

Sovereign Defaults and Public Investment (Capital)*

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Abstract

Sovereigns' public investment (capital) influences sovereign debt crises and resolutions. We compile a dataset on public expenditure composition in 1975–2020 for 75 countries. We show that during sovereign debt restructurings with private external creditors, public investment (i) experiences a severe decline and a slow recovery, (ii) differs from public consumption and transfers, and (iii) relates with restructuring delays. We develop a theoretical model of defaultable long-term debt that embeds endogenous public capital accumulation, expenditure composition, production, and multi-round debt renegotiations. The model quantitatively shows public investment dynamics delay debt settlement (i.e., “public capital accumulation delays”). Data support theoretical predictions.

JEL Classification Codes: F34, F41, H63

Key words: Sovereign Debt; Sovereign Default; Debt Restructuring; Public Investment; Public Capital.

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1 Introduction

Sovereigns’ public investment (capital) influences sovereign debt crises and resolutions. We compile a new dataset on public expenditure composition in 1975–2020 for 75 countries. We show that during sovereign debt restructurings with private external creditors, public investment (i) experiences a severe decline and a slow recovery, (ii) differs from public consumption and transfers, and (iii) relates with restructuring delays. To explain these facts, we develop a theoretical model of defaultable long-term debt that explicitly embeds endogenous public capital accumulation, expenditure composition, production and multi-round debt renegotiations with private external creditors. Our model quantitatively shows that public investment dynamics delay debt settlement, i.e., “public capital accumulation delays”. Data support theoretical predictions.

We start by presenting two new comprehensive datasets in 1975–2020 on (a) public expenditure composition and (b) sovereign debt restructurings with and without recovered debt payments in cash at settlement. Our first dataset is a balanced panel comprised of public consumption, investment, transfers and capital for 75 countries that have experienced sovereign debt restructurings with private external creditors. The dataset provides much wider coverage of countries, time-series and categories, and is thus superior to existing databases (e.g., the IMF World Economic Outlook). The dataset covers fully (i) 197 privately-held sovereign external debt restructurings and (ii) 325 non-debt crisis recessions in 1975–2020.

On our second dataset, we define a sovereign debt restructuring, with and without recovered debt payments in cash at settlement, applying four criteria. We compile a dataset covering 197 restructuring episodes with and without recovered debt payments in cash at settlement based on multiple sources (e.g., IMF 2021, WB 2000). The dataset fills a gap among existing datasets to provide a distinctive separation among 116 post-default restructurings with diverse characteristics—sovereigns default first and renegotiate defaulted debt later. We then merge our datasets with existing datasets on the duration, haircuts, and strategies of restructurings.

The consolidated datasets provide five new stylized facts on 116 post-default restructurings and 325 non-debt crisis recessions. Post-default restructurings are separated into those with and without recovered debt payments at settlement. This is because whether the sovereign pays recovered debt payments in cash or not at settlement also influences the degree to which public capital plays the role on sovereign debt crisis resolutions, and in turn, debt settlement. First, recovered debt payments are made in cash at settlement for half of the post-default restructurings. Second, post-default restructurings with recovered debt payments in cash at settlement are associated with longer duration and higher haircuts than those without recovered debt payments in cash at settlement. Third, public investment experiences a severe decline and a slow recovery around both types of post-default restructurings, while a short-lived decline and a quick recovery around non-debt crisis recessions. Fourth, public consumption and transfers experience a short-lived decline and a quick recovery or remain steady around both types of post-default restructurings and non-debt crisis recessions. Fifth, both severe declines and slow recoveries in public investment are associated with longer delays in both types of post-default restructurings, but with a weaker relation in those without recovered debt payments in cash at

settlement. We confirm these findings through both panel and cross-sectional regressions.

Our empirical findings unveil a new dimension of sovereign debt and default, which the literature has not fully explored yet. In particular, one question emerges from the facts: Why does public investment experience a severe decline and a slow recovery in debt crises, but public consumption and transfers do not? By answering this question, we raise a more fundamental question in the literature: What is the role of public capital (investment) on sovereign debt crises and resolutions under tight fiscal constraint? In this context, tight fiscal constraint is defined as the limited ability of the government to extract resources from the private sector, i.e., distortionary taxation and no lump-sum taxation.¹ Total size and composition of public expenditure is constrained by the government's limited resources. This is because, under tight fiscal constraint, public capital directly interacts with the sovereign's default, debt settlement and borrowing choice. These questions challenge the current understanding in the literature that public capital does not play any role in sovereign debt crises and resolutions.

To our knowledge, we are the first to shed light on the role of public capital (investment) on the sovereign debt crises and resolutions. To address these questions, we construct a theoretical model of sovereign long-term debt that explicitly embeds endogenous public capital accumulation, expenditure composition, production and post-default multi-round renegotiations with a risk averse sovereign and its risk-neutral foreign creditors. The model is built on the classical setup of Eaton and Gersovitz (1981) as in the recent quantitative analysis of sovereign debt.² In particular, our model follows two conventional frameworks in the literature: (i) one with a meaningful role for fiscal policy i.e., when private and public sectors are separated due to both distortionary taxation (and no lump-sum taxation) and two types of consumption (Cuadra et al. 2010; Arellano and Bai 2017) and (ii) one with multi-round debt renegotiations after default (Benjamin and Wright 2013; Bi 2008; Asonuma and Joo 2020).

The important theoretical innovation is incorporating endogenous public capital accumulation, expenditure composition, and production with public capital and labor in the model with long-term debt and endogenous defaults and renegotiations. We explicitly depart from two standard modeling approaches: an exogenous income process (e.g., Arellano 2008; Aguiar and Gopinath 2006) and endogenous production with only labor (e.g., Mendoza and Yue 2012; Cuadra et al. 2010). In each period, the sovereign chooses its expenditure composition (public consumption, investment and transfers) together with its choice of repayment and default, settlement and delay, and of external borrowing. Public capital is accumulated through public investment—net of both depreciation and adjustment costs.

We emphasize two novel predictions in our theoretical model, shown mainly quantitatively. First, the model predicts the role of public capital on the sovereign's choice of default, debt

¹In reality, the government often finds it difficult to extract resources from the private sector through both lump-sum taxation (without distortions) and an increase in the level of current distortionary taxation. These severely constrain the government's resource allocation choice, i.e., tight fiscal constraint. Ongoing work by Asonuma, Joo and Zhang (2023), with an empirical analysis on tax revenues, shows no empirical evidence on lump-sum taxation (without distortions) during debt restructurings.

²See Arellano (2008) and other studies covered in Aguiar and Amador (2014) and Aguiar et al. (2016).

settlement, and restructuring delays. *After default*, (i.e., during post-default restructuring), the sovereign is willing to delay renegotiations, *ceteris paribus*, when public capital is low. It opts to invest limited resources—owing to both financial exclusion and productivity loss—in public capital rather than use its resources for recovered debt payments given the high marginal product of public capital. As a result, debt settlement and delays are driven by the marginal product of public capital. This new driver is added on top of a conventional driver, i.e., recovery of repayment capacity (Benjamin and Wright 2013; Bi 2008). The new driver generates further delays, i.e., “public capital accumulation delays” differentiating our paper from previous studies.

Before default, the sovereign’s willingness to repay remains unchanged or decreases when public capital increases. On the one hand, higher public capital increases benefits of repayment by improving the sovereign’s repayment capacity (“smoothing channel”). On the other hand, higher public capital increases benefits of default by stabilizing household consumption in financial autarky (“autarky channel”) and achieving quick debt settlement (“renegotiation channel”). Effects from the latter two channels are equal to or weakly dominate those from the former channel. The renegotiation channel, newly introduced in our paper, differentiates our paper from Gordon and Guerron-Quintana (2018) in which the sovereign’s willingness to repay increases as aggregate capital increases.

Second, the model predicts the mechanism underlying the dynamics of public investment, and consumption and transfers. *At default*, a severe productivity shock interacts with the sovereign’s consumption-smoothing motive. The interaction of these factors results in a mild decline in public consumption and transfers, a severe decline in public investment, and a default. This is because the sovereign, with a consumption-smoothing motive, is willing to smooth household consumption by stabilizing public consumption and transfers, i.e., a mild decline in public consumption and transfers. As a result, under tight fiscal constraint, the sovereign cannot allocate enough resources to public investment and external debt payments, resulting in both a severe decline in public investment, and a default.³

During post-default restructuring, a combination of slow recovery of productivity, prohibition on external borrowing, and the sovereign’s consumption-smoothing motive generates a quick recovery in public consumption and transfers, slow public capital accumulation, and long restructuring delays. Public consumption and transfers recover quickly because the sovereign with a consumption-smoothing motive prioritizes household consumption smoothing. Public capital accumulation is slow both because external borrowing is unfeasible until the sovereign reaches a settlement with its foreign creditors, and because, under tight fiscal constraint, the sovereign has limited resources left after spending on public consumption and transfers. Debt renegotiations are delayed because with the limited resources left, the sovereign prioritizes public capital accumulation over recovered debt payments for settlement. This cycle continues until the sovereign accumulates public capital to a high level and reaches a settlement.

³Public investment is severely influenced by the sovereign’s consumption-smoothing motive because of limited resources under its tight fiscal constraint. This differs from private or aggregate investment which is influenced but less by its consumption-smoothing motive because of ample resources only under an economy-wide *resource constraint*, not its tight fiscal constraint.

Our theoretical predictions are supported by data: public investment dynamics delay debt settlement. The quantitative analysis calibrated to the Argentine defaults and restructurings in 2001–05 and 2019–20 replicates moment statistics, likelihood of debt settlement, and the five stylized facts: (i) a large number of restructurings with recovered debt payments in cash at settlement, (ii) higher haircuts and longer duration for restructurings with recovered debt payments in cash at settlement, (iii) a severe decline and a slow recovery of public investment, (iv) a short-lived decline and a quick recovery of public consumption and transfers, and (v) an association between public investment dynamics and restructuring delays.

Our model of endogenous public capital differs from a model of endogenous aggregate capital with no separation between private and public sectors (Gordon and Guerron-Quintana 2018; Park 2017) in three important aspects. First, our model of endogenous public capital explains slow public capital accumulation during restructurings, while the model of endogenous aggregate capital shows quick aggregate capital accumulation. Second, our model of endogenous public capital shows restructuring delays due to slow public capital accumulation, while the model of endogenous aggregate capital does not. This is because in the model of endogenous aggregate capital, the sovereign can extract resources from the private sector without distortion and allocate enough resources to aggregate investment quickly. Third, our model of endogenous public capital accounts for five facts, but the model of endogenous aggregate capital accounts for only one fact; (i) a large number of restructurings with recovered debt payments in cash at settlement.

Literature Review Our paper contributes to both theoretical and empirical literature on the role of public capital (investment) on business cycles.⁴ In the theoretical strand of literature, Baxter and King (1993) find that public investment has significant effects on private output and investment, resulting in a large supply side fiscal multiplier. Azzimonti (2015) shows that political re-election uncertainty triggers a reduction in public investment which, in turn, results in an economic downturn. In the empirical strand of literature, Aschauer (1989) finds evidence that private and public capital stocks are complementary inputs to private production technology in the US. Our paper contributes to the literature by showing different public investment dynamics between post-default restructurings and non-debt crisis recessions.

The paper is related to the theoretical literature exploring interactions between fiscal policy and the sovereign’s default and external borrowing choice (e.g., Cuadra et al. 2010; Arellano and Bai 2017; D’Erasmus and Mendoza 2016; Hatchondo et al. 2022; Bianchi et al. 2019).^{5,6} These studies explicitly embed different fiscal policy instruments on expenditure (e.g., public consumption or transfers) and on revenue (e.g., labor income tax or consumption tax) in the model with endogenous default and production with labor. Our paper differs from the existing literature in that with public investment newly introduced in the model, it explains the role of

⁴See also Leeper et al. (2010) and Ramey (2021) for the role of public investment on business cycles.

⁵Aguiar et al. (2009) and Mendoza et al. (2014) explore interactions between fiscal policy, i.e., different taxation method and external borrowing choice without the sovereign’s default choice.

⁶See also Roch and Uhlig (2018), Pouzo and Presno (2022), D’Erasmus and Mendoza (2021), Gonçalves and Guimaraes (2015), Fink and Scholl (2016), and Karantounias (2018). For empirical work on sovereign debt and fiscal policy, see Kaminsky et al. (2005), Ilzetzki (2011), Ilzetzki et al. (2013), and Frankel et al. (2013).

public capital on the sovereign’s choice of default, debt settlement and restructuring delays.

Lastly, the theoretical work on sovereign debt restructurings models the outcome of default and debt renegotiation as bargaining between a sovereign debtor and its creditors.⁷ With multi-round renegotiations, both Benjamin and Wright (2013) and Bi (2008) explain that recovery of the debtor’s repayment capacity generates delays, and Asonuma and Joo (2020) show that both the debtor’s repayment capacity and its risk averse foreign creditor’s consumption-smoothing motive interact and drive longer delays. On the contrary, Bai and Zhang (2012) find that delays arise due to information asymmetry between the debtor and its creditors. We fill a gap in the literature by explaining a new mechanism of delays, i.e., public capital accumulation delays.

2 Datasets and Stylized Facts

2.1 New Dataset on Public Expenditure Composition in 1975–2020

To explore explicitly the role of public capital on the sovereign debt crises and resolutions, we first code a new dataset of public expenditure—consumption, investment, transfers, and capital—in 1975–2020 for 75 countries experiencing privately-held sovereign external debt restructurings.

One main challenge for this coding exercise was a lack of high quality data on public expenditure composition satisfying criteria for (i) cross-country, (ii) times series, and (iii) category coverage simultaneously. The IMF World Economic Outlook (WEO) database provides annual data on government spending components, but it only meets the third criterion. Data are available only for limited years, i.e., since 2000 and for limited sample of countries, i.e., advanced countries. The World Bank (WB) Global Development Finance (GDF) database provides annual data on general government final consumption. The database meets both the first and second criteria. However, the database only covers a specific sub-category of public consumption and does not include compensation of general government employees (including employer contributions for government social insurance)—one of the main sub-categories of public consumption—resulting in an underestimation of total public consumption.

To have high quality data on categories of public expenditure, we therefore combine the limited annual data on public expenditure from IMF (2015), IMF WEO, and WB GDF with rich information from a new broad range of sources.⁸ Important quantitative sources for us in particular are twofold: the IMF Staff Reports from the IMF archives (Article IV consultations, requests and reviews for IMF-supported programs, statistical annexes, etc.) and reports from the country authorities (e.g., annual fiscal reports). For a detailed classification of public consumption, investment and transfers, we follow US BEA (2005)—explained in Table A2 in Appendix A.2. The coding outcome is documented in detail for 75 countries and backed by the exact sources used for coding. Table A3 in Appendix A.2 shows coding examples and the

⁷See also Bulow and Rogoff (1989), Kovrijnykh and Szentes (2007), Yue (2010), Arellano et al. (2017, 2023), D’Erasmus (2011), Hatchondo et al. (2014), Asonuma and Trebesch (2016), Pitchford and Wright (2012), Fernandez and Martin (2014), Dvorkin et al. (2021), and Asonuma (2016).

⁸Appendix A.1 explains how IMF (2015) constructs both public capital and investment series.

underlying sources for a few exemplary cases.

Table 1: Public Consumption, Investment, Transfers and Capital

A. Panel dataset in 1975–2020 ^{1/2/}						
	Observation	Mean	Median	Std. Dev.		
	Total	Average per country				
Country coverage ^{1/}	75					
	(Percent of GDP)					
Public consumption	2,640	35.2	16.7	15.7	7.4	
Public investment	3,097	41.3	5.2	3.7	5.5	
Public transfers	2,568	34.2	5.4	4.0	4.9	
Public capital	3,045	40.6	84.3	56.5	85.8	

B. Sovereign Debt Restructurings in 1975–2020 ^{2/}						
	Observation	Mean	Observation	Mean	Observation	Mean
Restructuring Episodes	197					
Restructuring Duration	3.2					
	Pre-restructuring period		Restructuring period		Post-restructuring period	
	<i>Percent of GDP</i>					
Public Consumption, average ^{3/}	163	16.4	163	16.7	160	16.4
Public Investment, average ^{3/}	179	4.6	177	3.7	175	4.1
Public Transfers, average ^{3/}	162	4.6	163	4.0	159	4.7
Public Capital, average ^{3/}	172	83.9	170	87.2	168	82.5

^{1/} 75 countries experiencing privately-held sovereign external debt restructurings.

