$  in general (Cong (2013a) discusses the optimization algorithm in more details).

Finally, type  $\theta$  can always do strictly better than  $\tilde{\theta} > \theta$  by using  $\tilde{\theta}$ 's strategy, thus  $\tilde{V}(C, S(\cdot), \theta)$  is decreasing in  $\theta$ . Similarly, it is non-decreasing in  $P_a$ . For  $P_t$  in the exercise region,  $\tilde{V}(C, S(\cdot), \theta) = P_t - S(P_t) - \theta - C - X$  is obviously continuous in  $\theta$ . For  $P_t$  in the continuation region, consider a change of  $\Delta\theta > 0$ ,  $0 < \tilde{V}(C, S(\cdot), \theta) - \tilde{V}(C, S(\cdot), \theta + \Delta\theta) \leq \Delta\theta$  because type  $\theta + \Delta\theta$  does weakly better than simply mimicking  $\theta$ 's strategy. As  $\Delta\theta \rightarrow 0$ ,  $\tilde{V}(C, S(\cdot), \theta + \Delta\theta) \rightarrow \tilde{V}(C, S(\cdot), \theta)$ . The case of  $\Delta\theta < 0$  is similar. Continuity in  $\theta$  follows.  $\square$

### A.3 Proof of Lemma 1

*Proof.* Since an upper threshold strategy has payoff  $(\frac{P_a}{P})^\beta [P - S(P) - \theta]$  for  $P \geq P_a$ , threshold  $\tilde{P}$  is optimal among all upper threshold strategies. I now verify that it is optimal among all stopping times by showing the expected value following any stopping time is bounded above by the expected value associated with the  $\tilde{P}$ -threshold strategy.

Let  $x_t = e^{-rt}\hat{W}(P_t)$ , where  $\hat{W}(P_t) = D(P_t; \hat{P})[\hat{P} - S(P_t, \hat{P}) - \theta]$  and  $\hat{P} = \max\{P_t, \tilde{P}\}$ . For  $P \leq \tilde{P}$ , using an extended version of Itô's formula (as, for example, in Karatzas and Shreve (1988), page 219),  $dx_t = e^{-rt}[\mathcal{D}\hat{W}(P_t) - r\hat{W}(P_t)]dt + e^{-rt}\hat{W}_P(P_t)\sigma P_t dB_t$ , where  $\mathcal{D}\hat{W}(P) = \hat{W}_P(P)\mu P + \frac{1}{2}\hat{W}_{PP}(P)\sigma^2 P^2$ .  $\hat{W}_P$  is bounded as seen by direct computation, thus by Proposition 5B in Duffie (2009) (also found in Protter (2004)), the last term in  $dx_t$  is a martingale under the current measure. The drift is  $\mathcal{D}\hat{W}(P) - r\hat{W}(P) = 0$  by the definition of  $\beta$  in (2). For  $P > \tilde{P}$ , apply Tanaka's Formula (Revuz and Yor (1999), also Karatzas and Shreve (1988)), the drift  $\mathcal{D}\hat{W}(P) - r\hat{W}(P) = \mu P[1 - S'(P)] - r[P - S(P) - \theta] - \frac{1}{2}\sigma^2 P^2 S''(P) < [\mu + \frac{1}{2}(\beta - 1)\sigma^2]P[1 - S'(P)] - r[P - S(P) - \theta] < [\mu\beta + \frac{1}{2}\sigma^2\beta(\beta - 1) - r][P - S(P) - \theta] = 0$ , using (Conv) and the definition of  $\beta$ . Since to the discounted occupancy measure, there is a discounted local time  $l$  (Stokey (2009)),

Theorems 3.6 and 3.7), the additional local time term in  $dx_t$  when  $S'(P)$  jumps is  $\frac{1}{2}\sigma^2 \int_{\mathbb{R}^+} l(p, t, r)\nu(dp)$ , where  $\nu((a, b]) \equiv \hat{W}'(b) - \hat{W}'(a)$ , is non-positive due to (Conv). Therefore,  $x_t$  is a super-martingale, implying for any stopping time  $\tau$ ,  $\hat{W}(P_0) = x_0 \geq \mathbb{E}[x_\tau] = \mathbb{E}[e^{-r\tau}\hat{W}(P_\tau)] \geq \mathbb{E}[e^{-r\tau}(P_\tau - S(s, P_\tau) - \theta)]$ . The equality holds for the first-hitting time with threshold  $\tilde{P}$ , establishing its optimality.  $\square$

#### A.4 Proof of Proposition 3

*Proof.* For  $s_1 < s_2$  and  $\theta_1 < \theta_2$ , because  $V(s, \theta)$  is absolutely continuous with derivative in  $s$  decreasing in  $\theta$ ,

$$\ln \left( \frac{V(s_1, \theta_1)V(s_2, \theta_2)}{V(s_1, \theta_2)V(s_2, \theta_1)} \right) = \int_{s_1}^{s_2} \frac{\partial V(s', \theta_2)}{\partial s} ds' - \int_{s_1}^{s_2} \frac{\partial V(s', \theta_1)}{\partial s} ds' < 0 \quad (16)$$

i.e.,  $V(s, \theta)$  is log-submodular, and thus strictly submodular. Let  $Q(s)$  be the probability of winning. Because  $s(\theta) \in \operatorname{argmax}_s Q(s)V(s, \theta) = \operatorname{argmax}_s \ln(Q(s)V(s, \theta))$ , by Topkis (1978),  $s(\theta)$  is non-increasing in  $\theta$ . If  $s(\theta) < s_H$  were constant on an interval, the bidder with the lower  $\theta$  can increase his bid marginally and increase his probability of winning (thus his payoff) by a discrete amount. Therefore  $s(\theta)$  must be decreasing in type for types bidding less than  $s_H$ . Therefore,  $Q(s(\theta)) = [1 - F(\theta)]^{N-1}$ . Note  $s$  is also continuous in  $\theta$ , lest a type right below a discontinuity could lower his bid marginally without affecting the chance of winning.

Next, by direct revelation,  $\theta \in \operatorname{argmax}_{\theta' \in [\underline{\theta}, \bar{\theta}]} Q(s(\theta'))V(s(\theta'), \theta)$ . For any  $\theta' < \theta$ ,

$$Q(s(\theta))V(s(\theta), \theta) \geq Q(s(\theta'))V(s(\theta'), \theta) = Q(s(\theta'))[V(s(\theta), \theta) + V_1(s^*, \theta)[s(\theta') - s(\theta)]]$$

for some  $s^*$  between  $s(\theta')$  and  $s(\theta)$ . Since  $V_1 < 0$ , the above expression can be written as

$$\frac{Q(s(\theta')) - Q(s(\theta))}{\theta' - \theta} \frac{V(s(\theta), \theta)}{-Q(s(\theta'))V_1(s^*, \theta)} \geq \frac{s(\theta') - s(\theta)}{\theta' - \theta}$$

Similarly, exchanging  $\theta$  and  $\theta'$ , for some  $s^{**}$  between  $s(\theta)$  and  $s(\theta')$ ,

$$\frac{Q(s(\theta')) - Q(s(\theta))}{\theta' - \theta} \frac{V(s(\theta'), \theta')}{-Q(s(\theta))V_1(s^{**}, \theta')} \leq \frac{s(\theta') - s(\theta)}{\theta' - \theta}$$

Taking the limit we get (7).

As  $V(s, \theta)$  is continuous in  $s$  over  $[s_L, s_H]$  and decreasing in  $\theta$ ,  $\max_s V(s, \theta)$  exists and  $\sup\{\theta \leq \bar{\theta} | [1 - F(\hat{\theta})]^{N-1} \max_s V(s, \hat{\theta}) \geq 0\}$  gives the cutoff type. In equilibrium,  $s(\hat{\theta}) = \sup\{s \in [s_L, s_H] | V(s, \hat{\theta}) \geq 0\}$ , otherwise bidding slightly more increases the winning probability discretely from zero while still breaking even upon winning. As  $V(s(\hat{\theta}), \hat{\theta}) \leq W(P_a; \hat{\theta}) - X$  and  $W(P_a; \hat{\theta}) - X = 0$  in cash auctions, the cutoff type for security bids is in general weakly smaller than that in cash auctions. With the absolute continuity assumption in the proposition, the cutoffs are the same as in cash auctions.

