Currency and Checking Deposits as Means of Payment

(very preliminary)

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Abstract

Casual observations suggest that cash is very often used for everyday small-value purchases while checks are used for larger-value payments. We use a search monetary model to study issues related to multiple means of payment, in which checking deposits pay interest but have a fixed record keeping cost. If the cost of using checking deposits for making payments is lower than the liquidity return obtained from financing big transactions but larger than that for small transactions, checks are used only in big transactions and currency is the only means of payment for small transactions. Inflation may have differential impacts on the terms of trade in transactions using different means of payment. Currency's liquidity value derives mainly from facilitating unexpected small transactions. As people are more likely to engage in cash transactions, the precautionary demand for money is higher and the balance of checking deposits is lower, resulting in a higher currency-deposit ratio.

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1 Introduction

Currency and checking deposits are two technologies for making payments. Casual observations suggest that cash is very often used for everyday small-value purchases while checks are used for larger-value payments.[1] The means-of-payment decisions of a person may be affected by the following features associated with currency and checking deposits. Carrying large amounts of cash is not good, because of the risk of loss or theft. The opportunity cost of holding cash is the interest forgone from holding other assets. Checking accounts may pay you interest but they often have fees, minimum balance requirements or a limit of how many checks you can write each month. The last feature makes buyers less willing to write checks for small amounts.

In this paper we use a search monetary model to study issues related to multiple means of payment. We explicitly consider the features that distinguish the two transaction technologies – in particular, a fixed record keeping cost incurred whenever a check (or debit card) is used as the means of payment.[2] The basic framework we use is Lagos and Wright (2005) with the addition of a banking sector. In this economy people trade goods in the market characterized by bilateral random matching, while they visit a centralized market periodically to adjust their asset holdings so that the distribution of balances of currency and checking deposits is analytically tractable. The banking system has a technology that keeps financial records of people but not transaction records in the goods market, and banks share financial information so that interbank settlement is possible. Therefore, individuals cannot issue trade credit; only cash and bank liabilities such as checks drawn on interest-bearing demand deposits are available means of payment.

The decentralized trading arrangement in this model makes money essential, which also allows us to explicitly depict the expected return provided by a means of payment in facilitating a transaction (called 'liquidity return'). We find that, if the cost of using checking deposits for making payments is lower than the liquidity return obtained from financing big transactions but larger than that for small transactions, checks are used only in big transactions and currency is the only means of payment for small transactions. Only when the transactions provide high

[1]For example, people may want to have enough cash to pay for things in the near future, such as a quick meal or an unexpected purchase.

[2]The recent rising trend of using debit cardsin US for payments is remarkable. Data show that in US debit card transactions grew from 8.3 billion in 2000 to 15.6 billion in 2003. The growth in debit card payments accounted for more than half the growth in electronic payments over the period.

enough payoff to compensate the cost would people choose to may payments with checks. In this equilibrium currency is used in all transactions while checks are used only in some transactions. A certain degree of 'illiquidity' associated with checks is necessary for the coexistence of both means of payment, since the interest-bearing feature implies a higher rate of return of checking deposits than currency.

The effects of inflation on the terms of trade in transactions using different means of payment depend on how it affects the deposit interest rate. If, for example, the interest rate rises more than the inflation rate does, a higher inflation reduces the terms of trade in cash transactions but raises that in transactions using checks. Thus, inflation may have differential impacts on people who use different means of payment.[3] The opportunity cost of holding currency is the forgone interest earnings. As long as a policy or a factor affects deposit interest rate, and hence the opportunity cost of holding currency, the liquidity return of currency must change in order for people to hold currency. Thus, a higher interest rate causes people to reduce currency holdings but increase balances in checking deposits and thus, induces a lower currency-deposit ratio. Currency's liquidity value derives mainly from facilitating unexpected small transactions. As people are more likely to engage in cash transactions, the precautionary demand for money is higher and the balance of checking deposits is lower, resulting in a higher currency-deposit ratio.

Search-theoretic models have been used to study competition among media of exchange and the resulting policy implications.[4] In this paper, we explicitly consider the features that distinguish checking deposits and currency to study agents' means-of-payment decisions and the effects of monetary policy. The arrangement studied in this model is sufficiently explicit

[3]Due to the minimum balance requirements to open and maintain a checking account, people with lower income or wealth may not have checking accounts, so they rely on cash to make transactions. If inflation has differential impacts on the prices of transactions using cash and checks, then it affects differentially people with different levels of income or wealth. A model with heterogenous agents can account for this implication.

