Assignment of Workers with Rigid Job Separation and Optimal Inflation

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September, 2025

Abstract

This paper studies the implication of worker's efficient job-to-job transition without unemployment for the optimal rate of inflation in New Keynesian models with the Calvo and menu-cost frictions. A matrix-valued job-to-job matching function whose two inputs are discrete type distributions of households and firms on the job market can be used to describe the assignment of workers to firms as a joint matching distribution of workers with different productivity levels and firms with different product prices allowing for rigid job separation. The aggregate labor productivity rises with inflation when high-productivity workers are assigned to high-output firms with relatively low product prices. The optimal rate of inflation is positive even with a realistic duration of employment relation while it falls as employment duration increases.

JEL classification: E31, E52, E58

Keywords: Assignment of Workers to Firms; Assignment Function; Job-to-Job Matching Function; Rigid Job Separation; Endogenous Frequency of Price Adjustment; Optimal Inflation; Relative Price Distortion

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E-mail address: tackyun@snu.ac.kr. I am grateful to Kosuke Aoki, Marianne Baxter, Ippei Fujiwara, Gabriel Lee, Ilhyock Shim, seminar participants of Keio University, the Bank of International Settlements (BIS, Basel), the University of Regensburg for helpful comments, and especially to Michael Woodford for his encouragement with detailed comments. This draft is a substantially modified version of previous drafts whose titles are "Assignment of Workers and Optimal Inflation" and "Heterogeneous Workers and Optimal Inflation" All errors are mine.

1 Introduction

A lot of academic and practical research works have been devoted to address the issue of what inflation rate is socially desirable while actual central banks usually set positive inflation targets. Meanwhile, a recent trend in macroeconomics is to emphasize that microeconomic heterogeneity should not be omitted in the equilibrium analysis of macroeconomics. For example, Kaplan, Moll, and Violante (2018) and Kaplan and Violante (2018) point out that Heterogenous Agent New Keynesian Models (hereafter HANK Models) provide rich theoretic frameworks for the quantitative analysis of interactions between cross-sectional distributions and aggregate dynamics leading to a better understanding of the transmission mechanism of monetary policy than Representative Agent New Keynesian Models (hereafter RANK Models).

In addition, the important role of the assignment of workers with different productivity levels to firms in the performance of individual firms is also a long-standing topic in economics as can be seen from works of Becker (1973), Sattinger (1975, 1993), Shimer and Smith (2000), and Shimer (2005). But little has been discussed on the implication of the assignment of workers to firms for the determination of the optimal inflation rate. A realistically plausible doubt about this motivation is associated with the issue of whether a significant assignment problem can arise between around 10 percent of firms that fix their prices for one year and around 10 percent of workers who find new workplaces reflecting the fact that workers move across firms only every eight years. Even with such a rigid job separation, the efficient assignment of workers to firms helps to obtain the optimality of a positive inflation. The reason for this argument is that a positive inflation is optimal when the productivity ratio of non-adjusting and price-adjusting firms is higher than one.

To fill such a gap, an assignment model of workers with different productivity levels are incorporated into an otherwise standard New Keynesian models such as Woodford (2003) with a modification that productivity levels of households are not homogeneous. In addition, ex-ate technologically homogeneous firms also can be different when nominal product prices are changed infrequently in a staggered fashion reflecting random fixed costs of price-adjustment in light of Dotsey, King, Wolman (1999), Nakamura and Steinsson (2010), Alvarez and Lippi (2022), and Auclert, Rigato, Rognlie and Straub (2024).

In order to avoid potential confusion, it would be desirable to address some issues associated with the set-up of the model. The first issue is the reconciliation of the absence of unemployment with the presence of an aggregate matching function that is comparable the aggregate matching function of Mortensen and Pissarides (1994) whose two input variables are numbers of unemployed workers and vacant jobs to get an outcome number of matched workers and firms. In the model of this paper, a matrix-valued job-to-job matching function is used to describe how workers with different productivity levels are assigned to firms with different product prices allowing for infrequent separation of their employment relation. The two inputs of the matrix-valued job-to-job matching function are distributions of worker's productivity levels and firm's product prices and the corresponding output is a joint distribution of productivity level and product price.

The second issue is associated with the following two facts. The first fact is that while the average job tenure of workers is more than 4 years from the BLS data, a substantial fraction of workers make job-to-job moves in the actual world. The second fact is that workers are on average relatively less productive at the time when they enter the labor market for the first time or relocate to unexperienced new industries and then become more productive reflecting learning-by-doing in their workplaces. In order to reflect these facts, the model of this paper allows for the possibility that households can alternate between high-productivity and low-productivity workers because they are subject to exogenous job separation and relocation shocks with the learning-by-doing accumulation of production knowledge. In addition, producers also can alternate between price-adjusting and non-adjusting firms, while they are ex-ante technologically homogeneous. In particular, such time-varying and alternating types of households and firms is different from the usual feature of assignment models where agents have permanently different productivity levels in their joint productions, as can be seen in works of Becker (1973), Sattinger (1975, 1993), Shimer and Smith (2000), and Shimer (2005).

For this reason, concepts of positive and negative assignments between workers and firms in the model of this paper are different from those used in assignment models. Specifically, a positive assortative matching in the literature on assignment models typically means that ex-ante high-productivity workers should match with ex-ante high-productivity firms. But a positive assignment in the model of this paper ends up with matchings between high-productivity workers and ex-post high-productivity firms because firms are ex-ate technologically identical. In this regard, while it is hard to say that this feature is exactly in line with the observed data, it is at least not inconsistent with empirical results of Haltiwanger, Hyatt, and McEntarfer (2018), which suggest that high-productivity workers move from low-productivity firms toward high-productivity firms through job-to-job transitions and end up with higher wages.

The third issue is associated with the feature that firm's output size is determined solely by its relative price following the model of Dixit and Stiglitz (1977). Hence in the model of this paper, product demands of price-adjusting firms are lower in a monopolistically competitive market under a positive rate of inflation than those of non-adjusting firms in the same way as is done in prototypical New Keynesian models where technologically identical firms are subject to the Calvo and menu-cost frictions. While it should be admitted that this feature may not be realistic because firm's output size is determined by various important factors in actual economies, the model of this paper continues to adopt the setup of New Keynesian models where the welfare cost of inflation arises in the presence of one-to-one correspondence between output dispersion and relative price dispersion. In this context, firm's output size is identical unless there is price dispersion. Hence the model of this paper abstracts from the firm-size effect on the frequency and size of price changes.

Given the set-up of the model discussed above, the optimal inflation is associated with the minimization of relative price distortion, markup distortion due to monopolistic competition in product markets, and menu-cost distortion of price adjustment as can be seen in Clarida, Gali and Gertler (1999), Woodford (2003), Yun (2005), Benigno and Woodford (2005, 2012), Nakov and Thomas (2014), Adam and Weber (2019, 2023) and e.t.c. In this vein, the following two results would be worthy of discussion. First, a zero inflation is optimal in both of the Calvo and stochastic menu-cost models without subsidies to eliminate the inefficient steady-state markup distortion as shown in Benigno and Woodford (2005, 2012) and Nakov and Thomas (2014). Second, a positive inflation is optimal in New Keynesian models with the Calvo and menu-cost frictions when firm-specific productivity levels of non-adjusting firms dominate those of price-adjusting firms as can be seen in Adam and Weber (2019, 2023).

An important distinction of this paper from these works is that productivity levels of price-adjusting and non-adjusting firms are determined as a labor market's assignment of workers to firms. The welfare consequence of this distinction lies in the welfare benefit of inflation that dominates welfare costs of inflation listed above. The reason for this result is that the aggregate labor productivity rises with inflation under the efficient assignment of workers to firms. For this reason, a positive inflation is optimal in New Keynesian models under the Calvo and stochastic menu-cost frictions, which is different from results of Benigno and Woodford (2005, 2012) and Nakov and Thomas (2014).

The rest of this paper is organized as follows. The discussion of section 2 is devoted to the review of the related literature. Section 3 demonstrates how high-productivity workers and low-productivity workers are hired by price-adjusting and non-adjusting firms and its consequences for the aggregate labor productivity in a two-period stochastic menu-cost model of Dotsey, King, and Wolman (1999). Section 4 analyzes the social planner's planner's problem and its implication for the optimal inflation on the basis of the two-period model presented in section 3. Section 5 discusses the robustness of the results obtained in the previous section on the basis of the Calvo model. Section 6 concludes.

2 Discussion of Related Literature

In this section, the literature review consists of two parts, The first part is the literature review of the optimal inflation to explain the reason why the assignment of heterogeneous workers to firms can change existing views of New Keynesian models about the welfare cost of inflation. The second part involves the literature review of assignment models that helps to justify the incorporation of assignment problem between households and firms into otherwise standard New Keynesian models as appropriate one for the analysis of the optimal inflation.

The welfare cost of inflation in New Keynesian models with staggered price-setting behaviors of firms mainly arises because inflation distorts the equilibrium relation between the aggregate output and aggregate labor by generating relative price dispersion as discussed in Woodford (2003). On top of this one, a zero inflation also minimizes menu-costs of price changes because profit-maximizing firms do not have incentives to adjust prices at a zero inflation as emphasized in Nakov and Thomas (2014). In particular, theses two results hold in many variants of New Keynesian models to the extent which both firms and households are technologically homogeneous.

But the welfare cost of inflation can be affected by the relaxation of the assumption that both firms and households are technologically homogeneous. For example, the welfare benefit of inflation can exist in the presence of technologically heterogeneous firms with different firm-specific productivity levels as shown in Adam and Weber (2019, 2023). A rough summary of their result is that the optimal rate of inflation is positive under the Calvo and menu-cost frictions when price-adjusting firms are less productive than non-adjusting firms.[1] An implication of their results is that the optimal rate of inflation can be negative when price-adjusting firms are more productive than non-adjusting firms.

The welfare benefit of inflation also can take place in models with nominal wage rigidity and search frictions such as Mortensen and Pissarides (1994). In this direction, Carlsson and Westermark (2016) demonstrates that when the aggregate employment and output remain at inefficiently low levels, the social planner can use a positive inflation to lower the real wage in an attempt to move up the aggregate employment and output toward their efficient levels in the presence of nominal wage rigidity in the labor market with search frictions. The difference of this paper from theirs abstracts from nominal wage rigidity and search frictions such as Mortensen and Pissarides (1994), while the welfare benefit of inflation relies on the labor market's role of assigning workers to firms in the model of this paper.

Turning to the second part, it is well-known that a lot of assignment models have successfully

[1] A real-world example of this mechanism is the product substitution between new and old products (of product life cycle) with a firm-productivity trend starting from relatively inefficient initial production of new products toward more efficient production of mature products as discussed in Adam and Weber (2019, 2023).

explained a lot of important actual phenomena in various direction since the work of Becker (1973). An important issue in this literature is how to implement the efficient matching between households and firms whose permanently fixed characteristics are heterogeneous as can be seen in works of Sattinger (1975, 1993), Shimer and Smith (2000), and Shimer (2005). Important features of assignment models also have been incorporated in dynamic search-matching equilibrium models for the analysis of aggregate consequences of skill mismatch observed in the actual labor market including Sahin, Song, Topa, and Violante (2014), Lise and Robin (2017), Vandeplas and Thum-Thysen (2019), Baley, Figueiredo, and Ulbright (2022), and Moscarini and Postel-Vinay (2023).[2]

While the incorporation of workers with different productivity levels into a general equilibrium model is not new, the model of this paper differs from many existing assignment models for the following three features. First, household's skill characteristic is not fixed over time. Second, firms are ex-ate technologically homogeneous even though their productivity levels become ex-post heterogenous depending on which workers are hired. Third, output sizes of firms is determined solely by their product prices reflecting a typical feature of New Keynesian models. In this economic environment, price-adjusting firms become less productive and have lower output levels than non-adjusting firms under a positive assignment while individual firms are ex-ante technologically identical.

While this feature originates from the set-up of New Keynesian models, it would generate curiosity about the empirical plausibility of the model in this paper reflecting the possibility that large and high-productivity firms tend to change more frequently than small and low-productivity firms. In fact, Goldberg and Hellerstein (2011) show that large firms change prices two or three times more frequently than small firms on the basis of 1987-2008 micro data collected by BLS for the producer price index (PPI). In this context, the first point is that the model of this paper does not have implication of the relation between ex-ante firm-size and frequency of price adjustment and the relation between ex-ante firm productivity and the frequency of price adjustment because firms are ex-ante identical when they have the same product price. But as the second point, the model of this paper predicts that the frequency of price-adjustment of large-output and high-productivity firms is higher than that of small-output and low-productivity firms, depending on which state-dependent pricing model is adopted.

In order to exemplify how this prediction works, firms set prices following a two-period model of Dotsey, King, and Wolman (1999). In this case, there are two groups of firms in period t. One is price-adjusting firms that choose P_t^* at period t and the other is non-adjusting firms that chose P_{t-1}^* at period $t-1$. The output of price adjusting firms is lower than that of non-adjusting firms

[2] For example Robin (2011) explains why low wages and high wages are more pro-cyclical than wages in the middle of the wage distribution. Lise and Robin (2017) analyzes how the reallocation of currently employed workers to more appropriate matches is related to the business cycle. Moscarini and Postel-Vinay (2023) shows that competition for employed workers transmits aggregate shocks to wages, thus leading to the aggregate inflation.

