Stable Lobbies: Lobbying or Free-Riding

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Abstract

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1 Introduction

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2 Lobby Formation Game

We will focus on interest groups' lobbying activities over government policies. We will consider a two stage game. In stage 1, interest groups decide if they join a lobbying process or not. In stage 2, the lobbying groups lobby over government policies. Having stated we assume a two stage game, we have to say that our solution concept is more like a pseudo-dynamic solution, since we consider a stationary state allocation. We start with the setup of a lobby formation game.

2.1 The Setup

There is a set of potential lobby participants (players), $N = \{1, ..., n\}$ and the government G. The government G can choose an agenda a from the set of agendas A. Each player i has utility function $v_i : A \to \mathbb{R}_+$, and similarly the government has utility function $v_G : A \to \mathbb{R}_+$. Each player i can choose if it participate in a lobby or not: if player i decides to participate in lobbying activities, then it can offer a menu of contributions $T_i : A \to \mathbb{R}_+$.

Suppose that subset S of N participates in the lobbying activities. Then, if the government chooses $a \in A$, then the government gets the payoff

$$u_G(a; (T_i)_{i\in S}) = v_G(a) + \sum_{i\in S} T_i(a),$$

and player i gets payoff

$$u_i(a; T_i(a)) = v_i(a) - T_i(a).$$

The government chooses a policy that maximizes u_G:

$$a^*(S) \in \arg\max_{a\in A} u_G(a; (T_i)_{i\in S}).$$

2.2 Lobbying

Given the set of participants S, we can apply the results in Bernheim and Whinston (1986, QJE). Although there are many Nash equilibria in this menu auction game, they show that the set of Nash equilibria in truthful strategies and the set of coalition-proof Nash equilibria are equivalent, and the set of resulting payoffs of these equilibria is characterized as follows. For each $T \subseteq S$, let

$$W(T) \equiv \max_{a\in A} \sum_{i\in T} v_i(a) + v_G(a),$$

and

$$W(\emptyset) \equiv \max_{a\in A} v_G(a),$$

The equilibrium agenda that the government chooses is

$$a^*(S) \in \arg\max_{a\in A} \left(\sum_{i\in S} v_i(a) + v_G(a) \right),$$

and firms' equilibrium payoff vector $u \in \mathbb{R}^S_+$ is an element of the Pareto frontier of set

$$Z \equiv \left\{ u' \in \mathbb{R}^S_+ : \sum_{i\in T} u_i \leq W(S) - W(S\backslash T) \text{ for all } T \subset S \right\}.$$

The inequality that Z satisfies can be interpreted that since the complement set $S\backslash T$ can achieve total payoff $W(S\backslash T)$ by themselves, T cannot ask more than $W(S) - W(S\backslash T)$.

2.3 Lobby Participation

We now consider a stationary state of lobby participation and players' payoffs. An **outcome** is a list $(S, a^*(S), u, u_G) \in 2^N \times \mathbb{R}^N \times \mathbb{R}_+$ such that (i) for all $i \notin S$, $u_i = v_i(a^*(S))$, (ii) for all $T \subset S$, $\sum_{i \in T} u_i \leq W(S) - W(S \backslash T)$, and (iii) $u_G = W(S) - \sum_{i \in S} u_i$.[1] An outcome (S, u, u_G) is **internally stable** if for all $i \in S$, $u_i \geq v_i(a^*(S \backslash \{i\}))$ holds. Internal stability of an outcome requires that no member of lobby can free ride unilaterally. We also say that a participant set S is an **internally stable lobby**, if there exists an internally stable outcome $(S, a^*(S), u, u_G)$. The readers may wonder about external stability. It is a bit harder to define, since when an outsider joins, we need to allocate some payoff to that player. However, we can imagine the following situation: once a new allocation is attained by including the new player, then the new allocation may not be internally stable. Thus, we may use the following: An outcome $(S, a^*(S), u, u_G)$ is **stable** if it is internally stable, and there is no internally stable outcome $(S', a^*(S'), u', u'_G)$ with $S' \supsetneqq S$ and $u'_i > u_i$ for all $i \in S'$.[2] We also say that a participant set S is a **stable lobby**, if there exists a stable allocation $(S, a^*(S), u, u_G)$. Note that there always exists a maximally internally stable outcome, since the set of internally stable participant sets is nonempty ($S = \emptyset$ is always trivially an internally stable lobby for any game) and there are only finite number of players.