[4]Some search-theoretic models study the competition of media of exchange; for example, Matsuyama et al. (1993), Head and Shi (2003) and Li and Matsui (2005) consider the competition of local currency and foreign currency, and Lagos and Rocheteau (2004) and Shi (2004) study the competition of money and other asset such as bonds or capital. Some studies, e.g., Calvacanti et al. (1999), Cavacanti and Wallace (1999), Williamson (1999) and Li (2005) incorporate a banking sector into the economy with decentralized markets to study the functions of private money and its competition with fiat money, or consider the safe-keeping role of banks (He et al. 2005).

that one can examine the costs and benefits associated with modifying the scheme – say, by imposing reserve requirements or technological improvement in the check clearing system. We hope this model may help us understand the consumer's choice of a means of payment and to make predictions on the development of financial intermediaries that provide those means of payment.

The rest of the paper is organized as follows. Section 2 presents the basic model. Section 3 discusses the optimal portfolio choices and the means-of-payment decisions. In section 4 we discuss the existence and properties of the equilibrium with the coexistence of currency and checking deposits. Section 5 concludes with suggestions for possible extensions.

2 The Environment

The basic framework we use is the divisible money model developed in Lagos and Wright (2005). This model allows us to introduce an idiosyncratic preference shock and incorporate a banking sector while keeping the distribution of balances of currency and checking deposits analytical tractable.

Time is discrete and there is a $[0, 1]$ continuum of infinitely-lived agents. Each period is divided into two subperiods, that differ in terms of economic activity. All consumption goods are nonstorable and perfectly divisible. In the first subperiod people specialize in production and consumption and there is no double coincidence of wants. There is a decentralized market in which agents meet anonymously according to a random bilateral matching process. When two agents meet, agent i wants something that agent j can produce but not vice versa with probability σ; agent j wants something agent i can produce but not vice versa with probability σ; and neither wants what the other produces with probability $1 - 2\sigma$, where $0 < \sigma < 1/2$. An idiosyncratic preference shock arrives to an agent that determines the utility from consuming goods. An agent consuming q units of his consumption good in period t gets utility $\varphi_t u(q)$, where φ_t is an i.i.d. preference shock with $\varphi_t \in \{\delta, 1\}$, $0 < \delta < 1$, and $\Pr[\varphi_t = 1] = \lambda$, $\Pr[\varphi_t = \delta] = 1 - \lambda$. Producers incur disutility $v(q)$ from producing q units of output. Assume $u(0) = v(0) = 0, u' > 0, v' > 0$, $u'(0) = \infty$, $u'' < 0, v'' \geq 0$. Trading histories of agents are private information to the agent. There is no commitment or public memory so all trade must be *quid pro quo*.

In the second subperiod there is a centralized market and all agents can produce and consume a consumption good (called 'general good'), getting utility $U(x)$ from x consumption, with $U'(x) > 0, U'(0) = \infty$, $U'(\infty) = 0$ and $U'(x) \leq 0$. Agents can produce one unit of the good with one unit of labor which generates one unit of disutility. The discount factor across dates is $\beta \in (0, 1)$.

Competitive banks open in the second subperiod. Banks accept deposits from agents and allow them to write checks to pay for purchases in the decentralized market. Those banks have a technology for record keeping on financial histories but not the trading histories in the goods markets of agents. We also assume that banks share the records so that interbank settlement is possible. If an agent accepts a check for payment in the decentralized market, he presents the check to his bank when arriving in the centralized market. The balance of the receiving party's checking account is credited while that of the agent who wrote a check is debited. Agents can adjust their balances in the checking accounts and currency holdings in the centralized market. Notice that in the decentralized market liabilities on banks can be used as a means of payment, while trade credit or individual issued IOU is not feasible, since banks keep records of financial transactions but not goods market transactions.[5]

Currency

A government is the sole issuer of fiat currency. For now we assume no costs associated with holding or using currency, but one can consider the costs of transportation, risk of loss, theft, and counterfeiting. We let currency stock evolve deterministically at a gross rate γ by means of lump-sum transfers: $M_t = \gamma M_{t-1}$, where $\gamma > 0$ and M_t denote the per capita money stock in period t. Agents receive lump-sum transfers $T_t = (\gamma - 1)M_{t-1}$ in the centralized market. Let ϕ_t denote the value of money in terms of the general good. We denote the real transfer $\tau_t = \phi_t T_t$. For notational ease variables corresponding to the next period are indexed by $+1$, and variables corresponding to the previous period are indexed by -1.

