A Theory of Man that Creates the World*

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April 13, 2007

Abstract

The present paper proposes a theory of man that tries to construct a model of the world in societal situations where people interact with each other. The present theory takes experiences, or chunks of impressions, as primitives as opposed to an "objective" game. A model is something that is constructed by an agent. Each model consists of structural and factual parts. The structural part is represented as a stochastic game. While the factual part is represented as a strategy profile of the game. In constructing a model, the agent uses some axioms. Examples of the axioms that the agent might use are coherence, according to which he can explain his own experiences, consistency with a solution concept adopted by the agent, and simplicity with respect to some measure, again adopted by him. The present paper does not assume the existence of an "objective" game, and different agents may construct different models of the world.

We are what we think.

All that we are arises with our thoughts.

With our thoughts we make the world.

"The Dhammapada: the saying of the Buddha"

1 Introduction

For more than a century, what used to be a common belief that human behavior is based on intelligence, while animals' behavior on instinct, has been challenged (see, e.g., Thorndike (1911/2000) for some earlier works). Many "intelligent" activities, especially those analyzed in Simon (1957), are now known to be shared not only by primates but also by a variety of animals. Many birds and mamals are known to use their intelligence to try to behave satisfactorily, if not optimally, in various situations. They too learn how to hunt, fly, and breed children. For example, it is commonly observed that birds

*Preliminary. Comments welcome. The author is indebted to Takashi Shimizu on extensive discussions on this project.

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bred by humans can neither fly nor breed their children by themselves.[1] Moreover, some bird was claimed to demonstrate numerical competence.[2] Needless to say, men, on the other hand, use their instinct like other animals to avoid danger and react to certain stimuli, of which tendencies have been extensively studied in behavioral economics (see, e.g., Camerer).

Still, homo sapiens is distinguished from other species in the way intelligence is used. One of the intellectual activities we often observe in human being, but not in other animals, is to construct a model of the world that explains their experiences.[3] The purpose of this paper is to propose a formal framework to study such an activity of *homo sapiens* in societal situations where people interact with each other.

Unlike the standard theory in economics, the present theory takes experiences, or chunks of impressions, as primitives as opposed to an "objective" game. A model is something that is constructed by an agent. Each model consists of structural and factual parts. The structural part is represented as a stochastic game. While the factual part is represented as a strategy profile of the game. In constructing a model, the agent uses some axioms. Examples of the axioms that agents might use are coherence, according to which an agent can explain his own experiences, consistency with a solution concept adopted by the agent, and simplicity with respect to some measure, again adopted by him. The present paper does not assume the existence of an "objective" game, and different agents may construct different models of the world.

Four applications are presented to show the basic working of the theory. The first application is concerning entry and predation. The failure of Air Do, an airline company, illustrates a working of the theory. The second one is on bullying. Through the activity of bullying in school, children may construct a specific way of viewing the situation. The third application is a simple repeated interaction between two agents, say, a wife and a husband. Two might view the situation quite differently after, say, a defection of the wife, and the difference may be the source of the impossibility of renegotiation. The fourth application is concerning the importance of pioneers.

The idea of the construction of models by agents based on experiences was initiated

[1] There are numerous reports on the difficulty of animals' returning to the wild. Many programs are designed to teach animals various skills to survive in the wild. See, e.g., Hendron (2000).

[2] For example, Pepperberg (1994) reported that an African gray parrot (Psittacus erithacus), Alex, trained to label vocally collections of 1-6 simultaneously presented homogeneous objects, correctly identified, without further training, quantities of targeted subsets in heterogeneous collections. For each test trial Alex was shown different collections of 4 groups of items that varied in 2 colors and 2 object categories (e.g., blue and red keys and trucks) and was asked to label the number of items uniquely defined by the conjunction of 1 color and 1 object category (e.g., "How many blue keys?"). The collections were designed to provide maximal confounds (or distractions).

