A Theory of Choice by Elimination*

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Abstract

We provide a descriptive model of *choice by elimination* that includes for its foundation. The basis of the model involves a decision procedure based on elimination rather than selecting an alternative. In this model, the alternative that cannot be eliminated by any of its comparables ends up being chosen. The necessary and sufficient condition for the model, which we call "axiom of choice by elimination" (ACE), reflects the idea of "bounded rationality." This condition is also normatively appealing since it is immune to "money pump" type of arguments despite the fact that it is weaker than the independent of irrelevant alternative (IIA). Our framework makes it possible not only to provide two characterizations for IIA and Simon's satisficing but also to accommodate endogenous reference-dependent choice.

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1 Introduction

Imagine an individual who needs to buy a house. Given the complexity of the problem, she solicits recommendation from a friend of hers. He suggests her to go for a specific house, say x. In order to decide whether she will go with his advice or not, she looks into properties that are comparable to x, in several aspects, such as price, location and size, through some real estate Web sites. For example, if x has three bedrooms and a big garage, she may include them into search criteria along with her budget. Then the Web site provides a list of houses which are comparable to x and remain within her budget. If she finds a house which dominates x in this list, she discards the advice. Otherwise, she is convinced to buy house x. If this is the case, house x is one of the acceptable suggestion (so it can be chosen).

In fact, this choice procedure is commonly used in real estate. Indeed, there is a term called "comparables" which is an abbreviation for "comparable properties."[1] Comparables are properties that have reasonably similar sizes, locations, and amenities and they are used for purposes of comparison between different houses. In the above example, "comparables" correspond to the list of houses provided by the web site. In real life, consumers have only a finite amount of time, knowledge, and/or attention to spend on a particular decision problem. Therefore, focusing on comparables (or the list) helps the decision maker to reduce her complex problem into a much more manageable one (Payne (1982) and Payne et al. (1988)).

What is more, these behaviors are not confined to real estate. Especially, after the explosion of internet, a consumer has been able to deploy the help of e-commerce sites (e.g., amazon.com, expedia.com) which construct sets of comparables (or shopper's list) for different choice problems, thereby narrowing down the decision-making process.[2] These sophisticated tools assist online customers by customizing the electronic

[1] In marketing literature, "awareness set," "consideration set," "relevant set," and "evoked set" are used to describe the similar idea of narrowing choice alternatives from many to few, see Alba and Chattopadhyay (1985) and Roberts and Lattin (1991).

[2] Haübl and Trifts (2000) conceptualize a *recommendation agent* (RA) as an interactive decision

shopping environment with respect to their individual preferences and budgets. Since the amount of time and effort spending on decision making is substantially reduced, the usage of online decision aids is inevitable and increasingly popular.

Returning to our customer, note that even if she decides to buy x, she may not have compared x against all attainable houses in the market. This is because the web site displays only properties that satisfy her criteria. Choice of her criteria depends not only on her budget but also on the properties of the house under consideration. Hence comparison is made within those properties which might actually be a strict subset of all attainable houses. That is why our model offers a better alternative to the standard utility maximization paradigm may not be able to accommodate her choice behavior.

On the other hand, if she discards a house, let's say y, she must have found at least one house which dominates y within the list provided by the web site. Even if she had compared house y against all other houses within her budget (without using the list), it would have been discarded anyway. Therefore, each elimination is perfectly rationalizable.

The aim of this paper is to provide a characterization of the choice behavior in which decision making is implemented by elimination in the restrictive fashion as exemplified above. Formally speaking, for a subset, S, of all alternatives set, X, we propose the following characterization:

$$C_{(E,\Omega)}(S) = \{x \in S| \nexists\ y \in \Omega(x, S) \text{ such that } yEx\}. \tag{1}$$

where E is an elimination order[3] on X and comparable sets Ω. The set $\Omega(x, S)$ represents the set of alternatives which are comparable to x in some respect within S, i.e., $\Omega(x, S) \subset S$.[4]

aid that assists consumers in the initial screening of the alternatives that are available in an online store.

[3]An elimination order is both asymmetric and negatively transitive. If one defines $(x, y) \in E^*$ if $(y, x) \notin E$. Then E is asymmetric and negatively transitive if and only if E^* is complete and transitive. Therefore, E is mathematically equivalent to a strict preference relation.

[4]In our model, both E and Ω will be endogenously derived from the decision maker choice

While the elimination order is *context-free*, $\Omega(x, S)$ depends on the budget set S, in other words it is context-dependent. One interpretation is that each elimination is made by the decision maker while the set $\Omega(x, S)$ is *exogenously* provided by the web site. As we mention above, comparables are a good example for Ω. If there is no house comparable to house x given S, then the house x will be chosen within S. If there are some comparable alternatives, $\Omega(x, S) \neq \emptyset$, the decision maker looks for an element *only* within $\Omega(x, S)$ which eliminates x. If she finds such an element y i.e. $y \in \Omega(x, S)$ and yEx, she discards x. Otherwise, x will be in the choice set, or is an *acceptable* recommedation ("choosable").

Since the suggested model is context-dependent (the set of comparables depends not only on x but also on the budget set S), it seems any given choice correspondence can be characterized by (1). The following example illustrates that this claim is false. Hence, the general view "everything goes with context-dependent model" is not quite right.

Example 1 (Strict Choice Cycle) Let $X := \{x, y, z\}$ and $C(\{x, y\}) = \{x\}$, $C(\{x, z\}) = \{z\}$, and $C(\{y, z\}) = \{y\}$. Assume that decision maker's choice behavior can be represented by (1). Then $y \notin C(\{x, y\})$ reveals that x eliminates y. Therefore, x must be in $\Omega(y, \{x, y\})$ and xEy. Similarly we have $xEyEzEx$ which contradicts that E is an asymmetric and negatively transitive order. Therefore, this choice behavior cannot be characterized by (1). Thus this type of cyclical choice behavior is ruled out by the representation.

To avoid any confusion, we need to clarify that yEx doesn't mean that y *always* eliminates x. yEx should be interpreted as y *may* eliminate x. That is, she eliminates x because of y if and only if y is comparable to x, i.e., $y \in \Omega(x, S)$. One might wonder

behavior. However, the questions regarding the determinants and/or source of Ω are beyond of this paper. Ω might be obtained through external aids or it is just a psychological constraint in the mind of the decision maker. See Awad *et al* (2004) for attempts to answer these questions in marketing literature.

whether an elimination order is simply a strict preference ordering. However, there might be a distinction between an elimination order and an usual preference ordering.