This establishes uniqueness of the equilibrium, whose existence follows from the sufficiency of bidders' F.O.C. - the quasiconcavity of  $\ln(Q(s)V(s, \theta))$ . For any  $s' \in (s(\theta), s(\theta'))$ ,  $\exists \theta' \in (0, \theta)$  such that  $s(\theta') = s'$ . Submodularity of  $V$  implies  $\frac{\partial}{\partial s} \ln[Q(s')V(s', \theta)] > \frac{\partial}{\partial s} \ln[Q(s')V(s', \theta')] = 0$ . Similarly,  $\frac{\partial}{\partial s} \ln[Q(s')V(s', \theta)] < 0$  for  $s' \in (s(\theta), s(\theta))$ . Therefore for every  $\theta$ , there exists a unique  $s$  maximizing  $Q(s)V(s, \theta)$ .  $\square$

#### A.5 Proof of Proposition 4

*Proof.* Since  $\Pi$  is a left-continuous map,  $V(s, \theta)$  is left-continuous in  $s$  by an argument similar to the one in Proposition 1 for  $V(s, \theta)$  to be continuous in  $\theta$ . Therefore  $s(\theta)$  is well-defined. Suppose a participating

bidder of type  $\theta$  bids  $s > s(\theta)$ , he benefits from decreasing  $s$  to reduce the states of the world in which he wins but receives negative payoff. Similarly, he wants to increase  $s$  when  $s < s(\theta)$ , assuming any indifference in bidding is resolved by bidding higher. As  $V(s, \theta)$  is decreasing in  $\theta$ , for  $\theta' > \theta$ ,  $V(s(\theta), \theta') < V(s(\theta), \theta) = 0 = V(s(\theta'), \theta')$ . Thus  $s(\theta) > s(\theta')$ , leading to  $s(\theta)$  being decreasing. The cut-off type is the same as in FPAs by an argument similar to that in the proof of Proposition 3.  $\square$

## A.6 Proof of Proposition 5

*Proof.* When  $P_a$  increases or  $X$  decreases,  $\hat{\theta}$  weakly increases, thus potentially a positive measure of originally non-participating bidders are bidding. Given the bidding strategy in SPAs and the fact that non-negative  $V$  is non-increasing in  $s$ , decreasing in  $X$  and increasing in  $P_a$ , for each original participant increasing  $P_a$  or decreasing  $X$  results in a bigger  $s$ . In FPAs, let  $\tilde{s}(\theta)$  denote the bidding strategy after  $P_a$  increases or  $X$  decreases. Then  $\tilde{s}(\theta) \geq s(\theta)$  at least for the original cutoff type  $\hat{\theta}_{old}$ . If  $\tilde{s}(\theta) = s(\theta)$  for any  $\theta \in [\underline{\theta}, \hat{\theta}_{old}]$ ,  $\tilde{s}'(\theta) > s'(\theta)$  by Proposition 3, thus  $\tilde{s}(\theta)$  stays above  $s(\theta)$  for a positive measure of types. Overall we have a positive measure of types bidding bigger  $s$ , and all types bidding weakly bigger  $s$ . For the same reason, the result holds when  $N$  increases in FPAs.  $\square$

## A.7 Proof of Lemma 2

*Proof.* The IC constraint can be written as  $\theta_i \in \operatorname{argmax}_{\tilde{\theta}_i \in [\underline{\theta}, \bar{\theta}]} U(\theta_i, \tilde{\theta}_i) \forall i$ . As  $U(\theta_i, \tilde{\theta}_i)$  is not necessarily differentiable in  $\tilde{\theta}_i$ , rewrite this as  $\tilde{\theta}_i \in \operatorname{argmax}_{\theta_i \in [\underline{\theta}, \bar{\theta}]} [U(\theta_i, \theta_i) - U(\theta_i)]$ . By bounded convergence theorem and envelope theorem,  $U(\theta_i, \tilde{\theta}_i)$  is differentiable in  $\theta_i$ , with

$$U'(\tilde{\theta}_i) = \left. \frac{\partial U(\theta_i, \tilde{\theta}_i)}{\partial \theta_i} \right|_{\theta_i = \tilde{\theta}_i} = \mathbb{E}_{\theta_{-i}} \left[ Q(\tilde{\theta}_i, \theta_{-i}) \mathbb{E}_P[-e^{-r\tau^*(\tilde{\theta}_i, \tilde{\theta}_i, \theta_{-i})}] \right] \quad (17)$$

which is uniformly bounded by 1. Then  $U(\theta_i)$  is Lipschitz continuous because it is the upper envelope of a family of Lipschitz continuous functions  $U(\theta_i, \tilde{\theta}_i)$  indexed by  $\tilde{\theta}_i$  with the same Lipschitz constant (Proposition 6.3 in Choquet (1966)).  $U(\theta_i)$  is thus absolutely continuous and differentiable almost everywhere (Corollary 6.3.7, Cohn (1980)). Writing it in the integral form concludes the proof.  $\square$

## A.8 Proof of Proposition 6

*Proof.* The ex-ante social welfare is  $N\mathbb{E}_{\bar{\theta}}[Q(\theta_i, \theta_{-i}) (\mathbb{E}_P[e^{-r\tau_i^*}(P_{\tau_i^*} - \theta_i)] - e^{-rt_a}(X + Y))]$ , and the seller's ex ante revenue is the social welfare less the agents' ex-ante utilities:  $N\mathbb{E}_{\bar{\theta}}[Q(\theta_i, \theta_{-i}) (\mathbb{E}_P[e^{-r\tau_i^*}(P_{\tau_i^*} - \theta_i)] - e^{-rt_a}(X + Y))] - N\mathbb{E}_{\bar{\theta}}[U(\theta_i)]$ . Using (9) and taking expectations over the winning bidder's type, it becomes

$$N\mathbb{E}_{\bar{\theta}}[Q(\theta_i, \theta_{-i}) (\mathbb{E}_P[e^{-r\tau_i^*}(P_{\tau_i^*} - z(\theta_i))] - e^{-rt_a}(X + Y))] - NU(\bar{\theta}). \quad (18)$$

With standard securities, a participant with the least cost wins, the proposition follows.  $\square$

## A.9 Proof of Proposition 7

*Proof.* To maximize expression (18), for every realization of the types and any allocation rule, the seller wants winner  $\theta_i$  to invest when  $P$  first hits  $P^*(z(\theta_i))$ . The proposed contingent payment achieves this outcome.

Moreover,  $U(\bar{\theta}) = 0$  and the project is only allocated to types that contribute positively to the revenue. With Uniform or Generalized Pareto,  $z$  is increasing in  $\theta$ , leading to the unique cutoff type  $\hat{\theta}$  proposed and allocation to a participant with the smallest  $\theta$ .

That  $U(\theta_i)$  is decreasing in  $\theta_i$  implies any mechanism satisfying the above meets IR of all types. Suppose  $\theta_i < \tilde{\theta}_i$ , Lemma 2 leads to  $U(\theta_i, \tilde{\theta}_i) = U(\tilde{\theta}_i) - \int_{\theta_i}^{\tilde{\theta}_i} U_1(\theta, \tilde{\theta}_i, \tau^*(\theta, z_i(\tilde{\theta}_i, \theta_{-i}, \cdot)))d\theta \leq U(\tilde{\theta}_i) - \int_{\theta_i}^{\tilde{\theta}_i} U_1(\theta, \theta, \tau^*(\theta, z_i(\theta, \theta_{-i}, \cdot)))d\theta = U(\theta_i)$ , where the inequality follows from (17) and the fact that reporting a higher investment cost leads to a lower probability of winning and a later investment. Similarly,  $U(\theta_i, \tilde{\theta}_i) \leq U(\theta_i)$  for  $\theta_i > \tilde{\theta}_i$ . Thus incentive compatibility holds.

Finally,  $C(\theta_i, \theta_{-i})$  and  $S(\tilde{\theta}_i, \theta_{-i}, \mathcal{I}_\tau, \tau^*)$  are such that Lemma 2 holds.  $\square$

## A.10 Proof of Proposition 8

*Proof.* The seller's expected utility for holding auction when  $P_a$  is first reached can be written as

$$D(P_0; P_a) \int_{\underline{\theta}}^{\hat{\theta}_z} Nf(\theta)[1 - F(\theta)]^{N-1}[W(P_a; z(\theta)) - X - Y]d\theta. \quad (19)$$

The derivative w.r.t.  $P_a$  has the same sign as the LHS of Equation(12), which is continuous in  $P$ . It is non-negative for  $P_a \leq \max\{\frac{\beta}{\beta-1}\underline{\theta}, P_{\underline{\theta}}\}$ , where  $P_{\underline{\theta}}$  solves  $W(P_{\underline{\theta}}; \underline{\theta}) = X + Y$  is the minimum cash flow at which there is non-trivial participation. Then for either Uniform or Generalized Pareto distribution, it becomes positive and changes sign only once as  $P_a$  increases. By the same argument as in the proof of Proposition 1, there is an optimal threshold strategy for holding the auction. FOC in Equation (12) gives the solution as (19) is quasi-concave. Notice optimal  $P_a \geq \frac{\beta}{\beta-1}\underline{\theta}$ , lest  $P^*(z(\theta)) = \frac{\beta}{\beta-1}z(\theta)$ , and LHS of (12) is either positive, or becomes positive when  $P_a$  increases further, making delaying profitable. Thus the seller never holds the auction when she expects no bidder to invest right away.  $\square$

## A.11 Proof of Proposition 9

*Proof.* It can be shown that for an efficient formal auction, there is also an optimal threshold strategy. I show that at the socially efficient timing, the LHS of (12) is positive for the seller, which implies the threshold for timing an optimal auction is higher.