Proposition 1. There is stable lobby and outcome.

3 The Case of No Conflict of Interest

First, we consider a simple case in which all players' interests are in the same direction, while the intensity of their interests are heterogeneous. A stylized public good model can be viewed as a special class of the above game. Agenda is a public good provision level, and is one-dimensional: $A = \mathbb{R}_+$, and the provision cost of public good is described by a C^2 cost function $C : A \to \mathbb{R}_+$

[1] An outcome contains $a^*(S)$ as an argument of its definition, since there may be multiple maximizing agendas.

[2] This definition of stability has a similarity with the ones in Conley and Konishi (2002) and Konishi and Ünver (2006).

with $C(0) = 0$, $C'(a) > 0$ and $C''(a) \geq 0$. Player i's utility function is quasi linear in private good net consumption x and is written as $v_i(a) - x$, where $v_i : A \to \mathbb{R}_+$ is $v_i(0) = 0$, $v_i'(a) > 0$ and $v_i''(a) \leq 0$. In order to guarantee the existence of solution, we assume the Inada condition on the cost function: $\lim_{a\to 0} C'(a) = 0$ and $\lim_{a\to\infty} C'(a) = \infty$. Laussel and Le Breton (1998, 2001) extensively studied the equilibrium payoff structures of common agency games on general versions of this public good problem, and obtained many interesting and useful results. Our analysis is built on theirs, but we consider possible free riding: our focus is the conflict between lobbying and free riding.

We will consider a common agency game of this economy. Let $T \subset N$. The efficient public good provision level for T is described by[3]

$$a^*(T) \equiv \arg\max_{a\in A} \sum_{i\in T} v_i(a) - C(a).$$

For a lobby group $S \subset N$, no free-riding incentive means that for all $i \in S$,

$$v_i(a^*(S)) - t_i \geq v_i(a^*(S\backslash\{i\}),$$

where t_i is player i's contribution level. A lobby participant $i \in S$ receives $u_i = v_i(a^*(S)) - t_i$. In this particular game, we have

$$W(S) = \sum_{i\in S} v_i(a^*(S)) - C(a^*(S)).$$

As Laussel and Le Breton (1998, 2001) note, this problem can be reinterpreted as a lobbying problem by letting $u_G(a) = -C(a)$. That is, the government does not want to provide public good, but if players offer enough contributions then public good is provided. Thus, participating in lobbying means consumers' voluntary contributions through menu auction, and not participating means their free riding. The following is a straightforward extension of a result of Laussel and Le Breton (1998, 2001).

Proposition 2. Consider a public good economy. For $S \subset N$, $u_S \in \mathbb{R}^S_+$ is an internally stable allocation if and only if:

1. $\sum_{i\in S} u_i = W(S) = \sum_{i\in S} v_i(a^*(S)) - C(a^*(S))$

[3] Under the assumptions, the maximizer is a singleton.

2. for all $T \subset S$, $\sum_{i \in T} u_i \geq W(T) = \sum_{i \in T} v_i(a^*(T)) - C(a^*(T))$

3. for all $i \in S$, $u_i \geq v_i(a^*(S \backslash \{i\})$.