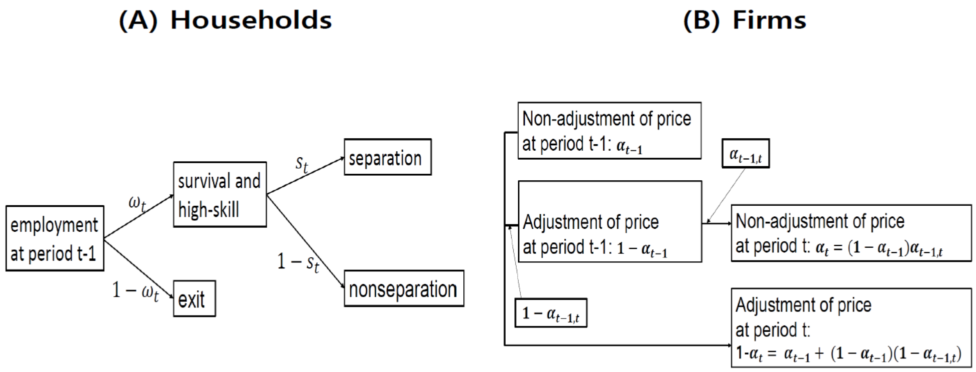
because $P^*_{t-1} < P^*_t$ tends to hold under a positive inflation. In addition, price adjusting firms are less productive than non-adjusting firms under a positive assignment. Hence non-adjusting firms are large-output and high-productivity firms, while adjusting firms are small-output and low-productivity firms.

In this set-up, the fraction of firms that choose to adjust prices between periods t and $t+1$ is higher in the group of non-adjusting firms than in the group of price-adjusting firms. The reason for this argument is that all of non-adjusting firms adjust prices between periods t and $t+1$, whereas only a fraction of price-adjusting firms $(1-\alpha_{t,t+1})$ do so between periods t and $t+1$ with $0 < \alpha_{t,t+1} \leq 1$. Hence the frequency of price-adjustment of firms with large-output and high-productivity is higher than that of firms with small-output and low-productivity in this set-up.

In fact, the model of this paper has no direct implication for the issue of how sizes and productivity levels of individual firms affect the frequency of price adjustment because these characteristics of individual firms are assumed to be ex-ante identical. Hence while it should be admitted that this example is not in line with the motivation of empirical studies on how sustainable output-sizes and productivity levels of individual firms affect their pricing behaviors for their products, it is at least useful to understand that the model is not at odds with actual data.

Turning to the literature on empirical studies on whether and how positive assignment is implemented in actual labor markets, the work of Haltiwanger, Hyatt, and McEntarfer (2018) demonstrates that a substantial fraction of workers move from less productive firms to more productive firms through job-to-job transitions. Fallick, Haltiwanger and McEntarfer (2012) emphasize that the fraction of job-to-job separations is slightly higher than 60% among all job separations for years of 1995, 1999, and 2001. In addition, works of Hahn, Hyatt, Janicki, and Tibbets (2017), Hyatt and McEntarfer (2012), and Fallick, Haltiwanger and McEntarfer (2012) underscore the importance of non-employment durations among job changers for their earnings outcomes, reflecting that job changers with short periods of non-employment tend to show increases in their earnings. Given these empirical observations of worker's job-to-job moves, the labor market in the model of this paper makes it possible for high-productivity workers to work at high-productivity firms through job-to-job transitions with a zero duration of non-employment through the positive assignment of high-productive workers to non-adjusting firms. In this light, the job-to-job moves generated by the positive assignment in the model of this paper is in line with the observed result of actual job-to-job moves.

Figure 1: Distributions of Households and Firms



3 A Model with Two Types of Firms and Workers

Households determine consumption demands and labor supplies and firms are monopolistically competitive suppliers of differentiated goods, following the prototypical set-up of New Keynesian models such as Woodford (2003) with a modification that productivity levels of households are not homogeneous. In addition, nominal product prices are changed infrequently in a staggered fashion reflecting random fixed costs of price-adjustment in light of Dotsey, King, Wolman (1999), Nakamura and Steinsson (2010), Alvarez and Lippi (2022), and Auclert, Rigato, Rognlie and Straub (2024). Hence technologically homogenous firms with different prices can have ex-post different productivity levels depending on which workers are assigned to which firms while productivity levels of households also can change over time. For this reason, the concept of assignment function $V_t(m,k)$ is used to represent the measure at period t of type m workers who work at type k firms, which is endogenously determined in the labor market.

In particular, the title of this section reflects a set of simplifying assumptions that there are only two different types of firms and workers whose measures are one respectively, which in turn facilitates to keep track of the aggregate consequences of interactions between temporarily heterogeneous characteristics of households and firms. For simplicity, firms are assumed to fix prices for at most two periods and households are identified as either high-productivity workers or low-productivity workers in this section (but this assumption is relaxed in Section 4 of this paper).

3.1 Economic Environment

The whole economy consists of a continuum of islands whose mass is one, following Woodford (2003) and Gertler and Leahy (2008). On each island, there is a continuum of households of mass unity. Households consume, supply labor and hold bonds. In addition, perfect consumption insurance is available across islands in the sense that it insulates consumption levels of individual households from island specific shocks. Profit flows of individual firms are redistributed lump-sum to households. Each period, a fraction of households $1-\omega_t$ relocate to a new island and the remaining fraction of households do not move, while households can supply labor only on the island they live on. Hence, on each island, a fraction of workers $1-\omega_t$ enters the labor market for the first time. All worker start as low-productivity workers in the initial period when they enter the labor market for the first time and then become high-productivity workers in the next period onward. For simplicity, workers are supposed to have only two different productivity levels with only one-period for a low-productivity worker to become a high-productivity worker.

Figure 1 illustrates how states of individual firms and households change over time. Panel (A) in the left-hand side shows the evolution of a worker's state and panel (B) in the right-hand side demonstrates how individual firms adjust prices. Panel (A) shows that each high-productivity worker either survives as a high-productivity worker with a probability of ω_t or exits out of the labor force with a probability of $(1-\omega_t)$. Conditional on survival, each worker makes job-to-job transition to a new job with a probability of s_t or does not separate from the current employer with a probability of $(1-s_t)$. The introduction of this feature into the model is intended to match the BLS report that the median tenure of employees is roughly four 4 years, which corresponds to setting $s_t = 0.75$. In addition, a fraction of low-skill workers $(1-\omega_t)$ enter the labor market. Hence the measure at period t of high-productivity workers $(=\Gamma_t)$ is determined as follows.

$$\Gamma_t = (1-\omega_{t-1})\omega_t + \omega_t\Gamma_{t-1}$$

The solution of this difference equation is $\Gamma_t = \omega_t$. Hence the fraction of low-productivity workers is $(1-\omega_t)$ and that of high-productivity workers is ω_t while their productivity levels can be affected by which firms they work for.

A survival shock for each worker takes place at the beginning of each period, which in turn determines the distribution of workers across firms at the beginning of each period. Before the survival shock, all workers become high-productivity workers and thus the measure of high-productivity workers is one. But after the survival shock, a fraction of workers $(1-\omega_t)$ move out of the labor market so that the remaining measure of high-productivity workers is $(1-\alpha_t)\omega_t$ for firms with low demands and $\alpha_t\omega_t$ for firms with high demands. Moreover, a fraction of workers $(1-s_t)$ do not change their jobs in each period, while the remaining fraction of workers s_t make job-to-job transition to new firms. Hence the measure of vacant jobs of firms with low demands is $(1-\alpha_t)(1-(1-s_t)\omega_t)$ and that of firms with high demands is $\alpha_t(1-(1-s_t)\omega_t)$. In order to fill vacant jobs, firms hire either high-productivity workers or low-productivity workers who are in the job market. The number of high-productivity workers who look for jobs is $s_t\omega_t$ and the number of low-productivity workers who look for jobs is $(1-\omega_t)$. It means that distributions of households who are on the job market can be summarized by a 2×1 column vector $[1-\omega_t, s_t\omega_t]'$ and the distribution of vacant jobs is given by a 2×1 column vector $[(1-\alpha_t)(1-(1-s_t)\omega_t), \alpha_t(1-(1-s_t)\omega_t)]'$.

The role of labor market in each island can be described by a matrix-valued matching function $M : R^2 \times R^2 \to R^{2\times 2}$ of the following form:

$$M(\begin{bmatrix} 1-\omega_t \\ s_t\omega_t \end{bmatrix}, \begin{bmatrix} (1-\alpha_t)(1-(1-s_t)\omega_t) \\ \alpha_t(1-(1-s_t)\omega_t) \end{bmatrix}) = \begin{bmatrix} V_t(0,0) & V_t(0,1) \\ V_t(1,0) & V_t(1,1) \end{bmatrix}$$

where $V_t(0,0)$ is the measure of low-productivity households who work at price-adjusting firms, $V_t(0,1)$ is the measure of low-productivity households who work at non-adjusting firms, $V_t(1,0)$ is the measure of high-productivity households who work at price-adjusting firms, and $V_t(1,1)$ is the measure of high-productivity households who work at non-adjusting firms. The left argument of function M records measures of households (with different productivity levels) who are on the job market and the right argument includes measures of vacant jobs posted by firms with different prices. The output of function M is a 2×2 matrix that records how low- and high-productivity workers are distributed between price-adjusting and non-adjusting firms. In fact, 4 elements in this 2×2 matrix correspond to values of the assignment function that satisfy the following restriction:

$$V_t(0,0) + V_t(0,1) + V_t(1,0) + V_t(1,1) = 1$$

It would be helpful to discuss the following two restrictions for the matrix-valued matching function $M : R^2 \times R^2 \to R^{2\times 2}$ described above. The first restriction is that matching function M should be homogeneous of degree one for its input vectors as follows.

$$\begin{aligned} & M(\begin{bmatrix} 1-\omega_t \\ s_t\omega_t \end{bmatrix}, \begin{bmatrix} (1-\alpha_t)(1-(1-s_t)\omega_t) \\ \alpha_t(1-(1-s_t)\omega_t) \end{bmatrix}) \\ = \quad & (1-\omega_t+s_t)M(\begin{bmatrix} \frac{1-\omega_t}{1-\omega_t+s_t} \\ \frac{s_t\omega_t}{1-\omega_t+s_t} \end{bmatrix}, \begin{bmatrix} \frac{(1-\alpha_t)(1-(1-s_t)\omega_t)}{1-\omega_t+s_t} \\ \frac{\alpha_t(1-(1-s_t)\omega_t)}{1-\omega_t+s_t} \end{bmatrix}) \end{aligned}$$

In the second-line representation, input vectors become stochastic vectors whose nonnegative elements sum up to one. The second restriction is that the output matrix of function M should be a joint probability matrix where sums of its rows correspond to household's distribution and sums of its columns correspond to firm's distribution. Moreover, it would be also helpful to show a matrix-valued job-to-job matching function for N_F types of firms and N_H types of workers without having detailed discussion while it could be a digression given the purpose of this section.

Table 3.1: Values of Assignment Function

A. Positive Assignment

	$1 < \frac{s_t\omega_t}{\alpha_t(1-\omega_t(1-s_t))}$	$1 > \frac{s_t\omega_t}{\alpha_t(1-\omega_t(1-s_t))}$
$V_t(0,0)$	$1-\omega_t$	$(1-\alpha_t)(1-\omega_t(1-s_t))$
$V_t(0,1)$	0	$\alpha_t(1-\omega_t(1-s_t)) - s_t\omega_t$
$V_t(1,0)$	$\omega_t - \alpha_t$	$(1-\alpha_t)(1-s_t)\omega_t$
$V_t(1,1)$	α_t	$(\alpha_t(1-s_t)+s_t)\omega_t$

B. Negative Assignment

	$1 < \frac{s_t\omega_t}{(1-\alpha_t)(1-\omega_t(1-s_t))}$	$1 > \frac{s_t\omega_t}{(1-\alpha_t)(1-\omega_t(1-s_t))}$
$V_t(0,0)$	0	$(1-\alpha_t)(1-\omega_t(1-s_t)) - s_t\omega_t$
$V_t(0,1)$	$1-\omega_t$	$\alpha_t(1-\omega_t(1-s_t))$
$V_t(1,0)$	$1-\alpha_t$	$((1-\alpha_t)(1-s_t)+s_t)\omega_t$
$V_t(1,1)$	$\omega_t+\alpha_t-1$	$\alpha_t\omega_t(1-s_t)$

Note: $0 < \alpha_t < 1$, and $0 < \omega_t < 1$.

Definition 3.1 (Matrix-Valued Job-to-Job Matching Function) For a pair of $N_H \times 1$ vector v_t^H and $N_F \times 1$ v_t^F whose elements satisfy $v_t^H(m) \geq 0$ for $m = 1, \cdots, N_H$ and $v_t^F(k) \geq 0$ for $k = 1, \cdots, N_F$, there exists a matrix-valued function of the following form:

$$M(v_t^H, v_T^F) = [V_t(m,k)]_{1 \leq m,k \leq (N_H \times N_F)}$$

with the following set of restrictions

$$\begin{array}{lcll} \sum_{m=1}^{N_H} v_t^H(m) & = & \sum_{k=1}^{N_F} v_t^F(k) & \\ \sum_{m=1}^{N_H} V_t(m,k) & = & m_t^F(k) & \text{for each } k = 1, \cdots, N_F \\ \sum_{k=1}^{N_F} m_t^F(k) & = & 1 & \\ \sum_{k=1}^{N_K} V_t(m,k) & = & m_t^H(m) & \text{for each } m = 1, \cdots, N_H \\ \sum_{m=1}^{N_H} m_t^H(m) & = & 1 & \\ V_t(m,k) & \geq & 0 & \text{for all } m \text{ and } k \end{array}$$

3.2 Assignment of Workers to Firms

The aim of this subsection is to show how values of assignment function is determined by using simple distributions of firms and households. Specifically, when measures of price-adjusting and non-adjusting firms is $(1-\alpha_t)$ and α_t respectively and measures of low-productivity and high-productivity workers are $(1-\omega_t)$ and ω_t respectively, there are only two different ways of assigning workers to firms. The first one is to assign high-productivity workers to high-output firms and low-productivity workers to low-output firms. The second one is to assign high-productivity workers to firms with low demands and low-productivity workers to firms with high demands. Table 3.1

summarizes how values of assignment function are determined depending on two possible different ways of assigning workers to firms. Table 3.1.A corresponds to the positive assignment that means the match between high(low)-productivity workers and high(low)-output firms. Table 3.1.B corresponds to the negative assignment that means the match between high(low)-productivity workers and low(high)-output firms.