Let  $\hat{\theta} \geq \hat{\theta}_z$  denote cutoff type of participation with efficient security designs (such as cash). If  $\hat{\theta}_z = \bar{\theta}$ , the integrand is weakly bigger with optimal security. It is strictly increasing for a positive measure of  $\theta$  as we increase  $z(\theta)$  because otherwise, the LHS of (12) is positive at  $z(\theta) = \theta$ , contradicting the fact that it is zero at the efficient timing. Hence, (12) is positive with optimal security design at the efficient timing.

If  $\hat{\theta}_z < \bar{\theta}$ , consider the following **Lemma:** *Suppose  $A(\theta)$  and  $\hat{A}(\theta)$  are positive with their ratio  $g(\theta)$  decreasing in  $\theta$ , and  $B(\theta)$  is weakly increasing in  $\theta$  and strictly increasing over some interval in  $[\theta_1, \theta_2]$ . Then  $\int_{\theta_1}^{\theta_2} A(\theta)B(\theta)d\theta = 0$  implies  $\int_{\theta_1}^{\theta_2} \hat{A}(\theta)B(\theta)d\theta > 0$ .*

To prove this, rescale  $A$  and  $\hat{A}$  so that they are pdf of random variables. Their ratio after re-scaling is still decreasing, implying  $\hat{A}$  has first-order stochastic dominance over  $A$ , the result then follows given  $B$  is weakly increasing in  $\theta$ . Q.E.D. Now at the socially efficient timing  $P_a$ ,

$$0 = \int_{\underline{\theta}}^{\hat{\theta}} \frac{\beta(X + Y + \theta') - (\beta - 1)P^*(\theta')}{[Nf(\theta')[1 - F(\theta')]^{N-1}]^{-1}} d\theta' = \int_{\underline{\theta}}^{z^{-1}(\hat{\theta})} \frac{\beta(X + Y + \theta') - (\beta - 1)P^*(\theta')}{[Nf(\theta')[1 - F(\theta')]^{N-1}]^{-1}} \frac{d\theta'}{d\theta} d\theta$$

where we have used change of variable  $\theta = z^{-1}(\theta')$ . Let  $A(\theta) = Nf(\theta')[1 - F(\theta')]^{N-1} \frac{d\theta'}{d\theta}$  and  $\hat{A}(\theta) =$



$Nf(\theta)[1 - F(\theta)]^{N-1}$ , and  $B(\theta) = \beta(X + Y + z(\theta)) - (\beta - 1)P^*(z(\theta))$ . Generalized pareto and uniform distributions have  $g(\theta) = \frac{dz(\theta)}{d\theta} \frac{f(z(\theta))[1 - F(z(\theta))]^{N-1}}{f(\theta)[1 - F(\theta)]^{N-1}}$  being decreasing in  $\theta$ . For example, exponential distribution with parameter  $\lambda_o$  has  $g'(\theta) = -\lambda_o^3 N e^{2\lambda_o\theta - \lambda_o N [e^{\lambda_o\theta} - 1]} < 0$ , and uniform distribution has  $g'(\theta) = -2 \frac{N-1}{\theta - \theta} \frac{[1 - F(2\theta - \theta)]^{N-2}}{[1 - F(\theta)]^{N-1}} < 0$ . Thus the lemma gives,

$$\int_{\theta}^{z^{-1}(\hat{\theta})} Nf(\theta)[1 - F(\theta)]^{N-1}[\beta(X + Y + z(\theta)) - (\beta - 1)P^*(z(\theta))]d\theta > 0. \quad (20)$$

This implies  $B(z^{-1}(\hat{\theta})) > 0$  as  $B(\theta)$  is increasing. Together with the fact  $\hat{\theta}_z \geq z^{-1}(\hat{\theta})$ , (12) at  $P_a$  has the same sign as  $\int_{\theta}^{\hat{\theta}_z} Nf(\theta)[1 - F(\theta)]^{N-1}[\beta(X + Y + z(\theta)) - (\beta - 1)P^*(z(\theta))]d\theta > 0$ . □

### A.12 Proof of Lemma 3

*Proof.* I first show that  $R(\Pi^i) = C^i + R_{\theta_i}(S^i)$ . This is obviously true if only one type uses  $\Pi^i$ . If more than one type use this bid, either it holds or one of the types  $\theta_1$  has  $R_{\theta_1}(S^i) + C^i \neq R(\Pi^i)$ . Then  $\exists \theta_2$  (potentially  $= \theta_1$ ) s.t.  $R(\Pi^i) < C^i + R_{\theta_2}(S^i)$ . Consider the deviation for bidder 2 to a cash bid equal to  $R(\Pi^i)$  and invest efficiently. This deviation is profitable because he creates weakly greater social surplus, pays less, and has the same marginal probability of winning. Thus by contradiction  $R(\Pi^i) = C^i + R_{\theta_i}(S^i)$  always.

Now suppose  $\tau_{\theta_i}^i \neq \tau_i^*$ , consider deviating to a cash bid  $C = R(\Pi^i)$ . The payoff from deviation  $\mathbb{E}[e^{-r\tau_i^*}(P_{\tau_i^*} - \theta_i)] - R(\Pi^i)$  dominates the original payoff  $\mathbb{E}[e^{-r\tau_{\theta_i}^i}(P_{\tau_{\theta_i}^i} - \theta_i - S^i(P_{\tau_{\theta_i}^i}))] - C^i = \mathbb{E}[e^{-r\tau_{\theta_i}^i}(P_{\tau_{\theta_i}^i} - \theta_i)] - R_{\theta_i}(S^i) - C^i$ . Thus the deviation is profitable and the claim follows. □

### A.13 Proof of Lemma 4

*Proof.* Suppose a non-singleton set  $\Theta_p$  of types pool to bid  $\Pi$ . The claim follows if there is always a profitable deviation by a type in this set.

From Lemma 3, a type  $\theta$  in expectation pays  $C + D(P_a; P^*(\theta))S(P^*(\theta))$ . Let  $\theta_k = \operatorname{argmax}_{\theta' \in \Theta_p} R_{\theta'}(S)$  where  $R_{\theta'}(S) = D(P_a; P^*(\theta'))S(P^*(\theta'))$ . Then  $R(\Pi) \leq C + D(P_a; P^*(\theta_k))S(P^*(\theta_k))$ . If the inequality is strict, type  $k$  can profitably deviate to cash bid  $R(\Pi)$ . Otherwise,  $R_{\theta_i}(S) = R_{\theta_j}(S) = R(\Pi) - C$ , for some  $\theta_i < \theta_j$  both in  $\Theta_p$ , but there is still a profitable deviation:

We first argue that  $\Theta_p$  contains a positive measure of types. For any  $\theta_n \in (\theta_i, \theta_j) \cap \Theta_p^c$ , call his bid  $\tilde{\Pi}$ . Let  $Q$  and  $\tilde{Q}$  be the probability of winning when bidding  $\Pi$  and  $\tilde{\Pi}$ . Since  $\theta_i$  does not want to deviate to cash bid  $R(\tilde{\Pi})$ ,  $Q[W(P_a; \theta_i) - R(\Pi) - X] \geq \tilde{Q}[W(P_a; \theta_i) - R(\tilde{\Pi}) - X]$ . Similarly,  $Q[W(P_a; \theta_j) - R(\Pi) - X] \geq \tilde{Q}[W(P_a; \theta_j) - R(\tilde{\Pi}) - X]$ . As  $\theta_i \neq \theta_j$ , the equality signs cannot hold simultaneously. Thus for  $\theta_n \in (\theta_i, \theta_j)$ ,  $Q[W(P_a; \theta_n) - R(\Pi) - X] > \tilde{Q}[W(P_a; \theta_n) - R(\tilde{\Pi}) - X]$ . This means  $\theta_n$  can profitably deviate to cash bid  $R(\Pi)$ . Therefore, it has to be that  $[\theta_i, \theta_j] \in \Theta_p$ .