^{2/} For all categories of public expenditure, our dataset covers both series in real and level (constant 2011 US dollars), and in percent of GDP.

^{3/} For each restructuring episode, we take an average of public expenditure component series for corresponding periods: (i) pre-restructuring period, i.e., 3 years before the start of restructurings; (ii) restructuring period, i.e., from the start to the end of restructurings; (iii) post-restructuring period, i.e., 3 years after the end of restructurings. Then, we take an average of the obtained statistics across restructuring observations.

Panel A in Table 1 summarizes our public expenditure composition dataset highlighting three main advantages compared to existing datasets, such as the IMF WEO and WB GDF. First of all and most importantly, it is the first comprehensive public expenditure composition dataset which covers a wide range of categories including transfers which have been rarely covered in existing datasets. Second, each expenditure category in our dataset covers long time series, i.e., 34–41 years of observations for each country on average. Third, each expenditure series is comprised of sub-categories. For example, public consumption series in our dataset include compensation of general government employees, and consumption of goods and services.

Our public expenditure composition dataset covers fully (i) 197 privately-held sovereign external debt restructurings and (ii) 325 non-debt crisis recessions in 1975–2020. Debt restructuring episodes are from Asonuma and Trebesch (2016) and non-debt crisis recession episodes are constructed based on four criteria: (i) start year; (ii) end year; (iii) magnitude and length; and (iv) no overlap with a restructuring event—definitions of the four criteria are provided in Appendix A.3. Panel B in Table 1 and panel A in Table A5 in Appendix A.3 emphasize two

novelties of our dataset specific to two types of episodes. First, each expenditure category in our dataset covers at least 80 percent of restructuring episodes (75 percent of non-debt crisis recession episodes). Second, each expenditure category covers three distinct time periods around restructuring (non-debt crisis recession) episodes: pre-restructuring, restructuring and post-restructuring periods (pre-recession, recession and post-recession periods).

2.2 New Dataset on Sovereign External Debt Restructurings with and without Recovered Debt Payments in Cash at Settlement in 1975–2020

Sovereign external debt restructurings are classified into two types, post-default restructurings (116 episodes)—the sovereign defaults first and renegotiates its debt—and preemptive restructurings (81 episodes)—renegotiations take place prior to a payment default. Post-default restructurings are much more diverse in terms of duration and haircuts than preemptive episodes (Asonuma and Trebesch 2016). This calls a need for an additional dimension of classification on top of the current classification of restructuring strategies. For this purpose, we present a new classification of debt restructurings by an exchange method, i.e., whether recovered debt payments are made in cash at settlement or not, complementing the existing classifications.⁹ The new classification is crucial for our paper because whether recovered debt payments are made in cash at settlement or not also directly influences the extent to which public capital plays the role on sovereign debt crisis resolutions, and in turn, settlement.

Definition 1: We define sovereign external debt restructuring with recovered debt payments in cash at settlement when at least one of the following four criteria is met:

- (i) a cash buyback at discount;
- (ii) a buyback at discount by a short-term debt instrument—maturity less than 1 year;
- (iii) cash is included in an exchange offer;
- (iv) a short-term debt instrument is included in an exchange offer.

Based on the definition, we compile new data on sovereign external debt restructurings with and without recovered debt payments in cash at settlement in 1975–2020. Our data are mostly based on six sources: (i) a comprehensive dataset on debt- and debt service-reduction operations (DDSROs) in 1980–2000 from IMF (2021), (ii) a comprehensive dataset on cash buybacks at discount in 1980–2000 from WB (2000), (iii) a dataset on buyback deals and restructurings with short-term debt included in 1978–2013 from Cruces and Trebesch (2013), (iv) a dataset on new debt instruments at exchange for restructurings in 1999–2020 in Asonuma, Niepelt and Ranciere (2023); (v) case studies of restructurings in 1999–2005 in Sturzenegger and Zettelmeyer (2006); (vi) the IMF Staff Reports (program requests and reviews).

⁹Trebesch and Zabel (2017) classify post-default restructurings into “hard” defaults and “soft” defaults applying two criteria; first on debtor behavior and negotiation and second on the size of haircuts.

We merge our newly-constructed data with the existing datasets on the duration and strategies (preemptive or post-default) of restructurings from Asonuma and Trebesch (2016) and on the haircuts from Cruces and Trebesch (2013) and Asonuma, Niepelt and Ranciere (2023).

2.3 Empirical Findings: Five Stylized Facts

Our empirical findings for post-default restructurings with and without recovered debt payments in cash, and non-debt crisis recessions in 1975–2020 are summarized in five stylized facts.¹⁰

- ***Stylized fact 1: Recovered debt payments are made in cash at settlement for half of the post-default restructurings.***

We classify 197 restructuring episodes as follows:

- Post-default restructurings (116 episodes)
 - 60 post-default restructurings with recovered debt payments in cash (52 percent)
 - 56 post-default restructurings without recovered debt payments in cash (48 percent)
- Preemptive restructurings (81 episodes)
 - 27 preemptive restructurings with recovered debt payments in cash (33 percent)
 - 54 preemptive restructurings without recovered debt payments in cash (67 percent)

We find that recovered debt payments are made in cash at settlement for half of the post-default restructurings. The remaining half of the post-default episodes are settled without recovered debt payments in cash. On the contrary, recovered debt payments are made in cash at settlement for only one third of the preemptive restructurings.

- ***Stylized fact 2: Post-default restructurings with recovered debt payments in cash at settlement are associated with longer duration and higher haircuts than those without recovered debt payments in cash at settlement.***

Table 2 reports both duration and haircuts for post-default restructurings with and without recovered debt payments in cash at settlement (together with non-debt crisis recessions). Asonuma and Trebesch (2016) define the start of restructurings as either a default or an announcement of restructuring, and end of restructurings as completion of debt exchange. We define the start and end of non-debt crisis recession as the first year when GDP deviation from the Hodrick-Prescott (HP) filtered trend turns negative and as the year before it recovers to positive, respectively. Average duration for post-default restructurings with recovered debt payments in cash at settlement (6.6 years) is longer than that for those without recovered debt

¹⁰Our findings also relate to empirical literature on sovereign debt restructurings. See Reinhart and Rogoff (2009), Kaminsky and Vega-Garcia (2016), and Reinhart and Trebesch (2016).

Table 2: Post-default Restructurings with and without Recovered Debt Payments in Cash at Settlement

	Number of Episodes	Duration (mean, years)	Haircut ^{1/} (mean, percent)
Post-default Restructuring			
with recovered debt payments in cash	60	6.6	58.5
without recovered debt payments in cash	56	2.9	34.7
Non-debt Crisis Recession	325	2.2	-

^{1/} Cruces and Trebesch (2013) cover episodes in 1975–2013 and Asonuma, Niepelt and Ranciere (2023) cover the remaining episodes in 2014–20.

payments in cash at settlement (2.9 years). Average haircut for post-default restructurings with recovered debt payments in cash at settlement (58.5 percent) is higher than that for those without recovered debt payments in cash at settlement (34.7 percent). Figure B1 in Appendix B.1 shows a scatter plot of duration and haircuts for two types of post-default restructurings. Duration and haircuts for post-default restructurings with recovered debt payments in cash at settlement are diverse ranging from 0 to 24 years and from 0 to 100 percent (panel (i)). On the contrary, duration and haircuts for a majority of post-default restructurings without recovered debt payments in cash at settlement are less than 8 years and lower than 50 percent (panel (ii)).

- ***Stylized fact 3: Public investment experiences a severe decline and a slow recovery around two types of post-default restructurings, while a short-lived decline and a quick recovery around non-debt crisis recessions.***¹¹

Figure 1 shows the dynamics of public investment around two types of post-default restructurings and non-debt crisis recessions.¹² In panels (i)–(iii), the start and end of the restructurings and non-debt crisis recessions are marked by gray and orange vertical bars, respectively. Public investment is presented in real and level terms and is normalized at the levels at the start of the restructurings and non-debt crisis recessions (=100). The blue solid lines represent the average for all post-default restructurings and non-debt crisis recessions for which public investment series is available in our dataset. The red dashed lines show the average public investment during the pre-restructuring (pre-recession) periods.

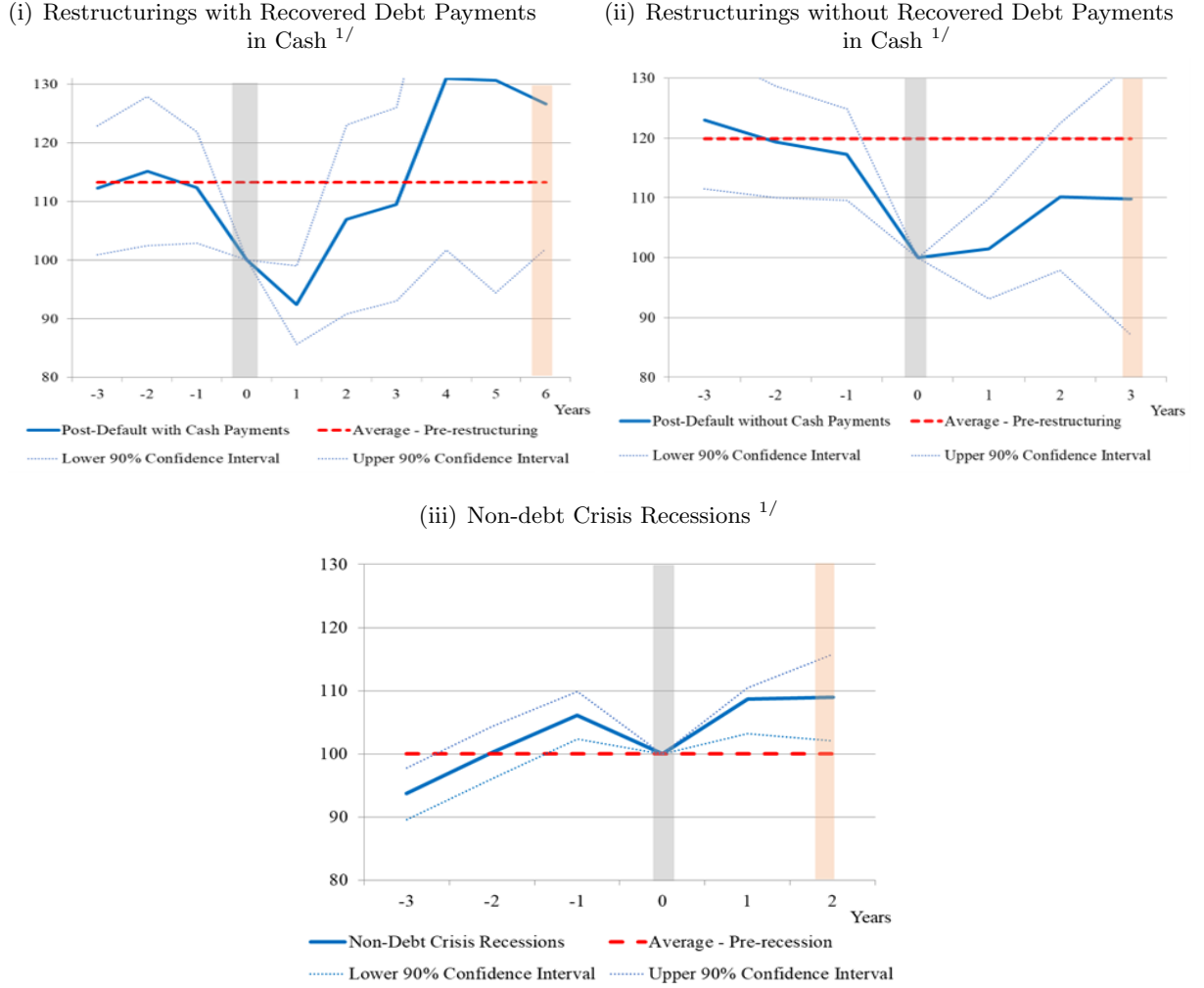
In post-default restructurings with recovered debt payments in cash at settlement (panel (i)), public investment declines severely at the onset of debt crises (year 0) and stays below

¹¹Our stylized facts are purely correlations or associations, not causal relationships. We do not identify any direct impacts of restructurings or recessions on fiscal indicators.

¹²Novelli and Barcia (2021) show a contraction in public investment during non-debt crisis recessions unless when debt is low and productivity shock is moderate. Izquierdo et al. (2019) find empirically that some European EMs with low initial stock of public capital have significantly higher public investment multipliers than those with high initial stock of public capital over 1987–2014.

the pre-crisis level in the subsequent years. Public investment only recovers to reach above the pre-crisis level in year 4, leading to the debt settlement in year 6.

Figure 1: Public Investment



^{1/} When we normalize public investment at the levels of pre-restructuring and pre-recession year (-1), levels and dynamics of public investment in pre-restructuring and pre-recession periods are similar.

In post-default restructurings without recovered debt payments in cash at settlement (panel (ii)), public investment also declines severely at the onset of debt crises (year 0) and stays below the pre-crisis level in the subsequent years. Before public investment recovers, the debt settlement takes place in year 3 largely due to no recovered debt payments in cash at settlement. A contrast between panels (i) and (ii) shows an identical public investment dynamics from year -3 to year 3 for two types of post-default restructurings.

In non-debt crisis recessions (panel (iii)), public investment declines mildly and temporarily at the onset of recessions (year 0). Immediately after, public investment recovers quickly and reaches above the pre-recession level in year 1. With lower debt level, the sovereigns maintain access to the international market, i.e., non-default, and issue new debt to finance public investment. A contrast between panels (i)–(ii) and (iii) shows a difference in public investment dynamics between post-default restructurings and non-debt crisis recessions: a severe decline and a slow recovery versus a short-lived decline and a quick recovery. When we measure public investment as percent of GDP, we observe the same dynamics of public investment-to-GDP ratio for both two types of post-default restructurings and non-debt crisis recessions as reported in Figure B2 in Appendix B.2.