[3] However, it is difficult to reject the hypothesis that animals do such an intelligent activity of constructing a model of the world in a broad sense.

by Kaneko and Matsui (1999), who examined a specific game called the festival game. Subsequent papers by Kaneko and Kline (2006) and Matsui and Shimizu (2006) are closely related to the present paper. Kaneko and Kline (2006) proposed the concept of information protocol and showed a correspondence between games expressed in extensive form and information protocols. While Kaneko and Kline (2006) tried to offer a comprehensive framework at the expense of the accessibility of the theory to the general reader, the present paper tries to minimize the amount necessary constructs and use the standard game theory terms whenever possible to make the theory more accessible to the reader without sacrificing its generality as a theory. We do not necessarily need a variety of concepts introduced in Kaneko and Kline (2006). Indeed, with a help of stochastic games, we are able to cope with fairly general situations.

Matsui and Shimizu (2006) confined their attention to the class of repeated games and sought conditions under which an objective game and a subjectively constructed model coincide. The present paper does not presume the existence of an objective game, and therefore, does not pay attention to the conditions under which agents can reconstruct the objective game from experiences.

The main inference rule agents use is induction in a broad sense. In this regard, the present paper shares a common spirit with a sequence of works by Gilboa and Schmeidler (1995, 2001) and Fudenberg and Levine (1993). However, a critical difference is that their focus is on the decision making process, and therefore, different from the main focus of the present paper, i.e., an activity of man who creates models of the world.

The rest of the paper is organized as follows. Section 2 presents the basic framework. Section 3 studies several applications. Section 4 concludes the paper.

2 The Framework

Let us first denote by $\mathcal{N}$ the set of all possible agents, by $\mathcal{A}$ the set of all possible acts, by $\mathcal{O}$ the set of all possible emotions, and by $\mathcal{I}$ the set of all possible impressions, of which meaning will be clear below. We assume that $\mathcal{N}$, $\mathcal{A}$, $\mathcal{O}$, and $\mathcal{I}$ are mutually disjoint.

2.1 Impressions and Experiences

Agents accumulate experiences. An experience of an agent is a chunk of impressions, which are sensed and felt by the agent. Let $\mathcal{I}$ be the set of impressions, which are primitives of the current framework.

Formally, an *experience* ε_i of Agent i ($i \in \mathcal{N}$) is a finite set of *impressions* $\omega_{i1}, \ldots, \omega_{iL} \in \mathcal{I}$, i.e., $\varepsilon_i = \{\omega_{i1}, \ldots, \omega_{iL}\}$. We denote by $\mathcal{E}$ the set of all possible experiences, i.e., all finite sets of impressions. This setup is sufficiently general since the set $\mathcal{I}$ of impressions is arbitrary. Yet, among various forms of impressions, the following forms along with their intended meanings are of special attention:

(i) $N \subset \mathcal{N}$: agents in N meet each other;

(ii) $(j : A_j) \in \mathcal{N} \times 2^{\mathcal{A}}$: Agent j has a set A_j of available acts;[4]

(iii) $(j : a) \in \mathcal{N} \times \mathcal{A}$: Agent j takes an act a;

(iv) $(j : \text{“}\textit{emotion}\text{”}) \in \mathcal{N} \times \mathcal{O}$: Agent j expresses or feels an “*emotion*”;

(v) $\varepsilon \succsim_i \varepsilon'$: Agent i weakly prefers experience ε to ε' (we also use $\varepsilon \succ_i \varepsilon'$ and $\varepsilon \sim_i \varepsilon'$ to mean strict preference and indifference, respectively).

(vi) $\emptyset$: a *null experience.*

We assume that these forms are in $\mathcal{I}$. We identify a sequence of experiences $(\varepsilon^1, \ldots, \varepsilon^{s-1}, \emptyset, \varepsilon^{s+1}, \ldots, \varepsilon^S)$ with $(\varepsilon^1, \ldots, \varepsilon^{s-1}, \varepsilon^{s+1}, \ldots, \varepsilon^S)$.[5]

Some examples of experiences are given below:

- $\varepsilon_i = (\{\{i, j, k\}, (i : A_i)\}, \{(i : a), (j : b)\}\}$: Agent i observed that Agents i, j, k met, that i has acts in A_i available, that i took a, and that j took b, but did not observe the act of k;

- $(\varepsilon_i^1, \varepsilon_i^2, \varepsilon_i^3)$ with $\varepsilon_i^1 = \{(i : a), (i : \text{“}\textit{pain}''\text{)}\}$, $\varepsilon_i^2 = \{(i : b), (i : \text{“}\textit{fun}'')\}$, $\varepsilon_i^3 = \{(i : a), (i : \text{“}\textit{calm}''), \varepsilon_i^1 \succ_i \varepsilon_i^2\}$: Agent i felt ”pain” when i took a, ”fun” when b, ”calm” when c, and i thought, when he was ”calm”, ε_i^1 was preferred to ε_i^2 in retrospect;

- $\varepsilon_i = \{\{j, k\}, (j : a), (k : ☹)\}$: Agent i observed that Agents j and k met, that j took a, and k expresses ☹.