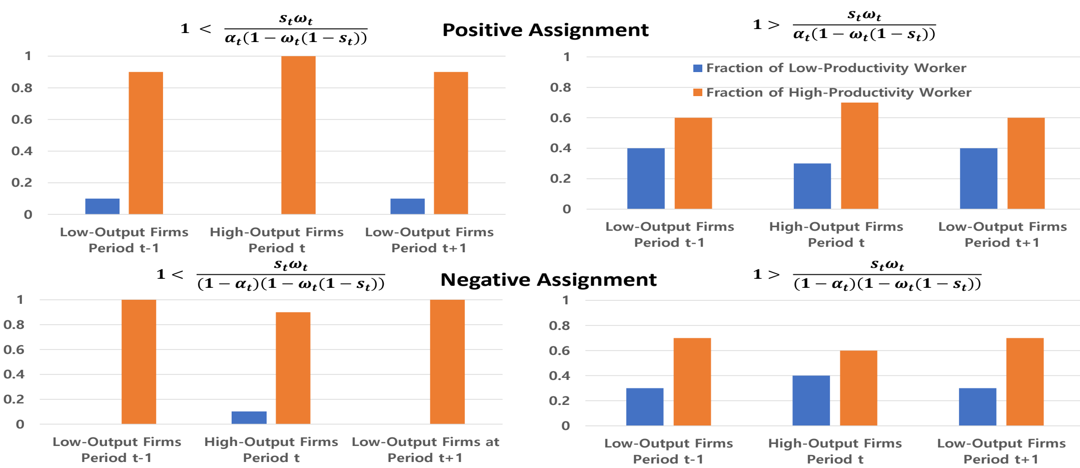
Table 3.1.A demonstrates two cases of the positive assignment. The left column corresponds to the case where job openings of non-adjusting firms are smaller than high-productivity job searchers: $\alpha_t(1 - s_t(1 - \omega_t)) < s_t\omega_t$. In this case, high-productivity workers are hired by both non-adjusting and adjusting firms: $V_t(1, 1) = \alpha_t$ and $V_t(1, 0) = \omega_t - \alpha_t$ and all low-productivity workers work at adjusting firms: $V_t(0, 0) = 1 - \omega_t$. The right column corresponds to the case where job openings of non-adjusting firms are larger than high-productivity job searchers: $\alpha_t(1 - s_t(1 - \omega_t)) > s_t\omega_t$. In this case, new employees of non-adjusting firms consist of both high- and low-productivity workers: $V_t(1, 1) = (1 - \alpha_t)(1 - \omega_t(1 - s_t))$ and $V_t(0, 1) = \alpha_t(1 - \omega_t(1 - s_t)) - s_t\omega_t$ and new employees of adjusting firms are all low-productivity workers: $V_t(0, 0) = (1 - \alpha_t)(1 - \omega_t(1 - s_t))$ and $V_t(h, a) = (1 - \alpha_t)(1 - s_t)\omega_t$.

Table 3.1.B demonstrates two cases of the negative assignment. The left column corresponds to the case where job openings of adjusting firms are smaller than high-productivity job searchers: $(1-\alpha_t)(1-s_t(1-\omega_t)) < s_t\omega_t$. In this case, high-productivity workers are hired by both non-adjusting and adjusting firms: $V_t(1, 1) = 1 - \alpha_t$ and $V_t(1, 0) = \omega_t + \alpha_t - 1$ and all low-productivity workers work at non-adjusting firms: $V_t(0, 1) = 1 - \omega_t$. The right column corresponds to the case where job opening of adjusting firms are larger than high-productivity job searchers: $(1 - \alpha_t)(1 - s_t(1 - \omega_t)) > s_t\omega_t$. In this case, new employees of adjusting firms consist of both high- and low-productivity workers: $V_t(1, 0) = ((1 - \alpha_t)(1 - s_t) + s_t)\omega_t$ and $V_t(0, 0) = (1 - \alpha_t)(1 - \omega_t(1 - s_t)) - s_t\omega_t$ and new employees of non-adjusting firms are all low-productivity workers: $V_t(0, 1) = \alpha_t(1 - \omega_t(1 - s_t))$ and $V_t(1, 1) = \alpha_t(1 - s_t)\omega_t$.

Figure 2 illustrates the implication of the assignment of workers to firms (summarized in Table 3.1) for the composition of employed workers as individual firms over time move between low-output and high-output firms. The orange bar represents the fraction of high-productivity workers and the blue bar is the fraction of low-productivity workers. The upper two panels of Figure 2 correspond to Table 3.1.A and the lower two panels of Figure 2 correspond to Table 3.1.B.

The important features of Figure 2 can be summarized as follows. First, heights of orange bars are higher than those of blue bars for all cases. It means that the fraction of high-productivity workers among all employees is always higher than that of low-productivity workers at all times regardless of types of firms reflecting the assumption that the number of experienced workers is larger than that of inexperienced workers. Second, a low-output firm at period t-1 becomes a high-

Figure 2: Evolution of Firm's State and Worker's Productivity Composition



output firm at period t and then a low-output firm at period $t+1$ again in order to emphasize that individual firms do not have permanently fixed characteristics. Third, a change in the assignment of workers to firms does not necessarily mean a radical change in the composition of high-productivity and low-productivity workers employed by individual firms.

Figure 2 demonstrates that a positive assignment makes the fraction of employed high-productivity workers higher when an individual firm becomes a high-output firm than when the firm becomes a low-output firm, whereas a negative assignment makes the fraction of employed high-productivity workers lower when an individual firm becomes a high-output firm than when the firm becomes a low-output firm. Hence high-output firms are more productive than low-output firms under positive assignments, while low-output firms are more productive than high-output firms under negative assignments. In particular, four panels in Figure 2 illustrates the possibility that the productivity difference between price-adjusting and non-adjusting firms can occur with only a slight change in the composition of high-productivity and low-productivity workers among their employees.

3.3 Assignment of Workers and Production Costs

The aim of this subsection is to discuss how production costs of firms are affected by which workers are assigned to which firms, which is in turn reflected in firm's preferences between high-productivity and low-productivity workers. For example, when the productivity effect of a high-productivity worker dominates the worker's wage effect, it means that the more high-productivity workers the lower production costs. In this case, firms prefer to hire high-productivity workers.

Figure 3: Assignment of Workers and Labor Costs

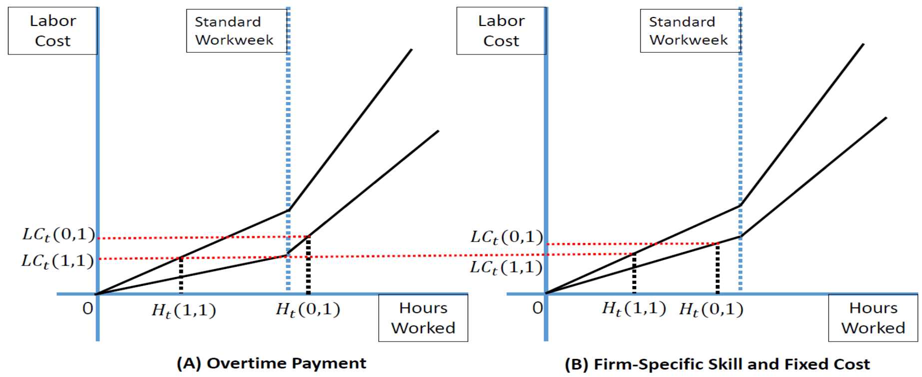


Figure 3 demonstrates how labor costs of firms are affected by the assignment of workers. Panel (A) corresponds to the case where high-productivity workers can complete work-burdens of high-output firms within regular hours but low-productivity workers cannot finish work-burdens of high-output firms within regular hours reflecting the following two facts. The first fact is that the completion of a given task tends to require more hours for unskilled workers than for skilled workers in many workplaces. The second fact is the existence of overtime pay for full-time workers in many countries.[3] The vertical dotted blue line marks the standard workweek. When high-productivity firms hire high-productivity workers, there is no overtime payment because all workers finish their tasks within the standard workweek. However, low-productivity workers who are employed by high-output firms have hours worked more than standard workweek, which in turn leads to overtime payment. Even though the regular wage rate of high-productivity workers is higher than that of low-productivity workers, the labor cost of high-productivity workers is lower than that of low-productivity workers for high-output firms because of overtime payment. It should be noted that this possibility can take place even when worker's skill is purely general.

Panel (B) allows for the possibility that worker's skill has both general and firm-specific features. In the presence of firm-specific skill, characteristics of both workers and firms together should interplay to determine the worker's marginal product of labor. It should be admitted that there are potentially many different ways for the interplay between workers and firms in actual economies.

[3]For example, a basic workweek in the U.S. is 40 hours, and non-exempt employees must receive overtime pay (at least one and a half times their regular rate) for hours worked over 40 hours in a workweek according to the Fair Labor Standards Act summarized in the website of U.S. Department of Labor.

But a relevant one in light of the model of this paper is to address the issue of whether the presence of firm-specific skill makes the match between high-productivity workers and high-output firms more efficient than the match between high-productivity workers and low-output firms. In this context, it is assumed that high-output firms can lower their labor costs by hiring high-productivity workers because the amount of hours worked by high-productivity workers is far lower than that of low-productivity workers, even if the regular wage rate of high-productivity workers is higher than that of low-productivity workers.

The two panels of Figure 3 suggest the following two features of worker's wage rate. First, the wage rate increases with worker's marginal product of labor. Second, the overtime payment exists for the amount of hours worked beyond the standard workweek. In order to incorporate these two features into the wage function, the wage rate for a pair of a type m worker and a type k firm (= $w_t(m,k)$) is defined to be $w_t(m,k) = (1 + d_t o_t(m,k) I_{\{o_t(m,k)>0\}})\ q_t(m,k)Z_t(m,k)w_t/(q_t Z_t)$ where w_t is the aggregate real wage, $(q_t(m,k)/q_t)$ is the wage premium factor, d_t is the overtime premium, Z_t is the aggregate productivity, and $o_t(m,k)$ is the ratio of overtime hours to total working hours with $I_{\{o_t(m,k)>0\}} = 1$ if $o_t(m,k) > 0$ and $I_{\{o_t(m,k)>0\}} = 0$ otherwise. In this case, the aggregate real wage w_t and aggregate wage premium q_t can be written as follows:

$$w_t = \sum_{k=0}^{1}\sum_{m=0}^{1} \omega_t(m,k) w_t(m,k),\ \ q_t = \sum_{k=0}^{1}\sum_{m=0}^{1} V_t(m,k)(1 + d_t o_t(m,k) I_{\{o_t(m,k)>0\}}) q_t(m,k)$$

where weights for the aggregate wage premium are defined as $\omega_t(m,k) = V_t(m,k)Z_t/Z_t(m,k)$ with $\sum_{k=0}^{1}\sum_{m=0}^{1} \omega_t(m,k) = 1$ and the aggregate productivity $Z_t = (\sum_{k=0}^{1}\sum_{m=0}^{1} V_t(m,k)Z_t(m,k)^{-1})^{-1}$. Hence the production cost of a pair of a type m worker and a type k firm (= $LC_t(m,k)$) can be written as

$$\begin{aligned} LC_t(m,k) &= mc_t(m,k)D_{k,t} \\ mc_t(m,k) &= (q_t(m,k)/q_t)(1 + d_t o_t(m,k) I_{\{o_t(m,k)>0\}})(w_t/Z_t) \end{aligned}$$

In sum, the two panels of Figure 3 imply that the production cost is lower when non-adjusting firms hire high=productivity workers than when they hire low-productivity workers and the number of hours high-productivity workers work at non-adjusting firms is lower than the number of hours low-productivity workers work at non-adjusting firms. It means that it is more profitable for non-adjusting firms to hire high-productivity workers than low-productivity workers. In other words, $LC_t(1,1) < LC_t(0,1)$ and $H_t(1,1) < H_t(0,1)$. Hence it will be shown in the next subsection that $v_t^F(1,1) \geq v_t^F(0,1)$ because of $\phi_t(1,1) \geq \phi_t(0,1)$.

3.4 Households

In order to describe the worker's choice of which firm to apply for jobs, the instantaneous utility function at period t of a household h whose skill level and workplace are indexed by (m,k) is represented by the following function

$$u(C_{h,t}(m,k), \bar{H} - H_{h,t}(m,k))$$

where $C_{h,t}(m,k)$ is the worker's consumption, $H_{h,t}(m,k)$ is the worker's hours worked at period t, and $\bar{H}$ is a constant time-endowment. The instantaneous utility function is continuously twice differentiable and strictly concave in its two arguments with positive marginal utilities of consumption and leisure. The worker's flow budget constraint at period t is

$$C_{h,t}(m,k) + B_{h,t}(m,k) = w_t(m,k)H_{h,t}(m,k) + A_{h,t-1}(m,k) + \Phi_t - T_t$$

where $w_t(m,k)$ is the real wage for a type m worker employed by a type k firm, $B_{h,t}(m,k)$ is the real value at period t of bond holdings, $A_{h,t-1}(m,k)$ is the real value of asset holdings at the beginning of period t, Φ_t is the dividend income and T_t is the lump-sum tax at period t.

It should be also noted that individual households with the same skill level and workplace have the same amount of consumption and hours worked at each period:

$$\begin{aligned} C_{h,t}(m,k) &= C_t(m,k) \quad \forall\ h \in [0,\ V_t(m,k)] \\ H_{h,t}(m,k) &= H_t(m,k) \quad \forall\ h \in [0,\ V_t(m,k)] \end{aligned}$$

The following three conditions are needed to obtain this result. First, the real wage is indexed by skill level and workplace i.e. $w_{h,t}(m,k) = w_t(m,k)\ \forall\ h \in [0,\ V_t(m,k)]$. Second, hours worked by individual households are set equal to labor demands of individual firms, so that they are indexed by only skill level and workplace: $H_{h,t}(m,k) = H_t(m,k)\ \forall\ h \in [0,\ V_t(m,k)]$. Third, the optimization condition for labor supply holds true as shown in the following discussion.