To inquire whether this is the case, we will use another example which can be modeled as in (1) even though it looks different from house search. Consider an individual who would like to impress his friend by his choice of wine. He always selects wine whose price is at least the median in the wine list of a restaurant. For example, there are three types of wine: Chteau Mouton (m, expensive), Jaboulet La Chapelle (j, moderate), and House wine (h, cheap). If all of them are available, then he picks either m or j, i.e., $C(\{m, j, h\}) = \{m, j\}$. If the house wine is not in the menu, he will order only m, i.e., $C(\{m, j\}) = \{m\}$. The revealed-preference approach dictates that the former reveals that m and j are indifferent. But the latter reveals that m is strictly preferred to j. So there is no preference ordering which is consistent with this choice behavior. However, in our interpretation, the latter implies that mEj. Note that mEj doesn't mean that m will eliminate j whenever m is available.[5] As long as m is not comparable to j when h is available, i.e. $m \notin \Omega(j, \{m, j, h\})$, j will be chosen from $\{m, j, h\}$. Notice that his choice behavior can be summarized as discarding wine whose price is below the median in the menu.[6]

Indeed, revealed preference suggests that if we have $x \in C(S)$ but $y \notin C(S)$ for some budget set S, then this means that x is strictly preferred to y. However, in our representation, it is possible that $\{x\} = C(\{x, y\})$ and $y \in C(T)$ but $x \notin C(T)$ for some budget set T including x. Hence it is impossible to have a preference ordering that represents such a behavior. That is why we interpret xEy as x may eliminate y (and y never eliminates x). Therefore, we call this representation as *choice by elimination*; an alternative that cannot be eliminated by any of its comparables will be chosen.

As we have seen above that our heroine on the housing market behaves perfectly

[5] mEj implies that j never eliminates m.

[6] In this example with three different type of wine, the decision maker uses price as a guide to quality of wines and follows the old adage "you shouldn't buy the cheapest wine on the menu." This reflects the idea that the cheapest wine on the menu is not drinkable.

rational as she is eliminating alternatives (there is always a good reason to eliminate). However, she might not discard as many alternatives as the classical choice theory would predict. When $\Omega(x, S)$ is a strict subset of S, x might not be eliminated even though there exists element $y \in S$ and yEx. Note that if $\Omega(x, S) = S$ for all x and S, this is nothing but the classical choice theory.

In the "wine" example above, the individual's choices are not in concert even with "Independence of Irrelevant Alternative (IIA)." That is, if one alternative is chosen from a set, S, it must also be chosen from any subset of S including x. Sen (1971) gives the following example to illustrate the axiom. If the world champion in a particular discipline is a Pakistani, he must also be a Pakistani champion (Sen, 1971). This is the very least condition which we impose on choice behavior before we call them rational. Hence, this condition has often been treated as the minimum rationality requirement. The "thin" theory of rationality of Elster (1983) is based on this assumption.[7]

Even though "wine" and the house search examples violate the normatively appealing axiom IIA, it is hard to claim that the reasoning used in those examples is irrational. We believe that there is room and need for a theory of this type of "bounded" rationality in which any elimination made by these two decision makers is perfectly rationalizable. Here we provide a theory of elimination based upon a proposal of minimal rationality conditions which we propose. This condition that we identify as "Axiom of Choice by Elimination" (ACE), resembles IIA but is weaker: for every set, S,

> there *exists* a chosen element from S which must be selected whenever it is available in any subset of S.

On the other hand, IIA requires

[7] However, May (1954), Tversky (1969), Tversky, Slovic, and Kahneman (1990), and Kahneman (2002) show that a startlingly broad range of choice behavior escapes the evaluative net of the thin theory. Especially, the evidence for intransitivity is usually observed either in a multi-dimensional context, in group decision making, or in the face of dynamic complications.

any chosen element from S must be selected whenever it is available in any subset of S.

ACE requires that consistency asked in IIA should hold *only for* some elements in $C(S)$. Even though this axiom doesn't impose as much as rationality IIA does, it still has a similar flavor to it. Here, for each choice problem, S, there is at least one "top" element which is never eliminated from S or any subset of S. Therefore, this axiom doesn't allow strict choice cycles as in Example 1. Indeed, ACE is not only a sufficient but also a necessary condition to rule out any strict choice cycles. Therefore, the decision maker whose choice behavior satisfies ACE is immune to "money pump" type of arguments that are often used against nontransitive preferences. That is why this axiom can be interpreted as "minimum requirement" for rationality. In other words, this is a minimal consistency in the sense of retaining IIA for at least one chosen element. We will prove that ACE is also the necessary and sufficient condition to have the representation as indicated by (1).

There are other examples in which a decision maker is perfectly rational, but her choice correspondence can violate the weak axiom of revealed preference (even IIA). For example, assume that a decision maker plays a non-cooperative game and discards all non-rationalizable strategies. If we can only observe her set of strategies and her choice, it is possible that her observed choice behavior violates IIA. However, almost all rationalizability satisfies ACE. Our approach is in line with Simon's "procedural rationality" or "bounded rationality." As Simon pointed out that there is a distinction between "standard" rationality (maximization paradigm) and "bounded rationality." Here, we are seeking a model in which decision makers have bounded rationality but do not exhibit strict choice cycles.[8]

Models of bounded rationality are introduced to capture the limitations of human

[8]There are some choice procedures which lie outside of our model, (1) such as the (u, v) procedure (Kalai, Rubinstein, and Spiegler (2002), the "second-best" procedure (Baigent and Gaertner (1996)) and the "median" procedure (Gaertner and Xu (1999)). The reason is that they all allow strict cycles which isn't permitted in our model.

minds. Ironically, these models are usually so complicated that people who follow those models should have essentially unlimited time and knowledge. As Todd and Gigerenzer (2000) pointed out that these models let the idea of perfect rationality to sneak in through the back door. However, the source of bounded rationality in our model may come from the use of e-commerce sites for instance. Since the set of comparables is provided by those sites, a consumer does not need to spend time and knowledge to figure out what to put in it. Unlike models referred in Todd and Gigerenzer (2000), our model exhibits bounded rationality without imposing extra cognitive load on the decision maker.

The outline of this paper is as follows: (i) we introduce the basic notations and definitions, (ii) discuss ACE and characterization of our representation, (iii) provide a characterization for IIA, (iv) cover Simon's Satisficing by introducing a new descriptive postulate, (v) elaborate on the relation between our model and preference modeling, (vi) show that our model can be interpret as endogenous formation of reference point, and finally (vii) conclude the paper. In appendix, we extend the model for cases where the domain is countable infinite, and state some of widely used consistency conditions, point out relationships between them.