[4]This type of element is less obvious than others. In reality, what I can observe is the fact that, say, someone opened the door of my office and walked toward me, and I do not observe that the person had an option of staying home and watched TV programs. I do not observe the latter, but based on my past experiences, I am convinced that he could have stayed home instead of coming to my office. Nonetheless, the subsequent setup assumes that this class of observation is also in $\mathcal{I}$ for the matter of convenience.

[5]If one would like to incorporate the notion of time, one can do it by, say, adding time to an experience.

2.2 Models

We use stochastic games as models of the world agents construct. Based on the set of experiences, each agent constructs a *model*, which represents his understanding of the situation in question. A *model* is generically given by

$$m = \langle (N, G, \mu), \sigma \rangle = \langle (N, G, \mu), (\sigma_i)_{i \in N} \rangle,$$

where (N, G, μ) is the *structural* part of the model, which is represented as a (modified) stochastic game, and σ is the *factual* part of the model, which is represented as a strategy profile of the stochastic game. Here, we have the following:

- $g = \langle N^g, (A_i^g)_{i \in N^g}, (u_i^g)_{i \in N}, (\varphi_i)_{i \in N} \rangle$ $(g \in G)$ is an augmented game in strategic form where
  - $N^g \subset N$ is the set of agents,
  - A_i^g is the set of acts of Agent $i \in N^g$;
  - $u_k^g : A^g \equiv \times_{j \in N^g} A_j^g \to \mathbb{R}$ is the payoff function of Agent $k \in N$;
  - φ_i^g $(i \in N)$ is an *experience function* of Agent i that maps $\{\emptyset\} \cup A^g$ into $\mathcal{E}$;
- μ is a *transition function* that maps each (g, a) to a probability distribution over G; and
- σ_i $(i \in N)$ is a *strategy* or a *behavior rule* of Agent i, which is a function of the past $\varphi_i(\cdot)$'s.

In this description, $\varphi_i^g(\emptyset)$ is an experience of i before g is played. The value $\varphi_i^g(\emptyset)$ typically, though not necessarily, contains the set of agents who meet to play the game and the set of available acts. Let $\mathcal{M}$ be the set of all such models.

Some models are of special interest. Here, we mention two classes of them. The first class is that of repeated game models. From sunrise to everyday work, one often views the situation he faces as if it would repeat indefinitely.

Model 1 A model $m = \langle (N, G, \mu), \sigma \rangle$ is an *infinitely repeated game model with discounting* if G is a singleton with $N^g = N$ (*a fortiori*, and μ is an identity map. In particular, it is a repeated game model with *perfect monitoring* if, for all $i \in N$, $\varphi_i^g(\emptyset) = \{N\}$ and $\varphi_i^g(a) = \{(j : a_j)\}_{j \in N}$.

Model 2 A model $m = \langle (N, G, \mu), \sigma \rangle$ is a *pairwise and uniform random matching model* (henceforth, *random matching model*) if G consists of g_{ij}'s $(i, j \in N)$ where $g_{ij} = \langle \{i, j\}, (A_i, A_j), (u_k)_{k \in N}, (\varphi_k)_{k \in N} \rangle$, $\mu(\cdot, \cdot)(g_{ij}) = 1/|N|(|N| - 1)$. In particular, it is a random matching model with *full observation* if each agent observes agents' identity and act pair taken for every game, i.e., $\varphi_i(g_{jk}) = \{\{j, k\}\}$ and $\varphi_i(g_{jk}, (a_j, a_k)) = \{(j : a_j), (k : a_k)\}$. On the other hand, it is a random matching model with *private observation* if each agent observes agents' identity and act pairs taken only for the games in which this agent participates, i.e.,

$$\varphi_i(g_{jk}) = \begin{cases} \{\{j, k\}\} & \text{if } i \in \{j, k\}, \\ \emptyset & \text{otherwise.} \end{cases}$$

and

$$\varphi_i(g_{jk}, (a_j, a_k)) = \begin{cases} \{(j : a_j), (k : a_k)\} & \text{if } i \in \{j, k\}, \\ \emptyset & \text{otherwise.} \end{cases}$$