- *Stylized fact 4: Public consumption and transfers experience a short-lived decline and a quick recovery or remain steady around post-default restructurings and non-debt crisis recessions.*

Figure 2: Public Consumption and Transfers

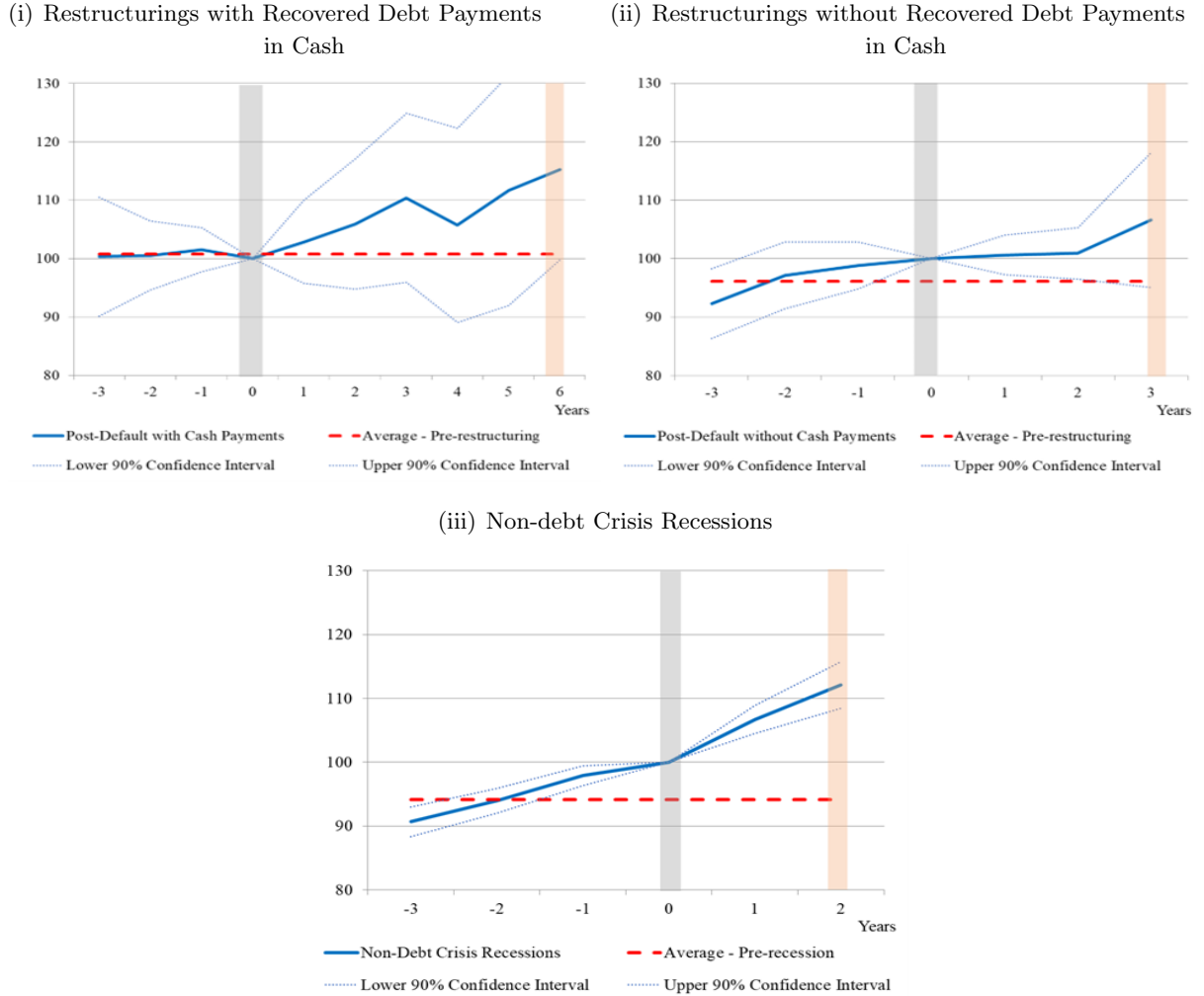


Figure 2 shows the dynamics of public consumption and transfers around two types of post-default restructurings and non-debt crisis recessions.¹³ We follow the same presentation approach as in Figure 1. In post-default restructurings with recovered debt payments in cash at settlement (panel (i)), public consumption and transfers fall temporarily at the onset of debt crises (year 0). Instantly after, public consumption and transfers recover quickly and reach above the pre-crisis level in year 1.

In post-default restructurings without recovered debt payments in cash at settlement (panel (ii)), public consumption and transfers remain steady at the onset of debt crises (year 0). Public consumption and transfers continue to be steady during restructurings. A contrast between panels (i) and (ii) shows an identical dynamics of public consumption and transfers from year -3 to year 3 for two types of post-default restructurings.

In non-debt crisis recessions (panel (iii)), public consumption and transfers remain steady at the onset of recession. Public consumption and transfers continue to be steady during restructurings. With lower debt level, the sovereigns maintain access to the international market, i.e., non-default, and issue more debt to finance public consumption and transfers. A contrast between panels (i)–(ii) and (iii) shows a similar dynamics of public consumption and transfers between two types of post-default restructurings and non-debt crisis recessions: a short-lived decline and a quick recovery or a steady trend.

- ***Stylized fact 5 : Both sharp declines and slow recoveries in public investment are associated with longer delays in both types of post-default restructurings, but with a weaker relation in those without recovered debt payment in cash at settlement.***

Panels (i) and (ii) in Figure 3 show scatter plots of the restructuring duration and the declines and recoveries in public investment during post-default restructurings with and without recovered debt payments in cash at settlement. The declines in public investment are measured as a percentage change of public investment-to-GDP ratio from the level in year -1 to the lowest level, i.e., the level at end of declining trend. The recoveries in public investment are measured as length of time (years) from the time at which public investment-to-GDP ratio is at the lowest level to the time at which it recovers to the pre-restructuring average.

In post-default restructurings with recovered debt payments in cash at settlement (panel (i)), restructurings are protracted when sovereign debtors experience both severe declines and slow recoveries in public investment. Regression lines in blue (results are reported in Table B2 in Appendix B.3) show both a significant and negative relationship between restructuring duration and a decline in public investment, and a significant and positive relationship between restructuring duration and length of public investment recoveries.

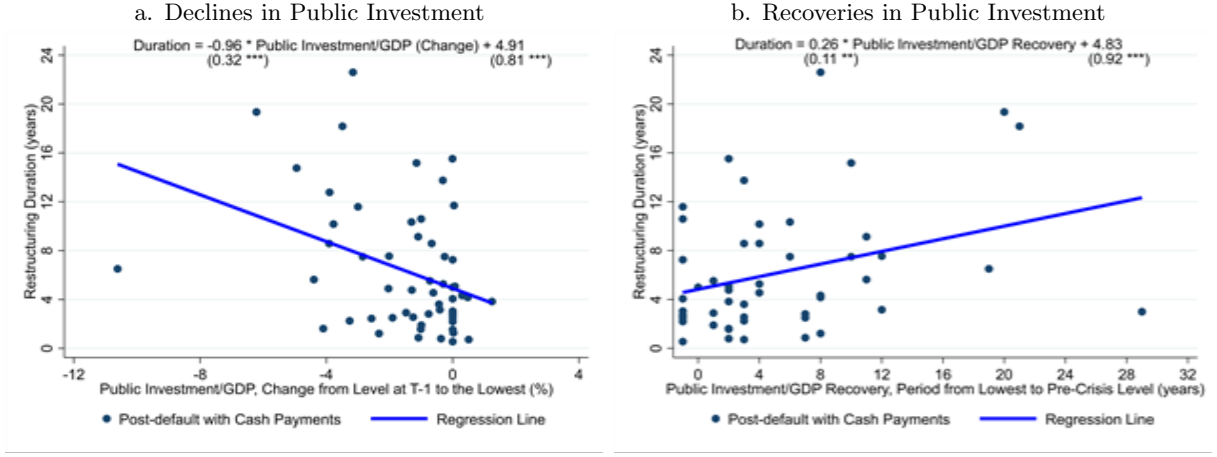
In post-default restructurings without recovered debt payments in cash at settlement (panel (ii)), both a negative relationship between restructuring duration and a decline in public investment and a positive relationship between restructuring duration and length of public investment

¹³Michaud and Rother (2018) find empirically that social transfers are procyclical and significantly contribute to procyclical government expenditure in emerging market countries.

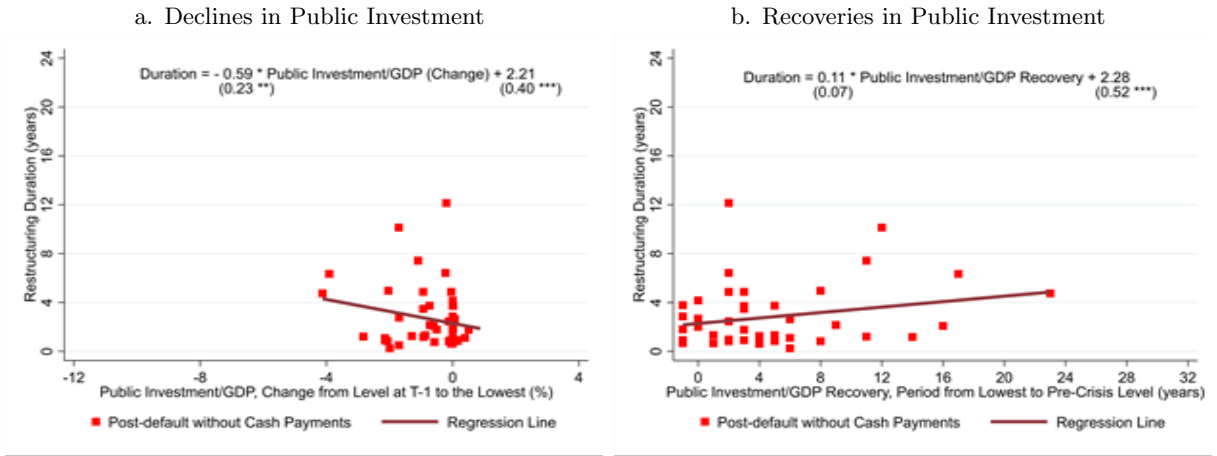
recoveries remain but become weaker (the brown regression lines). This is due to shorter duration for post-default restructurings without recovered debt payments in cash at settlement—observations are scattered below duration of 12 years. We also confirm the robustness of baseline results in Table B3 in Appendix B.3 when we include lagged PPG external debt (in percent of GDP), GDP deviation from the HP filtered trend, and export-to-debt service ratio as controls.

Figure 3: Restructuring Duration and Public Investment

(i) Restructurings with Recovered Debt Payments in Cash



(ii) Restructurings without Recovered Debt Payments in Cash



To conclude our empirical analysis, we assess the determinants of debt settlement using a logit model as in conventional empirical studies on debt restructurings (Asonuma and Joo 2020). Our dataset is an unbalanced panel comprised of 116 post-default restructuring episodes over the duration for each episode i.e., from the start of restructurings to the completion of exchanges. As in previous studies (Cruces and Trebesch 2013; Asonuma and Trebesch 2016), we treat each restructuring as an independent event when both exchanged debt instruments and dates of announcement and of settlement in one restructuring differ from those in other

Table 3: Likelihood of Debt Settlement

	Debt Settlement (binary, current)			
	Restructurings with recovered debt payments in cash		Restructurings without recovered debt payments in cash	
	(1)	(2)	(3)	(4)
	dy/dx / Delta-method se	dy/dx / Delta-method se	dy/dx / Delta-method se	dy/dx / Delta-method se
Public investment-to-GDP ratio in an upward trend (lagged) ^{1/}	0.047*** (0.022)	-	0.069** (0.033)	-
Public investment-to-GDP ratio in an upward trend (cumulative change from trough to a lagged period) ^{2/}	-	0.081** (0.041)	-	0.042 (0.040)
PPG external debt (lagged, percent of GDP) ^{3/}	-0.003*** (0.0003)	-0.003*** (0.0003)	-0.003*** (0.0006)	-0.003*** (0.0005)
GDP deviation from the trend (current, percent) ^{4/}	0.011* (0.006)	0.010 (0.006)	0.010 (0.012)	0.009 (0.012)
GDP trend growth rates (current, percent) ^{4/}	-0.003 (0.008)	-0.002 (0.007)	0.012 (0.017)	0.011 (0.014)
Episode-specific fixed effects	No	No	No	No
Number of default/restructuring episodes	53	53	35	35
Number of observations	339	339	154	154
Wald χ^2	70.71	77.08	25.06	35.35
Prob.> χ^2	0.000	0.000	0.000	0.000

Notes: The table shows results from random effects logit regressions. The dependent variable is debt settlement in the current year (binary). The main explanatory variables are public investment-to-GDP ratio in an upward trend measured as lagged or a cumulative change from trough to a lagged period. PPG external debt (percent of GDP) is lagged by one year. The other explanatory variables are in the current year. Significance levels are denoted by *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$, respectively. Robust standard errors (Delta-method standard errors) are in parentheses.

^{1/} A multiple of public investment-to-GDP ratio (lagged) and a dummy variable of an upward trend of public investment-to-GDP ratio (lagged, a binary variable).

^{2/} A multiple of public investment-to-GDP ratio (a cumulative change from trough to a lagged period) and a dummy variable of an upward trend of public investment-to-GDP ratio (lagged, a binary variable).

^{3/} Public and publicly guaranteed external debt.

^{4/} A deviation from the trend and a trend growth rate are a percentage deviation from the trend and an annual percent change of the trend obtained by applying a Hodrick-Prescott (HP) filter to annual GDP series with filter of 6.25.

restructurings. In this regard, there are overlapping observations included in our panel.

Following the convention in the literature (e.g., Struzenegger 2004, Asonuma and Trebesch 2016), our data are at an annual frequency due to the data availability of public investment and PPG external debt for the restructuring countries. The dependent variable captures whether restructurings are settled or not in the current year: 1 for completion of exchanges and 0 otherwise. Our main explanatory variables are public investment-to-GDP ratio in an upward trend—measured as lagged and a cumulative change from trough to a lagged period. Both explanatory variables are multiples of public investment-to-GDP ratio (as both lagged and a cumulative change from trough to a lagged period) and a dummy variable for an upward trend (lagged). We also include PPG external debt (in percent of GDP), and GDP deviation from the trend and GDP trend growth rates, which serve as proxies for productivity shocks.

Table 3 shows the logit regression results. We show that when the lagged public investment-to-GDP ratio is at a higher level in an upward trend, the sovereign is more likely to reach settlement in the current year for both restructurings with and without recovered debt payments in cash at settlement (columns 1 and 3). Quantitatively, a 1-percent increase in the lagged public investment-to-GDP ratio in an upward trend increases the probability of settlement by 4.7 and 6.9 percent, respectively. Furthermore, a cumulative public investment-to-GDP ratio (from trough to a lagged period) significantly increases the likelihood of debt settlement for restructurings with recovered debt payments in cash at settlement (column 2), but does not for those without recovered debt payments in cash at settlement (column 4). This is because for those without recovered debt payments in cash at settlement, debt settlement takes place before public investment recovers to the pre-restructuring level due to no recovered debt payments in cash at settlement (panel (ii) in Figure 1).

3 Theoretical Model

3.1 Summary of Theoretical Findings

Our theoretical model is built to shed light on the role of public capital (investment) on sovereign debt crises and resolutions. In particular, our model of sovereign debt embeds explicitly endogenous public capital accumulation, expenditure composition, production, and post-default multi-round renegotiations. To account for different economic situations for sovereign debtors, we take a two-step approach. At the first step, we use a conventional sovereign debt model with fiscal policy—private and public sectors are separated by distortionary consumption tax (and no lump-sum taxation) and two different consumption goods (Cuadra et al. 2010; Arellano and Bai 2017)—as benchmark and derive main results in Sections 3–5. At the second step, we incorporate additional assumptions used in the previous studies (e.g., Arellano and Bai 2017) in our framework and show the robustness of our model in Appendix C.

Our theoretical model makes two novel predictions shown mainly quantitatively. The first prediction is the role of public capital on the sovereign’s choice of default, debt settlement, and restructuring delays.¹⁴ *After default*, the sovereign is willing to delay renegotiations, *ceteris paribus*, when public capital is low. It opts to invest its limited resources—owing to both financial exclusion and productivity loss—in public capital rather than allocate those to recovered debt payments given the high marginal product of public capital. Therefore, the marginal product of public capital determines debt settlement and delays.¹⁵ *Before default*, the sovereign’s

¹⁴Public capital in our model differs from foreign reserves in Bianchi et al. (2018) in two aspects. First, output in the next period is endogenous depending on the current public capital in our model, while income in the next period is exogenous in their model. Second, public capital is long-term asset with state-dependent returns depending on both public capital and productivity shocks in our model, while foreign reserves are short-term (one-period) assets with fixed and exogenous returns in their model.

¹⁵There is no immediate settlement with new lending due to limited commitment (Benjamin and Wright 2013). This is because when the sovereign’s repayment capacity has not fully recovered due to low productivity, the creditors anticipate that the sovereign is more likely to default on newly issued debt immediately after settlement, and opt to delay the settlement until the sovereign’s repayment capacity recovers avoiding an immediate default.

willingness to repay remains unchanged or decreases when public capital increases. On the one hand, ex ante, higher public capital improves its repayment capacity (“smoothing channel”). On the other hand, ex post (after default), higher public capital smooths household consumption in financial autarky (“autarky channel”) and achieves quick debt settlement (“renegotiation channel”). Benefits from the latter two channels are equal to or weakly dominate those from the former channel due to introduction of the renegotiation channel.¹⁶

The second prediction is the mechanism underlying the dynamics of public investment, and consumption and transfers. *At default*, a severe productivity shock interacts with the sovereign’s consumption-smoothing motive, resulting in a mild decline in public consumption and transfers, a severe decline in public investment, and default. The sovereign prioritizes public consumption and transfers for household consumption smoothing over public investment and external debt payments. *During post-default restructuring*, a combination of slow recovery of productivity, prohibition on external borrowing, and the sovereign’s consumption-smoothing motive generates quick recovery in public consumption and transfers, slow public capital accumulation, and long restructuring delays. Public consumption and transfers recover quickly because the sovereign prioritizes household consumption smoothing. Public capital accumulation is slow both because external borrowing is unfeasible until debt settlement, and because under fiscal constraint, the sovereign has limited resources left after spending on public consumption and transfers.