Checking deposits

The means of payment associated with checking deposits include, for example, checks and debit cards. Although there are certain differences between both, they are liabilities of banks. We focus on two features of checking deposits: the interest payment and the cost of using checks

[5]We implicitly assume that banks have a commitment technology – they take deposits and settle financial transactions without defaulting on the interbank debt.

for payment. The opportunity cost of holding cash is the interest forgone from investing in other assets. We assume that the checking deposits pay interest at a rate $i_d > 0$. One can think that banks take deposits and invest in the international markets of which the returns are exogenously given. Competition among banks results in the deposit interest rate equal to the exogenous rate of return.[6]

Using checks to make payments incurs a fee p_c, which is paid when agents adjust portfolios in the centralized market. One can interpret p_c as the record keeping cost. From the stand-point of banks' customers, the fee is meant to capture the features that checking accounts often have fees, minimum balance requirements or a limit of how many checks people can write each month. We do not consider the private information problem regarding checks, though it is an important factor for whether they may be widely accepted. We assume that the technology of enforcement and punishment ensures that agents do not have their checks returned due to insufficient funds. Moreover, this is less a problem to debit cards, because the funds are immediately removed from buyers' accounts at the time of making payments.

Timing of events is as follows. At the beginning of the first subperiod, agents receive a preference shock. Then, agents meet at random and trade if there is single-coincidence of wants. Buyers choose to pay cash or checks for the purchases. In the second subperiod agents trade goods in the centralized market, settle financial claims with banks, receive lump-sum transfers, and adjust the balances of currency and checking deposits.

3 Equilibrium

In this economy the preference shock and random matching in the decentralized market result in different trading histories, which generates a nondegenerate distribution of portfolios within a period. Since agents can produce one unit of the general good with one unit of labor which generates one unit of disutility, they optimally redistribute their asset holdings uniformly so that all agents carry identical balances of cash and checking deposits out of the centralized market.[7]

[6] We do not consider banks' strategic behaviors such as competing for deposits, and how this competition affects deposit interest rates.

[7] One can consider ex ante heterogeneity among agents in preferences, discount factors and productivity. Thus, agents may choose different balances of money and deposits out of the centralized market; however, agents of identical type choose identical balances.

That is, under the quasi-linear utility assumption the distribution of asset holdings is degenerate at the beginning of a period.

We study equilibria in a stationary economy in which real value of agent's money holdings and checking account balances are constant. In particular, $\phi M = \phi_{-1} M_{-1}$, which implies $\frac{\phi_{-1}}{\phi} = \gamma$. A representative agent begins a period with real holdings (in terms of the general good) of currency and checking deposits denoted $z = (m, c)$. In a trading opportunity if the buyer has high (low) marginal utility and is willing to buy large (small) amounts of goods, then we call it a type h (l) transaction, or simply a big (small) transaction. An agent may encounter a single-coincidence meeting in which he is a seller or a buyer with high or low marginal utility. A buyer must choose the means to pay for the purchases. An agent may not have any trading opportunity. Consequently, due to different trading histories in the decentralized market, agents begin the second subperiod with different portfolios, denoted (in real terms) $z_{ij} = (m_{ij}, c_{ij})$, where $i = s, b, n$ identifies an agent who was a seller, buyer and non-trader in the decentralized market, respectively, and $j = h, l$ denotes the transaction type, but we use $j = 0$ for the non-traders. We let c_{ij} represent the balance of an agent's bank account when entering the second subperiod, including the interest earned over a period.

Let $I_j, j = h, l$, denote the indicator function, whose value is 1 if a buyer pays for a type j transaction in the decentralized market with checks, and 0, if not. Let d_{mj} and $d_{cj} I_j$ denote the amounts of currency and deposits transferred in a type j transaction, respectively. If a seller accepts checks for payments, his checking account balance is credited when presenting the checks to the banks. On the other hands, currency is physically changed hands at the point of sale.

Let $V(z)$ denote the expected life-time utility of a representative agent beginning a period with portfolio z, before the preference shock is realized. Let $W(z_{ij})$ be the expected life-time utility of an agent from entering the centralized market of the second subperiod with portfolio z_{ij}. In what follows we look at a representative period t and work backwards from the second to the first subperiod.