Proof. Since $W(\emptyset) = -C(0) = 0$, we have $W(S) - W(\emptyset) = W(S)$. As is known from Laussel and Le Breton (2001: Theorem 3.2), if $(W(T))_{T \subset S}$ exhibits convexity (i.e., for all $S \supset T' \supsetneqq T$ with $i \notin T'$, $W(T' \cup \{i\}) - W(T') \geq W(T \cup \{i\}) - W(T)$ holds), then there will not be any rent to the agent:[4] thus, $\sum_{i \in S} u_i = W(S) - W(\emptyset) = W(S)$.[5] This is condition 1. With this property, condition (ii) for the lobbying game outcome for S: for all $T \subset S$,

$$\sum_{i \in T} u_i \leq W(S) - W(S \backslash T),$$

is rewritten as,

$$\sum_{i \in S \backslash T} u_i \geq W(S \backslash T)$$

or by rereading $S \backslash T$ as T,

$$\sum_{i \in T} u_i \geq W(T)$$

for all $T \subset S$. Thus, we have condition 2. Condition 3 follows from the requirement of no free riding (singleton deviation is optimal in public good economy).■

An allocation that satisfies conditions 1 and 2, $u_S \in \mathbb{R}^S_+$, is in the core of a TU game $(W(T))_{T \subset S}$. Thus, an internally stable allocation requires group stabilities, and no free riding incentive. Conditions 2 and 3 are conflicting requirements. A high θ consumers have less free-riding incentives, while if they get together, they can form a strong deviating coalition. Thus, it is possible that for some S, there is no core allocation that satisfies condition 3, even if there is an allocation for S that satisfies codition 3 in general. We consider two special cases: (i) symmetric case where $v_i(a) = v_j(a)$ for all $a \in A$ and all $i, j \in N$, and (ii) quadratic cost case where $v_i(a) = \theta_i a$ and $C(a) = ka^2$ ($\theta_i > 0$ for all $i \in N$ and $k > 0$).

[4] Laussel and Le Breton (2001) calls this property "no rent property,"

[5] Moreover, Konishi, Le Breton and Weber (1999) show that coalition-proof Nash equilibrium is also stong Nash equilibrium with the no rent property.

3.1 Symmetric Players

In this subsection, we assume that all players' payoff functions are the same: i.e., $v_i(a) = v_j(a) = v(a)$ for all $i, j \in N$. The no free rider condition is

$$u_i = v(a^*(S)) - t_i \geq v(a^*(S\backslash\{i\})).$$

Since the core of TU game is a convex set, a symmetric cost sharing should be in the core unless the core is empty. We know that the core is nonempty in public good economy for all $S \subset N$. Thus, what is needed for internal stability is just the following symmetric condition:

$$v(a^*(S)) - \frac{C(a^*(S))}{|S|} \geq v(a^*(S\backslash\{i\})),$$

or

$$|S|v(a^*(S)) - C(a^*(S)) \geq |S|v(a^*(S\backslash\{i\})).$$

Proposition 3. In the symmetric case, there exists an internally stable allocation for S if and only if S satisfies $|S|v(a^*(S)) - C(a^*(S)) \geq |S|v(a^*(S\backslash\{i\}))$ (aggregated "no free riding conditions").

3.2 Linear Utility and Quadratic Cost Functions

In this subsection, we assume $v_i(a) = \theta_i a$ for all $i \in N$ and $C(a) = ka^2$, where $\theta_i > 0$ and $k > 0$ are parameters. In this case, despite of heterogeneous players, the same condition as above is necessary and sufficient for internal stability.

Proposition 4. In the quadratic public good problem, there exists an internally stable allocation for S if and only if S satisfies $\sum_{i \in S} \theta_i a^*(S) - k(a^*(S))^2 \geq \sum_{i \in S} \theta_i a^*(S\backslash\{i\})$ (aggregated "no free riding conditions").