3.2 Assumptions in the Model

There are four agents in the model: a household, a firm, a sovereign (government), and foreign creditors. The sovereign is risk averse and cannot affect the global risk-free interest rate (r^*). Foreign creditors are risk-neutral. They can borrow or lend as much as needed at the constant risk-free interest rate in the international capital market.

In each period, a stochastic productivity shock a_t materializes. It is stochastic, drawn from a compact set $A = [a_{min}, a_{max}] \subset R$. $\mu(a_{t+1}|a_t)$ is a probability distribution of a shock a_{t+1} conditional on its previous realization a_t . In addition, the sovereign has a credit record $h_t \in [0, 1]$, which indicates whether it has maintained access to the market ($h_t = 0$) or it has lost market access due to default ($h_t = 1$).

After observing the productivity shock, the sovereign receives consumption tax revenues (no lump-sum taxation allowed) and decides expenditure composition—public consumption, investment and transfers—and choice of repayment and default, settlement and delay, and of external borrowing. Consumption tax revenues are determined by the household’s optimal choice of private consumption given a constant consumption tax rate. Public consumption and transfers are provided to the household to improve his utility directly or indirectly by smoothing private consumption. Public capital rented to the firm is accumulated through net investment and is subject to both depreciation and adjustment costs. The household receives profits from the firm, and public consumption and transfers from the sovereign, respectively. He chooses private

¹⁶Hamann et al. (2018) find similar effects from two channels of oil reserves on the sovereign’s default choice.

consumption and labor supply, and pays consumption taxes to the sovereign. The firm receives public capital from the sovereign, chooses labor demand, and pays profits to the household.

The sovereign bond market is incomplete. Only the sovereign can borrow via long-term sovereign bonds at the market, while neither the household nor firm can. Following Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009), current outstanding debt (b_t) matures with probability λ and if it does not mature, it provides a coupon payment z . Its set is shown by $B = [b_{min}, 0]$ where $b_{min} \leq 0$. We set the lower bound for the sovereign's bond holding as the highest debt that the sovereign can repay. New debt issuance in the current period is shown by $-b_{t+1} + (1 - \lambda)b_t$. We assume $q(b_{t+1}, k_{t+1}^g, 0, a_t)$ to be price of sovereign bonds with the sovereign's debt b_{t+1} , public capital k_{t+1}^g , a good credit record ($h_t = 0$), and a productivity shock a_t . The bond price is determined in equilibrium.

We assume that the foreign creditors always commit to repay their debt. However, the sovereign is free to decide whether to repay its debt or to default. If the sovereign chooses to repay its debt, it will preserve access to the international capital market in the next period. On the contrary, if it chooses to default, it is then subject to exclusion from the international capital market, direct productivity loss, and accumulation of arrears.^{17,18}

After default, debt resolution (i.e., renegotiation on debt settlement) is done only via multi-round bargaining game between the sovereign and the foreign creditors. Debt settlement can be done with or without recovered debt payments in cash. At the renegotiation, one party, who is randomly selected with exogenous and constant probability, chooses whether to propose an offer with haircuts (recovery rates) or to pass its option. The other party decides whether to accept or reject the offer. If the offer with haircuts is proposed and accepted, then the sovereign pays recovered debt payments with cash and/or new debt and has a good credit record in the next period ($h_{t+1} = 0$). The foreign creditors receive recovered debt payments with cash and/or new debt. Otherwise, both parties continue the negotiation over debt in arrears in the next period.

In order to avoid permanent exclusion from the international capital market and direct productivity loss, the sovereign has an incentive to renegotiate and to pay recovered debt payments. The foreign creditors also have an incentive to renegotiate and receive the recovered debt payments because renegotiation is the only option to recoup losses on the defaulted debt.

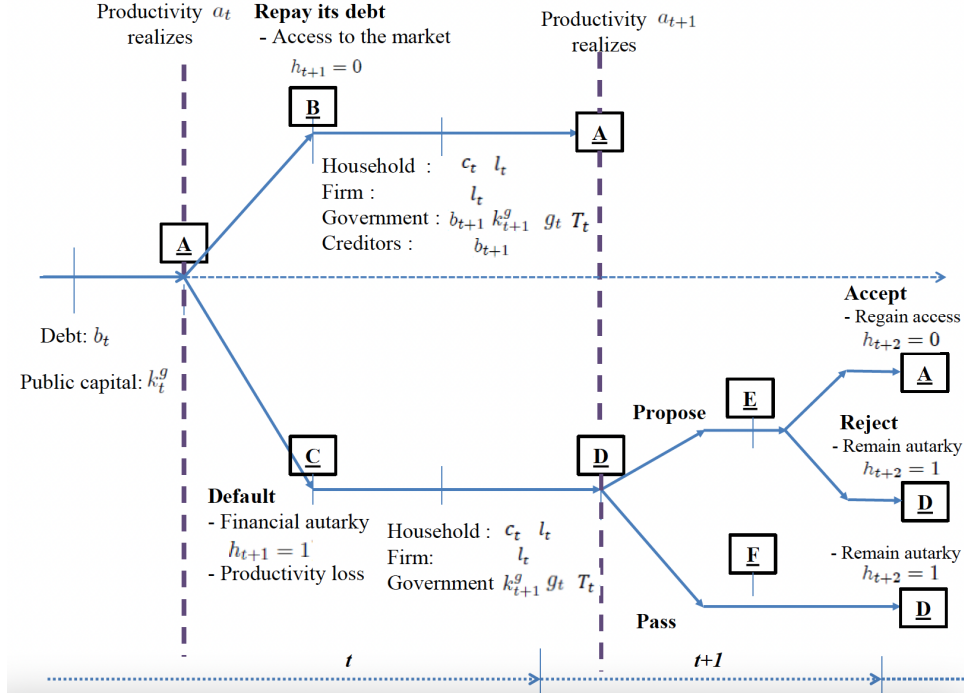
3.3 Timing of the Model

Figure 4 summarizes the timing of decisions within each period.

¹⁷The direct productivity loss assumption in our production model is conceptually equivalent to “output costs” assumption in conventional (exogenous) endowment models (e.g., Arellano 2008; Aguiar and Gopinath 2006; Yue 2010). In this regard, the direct production loss is widely accepted in the sovereign debt literature with endogenous production (Cuadra et al. 2010; Arellano and Bai 2017; Gordon and Guerron-Quintana 2018). Both assumptions are broadly in line with empirical estimates of output loss in defaults in general (Sturzenegger 2004; Tomz and Wright 2007; Levy-Yeyati and Panizza 2011) and those in post-default restructurings (Asonuma and Trebesch 2016; Asonuma et al. 2021).

¹⁸Mendoza and Yue (2012) theoretically explain that exclusion from the international capital market leads to declines in production efficiency due to a lack of imported inputs and labor reallocation away from final goods production.

Figure 4: Timing of Model



1. The sovereign starts the current period with initial debt and public capital. We are in node (A).
2. A productivity shock (a_t) realizes. The sovereign decides whether to repay its debt or to default.
3. (a) In node (B) (repayment node), if repayment is chosen, we move to the upper branch of a tree. The sovereign maintains market access ($h_{t+1} = 0$) and chooses debt in the next period, public consumption, capital and transfers. The foreign creditors choose the amount of sovereign bonds in the next period. The sovereign bond price is determined in the market. The household chooses his private consumption and labor supply, and the firm chooses labor demand. We proceed to node (A) in the next period.
(b) In node (C) (default node), if default is chosen, we move to the lower branch of a tree. The sovereign loses market access ($h_{t+1} = 1$), suffers the direct productivity loss, and chooses public consumption, capital and transfers. The household chooses his private consumption and labor supply, and the firm chooses labor demand.
4. A productivity shock (a_{t+1}) realizes.
5. In node (D) (default/post-default restructuring node), with constant probability, the sovereign has an opportunity to propose an offer to the foreign creditors. Otherwise, the

foreign creditors have an opportunity to propose an offer to the sovereign. The proposer decides whether to propose an offer or to pass.

6. (a) In node (E) (propose node), if the proposer chooses to propose an offer, the counterpart decides whether to accept or reject the offer. If the counterpart accepts the offer, the sovereign pays recovered debt payments with cash and/or new debt and has a good credit record in the next period ($h_{t+2} = 0$). We move to node (A) in the next period. Otherwise (if the counterpart rejects), the sovereign remains in autarky ($h_{t+2} = 1$). We move back to node (D).
- (b) In node (F) (pass node) if the proposer chooses to pass, the sovereign remains in autarky ($h_{t+2} = 1$). We move back to node (D).

4 Recursive Equilibrium

4.1 Household's Problem

This section defines the stationary recursive equilibrium of our model. A representative household's utility function is defined as:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t [(1 - \pi)u(c_t, l_t) + \pi v(g_t)]$$

where $0 < \beta < 1$ is a discount factor and c_t , l_t , g_t denote private consumption, labor supply and public consumption in period t , respectively. His period utility function is separable between a multiple of private consumption and labor supply, and public consumption. Both $u(\cdot)$ and $v(\cdot)$ are continuous, strictly increasing, strictly concave, and satisfy the Inada conditions. π denotes the weight on public consumption in the household's utility function.

Given the wage rate w_t , profits paid by the firm π_t^F , public transfers T_t , public consumption g_t and a consumption tax rate τ , the household chooses private consumption and labor supply.¹⁹ The household does not borrow directly from abroad, but the sovereign borrows, provides public consumption and transfers, and makes default decisions internalizing the household's utility.²⁰ The household's optimization problem is written as:

$$\max_{c_t, l_t} E_0 \sum_{t=0}^{\infty} \beta^t [(1 - \pi)u(c_t, l_t) + \pi v(g_t)] \quad (1)$$

¹⁹Introducing labor income tax in the model does not provide additional insights (Arellano and Bai 2017; Mendoza et al. 2014) as shown in panel B in Figure C2 in Appendix C. This is because, labor income tax and consumption tax are conceptually identical in that both affect the household's intra-temporal substitution between private consumption and labor (equation 3), but not the sovereign's inter-temporal substitution between consumption—public consumption and transfers—and saving (i.e., public investment).

²⁰Though the household lacks access to the international capital market as in conventional sovereign debt models, his utility can still be improved through three methods: (i) private consumption through transfers, (ii) public consumption, and (iii) labor supply.

$$s.t. \quad (1 + \tau)c_t = w_t l_t + \pi_t^F + T_t \quad (2)$$

The consumption tax rate is assumed to be constant (Arellano and Bai 2017)—also supported by empirical findings on value-added taxes in developing countries in Gunter et al. (2017). The optimality condition of the household is shown as follows:

$$\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \frac{w_t}{1 + \tau} \quad (3)$$

4.2 Firm's Problem

A representative firm chooses labor l_t for goods production given productivity shock a_t , public capital k_t^g and fixed private capital stock ($\bar{k}^p = 1$) following Mendoza and Yue (2012) and Azzimonti (2015). The production function is Cobb-Douglas:

$$y_t = a_t(l_t)^{\alpha_l}(k_t^g)^{\alpha_k}(\bar{k}^p)^{1-\alpha_l-\alpha_k} \quad (4)$$

where α_l and α_k denote labor and public capital income share. The firm's optimization problem is written as:

$$\max_{l_t} \pi_t^F = a_t(l_t)^{\alpha_l}(k_t^g)^{\alpha_k}(\bar{k}^p)^{1-\alpha_l-\alpha_k} - w_t l_t \quad (5)$$

The optimality condition of the firm is shown as follows:

$$w_t = \alpha_l a_t(l_t)^{\alpha_l-1}(k_t^g)^{\alpha_k}(\bar{k}^p)^{1-\alpha_l-\alpha_k} \quad (6)$$

4.3 Sovereign's (Government's) Problem

The sovereign maximizes its expected lifetime utility and its value function is denoted by $V(b_t, k_t^g, h_t, a_t)$. First, we start with its problem when the sovereign has a good credit record ($h_t = 0$). It decides whether to repay or to default after observing its productivity shock. If the sovereign chooses to pay its debt, it receives tax revenues from the household and determines public consumption, capital, transfers, and debt in the next period. In contrast, if it chooses to default, it will be excluded from the international capital market and its credit record deteriorates to $h_{t+1} = 1$, with debt in arrears $b_{t+1} = (1 + r^*)b_t$ in the next period where r^*b_t corresponds to accumulation of arrears. After suffering the direct productivity loss and receiving tax revenues, it determines public consumption, capital and transfers.

$$V(b_t, k_t^g, 0, a_t) = \max [V^R(b_t, k_t^g, 0, a_t), V^D(b_t, k_t^g, 0, a_t)] \quad (7)$$

$V^R(b_t, k_t^g, 0, a_t)$ is its value associated with repayment:

$$V^R(b_t, k_t^g, 0, a_t) = \max_{g_t, b_{t+1}, k_{t+1}^g, T_t} (1 - \pi)u(c_t, l_t) + \pi v(g_t) + \beta \int_A V(b_{t+1}, k_{t+1}^g, 0, a_{t+1}) d\mu(a_{t+1}|a_t) \quad (8)$$

$$s.t. \ g_t + k_{t+1}^g + T_t + q(b_{t+1}, k_{t+1}^g, 0, a_t)(b_{t+1} - (1 - \lambda)b_t) = \tau c_t + (1 - \delta^k)k_t^g - \frac{\Omega}{2} \left(\frac{k_{t+1}^g - k_t^g}{k_t^g} \right)^2 k_t^g + (\lambda + (1 - \lambda)z)b_t \quad (9)$$

$$T_t \geq 0 \quad (10)$$

$$\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_l a_t (l_t)^{\alpha_l - 1} (k_t^g)^{\alpha_k} (\bar{k}^p)^{1 - \alpha_l - \alpha_k}}{1 + \tau} \quad (11)$$

$$(1 + \tau)c_t = y_t + T_t \quad (12)$$

where equation (9) is the budget constraint for the sovereign where it receives consumption tax revenues τc_t , public capital stock net of depreciation and adjustment costs $(1 - \delta^k)k_t^g - \frac{\Omega}{2} \left(\frac{k_{t+1}^g - k_t^g}{k_t^g} \right)^2 k_t^g$ —non-linear adjustment costs are assumed²¹ and δ^k is the depreciation rate of public capital—and repays debt payments due in the current period $(\lambda + (1 - \lambda)z)b_t$. It allocates to public consumption g_t , capital k_{t+1}^g , transfers T_t and issues new debt in the current period $q(b_{t+1}, k_{t+1}^g, 0, a_t)(b_{t+1} - (1 - \lambda)b_t)$. Equation (10) is the “no lump-sum taxation constraint”—lump-sum taxation is not allowed. A combination of distortionary consumption taxation and no lump-sum taxation constraint corresponds to “fiscal constraint” such as limitations of the sovereign from transferring resources to and from the private sector (Arellano and Bai 2017). Mechanically, the sovereign can freely transfer positive net borrowing through transfers, but cannot extract more resources from the private sector beyond the distortionary consumption tax revenues. Equations (11) and (12) denote the combined optimality condition and budget constraint for both the household and the firm, respectively.

$V^D(b_t, k_t^g, 0, a_t)$ is its value associated with default:

$$V^D(b_t, k_t^g, 0, a_t) = \max_{g_t, k_{t+1}^g, T_t} (1 - \pi)u(c_t, l_t) + \pi v(g_t) + \beta \int_A V((1 + r^*)b_t, k_{t+1}^g, 1, a_{t+1}) d\mu(a_{t+1} | a_t) \quad (13)$$

s.t. (10) and

$$g_t + k_{t+1}^g + T_t = \tau c_t + (1 - \delta^k)k_t^g - \frac{\Omega}{2} \left(\frac{k_{t+1}^g - k_t^g}{k_t^g} \right)^2 k_t^g \quad (9a)$$

$$\frac{u_l(c_t, l_t)}{u_c(c_t, l_t)} = \frac{\alpha_l \tilde{a}_t (l_t)^{\alpha_l - 1} (k_t^g)^{\alpha_k} (\bar{k}^p)^{1 - \alpha_l - \alpha_k}}{1 + \tau} \quad (11a)$$

$$(1 + \tau)c_t = \tilde{y}_t + T_t \quad (12a)$$

where $\tilde{y}_t = \tilde{a}_t (l_t)^{\alpha_l} (k_t^g)^{\alpha_k} (\bar{k}^p)^{1 - \alpha_l - \alpha_k}$ indicating output with the direct productivity loss \tilde{a}_t .