2.3 Axioms

Axioms are the criteria which agents use to construct models of the world. There is no axiom that *ought* to be used *a priori*. Axioms themselves may be in flux in human mind, just like a researcher adopting different axioms from time to time. However, there are some that are considered plausible. The first of such axioms is *coherence*, which requires that a model be able to explain one's experiences.

Axiom 1 (Coherence) Given a model $m = \langle (N, G, \mu), \sigma \rangle$ and a sequence $\tilde{\varepsilon}_i = (\tilde{\varepsilon}_i^1, \ldots, \tilde{\varepsilon}_i^K)$ of experiences, m is said to be coherent with $\tilde{\varepsilon}$ if there exist $\varepsilon_i = (\varepsilon_{ia}^t, \varepsilon_{ip}^t)_{t=0}^T$ that is equivalent to $\tilde{\varepsilon}_i$, $(g^0, g^1, \ldots, g^T)$, and $(a^0, a^1, \ldots, a^T)$ such that the following conditions hold:

1. $\mu(g^{t-1}, a^{t-1})(g^t) > 0$, $t = 1, \ldots, T$;
2. $\sigma_j(\varphi_j^{g^0}(\emptyset), \varphi_j^{g^0}(a^0), \ldots, \varphi_j^{g^{t-1}}(a^{t-1}), \varphi_j^{g^t}(\emptyset))(a_j^t) > 0$, $t = 1, \ldots, T$, $j \in N$;
3. $\varphi_i^{g^t}(\emptyset) = \varepsilon_{ia}^t$, and $\varphi_i^{g^t}(a^t) = \varepsilon_{ip}^t$, $t = 0, \ldots, T$.
4. $u_i^{g^t}(a^t) \geq u_i^{g^{t'}}(a^{t'})$ if there exists $\tau = 0, 1, \ldots, T$ such that $(\varepsilon^t \succeq_i \varepsilon^{t'}) \in \varepsilon^\tau$ holds where "$>$" and "$=$" hold for "$\succ_i$" and "$\sim_i$", respectively.

Also, we may add another condition to consider the notion of statiscal coherence. Given a set of statistical tests and a sequence of experiences, a model is *statistically* *coherent* if, in addition to the four conditions of Axiom 1, the null hypothesis that the system is governed by μ is not rejected by these tests. We do not define this axiom more rigorously as the way it is defined depends upon the set of statistical tests to be used. We do not use statistical coherence in the subsequent applications.

A *solution concept* is a correspondence ψ that maps a stochastic game to a set of strategy profiles (possibly empty for some games). It is defined without referring to experiences.

Axiom 2 (Solution) Given a solution concept ψ, a model $m = \langle (N, G, \mu), \sigma \rangle$ conforms to the behavior rule ψ if $\sigma \in \psi(N, G, \mu)$.

An example of solution concepts is Nash equilibrium. Another is solution by backward induction. Other concepts induced by, say, some behavior rules can be represented as a ψ, too.

Axiom 3 (Uniqueness of Outcome/Solution) Given a model $m = \langle (N, G, \mu), \sigma \rangle$, a solution concept ψ induces the *unique outcome* if all $\tilde{\sigma}$'s in $\psi(N, G, \mu)$ induce the same stochastic process of outcome. ψ induces the *unique solution* if $\psi(N, G, \mu) = \{\sigma\}$.

The following two axioms are controversial in science. Nonetheless, there are tendencies to use them in reality by scientists as well as by laymen. The first one is the principle of simplicity and the second is that of observability.

Given $M \subset \mathcal{M}$, let $\geq_M$ denote a binary relation on M. We write $m >_M m'$ if $m \geq_M m'$ holds, but not $m' \geq_M m$.