2 Model

2.1 Notations

Throughout this paper X will stand for an arbitrary non-empty finite set and Θ_X will stand for all subset of $2^X \setminus \{\emptyset\}$. A binary relation, R, on a set X is a set of ordered pairs (x, y) with $x \in X$ and $y \in X$. If (x, x) belongs to R for all $x \in X$, we say that R is reflexive. A binary relation R is transitive if for all $x, y, z \in X$, xRy and yRz implies that xRz, and is complete if for all $x, y \in X$, either xRy or yRx (or both). We call any binary relation which is complete and transitive as **preference relation** and denote by $\succsim$. A binary relation, R, asymmetric if xRy implies not yRx for all $x, y \in X$ and is negatively transitive if not xRy and not yRz implies not xRz for all

$x, y, z \in X$. I_R and P_R denote the symmetric and asymmetric parts of any reflexive binary relation R, respectively. We define an elimination order as follows:

Definition 1 *A binary relation E on X is called an **elimination order** if it is asymmetric and negatively transitive.*

A choice or plan assigns a chosen set to every non-empty feasible set. This choice can be represented by a **choice correspondence** on Θ_X, $C : \Theta_X \to \Theta_X$, such that $\emptyset \neq C(S) \subset S$ for every $S \in \Theta_X$. The set $C(S)$ is called the **choice set** of the individual for the problem $S \in \Theta_X$. We tend to interpret $x \in C(S)$ as "x is revealed to be at least as good as all other alternatives in S " in the classical theory of revealed preference. Here, as we mentioned before, it is possible that the decision maker does not have any preference ordering. Sen (1993) says that "... it may be useful to interpret $C(S)$ as the set of "choosable" elements - the alternatives that can be chosen." We believe that this is the right interpretation for our paper. At the same token, $x, y \in C(S)$ doesn't necessarily imply that x is indifferent to y, it simply means that both x and y are choosable.

Finally, for elimination order E, we denote the set of E-maximal elements of S by

$$C_E(S) = \{x \in S| \nexists y \in S \ \ yEx\}. \tag{2}$$

2.2 Axiom of Choice by Elimination

In the introduction, we have provided two examples in which the choice behavior cannot be rationalizable by any binary relation. To elaborate on this, we would like to present another example from game theory. First, consider the notion of iterative eliminations of strictly dominated strategies. Suppose, there is a decision maker who decides among three strategies, say u, m, and d. However, we can only observe her choice of strategy even though she is playing a game against another player who has two only two strategies, l, r. The payoffs are given below, the first number in each box referring to DM's payoff which are also not observable to us.

	l	r
u	$4, 2$	$1, 0$
m	$1, 2$	$4, 0$
d	$3, 0$	$3, 2$

If all the strategies are available to DM, since there is no strict domination, all of them are choosable. However, if u and m are the only available strategies, r strictly dominates l for the second player in the game. Therefore, she will choose u when u and m are available, i.e. $\{u\} = C(\{u, m\})$. Her choice of m from $\{u, m, d\}$ and of only u from $\{u, m\}$ violates IIA, which is a weaker consistency requirement than WARP. Note that there is no elimination for two other pairs. Hence, the only time she eliminates something is when only u and m are available. Since u eliminates m when $S = \{u, m\}$, we should impose uEm so that $C(\{u, m\}) = C_E(\{u, m\})$. Then we have

$$\{u, m, d\} = C(\{u, m, d\}) \neq C_E(\{u, m, d\}) = \{u, d\}.$$

Therefore, it cannot be represented as in (2).[9]

Lets consider the case where IIA is violated in the above example, $C(\{u, m, d\}) = \{u, m, d\}$ and $C(\{u, m\}) = \{u\}$. Even though the normatively appealing axiom IIA is not satisfied, is there any kind of consistency within her choice behavior? Note that u is always choosable whenever it is available in any subset of $\{u, m, d\}$. Of course, having only one alternative is not enough to satisfy IIA which dictates that for all choosable alternative should be choosable from any smaller choice problem whenever they are available. We claim that if we are interested in modeling bounded rationality, IIA is too restrictive. Therefore, we offer a minimal consistency in the sense of retaining IIA for at least one choosable element.

[9]In general, for given game, $G(S_1, S_2, u_1, u_2)$, if, for any $S'_1 \subset S_1$, $G(S'_1, S_2, u_1, u_2)$ has a pure nash equilibrium, then player 1's choice correspondence induced by iterative eliminations os strictly dominated strategies satisfies ACE.

Axiom of Choice by Elimination (ACE): For any $S \in \Theta_X$, there exists an element, $x \in C(S)$ such that if $x \in T \subset S$ then $x \in C(T)$.

Even though this axiom doesn't impose as much as rationality IIA does, it still has a similar flavor IIA has. Here, for each choice problem, S, there is at least one "top" element which is never eliminated from S or any subset of S. Having such alternative does not allow strict choice cycles as in Example 1, i.e $C(\{x, y\}) = \{x\}$, $C(\{x, z\}) = \{z\}$, and $C(\{y, z\}) = \{y\}$. If we write the negation of ACE, it says that there exists some subset of X, say S, such that for all $x \in S$ there exists $T_x \subset S$, $x \notin C(T_x)$ even though $x \in T_x$. If we have such S, we can create a strict cycle by using $\{T_x\}_{x \in S}$.

In the literature, there are some arguments against having strict choice cycle. Money pump is one of the most convincing arguments. If ACE is violated then one can argue that money pump is possible by using elements of S. When the decision maker currently owns $x \in S$, being offered $T_x \subset S$ she is willing to pay a small amount of her wealth to discard x because $x \notin C(T_x)$. Whichever element $y \in C(T_x)$ she exchanges with x, she is still willing to pay a small amount to discard y when she is offered $T_y \subset S$. Given the cycle we have above, it is possible that a decision maker find herself where she was to begin with, but poorer. Of course, this can conceivably be repeated. Given sufficiently many iterations, we can extract everything the decision maker owns.

However, ACE does not allow such money pumping. To see this, assume one who tries to make money pumping from her by using elements of some subset of X, say S. ACE implies that there exists at least one top element of S, say x. Whenever x is available, the decision maker may pick x after which she is never willing to pay to switch from it because x is choosable from any subset of S. Therefore, x should not be included in any offer of money pumping. Given that, only elements of $S \setminus \{x\}$ can be used to extract money from her. However, ACE guarantees that there is a top element of $S \setminus \{x\}$, which cannot be used for money pumping for the same reason above. Then recursively, the decision maker cannot be a victim of money pumping.[10]

Before we will prove that ACE is also the necessary and sufficient condition to have the representation as (1), we would like to define the set comparables.

Definition 2 *For $x \in S \in \Theta_X$, $\Omega(x, S) \subset S$ is called the* ***comparable*** *set (of x under choice problem S).*

Theorem 1 *A choice correspondence, C on Θ_X satisfies ACE if, and only if, there exists an elimination order, E on X and comparable sets Ω such that*

$$C(S) = \{x \in S | \nexists \ y \in \Omega(x, S) \text{ such that } yEx\}.$$

The representation is consistent with the house search example discussed in the introduction. The decision maker discards x from choice problem S if and only if she finds another available element y, from the comparable set of x, which eliminates x. Otherwise, x is in the choice set, or is *choosable*.

Our frame work suggests that decision maker uses two-stage processes to reach her decision. Consider the case where $x \in C(S)$. First $\Omega(x, S)$ has been constructed. Since she might have used the help of an external (or online) decision aid, this stage is almost costless for her. At the second stage, she has performed detailed binary comparisons between the alternative under consideration and alternatives which are comparable to it. Since $x \in C(S)$, there was no alternative which dominates x with respect to the context-free elimination order, E.