Proof. By Condition 3, if the above request is violated, there is no allocation that satisfies no free riding for S. Thus, we only need to show that if the above request is satisfied then we can find a core allocation that satisfies condition 3. To be instructive, we will not explicitly solve $a^*(T)$ for a while. The strategy we take is to construct an allocation, and verify that it is in the core. Let $u_S \in \mathbb{R}^S_+$ be such that for all $i \in S$

$$u_i = \theta_i a^*(S\backslash\{i\}) + \frac{\theta_i}{\sum_{j \in S} \theta_j} \left(\sum_{i \in S} \theta_i a^*(S) - k(a^*(S))^2 - \sum_{j \in S} \theta_j a^*(S\backslash\{j\}) \right).$$

Notice that the contents of the parenthesis is the aggregated "no free riding" surplus: given the no free riding conditions, the most surplus the lobby group S can distribute for their members. The above formula distribute this surplus proportionally according to members' willingnesses-to-pay θs. Obviously, we have $\sum_{i\in S} u_i = W(S) = \sum_{i\in S} \theta_i a^*(S) - k(a^*(S))^2$, and $u_i \geq \theta_i a^*(S\backslash\{i\})$. Thus, we only need to check condition 2. For a coalition $T \subsetneqq S$, we have

$$\begin{aligned}
&\sum_{i\in T} u_i - W(T) \\
= \; & \sum_{i\in T} \theta_i a^*(S\backslash\{i\}) + \frac{\sum_{i\in T}\theta_i}{\sum_{j\in S}\theta_j}\left(\sum_{j\in S}\theta_j a^*(S) - k(a^*(S))^2 - \sum_{j\in S}\theta_j a^*(S\backslash\{j\})\right) \\
& -\left(\sum_{i\in T}\theta_i a^*(T) - k(a^*(T))^2\right) \\
= \; & \frac{\sum_{i\in T}\theta_i}{\sum_{j\in S}\theta_j}\left(\sum_{j\in S}\theta_j a^*(S) - k(a^*(S))^2\right) - \left(\sum_{i\in T}\theta_i a^*(T) - k(a^*(T))^2\right) \\
& + \sum_{i\in T}\theta_i a^*(S\backslash\{i\}) - \frac{\sum_{i\in T}\theta_i}{\sum_{j\in S}\theta_j}\sum_{j\in S}\theta_j a^*(S\backslash\{j\}).
\end{aligned}$$

We want this to be nonnegative for all $T \subset S$. Now, we use quadratic cost and linear utility. The first order condition for optimal public good provision is

$$\sum_{i\in S}\theta_i - 2ka = 0,$$

or

$$a^*(S) = \frac{\sum_{i\in S}\theta_i}{2k}.$$

Thus, we have

$$\sum_{i\in S}\theta_i a^*(S) - k(a^*(S))^2 = \frac{\left(\sum_{i\in S}\theta_i\right)^2}{2k} - \frac{\left(\sum_{i\in S}\theta_i\right)^2}{4k} = \frac{\left(\sum_{i\in S}\theta_i\right)^2}{4k},$$

and

$$\theta_i a^*(S\backslash\{i\}) = \frac{\theta_i\left(\sum_{j\in S}\theta_j - \theta_i\right)}{2k}.$$