Axiom 4 (Minimality) Given $M \subset \mathcal{M}$ and a binary relation $\geq_M$ over M, a model m is said to be *minimal* with respect to $\geq_M$ on M if there exists no $m' \in M$ satisfying $m >_M m'$.

Different agents may use different binary relations. A confused agent may have an intransitive binary relation, but we may assume that $\geq_M$ is a partial preorder.[6]

The next axiom is a principle of observability, according to which one tends to choose a model that contains only observables.

Axiom 5 (Observability) Given a sequence $(\varepsilon_i^1, \ldots, \varepsilon_i^K)$ of experiences, a model $m = \langle (N, G, \mu), \sigma \rangle$ satisfies the *principle of observability* if u^g $(g \in G)$ and σ are functions of observables.

[6] A binary relation $\geq_M$ is a *partial preorder* if it satisfies reflexivity and transitivity, i.e., $[\forall x \in M (x \geq_M x)]$ and $[\forall x, y, z \in M (x \geq_M y \& y \geq_M z \Rightarrow x \geq_M z)]$, respectively.

2.4 Prior Beliefs

Prior to the construction of a model based on experiences, agents may have held a certain belief of the situation. At this point, we do not care where this belief comes from, e.g., whether it comes from pure reasoning or from prior experiences. This belief may take various forms. A possible representation of such a belief is to restrict a possible class of models, at least from the viewpoint of researchers. Let $M \subset \mathcal{M}$ be the subset of games in extensive form. An agent's a priori knowledge can be represented by such an M.

3 Applications

3.1 Predation

> Experience is a dear teacher, but fools will learn at no other.
>
> –Benjamin Franklin

In 1998, Air Do entered the Japanese domestic airline market, raising money from the general public, after the deregulation of the airline industry in 1990's. Air Do was one of Japan's first low-fare airlines, operating between Chitose, Hokkaido and Haneda, Tokyo. It was originally called "*Do-min no Tsubasa* (The wing of Hokkaido-residents)," providing its passengers with low fare flights between Tokyo and Hokkaido. It competed with Japan's major domestic carriers (All Nippon Airways, Japan Airlines, and Japan Air System), who lowered their fares to Air Do's level without compromising corporate profits too much. After two years of losses in spite of a series of financial support from the local government of Hokkaido, Air Do went bankrupt, retired all of its stocks, and made a code-sharing agreement with ANA. Air Do not only lost its money, but also lost a dream to become "the wing of Hokkaido-residents," adopting the same general fare structure as the majors. ANA seems to be the winner of this predation game as it acquired Air Do as a low cost airline. Indeed, ANA has had Air Do expand its routes from only one (Haneda-Chitose) to four (Haneda-Chitose, -Asahikawa, -Hakodate, and -Memanbetsu).

To understand this situation, suppose that a potential entrant E considers whether or not to enter a market monopolized by an incumbent I before deregulation. From some other markets of similar characteristics, say, US airline market, E learns that once an entrant enters, an incumbent often acquiesces, and the two firms share the market accordingly. Formally, assume that an Entrant E's (indirect) experiences are:

$$(\textit{Regulation}, (E' : \{\textit{not}\}), (E' : \textit{not}), (I' : \pi^m_{I'}), (E' : 0)),$$

before deregulation, and

$$(\textit{Deregulation}, (E' : \{\textit{not}, \textit{enter}\}), (E' : \textit{enter}), (I' : \{p_H, p_L\}), (I' : p_H),$$
$$(I' : \pi^d_{I'}), (E' : \pi^d_{E'}), (\pi^m_{I'} > \pi^d_{I'}), (\pi^d_{E'} > 0)),$$

after deregulation, where p_H (resp. p_L) stands for a high (resp. low) price, and π^ρ_i ($i = I', E'$, $\rho = m, d$) is a profit. Having observed them, E may construct a structural model (N, G, μ) after deregulation as follows:

- $N = \{I, E\}$;
- $G = \{g^m, g^d\}$ where m and d stand for monopoly and duopoly, respectively;
  - $N^m = \{E\}$, $N^d = \{I\}$;
  - $A^m_E = \{\textit{enter}, \textit{not}\}$;
  - $A^d_I = \{p_H, p_L\}$;
  - $u^m_I(\cdot) = 4,\ u^m_E(\cdot) = 0$;
  - $u^d_I(p_H) = 2,\ u^d_E(p_H) = 1$;
  - $u^d_I(p_L) = v,\ u^d_E(p_L) = w$ for some $v, w \in \mathbb{R}$;
- $\mu(g^m, a) = \begin{cases} 1_{g^m} & \text{if } a_E = \textit{not}, \\ 1_{g^d} & \text{if } a_E = \textit{enter}, \end{cases}$

  where 1_g is a probability distribution that assigns one to g;
- $\mu(g^d, a) = 1_{g^d},\ \forall a \in A^d_I$.