Note that the bigger the set $\Omega(x, S)$ is, the more likely the elimination happens. However, elimination takes place only in the second stage. First stage makes harder to discard an alternative since it restricts the comparable set.

[10] Here, it is allowed to money pump for finite round. The decision maker may choose something other than the top element of S when the cardinality of choice set is bigger than one. However if the decision maker never chooses the top element, this is observatively to equivalent to the violation of ACE, i.e., there is no top element.

We can also interpret this representation as an equilibrium argument or an ex-post reasoning. If x is discarded from the budget set S, then she can rationalize her decision by claiming that she found an alternative that eliminates x within the set of comparables to x. In other words, if there exists $y \in \Omega(x, S)$ with yEx, then, she is aware of a dominating alternative so she will never stay with x. Hence not choosing x is perfectly rational. On the other hand, she keeps x if there is no dominating alternative which is comparable to x.

As we mentioned before, E may not be a preference ordering. That is why we cannot concentrate only on the binary comparisons for the construction of E. Assume the decision maker chooses only x from $\{x, y, z\}$ and everything from any strict subset of $\{x, y, z\}$. Then focusing on binary comparisons results in no elimination relationships at all which cannot explain the choice of singleton x from $\{x, y, z\}$. Therefore, binary comparison does not work for our representation.

However ACE guarantees the existence of "top" element in any choice problem, particularly X. Since top element(s) is never eliminated, it is natural to put them on the top of the elimination order. Then in the next step, we will extract all the top elements of X from X. Now consider the top element(s) of this strictly smaller set. These second top elements may be eliminated only by the top element of X. This idea is formally illustrated next. We recursively define E starting from the grand set, X and construct a partition of X. Define $X = X_0$ and

$$I_0 = \{x \in X_0 |\ x \in C(T) \text{ for all } T \text{ s.t. } x \in T \subset X_0\}.$$

Note that I_0 is the set of top elements of X_0. ACE implies that I_0 is non-empty. Define $X_1 = X_0 \setminus I_0$. If it is non-empty, define

$$I_1 = \{x \in X_1 |\ x \in C(T) \text{ for all } T \text{ s.t. } x \in T \subset X_1\}.$$

ACE also implies that I_1 is non-empty. Then define recursively,

$$I_k = \{x \in X_k |\ x \in C(T) \text{ for all } T \text{ s.t. } x \in T \subset X_k\}$$

where $X_k = X_{k-1} \setminus I_{k-1}$ until $\cup I_k = X$. Note that $\{I_n\}$ is a partition of X, and X_n's are nested. Given the partition $\{I_n\}$, we can define E

$$xEy \text{ if } x \in I_k,\ y \in I_n \ \text{ and } k < n.$$

Since X is finite, this recursive process will end in finite time.[11] Given the construction, E is asymmetric and negatively transitive.

Proof of Theorem 1: Let E and Ω be as introduced in (1). To show that C then satisfies ACE, take any $S \in \Theta_X$. Since S is finite and E is an elimination order, there exists x such that there is no yEx in S. Therefore, for any $T \subset S$ with $x \in T$, $x \in C(T)$. Therefore, C satisfies ACE.

Now, we show that any C satisfying ACE has a represenation in (1). We will use the elimination order constructed as above.

Claim: If $y \notin C(S)$ then $\exists\ x \in S$ such that xEy.

Proof: Suppose $y \in X_k$. Then, by construction, $y \in C(T)$ for any $T \subset X_k$ with $y \in T$. Therefore, there exists $x \in S$ such that $x \notin X_k$. By construction, xEy.

Now, we define also the comparable sets as follows:

$$\Omega(x,S) = \begin{cases} S & \text{if } x \notin C(S) \\ X_k \cap S & \text{if } x \in C(S) \end{cases}$$

where $x \in I_k$. Then when $x \notin C(S)$, by the previous claim, $\Omega(x,S)(= S)$ contains some element yEx. When $x \in C(S)$, $\Omega(x,S)$ does not include any yEx by construction. Therefore, the pair of (E,Ω) represents C.

2.3 Independence of Irrelevant Alternatives

The following example from game theory satisfies IIA. Again, the decision maker is playing a game with another player but we can observe only her set of strategies and

[11] We will discuss for infinite case later.

choices. Here, she discards only dominated strategies (without iterative eliminations). Consider the following game (player 2's payoffs are omitted):

	l	r
u	10	0
m	0	10
d	4	4

Then, she eliminates no strategy except when all of u, m, d are available. In this case, d is discarded because it is strictly dominated by $(1/2)u + (1/2)m$. It is routine to verify IIA for this example. Indeed, this procedure satisfies IIA for any given game.

Since ACE is weaker than Independence of Irrelevant Alternatives (IIA), one may wonder how the characterization provided in Theorem 1 changes when IIA is assumed. The following theorem provides a characterization for IIA.[12]

Theorem 2 *A choice correspondence, C on Θ_X satisfies IIA if, and only if, there exists an elimination order, E on X and comparable sets Ω such that*

$$C(S) = \{x \in S | \nexists\ y \in \Omega(x, S) \text{ such that } yEx\}.$$

and if $T \subset S$ then $\Omega(x, T) \subset \Omega(x, S)$.

Remember the house choice problem. If the website, given a fixed criteria (i.e. house x), provides more houses when her budget increases, then her choice behavior satisfies IIA. Then theorem B implies it because the comparable set created by the website satisfies the monotonicity condition: $\Omega(x, T) \subset \Omega(x, S)$ when $T \subset S$.

Proof of Theorem 2: Suppose C has a representation given in the theorem. If $x \notin C(T)$ for some $T \ni x$, then there exists yEx within $\Omega(x, T)$. Therefore for any $S \supset T$, $y \in \Omega(x, S)$ so $x \notin C(S)$, which proves IIA.

[12]For another characterization see Ok (2004).

Now suppose C satisfies IIA. Let us construct E and Ω exactly in the same manner as in Theorem 1. Since IIA is stronger than ACE, this pair represents C by Theorem 1. Thus, we only need to show the monotonicity of Ω in S.