Next, note  $W(P_a; \theta_i) - X - R_{\theta_i}(S) - C > W(P_a; \theta_j) - X - R_{\theta_j}(S) - C \geq 0$ . Type  $\theta_i$  can deviate profitably to cash bid  $\epsilon + R(\Pi)$  which reduces his payoff by  $\epsilon$  upon winning but increases his marginal chance of winning by a discrete amount (because he separates from a positive measure of types). □

### A.14 Proof of Proposition 10

*Proof.* Consider the bidding strategy from a PFA in cash. The valuations for the bids are simply the cash amounts. I show there exists a belief that supports an equilibrium with this bidding strategy in the informal

auction. First, there would not be any deviation to another cash amount since the bidding strategy comes from the equilibrium in FPA cash auction. Next, for beliefs such that upon seeing an out-of-equilibrium bid  $\Pi^i$ , the auctioneer believes it comes from  $\tilde{\theta}_i = \operatorname{argmin}_{\theta \in [\underline{\theta}, \bar{\theta}]} [R_\theta(S^i) + C^i]$  and gives it a valuation  $\tilde{R}$ . If bidder  $i$  finds this deviation attractive (yielding an expected payoff more than the original cash amount he is paying), then he also finds deviating to cash bid  $\tilde{R}$  weakly more attractive, contradicting the fact that no deviation to another cash amount is profitable. Thus the equilibrium from a first-price cash auction is an equilibrium in the informal auction. The argument also applies to cash-like bid  $\Pi$  such that  $R(\Pi)$  is independent of the seller's belief on the bidders' types.

Next I show any equilibrium in the informal auction has the same expected payoffs as cash auctions. The seller forms correct beliefs about types since Lemma 4 rules out pooling. Bidder  $i$ 's bid  $S^i$  can be replaced by an equivalent cash bid. Note  $\tau_i = \tau_i^*$  from Lemma 3. This would not change the marginal probability of winning, neither does it change the payoff upon winning. Since the bidders face the same maximization problem as in a FPA with cash, . Thus in terms of expected payoff, almost every bid is cash-like.  $\square$

## A.15 Proof of Proposition 11

*Proof.* The derivative of the seller's payoff has the same sign as

$$\int_{\underline{\theta}}^{\theta_c} f(\theta) F(\theta) [1 - F(\theta)]^{N-2} [\beta(X + Y + \theta) - (\beta - 1)P^*(\theta)] d\theta, \quad (21)$$

where  $\theta_c$  is the cutoff type in cash auction with reserve price  $Y$ . The derivative changes sign only once, implying a unique optimal threshold strategy for timing the auction. At the efficient timing, apply the lemma in the proof of Proposition 9 and note  $g(\theta) = \frac{1-F(\theta)}{(N-1)F(\theta)}$  is decreasing in  $\theta$ , we get that the sign of the derivative is positive. Given the uniqueness of the optimal timing, the threshold for having the auction is higher - the seller waits inefficiently longer in timing a cash auction.  $\square$

## A.16 Proof of Proposition 12

*Proof.* Conjecture that in equilibrium bidder  $\theta$  initiates the auction with threshold  $P_I(\theta)$ , which is potentially non-monotone, and the seller initiates with a threshold  $P_S$ . Let the probability of auction initiation before  $P$  is reached be  $\tilde{F}(P)$  in equilibrium. Consider SPAs first. The expected payoff to the bidder  $\theta$  following initiation threshold  $P_a \leq P_S$  is

$$\begin{aligned} & \int_{P_0}^{P_a} d\tilde{F}(P') \int_{\Theta_H(P')} d\theta' \left(\frac{P_0}{P'}\right)^\beta \left[ [W(P'; \theta) - X - Y]^+ - [W(P'; \theta') - X - Y]^+ \right]^+ \\ & + \int_{P_a}^{\infty} d\tilde{F}(P') \int_{\Theta_H(P_a)} d\theta' \left(\frac{P_0}{P_a}\right)^\beta \left[ [W(P_a; \theta) - X - Y]^+ - [W(P_a; \theta') - X - Y]^+ \right]^+ \end{aligned} \quad (22)$$

where  $\Theta_H(P)$  is the set of types that initiate with thresholds higher than  $P$ ; payoff when  $P > P_S$  is

$$\begin{aligned} & \int_{P_0}^{P_S} d\tilde{F}(P') \int_{\Theta_H(P')} d\theta' \left(\frac{P_0}{P'}\right)^\beta \left[ [W(P'; \theta) - X - Y]^+ - [W(P'; \theta') - X - Y]^+ \right]^+ \\ & + \int_{P_S}^{\infty} d\tilde{F}(P') \int_{\Theta_H(P_S)} d\theta' \left(\frac{P_0}{P_S}\right)^\beta \left[ [W(P_S; \theta) - X - Y]^+ - [W(P_S; \theta') - X - Y]^+ \right]^+ . \end{aligned}$$

Denote the solution to  $W(P_a, \theta) = X + Y$  by  $\hat{P}(\theta)$ . Note when  $P_a \leq P_S$ ,  $(P_0/P_a)^\beta ([W(P_a; \theta) - X - Y]^+ - [W(P_a; \theta') - X - Y]^+)$ , if positive, is decreasing when  $P_a > P^*(X + Y + \theta)$ , increasing at  $\hat{P}$ , and constant for  $P_a < \hat{P}$ . Differentiating (22) and applying Leibniz's formula gives that in equilibrium  $\hat{P} \leq P_I(\theta) \leq P^*(\theta + X + Y)$ . Now for the seller, if she uses threshold  $P_a$ , the expected payoff is,

$$\begin{aligned} & \int_{P_0}^{P_a} d\tilde{F}(P') \int_{\Theta_H(P')} d\theta' \left(\frac{P_0}{P'}\right)^\beta [W(P'; \theta') - X - Y]^+ f(\theta') F(\theta') [1 - F(\theta')]^{N-2} N(N-1) \\ & + \int_{P_a}^{\infty} d\tilde{F}(P') \int_{\Theta_H(P_a)} d\theta' \left(\frac{P_0}{P_a}\right)^\beta [W(P_a; \theta') - X - Y]^+ f(\theta') F(\theta') [1 - F(\theta')]^{N-2} N(N-1). \end{aligned} \quad (23)$$

Suppose  $\Theta_H(P_a)$  contains positive measure of types. For any  $\theta' \in \Theta_H(P_a)$ , the earlier argument leads to  $P_a < P^*(\theta' + X + Y)$ , for otherwise  $\theta'$  would initiate earlier than  $P_a$  - a contradiction. The derivative of (23) is thus positive for any  $P_S$  unless  $\Theta_H(P_S)$  is measure-zero. Thus almost surely the seller never initiates.

Now the bidder's problem is reduced to expression (22). The derivative at  $P_I(\theta)$  is

$$\left[1 - \tilde{F}(P_I)\right] \int_{\Theta_H(P_I)} d\theta' \frac{d}{dP} \left[ \frac{[W(P; \theta) - X - Y]^+ - [W(P; \theta') - X - Y]^+}{P^\beta} \right] \Bigg|_{P=P_I}, \quad (24)$$

which is positive at  $\hat{P}(\theta)$  and non-positive at  $P^*(\theta + X + Y)$ . The integrand is weakly monotone in  $P_I$  path-by-path, thus (24) changes sign at a unique  $P_I$ .

Given (22) is concave in  $P_a$  with non-negative cross-partial in  $P_a$  and  $\theta$ , and there exists unique maximizer  $P_I(\theta)$ , Implicit Function Theorem gives that  $P_I(\theta)$  is non-decreasing. This ensures (24) is continuous, establishing the optimality of  $P_I$  and the FOC in the proposition for SPAs. There could be multiple equilibria with different initiation thresholds below  $P_0$ , but in terms of initiation outcome and payoffs, they are all equivalent, making the proposed equilibrium essentially unique.

Now consider the FPA equilibria with increasing initiation thresholds. Having seen no initiation up to  $P$  allows the bidders to truncate the support of the distribution of types to  $[P_I^{-1}(P), \bar{\theta}]$ . Fortunately, the bidding strategy of type  $\theta$  only depends on his belief of types that are worse than her, i.e., types distributed in  $(\theta, \bar{\theta}]$ , thus the payoff in symmetric bidding strategy is equivalent to SPA in terms of revenue and allocation conditional on the auction time  $P_a$ . The analysis carries through and the same initiation equilibrium results.