3.1 The second subperiod

In the second subperiod, there is a standard centralized market. Agents produce h goods and consume x, have their account balances credited or debited with banks, and adjust their balances of currency and deposits. An agent enters the centralized market with portfolio z_{ij} has expected life-time utility

$$
\begin{aligned}
W(z_{ij}) &= \max_{x,h_{ij},z_{+1}\geq 0} \{U(x) - h_{ij} + \beta V(z_{+1})\} \\
\text{s.t. } x &= h_{ij} + (m_{ij} + c_{ij} + \tau) - p_c I_j - \gamma(m_{+1} + c_{+1})
\end{aligned}
\tag{1}
$$

where m_{+1} and c_{+1} are the real balances of money and deposits, respectively, taken into period $t+1$. The factor γ multiplies m_{+1} and c_{+1} because the budget constraint lists the current real value. Note that the cost p_c must be paid if an agent make a payment using checking deposits in the decentralized market.

Substituting h_{ij} from the budget constraint, (1) is rearranged as

$$
W(z_{ij}) = m_{ij} + c_{ij} + \tau - p_c I_j + \max_{x,z_{+1}\geq 0} \{U(x) - x - \gamma(m_{+1} + c_{+1}) + \beta V(z_{+1})\}
$$

The first-order condition for x is $U'(x) = 1$, which implies $x = x^*$ for all agents. Also,

$$
\gamma \geq \beta \frac{\partial V(z_{+1})}{\partial m_{+1}}, \quad = \text{ if } m_{+1} > 0 \tag{2}
$$

$$
\gamma \geq \beta \frac{\partial V(z_{+1})}{\partial c_{+1}}, \quad = \text{ if } c_{+1} > 0. \tag{3}
$$

Conditions (2) and (3) determine z_{+1}, independent of x and z_{ij}. That is, the optimal choice of z_{+1} is independent of the initial portfolio when entering the centralized market. The envelope conditions are

$$
\frac{\partial W(z_{ij})}{\partial m_{ij}} = 1 \tag{4}
$$

$$
\frac{\partial W(z_{ij})}{\partial c_{ij}} = 1. \tag{5}
$$

The marginal values of currency and checking deposits simply reflect the price of real balances, which is 1, due to the linearity of production disutility and competitive pricing in the centralized market.

The model allows us to rewrite $W(z_{ij}) = W(m_{ij} + c_{ij})$. That is, when entering the centralized market, agents care only about the total value of their real portfolios, not the composition. Moreover, we can disentangle the agents' portfolios from their trading histories since $W(z_{ij}) = W(0) + m_{ij} + c_{ij}$. This implies that agents exit the centralized market choose identical portfolios, independent of their trading histories.

3.2 The first subperiod

Agents enter the decentralized market at the beginning of a period, in which each meeting is bilateral and at random. The terms of trade are determined by bargaining. In any pairwise meetings, the terms of trade depend on the agents' portfolios. Consider a single-coincidence meeting, in which the buyer has portfolio z and the seller has portfolio z'. The terms of trade are $[q_j(z,z'), d_j(z,z')]$, where $q_j(z,z') \in \mathbb{R}_+$ is the quantity of good traded and $d_j(z,z') = (d_{jm}, d_{jc}I_j) \in \mathbb{R}^2_+$ represents the transfers of real money holdings and checking deposits from the buyer to the seller.

The value function $V(z)$ satisfies the following Bellman equation:

$$\begin{aligned} V(z) \;\; = \;\; & \sigma\lambda\{u[q_h(z,z')] + W[z - d_h(z,z')]\}dH(z') \\ & +\sigma(1-\lambda)\{\delta u[q_l(z,z')] + W[z - d_l(z,z')]\}dH(z') \\ & +\sigma\lambda\{-v[q_h(z',z)] + W[z + d_h(z',z)]\}dH(z') \\ & +\sigma(1-\lambda)\{-v[q_l(z',z)] + W[z + d_l(z',z)]\}dH(z') \\ & +(1-2\sigma)W(z) + i_d c, \end{aligned} \tag{6}$$

where the first two terms represent the expected payoffs to buying in a big and small transaction, the third and forth terms represent the expected payoffs to selling and the fifth term represents the expected payoff of non-trading. Note that by linearity of $W(\cdot)$ we have the last term in (6) represent the interest payments earned over the period from the checking account.