Take any $x \in T \subset S$. If $x \in C(S)$, then IIA implies $x \in C(T)$ so $\Omega(x,T) = X_k \cap T \subset X_k \cap S = \Omega(x,S)$. If $x \notin C(S)$, then by the construction of Ω, $\Omega(x,S) = S \supset T \supset \Omega(x,T)$.□

3 Choice by Satisfaction

> "...(D)ecision makers can satisfice either by finding optimum solutions for a simplified world, or by finding satisfactory solutions for a more realistic world." Simon's Nobel Prize Lecture, (1978)[13]

The idea of satisficing is to choose alternatives which exceeds some level of satisfaction, which might depend on the choice problem under consideration. This can be interpreted as elimination by dissatisfaction. This leads us the following representation where the comparable set is independent of the alternative under consideration.

$$C(S) = \{x \in S | \nexists\, y \in \bar{\Omega}(S) \text{ such that } yEx\}$$

where $\bar{\Omega}(S)$ is the comparable set, which is a subset of S. One of E-best elements, say x_S, in $\bar{\Omega}(S)$ can be considered as the minimum level of satisficing level for choice problem S. Any alternative strictly below x_S with respect to E will be eliminated. Here, given our interpretation, E can be called satisfaction order. The following axiom is the necessary and sufficient for the representation.

> Axiom of Choice by Satisfaction (ACS) : For any $S \in \Theta_X$, there exists $x \in C(S)$ such that for all $y \in S$ and $T \ni x$, $y \in C(T)$ implies $x \in C(T)$.

[13]Simon (1955) coined the term "satisficing" which is a combination of "satisfy" and "suffice." It means that a decision maker will take an alternative, which is "good" enough, instead of searching for the best element.

Notice that ACS imposes that for each S there exists at least one element, say x, which serves as a *most frequently satisficing* element. x is still most frequently satisficing one in any smaller choice problem, so it must be chosen. Actually, any top element also satisfies this property, i.e., most frequently satisficing element is always a top element. However, unlike top elements, most frequently satisficing elements have one more property. That is, if x is discarded some other choice problem (must not be a subset of S), any other elements in S must be also eliminated from the choice set because they are less satisficing than x. Therefore, ACS implies ACE. However, being top element does not imply being most frequently satisficing because of the additional requirement. ACS requires the existence of such element in any choice problem. Therefore, ACS is stronger axiom than ACE.[14]

The "wine" example discussed in the introduction satisfies ACS. Any choice behavior choosing any alternative weakly above median with respect to the elimination order (average, if alternatives have numerical scores) for instance also meets the requirement of ACS.

One of examples satisfying ACE but not ACS is a choice by iterative eliminations of strictly dominated strategies discussed in the section of ACE.

Another example, which is similar to the wine example is the following: There are three red wines r_1, r_2, r_3 and three white wine w_1, w_2, w_3 where r_1 (w_1) is the most expensive red (white) wine and r_3 (w_3) is the cheapest red (white) wine. The decision maker discards the cheapest red wine and white wine. It is easy to see that this choice behavior satisfies ACE. However, it violates ACS. To illustrate this, consider $S = \{r_2, w_2\}$. Neither of them satisfies the condition in ACS because

$$C(\{r_1, r_2, w_2\}) = \{r_1, w_2\} \text{ but } C(\{r_2, w_1, w_2\}) = \{r_2, w_1\}.$$

Before we provide the proof of the theorem, we would like to point out that the construction we used for the elimination order does not work in general. To illustrate

[14] ACE can be thought as a contraction axiom as IIA but ACS is not only contraction but also expansion.

that assume $X := \{x, y, z\}$ and

$$C(\{x,y,z\}) = \{x,z\},\ C(\{x,y\}) = \{x\},\ C(\{y,z\}) = \{y,z\},\ C(\{x,z\}) = \{x\}.$$

If we follow top element construction, we have xEy and xEz. This means that neither y eliminates x nor x eliminates y. Hence, given the satisfaction level, either both of them are satificising or none of them. Therefore, the choice of $C(\{x,y,z\}) = \{x,z\}$ cannot be explained by satisfaction with E. However, if we assume $xEzEy$ (instead of xEy and xEz) then the choice behavior is in line with the satisfaction story.

ACS hints that E can be constructed through binary comparisons, namely

$$xEy \ \text{ iff } \ y \in C(S) \Rightarrow x \in C(S).$$

Unfortunately, this does not work either.[15] Instead, we construct E here based on *most frequently satisficing element* as follows:

Define $X = Y_0$ and

$$J_0 = \{x \in Y_0 | \forall\ y \in Y_0 \text{ and } \forall\ T \ni \{x,y\},\ y \in C(S) \Rightarrow x \in C(S)\}.$$

Here J_0 is the set of most frequently satisficing elements of Y_0. ACS implies that I_0 is non-empty. Define $Y_1 = Y_0 \setminus J_0$. If it is non-empty, define

$$J_1 = \{x \in Y_1 | \forall\ y \in Y_1 \text{ and } \forall\ T \ni \{x,y\},\ y \in C(S) \Rightarrow x \in C(S)\}.$$

ACS also implies that J_1 is non-empty. Then define recursively,

$$J_k = \{x \in Y_k | \forall\ y \in Y_k \text{ and } \forall\ T \ni \{x,y\},\ y \in C(S) \Rightarrow x \in C(S)\}$$

where $Y_k = Y_{k-1} \setminus J_{k-1}$ until $\bigcup J_k = X$. Note that $\{J_n\}$ is a partition of X, and Y_n's are nested. Given the partition $\{J_n\}$, we can define E,

$$xEy \text{ if } x \in J_k,\ y \in J_n \ \text{ and } k < n.$$

[15] Here is the example: $C(\{\alpha,x,y,z\}) = \{\alpha\}$, $C(\{\alpha,x,y\}) = \{\alpha,x,y\}$, $C(\{\alpha,y,z\}) = \{\alpha,y\}$, $C(\{\alpha,x,z\}) = \{\alpha,z\}$, $C(\{x,y,z\}) = \{x,y,z\}$, $C(\{x,y\}) = \{x,y\}$, $C(\{y,z\}) = \{y\}$, $C(\{x,z\}) = \{x,z\}$.

Since X is finite, this recursive process will end in finite time. Given the construction, E is asymmetric and negatively transitive.

Theorem 3 *A choice correspondence, C on Θ_X satisfies ACS if, and only if, there exists an elimination order, E on X and comparable sets $\bar{\Omega}$, which is independent of x such that*

$$C(S) = \{x \in S | \nexists\ y \in \bar{\Omega}(S) \text{ such that } yEx\}.$$

Proof of Theorem 3: The proof of the "if" part is left to readers. Now suppose C satisfies ACS. The elimination order is constructed by using the most frequently satisficing elements as described above, so we only need to define $\Omega(S)$.

For each S, define

$$k^S = \max\{k \mid C(S) \cap J_k \neq \emptyset\}$$

and

$$\bar{\Omega}(S) = C(S) \cap J_{k^S}.$$

Now we show that the pair of E and $\bar{\Omega}$ represents C. Take any $x \in J_k$. If $x \in C(S)$ then by construction of k^S, $k \leq k^S$ so there is no $y \in \bar{\Omega}(S)$ which eliminates x.