Given that a bidder's threshold for holding the auction is lower than his threshold if he were maximizing social welfare, the initiation is accelerated in the ex post sense. Moreover, he would invest in the project right away, making the exercise of the real option faster than in seller-initiated auctions where the realized winning type might still wait after the auction.  $\square$

## A.17 Proof of Proposition 13

*Proof.* For simplicity, consider pure contingent securities. Extension to include cash is straightforward. Conditional on an auction timing, cash auctions lead to efficient investments and obviously dominate in terms of welfare. For the seller's revenue, first consider SPAs. The revenue is  $\mathbb{E}[e^{-r\tilde{\tau}} S(s(\theta_{(2)}), P_{\tilde{\tau}}) \mathbb{1}_{\{\theta_{(2)} \leq \hat{\theta}\}}]$   $= \mathbb{E}[(e^{-r\tilde{\tau}}(P_{\tilde{\tau}} - \theta_{(1)}) - U(\tilde{\tau}, s(\theta_{(2)}), \theta_{(1)})) \mathbb{1}_{\{\theta_{(2)} \leq \hat{\theta}\}}] \leq \mathbb{E}[(e^{-r\tilde{\tau}}(P_{\tilde{\tau}} - \theta_{(1)}) - X) \mathbb{1}_{\{\theta_{(1)} \leq \hat{\theta}\}}] \equiv R_0$ , where  $\tilde{\tau} = \operatorname{argmax}_\tau U(\tau, s(\theta_{(2)}), \theta_{(1)})$  and  $U(\tau, s, \theta) = \mathbb{E}[e^{-r\tau}(P_\tau - S(s, P_\tau) - \theta)]$ . Similarly in FPAs, the revenue is bounded above by  $R_0$  with  $\tilde{\tau} = \operatorname{argmax}_\tau U(\tau, s(\theta_{(1)}), \theta_{(1)})$ . Let  $s_w$  denote the index the winning bidder pays in general. Then in FPAs and SPAs, the revenue is bounded above by  $R_0$  with  $\tilde{\tau} = \operatorname{argmax}_\tau U(\tau, s_w, \theta_{(1)})$

The revenue from cash auction would be the expected second highest valuation  $R_2 \equiv \mathbb{E}[(W(P_a; \theta_{(2)}) -$

$X)\mathbb{1}_{\{\theta_{(2)} \leq \hat{\theta}\}}$ . When  $N \rightarrow \infty$ ,  $\theta_{(2)} - \theta_{(1)} \xrightarrow{a.s.} 0$ . Thus  $W(P_a; \theta_{(2)}) - W(P_a; \theta_{(1)}) \xrightarrow{a.s.} 0$ . Now  $\mathbb{1}_{\{\theta_{(2)} \leq \hat{\theta}\}}$  and the above are bounded, by bounded convergence,  $R_2$  converges a.s. to  $R_1 \equiv \mathbb{E}[(W(P_a; \theta_{(1)}) - X)\mathbb{1}_{\{\theta_{(1)} \leq \hat{\theta}\}}]$ .

If  $R_1 - R_0$  converges to a quantity bounded below by a positive constant, the claims follow. First note  $U(\tau, s_w, \theta_{(1)})$  admits an optimal stopping solution involving threshold strategies. To see this, write  $U(\tau, s_w, \theta_{(1)}) = D(P_a; P)[P - \theta_{(1)} - \sum_{i \in I} a_i(s_w)[P - b_i(s_w)]^+]$ , which admits a maximizer  $\tilde{P}(\theta_{(1)})$ . Then use that as an investment trigger and apply the standard verification argument. Next, as  $\theta_{(1)} - \underline{\theta} \xrightarrow{a.s.} 0$ , the investment trigger in cash auctions converges to  $P^* = \frac{\beta}{\beta-1}\underline{\theta}$ , and  $\tilde{P}(\theta_{(1)})$  to  $\tilde{P}^* = \tilde{P}(\underline{\theta})$ . Whether  $\tilde{P}^* \in [b_m, b_{m+1})$  or not,  $|\tilde{P}^* - P^*| \geq M$ . Since  $P^*$  is the optimal trigger for  $\mathbb{E}[e^{-r\tau}(P_\tau - \underline{\theta})]$ ,  $R_1 - R_0 \xrightarrow{a.s.} \epsilon$  for some  $\epsilon > f(M)$ , where  $f(M)$  is a function of  $M$  that is positive and independent of  $N$ . Therefore as  $N$  becomes big,  $R_2$  converges to  $R_1$  which dominates  $R_0$  in the limit. Thus cash auctions yield higher revenue than the security-bid auctions.  $\square$

## B Technical Requirements for “Well-behaved” Distributions

To make the design problem “regular” and avoid discussing “ironing” techniques, I require  $z(\theta) = \theta + F(\theta)/f(\theta)$  to be increasing in  $\theta$ . A standard assumption in the economics literature is that the “Inverse Hazard Rate” being increasing. In our case, it is  $F(\theta)/f(\theta)$  being increasing.

To have single threshold strategies for optimal auction timing, I require  $z(\theta)$  to be differentiable and invertible. Moreover, for  $\theta$ s that solve  $W(P; \theta) = X + Y$  and  $W(P; z(\theta)) = X + Y$ , where  $W(P; \theta)$  is given in (3), the marginal increase in participation expected from waiting for higher cash flow level  $f(\theta)d\theta/dP$  is singly-peaked as a function of  $P$ . Finally, I require  $\frac{dz(\theta)}{d\theta} \frac{f(z(\theta))}{f(\theta)} \left[ \frac{1-F(z(\theta))}{1-F(\theta)} \right]^{N-1}$  being decreasing in  $\theta$ .

Note these requirements are satisfied by continuous distributions commonly used in the economics and finance literature, such as Uniform Distribution, Exponential Distribution, and Pareto Distribution.

## C Institutional Details and Empirical Evidence

### C.1 Oil and Gas Auction and Drilling in the Gulf of Mexico

Offshore drilling activities in the gulf of Mexico date back to the 1940s. The US Congress passed the Outer Continental Shelf Lands Act (OCSLA) in 1953 to grant the Department of the Interior the authority for conducting lease auctions, collecting royalties, and overseeing all activities associated with the drilling in federal waters. The Minerals Management Service (MMS) traditionally conducted the lease auctions, but due to a reorganization in response to the DeepWater Horizon oil spill in 2010, it was replaced by the Bureau of Ocean Energy Management (BOEM) and the Bureau of Safety and Environmental Enforcement (BSEE). Most leases were sold in “Bonus-bid” auctions, where the royalty rate on future revenue is fixed and the bidders bid upfront cash. The current royalty rate is standardized at 18.75%, but has historically taken on different values at various times and for different leases.

The empirical tests in this paper utilize the following two policy changes:

(1) The introduction of Area Wide Leasing (AWL) in May 1983 marks an important break in the lease auction and drilling environment for offshore tracts. Prior to AWL, potential bidders could nominate most nearshore tracts (less than 200 meters of water depth) and certain deepwater tracts (exceeding 200 meters of water depth) to be auctioned. Following comments by other interested parties, such as fishery and environmental interests, BOEM carried out lease sales which were typically on the order of a few hundred tracts. AWL eliminated the nomination process and made most of the offshore blocks in a region available

in each sale, including thousands of tracts in deep water areas. Moreover, some lease tenures were increased from 5 or 8 years to 10 years, and the royalty rates on tracts with water depth of more than 400 meters were lowered from 1/6 to 1/8. Opaluch, Grigalunas, Anderson, Trandafir, and Jin (2010) give more details.

(2) Common incentives for offshore oil and gas development include certain forms of royalty relief. The OCSLA authorized the Secretary of Interior to grant royalty relief to promote increased production in oil and gas (43 U.S.C. 1337). The Deep Water Royalty Relief Act of 1995 (DWRRA) expanded the Secretary's royalty relief authority in the Gulf of Mexico outer continental shelf. Eligible leases are those issued in the Gulf of Mexico between 1996 and 2000 at depths greater than 200 meters located wholly west of 87 degrees, 30 minutes West longitude. Interest in deep water surged after the enactment of the act (August 8, 1995), with 3,000 deepwater leases bid between 1996 and 1999. There is also significant increase in annual deepwater oil production. Greater details on various royalty relief programs can be found at BOEM's official website: <http://www.boem.gov/Oil-and-Gas-Energy-Program/Energy-Economics/Royalty-Relief/Index.aspx>.

## C.2 Data Construction

Data on the lease auctions, drilling, and mineral production are from the Minerals Management Service of the Department of Interior. I observe detailed lease-level variables, borehole-level variables, lease sale data, ownership data, and production data. For the cost of drilling and equipping a borehole that varies by year, region, well type, and well depth, I use John Beshears's inflation-adjusted estimates based on annual surveys by the American Petroleum Institute (API) and GDP implicit price deflator index from the Bureau of Economic Analysis. The detailed description of various variables are in Beshears (2011). The estimation related to AWL uses leases auctioned in 1978-1989 with a total of 5399 leases. The estimation related to DWRRA uses leases auctioned in 1991-2000 with a total of 7858 leases.