The sovereign’s default choice can be characterized by default set $D(b_t, k_t^g, 0) \subset A$. It is a set of productivity shocks a_t at which default is optimal:

$$D(b_t, k_t^g, 0) = \{a_t \in A : V^R(b_t, k_t^g, 0, a_t) < V^D(b_t, k_t^g, 0, a_t)\} \quad (14)$$

²¹Non-linear adjustment costs are assumed to replicate to smooth investment dynamics.

Second, we continue with the sovereign's problem with a bad credit record and debt in arrears ($h_t = 1$ & $b_t < 0$). The sovereign is currently excluded from the international capital market, suffers the direct productivity loss, and may reach a settlement through renegotiations with the foreign creditors. Its value, denoted by $V(b_t, k_t^g, 1, a_t)$, is an expected payoff that the sovereign obtains from the bargaining which starts in period t :

$$V(b_t, k_t^g, 1, a_t) = \Gamma(b_t, k_t^g, a_t) \quad (15)$$

4.4 Foreign Creditors' Problem

Foreign creditors are risk-neutral and can borrow from the international capital market with the global risk-free interest rate (r^*). When the sovereign has a good credit record ($h_t = 0$), given the sovereign bond price, the foreign creditors choose the amount of debt in the next period (b_{t+1}) to maximize the expected profit:

$$\begin{aligned} \pi^c(b_{t+1}, k_{t+1}^g, 0, a_t) = & \left[\frac{(1 - p^D(b_{t+1}, k_{t+1}^g, 0, a_t))\{\lambda + (1 - \lambda)[z + q(b_{t+2}, k_{t+2}^g, 0, a_{t+1})]\}}{1 + r^*} \right. \\ & + \frac{p^D(b_{t+1}, k_{t+1}^g, 0, a_t) \int_A \gamma(b_{t+1}, k_{t+1}^g, 1, a_{t+1}) d\mu(a_{t+1}|a_t)}{1 + r^*} \left. \right] (-b_{t+1}) \\ & - q(b_{t+1}, k_{t+1}^g, 0, a_t)(-b_{t+1}) \end{aligned} \quad (16)$$

where $p^D(b_{t+1}, k_{t+1}^g, 0, a_t)$ and $\gamma(b_{t+1}, k_{t+1}^g, 1, a_{t+1})$ are the probability of default and expected recovery rates conditional on the sovereign's default choice (defined in equations 37 and 39).

Since we assume that the sovereign bond market is competitive, the foreign creditors' expected profit is zero in equilibrium. Using a zero expected profit condition, we get

$$\begin{aligned} q(b_{t+1}, k_{t+1}^g, 0, a_t) = & \frac{(1 - p^D(b_{t+1}, k_{t+1}^g, 0, a_t))\{\lambda + (1 - \lambda)[z + q(b_{t+2}, k_{t+2}^g, 0, a_{t+1})]\}}{1 + r^*} \\ & + \frac{p^D(b_{t+1}, k_{t+1}^g, 0, a_t) \int_A \gamma(b_{t+1}, k_{t+1}^g, 1, a_{t+1}) d\mu(a_{t+1}|a_t)}{1 + r^*} \end{aligned} \quad (17)$$

There is default risk and the bonds are priced to compensate the foreign creditors for the risk. Since $0 \leq p^D(b_{t+1}, k_{t+1}^g, 0, a_t) \leq 1$ and $0 \leq \gamma(b_{t+1}, k_{t+1}^g, 1, a_{t+1}) \leq 1$, the sovereign bond price $q(b_{t+1}, k_{t+1}^g, 0, a_t)$ lies in $[0, \frac{1}{1+r^*}]$.

4.5 Debt Renegotiation

The debt renegotiation takes the form of a two-player stochastic bargaining game with complete information as in Merlo and Wilson (1995).²² It is a multi-round stochastic bargaining game

²²While the bargaining game between two parties can be modeled in other different forms, we follow the conventional bargaining game in Merlo and Wilson (1995) for their simplicity and tractability.

in that both the productivity process of the sovereign and the identity of the proposer are stochastic. The foreign creditors' incentive to delay settlement is identical to that in previous studies on multi-round renegotiations (Benjamin and Wright 2013; Bi 2008): the risk-neutral creditors with constant discount rate, who care only about recovery rates in present value terms, prefer to wait for the sovereign's capacity to repay high recovered debt payments.

More importantly, however, the sovereign's incentive to delay settlement clearly differentiates our model from the previous studies: in their models, the sovereign is willing to wait for recovery of repayment capacity, (i.e., output) which follows an exogenous process. In contrast, in our model, what determines the sovereign's choice of settlement and delay are not only the recovery of repayment capacity (i.e., productivity) but also state-dependent benefits and costs of public investment (i.e., the marginal product of public capital). The sovereign opts to delay settlement because it prioritizes investing limited resources—owing to both financial exclusion and productivity loss—in public capital over debt settlement with its foreign creditors until public capital reaches a high level (low marginal product of public capital).

At settlement, the sovereign repays recovered debt payments either by both cash and new debt (Bi 2008) or by only new debt (Benjamin and Wright 2013).²³

In every round, a state is realized and the proposer is randomly selected. For simplicity, each player has a constant probability of being selected as the proposer in each round of the negotiation. That is, the identity of the proposer is independent of the sovereign's productivity process. Let ϕ denote the probability that the borrower (sovereign), B, can propose and $1-\phi$ denote the probability that the lender (foreign creditors), L, can propose. The probability with which one of the players is selected as the proposer is a parsimonious way to reflect the bargaining power obtained through one's ability to enjoy the first-mover advantage. The proposer chooses to propose recovery rates (haircuts) or to pass. If he proposes, then the counterpart chooses to accept or to reject the proposal.²⁴ If the proposal is accepted, then the sovereign repays its reduced debt arrears and resumes access to the international capital market in the next period. If the proposal is rejected, both parties repeat the bargaining game in the next period. If the proposer passes, both parties also repeat the bargaining game in the next period.

We define some basic concepts of the game. A stochastic bargaining game is denoted by $(C, \beta, 1/(1+r^*))$, where for each productivity process $a \in A$, $C(a)$ is the set of feasible utility vectors that may be agreed upon in that state. β and $1/(1+r^*)$ are the discount factors for B and L, respectively.²⁵ A payoff function is an element $\Delta(a) \in C(a)$, where $\Delta_i(a)$ is the utility

²³Extending the model to consider endogenous choice of recovered debt payment methods does not provide additional insights in our model but increases technical difficulty to solve the model.

²⁴We assume that the proposer makes an offer that the counterpart accepts when the value of proposing is higher or equal to the value of passing, and passes otherwise. This assumption can get rid of trivial sources of multiplicity. See Merlo and Wilson (1995) for the same treatment.

²⁵Merlo and Wilson (1995) assume a common discount factor between the two players. However, they explain that "there is no real restriction implied by the assumption that players discount utility at a common constant rate. So long as the discounted size of the "cake" converges uniformly to 0. . . player-dependent discount factors can always be represented by a different "cake" process with a common fixed discount factor". Our model assumes asymmetric discount factors between the borrower and the lender.

to player i for $i = B, L$.

As in Merlo and Wilson (1995), we focus on a game with stationary strategies, that is, the players' actions depend only on the current state $(b_t, k_t^g, 1, a_t)$ where $h_t = 1$ and the current offer. In equilibrium, the proposer's strategy is to propose when the counterpart would accept for certain and to pass otherwise. In contrast, the counterpart's strategy is to accept when the proposal is made and to reject otherwise. Therefore, we can denote the proposer i 's and the counterpart j 's equilibrium strategies as follows: (a) $\theta_i(b_t, k_t^g, 1, a_t) = 1$ (propose) when the proposer i proposes and $\theta_j(b_t, k_t^g, 1, a_t) = 1$ (accept) when the counterpart j accepts the offer, or (b) $\theta_i(b_t, k_t^g, 1, a_t) = 0$ (pass) when the proposer i passes and $\theta_j(b_t, k_t^g, 1, a_t) = 0$ (reject) when the counterpart j rejects the offer.²⁶

A stationary subgame perfect (SP) equilibrium is defined as the players' equilibrium stationary strategies θ and θ^* , and the payoff functions, Γ and Γ^* associated with these strategies for player B and L. The expected payoffs for the borrower B and lender L in period t , are shown as:

$$\Gamma(b_t, k_t^g, a_t) = \phi \Gamma^B(b_t, k_t^g, a_t) + (1 - \phi) \Gamma^L(b_t, k_t^g, a_t) \quad (18)$$

$$\Gamma^*(b_t, k_t^g, a_t) = \phi \Gamma^{*B}(b_t, k_t^g, a_t) + (1 - \phi) \Gamma^{*L}(b_t, k_t^g, a_t) \quad (19)$$

Here, the superscript denotes the identity of the proposer: $\Gamma^B(\Gamma^{*B})$ represents the borrower's (lender's) payoff when the borrower is the proposer and $\Gamma^L(\Gamma^{*L})$ refers to the borrower's (lender's) payoff when the lender is the proposer.

First, we start with the case when the borrower B is the proposer. We denote the proposed debt recovery rates as δ_t^B , the borrower's values of proposing and passing as V^{PRO} and V^{PASS} , and the lender's values of accepting and rejecting as V^{*ACT} and V^{*REJ} , respectively. When the borrower B proposes and the proposal is accepted, the sovereign repays reduced debt arrears (i.e., recovered debt payments) $-\delta_t^B b_t$ with either a combination of cash and new debt or only new debt, and resumes access to the international capital market ($h_{t+1} = 0$) in the next period.

$$V^{PRO}(b_t, k_t^g, a_t) = \max_{g_t, k_{t+1}^g, T_t} (1 - \pi)u(c_t, l_t) + \pi v(g_t) + \beta \int_A V(b_{t+1}, k_{t+1}^g, 0, a_{t+1}) d\mu(a_{t+1}|a_t) \quad (20)$$

s.t. (10), (11a), (12a), and

$$g_t + k_{t+1}^g + T_t + q(b_{t+1}, k_t^g, a_t)b_{t+1} = \tau c_t + (1 - \delta^k)k_t^g - \frac{\Omega}{2} \left(\frac{k_{t+1}^g - k_t^g}{k_t^g} \right)^2 k_t^g + \delta_t^B b_t \quad (9b)$$

$$(i) q(b_{t+1}, k_t^g, a_t)b_{t+1} \geq \delta_t^B b_t \text{ or } (ii) q(b_{t+1}, k_t^g, a_t)b_{t+1} < \delta_t^B b_t \quad (21)$$

where equation (21) defines whether recovered debt payments are paid with a combination of

²⁶Benjamin and Wright (2013) theoretically prove both existence and uniqueness of the equilibrium in the multi-round bargaining over defaulted debt.

cash and new debt (case (i)) or only new debt (case (ii)).

$$V^{*ACT}(b_t, k_t^g, a_t) = -\delta_t^B b_t \quad (22)$$

When the borrower B passes, both parties proceed to the next period with accumulated arrears $(1 + r^*)b_t$.

$$V^{PASS}(b_t, k_t^g, a_t) = \max_{g_t, k_{t+1}^g, T_t} (1 - \pi)u(c_t, l_t) + \pi v(g_t) + \beta \int_A V((1 + r^*)b_t, k_{t+1}^g, 1, a_{t+1}) d\mu(a_{t+1}|a_t) \quad (23)$$

s.t. (9a), (10), (11a), and (12a)

$$V^{*REJ}(b_t, k_t^g, a_t) = \frac{1}{1 + r^*} \int_A \Gamma^*((1 + r^*)b_t, k_{t+1}^g, a_{t+1}) d\mu(a_{t+1}|a_t) \quad (24)$$

In equilibrium where off-equilibrium paths are eliminated, the agreed recovery rates δ_t^{B*} satisfy the following:

$$\begin{aligned} \delta_t^{B*} &= \operatorname{argmax} V^{PRO}(b_t, k_t^g, a_t) \\ \text{s.t. } V^{PRO}(b_t, k_t^g, a_t) &\geq V^{PASS}(b_t, k_t^g, a_t) \\ V^{*ACT}(b_t, k_t^g, a_t) &\geq V^{*REJ}(b_t, k_t^g, a_t) \end{aligned} \quad (25)$$

If both parties reach an agreement, the two parties' payoffs are as follows:

$$\Gamma^B(b_t, k_t^g, a_t) = V^{PRO}(b_t, k_t^g, a_t) \quad (26)$$

$$\Gamma^{B*}(b_t, k_t^g, a_t) = V^{*ACT}(b_t, k_t^g, a_t) \quad (27)$$

Otherwise,

$$\Gamma^B(b_t, k_t^g, a_t) = V^{PASS}(b_t, k_t^g, a_t) \quad (26a)$$

$$\Gamma^{B*}(b_t, k_t^g, a_t) = V^{*REJ}(b_t, k_t^g, a_t) \quad (27a)$$

The debt settlement can be characterized by settlement set $R^B(b_t, k_t^g) \subset A$. It is a set of productivity shocks a_t at which both parties reach an agreement:

$$R^B(b_t, k_t^g) = \left\{ a_t \in A : \begin{aligned} &V^{PRO}(b_t, k_t^g, a_t) \geq V^{PASS}(b_t, k_t^g, a_t) \\ &V^{*ACT}(b_t, k_t^g, a_t) \geq V^{*REJ}(b_t, k_t^g, a_t) \end{aligned} \right\} \quad (28)$$

Second, we consider the case when the lender L is the proposer. We denote the proposed debt recovery rates as δ_t^L , the borrower's values of accepting and rejecting as V^{ACT} and V^{REJ} , and the lender's values of proposing and passing as V^{*PRO} and V^{*PASS} , respectively. When the

lender L proposes and the proposal is accepted,

$$V^{*PRO}(b_t, k_t^g, a_t) = -\delta_t^L b_t \quad (29)$$

$$V^{ACT}(b_t, k_t^g, a_t) = \max_{g_t, k_{t+1}^g, T_t} (1 - \pi)u(c_t, l_t) + \pi v(g_t) + \beta \int_A V(b_{t+1}, k_{t+1}^g, 0, a_{t+1}) d\mu(a_{t+1}|a_t) \quad (30)$$

s.t. (10), (11a), (12a), and

$$g_t + k_{t+1}^g + T_t + q(b_{t+1}, k_t^g, a_t)b_{t+1} = \tau c_t + (1 - \delta^k)k_t^g - \frac{\Omega}{2} \left(\frac{k_{t+1}^g - k_t^g}{k_t^g} \right)^2 k_t^g + \delta_t^L b_t \quad (9c)$$

$$(i) \ q(b_{t+1}, k_t^g, a_t)b_{t+1} \geq \delta_t^L b_t \text{ or } (ii) \ q(b_{t+1}, k_t^g, a_t)b_{t+1} < \delta_t^L b_t \quad (21a)$$

When the lender L passes,

$$V^{*PASS}(b_t, k_t^g, a_t) = \frac{1}{1 + r^*} \int_A \Gamma^*((1 + r^*)b_t, k_{t+1}^g, a_{t+1}) d\mu(a_{t+1}|a_t) \quad (31)$$

$$V^{REJ}(b_t, k_t^g, a_t) = \max_{g_t, k_{t+1}^g, T_t} (1 - \pi)u(c_t, l_t) + \pi v(g_t) + \beta \int_A V((1 + r^*)b_t, k_{t+1}^g, 1, a_{t+1}) d\mu(a_{t+1}|a_t) \quad (32)$$

s.t. (9a), (10), (11a), and (12a)

In equilibrium, the agreed recovery rates δ_t^{L*} satisfy the following:

$$\begin{aligned} \delta_t^{L*} &= \operatorname{argmax} V^{*PRO}(b_t, k_t^g, a_t) \\ \text{s.t. } V^{*PRO}(b_t, k_t^g, a_t) &\geq V^{*PASS}(b_t, k_t^g, a_t) \\ V^{ACT}(b_t, k_t^g, a_t) &\geq V^{REJ}(b_t, k_t^g, a_t) \end{aligned} \quad (33)$$

If both parties reach an agreement, the two parties' payoffs are as follows:

$$\Gamma^{*L}(b_t, k_t^g, a_t) = V^{*PRO}(b_t, k_t^g, a_t) \quad (34)$$

$$\Gamma^L(b_t, k_t^g, a_t) = V^{ACT}(b_t, k_t^g, a_t) \quad (35)$$

Otherwise,

$$\Gamma^{*L}(b_t, k_t^g, a_t) = V^{*PASS}(b_t, k_t^g, a_t) \quad (34a)$$

$$\Gamma^L(b_t, k_t^g, a_t) = V^{REJ}(b_t, k_t^g, a_t) \quad (35a)$$

The debt settlement can be characterized by settlement set $R^L(b_t, k_t^g) \subset A$. It is a set of productivity shocks a_t at which both parties reach an agreement:

$$R^L(b_t, k_t^g) = \left\{ a_t \in A : \begin{array}{l} V^{*PRO}(b_t, k_t^g, a_t) \geq V^{*PASS}(b_t, k_t^g, a_t) \\ V^{ACT}(b_t, k_t^g, a_t) \geq V^{REJ}(b_t, k_t^g, a_t) \end{array} \right\} \quad (36)$$

4.6 Equilibrium

A recursive equilibrium is defined as a set of functions for (a) the sovereign's value function, public consumption, capital, transfers, debt, default set, (b) the household's private consumption, labor supply, (c) the firm's labor demand, (d) the sovereign bond price, and (e) the sovereign's and the foreign creditors' decision functions, payoffs, recovery rates, debt settlement sets (all depending on who is the proposer) such that

[1]. the sovereign's value function, public consumption, capital, transfers, debt, and default set satisfy its optimization problem (7)–(15);

[2]. the household's private consumption and labor supply satisfy his optimization problem (1)–(3);

[3]. the firm's labor demand satisfies its optimization problem (4)–(6);

[4]. the sovereign bond price satisfies the foreign creditors' optimization problem (16)–(17);

[5]. both parties' decisions, payoffs, recovery rates, and debt settlement sets solve the multi-round debt renegotiation problem (18)–(36).