Suppose $x \notin C(S)$. By definition of J_k, x is one of most frequently satisficing elements of Y_k. Therefore, for any $k' \geq k$ and any $y \in J_{k'}$, CAS requires $y \notin C(S)$. Since C is always non-empty, there must exists $k'' < k$ such that $z \in J_{k''}$ and $z \in C(S)$. Therefore, we conclude that $k^S < k$ so $\bar{\Omega}(S)$ has an element $a \in S$ such that aEx.□

4 Context-free Comparable Set and Preferences

In the above section, we provide two characterization theorems in which the comparable sets depend on both the alternative under consideration and the budget set.

In other words, the comparable set is context dependent, $\Omega(x, S)$. In this section, we will investigate under what conditions the comparable set is context free, i.e. $\Omega(x, S) = \Omega^*(x) \cap S$.

The classical choice theory assumes that a decision maker behaves in an internally consistent way: if the agent is willing to choose x in some budget set in which x and y are offered, then, in any other budget set also containing x and y, if the agent is willing to choose y, he must also be willing to choose x. This is so-called "Weak Axiom of Revealed Preference (WARP)." This axiom guarantees that choice behavior can be characterized by maximization of a well defined preference ordering. Let us state the contrapositive of WARP here, which will make it easy to compare WARP with other axioms discussed later.

> Weak Axiom of Revealed Preference (WARP): For any $S \in \Theta_X$ with $y \in S$, if $y \notin C(S)$, then for any $x \in C(S)$, $x \in T$ implies $y \notin C(T)$.

It is well known that WARP is the necessary and sufficient condition for the decision maker to maximize a preference relation, which is complete and transitive.

Obviously, WARP is a special case of ACE, which leads a representation in (1) with $\Omega(x, S) = S$ or $\Omega^*(x) = X$.

WARP suggests that whenever y is discarded from S, the availability of *any* chosen elements from S eliminates y. This means any chosen element from S eliminates y from any choice problem not limited to S. To see how strong WARP actually is, consider the following example: Let $x = (3, 1)$, $y = (2, 0)$, and $z = (1, 3)$ and

$$C(\{x, y, z\}) = \{x, z\},\ C(\{x, y\}) = \{x\},\ C(\{y, z\}) = \{y, z\},\ C(\{x, z\}) = \{x, z\}$$

Note that she discards y whenever x is available since x dominates y in both components. However, there is no direct comparison between y and z. Although she discards y and picks z from $\{x, y, z\}$, the availability of z does not eliminate y. Therefore, her choice behavior violates WARP even though her reasoning makes sense.

In this example, *only* one of the chosen element from $\{x, y, z\}$, which is x, eliminates y. This suggest the following axiom, which is weaker than WARP:

> WEAK AXIOM OF REVEALED NON-INFERIORITY (WARNI) : For any $S \in \Theta_X$ with $y \in S$, if $y \notin C(S)$, then there exists $x \in C(S)$, $x \in T$ implies $y \notin C(T)$.

Bandyopadhyay and Sengupta (1991) showed that WARNI[16] is equivalent to maximization of an *incomplete* but transitive preference.[17]

WARNI claims that whenever y is eliminated from S, at least one of chosen element always eliminates y. However, the following example suggests that y can be eliminated only by an unchosen element.

$$C(\{x, y, z\}) = \{x\},\ C(\{x, y\}) = \{x, y\},\ C(\{y, z\}) = \{z\},\ C(\{x, z\}) = \{x\}$$

Although y is eliminated from $\{x, y, z\}$, the unique chosen element x does not eliminate y in the smaller set. but the other unchosen element z eliminates y whenever both of them are available. Such a choice behavior sounds awkward but is generated by the following story. "There are three houses located in the order of x, z, y. House x is the best and y is the worst. The decision maker compares only houses next to the one under consideration (possibly because the real estate broker shows only the immediate neighborhoods). She discards a house if there is a affordable better house in its neighborhood." The following axiom accommodates the choice behavior above.

> STRONG AXIOM OF CHOICE BY ELIMINATION (SACE) : For any $S \in \Theta_X$ with $y \in S$, if $y \notin C(S)$, then there exists $x \in S$, $x \in T$ implies $y \notin C(T)$.

[16] It is named by Eliaz and OK (2006).

[17] The incomplete preference is first axiomatized by Sen(1971) by using three different conditions. Jamison and Lau (1973), Plott (1973), Fishburn (1975) and Schwartz (1976) propose different set of axioms to characterize the imcomplete preference. By relaxing finiteness assumption, Eliaz and Ok (2006) is made the theorem applicable to many choice situations.

Bandyopadhyay and Sengupta (1991) also showed that SACE is equivalent to maximization of an *acyclical* preference, possibly intransitive.[18]

In the above story, the decision maker is aware of next houses to the one she is contemplating regardless of her budget set (even though they are not affordable). This suggests the comparable set is independent of her budget. The following theorem makes this reasoning clearer.

Theorem 4 *A choice correspondence, C on Θ_X satisfies SACE if, and only if, there exists an elimination order, E on X and comparable sets Ω^*, which is independent of S such that*

$$C(S) = \{x \in S |\ \nexists\ y \in \Omega^*(x) \cap S \textit{ such that } yEx\}.$$

With WARNI, which is stronger than SACE, we provide the characterization as in Theorem 4 with the condition: if $x \in \Omega^*(y)$ then $\Omega^*(x) \subset \Omega^*(y)$.

Theorem 5 *A choice correspondence, C on Θ_X satisfies WARNI if, and only if, there exists an elimination order, E on X and comparable sets Ω^*, which is independent of S such that*

$$C(S) = \{x \in S |\ \nexists\ y \in \Omega^*(x) \cap S \textit{ such that } yEx\}.$$

Furthermore, if $x \in \Omega^*(y)$ then $\Omega^*(x) \subset \Omega^*(y)$.

Proofs of Theorem 4 and 5: The proofs of the "if" parts are left to reader. Now suppose C satisfies SACE. The elimination order is constructed as before. Define $\Omega^*(x)$ as follows:

$$\Omega^*(x) = \{y \in X |\ x \notin C(\{x, y\})\}.$$

Suppose if $x \in C(S)$ then we argue that SACE implies that $\Omega^*(x) \cap S = \emptyset$. To see this, if there exists $y \in \Omega^*(x) \cap S$, then $x \notin C(\{x, y\})$. This contradicts to IIA which is a weaker condition than SACE.

[18]The acyclical preference is also axiomatized by Sen(1971), Jamison and Lau (1973), and Schwartz (1976) with different sets of axioms.

If $x \notin C(S)$ then SACE implies that $\exists y \in S$ such that $x \notin C(\{x, y\})$. Hence $y \in \Omega^*(x) \cap S$. Now we need to show yEx. Suppose that $y \in I_k$. By SACE, $x \notin C(T)$ if $y \in T$. Therefore $x \notin C(X_{k'})$ for any $k' \leq k$. By construction of E, yEx which completes the proof of Theorem 4.

