Inspecting the Mechanism of Diagnostic Expectations: An Analytical Approach

Jinill Kim*

Department of Economics, Korea University, Seoul, Korea

(JINILLKIM@korea.ac.kr)

September 3, 2024

Abstract

In line with recent surge of behavioral economics, the diagnostic expectations (DE) paradigm has been adopted by many papers in macroeconomic and international literature: Under DE, new information influences expectations more strongly than under rational expectations. This paper contributes to a better understanding of diagnostic expectations by analytically solving some dynamic models. It starts with a canonical adjustment cost model to illustrate ways to deal with endogenous variables appearing in leads and lags. We also discuss a DE model with nominal rigidities—when prices are predetermined one period in advance—and show that the model builds in a moving-average behavior.

Keywords: Diagnostic Expectations, Rational Expectations.

JEL classification: E.

*Comments from Donghoon Yoo and Seunghoon Na are appreciated. The remaining errors are the author's.

1 Introduction

(Introduction to be written)

Related literature: Having emerged as an important departure from rational expectations, diagnostic expectations (DE) have gained traction in the literature. Based on influential work on the representativeness heuristic by Kahneman and Tversky (1972), DE have been adapted in macroeconomics contexts in recent studies by Bordalo, Gennaioli, Ma, and Shleifer (2020), D'Arienzo (2020), Maxted (2020), Bordalo, Gennaioli, Shleifer, and Terry (2021), Bianchi, Ilut, and Saijo (2024), L'Huillier, Singh, and Yoo (2023), Na and Yoo (2024), among others. For a comprehensive review, see Gennaioli and Shleifer (2018), and Bordalo, Gennaioli, and Shleifer (2022).

Maxted (2020) and Bordalo et al. (2021) incorporate DE in macro-finance frameworks. Specifically, Maxted (2020) shows that incorporating DE into a macro-finance framework can reproduce several facts surrounding financial crises whereas Bordalo et al. (2021) show that DE can quantitatively generate countercyclical credit spreads in a heterogeneous firms business-cycle model. D'Arienzo (2020) investigates the ability of DE to reconcile the over-reaction of expectations of long rates relative to the expectations of short rates to news in bond markets. Ma, Ropele, Sraer, and Thesmar (2020) quantify the costs of managerial biases. Bianchi et al. (2024) and L'Huillier et al. (2023) propose a solution to solve a linear DSGE model under DE, with Bianchi et al. (2024) focusing on distant memory, where agents' reference distributions extend beyond one period. In an open-economy context, Na and Yoo (2024) introduce DE into small open economy (SOE) business cycle models to discuss macroeconomic fluctuation of the external balance.

2 A Primer on Rational Expectations

Before delving into the discussion on diagnostic expectations, it is worthwhile to set up a simple model of adjustment costs under rational expections. Even more, we start with a determinstic version of rational expectations: perfect foresight.

2.1 Perfect Foresight

A deterministic version of adjustment cost model chooses the process of k_s for $s = t, t+1, t+2, ...$ to minimize the following objective function:

$$\frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}\left[(k_s - k_s^*)^2 + \alpha(k_s - k_{s-1})^2\right]. \tag{1}$$

The first-order condition of this optimization problem (for a representative k_t) is

$$\begin{aligned} 0 &= k_t - k_t^* + \alpha(k_t - k_{t-1}) - \alpha\beta(k_{t+1} - k_t) \\ &= -\alpha\beta k_{t+1} + (1+\alpha+\alpha\beta)k_t - \alpha k_{t-1} - k_t^* \\ &= -\left[\alpha\beta L^{-1} - (1+\alpha+\alpha\beta) + \alpha L\right] k_t - k_t^*, \end{aligned}$$

whose solution can be expressed as follows:

$$k_t = \lambda k_{t-1} + \alpha^{-1}\lambda \sum_{s=t}^{\infty} \left[(\beta\lambda)^{s-t} k_s^*\right], \tag{2}$$

where

$$\lambda = \frac{(1+\alpha+\alpha\beta) - \sqrt{(1+\alpha+\alpha\beta)^2 - 4\alpha^2\beta}}{2\alpha\beta}. \tag{3}$$