At the same time, the factual part of the model is given as follows:

- $\sigma_I(\cdot, g^d) = p_H$,
- $\sigma_E(\cdot, g^m) = \textit{enter}$.

To intuitively understand the above model, it may be helpful to consider an "isomorphic" game in extensive form, though the term "isomorphic" is not formally defined. In order to construct a model "isomorphic" with the one created by E, let us assume, for the moment, that I has two options, p_L-*forever* and p_H-*forever* labelled *fight* and *acquiesce*, respectively.

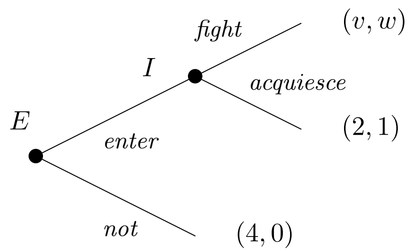

Figure 1: Predation Game

This game is coherent with E's experiences no matter what v and w may be. Also, suppose that $\geq_{|\cdot|}$ is a partial order with respect to the sizes of N, G, and A_i^g's.[7] Then this model is a minimal model with respect to $\geq_{|\cdot|}$.

Suppose further that ψ complies with backward induction, then E's experience (*enter*, *acquiesce*) implies that $2 > v$ holds, which leads to a positive profit for E even if w is negative.

On the other hand, I constructed a different structural model which is the same as E's structural model except that g^d moves to g^p if "fight" is chosen by I, and in g^p, E decides whether to "exit" or "stay". In this new model, $A_E^{g^p} = \{stay, exit\}$ and

$$\mu(g^p, a) = \begin{cases} 1_{g^p} & \text{if } a_E = stay, \\ 1_{g^m} & \text{if } a_E = exit, \end{cases}$$

in place of $\mu(g^d, a) = 1_{g^d}$ with $u_I^{g^p}(\cdot) = v$ and $u_E^{g^p}(\cdot) = w$.

An "isomorphic" game with the model created by I can be constructed if we assume that after I takes *fight*, E has an option of *exit*, and otherwise *stay*.

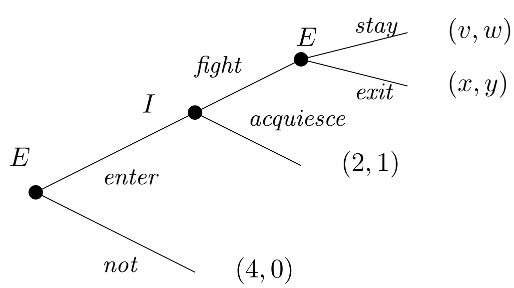

Figure 2: Predation Game with an Exit Option

[7] Precisely speaking, one may write $(N, G, \mu; \sigma) \geq (N', G', \mu', \sigma')$ if $|N| \geq |N'|$, $|G| \geq |G'|$, and there exist one-to-one (but not necessarily onto) correspondences $\varphi : N' \to N$ and $\rho : G' \to G$ such that for all $g \in G'$ and all $i \in N'^g [|A_{\varphi(i)}^{\rho(g)}| \geq |A_i^g|]$.

In this game, E has to be cautious and in fact refrain from entry since it should expect a negative profit if it believes that ψ complies with backward induction.

In the case of Air Do, it could not bear the loss caused by the predatory pricing of the two incumbents, ANA and JAL, and exited the market, or to be precise, retired its capital and reached a code-share agreement with ANA.