Without loss of generality, we can set $k = \frac{1}{2}$. Then, we have

$$
\begin{aligned}
&\sum_{i \in T} u_i - W(T) \\
= \; & \frac{\sum_{i \in T} \theta_i}{2 \sum_{j \in S} \theta_j} \left(\sum_{j \in S} \theta_j \right)^2 - \frac{1}{2} \left(\sum_{i \in T} \theta_i \right)^2 + \sum_{i \in T} \theta_i \sum_{j \neq i, j \in S} \theta_j - \frac{\sum_{i \in T} \theta_i}{\sum_{i \in S} \theta_i} \sum_{i \in S} \theta_i \sum_{j \neq i, j \in S} \theta_j \\
= \; & \frac{1}{2} \left(\sum_{i \in T} \theta_i \right) \left(\sum_{j \in S} \theta_j \right) + \sum_{i \in T} \theta_i \left(\sum_{j \in S} \theta_j - \theta_i \right) - \frac{\sum_{i \in T} \theta_i}{\sum_{i \in S} \theta_i} \sum_{i \in S} \theta_i \left(\sum_{j \in S} \theta_j - \theta_i \right) \\
= \; & \frac{1}{2} \left(\sum_{i \in T} \theta_i \right) \left(\sum_{j \in S} \theta_j \right) + \sum_{i \in T} \theta_i \left(\sum_{j \in S} \theta_j \right) - \sum_{i \in T} \theta_i^2 - \sum_{i \in T} \theta_i \left(\sum_{j \in S} \theta_j \right) + \frac{\sum_{i \in T} \theta_i}{\sum_{i \in S} \theta_i} \sum_{i \in S} \theta_i^2 \\
= \; & \frac{1}{2} \left(\sum_{i \in T} \theta_i \right) \left(\sum_{j \in S} \theta_j \right) - \sum_{i \in T} \theta_i^2 + \frac{\sum_{i \in T} \theta_i}{\sum_{i \in S} \theta_i} \sum_{i \in S} \theta_i^2 \\
= \; & \left(\sum_{i \in T} \theta_i \right) \left[\frac{\sum_{j \in S} \theta_j}{2} - \frac{\sum_{i \in T} \theta_i^2}{\sum_{i \in T} \theta_i} + \frac{\sum_{i \in S} \theta_i^2}{\sum_{i \in S} \theta_i} \right] \\
= \; & \left(\sum_{i \in T} \theta_i \right) \left[\frac{\sum_{j \in S} \theta_j}{2} - \sum_{j \in T} \frac{\theta_j}{\sum_{i \in T} \theta_i} \times \theta_j + \sum_{j \in S} \frac{\theta_j}{\sum_{i \in S} \theta_i} \times \theta_j \right].
\end{aligned}
$$

The second term is the only negative term, and it takes maximum absolute value when T is composed by the players with the highest values of θ_j. Let us call such value $\theta_{\max}$. Suppose that $\sum_{i \in S} u_i - W(T) < 0$. Then, by focusing the first two terms, we know $\theta_{\max} > \frac{1}{2} \sum_{i \in S} \theta_i$. However, if it is the case, we have

$$
\begin{aligned}
& \frac{\sum_{j \in S} \theta_j}{2} - \sum_{j \in T} \frac{\theta_j}{\sum_{i \in T} \theta_i} \times \theta_j + \sum_{j \in S} \frac{\theta_j}{\sum_{i \in S} \theta_i} \times \theta_j \\
\geq \; & \frac{\sum_{j \in S} \theta_j}{2} - \theta_{\max} + \sum_{j \in S} \frac{\theta_j}{\sum_{i \in S} \theta_i} \times \theta_j \\
\geq \; & \frac{\theta_{\max}}{2} - \theta_{\max} + \frac{\theta_{\max}}{\sum_{i \in S} \theta_i} \times \theta_{\max} \\
> \; & \frac{\theta_{\max}}{2} - \theta_{\max} + \frac{1}{2} \times \theta_{\max} = 0.
\end{aligned}
$$

This is a contradiction. Therefore, all conditions 1, 2 and 3 are satisfied for an allocation u_S.■

It is obvious that a maximal internally stable lobby achieves a Pareto efficient internally stable outcome in public good economies. Note, however, that all Pareto efficient internally stable allocation is achieved by a maximal internally stable lobby. For example, suppose that both S and S' are internally stable lobbies and $S' \supset S$ is maximal. If allocation u under S satisfies $u_j = v_j(a^*(S\backslash\{j\})$ for all $j \neq i \in S$, and $u_i = \sum_{j\in S} v_j(a^*(S)) - C(a^*(S)) - \sum_{j\neq i} v_j(a^*(S\backslash\{j\}))$, then there may not be an internally stable allocation u' under S' with $u'_i \geq u_i$, since everybody besides i need more in order to satisfy internal stability under S' than under S. For this, i needs to have a high θ_i so that condition 2 does not bind.