The utility-maximization condition for labor supply can be written as follows.

$$\frac{u_2(C_t(m,k), \bar{H} - H_t(m,k))}{u_1(C_t(m,k), \bar{H} - H_t(m,k))} = w_t(m,k) \tag{3.1}$$

The application of the implicit function theorem to the optimization condition specified in equation (3.1) enables one to express each household's consumption as a function of real wage and leisure:

$$C_t(m,k) = C(w_t(m,k), \bar{H} - H_t(m,k)) \tag{3.2}$$

where $C(w_t(m,k), \bar{H}-H_t(m,k))$ is a non-decreasing function of its two arguments. The substitution of equation (3.2) into the instantaneous utility function then leads to the following representation of the instantaneous utility function.

$$u_t(m,k) = u(C(w_t(m,k), \bar{H} - H_t(m,k)), \bar{H} - H_t(m,k)) \tag{3.3}$$

It follows from equation (3.3) that $u_t(m,k)$ can be interpreted as the value of the instantaneous utility function evaluated under the equality between the real wage and marginal rate of substitution between consumption and leisure. Given the existence of an instantaneous utility function of individual workers $\bar{u}_t(m,k)$ derived above, the value function of individual workers is defined as the expected present value of his or her current and future values of the instantaneous utility function:

$$\begin{aligned} v_t^H(m,k) &= \bar{u}_t(m,k) + \beta E_t[\omega_{t+1}((1-s_{t+1})v_{t+1}^H(1,k) \\ &+ s_{t+1}\textstyle\sum_{k=0}^{1}\gamma_{t+1}(1,k)v_{t+1}^H(1,k)) \\ &+ (1-\omega_{t+1})\textstyle\sum_{k=0}^{1}\gamma_{t+1}(0,k)v_{t+1}^H(0,k))] \end{aligned}$$

for $m = 0,\,1$ and $k = 0,\,1$, and where $\gamma_{t+1}(1,k)$ $(= V_{t+1}(1,k)/\sum_{k=0}^{1} V_{t+1}(1,k))$ is the probability at period $t+1$ that a high-productivity worker finds a job at a type k firm. Individual workers then choose between high-output and low-output firms on the basis of the value function $(= v_t^H(m,k))$.

3.5 Firms

Each period, individual firms make a sequence of decisions on whether to adjust prices and which type of workers to hire. Each period, firms make commitment on their nominal product prices without knowing which type of workers to hire. Hence firm's pricing decision is made ahead of its employment decision. Turning to panel (B) of Figure 1, all firms whose prices are adjusted at period $t-1$ can choose at period t between adjustment and non-adjustment of prices. Given that $\alpha_{t-1,1}$ is the non-adjusting fraction of these firms, the measure at period t of non-adjusting firms is determined as follows.

$$\alpha_t = (1-\alpha_{t-1})\alpha_{t-1,t} \tag{3.4}$$

Hence the fraction of firms whose nominal price is P_{t-1}^* at period t is α_t and the fraction of firms whose nominal price is P_t^* at period t is $1-\alpha_t$ reflecting the restriction that the maximum time-length of fixing prices is two-period. Following the model of Dixit and Stiglitz (1977), product demands of individual firms are determined as follows.

$$D_{k,t} = (\frac{P_{t-k}^*}{P_t})^{-\epsilon} D_t$$

with $k = 0$ for adjusting firms and $k = 1$ for non-adjusting firms. In addition, D_t is the aggregate demand at period t and P_t is the aggregate price index at period t:

$$\begin{aligned} D_t &= ((1-\alpha_t)D_{0,t}^{\frac{\epsilon-1}{\epsilon}} + \alpha_t D_{1,t}^{\frac{\epsilon-1}{\epsilon}})^{\frac{\epsilon}{\epsilon-1}} \\ P_t &= ((1-\alpha_t)(P_t^*)^{1-\epsilon} + \alpha_t(P_{t-1}^*)^{1-\epsilon})^{\frac{1}{1-\epsilon}} \end{aligned} \tag{3.5}$$

where ϵ is a positive constant greater than one.

The output of consumption goods produced by a pair of a type m worker and a type k firm is determined by a linear production function of the form: $Y_t(m,k) = Z_t(m,k)H_t(m,k)$ where $Y_t(m,k)$ is the output level, $H_t(m,k)$ is the number of hours worked by a type m worker and $Z_t(m,k)$ is the corresponding marginal product of labor that depend on types of both workers and firms.[4]

[4]The marginal product of labor is specified as $Z_t(m,k) = Z_{m,t}^{\delta(m,k)}$ where $\delta(m,k)$ represents the output elasticity of

The instantaneous profit function at period t of a type k firm that hires a type m worker at period t $(= \phi_t(m,k))$ is defined as $\phi_t(m,k) = ((P^*_{t-k}/P_t) - mc_t(m,k))D_{k,t}$ where $mc_t(m,k)$ is the marginal cost of production at period t. On top of production costs, firms should pay fixed costs of price adjustment when they choose to change prices. Following Dotsey, King, Wolman (1999), Golosov and Lucas (2007), Nakamura and Steinsson (2010), Alvarez and Lippi (2022), and Auclert, Rigato, Rognlie and Straub (2024), fixed costs of price-adjustment for individual firms are proportional to the aggregate real wage $a_t w_t$ where a_t is an idiosyncratic and independent random variable whose cumulative distribution function $\Gamma(a_t)$ defined over a closed and bounded interval $[0,\, a_{\max}]$ with probability density function is $g(a_t)$.

In order to describe how firms change prices, the ex-post value function of individual firms $(=v^F_t(m,k))$ is defined as the expected present value of current and future profit flows. Each period, the criteria for the distinction between ex-post and ex-ate value functions is the employment adjustment of workers. In other words, the distinction between ex-post and ex-ate value functions depends on whether individual firms know which workers are assigned to which firms. In order to compute the ex-post value function of individual firms, one should take into account the following two points. First, the current period's employment relation between firms and workers will remain the same in the next period unless they will be separated. Second, firms fix prices at most for two periods. Given these two points, the ex-post value function at period t of price-adjusting firms can be written as

$$\begin{aligned} v^F_t(m,0) &= \phi_t(m,0) + E_t[\Lambda_{t,t+1}(\alpha_{t,t+1}v^F_{1,t+1} + (1-\alpha_{t,t+1})v^F_{0,t+1} - \bar{c}hi_{t+1})] \\ \bar{\chi}_{t+1} &= w_{t+1}\int_0^{w^{-1}_{t+1}(v_{0,t+1}-v_{0,t+1})} a_{t+1}g(a_{t+1})da_{t+1} \end{aligned}$$

for $m = 0,\, 1$, and where $\bar{\chi}_{t+1}$ represents average costs of price adjustment at period $t+1$. In this representation, ex-ate value functions $\{v^F_{k,t}\}^1_{k=0}$ for price-adjusting and non-adjusting firms can be written as follows.

$$\begin{aligned} v^F_{k,t} &= \sum_{m=0}^{1} \gamma^F_t(m,k)v^F_t(m,k) \\ \gamma^F_t(m,k) &= \gamma_t(m,k)s_t + I_{\{m=1\}}(1-s_t) \end{aligned}$$

where $I_{\{m=1\}}$ is the characteristic function whose value is 1 if $m = 1$ and 0 otherwise. The ex-post value function at period t of price-adjusting firms can be written as

$$v^F_t(m,1) = \phi_t(m,1) + E_t[\Lambda_{t,t+1}v^F_{0,t+1}]$$

for $m = 0,\, 1$. In this representation, it is assumed that menu costs are not paid when firms fix

worker's production skill that determines the magnitude of skill's contribution to the level of output given amounts of hours worked by households. The specification of $\delta(m,k)$ reflects whether or not worker's production skill is general. For example, when workers have purely general skills so that their skills are equally useful for all firms, $\delta(m,k) = \delta$ (> 0) for all m and k. But workers in actual economies tend to have both general and firm-specific skills in light of distinctions made in Acemoglu and Pischke (1999) and Lazear (2009).

their prices for two periods, the maximum number of time periods that individual firms fix nominal product prices.

It follows from these ex-post value functions that preferences of individual firms between low-productivity and high-productivity workers are determined by relative sizes of their instantaneous profit functions: $v_t^F(0,0) - v_t^F(1,0) = \phi_t(0,0) - \phi_t(1,0)$ and $v_t^F(0,1) - v_t^F(1,1) = \phi_t(0,1) - \phi_t(1,1)$. It should be noted that this result is derived from the following two features. First, low-productivity workers become high-productivity workers in the next period. Second, individual firms fix prices at most for two periods (this assumption will be relaxed in a later section). On the basis of ex-post value functions described above, individual firms with the same output level act collectively like a single entity when they hire workers.

Turning to optimization conditions of firms, firms are supposed to adjust prices before the employment adjustment of workers takes place. Hence firms adjust prices at period t only when $v_{0,t}^F \geq v_{1,t}^F + \bar{\chi}_t$ where $v_{0,t}^F$ is the ex-ate value function of price-adjusting firms and $v_{1,t}^F$ is the ex-ate value function of non-adjusting firms. The ex-ate value functions of price-adjusting and non-adjusting firms are determined as follows.

$$\begin{aligned} v_{0,t}^F &= \max_{P_t^*}\{\phi_{0,t} + E_t[\Lambda_{t,t+1}(\alpha_{t,t+1}v_{1,t+1}^F + (1-\alpha_{t,t+1})v_{0,t+1}^F - \bar{\chi}_{t+1})]\} \\ v_{1,t}^F &= \phi_{1,t} + E_t[\Lambda_{t,t+1}v_{0,t+1}^F]. \end{aligned} \tag{3.6}$$

In this representation, $\phi_{0,t}$ is the ex-ate instantaneous profit flow at period t of price-adjusting firms and $\phi_{1,t}$ is the ex-ate instantaneous profit flow at period t of non-adjusting firms: $\phi_{k,t} = (P_{t-k}^*/P_t$ - $e_{k,t}mc_t)D_{k,t}$ and $e_{k,t} = \sum_{m=0}^{1}\gamma_t^F(m,k)mc_t(m,k)/mc_t$ for $k = 0$ and 1. It also follows that ex-ate marginal costs of production differ between price-adjusting and non-adjusting firms. The optimization condition at period t for P_t^* can be written as follows.

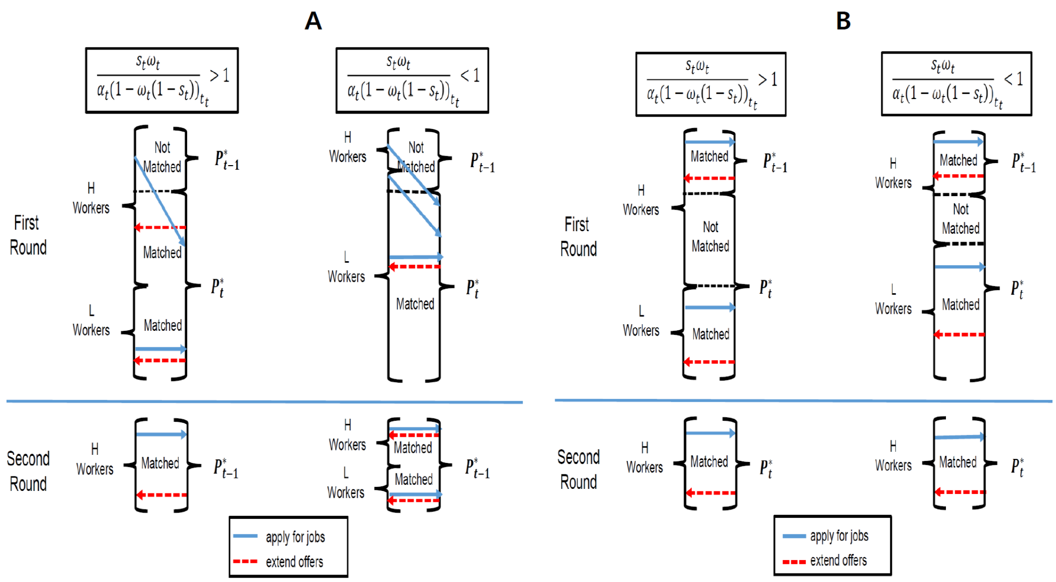
$$\frac{P_t^*}{P_t} = \frac{\epsilon}{\epsilon - 1}\frac{e_{0,t}mc_tY_t + E_t[\alpha_{t,t+1}\Lambda_{t,t+1}\Pi_{t+1}^{\epsilon}e_{1,t+1}mc_{t+1}Y_{t+1}]}{Y_t + E_t[\alpha_{t,t+1}\Lambda_{t,t+1}\Pi_{t+1}^{\epsilon-1}Y_{t+1}]} \tag{3.7}$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor that is used to compute the value at period t of one unit of consumption goods at period $t+1$.

3.6 Positive Assignment as a Labor-Market Outcome

The positive assignment can be attained in a labor market equilibrium given preferences of workers and firms described above. In order to show this result, this subsection begins with the discussion of how workers and firms interact for job applications and offers in the labor market. Specifically, interactions between workers and firms proceed under the following three conditions. The first one is that individual workers are free to send job applications to firms multiple times until they are hired. The second one is that individual workers and firms have perfect information about the determination of wages and decision-makings of all labor-market participants. The third one is

Figure 4: Implementation of Positive Assignment in the Labor Market



that multiple rounds of assigning workers to firms can take place without incurring costs until all jobs are filled and all workers are hired. As a result, the model of this paper abstracts from the existence of unemployed workers and vacant jobs.

Figure 4 illustrates how households and firms interact in the labor market. Case A corresponds to the case where wages of individual workers do not depend on workplaces but the ratio of high-productivity worker's wage to the worker's marginal product of labor is higher than that of low-productivity worker's wage to the worker's marginal product of labor.