In equilibrium, the probability of default and settlement is defined by using the sovereign's default set and the debt settlement sets, respectively:

$$p^D(b_{t+1}, k_{t+1}^g, 0, a_t) = \int_{D(b_{t+1}, k_{t+1}^g)} d\mu(a_{t+1}|a_t) \quad (37)$$

$$p^R(b_{t+1}, k_{t+1}^g, 1, a_t) = \phi \int_{R^B(b_{t+1}, k_{t+1}^g)} d\mu(a_{t+1}|a_t) + (1 - \phi) \int_{R^L(b_{t+1}, k_{t+1}^g)} d\mu(a_{t+1}|a_t) \quad (38)$$

The expected recovery rates conditional on the sovereign's default choice in period $t+1$ are defined as:

$$\gamma(b_{t+1}, k_{t+1}^g, 1, a_{t+1}) = \int_A \left[\begin{array}{l} \phi \mathbb{1}_{a_{t+2} \in R^B(b_{t+2}, k_{t+2}^g)} \delta^{B*}(b_{t+2}, k_{t+2}^g, a_{t+2}) \\ + (1 - \phi) \mathbb{1}_{a_{t+2} \in R^L(b_{t+2}, k_{t+2}^g)} \delta^{L*}(b_{t+2}, k_{t+2}^g, a_{t+2}) \\ + \left(\begin{array}{l} \phi \mathbb{1}_{a_{t+2} \notin R^B(b_{t+2}, k_{t+2}^g)} \\ + (1 - \phi) \mathbb{1}_{a_{t+2} \notin R^L(b_{t+2}, k_{t+2}^g)} \end{array} \right) \gamma(b_{t+2}, k_{t+2}^g, 1, a_{t+2}) \end{array} \right] d\mu(a_{t+2}|a_{t+1}) \quad (39)$$

where the third term inside the bracket on the right hand side of equation (39) reflects both no settlement (delays) in period $t+2$ and expected recovery rates in the future periods.

Following Chatterjee and Eyigungor (2012), sovereign’s bond spread is defined as the difference between an annualized “internal rate of return”—an \tilde{r} satisfying $q(b_{t+1}, k_{t+1}^g, 0, a_t) = (\lambda + (1 - \lambda)z)/(\lambda + \tilde{r}(b_{t+1}, k_{t+1}^g, 0, a_t))$ —and the annualized global risk-free interest rate.

$$s(b_{t+1}, k_{t+1}^g, 0, a_t) = (1 + \tilde{r}(b_{t+1}, k_{t+1}^g, 0, a_t))^4 - (1 + r^*)^4 \quad (40)$$

5 Quantitative Analysis

The quantitative analysis of the model is applied to two Argentine post-default restructurings: (i) 2001–05 with recovered debt payments in cash at settlement; (ii) 2019–20 without recovered debt payments in cash at settlement. There are four main findings. First, our model predicts that after default (ex post), the sovereign is willing to delay renegotiations, *ceteris paribus*, when public capital is low. Second, we also predict that before default (ex ante), the sovereign’s willingness to repay remains unchanged or decreases, *ceteris paribus*, when public capital increases. Third, our model provides a mechanism underlying the dynamics of public investment, and consumption and transfers. Fourth, our quantitative analysis of the model successfully replicates moment statistics, likelihood of debt settlement, and the five stylized facts.

5.1 Parameters and Functional Forms

The parameter values and functional forms follow closely those in previous studies on sovereign debt and fiscal policy. We assume the following constant relative risk aversion (CRRA) functions for private consumption and labor, and for public consumption:

$$u(c_t, l_t) = \frac{(c_t - \frac{l_t^{1+\psi}}{1+\psi})^{1-\sigma}}{1-\sigma}, \quad v(g_t) = \frac{g_t^{1-\sigma_g}}{1-\sigma_g} \quad (41)$$

As in conventional sovereign debt models (e.g., Mendoza and Yue 2012; Cuadra et al. 2010), $u(\cdot)$ follows Greenwood et al. (1988)’s specification, which provides the marginal rate of substitution between private consumption and labor orthogonal to the level of private consumption. This implies no wealth effects on labor supply. We assume symmetric degree of risk aversion between private and public consumption $\sigma = \sigma_g = 3$, as in previous studies (Cuadra et al. 2010; Arellano and Bai 2017) to maintain the same degree of consumption-smoothing between two types of consumption.²⁷ The risk-free interest rate is $r^* = 0.01$ corresponding to the average quarterly interest rate on the 3-month US Treasury bills (Aguiar et al. 2016). Labor elasticity ψ is set to 0.48 following Mendoza (1991). Labor and public capital income share is assumed to be 0.64 and 0.058 based on Gordon and Guerron-Quintana (2018) and public capital income share in Argentina in 1993–2020 from our dataset. Public capital depreciation rate is set to

²⁷Hatchondo et al. (2022) assume asymmetric degree of risk aversion between two types of consumption because there are no public transfers in their model. However, with public transfers included in our model, the same degree of risk aversion to improve household utility is necessary to have both public consumption and transfers available for the sovereign (Cuadra et al. 2010; Arellano and Bai 2017).

Table 4: Model Parameters

Parameter	Value	Source
Risk aversion for private consumption	$\sigma = 3$	Cuadra et al. (2010), Arellano and Bai (2017)
Risk aversion for public consumption	$\sigma_g = 3$	Cuadra et al. (2010), Arellano and Bai (2017)
Risk-free interest rate	$r^* = 0.01$	Aguiar et al. (2016) - US Treasury bill rate
Labor elasticity	$\psi = 0.48$	Mendoza (1991)
Labor income share	$\alpha^l = 0.64$	Gordon and Guerron-Quintana (2018) - Argentina
Public capital income share	$\alpha^k = 0.058$	Our dataset - Argentina
Public capital depreciation rate	$\delta^k = 0.04$	US BEA (1999)
Effective consumption tax rate	$\tau = 0.33$	Computed - Asonuma et al. (2023)
Auto-correlation of productivity shock	$\rho = 0.95$	Computed - Argentine GDP (MECON / INDEC)
Standard deviation of productivity shock	$\sigma^a = 0.017$	Computed - Argentine GDP (MECON / INDEC)
Discount rate	$\beta = 0.95$	Computed
Bargaining power	$\phi = 0.72$	Yue (2010) - Argentina 2001–05
Coupon payment	$z = 0.03$	Chatterjee and Eyigungor (2012)
Probability of maturity (long-term debt)	$\lambda = 0.05$	Chatterjee and Eyigungor (2012)
Weight on public consumption	$\pi = 0.2$	Computed
Public capital adjustment costs	$\Omega = 14$	Computed
Direct productivity loss	$\kappa_0 = -0.18, \kappa_1 = 0.24$	Computed

0.04 following US BEA (1999). Effective consumption tax rate $\tau = 0.33$ is computed from tax revenues in Argentina in 1993–2020 from Asonuma et al. (2023).

The productivity process is calibrated to match quarterly seasonally adjusted GDP data from the Ministry of Economy and Production (MECON) and the National Institute of Statistics and Censuses (INDEC) in Argentina. As in previous work (Gordon and Guerron-Quintana 2018), we assume the productivity process follows a log normal AR (1) process,

$$\log(a_t) = \rho \log(a_{t-1}) + \epsilon_{a,t} \quad (42)$$

where a productivity shock $\epsilon_{a,t}$ is *i.i.d* $N(0, \sigma^{a,2})$. We obtain auto-correlation and standard deviation of the productivity shock $\rho = 0.95$ and $\sigma^a = 0.017$. We approximate the stochastic process as a discrete Markov chain of equally spaced grids by using the quadrature method in Tauchen (1986).

The productivity loss due to default follows a functional form in both Chatterjee and Eyigungor (2012) and Gordon and Guerron-Quintana (2018):

$$\tilde{a}_t = (1 - \kappa(a))a_t \quad (43)$$

$$\kappa(a) = \min[\max(\kappa_0 + \kappa_1 a, 0), 1]$$

Sturzenegger and Zettelmeyer (2006) and Asonuma and Trebesch (2016) report that Argentina experienced 7 sovereign external debt restructurings in 1820–2020. Sturzenegger and Zettelmeyer (2008) and Asonuma, Niepelt and Ranciere (2023) find the recovery rates (haircuts) in Argentina 2001–05 and 2019–20 debt restructurings were 25.0% (75.0%) and 63.8% (36.2%),

respectively. We set the sovereign’s discount rate $\beta = 0.95$ to generate average default frequency of 3.50%. Bargaining power is set $\phi = 0.72$ as in Yue (2010) which applies to Argentine 2001–05 debt restructuring. On long-term debt structure in Argentina, we follow Chatterjee and Eyingungor (2012) to set coupon payment of 3% ($z = 0.03$) and probability of debt maturity of 5% ($\lambda = 0.05$).

We specify productivity loss due to default $\kappa_0 = -0.18$ and $\kappa_1 = 0.24$, public capital adjustment costs $\Omega = 14$, and weight on public consumption $\pi = 0.2$ to replicate the average default spread of 7.2(%), the standard deviation of the spread of 3.40(%), public investment standard deviation relative to output of 2.7, and average public consumption and transfers-to-GDP ratio of 22.5(%). Table 4 summarizes the model parameters which are applied symmetrically to both two Argentine defaults and post-default restructurings. Our computation algorithm is reported in Appendix D.

5.2 Numerical Results on Equilibrium Properties

We start from presenting the qualitative equilibrium properties of our theoretical model for the case when the sovereign proposes. Similarly, Appendix E.2 discusses those for the case when the creditors propose—underlying mechanisms apply symmetrically and generate identical results.

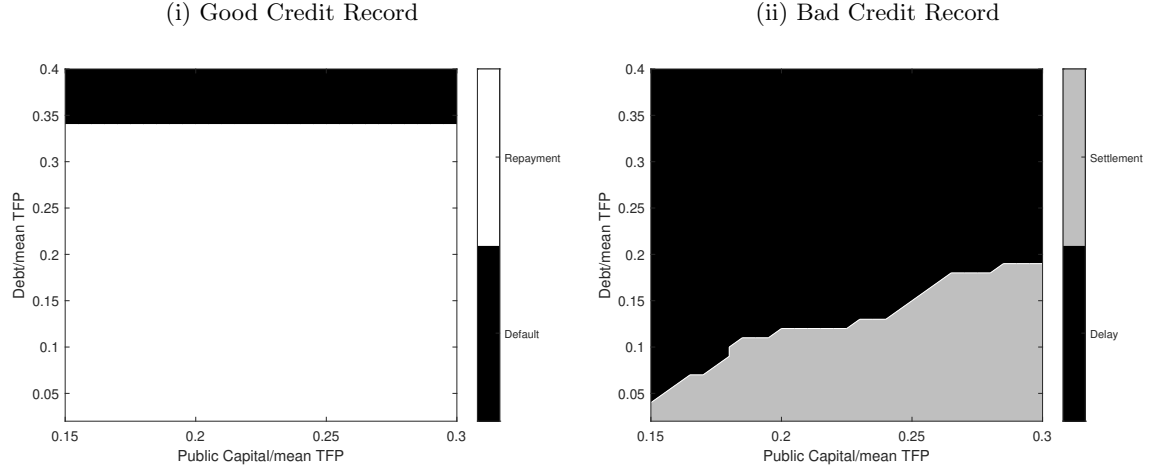
Figure 5 reports the sovereign’s choice between repayment and default, and between settlement and delay when the debtor TFP is fixed at the mean level—those when public capital is fixed at the mean level are reported in Figure E1 in Appendix E.1. The horizontal and vertical axes are public capital-to-mean TFP ratio and debt-to-mean TFP ratio, respectively.

First, we focus on a debt restructuring with recovered debt payments in cash at settlement (panel A, Argentina 2001–05). More importantly, on its choice between settlement and delay reported in panel A-(ii), what our model explains newly is that the sovereign opts to delay (settle), *ceteris paribus*, when public capital is low (high). A key driver is choice between investment in public capital and use of resources for debt settlement which determines the sovereign’s choice between settlement and delay in our model. This new driver differentiates our model from previous studies in which public capital does not play any role on the sovereign’s choice between settlement and delay. In the case of low public capital, the sovereign opts to invest limited resources—due to both financial exclusion and productivity loss—in public capital and refrains from using them for recovered debt payments given the high marginal product of public capital (i.e., high shadow value of public capital). The sovereign’s willingness to delay is reflected in the enlarged “delay” region in black color.