It should be noted though that there is tendency that a Pareto efficient maximal internally stable allocation tends to be more equal than any other Pareto efficient allocation. It is because under maximal internally stable lobby S', the surplus of lobby defined by

$$\Phi(S') \equiv \sum_{j\in S'} v_j(a^*(S')) - C(a^*(S')) - \sum_{j\in S'} v_j(a^*(S'\backslash\{j\}))$$

must be rather small (otherwise, it can still expand by adding more members). Thus, there is only small room for giving extra to some members of S' in addition to their participation constraint.

Even in quadratic cost case, we can make an interesting observation.[6]

Proposition 5. Even in the quadratic cost case, a maximal internally stable lobby may not be consecutive.

Proof. By the following an example.

Example. Let $v(a) = a$, and $C(a) = \frac{1}{2}a^2$. Suppose that $N = \{1, 2, 3, 5, 11\}$ with $\theta_i = i$ for each $i \in N$. In this case, $a^*(S) = \sum_{i\in S} i$, since for group S, the marginal benefit and cost are $\sum_{i\in S} i$ and a, respectively. Suppose that $S_1 = \{3, 5, 11\}$. Then, $a^*(S_1) = 19$ and $W(S_1) = 180.5$. Now, $11v(a^*(S_1\backslash\{11\})) = 88$, $5v(a^*(S_1\backslash\{5\}) = 70$ and $3v(a^*(S_1\backslash\{3\})) = 48$. Since $88+70+48 > 180.5$, there is no internally stable allocation for S_1. Similarly, there is no internally stable allocation for $S_2 = \{2, 5, 11\}$. However, for $S_3 = \{1, 5, 11\}$, $a^*(S_3) = 17$

[6] Although the context and approach are very different, in political science and sociology, formation of such non-consecutive coalitions is of a tremendous interest. For a game theoretical treatment of this line of literature (known and "Gamson's law"), see Le Breton et al. (2007).

and $W(S_3) = 144.5$. Now, $11v(a^*(S_3\backslash\{11\})) = 66$, $5v(a^*(S_3\backslash\{5\}) = 60$ and $1v(a^*(S_3\backslash\{1\})) = 16$, thus any (u_{11}, u_5, u_1) with $u_{11}+u_5+u_1 = 144.5$ satisfies conditions 2 and 3. Condition 3 requires $u_{11} \geq 66$, $u_5 \geq 60$ and $u_1 \geq 16$. Condition 2 also requires $u_{11} + u_5 \geq 128$, $u_{11} + u_1 \geq 72$ and $u_5 + u_1 \geq 18$ (only the first one can bind given condition 3). As Proposition 4 asserts, $(u_{11}, u_5, u_1) = (66 + 2.5 \times \frac{11}{17}, 60 + 2.5 \times \frac{5}{17}, 16 + 2.5 \times \frac{1}{17})$ for S_3 satisfies all the above conditions, thus is internally stable, and is maximal. This example shows that the maximal internally stable lobby may not be consecutive.■

The intuition is simple. Suppose $\Phi(S)$ is positive. Then by Proposition 1, there is an internally stable allocation for S. Thus, let us find $S' \supset S$ such that $\Phi(S') \geq 0$. If the value of $\Phi(S)$ is not so high, adding high θ player(s) increases $a^*(S')$ a lot. This makes free-riding problem severer, and $\Phi(S') < 0$ may occur by that. However, if low θ player(s) are added, the free-rider problem does not become too severe, and $\Phi(S') \geq 0$ may be more easily satisfied.