3.2 Bullying

> "You will probably be bullied wherever you may go unless you have some fighting spirit." –Shintaro Ishihara[8]

Suppose that there are four children, A, B, C and D, which has been already part of prior knowledge. Also, every child has prior knowledge that each time two children meet in pair, they simultaneously decide whether they play friendly or not. Now, these children have observed that each time two of A, B and C meet in pair, they play friendly (F) and look happy, while when they meet with D and form a pair, they play unfriendly (U), while D plays both friendly and unfriendly from time to time, and the two looked unhappy. In addition to these impressions, the children observed some attributes of each other, e.g., a color of skin, height, body shape, face, etc. Let ε_j denote such an experience of Child j ($j = A, B, C, D$).

There are plenty of models coherent with the above set of experiences even if we restrict our attention to random matching models with full observation. Here, we consider two classes. The first one assumes some intrinsic differences between the children, while the second does not. In both models, Nature determines a pair of children to be matched with randomly. After two children are matched with, they play a simultaneous move game where both of them have two available acts F (Friendly) and U (Unfriendly), which is also known by the children.

In the first model of Child $i = A, B, C$, a utility function can be of the following form:

$$u_i(a_i, a_j; j) = \begin{cases} 1 & \text{if } a_i = a_j = F \text{ and } j \neq D, \\ 0 & \text{otherwise, } (i = A, B, C). \end{cases} \tag{1}$$

Behavior rules of A, B, and C are given by:

$$\sigma_i(\{i, j\}) = \begin{cases} F & \text{if } j \neq D, \\ U & \text{if } j = D, \ (i = A, B, C). \end{cases}$$

[8] A remark at press conference on Nov. 10, 2006; translated by the author

This model of Child i is coherent with ε_i, and their strategy profile constitutes a Nash equilibrium. Moreover, under some "reasonable" criteria of minimality such as the one that counts "complexity" by the number of acts and payoff values, this model becomes minimal.

If it happens to be the case that D is taller than the other three children, then children may construct a model in such a way that they do not have fun if they play with a tall child. To construct such a model, suppose that h_j $(j = A, B, C, D)$ is the height of Child j, and that $h_j < \bar{h}$ for $j \neq D$, while $h_D > \bar{h}$ where $\bar{h}$ is a threshold value. In this case, we have

$$u_i(a_i, a_j; h_j) = \begin{cases} 1 & \text{if } a_i = a_j = F \text{ and } h_j < \bar{h}, \\ 0 & \text{otherwise, } (i = A, B, C). \end{cases} \tag{2}$$

in place of (1). The point of this analysis is to show that any attribute can be a reason for bullying.

In the second model, each child obtains one as a payoff if both choose F, and zero otherwise.

$$u_i(a_i, a_j) = \begin{cases} 1 & \text{if } a_i = a_j = F, \\ 0 & \text{otherwise, } (i = A, B, C, D). \end{cases} \tag{3}$$

Each child plays a repeated game strategy according to which they determine a "target" and play U whenever a child meets the target child, and they continue to do so until someone takes F against the target, after which the one who took F now becomes a new target. This strategy profile is a subgame perfect equilibrium of the constructed repeated game.

3.3 Spilt Water

Fukusui Bon-ni Kaerazu.[9]

–An old saying from the story of Tai Kung-Wang (*Taikobo*)

A well known chinese historical character, Tai Kung-Wang, or *Taikobo*, was left by his wife after reading and fishing day by day in spite of her devotion to her husband. When he became a local lord, the ex-wife tried to be reconciled with him. He said he would reinstate her if she could put spilt water back into the tray.

[9] Spilt water never returns to its tray, corresponding to the English saying, "There is no use crying over spilt milk."

To study this story, we consider the following situation. In a repeated interaction, one defection sometimes devastates the relationship. To interpret such a situation, a game theorist would build a repeated game model of prisoners' dilemma given by Table 1 and assume that the players take the grim trigger strategies according to which one keeps cooperating (C) till the opponent defects (D), which triggers defection forever. Yet, some other people think that the game they play changes after they encounter defection.

		husband		
		C	D	R
wife	C	2, 2	-1, 3	0, -1
	D	3, -1	0, 0	0, -1
	R	-1, 0	-1, 0	-1, -1

Table 1: Prisoners' Dilemma

Suppose that there is a couple who get along well with each other. The wife thinks that the situation they are in can be represented by the repetition of the prisoners' dilemma given by Table 1. So far, their experiences were $((C, C), \ldots, (C, C))$. One day, the wife took D. He thought the husband would forgive him if he repented, which he actually did. However, it turns out that the husband claims "The game has been changed," and never takes C thereafter.