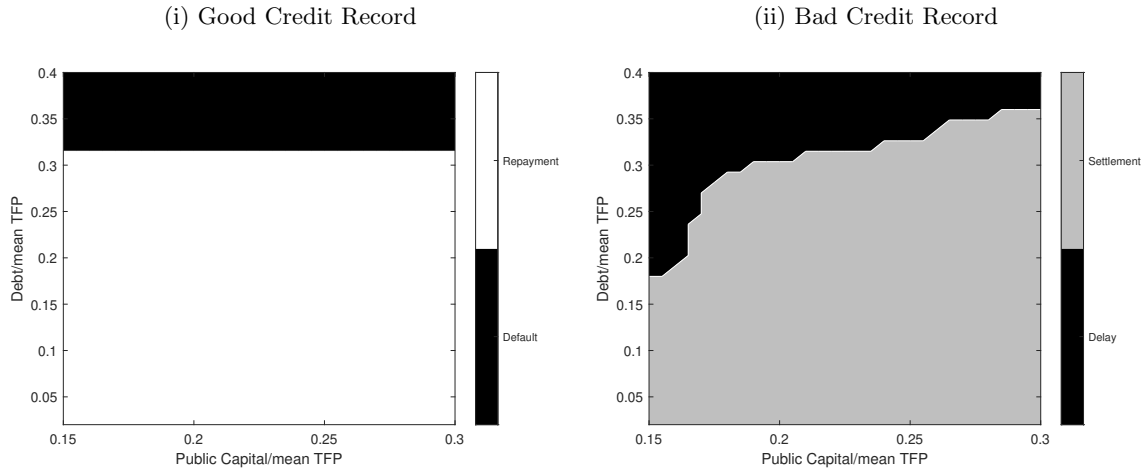
Moreover, on its choice between repayment and default reported in panel A-(i), our new finding is that the sovereign’s willingness to repay remains unchanged or is weakly decreasing when public capital increases—presented in unchanged or slightly enlarged “default” region in black color. On the one hand, higher public capital increases benefits of repayment by improving the sovereign’s repayment capacity, i.e., an increase in value of repayment defined in equation 8 (“smoothing channel”). On the other hand, higher public capital also increases benefits of default, an increase in value of default shown in equation 13 by stabilizing household

Figure 5: Debtor’s Choice between Repayment and Default, and between Settlement and Delay

A: Restructuring with Recovered Payments in Cash (Argentina 2001–05)—Mean TFP



B: Restructuring without Recovered Debt Payments in Cash (Argentina 2019–20)—Mean TFP



consumption in financial autarky (“autarky channel”) and achieving the quick debt settlement (“renegotiation channel”). What is newly introduced in our model is the renegotiation channel which makes benefits of default equivalent to or slightly higher than benefits of repayment. This differentiates our finding from a conventional finding that the sovereign’s willingness to repay increases as aggregate capital increases (Gordon and Guerron-Quintana 2018).²⁸

Second, we move on to a debt restructuring without recovered debt payments in cash (panel B, Argentina 2019–20). On its choice between settlement and delay reported in panels B-(ii), the sovereign is more willing to delay (settle), *ceteris paribus*, when public capital is low (high). The pattern is qualitatively similar to that in the case of the debt restructuring with recovered

²⁸Appendix E.3 discusses the sovereign’s choice of repayment and default in models with no renegotiations, i.e., exogenous re-entry and with one-round negotiation which lack the renegotiation channel of public capital.

debt payments in cash in panel A-(ii). A difference between panels A-(ii) and B-(ii) is that given public capital, the sovereign is more willing to settle in the debt restructuring without recovered debt payments in cash at settlement—presented in larger “settlement” region in gray color in panel B-(ii). This is because the sovereign pays recovered debt payments by only bonds, not by a combination of cash and bonds at settlement. As a result, it can achieve both settlement at a slightly higher level of debt and on higher recovery rates (low haircuts).

On its choice between repayment and default reported in panel B-(i), the sovereign’s willingness to repay remains unchanged or is weakly decreasing when public capital increases. The pattern is qualitatively the same with that in the case of the debt restructuring with recovered debt payments in cash in panel A-(i). A difference between panels A-(i) and B-(i) is that the sovereign is more willing to default at lower level of debt in the debt restructuring without recovered debt payments in cash at settlement—presented in larger “default” region in black color in panel B-(i). In this case, the effective costs of a default are small due to a shorter period of financial exclusion because the sovereign is more likely to settle quickly conditional on a default. Section 5.5 explores in detail the role of public capital and provides more insights.

5.3 Simulation Exercise

We provide simulation results to show how precisely our theoretical model predicts the two Argentine defaults and post-default restructurings: (i) 2001–05 with recovered debt payments in cash at settlement: (ii) 2019–20 without recovered debt payments in cash at settlement. We focus on three parts of simulation results: moment statistics, simulated stylized facts, and likelihood of debt settlement. Following a conventional approach, we apply 1000 rounds of simulations, with 2000 periods per round and extracts the last 200 observations. In the last 200 samples, we withdraw 40 observations before and observations during the last default and restructuring event.²⁹

For private sector data for Argentina, output, consumption and the trade balance are at quarterly frequency and all seasonally adjusted from the MECON for 1993Q1–2013Q3 and the INDEC for 2013Q4–20Q3. The trade balance is measured as a percentage of GDP. For public sector data for Argentina, consumption, investment, transfers and capital are at annual frequency from our dataset for 1993–2020. Argentine external debt data are from the IMF WEO for 1993–2020. Average external debt is also measured as a percentage of GDP. Bond spreads are from the J.P. Morgan’s Emerging Markets Bond Index Global (EMBIG) for 1997Q1–2020Q3. For (i) 2001–05 restructuring with recovered debt payments in cash at settlement, we set 1993–2000 (1993Q1–2000Q4) as pre-default periods and 2001–05 (2001Q4–05Q2) as renegotiation periods. For (ii) 2019–20 restructuring without recovered debt payments in cash at settlement, we set 2006–18 (2006Q1–18Q4) as pre-default periods and 2019–20 (2019Q4–20Q3) as renegotiation periods.

Moment Statistics We compare non-target statistics of our baseline models with the data, those in recalibration results of Cuadra et al. (2010) and Gordon and Guerron-Quintana

²⁹See Arellano (2008) and Yue (2010) for this treatment of simulation.

(2018) in Table 5. We also report additional recalibration results of previous studies of (c) sovereign debt and fiscal policy and (d) debt renegotiations in Table F2 in Appendix F.2. In the recalibration analysis of previous studies, we recalibrate these models to match their targeted statistics leaving the remaining parameter values unchanged with our baseline model of endogenous public capital.

Separately, we report quantitative analysis results of (a) a model of exogenous public capital (Arellano and Bai 2017; Cuadra et al. 2010) and (b) a model of endogenous public capital (Gordon and Guerron-Quintana 2018; Park 2017) in Table F3 in Appendix F.3. In this quantitative analysis, we introduce additional specific features and parameter values in our baseline model of endogenous public capital.

Panels (i) and (ii) in Table 5 report business cycle statistics for public sector and non-business cycle statistics—business cycle statistics for private sector are reported in Table F1 in Appendix F.1. For public sector statistics, our simulated moments fit the data well. Both baseline models with and without recovered debt payments in cash at settlement successfully replicate notable public sector characteristics in emerging market countries: procyclical and volatile public consumption and transfers. This is in line with previous studies of sovereign debt with fiscal policy (Arellano and Bai 2017; Cuadra et al. 2010).

Most importantly, our calibration results provide four novelties contributing to the literature. First of all, both baseline models with and without recovered debt payments in cash at settlement successfully replicate lower average public investment in the renegotiation periods than that in the pre-default periods as observed in the data (1.85 vs. 2.30 percent of GDP for model with recovered debt payments in cash at settlement; 1.33 vs. 2.72 percent of GDP for model without recovered debt payments in cash at settlement). Simultaneously, both baseline models with and without recovered debt payments in cash at settlement account for lower investment share in public expenditure in the renegotiation periods than that in the pre-default periods (8.2 vs. 9.3 percent for model with recovered debt payments in cash at settlement; 5.7 vs. 12.2 percent for model without recovered debt payments in cash at settlement). On the contrary, the model of exogenous public capital—reported in Table F3 in Appendix F.3—generates both higher average public investment and higher investment share in public expenditure in the renegotiation periods than that in the pre-default periods because of both fixed public investment level and endogenous output dynamics. Neither the model of endogenous aggregate capital—reported in Table F3 in Appendix F.3—nor recalibrations of Cuadra et al. (2010) or Gordon and Guerron-Quintana (2018) replicate any of these features because of no replication of public investment.

Second, both baseline models with and without recovered debt payments in cash at settlement replicate both a negative relationship between a decline in public investment and duration, and a positive relationship between a recovery in public investment and duration as observed in the data (-0.10 vs. 0.58 for model with recovered debt payments in cash at settlement; -0.52 vs. 0.14 for model without recovered debt payments in cash at settlement). Neither models of exogenous public capital or endogenous aggregate capital, nor replications of Cuadra et al. (2010) or Gordon and Guerron-Quintana (2018) replicate any of these features because of fixed

public investment level or no replication of public investment.

Third, we replicate longer average restructuring duration for the baseline model with recovered debt payments in cash at settlement than for that without recovered debt payments in cash at settlement (11.5 vs. 5.0 quarters) as in the data. In our baseline model with recovered debt payments in cash at settlement, what generate longer average restructuring duration are both endogenous public capital accumulation (“public capital accumulation delays”) and productivity recovery (“productivity recovery delays”)—see decomposition of delays in Section 5.4. On the contrary, both models of exogenous public capital and endogenous aggregate capital replicate only short restructuring duration (4.8 and 4.5 quarters). Recalibrations of Cuadra et al. (2010) or Gordon and Guerron-Quintana (2018) do not replicate any restructuring duration because of no renegotiations, i.e., exogenous re-entry.

Fourth, we generate lower average recovery rate (higher average haircut) for our baseline model with recovered debt payments in cash at settlement than for that without recovered debt payments in cash at settlement (22.5 vs. 51.0 percent) consistent with the data. In our baseline model with recovered debt payments in cash at settlement, what generate lower average recovery rate (higher average haircut) are recovered debt payments in cash at settlement.

As in conventional models in the literature, both baseline models with and without recovered debt payments in cash at settlement replicate high average debt-to-GDP ratio in pre-default periods, average and standard deviation of bond spreads, and countercyclical bond spreads due to long-term debt (Hatchondo and Martinez 2009; Chatterjee and Eyigungor 2012) as observed in the data.

Table 5: Moment Statistics from Simulation Results

(i) Public Sector Business Cycle Statistics						
	Recovered Debt Payments in Cash		No Recovered Debt Payments in Cash			
	Argentine 2001-05		Argentine 2019-20			
	Data	Baseline Model	Data	Baseline Model	Cuadra et al. (2010) Recalibration ^{1/}	Gordon and Guerron-Quintana (2018) Recalibration ^{2/}
Target statistics						
Pre-default periods ^{3/}						
Average public consumption & transfers/GDP ratio (%)	22.5	22.5	22.5	24.7	24.8	-
Public investment (std. dev.)/output (std. dev.)	2.7	2.41	2.7	2.90	-	-
Public consumption & transfers (std. dev.)/output (std. dev.)	1.26	-	1.50	-	1.01	-
Non-target statistics						
Pre-default periods						
Public sector						
Public consumption & transfers (std. dev.)/output (std. dev.)	1.26	1.10	1.50	1.01	-	-
Corr.(public consumption & transfers, output)	0.77	0.88	0.25	0.78	0.98	-
Average public investment/GDP ratio (%)	1.31	2.30	2.40	2.72	-	-
Average public investment/public expenditure ratio (%)	6.2	9.3	11.7	12.2	-	-
Renegotiation periods						
Public sector						
Public consumption & transfers (std. dev.)/output (std. dev.)	0.99	1.43	2.34	2.80	1.00	-
Corr.(public consumption & transfers, output) ^{4/}	0.97	0.79	n.a	0.10	0.99	-
Average public consumption & transfers/GDP ratio (%)	20.2	22.5	19.4	23.2	24.8	-
Average public investment/GDP ratio (%)	1.19	1.85	1.80	1.33	-	-
Average public investment/public expenditure ratio (%)	5.7	8.2	8.7	5.7	-	-

(ii) Non-business Cycle Statistics						
	Recovered Debt Payments in Cash		No Recovered Debt Payments in Cash			
	Argentine 2001-05		Argentine 2019-20			
	Data	Baseline Model	Data	Baseline Model	Cuadra et al. (2010) Recalibration ^{1/}	Gordon and Guerron-Quintana (2018) Recalibration ^{2/}
Target statistics						
Pre-default periods ^{3/}						
Default probability (%)	3.50	3.80	3.50	3.20	-	3.51
Bond spreads: average (%)	7.2	8.2	7.2	7.9	-	9.2
Bond spreads: std. dev. (%)	3.40	3.20	3.40	4.7	-	4.41
Average debt/GDP ratio (%)	32.6	-	58.9	-	4.5	-
Non-target statistics						
Pre-default periods						
Default probability (%)	3.50	-	3.50	-	3.50	-
Bond spreads: std. dev. (%)	3.40	-	3.40	-	1.50	-
Average debt/GDP ratio (%)	32.6	55.4	58.9	38.0	-	45.4
Corr.(output, spreads)	-0.88	-0.20	-0.63/-0.28	-0.35	-0.41	-0.28
Corr.(debt/GDP, spreads)	0.92	0.20	0.36	0.27	0.24	0.33
Renegotiation periods						
Average debt/GDP ratio (%)	109.6	88.7	85.7	75.8	5.5	54.3
Average restructuring duration (quarters)	14.6	11.5	3.00	5.0	-	-
Average recovery rate (%)	25.0	22.5	63.8	51.0	-	-
Corr.(decline in public investment, duration) ^{5/}	-0.25	-0.10	-0.37	-0.52	-	-
Corr.(recovery in public investment, duration) ^{6/}	0.22	0.58	0.18	0.14	-	-

Sources: Datastream, IMF WEO, INDEC and MECON.

Notes: ^{1/} Cuadra et al. (2010) recalibration corresponds to calibration results of one-period (short-term) debt with three target statistics (i) debt service-to-GDP ratio, (ii) ratio between public consumption and transfers and private consumption, and (iii) ratio between standard deviation of public consumption and standard deviation of output. In the model, debt-to-GDP ratio corresponds to debt service-to-GDP ratio. ^{2/} Gordon and Guerron-Quintana (2018) recalibration corresponds to calibration results with four target statistics (i) average bond spreads, (ii) standard deviation of bond spreads, (iii) ratio between standard deviation of total investment and standard deviation of output, and (iv) excess consumption volatility.

^{3/} Over 1993–2000 (1993Q1–2000Q4) and 2006–18 (2006Q1–18Q4).

^{4/} Correlation for Argentine 2019–20 episode is not available due to limited number of observations at an annual frequency.

^{5/} Decline in public investment is measured as percent change of public investment-to-GDP ratio from level in t-4 (quarter) to the lowest level, i.e., the level at end of declining trend.

^{6/} Recovery in public investment is measured as length of time (quarters) from the time which public investment-to-GDP ratio is at the lowest level to the time which it recovers to the the level in the pre-default periods.

Simulated Stylized Facts Figure 6 shows simulation results on restructuring duration, haircuts, public investment, public consumption and transfers. For panels (i)–(v), we follow the same presentation approaches as in Figure B1 in Appendix B.1, Figures 1, 2, and 3, respectively.

First, panels (i)-a and (i)-b show that our two baseline models replicate a large number of post-default restructurings with recovered debt payments in cash at settlement as well as a large number of post-default restructurings without recovered debt payments in cash at settlement. Panels (i)-a and (i)-b also show those with recovered debt payments in cash at settlement are associated with longer restructuring duration and higher haircuts than those without recovered debt payments in cash at settlement. Both results are consistent with the data (Table 2 and Figure B1 in Appendix B.1).

Second and most importantly, for post-default restructurings with recovered debt payments in cash at settlement, panel (ii)-a shows that our baseline model (red dashed line) replicates both a sharp decline at the onset of the restructuring and a gradual recovery of public investment to the pre-restructuring level during the restructuring as observed in the data (blue solid line). This is one of the main drivers of longer restructuring duration in our baseline model (11.5 quarters), which is close to the data (14.6 quarter).

For post-default restructurings without recovered debt payments in cash at settlement, panel (ii)-b shows that our baseline model (red dashed line) replicates a sharp decline at the onset of the restructuring as observed in the data (blue solid line). Our baseline model replicates shorter restructuring duration (5.0 quarters) and only partial recovery of public investment to the pre-restructuring level during the restructuring.

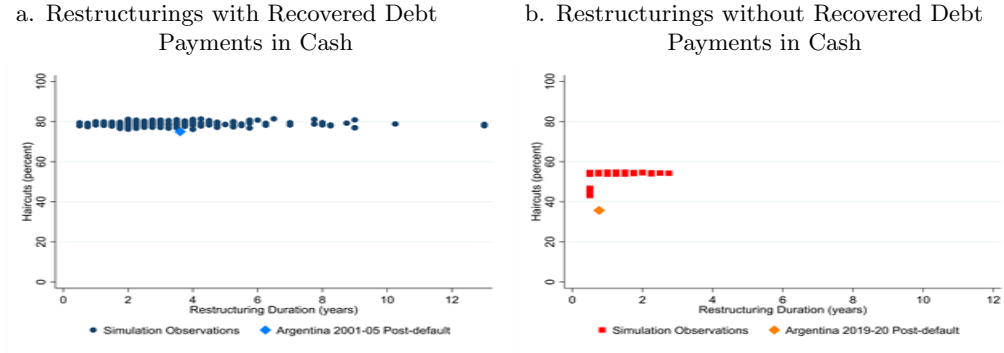
Third, for both post-default restructurings with and without recovered debt payments in cash at settlement, panels (iii)-a and (iii)-b show that our baseline model (red dashed line) replicates a small decline and a quick recovery in public consumption and transfers as observed in the data (blue solid line). The dynamics of public consumption and transfers differ significantly from those of public investment. Panels (i)–(ii) in Figure F1 in Appendix F.1 show that our baseline model (red dashed line) replicates dynamics of both public investment, and consumption and transfers for non-debt crisis recessions as observed in the data (blue solid line). Public investment dynamics in non-debt crisis recessions differ from those in our baseline model because with low debt, the sovereign can borrow to finance public investment in non-debt crisis recessions.