Monthly prices for oil and natural gas are obtained from the World Bank Commodity Price Data (Index Mundi Data Set and the Energy Information Administration (EIA) also contain futures prices, but are not monthly). The prices are inflation-adjusted using monthly CPI data from the Federal Reserve Bank in St. Louis. ([http://research.stlouisfed.org/fred2/graph/?s\[1\]\[id\]=CPIAUCSL](http://research.stlouisfed.org/fred2/graph/?s[1][id]=CPIAUCSL)). The discussion in this paper uses average spot prices and the results are robust to using different categories of crude oil and natural gas (such as West Texas Intermediate, Brent Crude Oil, etc.).

The key variables in the empirical exercises are listed below:

**Event:** The event is first exploratory drill. For this, I take the spud date for the first exploratory borehole drilled in each lease tract.

**AWL:** Dummy for the implementation of AWL policy. It takes the value 0 for leases auctioned before May 1983 and 1 afterwards.

**DEEP:** Dummy for leases with water depth greater than 200 meters in areas west of the 87 degrees 30 minutes West longitudinal line.

**RELIEF:** Dummy for the implementation of DWRRA. It takes the value 0 for leases auctioned before August 1995 and 1 afterwards.

**DEEP\*RELIEF:** Interaction term for the DEEP and RELIEF dummies. The coefficient for this variable is of interests in the diff-in-diff test.

**RTY:** The royalty rate specified in the lease agreement. It is typically 1/6 or 1/8 in this data set.

**DUR:** The number of days from lease auction to expiration. Most lease terms are 5-10 years.

**DEPTH:** The water depth of the leased tract. The results reported use minimum water depth, and are robust to alternative specifications using maximum water depth or average water depth.

**SIZE:** The area covered under each lease. Most leases had an area of approximately 5,000 acres, though some were smaller.

**MKT:** The number of leases sold in the same sale as a proxy for the market demand for oil and gas leases. When the market demand is high, the quality of the marginal lease sold may be low in the sense that the reserve quantity is small or there is huge uncertainty, which in turn affect drilling decisions.

**P(t):** Average spot price for oil and gas. The results are robust to the inclusion of oil and gas prices separately, or prices lagged by 1-5 months.

**VOL(t):** Average trailing-12-month volatilities for oil and gas spot prices. The results are robust to the inclusion of oil and gas price volatilities separately and to variations within one year of the trailing window.

**COST(t):** The industry average drilling cost for dry, oil, and gas boreholes. The drilling cost is the initial cost plus the equipping cost.

**Firm f.e.:** I control for firm-specific characteristics by including the firm ID as a factor. Over the years, it tends to be the same group of firms that bid for leases. For jointly-owned leases, I use the largest shareholder. The results are robust to using the second-largest shareholder.

**Info.Ext.:** I control for information externality by using subsamples with lease areas exceeding 4,000 acres and 5,000 acres (full control) respectively. Prior studies (Lin (2009, 2012)) have shown that information externality is not significant for exploratory drills, and decreases with the size of the tract. Even for development drills, information externality is negligible for tracts larger than 5,000 acres.

### C.3 Empirical Models and Tests

I employ a Cox proportional hazard model to test the following two hypotheses.

**Null Hypothesis I:** Auction initiation has no impact on how quickly tracts are explored.

**Null Hypothesis II:** A lower royalty rate does not lead to greater likelihood of exploratory drill.

Cox hazard models probably represent the state of the art in survival analysis with reduced-form models. They make the assumption that the hazard rate  $\kappa(t)$  of exploratory drill at time  $t$  conditional on lack of drill until time  $t$  is  $\kappa(t) = \psi(t)[\exp(\vec{X}(t)^T \vec{\gamma})]$ , where  $\psi(t)$  is the baseline hazard rate that is completely unrestricted, and  $X(t)$  is a vector of independent variables listed earlier (*AWL* is only for the first hypothesis where as *DEEP*, *RELIEF*, *DEEP\*RELIEF* are only for the second). This specification handles censoring of observations and allows time-varying covariates. There is no survivorship bias or response bias because the leases are sampled at birth (the auction), and all leases are recorded by the Department of Interior.

Hypothesis I is rejected and the results are reported in Section 5 of the paper and in Table 2 and Figure 4. Hypothesis II is rejected only at low significance level, and the estimations are sensitive to the time window of the analysis. Table 3 gives one analysis using leases auctioned in 1991-2000. The coefficient of *DEEP \* RELIEF* indicates that the implementation of DWRRA led to an increase in the difference in exploration likelihood between treated and untreated groups. Cong (2013b) carries out more detailed studies on the effect of changing royalty rate using different approaches.

## Tables and Figures

Table 2: Exploration of Oil and Gas Tracts

This table presents estimates from a Cox regression with time-varying covariates. The dependent variable is time-to-exploratory-drill, which measures the number of days from the lease auction to the first exploration. The independent variables are the variable of interest *AWL* indicating the absence of bidder initiation, oil and gas price measure  $P(t)$ , price volatility  $VOL(t)$ , drilling cost  $COST(t)$ , royalty rate *RTY*, water depth *DEPTH*, lease length *DUR*, tract size *SIZE*, market demand *MKT*, firm fixed effects *FIRM f.e.*, and control for information externality *INFO*. If the dependent variable is observed without any realization, it is treated as a censored event. Model  $\chi^2$  reports the joint significance of the estimates. Hazard ratio indicates the impact of *AWL* on likelihood to drill.

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
<i>AWL</i>	-0.5674*** (0.0698)	-0.3875*** (0.0820)	-0.1856* (0.1123)	-0.3310*** (0.1008)	-0.2329* (0.1316)	-0.3558* (0.1839)	-0.1623* (0.0889)
<i>MKT</i>		0.0118** (0.0054)		0.0089 (0.0061)	0.0144*** (0.0055)	0.9153*** (0.3169)	0.0071 (0.0061)
<i>RTY</i>		-0.0200 (0.0227)		-0.0591* (0.0318)		-0.0341 (0.0588)	-0.0611* (0.0315)
<i>DUR</i>		-0.0073* (0.0043)		-0.0106** (0.0046)		-0.2728 (82.07)	-0.0108** (0.0046)
<i>DEPTH</i>		-0.0005*** (0.0001)		0.0004*** (0.0001)		-0.0002 (0.0004)	-0.0004*** (0.0001)
<i>SIZE</i>		-0.0003 (0.0002)		-0.0022*** (0.0005)			-0.0022*** (0.0005)
$P(t)$			0.0214*** (0.0058)		0.0212*** (0.0065)	-0.0495 (0.1019)	0.0144** (0.0068)
$VOL(t)$			0.0037 (0.0159)		-0.0076 (0.0170)	-0.3401* (0.2053)	-0.0277 (0.0187)
$COST(t)$			0.0004 (0.0004)		-0.0003 (0.0004)	-0.0112 (0.0073)	0.0002 (0.0004)
<i>Info.Ext.</i>	No	No	No	Yes	Yes	Yes (Full)	Yes
<i>Firm f.e.</i>	No	No	No	Yes	Yes	No	Yes
<i>Hazard Ratio</i>	0.5670	0.6787	0.8306	0.7182	0.7922	0.7006	0.8502
<i>No. of Leases</i>	5399	5399	5399	4983	4983	3266	4983
Model $\chi^2$	66.05	210.7	90.43	445.1	437.7	23.92	488.1

Standard errors in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table 3: Exploration of Oil and Gas Tracts

This table presents estimates from a Cox regression with time-varying covariates. The dependent variable is time-to-exploratory-drill, which measures the number of days from the lease auction to the first exploration. The independent variables are *DEEP* indicating the DWRRA-treated group, *RELIEF* indicating whether DWRRA is implemented, the variable of interest *DEEP\*RELIEF* capturing the interaction, oil and gas price measure  $P(t)$ , price volatility  $VOL(t)$ , drilling cost  $COST(t)$ , royalty rate *RTY*, water depth *DEPTH*, lease length *DUR*, tract size *SIZE*, market demand *MKT*, firm fixed effects *FIRM f.e.*, and information externality *INFO*. If the dependent variable is observed without any realization, it is treated as a censored event. Model  $\chi^2$  reports the joint significance of the estimates.