2.2 Rational Expectations

To introduce rational expectations in a simple fashion, let us assume that the target process is stochastic. Then our objective function is

$$\frac{1}{2}\mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \left[(k_s - k_s^*)^2 + \alpha(k_s - k_{s-1})^2\right]\right], \tag{4}$$

which can—under the assumption of rational expectations—be expressed as follows:

$$\frac{1}{2}\left\{(k_t - k_t^*)^2 + \alpha(k_t - k_{t-1})^2 + \mathbb{E}_t \left[\sum_{s=t+1}^{\infty} \beta^{s-t} \left[(k_s - k_s^*)^2 + \alpha(k_s - k_{s-1})^2\right]\right]\right\}. \tag{5}$$

Under rational expectations, the first order condition is

$$0 = k_t - k_t^* + \alpha(k_t - k_{t-1}) - \alpha\beta\left(\mathbb{E}_t\left[k_{t+1}\right] - k_t\right), \tag{6}$$

and its solution is

$$k_t = \lambda k_{t-1} + \alpha^{-1}\lambda\mathbb{E}_t \left[\sum_{s=t}^{\infty} (\beta\lambda)^{s-t} k_s^*\right]. \tag{7}$$

From now on—for the sake of notational simplicity—we assume that the exogenous target follows an autoregressive process,

$$k_t^* = \rho k_{t-1}^* + \varepsilon_t, \qquad \varepsilon_t \overset{iid}{\sim} N(0,1). \tag{8}$$

Under the AR(1) process, the rational expectatios solution is reduced as follows:

$$k_t = \lambda k_{t-1} + \frac{\alpha^{-1}\lambda}{1-\beta\lambda\rho} k_t^*. \tag{9}$$

If we consider the case of random walk ($\rho = 1$), it is intuitive to guess and straightforward to show that

$$1 = \lambda + \frac{\alpha^{-1}\lambda}{1-\beta\lambda}. \tag{10}$$

3 Three Versions of Diagnostic Expectations

The key equation capturing the essence of diagnostic expectations modifies rational expectations as follows:

$$\mathbb{E}_t^\theta [X_t] = \mathbb{E}_t [X_t] + \theta \left(\mathbb{E}_t [X_t] - \mathbb{E}_{t-1} [X_t] \right). \tag{11}$$

In applying diagnostic expectations to a dynamic problem such as adjustment costs, three versions have been considered. This paper—to differentiate them from one another—applies three adjetives: careful, casual and cavalier. The former two versions start from two objective functions that are slightly from each other while being equivalent under rational expectations. The last version—instead of basing itself on an objective function—modifies the first order condition from rational expectations.

3.1 The Careful

After considering how to apply diagnostic expectations to a dynamic optimization setting with due care, BIS and LSY set the diagnostic expectations version of an objective function as specified in (5):

$$\frac{1}{2}\left\{ (k_t - k_t^*)^2 + \alpha (k_t - k_{t-1})^2 + \mathbb{E}_t^\theta \left[\sum_{s=t+1}^{\infty} \beta^{s-t} \left[(k_s - k_s^*)^2 + \alpha (k_s - k_{s-1})^2 \right] \right] \right\}. \tag{12}$$

Noting that k_t appears three times in this objective function—twice as it is and once in the diagnostic expectations operator—careful derivation of the first order condition yields the following optimality condition:

$$0 = k_t - k_t^* + \alpha(k_t - k_{t-1}) - \alpha\beta\left(\mathbb{E}_t^\theta\left[k_{t+1} - k_t\right]\right). \tag{13}$$

3.2 The Casual (or The Offhanded)

Some papers did not give enough care in setting up a dynamic problem and started with the following objective function that is analogous to (4):

$$\frac{1}{2}\mathbb{E}_t^\theta\left[\sum_{s=t}^{\infty}\beta^{s-t}\left[(k_s - k_s^*)^2 + \alpha(k_s - k_{s-1})^2\right]\right], \tag{14}$$

whose first order condition would be

$$0 = \mathbb{E}_t^\theta\left[k_t - k_t^*\right] + \alpha(\mathbb{E}_t^\theta\left[k_t\right] - k_{t-1}) - \alpha\beta\left(\mathbb{E}_t^\theta\left[k_{t+1} - k_t\right]\right). \tag{15}$$

3.3 The Cavalier

Instead of starting from an optimizing problem, one could simply (or offhandedly) replace the DE operator for the RE operator in (6) as follows:

$$0 = k_t - k_t^* + \alpha(k_t - k_{t-1}) - \alpha\beta\left(\mathbb{E}_t^\theta\left[k_{t+1}\right] - k_t\right). \tag{16}$$

4 Conclusion

Do we choose by theory or by empirics?