1. $v_t^H(0,1) < v_t^H(0,0)$ and $v_t^H(1,1) < v_t^H(1,0)$.
2. $v_t^F(1,0) < v_t^F(0,0)$ and $v_t^F(1,1) < v_t^F(0,1)$

In the first panel, the measure of high-productivity workers who are on the job market exceeds the number of vacant jobs posted by non-adjusting firms. In the second panel, the measure of high-productivity workers who are on the job market is less than the number of vacant jobs posted by non-adjusting firms. The first panel works as follows. In the first round, a low-productivity worker applies for low-output firms under the anticipation of having job offers for sure and a high-productivity worker applies for high-output firms under the anticipation of having job offers with a probability of $\alpha_t(1-(1-s_t)\omega_t)/(s_t\omega_t)$. Given the first-round application, low-productivity firms hire all of low-productivity workers, while high-output firms hire only a subset of high-productivity workers $(\alpha_t(1-(1-s_t)\omega_t)$. In the second round, low-output firms hire the rest of high-productivity workers who do not have job offers in the first round.

The mechanism of the second panel can be described as follows. In the first round, a productivity worker applies for low-output firms under the anticipation of having job offers with a probability of $(1-\alpha_t)(1-(1-s_t)\omega_t)/(1-\omega_t)$ and a high-productivity worker applies for non-adjusting firms under the anticipation of having job offers for sure. Given the first-round application, price-adjusting firms hire only a subset of low-productivity workers $((1-\alpha_t)(1-(1-s_t)\omega_t))$, while non-adjusting firms hire all of high-productivity workers. In the second round, non-adjusting firms hire the rest of low-productivity workers who do not have job offers in the first round.

Case B corresponds to the case where non-adjusting firm's marginal production cost is lower when it hires high-productivity workers than low-productivity workers because wage premiums of high-productivity workers are high enough. In addition, high workers prefer non-adjusting firms and low workers prefer price-adjusting firms.

1. $v_t^H(1,1) > v_t^H(1,0)$ and $v_t^H(0,1) < v_t^H(0,0)$
2. $v_t^F(1,1) > v_t^F(0,1)$ and $v_t^F(1,0) < v_t^F(0,0)$

3.7 Aggregation and Efficient Assignment

The first-step in the derivation of the aggregate production function is to define relative price distortion that is forgone because of price dispersion. Specifically relative price distortion ($=\Delta_t$) is defined as a weighted average of relative outputs of individual firms in the same as is done in existing New Keynesian models:

$$\Delta_t = \sum_{k=0}^{\bar{k}} \sum_{m=0}^{\bar{m}} \omega_t(m,k) \frac{D_{k,t}}{D_t}$$

where $\omega_t(m,k) = (V_t(m,k)Z_t)/Z_t(m,k)$. The restriction that the sum of weights must be one is equivalent to the definition of the aggregate labor productivity ($= Z_t$) that is a weighted harmonic mean of productivity levels of individual firms as can be seen below.

$$\sum_{k=0}^{\bar{k}} \sum_{m=0}^{\bar{m}} \omega_t(m,k) = 1 \rightarrow Z_t = (\sum_{k=0}^{\bar{k}} \sum_{m=0}^{\bar{m}} V_t(m,k) Z_t(m,k)^{-1})^{-1}$$

The aggregate labor input ($= H_t$) and the aggregate output ($= Y_t$) are defined as

$$H_t = \sum_{k=0}^{1} \sum_{m=0}^{1} V_t(m,k) H_t(m,k) \quad Y_t = \sum_{k=0}^{1} \sum_{m=0}^{1} V(m,k)(P_{k,t}/P_t) Y_t(m,k)$$

The second step is the substitution of firm's production functions into the definition of the aggregate labor input to eliminate labor inputs of individual firms. As a result, the aggregate output can be

Table 3.2: Distribution of Hours Worked for $H_{n,t} - H_{p,t}$

	(PL, NR)	(PL, NL)	(PR, NL)	(PR, NR)
$H_t(0,0)$	$-\alpha_t(1-(1-s_t)\omega_t)$	$-(1-\omega_t)$	$-(1-\alpha_t)(1-(1-s_t)\omega_t)$	$-s_t\omega_t$
$H_t(0,1)$	$\alpha_t(1-(1-s_t)\omega_t)$	$1-\omega_t$	$(1-\alpha_t)(1-(1-s_t)\omega_t)$	$s_t\omega_t$
$H_t(1,0)$	$\alpha_t(1-(1-s_t)\omega_t)$	$1-\omega_t$	$(1-\alpha_t)(1-(1-s_t)\omega_t)$	$s_t\omega_t$
$H_t(1,1)$	$-\alpha_t(1-(1-s_t)\omega_t)$	$-(1-\omega_t)$	$-(1-\alpha_t)(1-(1-s_t)\omega_t)$	$-s_t\omega_t$

Note: NR means $1 > \frac{s_t\omega_t}{(1-\alpha_t)(1-(1-s_t)\omega_t)}$, NL means $1 < \frac{s_t\omega_t}{(1-\alpha_t)(1-(1-s_t)\omega_t)}$, PR means $1 > \frac{s_t\omega_t}{\alpha_t(1-(1-s_t)\omega_t)}$ and PL means $1 < \frac{s_t\omega_t}{\alpha_t(1-(1-s_t)\omega_t)}$. In addition, $0 < \alpha_t < 1$, and $0 < \omega_t < 1$.

written as a function of the aggregate labor input, the aggregate labor productivity, and relative price distortion:

$$Y_t = \Delta_t^{-1} Z_t H_t \tag{3.8}$$

In addition, the aggregate market-clearing condition is

$$Y_t = C_t + \bar{\mathcal{X}}_t$$

where C_t is the aggregate consumption, and $\bar{\mathcal{X}}_t$ is the aggregate fixed costs of price changes that will be specified later in detail.[5] The government budget constraint at period t is

$$P_t T_t + B_{t+1}/(1+i_t) = B_t$$

where B_{t+1} is the nominal value of one-period government bond issued at period t and i_t is the nominal interest rate.

Turning to the aggregate consequences of how workers with different productivity levels are assigned to firms with different prices, it should be noted that the assignment of workers to firms affects aggregate variables through its impact on assignment function. In this context, conditional on that a positive assignment is the labor-market's equilibrium outcome as discussed above, it is possible to ask if positive assignment is efficient than negative assignment.

In order to answer this question, it suffices to show that $H_{n,t} > H_{p,t}$ because the aggregate demand is independent of the assignment of workers to firms where $H_{p,t}$ is the aggregate labor under the positive assignment and $H_{n,t}$ is the aggregate labor under the negative assignment. Table 3.2 summarized difference between values of assignment functions under negative and positive assignments for $H_t(0,0)$, $H_t(0,1)$, $H_t(1,0)$, and $H_t(1,1)$. In addition, four columns of this table reflect that there are two cases for positive assignment and two cases for negative assignment as

[5] It will be shown later that $\bar{\mathcal{X}}_t = \varphi C_t \chi(\Delta_{1,t})$ where $\Delta_{1,t}$ reflects the difference of value functions of price-adjusting and non-adjusting firms.

can be seen in Table 3.1. On the basis of Table 3.2, the difference between aggregate labor inputs under negative and positive assignments can be written as

$$(H_t(1,0) - H_t(1,1)) + (H_t(0,1) - H_t(0,0)) > 0 \quad \rightarrow \quad H_{n,t} > H_{p,t} > 0 \tag{3.9}$$

In this equation, the first condition in left-hand side of the arrow can be regarded as the sufficient condition for the positive assignment to be efficient. In order to show how this sufficient condition is satisfied, it should be noted that $H_t(0,1) > H_t(0,1)$ as can be confirmed in Figure 3. Hence to the extent which the size of $H_t(0,0) - H_t(1,0)$ does not dominate that of $H_t(0,1) - H_t(0,1)$, the sufficient condition for $H_{n,t} > H_{p,t}$ holds. For this reason, the analysis of this paper proceeds with the assumption of either the absence of overtime hours of low-productivity workers when they work at low-output firms in light of Figure 3 or no productivity difference between high-productivity workers and low-productivity workers when they work at low-output firms.

4 Optimal Allocation and Optimal Inflation

The aim of this section is the analysis of the optimal allocation for the economy described in the previous section and its implication for the optimal inflation. The optimal allocation is defined as the social planner's optimization subject to a set of feasible allocation that is identified by a set of implementation constraints. While the optimal inflation is obtained as a part of the optimal allocation problem, it is also associated with the minimization of the following four distortions involved in the model of this paper. The first one is relative price distortion. The relative price distortion arises because inflation increases price dispersion and thus wedges between MPL and MRS in staggered price-setting models with technologically homogenous firms. The second one is menu-cost distortion. In this case, inflation increases the frequency of price adjustment and thus the amount of real resources needed for price adjustment. The third one is composition effect. The composition effect takes place because inflation increases the fraction of firms whose prices are reset in the current period and thus makes the Phillips curve steeper. The fourth one is markup distortion. Hence inflation can be used as a social planner's tool to reduce the inefficient output gap of monopolistic competition in goods market.

In fact, theses distortions are reflected in constraints of the social planner's optimization problem to find the optimal allocation. First, the relative price distortion affects the aggregate production function.

$$Y_t = \frac{Z_t}{\Delta_t} H_t \;\; \text{with} \;\; \Delta_t = \frac{(1-\alpha_t) Z_t}{\bar{Z}_{0,t}} s_{0,t} + \frac{\alpha_t Z_t}{\bar{Z}_{1,t}} s_{1,t} \tag{4.1}$$

where $s_{0,t}$ and $s_{1,t}$ are relative outputs of price-adjusting and non-adjusting firms. The definition of the aggregate price index is also a constraint for relative outputs as can be seen below.

$$1 = (1-\alpha_t)s_{0,t}^{(\epsilon-1)/\epsilon} + \alpha_t s_{1,t}^{(\epsilon-1)/\epsilon} \tag{4.2}$$

Second, the menu-cost distortion is included in the aggregate resource constraint

$$Y_t = C_t(1+\varphi\chi(\Delta_{1,t})) \tag{4.3}$$

where $\chi(\Delta_{1,t})$ is the aggregate menu-cost. Third, the composition effect is associated with the endogenous variation of the frequency of price adjustment. For this reason, the composition effect is reflected in the following two equations. The first one is the endogenous determination of the fraction of non-adjusting firms:

$$\alpha_t = (1-\alpha_{t-1})(1-\Gamma(\Delta_{1,t})) \tag{4.4}$$

The second one is the determination of the difference between value functions of price-adjusting and non-adjusting firms $\Delta_{1,t}$ $(= (v_{0,t}^F - v_{1,t}^F)/w_t)$:

$$\Delta_{1,t} = \frac{((s_{0,t}^{\frac{\epsilon-1}{\epsilon}} - s_{1,t}^{\frac{\epsilon-1}{\epsilon}}) - (e_{0,t}s_{0,t} - e_{1,t}s_{1,t})mc_t)Y_t}{mc_t z_t} - \beta E_t[((1-\Gamma(\Delta_{1,t+1}))\Delta_{1,t+1} + \chi(\Delta_{1,t+1}))]. \tag{4.5}$$

Fourth, the markup distortion is reflected in the labor-market equilibrium condition and profit-maximization condition of firms. Given that the household's instantaneous utility function is $U(C,H) = \log C$ - φH, the labor-market equilibrium condition can be written as follows.

$$\varphi C_t = mc_t Z_t \tag{4.6}$$

The left-hand side of this equation corresponds to the marginal rate of substitution between consumption and leisure and the right-hand side is the real wage expressed in terms of marginal cost of production and the aggregate labor productivity level. The profit-maximization condition of price-adjusting firms is

$$s_{0,t}^{-\frac{1}{\epsilon}} = \frac{\epsilon(e_{0,t}mc_t\mathcal{X}_t(\Delta_{1,t}) + \beta E_t[(1-\Gamma(\Delta_{1,t+1}))\Pi_{t+1}^{\epsilon}e_{1,t+1}mc_{t+1}\mathcal{X}_t(\Delta_{1,t+1})])}{(\epsilon-1)(\mathcal{X}_t(\Delta_{1,t}) + \beta E_t[(1-\Gamma(\Delta_{1,t+1}))\Pi_{t+1}^{\epsilon-1}\mathcal{X}_{t+1}(\Delta_{1,t+1})])} \tag{4.7}$$

In sum, 10 endogenous variables such as $\{Y_t, H_t, C_t, \Delta_t, s_{0,t}, s_{1,t}, mc_t, \alpha_t, \Delta_{1,t}, \Pi_t\}$ are included in the representation of aggregate equilibrium conditions in this section. It is also possible to obtain 9 equilibrium conditions except the determination of the aggregate inflation Π_t set by the government. Hence the number of endogenous variables is equal to that of equilibrium conditions.[6]

[6] The number of equilibrium conditions can be counted as follows: 7 equations from equation (4.1) through equation (4.7), one additional equation from equation (4.1) for Δ_t, one additional equation (4.7) for $s_{1,t}^{-1/\epsilon}\Pi_t$ on the basis of period $(t-1)$ profit-maximization condition.