One may claim that what has been changed is the history, not the game. However, the following model m justifies the husband's claim that "the game has been changed". Denote by g^{PD} the game given by Table 1, and by g^B the game where the one who was betrayed has only one act D, which is given by Table 2.

		husband
		D
wife	C	-1, 3
	D	0, 0
	R	-1, 0

Table 2: Aftermath of Defection by Wife

Both models are coherent with their experiences. The two models are different, however, in terms of the possibility of the husband's changing his behavior to cooperate again after the defection of the wife. In the repeated game model, what the wife should do is to persuade the husband to take C again. Indeed, the husband's behavior is not "renegotiation-proof" in a loose sense. On the other hand, in the second model, the wife has to change the view of the world of the husband: his behavior together with D

by the wife in Table 2 forms a renegotiation-proof Nash equilibrium.

3.4 Pioneers

> "I think the importance of being a pioneer is that you have to be successful, ... Being successful leads to the next player, and the next player and so on."
>
> –Don Nomura, the agent of Hideo Nomo[10]

In 1995, Hideo Nomo, a Japanese pitcher, signed a contract with Los Angeles Dodgers after a contract dispute with Kintetsu Buffaloes. He was the second Japanese baseball player to make the Major League debut, only after almost forgotten Masanori Murakami. Nomo's games were regularly broadcast in Japan. And unlike Murakami, Nomo exceeded the expectations of Japanese media and fans. His success inspired many baseball stars like Ichiro and Matsuzaka to come to the United States, too. Before Nomo, neither players nor club teams had a dream of success of Japanese in the Major League Baseball. Nobody ever predicted before 1995 that Japanese players can compete with MLB players. Moving to MLB was not even in their scope. After 1995, there opened a door to MLB all the sudden.

Pioneering works have something in common. They all change the scope of people. Rather, it is almost the definition of being a "pioneer." After observing (play in Japan, success) and (play in Japan, failure) thousands of times with no observation of (play in US, success) and only one case of (play in US, failure), which had been forgotten, it is not difficult to imagine that people construct a model where there is no option of playing in US.

4 Conclusion

The present paper proposes a theory of man that tries to construct a model of the world in societal situations where people interact with each other. The present theory takes experiences, or chunks of impressions, as primitives as opposed to an "objective" game. A model is something that is constructed by an agent. Each model consists of structural and factual parts. The structural part is represented as a stochastic game. While the factual part is represented as a strategy profile of the game. In constructing a model, the agent uses some axioms. Examples of the axioms that the agent might use are coherence, according to which he can explain his own experiences, consistency

[10] Quoted in the article "Wally Yonamine" by Rob Smaal in English edition of Asahi.com, Jan. 2, 2007.

with a solution concept adopted by the agent, and simplicity with respect to some measure, again adopted by him. The present paper does not assume the existence of an "objective" game, and different agents may construct different models of the world.

If we say a "model of the world," it sounds as though there existed an object called "the world." We do not know whether there exists a situation called an objective world or not. The concept of objective game itself is a creation of researchers. An "objective game" is constructed by a researcher in order to understand our experiences/impressions better than otherwise. However, in the present framework, we do not have to presume the existence of such an objective world. Nor, do we have to take a position of denying it. Even without entering such a metaphysical discourse, the present framework can be used to address issues the current society confronts.

Different individuals create different worlds. This idea can be seen in *Vijñānavāda* (doctrine of consciousness), a school of *Mahāyāna* (greater vehicle) founded by Asanga and Vasubandhu (5th century AD), who used the parable "*Issui-Shiken*," or "One water, four lookings": what humans view as "water" may be viewed as "bloody sea" by *gaki* (hungry ghosts), as "residence" by fish, and as "treasure land" by heavenly beings.

Our experiences are limited in various ways. We cannot feel what others feel. All we can do is to infer others' feelings from circumstances and their facial and other expressions. When we do it, we have already constructed a model of others. In this sense, it may well be the case that animals other than human have some ability to construct a model. After all, God "created man in his own image" (Genesis 1:27) in the western tradition, while humans and animals transmigrate into each other in the eastern tradition.

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