4 The Case of Conflict of Interest

There are two agendas $A = \{-1, 1\}$, and there are two groups of players $N = N^+ \cup N^-$: each player $i \in N^+$ has utility function $v_i(a) = \theta_i a$, and each player $j \in N^-$ has utility function $v_j(a) = -\theta_j a$, where $\theta_i, \theta_j > 0$. That is, players in group N^+ prefer $a = 1$ to $a = -1$, while ones in N^- prefer $a = -1$ to $a = 1$. The government's utility function is $u_G(a) = 0$ for all $a \in \{-1, 1\}$ so that the government has no policy preference. For $S \subset N$, we have

$$a^*(S) = \begin{cases} -1 & \text{if } \sum_{i \in N^+ \cap S} \theta_i - \sum_{j \in N^- \cap S} \theta_j < 0 \\ \{-1, 1\} & \text{if } \sum_{i \in N^+ \cap S} \theta_i - \sum_{j \in N^- \cap S} \theta_j = 0 \\ 1 & \text{if } \sum_{i \in N^+ \cap S} \theta_i - \sum_{j \in N^- \cap S} \theta_j > 0 \end{cases}$$

The government gets contributions of the losing groups' utility sum in each case.

In this problem, a maximal internally stable allocation is described in a simple manner. When $\sum_{i \in N^+} \theta_i > \sum_{j \in N^-} \theta_j$, subset $S \subset N$ is a maximal internally stable lobby if $N^- \cap S = N^-$, $\sum_{i \in N^+ \cap S} \theta_i > \sum_{j \in N^-} \theta_j$, and for any $i' \in N^+ \cap S$, $\sum_{i \in N^+ \cap (S \backslash \{i'\})} \theta_i < \sum_{j \in N^-} \theta_j$ holds. Symmetrically, when $\sum_{i \in N^+} \theta_i < \sum_{j \in N^-} \theta_j$, subset $S \subset N$ is a maximal internally stable lobby if

$N^+ \cap S = N^+$, $\sum_{i \in N^+} \theta_i > \sum_{j \in N^- \cap S} \theta_j$, and for any $j' \in N^- \cap S$, $\sum_{i \in N^+} \theta_i < \sum_{j \in N^- \cap (S \backslash \{j'\})} \theta_j$ holds. That is, the group that has majority wins, but if some player in the majority group knows that her group can win without her, then she free-rides. And "maximal" has a good justification here. If a player is in a minority group and is a free-rider, she has an incentive to join the lobby to reverse the result. In the end, the minority group pays nothing, so they lose nothing by joining the lobby. Thus, maximal internally stable lobby is well-motivated in this particular problem, too.

5 Lobby Formation Game

In this section, we define a coalition-proof lobby. Note that we are not only talking about coalition-proof Nash equilibrium allocation in the menu auction stage. We also require that the lobby group formation itself is coalition-proof as well. We first define the first stage lobby formation game assuming that the outcome of each possible lobby S is a coalition-proof Nash equilibrium of a menu auction game played by S. A **lobby formation game** is a list $G(N, \{0,1\}, (u_i)_{i \in N})$, where players' strategy sets are common $\{0,1\}$: 0 and 1 represent "not participating" and "participating" in a lobby, respectively. A strategy profile $\sigma = (\sigma_1, ..., \sigma_n) \in \{0,1\}^N$. Let $S(\sigma) = \{i \in N : \sigma_i = 1\}$. The payoff profile when S is formed as a lobby is an outcome of the common agency game with $S(\sigma)$ being the lobby: the payoff profile $u(\sigma) \in \mathbb{R}^N_+$ is on the Pareto frontier of $\{u' \in \mathbb{R}^N_+ : u'_i = v_i(a^*(S(\sigma)))$ for all $i \in S(\sigma)$, and $\sum_{j \in T} u'_j \leq W(T) - W(S \backslash T)$ for all $T \subseteqq S(\sigma)\}$. For any $V \subseteq N$, any $\sigma \in \{0,1\}^N$ and any $\sigma'_V \in \{0,1\}^V$, the pair (V, σ'_V) is a **strategic coalitional deviation from** σ if $u_i(\sigma'_V, \sigma_{-V}) > u_i(\sigma)$ for every $i \in V$. A strategy profile $\sigma^* \in \{0,1\}^N$ is a **strong Nash equilibrium** of $G(N, \{0,1\}, (u_i)_{i \in N})$ if there exists no strategic coalitional deviation from σ^* (Aumann 1959). This is a very strong requirement, since strong equilibrium is necessarily Pareto efficient. Indeed, in public good provision game, strong Nash equilibrium does not exist in the presence of free riding incentive.