In a single-coincidence meeting, the terms of trade are determined by generalized Nash bargaining, in which the buyer has bargaining power $\theta > 0$, and threat points are given by the continuation values. Consider a meeting in which the buyer has high marginal utility of consumption. Then (q_h, d_h) solves

$$\max_{q_h, d_h \le z} [u(q_h) + W(z - d_h) - W(z)]^{\theta}[-v(q_h) + W(z' + d_h) - W(z')]^{1-\theta}.$$

Given $W(z + d_j) = W(z) + d_{jm} + d_{jc}I_j$ and let a_j denote $d_{jm} + d_{jc}I_j$, the bargaining problem can be rewritten as

$$\max_{q_h, d_h \le z} [u(q_h) - a_j]^{\theta}[-v(q_h) + a_j]^{1-\theta}.$$

Solving the bargaining problem we find that the total value of assets that the buyer needs to transfer to the seller in exchange for quantity $q_h \in [0, q_h^*]$ of good is $b_h(q_h)$, where

$$b_h(q_h) = \frac{\theta v(q_h)u'(q_h) + (1-\theta)u(q_h)v'(q_h)}{\theta u'(q_h) + (1-\theta)v'(q_h)}$$

and q_h^* solves $u'(q) = v'(q)$. Similarly, for small transactions, the buyer spends $b_l(q_l)$ in exchange for $q_l \in [0, q_l^*]$, where

$$b_l(q_l) = \frac{\theta\delta v(q_l)u'(q_l) + (1-\theta)\delta u(q_l)v'(q_l)}{\theta\delta u'(q_l) + (1-\theta)v'(q_l)}$$

and q_l^* solves $\delta u'(q) = v'(q)$. Note that $b_h(q) > b_l(q)$; to buy the same quantity of goods people pay more when they have higher marginal utility.

The bargaining solutions can be rewritten as follows.

$$q_j(a_j) = \begin{cases} q_j^* & \text{if } \ a_j \geq b_j(q_j^*) \\ b_j^{-1}(a_j) & \text{if } \ a_j < b_j(q_j^*) \end{cases} \tag{7}$$

Note that $b_j'(q_j) > 0$ and $\frac{\partial q_j}{\partial m} = \frac{\partial b_j^{-1}(a_j)}{\partial m} = \frac{1}{b_j'(q_j)}$ is the change in the terms of trade if the buyer brings an additional unit of real balance (cash, or deposits if $I_j = 1$) to the market. Also note that the bargaining solutions are independent of the seller's portfolio and the composition of currency and deposits in buyer's portfolio.

Given the bargaining solution (7), we rewrite (6) as

$$\begin{aligned} V(z) \quad = \quad & \sigma\lambda\{u[q_h(a_h)] - b_h[q_h(a_h)]\} + \sigma(1-\lambda)\{\delta u[q_l(a_l)] - b_l[q_l(a_l)]\} \\ & +\sigma\lambda\{-v[q_h(a_h')] + a_h'\} + \sigma(1-\lambda)\{-v[q_l(a_l')] + a_l'\} \\ & +W(z) + i_d c. \end{aligned} \tag{8}$$

In what follows, we assume that $\frac{u'(q)}{b_h'(q)}$ is strictly decreasing in q, so that we have $a_h < b_h(q_h^*)$. The total value of agents portfolio is less than the amount that is required to buy the socially efficient quantity. This also implies that a buyer will spend all his asset in a big transaction. We will also show below that in equilibrium if checks are used to make payments in big transactions, currency is the only means to pay for the small purchases. We assume that $\frac{u'(q)}{b_l'(q)}$ is strictly decreasing in q so that a buyer spends all his currency holding in small transactions.

3.3 The choice of portfolios and means of payment

Our interest is to find the conditions under which checks are used only in big transactions while currency is used as the only means of payment in small transactions. We thus work under the conjecture that buyers do not use checks to pay for small purchases (we will show below this holds in equilibrium) to focus on whether agents use checking deposits to pay for the purchases in big transactions, and derive the optimal portfolios. In the following discussion, we set $I_l = 0$ and check whether $I_h = 1$.

To find the optimal portfolio of an agent, we calculate the expected marginal values of each asset, $\frac{\partial V(z)}{\partial m}$ and $\frac{\partial V(z)}{\partial c}$. Taking derivatives of $V(z)$ in (8) yields

$$\begin{aligned}
\frac{\partial V(z)}{\partial m} &= \sigma\lambda\{u'[q_h(a_h)] - b_h'[q_h(a_h)]\}\frac{\partial q_h(a_h)}{\partial m} \\
&\quad +\sigma(1-\lambda)\{\delta u'[q_l(m)] - b_l'[q_l(m)]\}\frac{\partial q_l(m)}{\partial m} + \frac{\partial W(z)}{\partial m} \\
\frac{\partial V(z)}{\partial c} &= \sigma\lambda\{u'[q_h(a_h)] - b_h'[q_h(a_h)]\}\frac{\partial q_h(a_h)}{\partial c} + \frac{\partial W(z)}{\partial c} + i_d
\end{aligned}$$