Table 4.1: Social Planner's Problem

$$
\begin{aligned}
&\mathcal{L}_0 = \sum_{t=0}^{\infty} \beta^t E_0[\log Y_t - \varphi \tfrac{Y_t \Delta_t}{Z_t} - \log \mathcal{X}(\Delta_{1,t}) \\
&+\Lambda_{1,t}((1-\alpha_t)s_{0,t}^{(\epsilon-1)/\epsilon} + \alpha_t s_{1,t}^{(\epsilon-1)/\epsilon} - 1) \\
&+\Lambda_{2,t}(\tfrac{(1-\alpha_t)Z_t}{Z_{0,t}} s_{0,t} + \tfrac{\alpha_t Z_t}{Z_{1,t}} s_{1,t} - \Delta_t) \\
&+\Lambda_{3,t}(s_{0,t}^{-\frac{1}{\epsilon}} \mathcal{D}(\Delta_{1,t}, \Delta_{1,t+1}, \Pi_{t+1}) - \mathcal{F}(Y_t, Y_{t+1}, \Delta_{1,t}, \Delta_{1,t+1}, \Pi_{t+1})) \\
&+\Lambda_{4,t}(\alpha_t - (1-\alpha_{t-1})(1-\Gamma(\Delta_{1,t}))) \\
&+\Lambda_{5,t}(\mathcal{B}(s_{0,t}, s_{1,t}, \Delta_{1,t}) + \tfrac{(e_{0,t}s_{0,t} - e_{1,t}s_{1,t})Y_t}{Z_t} + \beta \mathcal{A}(\Delta_{1,t+1}))]
\end{aligned}
$$

with the following definitions of the functions used in the Lagrangian

$$
\begin{aligned}
&\mathcal{X}(\Delta_{1,t}) = 1 + \varphi \int_0^{\Delta_{1,t}} a g(a) da \\
&\mathcal{A}(\Delta_{1,t}) = (1-\Gamma(\Delta_{1,t}))\Delta_{1,t} + \int_0^{\Delta_{1,t}} a g(a) da \\
&\mathcal{B}(s_{0,t}, s_{1,t}, \Delta_{1,t}) = \Delta_{1,t} - (\varphi \mathcal{X}(\Delta_{1,t}))^{-1}(s_{0,t}^{\frac{\epsilon-1}{\epsilon}} - s_{1,t}^{\frac{\epsilon-1}{\epsilon}}) \\
&\mathcal{D}(\Delta_{1,t}, \Delta_{1,t+1}, \Pi_{t+1}) = (\epsilon-1)(\mathcal{X}_t(\Delta_{1,t}) + \beta(1-\Gamma(\Delta_{1,t+1}))\Pi_{t+1}^{\epsilon-1}\mathcal{X}_{t+1}(\Delta_{1,t+1})) \\
&\mathcal{F}(Y_t, Y_{t+1}, \Delta_{1,t+1}, \Pi_{t+1}) = \epsilon(\tfrac{e_{0,t}Y_t}{Z_t} + \tfrac{\beta(1-\Gamma(\Delta_{1,t+1}))e_{1,t+1}Y_{t+1}\Pi_{t+1}^{\epsilon}}{Z_{t+1}})
\end{aligned}
$$

Note: The household's instantaneous utility function is $U(C, H) = \log C$ - φH.

4.1 Social Planner's Problem

The benevolent social planner maximizes the expected discounted sum of the representative household's instantaneous utility function subject to a set of constraints discussed above. Table 4.1 demonstrates the Lagrangian of the social planner's optimization problem. The social planner's objective function is obtained by the substitution of the labor-market equilibrium condition and the aggregate market clearing condition with the aggregate production function into the representative household's instantaneous utility function. Hence the social planner's objective function can be written as a function of the aggregate output, relative price distortion, and the difference between value functions of price-adjusting and non-adjusting firms. In this representation, the initial time period of the social planner's optimization problem is period 0, while the social planner did not make any commitment before period 0 following the usual assumption of the Ramsey problem about the initial period as discussed in Woodford (2003).[7]

Table 4.1 also indicates that there are five constraints in the social planner's optimization problem. The first and second constraints are definitions of the aggregate price index and relative price distortion. The third constraint is the profit maximization condition of price-adjusting firms

[7] It is also possible to incorporate the timeless perspective of Benigno and Woodford (2005, 2012) into the social planner's optimization problem specified in Table 4.1. In order to do so, the social planner's optimization problem should be rewritten for the inclusion of lagged multipliers attached to several terms that involve future values such as $\Delta_{1,t+1}$, Y_{t+1}, and Π_{t+1}.

Table 4.2: First-Order Condition of the Social Planner's Problem

$$\frac{Y_t^{-1}-\varphi\frac{\Delta_t}{Z_t}}{\mathcal{F}_1(Y_t,Y_{t+1},\Delta_{1,t+1},\Pi_{t+1})} = \Lambda_{3,t} + \frac{\Lambda_{3,t-1}\mathcal{F}_2(Y_{t-1},Y_t,\Delta_{1,t},\Pi_t)}{\beta\mathcal{F}_1(Y_t,Y_{t+1},\Delta_{1,t+1},\Pi_{t+1})}$$

$$s_{0,t}^{-\frac{1}{\epsilon}} = \frac{\mathcal{F}_4(Y_t,Y_{t+1},\Delta_{1,t+1},\Pi_{t+1})}{\mathcal{D}_3(\Delta_{1,t},\Delta_{1,t+1},\Pi_{t+1})}$$

$$s_{0,t}/s_{1,t} = (m_t\bar{Z}_{1,t}/\bar{Z}_{0,t})^{-\epsilon}$$

$$\Lambda_{4,t} = -\frac{\beta E_t[\alpha_{t+1}\Lambda_{4,t+1}]}{1-\alpha_t} - \Lambda_{2,t}s_{0,t}^{\frac{\epsilon-1}{\epsilon}}((\frac{s_{1,t}}{s_{0,t}})^{\frac{\epsilon-1}{\epsilon}} - 1) - \frac{s_{0,t}Z_t\Lambda_{3,t}(\frac{\bar{Z}_{0,t}s_{1,t}}{\bar{Z}_{1,t}s_{0,t}}-1)}{\bar{Z}_{0,t}}$$

$$\frac{\mathcal{X}'(\Delta_{1,t})}{\mathcal{X}(\Delta_{1,t})} = n_t + \frac{\beta\Lambda_{1,t}\mathcal{D}_1(\Delta_{1,t},\Delta_{1,t+1},\Pi_{t+1})+\Lambda_{1,t-1}(\mathcal{D}_2(\Delta_{1,t-1},\Delta_{1,t},\Pi_t)-s_{0,t-1}^{1/\epsilon}\mathcal{F}_3(Y_{t-1},Y_t,\Delta_{1,t},\Pi_t))}{\beta s_{0,t-1}^{1/\epsilon}}$$

where m_t and n_t are defined as follows.

$$m_t = \frac{(1+\frac{\Lambda_{5,t}\mathcal{X}(\Delta_{1,t})}{\alpha_t\Lambda_{2,t}})(1+\frac{e_{0,t}\bar{Z}_{0,t}Y_t\Lambda_{5,t}}{\Lambda_{3,t}(1-\alpha_t)Z_t^2})}{(1+\frac{e_{1,t}\bar{Z}_{1,t}Y_t\Lambda_{5,t}}{\Lambda_{3,t}\alpha_tZ_t^2})(1-\frac{\Lambda_{5,t}\mathcal{X}(\Delta_{1,t})}{(1-\alpha_t)\Lambda_{2,t}}-\frac{\Lambda_{3,t}\mathcal{D}(\Delta_{1,t},\Delta_{1,t+1},\Pi_{t+1})}{(\epsilon-1)s_{0,t}(1-\alpha_t)\Lambda_{2,t}})}$$

$$n_t = \Lambda_{5,t} - \beta^{-1}\Lambda_{5,t-1}\mathcal{A}'(\Delta_{1,t}) + (1-\alpha_{t-1})\Lambda_{4,t}\Gamma'(\Delta_{1,t})$$

that reflects the markup distortion under pricing friction. The fourth and fifth constraints determine the profit-maximizing frequencies of price adjustment that is associated with the composition effect and menu-cost distortion. In sum, five constraints of the social planner's problem are associated with four distortions involved in the model of this paper.

Table 4.1 also indicates that there are five constraints in the social planner's optimization problem. The first and second constraints are definitions of the aggregate price index and relative price distortion. The third constraint is the profit maximization condition of price-adjusting firms that reflects the markup distortion under pricing friction. The fourth and fifth constraints determine the profit-maximizing frequencies of price adjustment that is associated with the composition effect and menu-cost distortion. In sum, five constraints of the social planner's problem are associated with four distortions involved in the model of this paper.

Table 4.2 summarizes the first-order conditions of the social planner's problem specified in Table 4.2. The first line is the first order condition with respect to the aggregate output. The second line is the first order condition with respect to future inflation. The third line is a simplified equation of two first order conditions with respect to relative outputs of price-adjusting and non-adjusting firms. The fourth line is the first order condition with respect to the fraction of non-adjusting firms. The fifth line is the first order condition with respect to the difference between value functions of price-adjusting and non-adjusting firms.

Having described the optimization conditions of the social planner's problem, the next topic is the discussion of how the aggregate inflation is determined. In this context, the third line of Table

Table 4.3: Social Planner's Problem: First-Best Allocation

$\mathcal{L}_0 = \sum_{t=0}^{\infty} \beta^t E_0[\log Y_t - \varphi \frac{Y_t \Delta_t}{Z_t} - \log \mathcal{X}(\Delta_{1,t})$
$+\Lambda_{1,t}((1-\alpha_t)s_{0,t}^{(\epsilon-1)/\epsilon} + \alpha_t s_{1,t}^{(\epsilon-1)/\epsilon} - 1)$
$+\Lambda_{2,t}(\frac{(1-\alpha_t)Z_t}{\bar{Z}_{0,t}} s_{0,t} + \frac{\alpha_t Z_t}{\bar{Z}_{1,t}} s_{1,t} - \Delta_t)]$
with the following optimization conditions
$\varphi Y_t \Delta_t / Z_t = 1$
$s_{0,t}/s_{1,t} = (\bar{Z}_{1,t}/\bar{Z}_{0,t})^{-\epsilon}$
$\mathcal{X}'(\Delta_{1,t}) = 0$
$\Lambda_{1,t} = \frac{\epsilon \varphi Y_t}{(\epsilon-1)\bar{Z}_{0,t}} s_{0,t}^{1/\epsilon}$
$\Lambda_{2,t} = -\varphi Y_t / Z_t$

4.2 can be used to show that the aggregate inflation is determined by the following equation:

$$\Pi_t = m_t \frac{p^*_{t-1}}{p^*_t} \frac{\bar{Z}_{1,t}}{\bar{Z}_{0,t}}$$

It follows from this condition that the steady-state inflation is the multiplication of steady-state value of m and the productivity ratio between non-adjusting and adjusting firms: $\Pi = m\bar{Z}_1/\bar{Z}_0$. The steady-state value of m therefore plays an important role of the steady-state inflation that is implied by the social planner's problem specified in Table 4.1. It also follows the definition of m_t (specified in Table 4.2) that $m_t = 1$ when $\Lambda_{1,t} = 0$ and $\Lambda_{5,t} = 0$ where $\Lambda_{1,t}$ and $\Lambda_{5,t}$ are Lagrange multipliers of firm's profit-maximizing conditions regarding the size and adjustment timing of nominal product price. The condition of $m = 1$ implies that $\Pi = \bar{Z}_1/\bar{Z}_0$. In this case, the steady-state inflation turns out to be the productivity ratio between non-adjusting and adjusting firms.

Turning to the definition of the optimal inflation, it would be necessary to address the issue of whether the optimal inflation can be obtained as the solution to the social planner's optimization problem specified in Table 4.1 that is not consistent with the first-best allocation. The motivation of this issue is that the benevolent social planner's optimization problem leads to the first-best allocation in the absence of the steady-state markup distortion that can be eliminated by government's fiscal policies as discussed in Woodford (2003).

In relation to this issue, Table 4.3 illustrates the social planner's optimization problem whose solution attains the first-best allocation. In this representation, the social planner takes as given firm's profit-maximizing choices regarding the size and adjustment timing of its nominal price, while the social planner is allowed to choose the aggregate menu cost to maximize the social welfare given the aggregate market-clearing condition. The optimization condition of the social planner's problem specified in Table 4.3 is $P^*_t/P^*_{t-1} = \bar{Z}_{1,t}/\bar{Z}_{0,t}$. As mentioned above, this condition also holds when $m_t = 1$ in the case of the social planner's problem specified in Table 4.1. Hence the optimal steady-state inflation is defined as $\Pi^* = \bar{Z}_1/\bar{Z}_0$ where Π^* denotes the optimal inflation.

4.2 Implementation of the Optimal Allocation

The main discussion of this subsection is to see if and how the first-best allocation (with $m_t = 1$) can be attained at a decentralized market equilibrium. The key argument of this subsection is that the government should make its commitment of $\Pi_{t+1} = e_{0,t}mc_t/(e_{1,t+1}mc_{t+1})$ in order to attain the first-best allocation at a decentralized market equilibrium.

In order to show that this argument is correct, the first point is that the substitution of $\Pi_{t+1} = e_{0,t}mc_t/(e_{1,t+1}mc_{t+1})$ into the profit maximization condition of price-adjusting firms leads to the following condition: $P^*_t = \mu e_{0,t}MC_t$ where $\mu = \epsilon/(\epsilon - 1)$. The former is the determination of inflation when the government can attain its inflation target. The latter is determination of nominal product prices set by price-adjusting firms. In particular, the latter is the static profit maximization condition that holds in the absence of menu-costs. In this case, the profit-maximizing condition of price-adjusting firms under a menu-cost friction is then reduced to the static profit-maximization condition under fully flexible adjustment of prices. In other words, all price-adjusting firms adopt a constant markup policy for their nominal product prices: $P^*_t/(e_{0,t}MC_t) = \mu$.