Variables	Hazard Ratio	Coefficient	Standard Error
<i>DEEP*RELIEF</i>	1.1080 <sup>.....</sup>	0.1024 <sup>.....</sup>	0.6890
<i>DEEP</i>	1.0830	0.0794	0.1337
<i>RELIEF</i>	0.5389 <sup>***</sup>	-0.6183 <sup>***</sup>	0.1958
<i>MKT</i>	0.9989	-0.0011 <sup>***</sup>	0.0002
<i>DEPTH</i>	0.9997 <sup>***</sup>	-0.0003 <sup>***</sup>	0.0001
<i>SIZE</i>	1.0000 <sup>**</sup>	0.0000 <sup>**</sup>	0.0000
<i>RTY</i>	1.0120	0.0117	0.0517
<i>DUR</i>	1.0040	0.0039	0.0047
$P(t)$	1.0790 <sup>***</sup>	0.0764 <sup>***</sup>	0.0277
$VOL(t)$	1.0150	0.0152	0.0172
$COST(t)$	0.9996	-0.0004	0.0010
No. of leases = 7858	Model $\chi^2 = 451$	<i>FIRM f.e.</i> Yes	<i>INFO</i> Yes

.....  $p < 0.15$ , \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

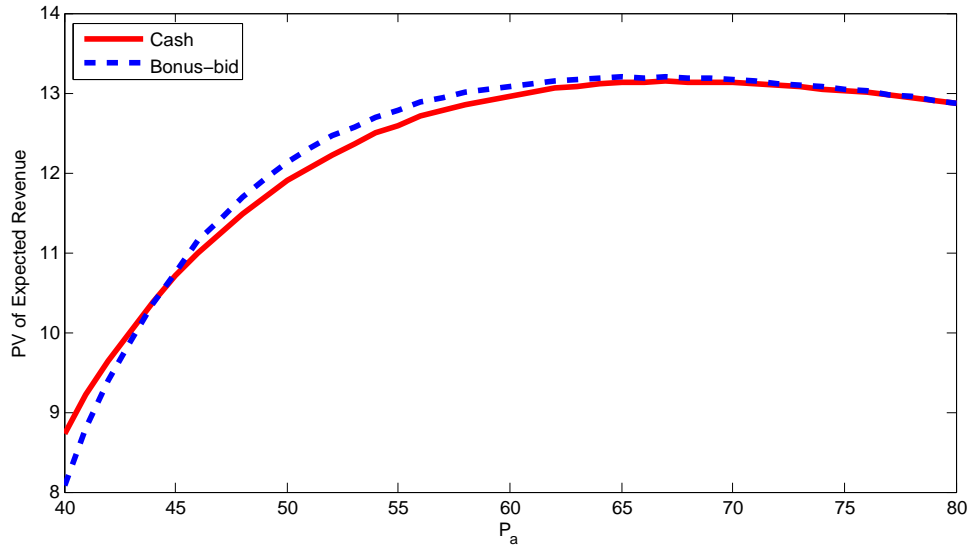


Figure 2: Revenues (normalized) for cash and bonus-bid auctions. 100,000 simulations for  $\theta \sim Unif[10, 40]$ ,  $\beta = 1.9$ ,  $X = 12$ ,  $Y = 0$ ,  $N = 5$ ,  $P_0 = 40$ , and  $\phi = 1/3$ .



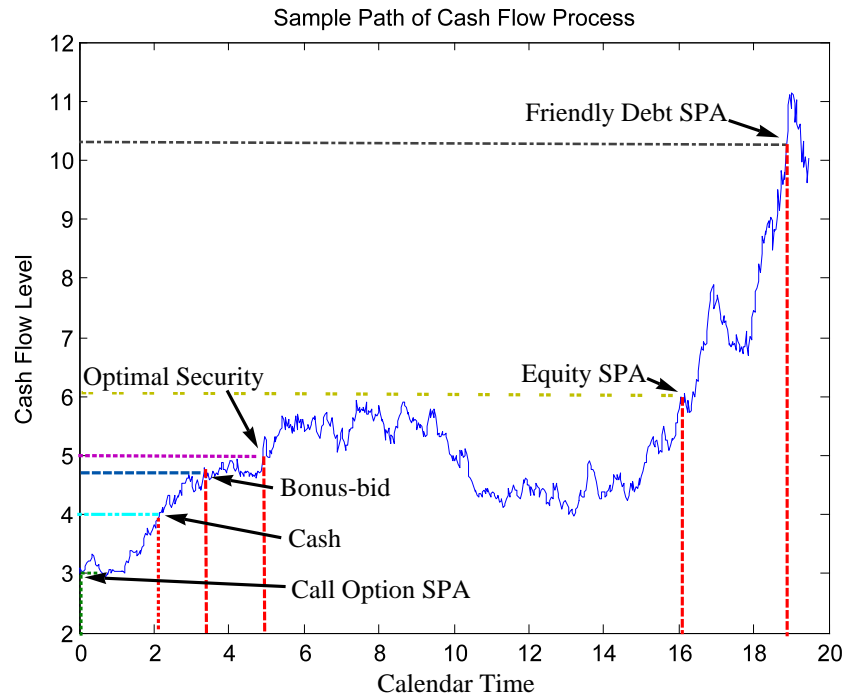


Figure 3: Investment thresholds under various security designs. Simulated with  $\mu = 0.06$ ,  $\sigma = 0.2$ ,  $r = 0.16$ ,  $\theta \sim Unif[1.5, 5]$ ,  $X = 0.4$ ,  $Y = 0$ ,  $P_a = 3$ . Horizontal lines indicate investment thresholdss, vertical lines indicate the calendar time of investment.

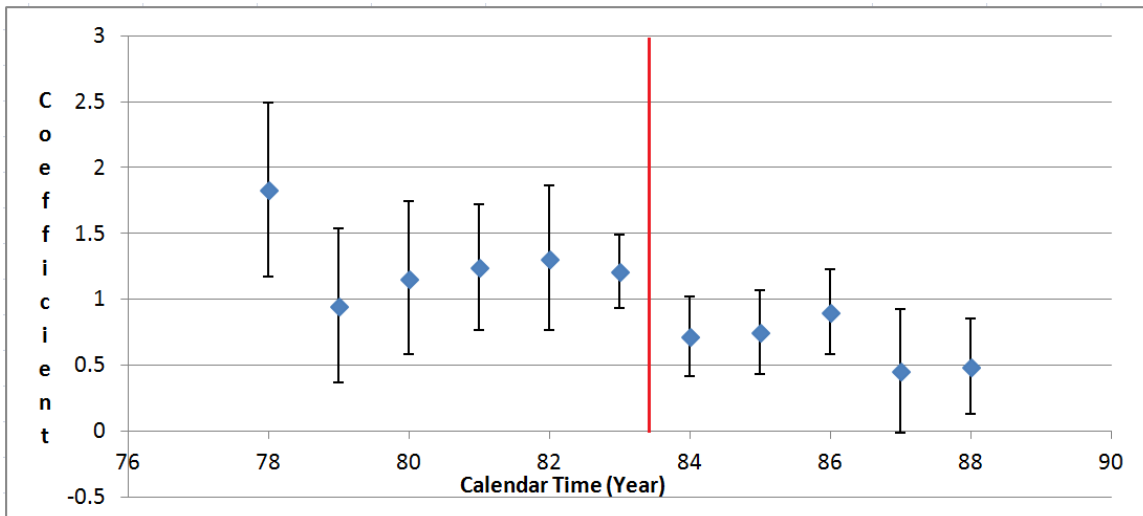


Figure 4: Coefficients for year dummies in the Cox estimation after controlling for all observable covariates, firm fixed effect, and information externality. The estimates are reported with 95% confidence interval. The red line marks the commencement of Area Wide Leasing (AWL).