Recall that $\frac{\partial q_j}{\partial a_j} = \frac{\partial b_j^{-1}(a_j)}{\partial a_j} = \frac{1}{b_j'(q_j)}$. Let $r_h(a_h) = \frac{u'[q_h(a_h)]}{b_h'[q_h(a_h)]} - 1$ and $r_l(m) = \frac{\delta u'[q_l(m)]}{b_l'[q_l(m)]} - 1$ denote the liquidity return of an asset from conducting big and small transactions, respectively. Given that the real cost of m and c is 1, i.e., $\frac{\partial W(z)}{\partial m} = \frac{\partial W(z)}{\partial c} = 1$, the first-order conditions can be written as

$$\frac{\partial V(z)}{\partial m} = \sigma\lambda r_h(a_h) + \sigma(1-\lambda)r_l(m) + 1 \tag{9}$$

$$\frac{\partial V(z)}{\partial c} = \sigma\lambda I_h r_h(a_h) + 1 + i_d. \tag{10}$$

Using (2), (3), (9) and (10), the representative agent's optimal portfolio choices must satisfy

$$1 \geq \frac{\beta}{\gamma}[1 + \sigma\lambda r_h(a_h) + \sigma(1-\lambda)r_l(m)], \quad = \text{ if } m > 0 \tag{11}$$

$$1 \geq \frac{\beta}{\gamma}[1 + \sigma\lambda I_h r_h(a_h) + i_d], \quad = \text{ if } c > 0. \tag{12}$$

Condition (11) states that if people choose to hold currency, the cost of acquiring an additional unit of real money balance must equal the expected discounted payoff from facilitating all kinds of transactions in the decentralized market. Condition (12) states that if checking deposits are to be held, the marginal cost of acquiring an additional unit of real balance in the checking deposits must equal the interest rate plus the expected discounted payoff from facilitating big transactions if people choose to make payments out of the checking deposits.

We now determine agents' means-of-payment decisions. Since using checks incurs a fee p_c, agents will choose to make purchases out of the checking deposits only if the expected payoff from financing the transactions is larger than the cost. From (8), if in a big transaction an agent uses only currency to buy goods, the expected payoff he gets is $u[q_h(m)] - b_h[q_h(m)]$ plus the continuation value of holding the deposits c, $W(c)$. If he spends all the currency and deposits, the net expected payoff he gets is $u[q_h(m+c)] - b_h[q_h(m+c)] + W(0) - p_c$. The difference between both is

$$\triangle = \{u[q_h(m+c)] - u[q_h(m)]\} - \{b_h[q_h(m+c)] - b_h[q_h(m)]\} - c - p_c.$$

Using the notion of the liquidity return $r_h(a_h)$, we have

$$I_h = 1 \text{ if } p_c \leq \int_m^{a_h} r_h(a_h) dc - c. \tag{13}$$

A similar argument can be applied to find the condition on I_l. However, note that the bargaining solution (7) says that the buyer will not spend more than what is needed to buy q_l^*; i.e., at most he will spend $b_l(q_l^*)$ in a small transaction. Let $c_l = \min\{c, b_l(q_l^*) - m\}$ and $a_l = c_l + m$. We have

$$I_l = 1 \text{ if } p_c \leq \int_m^{a_l} r_l(a_l) dc - c_l. \tag{14}$$

Given the cost p_c, if δ is sufficiently small, it is possible that the expected payoff from using checks to finance small transactions is not enough to compensate the cost, and so agents will use only currency to pay for the purchases.

Definition 1 *A stationary equilibrium is a list of value functions* (V, W), *individuals' choices* (m, c, I_h, I_l, x, h), *terms of trade* (q_h, q_l, d_h, d_l), *and a sequence of prices* $\{\phi_t\}$ *that solve (1) and (6), satisfy the bargaining solution (7), the optimal portfolio choices (11) and (12), the spending strategies (13) and (14), and* $m = \phi M$.

We first rule out the case with $I_h = 1$ and $I_l = 0$.

Lemma 1 *In a monetary equilibrium if* $I_h = 1$ *then* $I_l = 0$.

Proof. Suppose not, then both means of payment have identical return from facilitating transactions. However, Comparing (11) and (12) one finds that checking deposits earn interest whereas currency does not, but the real price of both means of payments in the centralized market are identical, a contradiction.■

From (12), since $r_h(a_h) \geq 0$, in equilibrium we must have $\gamma \geq \beta(1 + i_d)$ so that agents do not demand infinite amount of deposits. Intuitively, when inflation is high to offset what would have earned from working harder today for more savings, people keep their deposits only for transaction rather than for store of value. Obviously, in an economy with strictly positive deposit interest rate, Friedman rule is not sustainable. The intuition is simple. Given that people can make savings with strictly positive interest rate, the lowest possible opportunity cost of holding money should be higher than what implied by the time preference rate. In the following discussion we consider only the case with $\gamma \geq \beta(1+ i_d)$.