The second point is that $\Delta_{1,t} = \Delta_{1,t+1} = 0$ under the solution of the social planner's problem specified in Table 4.3. In order to explain how this result can be obtained, it should be noted that the endogenous frequency of price adjustment reflects the following equation for the difference between value functions of price-adjusting and non-adjusting firms.

$$\Delta_{1,t} = \max\{\frac{\phi_{0,t} - \phi_{1,t}}{w_t} - \beta E_t[((1 - \Gamma(\Delta_{1,t+1}))\Delta_{1,t+1} + \chi(\Delta_{1,t+1}))], 0\} \tag{4.8}$$

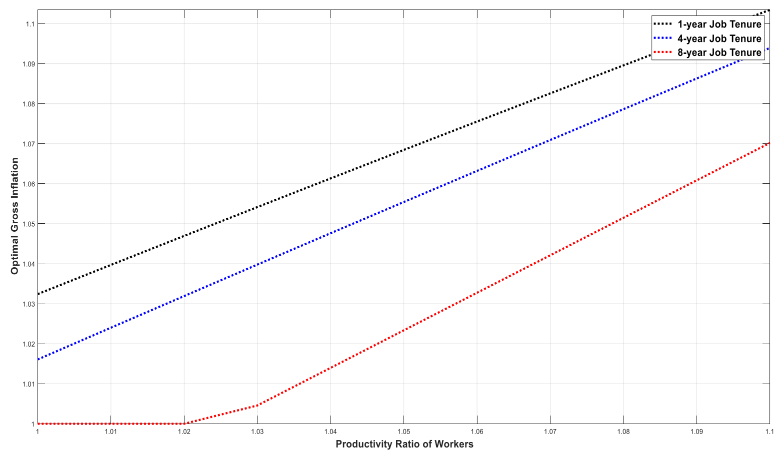
where $\phi_{0,t}$ and $\phi_{1,t}$ are defined as follows.

$$\begin{aligned} \phi_{0,t} &= (s_{0,t}^{\frac{\epsilon-1}{\epsilon}} - e_{0,t}mc_t s_{0,t})Y_t \\ \phi_{1,t} &= (s_{1,t}^{\frac{\epsilon-1}{\epsilon}} - e_{1,t}mc_t s_{1,t})Y_t \end{aligned}$$

It follows from the first term in the right-hand side of equation (4.8) that a positive value of $\Delta_{1,t}$ is consistent with the condition of $\phi_{0,t} > \phi_{1,t}$. But such a condition does not take place under the solution of the social planner's problem specified in Table 4.3. In fact, $\phi_{0,t} < \phi_{1,t}$ is consistent with the solution of the social planner's problem specified in Table 4.3. Hence $\Delta_{1,t} = \Delta_{1,t+1} = 0$ under the solution of the social planner's problem specified in Table 4.3.

The substitution of $\Delta_{1,t} = 0$ into the fourth constraint of the social planner's problem specified in Table 4.1 results into the condition of $\alpha_t = 1 - \alpha_{t-1}$. The steady-state solution of this condition is $\alpha_t = \alpha_{t-1} = 1/2$. As a result, $\Lambda_{3,t} = \Lambda_{4,t} = \Lambda_{5,t} = 0$ under the solution of the social planner's

Figure 5: Job Separation Rate and Optimal Inflation



problem specified in Table 4.3. In particular, this result can be regarded as a justification that the third, fourth, and fifth constraints of the social planner's problem specified in Table 4.1 are not involved in the social planner's problem specified in Table 4.3.

The final discussion of this subsection is to confirm that $\Pi_{t+1} = e_{0,t}mc_t/(e_{1,t+1}mc_{t+1})$ satisfies the optimization condition of the social planner's problem specified in Table 4.1 when $\Lambda_{3,t} = \Lambda_{4,t} = \Lambda_{5,t} = 0$ holds. The third optimization condition of Table 4.2 can be rewritten as $P_t^*/P_{t-1}^* = m_t\bar{Z}_{1,t}/\bar{Z}_{0,t}$. The substitution of $P_t^* = \mu e_{0,t}MC_t$ together with $e_{0,t} = Z_t/\bar{Z}_{0,t}$ and $e_{1,t} = Z_t/\bar{Z}_{1,t}$ into the third optimization condition of Table 4.2 leads to the following relation.

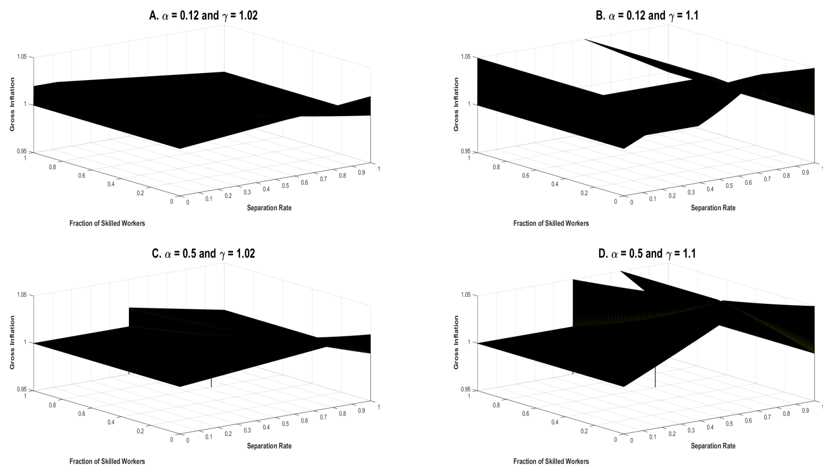
$$\frac{e_{0,t}MC_t}{e_{0,t-1}MC_{t-1}}\frac{\bar{Z}_{0,t}}{\bar{Z}_{1,t}} = m_t \quad \rightarrow \quad \frac{e_{1,t}mc_t}{e_{0,t-1}mc_{t-1}}\Pi_t = m_t$$

It follows from this equation that $\Pi_{t+1} = e_{0,t}mc_t/(e_{1,t+1}mc_{t+1})$ satisfies the optimization condition of the social planner's problem specified in Table 4.1 when $\Lambda_{3,t} = \Lambda_{4,t} = \Lambda_{5,t} = 0$ holds in each period $t = 0, 1, \cdots$.

4.3 Numerical Solution

Since the optimal steady-state inflation is $\Pi^* = \bar{Z}_1/\bar{Z}_0$, the computation of the optimal inflation requires the fact that the assignment of workers to firms affects average productivity levels of price-adjusting and non-adjusting firms. It means that the computation of the optimal inflation should take into account results of Table 3.1. In sum, the steady-state optimal inflation is determined as

Figure 6: Household's Productivity Distribution and Optimal Inflation



follows.

$$\Pi^* = \begin{cases} \left(\frac{1-\omega}{1-\alpha}\right)\frac{Z(1,1)}{Z(0,0)} + \left(\frac{\omega-\alpha}{1-\alpha}\right)\frac{Z(1,1)}{Z(1,0)} & \text{if} \quad \alpha < \frac{s\omega}{1-(1-s)\omega} \\ \frac{1-(1-s)\omega(1-\frac{Z(0,0)}{Z(0,1)})}{(1-((1-s)\omega+\frac{s\omega}{\alpha}))\frac{Z(0,0)}{Z(0,1)}+((1-s)\omega+\frac{s\omega}{\alpha})\frac{Z(0,0)}{Z(1,1)}} & \text{if} \quad \alpha > \frac{s\omega}{1-(1-s)\omega} \end{cases}$$

The first case is that the number of separated skilled workers is larger than the job-opening number of non-adjusting firms. Hence all jobs of non-adjusting firms can be filled with skilled workers. In this case, if all unskilled workers are separated each period, separation rate does not affect the firm's productivity distribution. Hence optimal inflation is not affected by separation rate. The second case is that the number of separated skilled workers is smaller than the job-opening number of non-adjusting firms. An increase in the number of separated skilled workers raises the average productivity of non-adjusting firms under positive assignment. In this case, optimal inflation increases as more skilled workers separate.

Figure 5 presents the relation between job separation rate and optimal inflation on the basis of numerical solutions. The horizontal axis represents the productivity ratio between high-productivity and low-productivity workers, while the vertical axis represents the optimal inflation. In order to see the impact of job separation rate on optimal inflation, there are three different values of job tenure. Specifically, the black line corresponds to one-year job tenure, the blue line corresponds to four-year job tenure and the red line corresponds to eight-year job tenure. Given the BLS report that average job tenure of typical U.S. workers is 4-5 years, optimal inflation is positive within a plausible range of job tenure.[8]

[8] In this numerical example, $\alpha = 0.5$ from the solution of Table 4.3 and $\omega = 0.31$ from the result of Yun (2023).

Table 5.1: Social Planner's Problem: Stochastic Menu-Cost Model

$$\mathcal{L}_0 = \sum_{t=0}^{\infty} \beta^t E_0[\log Y_t - \varphi \frac{Y_t \Delta_t}{Z_t} - \log \mathcal{X}(\Delta_{1,t}, \cdots, \Delta_{K-1,t})$$
$$+\Lambda_{1,t}((1-\alpha_t) s_{0,t}^{(\epsilon-1)/\epsilon} + \sum_{k=1}^{K-1} (1-\alpha_{t-k})(\prod_{i=1}^{k} \alpha_{t-k,t-k+i}) s_{k,t}^{(\epsilon-1)/\epsilon} - 1)$$
$$+\Lambda_{2,t}(\frac{(1-\alpha_t) s_{0,t} Z_t}{\bar{Z}_{0,t}} + \sum_{k=1}^{K-1} (1-\alpha_{t-k}) \frac{(\prod_{i=1}^{k} \alpha_{t-k,t-k+i}) s_{k,t} Z_t}{\bar{Z}_{k,t}} - \Delta_t)]$$

with the following optimization conditions

$$\varphi Y_t \Delta_t / Z_t = 1$$
$$s_{0,t}/s_{k,t} = (\bar{Z}_{k,t}/\bar{Z}_{0,t})^{-\epsilon} \quad \text{for } k = 1, \cdots, K-1$$
$$\mathcal{X}_k(\Delta_{1,t}, \cdots, \Delta_{K-1,t}) = 0 \quad \text{for } k = 1, \cdots, K-1$$
$$\Lambda_{1,t} = \frac{\epsilon \varphi Y_t}{(\epsilon-1)\bar{Z}_{0,t}} s_{0,t}^{1/\epsilon}$$
$$\Lambda_{2,t} = -\varphi Y_t / Z_t$$

Note: $\mathcal{X}(\Delta_{1,t}, \cdots, \Delta_{K-1,t}) = 1 + \varphi \sum_{k=1}^{K-1} \chi(\Delta_{k,t})$ and $\mathcal{X}_k(\Delta_{1,t}, \cdots, \Delta_{K-1,t})$ is the partial derivative of $\mathcal{X}(\Delta_{1,t}, \cdots, \Delta_{K-1,t})$ with respect to $\Delta_{k,t}$.

Figure 6 demonstrates the impact of household's productivity distribution on the optimal inflation with rigid job separation. The x-axis is the fraction of high-productivity workers, y-axis is the separation rate and z-axis is the optimal inflation. The four panels have different values of nominal rigidity and productivity ratio between high-productivity and low-productivity workers. The implication of Figure 6 can be summarized as follows. First, the optimal inflation tends to increase as the fraction of high-productivity workers rises, which may depend on other parameter values. Second, the optimal inflation tends to increase as separation rate increases, which may depend on other parameter values.

In sum, numerical solutions imply the following three points. First, optimal inflation increases as the productivity ratio of workers increases for all separation rates. Second, optimal inflation decreases as the rigidity of job separation increases. Third, optimal inflation is positive only when productivity difference between high-productivity and low-productivity workers is sufficiently high in the presence of significant workplace effect for worker's productivity and rigid job separation.

5 Robustness of Results under Different Pricing Frictions

The first topic of this section is the discussion of whether the previous section's result on the optimal inflation is robust to a change in the number of time periods of fixing nominal product prices.

Table 5.1 summarizes the social planner's optimization problem when the maximum number of time periods of fixing nominal product prices is K where K is a positive finite integer. The social

Table 5.2: Social Planner's Problem: Calvo Model

$$\mathcal{L}_0 = \sum_{t=0}^{\infty} \beta^t E_0[\log Y_t - \varphi \frac{Y_t \Delta_t}{Z_t}$$
$$+\Lambda_{1,t}((1-\alpha)s_{0,t}^{(\epsilon-1)/\epsilon} + \sum_{k=1}^{\infty}(1-\alpha)\alpha^k s_{k,t}^{(\epsilon-1)/\epsilon} - 1)$$
$$+\Lambda_{2,t}(\frac{(1-\alpha)s_{0,t}Z_t}{\bar{Z}_{0,t}} + \sum_{k=1}^{\infty}\frac{(1-\alpha)\alpha^k s_{k,t}Z_t}{\bar{Z}_{k,t}} - \Delta_t)]$$

with the following optimization conditions

$$\varphi Y_t \Delta_t / Z_t = 1$$
$$s_{0,t}/s_{k,t} = (\bar{Z}_{k,t}/\bar{Z}_{0,t})^{-\epsilon} \qquad \text{for } k = 1, \cdots, \infty$$
$$\Lambda_{1,t} = \frac{\epsilon \varphi Y_t}{(\epsilon-1)\bar{Z}_{0,t}} s_{0,t}^{1/\epsilon}$$
$$\Lambda_{2,t} = -\varphi Y_t / Z_t$$

planner's optimization problem specified in Table 4.3 corresponds to the case of $K = 2$, which means that it is a special case of the optimization problem specified in Table 5.1. In this optimization problem, the social planner takes as given firm's profit-maximizing choices regarding the size and adjustment timing of its nominal price, while the social planner is allowed to choose the aggregate menu cost to maximize the social welfare given the aggregate market-clearing condition.

The second optimization condition of the social planner's problem specified in Table 5.1 implies that the ratio of relative outputs of price-adjusting firms and non-adjusting firms should be an increasing function of the corresponding productivity ratio. It follows from definitions of relative outputs of price-adjusting firms and non-adjusting firms that the ratio of relative outputs of price-adjusting firms and non-adjusting firms is determined by the price ratio of price-adjusting firms and non-adjusting firms. Hence the second optimization condition of the social planner's problem specified in Table 5.1 implies that the price ratio of price-adjusting firms and non-adjusting firms is the inverse of their productivity ratio. Specifically, $P_t^*/P_{t-k}^* = \bar{Z}_{k,t}/\bar{Z}_{0,t}$ for $k = 1, \cdots, K$. Moreover, this condition can be regarded as a generalization of the optimal condition of $P_t^*/P_{t-1}^* = \bar{Z}_{1,t}/\bar{Z}_{0,t}$ derived in the previous section where $K = 2$.