Since WARNI is stronger than SACE, we only need to verify the condition, $y \in \Omega(x)$ implies $\Omega(y) \subset \Omega(x)$, to prove Theorem 5. If $y \in \Omega(x)$ then by definition $\{y\} = C(\{x, y\})$. Assume that there exists an element $z \in X$ such that $z \in \Omega(y) \backslash \Omega(x)$ which implies $x \in C(\{x, z\})$ and $\{z\} = C(\{y, z\})$. Consider the set $\{x, y, z\}$. Since $x \notin C(\{x, y\})$ and $y \notin C(\{y, z\})$, IIA implies $C(\{x, y, z\}) = \{z\}$. This contradicts the fact that c satisfies WARNI since $x \in c(\{x, z\})$ and $C(\{x, y, z\}) = \{z\}$. $\square$

5 Endogenous Reference-Dependent Choice

In this section, we illustrate that our model can be interpreted as endogenous formation of reference points. To do this, we discuss two reference-dependent models which are provided in Masatlioglu and Ok (2005) and Kösezgi and Rabin (2006).

Kösezgi and Rabin (2006) propose a model of reference-dependent in which reference point is determined endogenously. To do this, they define a personal equilibrium as follows:

$$PE_U(S) = \{x \in S | \; U(x|x) \geq U(y|x) \;\; \text{for all } y \in S\}$$

where $U(x|y)$ is the reference-dependent utility, that is, the utility of x when the reference point is y. In this model, x will be discarded if there is another alternative within the budget set which has a higher utility than x when x itself is the reference point. In other words, if the a decision maker expects to choose an alternative, say x, (x becomes her reference point) and she is willing to choose x given that x is her reference point, then indeed x will be chosen. It basically means that x is a self-fulfilling plan. Therefore, $PE_U(S)$ is the set of self-fulfilling plans.

To make the connection with our model, assume that $x \notin PE_U(S)$.[19] It must be

[19]Under certain condition, the non-emptiness could be guaranteed.

the case that there exists an alternative y such that $U(y|x) > U(x|x)$. Then it is clear that the presence of y *always* eliminates x. Therefore, PE_U satisfies SACE.[20] Since it satisfies SACE, it can be written as in Theorem 4, that is,

$$PE_U(S) = \{x \in S | \nexists\ y \in \Omega^*(x) \cap S \text{ such that } yEx\}.$$

where $\Omega^*(x) = \{y \in X | U(y|x) > U(x|x)\}$ and E is constructed as in Theorem 1.

Another closely related reference-dependent model is provided in Masatlioglu and Ok (2005). Their main concern is to investigate the notion of status quo bias when status quo point is observable. In other words, unlike Kösezgi and Rabin (2006), the reference point is exogenously given. In this model, for each status quo, in the first stage decision maker eliminates alternatives which does not dominate the status quo according to a partial order, and she uses a complete preference ordering to finalize her decision at the second stage. Their model can be summarized as follows:

$$C_{MO}(S,x) = \{y \in \Omega^*(x) \cap S | \nexists\ z \in \Omega^*(x) \cap S \text{ such that } zEy\}^{21}$$

where if $x \in \Omega^*(y)$ then $\Omega^*(x) \subset \Omega^*(y)$. In this model, $\Omega^*(x)$ is interpreted as the psychological constraint imposed by the status quo point x and E is the material preference or preference without status quo.[22]

There are two main important differences between our model and the model of Masatlioglu and Ok (2005). First, unlike our model, the status quo point is exogenously given in their model, i.e., $C_{MO}(S,x)$. Indeed, our model can be also interpreted as the endogenous formation of reference point in their setup. As in Kösezgi and Rabin (2006), if the a decision maker expects to choose an alternative, say x, (x becomes her status quo) and she is willing to choose x given that x is her status quo, then indeed x will be one of the "choosable" alternatives in our model;

$$x \in C_{MO}(S,x) \text{ if, and only if, } x \in C_{(E,\Omega)}(S).$$

[20] See Proposition 1 in Gul and Pesendorfer (2006) for the formal proof.

[21] C_{MO} denotes the choice correspondence defined in Masatlioglu and Ok (2005).

[22] The same authors also provide another model of reference-dependent choice in which the first stage involves several rounds (Masatlioglu and Ok (2006)).

It is possible that if she expects to choose x from the choice problem S, she chooses x and, if she expects to choose y from the choice problem S, she chooses y. In other words, both x and y are choosable depending on the decision maker expectations. This formulation is in line with the personal equilibrium of Kosezgi and Rabin (2006). If you expect to choose an element and actually choose it, your actual choice fulfills rational expectation, and the set of all choosable elements consists of such elements. Note that Theorem 5 implies that the endogenous reference-dependent choice in the sense of Masatlioglu and Ok (2005) satisfies WARNI.

While our model can capture endogenous formation of reference point in their setup, it allows more general framework in which the phycological constraint depends not only status quo x but also the choice problem. To see this, we need define a general model of reference-dependent choice by

$$C_{RDC}(S,x) = \{y \in \Omega(x,S) | \nexists\, z \in \Omega(x,S) \text{ such that } zEy\}.$$

Here the psychological constraint depends on both the reference point and the budget set. Then the endogenous reference-dependent choice is defined by

$$x \in C_{(E,\Omega)}(S) \text{ if, and only if, } x \in C_{RDC}(S,x).$$

Therefore, if a choice correspondence satisfies ACE, it can be interpret as the endogenous reference-dependent choices: any element of $C_{(E,\Omega)}(S)$ is a self-fulfilling plan with S.

6 Concluding Remarks

Motivated by real life decision problem, we provide a descriptive model of choice by elimination in which decision procedure involves elimination rather than selecting an alternative. In this model, bounded rationality is captured by idea that an alternative under consideration might not be compared by all available alternatives in the budget set. The comparison takes place in a subset of the budget due to, for example, the complexity of the problem, the limitation an time and cognitive ability, or the usage of some particular e-commerce site. Of course, this decreases the cognitive load on the decision maker. While the elimination is based on a context free ordering, the comparison set might depends on the budget set and the alternative under consideration.

On the normative ground, we provide a necessary and sufficient condition for the model. The "axiom of choice by elimination" (ACE) assumes that the decision maker has at least one alternative for each budget set which is always choosable whenever it is available from a subset of the original budget set. As in the representation, this reflects the idea of bounded rationality. This condition is also normatively appealing since it is immune to money pump type of arguments despite the fact that it is weaker than the independent of irrelevant alternative (IIA).

The richness of our framework allows to provide a characterization for IIA which is impossible to represented by any binary relation. Moreover, our model delivers a unifying structure for existing different kind of preference representations. Finally, we present a model for Simon's satisficing in which the decision maker picking an alternative if it is above a certain level depending on the budget set. This model lies also outside of preference modeling.