4 Coexistence of currency and checking deposits

We first study the economy in which currency and checking deposits are used as means of payment.

Proposition 1 *In a monetary equilibrium, if p_c is not too large or too small, currency is the only means of payment for small transactions and checks are used only in big transactions.*

Notice that in this equilibrium the expected payoff from small transactions is not high enough to compensate the cost of using checks, and so agents will use only currency to pay for the purchases. Checking deposits are used only when its expected payoff from facilitating transactions are high enough to offset the cost of writing checks. We establish some properties of this equilibrium in the following proposition.

Proposition 2 *In the equilibrium in which currency is the only means of payment for small transactions and checks are used only in big transactions:*

1. *In big transactions, $a_h = m + c$, and q_h solves $1 + \frac{\frac{\gamma}{\beta}-(1+i_d)}{\sigma\lambda} = \frac{u'(q)}{b'_h(q)}$. In small transactions, $a_l = m$ and q_l solves $1 + \frac{i_d}{\sigma(1-\lambda)} = \frac{\delta u'(q)}{b'_l(q)}$.*
2. *An increase in the deposit interest rate reduces q_l but raises q_h, resulting a lower currency deposit ratio.*
3. *Inflation has differential impacts on the quantities traded with different means of payment: $\frac{\partial q_h}{\partial \gamma} < 0$ and $\frac{\partial q_l}{\partial \gamma} = 0$.*
4. *Welfare is reduced by inflation.*

Proof. 1. Conditions (11) and (12) hold with equality if currency and checking deposits are to be held. From the two conditions on can solve for q_h and q_l.

2. Note $b_h(q_h) = m + c$ and $b_l(q_l) = m$. From the solutions to q_h and q_l, one finds that $\frac{\partial q_h}{\partial i_d} > 0$ because $\frac{u'(q)}{b'_h(q)}$ is strictly decreasing in q, and $\frac{\partial q_l}{\partial i_d} < 0$. Since $b'_h(q_h) > 0$ and $b'_l(q_l) > 0$, we have $\frac{\partial c}{\partial i_d} > 0$ and $\frac{\partial m}{\partial i_d} < 0$, resulting a lower $\frac{m}{c}$.

3. From the solutions to q_h and q_l, one immediately see $\frac{\partial q_h}{\partial \gamma} < 0$ and $\frac{\partial q_l}{\partial \gamma} = 0$.

4. Let ω denote the welfare, then $(1-\beta)\omega = \sigma\lambda[u(q_h) - v(q_h)] + \sigma(1-\lambda)[\delta u(q_l) - v(q_l)]$. Since q_h is reduced by inflation, so does welfare.■

Conditions (11) and (12) say that in this equilibrium $\sigma(1-\lambda)r_l(m) = 1 + i_d$; that is, the expected liquidity return from holding currency must equal its opportunity cost, which is the interest earned from checking deposits. Note that the deposit interest rate i_d affects the opportunity cost of holding money, and therefore the composition in the portfolios, as described in the proposition. Thus, any policies or factors that affect the deposit interest rate will change the relative prices in different types of transactions. It also implies that those policies or factors have differential impacts on welfare of people who are involved in different transaction with different frequencies. Our analysis is conducted under the assumption that the deposit interest rate is given. However, if inflation affects the deposit interest rate, then we have the following results. If $0 < \frac{\partial i_d}{\partial \gamma} < 1$, then $\frac{\partial q_h}{\partial \gamma} < 0$, $\frac{\partial q_l}{\partial \gamma} < 0$. If $\frac{\partial i_d}{\partial \gamma} > 1$, then $\frac{\partial q_h}{\partial \gamma} > 0$, $\frac{\partial q_l}{\partial \gamma} < 0$. The effects of inflation on the terms of trade in different transactions and thus, welfare, depend on how it affects deposit interest rate.

We now determine the value of money, ϕ. Given the money supply M, the value ϕ satisfies $m = \phi M$. Given q_l, one can solve $m = b_l(q_l)$. So $\phi = \frac{b_l(q_l)}{M}$.