Next we define a weaker solution concept based on credibility of strategic coalitional deviations: coalition-proof Nash equilibrium (Bernheim, Peleg, and Whinston, 1987). Fix game $G(N, \{0,1\}, (u_i)_{i \in N})$. For $V \subseteq N$, consider a **reduced game** $\bar{G}(V, \sigma_{N \backslash V})$ that is a strategic-form game with players in V by letting players in $N \backslash V$ passive players in $G(N, \{0,1\}, (u_i)_{i \in N})$, who always play $\sigma_{N \backslash V}$. A **coalition-proof Nash equilibrium** (**CPNE**) is recursively defined as follows:

(a) For any $i \in N$ and any $\sigma_{-i} \in \{0,1\}^{N\setminus\{i\}}$, strategy $\sigma_i^* \in \{0,1\}$ is a **CPNE** of reduced game $\bar{G}(\{i\}, \sigma_{-i})$ if there is no $\sigma_i' \in \{0,1\}$ with $u_i(\sigma_i', \sigma_{-i}) > u_i(\sigma_i^*, \sigma_{-i})$.

(b) Pick any positive integer $r < |N|$. Let all CPNEs of a reduced game $\bar{G}(W, \sigma_{N\setminus W})$ be defined for any $W \subset N$ with $|W| \leq r$ and any $\sigma_{N\setminus W} \in \{0,1\}^{N\setminus W}$. Then,

   (i) for any $V \subseteq N$ with $|V| = r+1$, σ_V^* is **self-enforcing** in reduced game $\bar{G}(V, \sigma_{N\setminus V})$ if for every $W \subset V$ we have σ_W^* is a CPNE of reduced game $\bar{G}(W, \sigma_{V\setminus W}^*)$ of $\bar{G}(V, \sigma_{N\setminus V})$, and

   (ii) for any $V \subseteq N$ with $|V| = r+1$, σ_V^* is a **CPNE** of reduced game $\bar{G}(V, \sigma_{N\setminus V})$ if σ_V^* is self-enforcing in reduced game $\bar{G}(V, \sigma_{N\setminus V})$, and there is no other self-enforcing σ_V' such that $u_i\left(\sigma_V', \sigma_{N\setminus V}\right) > u_i\left(\sigma_V^*, \sigma_{N\setminus V}\right)$ for every $i \in V$.

For any $V \subseteq N$ and any strategy profile σ, let $CPNE(G(V, \sigma_{N\setminus V}))$ denote the set of CPNE strategy profiles on V for the game $\bar{G}(V, \sigma_{N\setminus V})$. For any strategy profile σ, a strategic coalitional deviation (V, σ_V') from σ is **credible** if $\sigma_V' \in CPNE(\bar{G}(V, \sigma_{N\setminus V}))$. A CPNE is *a strategy profile that is immune to any credible strategic coalitional deviation*.

We claim the following.

Conjecture. In both games without conflict and with conflict, an allocation $(S, a^*(S), u, u_G)$ is stable if and only if $G(N, \{0,1\}, (u_i)_{i\in N})$ with $u_i(\sigma(S)) = u_i$ for all $i \in N$. Moreover, S is maximal internally stable lobby. In games without conflict, $u_G = 0$, while in games with conflict, $u_G > 0$.

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