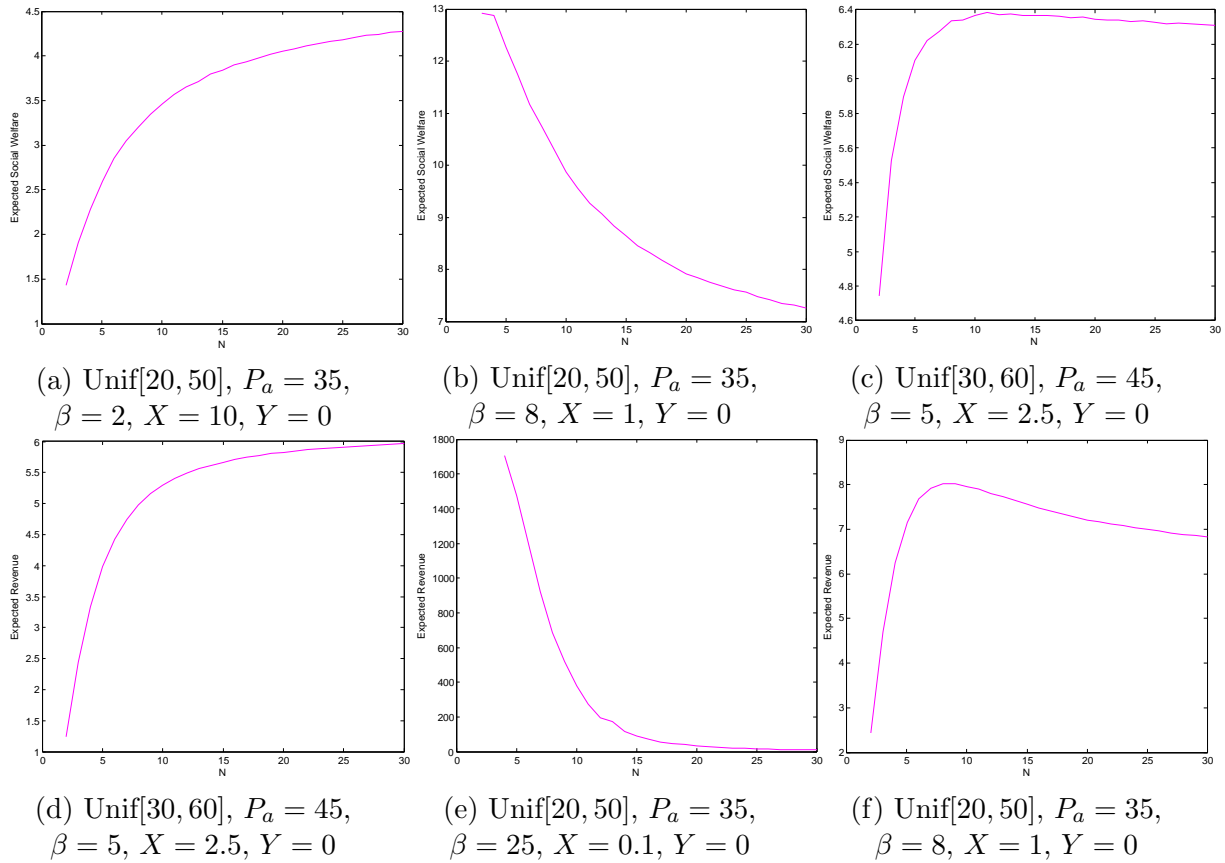


Figure 5: Plots of expected social welfare (a)(b)(c) and seller's revenue (d)(e)(f) against number of bidders  $N$ . One million simulations in SPA with equity bids and uniformly distributed  $\theta$ .

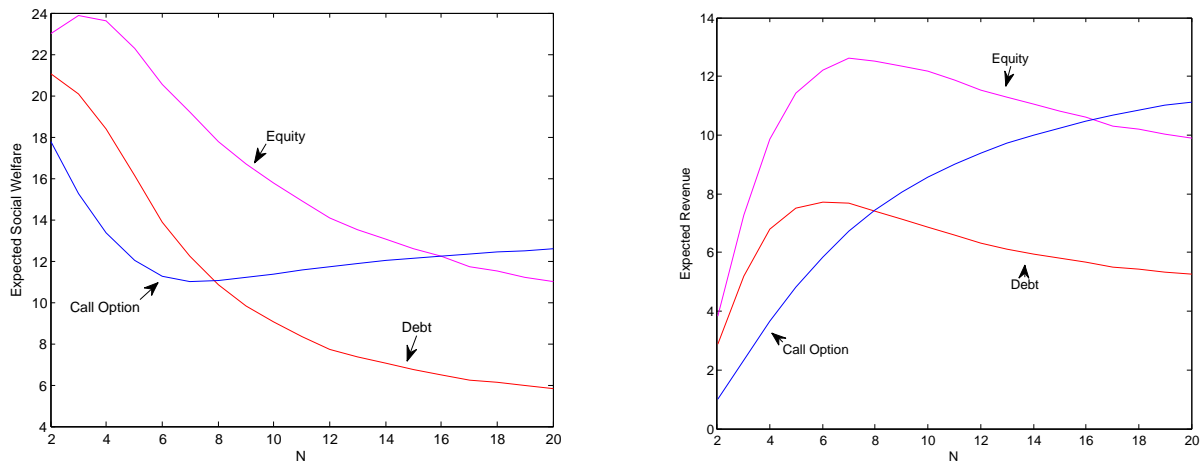
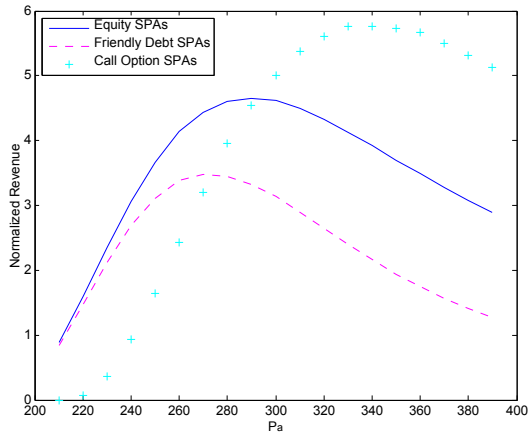
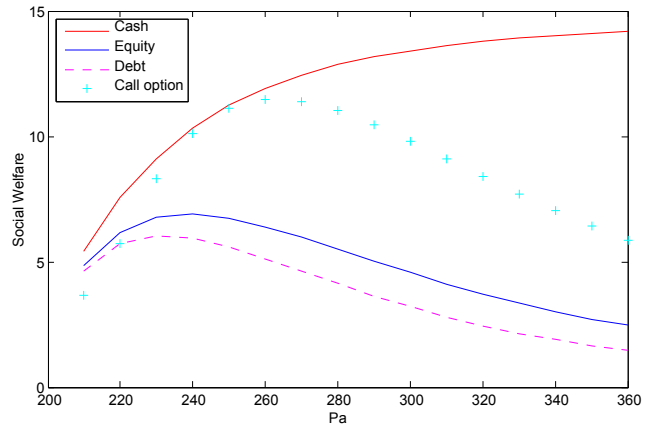


Figure 6: Plots of expected social welfare and seller's revenue against number of bidders  $N$  for SPAs with equities, friendly debts and call options. One million simulations with  $\theta$  uniformly distributed in  $[20, 50]$ ,  $P_a = 35$ ,  $\beta = 10$ , and  $X + Y = 1$ .



(a)  $\beta = 5, X = 10, Y = 0, N = 5$



(b)  $\beta = 6, X = 8, Y = 0, N = 30$

Figure 7: Plots of expected seller's revenue and social welfare against the auction threshold for SPAs with equities, friendly debts and call options. One million simulations for  $\theta$  uniformly distributed in  $[200, 500]$ ,  $P_0 = 210$ .

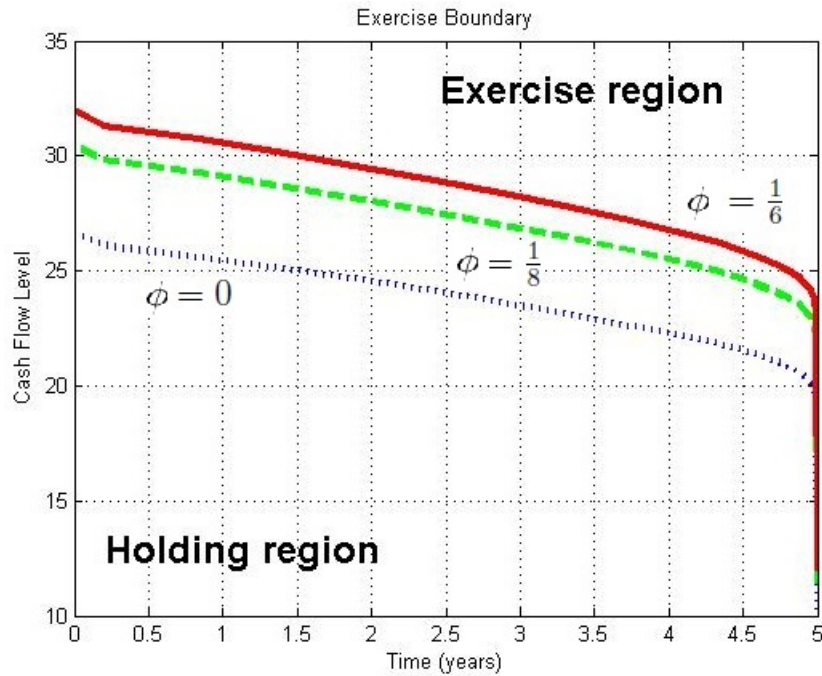


Figure 8: Investment Thresholds for Real Options with Expirations. The plots are for  $\theta = 10$ ,  $\mu = 0.02$ ,  $r = 0.04$ ,  $\sigma = 0.2$ ,  $T = 5$ , and fixed royalty rates  $\phi = 0, \frac{1}{8}, \frac{1}{6}$  respectively.