Corollary 1 *The quantities q_h and q_l are less than the quantities that maximize buyer's surplus in the decentralized markets.*

Proof. To get the quantities that maximize buyer's surplus in the decentralized markets, we need $\frac{u'[q_h(a_h)]}{b'_h[q_h(a_h)]} = 1$ and $\frac{\delta u'[q_l(m)]}{b'_l[q_l(m)]} = 1$, which requires $\gamma = \beta$, a contradiction to (12) with $i_d > 0$.■

The implications from lemma 3 are that $\frac{u'[q_h(a_h)]}{b'_h[q_h(a_h)]} > 1$ and $\frac{\delta u'[q_l(m)]}{b'_l[q_l(m)]} > 1$.

4.1 The precautionary demand for money

In this equilibrium fiat money is dominated by checking deposits in the rate of returns, yet both coexists. If the cost of using checking deposits is higher than the expected return from financing small transactions, people are willing to forgo some interest earnings by holding currency in order to pay for the unexpected small-value purchases. The forgone interest earnings may be interpreted as the premium to pay for the insurance for not using checks to make payment for the unexpected purchases. Currency's liquidity value thus derives mainly from facilitating the unexpected small transactions, and we call it the precautionary demand for money. We show in the following proposition that as the possibility of the unexpected purchases is higher, so is the precautionary demand for money.

Proposition 3 *In the equilibrium with currency and checking deposits as means of payment, if agents are more likely to conduct the transactions facilitated by currency, the demand for money is higher and checking deposits lower, resulting a higher currency-deposit ratio.*

Proof. agents are more likely to conduct small transactions if λ is lower. We have $\frac{\partial r_h(a_h)}{\partial \lambda} < 0$ and $\frac{\partial r_l(m)}{\partial \lambda} > 0$, which imply $\frac{\partial q_h}{\partial \lambda} > 0$ and $\frac{\partial q_l}{\partial \lambda} < 0$.■

The currency-deposit ratio in a country has a long-run trend and short-run fluctuations, and it varies across countries. Data show that this ratio rises during inflation and the time when the marginal tax rate is raised. Inflation raises the marginal tax rate. Since there is no record of the payment by using currency, people opt for more cash transactions to reduce the tax burden. Though our steady-state model is not designed for studying fluctuations in the inflation rate and tax rates, it is suited for the study of smoothed data series for a given economy or for cross-country, time-averaged data. One can think of the cash transactions as in the informal sector in which government can not observe transactions and enforce taxes, and transactions using checking deposits as in the formal sector in which transactions are taxable. One can modify this model to show that as tax rate is higher, people may try to avoid taxes by conducting more cash transactions in the informal economy.[8]

[8] Some empirical evidences in Gordon and Li (2005) show that in developing countries taxes on capital, tariffs and seigniorage are important sources of revenues. Inflation policy has the effect of taxing the informal sector.

5 Conclusion

This paper uses a search monetary model to study issues related to multiple means of payment, in which there is a record keeping cost associated with the use of checking deposits. Agent's means-of-payment decisions depend on whether the liquidity return from financing the transactions using checks is higher than the associated cost. Due to the lower expected payoff from small-value transactions, agents tend to use currency to make payments to avoid the cost of using checks. Currency's liquidity value in this economy derives mainly from facilitating unexpected small transactions. Thus we provide a rationale for the precautionary demand for money.

One can incorporate many additional features in the model to study other issues related to multiple means of payment. For example, we assume no costs of holding or using currency, but one can consider the costs of transportation, risk of loss, theft, and counterfeiting. This model assumes that banks operate with zero reserve requirements. Since interest is usually not paid on reserves, a reserve requirement works like an implicit taxation on banks. Therefore, the deposit interest rate would be reduced by reserve requirements, and thus has adverse effects on the terms of trade in cash transactions and currency-deposit ratio. Furthermore, one can introduce some features into the environment that would give rise to the use of credit as well as the use of currency and checking deposits.

We do not consider merchant's incentives to accept different means for payment. The incentives to accept checks for payment may be affected by the time cost of check clearing, and the possibility that checks may be returned due to insufficient funds in the checking account. There is usually a day or two between when a merchant receives a check and when the funds in the checking account are actually deducted for payment. But this problem is less important now under the rapid development of debit cards and other types of electronic payments based on checking deposits, since the amounts are deducted immediately from payers' accounts.[9]

[9] The Check Clearing for the 21st Century Act was passed in October 2003 in the U.S., which speeds up the check-clearing process. The law permits banks to clear funds electronically instead of waiting for paper checks to make their way around the country, thus eliminating the three- to four-day "float" many consumers have come to count on. Check 21 is intended to increase the speed of check clearing, lower clearing system costs, and reduce the financial system's vulnerability to problems with air and ground travel.

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