We would like to end by discussing the model of Manzini and Mariotti (2007). They have proposed a model in which the decision maker uses two rationales (possibly more) to make decision. In the first step, she shortlists the non-dominated alternatives using the first rationale. In the second step, she considers only this shortlist and select the non-dominated alternatives using a second rationale. Unlike our model, their model allows strict choice cycles even when the choice correspondence is single valued.[23] This is because of the interaction of two rationales. However, our model is not a special case of theirs. For example, the following choice behavior,

$$C(\{x,y,z\}) = \{x,y,z\},\ C(\{x,y\}) = \{x\},\ C(\{y,z\}) = \{y\},\ C(\{x,z\}) = \{x\},$$

satisfies ACE. However, no matter how many rationales we use, it cannot be captured

[23]Note that if we work with choice function instead of choice correspondence, our weakest axiom, ACE, is equivalent to the standard preference maximization.

by the model of Manzini and Mariotti (2007). Therefore, neither of these two models imply each other.

On the other hand, one can wonder whether the model of Choice by Satisfaction is a special case of their model if we use the first rationale to construct $\bar{\Omega}(S)$ for each choice problem. Since the above example also satisfies ACS, there is no implication between these models. Even though our model shares a similar procedural feature, they are completely different. The reason is that the first stage is used as a filtration (or another level of elimination) in their model. However, the first stage of our model just describes the set of comparables without eliminating any alternatives.

7 Appendix

7.1 Countable Domain

Our construction of the elimination order E heavily depends on the finiteness assumption of the domain, X. Here, we extend our results to the case where X is countable.

In this section, we assume that X is a countably infinite set and a choice correspondence C is defined over the collection of all finite subsets of X, which is denoted by Θ_X.

Theorem 6 *Theorem 1-5 hold when X is a countably infinite set and Ω_X is a collection of all finite subsets of X.*

Proof of Theorem 6: Let

$$X = \{x_1, x_2, x_3, \ldots\}$$

For each

$$X_n = \{x_1, x_2, \ldots, x_n\}$$

define an elimination order over X_n, denoted by E_n as we did for the case of finitely many elements and let

$$u_n(x_m) = \sum_{i \leq n \text{ and } x_m E_n x_i} \left(\frac{1}{2}\right)^i$$

for $m \leq n$.

Since $u_n(x_m) \in [0,1]$, for each x_m, we can define

$$u_n(x_m) = \limsup_{n \to \infty} u_n(x_m)$$

and define E, which is an asymmetric and negatively transitive order on X as

$$x_m E x'_m \text{if and only if } u(x_m) > u(x'_m)$$

Then, we argue that whenever $x_i \notin C(S)$, there exists an element within S which eliminates x_i. That is, $x^* E x_i$ for some $x^* \in S$. To see this, consider large n so that all elements in S are included in X_n. Since E_n is the elimination order which represents C (with an appropriate Ω function) with X_n being a grand set, there exists $x \in S$ such that $x E_n x_i$. Notice that x may depend on n.

Therefore, for each n, we have

$$\max_{x \in S} u_n(x) \geq u_n(x_i) + \frac{1}{2^i}$$

Since S is a finite set, we have

$$\max_{x \in S} \limsup_{n \to \infty} u_n(x) \geq \limsup_{n \to \infty} u_n(x)$$

Given E, we can construct Ω exactly in the same manner as when X is finite.

7.2 Minimal Consistency

A consistency condition is a postulate which is imposed on choice correspondences to rule out some type of irrational choice behavior (at least it seems irrational). In the literature, some of these consistency conditions are perceived weaker then others, namely IIA, Sugden's Minimal Consistency, Fishburn's A5, and Sen's $\alpha(-)$. We will discuss relationship between ACE and those. In a nutshell, the common feature of consistency conditions which are weaker then our axiom is to allow strict cyclical choice behavior. We will elaborate on this. We first start describing Sugden's Minimal consistency axiom.

Sugden's Minimal consistency: If $\{x\} = C(\{x, y\})$ then for all feasible sets $S = \{x, y, \dots\}$, $\{y\} \neq C(S)$.

This axiom argues that if one chooses uniquely x from a set limited to x and y, y should not be equal to her choice set from a larger set including x, y and some other elements. Sugden (1985) illustrates his point with the case of regret in which we observe cyclical choice behavior as in example 1. Therefore, This axiom doesn't imply ACE. On other hand, ACE implies this axiom.

Sen's $\alpha(-)$: For each S, for some $x \in c(S)$, x is in $C(\{x, y\})$ for all $y \in S$.

In words, every chosen set $C(S)$ must contain at least one element x such that $x \in C(\{x, y\})$ is true for all $y \in S$. It is routine to verify that ACE implies Sen's $\alpha(-)$, but not the other way around. The latter can be seen in the example: $X = \{x, y, z, t\}$ and $C(S) = S$ except $\{x\} = C(\{x, y, z, t\})$ and $\{y\} = C(\{x, y, z\})$, which satisfies $\alpha(-)$ but not ACE. Note that for this example, the money pump argument can be implemented by offering $\{y, z\}$ when the decision maker has x and $\{x, z, t\}$ when she owns y. Also note that Sugden's minimal consistency is weaker than IIA, Sen's $\alpha(-)$, and ACE.

Fishburn's (A5): If S contains more than two alternatives and x is in $C(S)$, then x is in $C(\{x, y\})$ for some $y \neq x$ in S.

Note that neither does $\alpha(-)$ imply Fishburn's A5, nor does the latter imply $\alpha(-)$. The former is seen by considering the example: $\{x\} = C(\{x, y\})$, $\{y\} = C(\{y, z\})$, $\{x\} = C(\{x, z\})$, and $\{x, z\} = C(\{x, y, z\})$ which satisfies $\alpha(-)$ but not Fishburn's A5. The latter is seen in the example: $\{x\} = C(\{x, y\})$, $\{y\} =$

$C(\{y,z\})$, $\{z\} = C(\{x,z\})$, and $\{x\} = C(\{x,y,z\})$, which satisfies Fishburn's A5 but not $\alpha(-)$. These examples also suggest that neither does ACE imply Fishburn's A5, nor does the latter imply $\alpha(-)$.

Independence of Irrelevant Alternatives (IIA): For any $S \in \Theta_X$ and any $x \in C(S)$, if $x \in T \subset S$ then $x \in C(T)$.

If x is chosen from a set containing only x and y, then introducing a third alternative z, thus expanding the choice set to $\{x,y,z\}$, must not make y a chosen element. In other words, whether x is going to be chosen should not depend on the availability an alternative which is irrelevant.

Plott's Path Independence: For all S, T, $C(C(T) \cup C(S)) = C(T \cup S)$.

For the best description of this axiom, we refer to Plott (1973):

> "The alternatives are split up into smaller sets, a choice is made over each of these sets, the chosen elements are collected, and then a choice is made from them. Path independence, in this case, would mean that the final result would be independent of the way the alternatives were initially divided up for consideration"

It is easily checked that Plott's path independence implies IIA. Indeed, Parks (1974) shows that IIA is equivalent to a part of path independence, $C(T \cup S) \subset C(C(T) \cup C(S))$.

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