Turning to the Calvo model, the substitution of the Calvo friction such as $(\prod_{i=1}^{k} \alpha_{t-k,t-k+i}) = \alpha^k$ and $\alpha_t = \alpha$ with $K = \infty$ into the social planner's optimization problem specified in Table 5.1 leads to the social planner's optimization problem specified in Table 5.2. The social planner's optimization problem specified in Table 5.2 can be regarded as the social planner's optimization problem for the Calvo model. The difference of the Calvo model from the menu-cost model specified in Table 5.1 is that the social planner's objective function is not affected by the aggregate menu cost and the adjustment timing of nominal prices is exogenously determined. But this difference is not reflected in the social planner's optimization condition for the aggregate inflation as will be seen below.

The derivation of the optimization condition for the aggregate inflation in the Calvo model begins with the second optimization condition of the social planner's problem specified in Table 5.2, which in turn implies that the ratio of relative outputs of price-adjusting firms and non-adjusting firms should be an increasing function of the corresponding productivity ratio. As mentioned above, it follows from definitions of relative outputs of price-adjusting firms and non-adjusting firms that the ratio of relative outputs of price-adjusting firms and non-adjusting firms is determined by the price ratio of price-adjusting firms and non-adjusting firms. Hence the second optimization condition of the social planner's problem specified in Table 5.2 also implies that the price ratio of price-adjusting firms and non-adjusting firms is the inverse of their productivity ratio. Specifically, the optimization condition of the social planner's problem specified in Table 5.2 is $P_t^*/P_{t-k}^* = \bar{Z}_{k,t}/\bar{Z}_{0,t}$ for $k = 1, \cdots, \infty$.

The common feature of the optimal inflation that is implied by both of the stochastic menu-cost and Calvo models is that the stationarity of the steady-state optimal inflation is determined by the stationarity of the productivity ratio of price-adjusting firms and non-adjusting firms. For example, if the productivity ratio of non-adjusting and price-adjusting firms is given by $\bar{Z}_k/\bar{Z}_0 = \gamma^k$ with a positive constant γ greater than one, then the optimal steady-state inflation is determined as $\Pi^* = \gamma$ (> 1). In this case, the optimal steady-state inflation is set by $\Pi^* = \bar{Z}_1/\bar{Z}_0(= \gamma)$ in the same way as is done in the previous section, although the productivity ratio of the present section can be different from that of the previous section as households and firms have different productivity and price distributions.[9]

6 Conclusion

In the concluding part of this paper, it would be helpful to emphasize two features of the model in this paper. The first feature is that characteristics of firms and workers change over time, which is different from many existing assignment models. Hence optimizing behaviors of firms and workers in the model of this paper reflect the possibility that their future characteristics are not equal to their current characteristics. The second one is the presence of ex-ante technologically homogeneous firms. Since productivity levels of all firms are ex-ante identical, it should be mentioned that the assignment of workers to firms in this paper differs from the labor market's mismatch that happens

[9] For example, Yun (2023) shows that the optimal inflation is $\Pi^* = \gamma_g^\sigma$ in the Calvo model where γ_g is the annual average growth rate of worker's productivity and $\sigma = \log\alpha/\log\omega$ under the assumption of a discrete distribution of worker's productivity with $\Gamma_m = (1-\omega)\omega^m$ for the measure of workers whose productivity is Z_m for $m = 0, \cdots \infty$. Hence the value of γ in the text should be replaced by $\gamma = \gamma_g^\sigma$ under a different specification of worker's productivity distribution.

when technologically permanent characteristics of firms and workers are not aligned.

A potential criticism about the set-up of the model in this paper is associated with the issue of whether the efficient assignment of workers to firms requires a substantial fraction of workers who make job-to-job transition between different workplaces in each period. In order to attain the efficient assignment of workers to firms in actual economies, it would be unavoidable to entail the physical relocation of workers to new workplaces that can be rigid reflecting a variety of actual relocation costs. Hence if rigid job separation can hinder the attainment of the efficient assignment of workers to firms, a plausible criticism is that the policy prescription of this paper is based on a unrealistic model. In line with this criticism, it would be possible to ask if a positive inflation through the assignment of workers to firms can be attained when the average job tenure of typical workers is loner than 5 years in actual economies. The answer to this equation lies in the fact that only a small fraction of firms do not change price in each year. Hence the productivity level of non-adjusting firms can be higher than that of price-adjusting firms even if there are relatively a small fraction of workers who make job-to-job transition between different workplaces.

The other potential criticism about the set-up of the model is the absence of unemployment. While it should be admitted that the explicit inclusion of unemployment is a desirable extension of the present paper, the current focus is given on worker's job-to-job transition without non-employment as a tool for the assignment of workers to firms. It should be mentioned that the importance of worker's job-to-job transition has been emphasized in recent empirical studies on the labor market. For example, Haltiwanger, Hyatt, and McEntarfer (2018) report that job-to-job separations account for a substantial fraction of workers who are relocated from less productive firms to more productive firms with the finding of Fallick, Haltiwanger and McEntarfer (2012) that the fraction of job-to-job separations is slightly higher than 60% among all job separations for years of 1995, 1999, and 2001. In addition, Hahn, Hyatt, Janicki, and Tibbets (2017), Hyatt and McEntarfer (2012), and Fallick, Haltiwanger and McEntarfer (2012) emphasize the importance of non-employment durations among job changers for their earnings outcomes, reflecting that job changers with short periods of non-employment tend to show increases in their earnings.

In this paper, the welfare gain from a positive rate of inflation is associated with the model's feature that the aggregate labor productivity rises with the aggregate inflation under the efficient assignment of workers to firms. In this light, the model of this paper implies that inflation can grease the wheels of the aggregate economy under the efficient assignment of workers to firms in models of staggered price-setting behaviors of firms even without nominal wage rigidity.[10]

[10] A well-known hypothesis of Tobin (1972) for the optimality of a positive inflation rate in macroeconomics textbooks is that inflation can reduce real wage and thus ease labor market adjustments in the presence of downward nominal-wage rigidity. I am indebted to Lawrence J. Christiano for making this point.

The final paragraph of this paper is to ask if the introduction of heterogeneous workers with different productivity levels into an otherwise New Keynesian model is crucial for the main argument of the present paper that inflation can improve the allocation of labor inputs and thus the social welfare. The answer to this question is that such an introduction of heterogeneous workers with different productivity levels is crucial in the implementation of the social planner's efficiency principle of "the higher relative productivity, the higher relative output" as can be seen in the second optimization condition in Table 4.3. Specifically, the social planner's optimization condition of $s_{0,t}/s_{1,t} = (\bar{Z}_{1,t}/\bar{Z}_{0,t})^{-\epsilon}$ implies that either if $\bar{Z}_{1,t} > \bar{Z}_{0,t}$, then $s_{1,t} > s_{0,t}$ or if $s_{1,t} > s_{0,t}$, then $\bar{Z}_{1,t} > \bar{Z}_{0,t}$. In models where firms have ex-ante identical production technologies, the social planner's efficiency principle implies that inflation does not improve the allocation of labor inputs and the social welfare as well if $\bar{Z}_{1,t} = \bar{Z}_{0,t}$ in the absence of heterogeneous workers with different productivity levels. In conjunction with this argument, the matrix-valued job-to-job matching function summarizes the labor market's role in the determination of firm's productivity distribution through its assignment of workers to firms.

References

Acemoglu, Daron and Jörn-Steffen Pischke, 1999. "The Structure of Wages and Investment in General Training." Journal of Political Economy Vol. 107 (3): 539-572.

Adam, Klaus and Henning Weber. 2019. "Optimal Trend Inflation." American Economic Review Vol. 109 (2): 702-737.

Adam, Klaus and Henning Weber. 2023. "Estimating the Optimal Inflation Target from Trends in Relative Prices." American Economic Journal: Macroeconomics Vol. 15 (3): 1-42.

Alvarez Fernando and Francesco Lippi. 2022. "The Analytic Theory of A Monetary Shock." Econometrica. Vol. 90 (4): pp. 1655-1680.

Auclert, Adrien, Rodolfo Rigato, Matthew Rognlie and Ludwig Straub. 2024. "New Pricing Models, Same Old Phillips Curve?" Quarterly Journal of Economics. Vol. 139 (1): 121-186.

Baley, Issac, Ana Figueiredo, and Robert Ulbright. 2022. "Mismatch Cycles." Journal of Political Economy. Vol. 130. (11): 2943-2984.

Becker Gary. 1973. "A Theory of Marriage: Part I." Journal of Political Economy Vol. 81 (4): 813-846.

Benigno, Pierpaolo, and Michael Woodford. 2005. "Inflation Stabilization and Welfare: The Case of a Distorted Steady-State." Journal of European Economic Association Vol. 3 (6): 1185-1236.

Benigno, Pierpaolo, and Michael Woodford. 2012. "Linear-Quadratic Approximation of Optimal Policy Problems." Journal of Economic Theory Vol. 147: 1–42.

Calvo, Guillermo A. 1983. "Staggered Prices in a Utility-Maximizing Framework." Journal of Monetary Economics 12 (3): 383-398.

Carlsson, Mikael, and Andreas Westermark. 2016. "Labor Market Frictions and Optimal Steady-State Inflation." Journal of Monetary Economics 78: 67–79.

Dixit, Avinash K. and Joseph E. Stiglitz. 1977. "Monopolistic Competition and Optimum Product Diversity." American Economic Review, Vol. 67 (3): 297-308.

Dotsey, Michael, Robert G. King and Alexander L. Wolman. 1999. "State-Dependent Pricing and the General Equilibrium Dynamics of Money and Output." Quarterly Journal of Economics Vol. 114 (2): 655-690.

Hahn, Joyce; Henry Hyatt; Hubert Janicki, and Stephen Tibbets. 2017. "Job-to-Job Flows and Earnings Growth." American Economic Review Vol. 107 (5): 358-363.

Haltiwanger John, Henry Hyatt, and Erika McEntarfer. 2018. "Who Moves Up the Job Ladder?" Journal of Labor Economics vol. 36 (S1): 301-336.

Hyatt, Henry and Erika McEntarfer. 2012. "Job-to-Job Flows in the Great Recession." American Economic Review Vol. 102 (3): 580-583.

Fallick, Bruce, John Haltiwanger, and Erika McEntarfer. 2012. "Job-to-Job Flows and the Consequences of Job Separations." Finance and Economics Discussion Series 2012-73, Federal Reserve Board.

Gertler, Mark and John Leahy. 2008. "A Phillips Curve with an Ss Foundation." Journal of Political Economy Vol. 116 (3): 533-572.

Goldberg Pinelopi Koujianou and Rececca Hellerstein. 2011. "How Rigid Are Producer Prices?" Federal Reserve Bank of New York. Staff Report No. 407.

Golosov, Mikhail and Robert E. Lucas Jr. 2007. "Menu Costs and Phillips Curves." Journal of Political Economy Vol. 115 (2): 171-199.

Kaplan Greg and Giovanni L. Violante. 2018. "Microeconomic Heterogeneity and Macroeconomic Shocks." Journal of Economic Perspectives. Vol. 32 (3): 167-194.

Kaplan Greg, Benjamin Moll, and Giovanni L. Violante. 2018. "Monetary Policy According to HANK." American Economic Review Vol. 108 (3): 697-743.

Lazear, Edward. 2009. "Firm-Specific Human Capital: A Skill-Weights Approach." Journal of Political Economy Vol. 117 (5): 914-940.

Mortensen, Dale T. and Christopher A. Pissarides. 1994. "Job Creation and Job Destruction in the Theory of Unemployment." Review of Economic Studies Vol. 61 (3): 397-415.

Nakamura, Emi and Jon Steisson. 2010. "Monetary Non-neutrality in a Multi-sector Menu Cost Model." Quarterly Journal of Economics Vol. 125 (3): 961-1013.

Nakov, Anton and Carlos Thomas, 2014, "Optimal Monetary Policy with State-Dependent Pricing." International Journal of Central Banking, Vol. 10 (3): pp. 49-94.

Sahin, Aysegul, Joseph Song, Giorgio Topa, and Giovanni L. Violante. 2014. "Mismatch Unemployment." American Economic Review. Vol. 104 (11): 3529-3564.

Sattinger, Michael B. 1975. "Comparative Advantage and the Distribution of Earnings and Abilities." Econometrica Vol. 43 (3): 455-468.

Sattinger, Michael B. 1993. "Assignment Models of the Distribution of Earnings." Journal of Economic Literature, Vol. 31 (2): pp. 831-880.

Shimer, Robert and Lones Smith. 2000. "Assortative Matching and Search." Econometrica Vol. 68 (2): pp. 343-369.

Shimer, Robert. 2005. "The Assignment of Workers to Jobs in an Economy with Coordination Frictions." Journal of Political Economy Vol. 113 (2): pp. 996-1025.

Tobin, James. 1972. "Inflation and Unemployment." American Economic Review. Vol. 62 (1): 1-18.

Taylor, John B. 1980. "Aggregate Dynamics and Staggered Contracts." Journal of Political Economy Vol. 88 (1): 1-23.

Vandeplas, Anneleen and Anna Thum-Thysen. 2019. "Skills Mismatch and Productivity in the EU." European Economy Discussion Paper 100. European Commission.

Woodford, Michael. 2003. "Interest and Prices." Princeton: Princeton University Press.

Yun, Tack. 2005. "Optimal Monetary Policy with Relative Price Distortions." American

Economic Review Vol. 95 (1): 89-109.

Yun, Tack. 2023. “Heterogeneous Workers and Optimal Inflation.” Unpublished Manuscript, Department of Economics, Seoul National University.