Fourth, for post-default restructurings with recovered debt payments in cash at settlement, panel (iv) shows that our baseline model replicates both a negative relationship between restructuring duration and declines in public investment, and a positive relationship between restructuring duration and recoveries in public investment which are consistent with the data (Figure 3 panel (i)). For post-default restructurings without recovered debt payments in cash at settlement, panel (v) shows that our baseline model replicates both the negative relationship between restructuring duration and declines in public investment, and the positive relationship between restructuring duration and recoveries in public investment but with weaker relationships than those in post-default restructurings with recovered debt payments in cash at settlement as observed in the data (Figure 3 panel (ii)). Sections 6.1 and 6.2 discuss simulated stylized facts

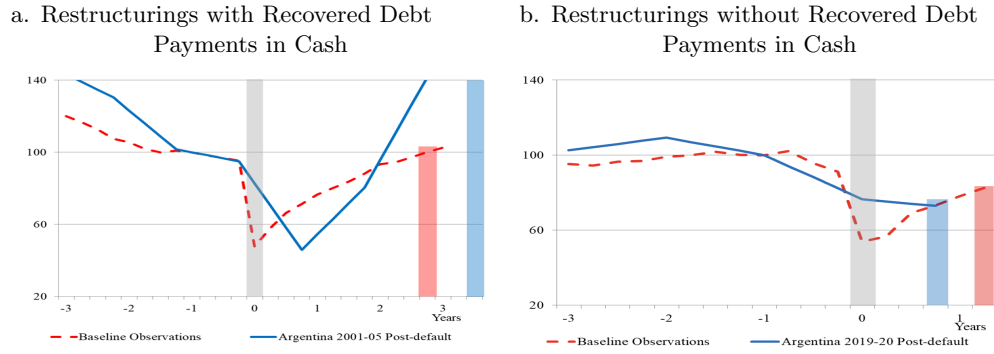
in models of exogenous public capital and endogenous aggregate capital.

Figure 6: Restructuring Duration, Haircuts, Public Investment, Consumption and Transfers

(i) Restructuring Duration and Haircuts



(ii) Public Investment



(iii) Public Consumption and Transfers

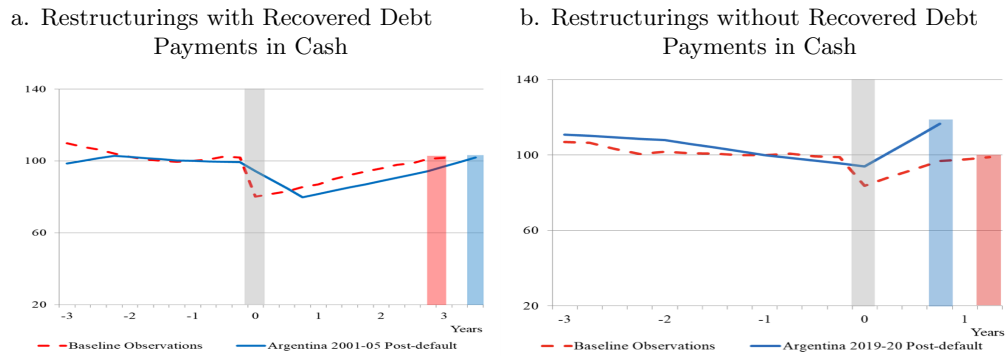
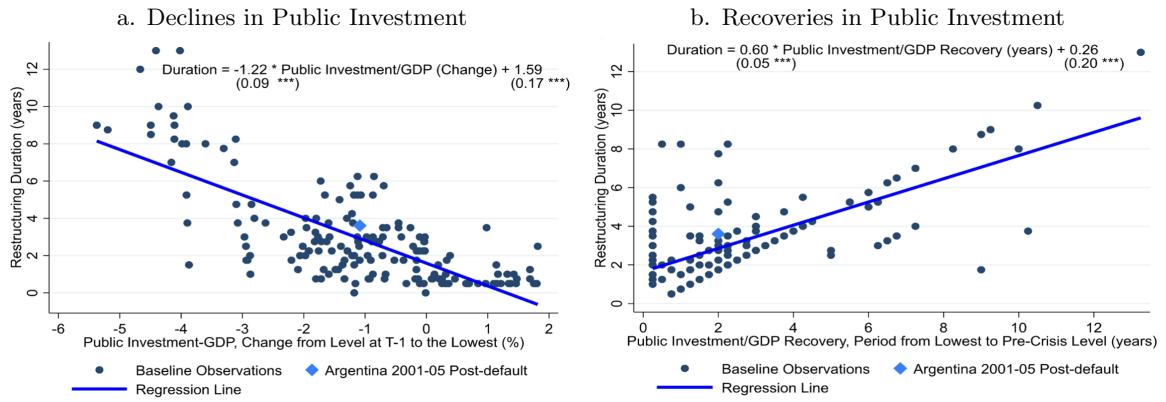
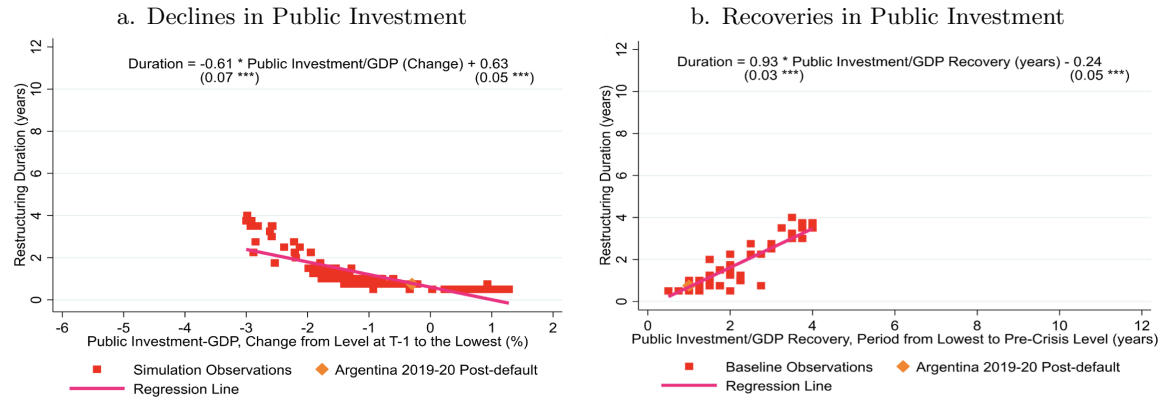


Figure 6: Restructuring Duration, Haircuts, Public Investment, Consumption and Transfers (cont.)

(iv) Restructuring Duration and Public Investment—Restructurings with Recovered Debt Payments in Cash ^{1/}



(v) Restructuring Duration and Public Investment—Restructurings without Recovered Debt Payments in Cash ^{1/}



1/. Our baseline model successfully replicates a larger variation in restructuring duration as in Figure 3 when it is calibrated to post-default restructuring episodes with longer restructuring duration than that of the Argentine 2001–05 and 2019–20 episodes.

Likelihood of Debt Settlement We use simulated data series obtained from our two baseline models and apply a logit regression on debt settlement (binary) reported in Table 6. Our main explanatory variables are public investment-to-GDP ratio in an upward trend—measured as both lagged and a cumulative change from trough to a lagged period. Both explanatory variables are multiples of public investment-to-GDP ratio (as both lagged and a cumulative change from trough to a lagged period) and a dummy variable for an upward trend (lagged). We show that when the lagged public investment-to-GDP ratio is at a higher level in an upward trend, the sovereign is more likely to reach settlement in the current quarter for both post-default restructurings with and without recovered debt payments in cash at settlement (columns 1 and 3). Furthermore, the cumulative change of public investment-to-GDP ratio, from trough to a lagged period, significantly increases the likelihood of debt settlement for post-default restructurings with and without recovered debt payments in cash at settlement (columns 2 and 4).

Table 6: Likelihood of Debt Settlement

	Debt Settlement (binary, current)			
	Recovered Debt Payments in Cash		No Recovered Debt Payments in Cash	
	(1)	(2)	(3)	(4)
	dy/dx / Delta-method se	dy/dx / Delta-method se	dy/dx / Delta-method se	dy/dx / Delta-method se
Public investment-to-GDP ratio in an upward trend (lagged) ^{1/}	2.457*** (0.608)	-	7.271*** (1.700)	-
Public investment-to-GDP ratio in an upward trend (a cumulative change from trough to a lagged period) ^{2/}	-	1.483** (0.642)	-	3.403** (1.473)
Debt-to-GDP ratio (lagged)	-0.002*** (0.0002)	-0.001*** (0.0001)	-0.002*** (0.0002)	-0.001*** (0.0002)
GDP deviation from the trend (current, percent)	0.009*** (0.007)	0.009*** (0.001)	0.025*** (0.003)	0.025*** (0.003)
Episode-specific fixed effects	No	No	No	No
Number of default/restructuring episodes	132	132	147	147
Number of observations	1,901	1,901	658	658
Wald χ^2	240.34	169.30	180.24	180.30
Prob.> χ^2	0.000	0.000	0.000	0.000

The table shows results from random effects logit regressions. The dependent variable is debt settlement in the current quarter (binary). The main explanatory variables are public investment-to-GDP ratio in an upward trend measured as both lagged and a cumulative change from trough to a lagged period. Debt-to-GDP ratio is lagged by one quarter. The other explanatory variables are in the current quarter. Significance levels are denoted by *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$, respectively. Robust standard errors (Delta-method standard errors) are in parentheses.

^{1/} A multiple of public investment-to-GDP ratio (lagged) and a dummy variable of an upward trend of investment-to-GDP ratio (lagged, a binary variable).

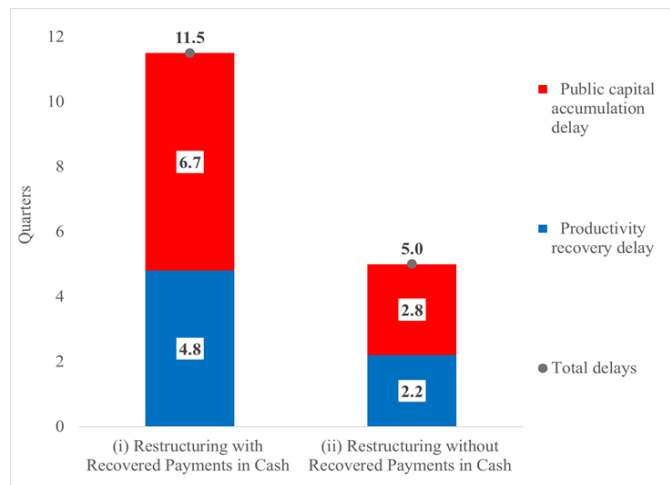
^{2/} A multiple of public investment-to-GDP ratio (a cumulative change from trough to a lagged period) and a dummy variable of an upward trend of public investment-to-GDP ratio (lagged, a binary variable).

5.4 Decomposition of Delays

Both slow productivity recovery and slow public capital accumulation interact and generate delays in our baseline model. Figure 7 shows how these two drivers contribute to total delays of 11.5 quarters and 5.0 quarters in two types of post-default restructurings. Decomposition of delays is based on both calibration results of our baseline model of endogenous public capital and quantitative analysis results of the model of exogenous public capital reported in Table F3 in Appendix F.3. In our baseline model, productivity recovery generates delays of 4.8 quarters for a post-default restructuring with recovered debt payments in cash at settlement and those of 2.2 quarters for a post-default restructuring without recovered debt payments in cash at settlement, respectively, i.e., “productivity recovery delay” (blue bars in (i)-(ii)). Moreover, public capital accumulation generates delays of 6.7 quarters for a post-default restructuring with recovered debt payments in cash at settlement and those of 2.8 quarters for a post-default restructuring without recovered debt payments in cash at settlement, respectively, i.e., “public capital accumulation delays” (red bars in (i)-(ii)).

On the contrary, both recalibrations of Arellano and Bai (2017) and Benjamin and Wright (2013) replicate only short restructuring duration (2.0 and 6.0 quarters) in Table F2 in Appendix F.2. Both models take into account only productivity recovery because there is no public capital (corresponding to exogenous public capital). As a result, total restructuring delays in two models are shorter than those in our baseline model. Furthermore, Arellano and Bai (2017) lack multi-round debt renegotiations and generates much shorter delays (2.0 quarters).

Figure 7: Decomposition of Delays



5.5 Roles of Public Capital

We follow Gordon and Guerron-Quintana (2018) approach and explore three roles of public capital: (i) smoothing channel; (ii) autarky channel; and (iii) renegotiation channel.

First, we analyze how the autarky channel and the renegotiation channel of public capital play role on sovereign's choice of proposing and passing. In order to provide insights on these two channels, we take a first derivative for objective functions of proposing and passing (equations 20 and 23) with respect to public capital in the next period (k_{t+1}^g) and evaluate two derivatives given $c_t^{PRO} = c_t^{PASS} + \delta_t^* b_t < c_t^{PASS}$, $g_t^{PRO} = g_t^{PASS}$, and $k_{t+1}^{g,PRO} = k_{t+1}^{g,PASS}$ and $\tilde{y}_{t+1} < y'_{t+1}$ where y'_{t+1} and \tilde{y}_{t+1} denote output in the next period with good credit record after debt settlement and with bad credit record due to no settlement. This implies that only two differences between two derivatives are that the sovereign reduces current private consumption by the amount of the agreed recovered debt payments ($-\delta^* b_t$) for debt settlement in the case of proposing and that output in the next period after debt settlement is higher. For simplicity, we also assume (i) neither leisure nor public consumption is valued; (ii) there are no public capital adjustment costs, (iii) net bond issuance at settlement is zero ($b_{t+1} = 0$). The obtained expressions are the follows:

$$(1 - \pi) \frac{\partial u(c_t^{PRO}, l_t)}{\partial c_t^{PRO}} (-1) + \beta \int_A (1 - \pi) \frac{\partial u(c_{t+1}^R, l_{t+1})}{\partial c_{t+1}^R} \left\{ \frac{\partial y'_{t+1}}{\partial k_{t+1}^g} + (1 - \delta^k) \right\} d\mu(a_{t+1}|a_t)$$

$$(1 - \pi) \frac{\partial u(c_t^{PASS}, l_t)}{\partial c_t^{PASS}} (-1) + \beta \int_A (1 - \pi) \frac{\partial u(c_{t+1}^{PASS}, l_{t+1})}{\partial c_{t+1}^{PASS}} \left\{ \frac{\partial \tilde{y}_{t+1}}{\partial k_{t+1}^g} + (1 - \delta^k) \right\} d\mu(a_{t+1}|a_t)$$

A difference between the two expressions (divided by $(1 - \pi)$) can be expressed as follows:

$$\left[\frac{\partial u(c_t^{PRO}, l_t)}{\partial c_t^{PRO}} - \frac{\partial u(c_t^{PASS}, l_t)}{\partial c_t^{PASS}} \right] (-1) + \beta \int_A \left[\frac{\partial u(c_{t+1}^R, l_{t+1})}{\partial c_{t+1}^R} \left\{ \frac{\partial y'_{t+1}}{\partial k_{t+1}^g} + (1 - \delta^k) \right\} - \frac{\partial u(c_{t+1}^{PASS}, l_{t+1})}{\partial c_{t+1}^{PASS}} \left\{ \frac{\partial \tilde{y}_{t+1}}{\partial k_{t+1}^g} + (1 - \delta^k) \right\} \right] d\mu(a_{t+1}|a_t) \quad (44)$$

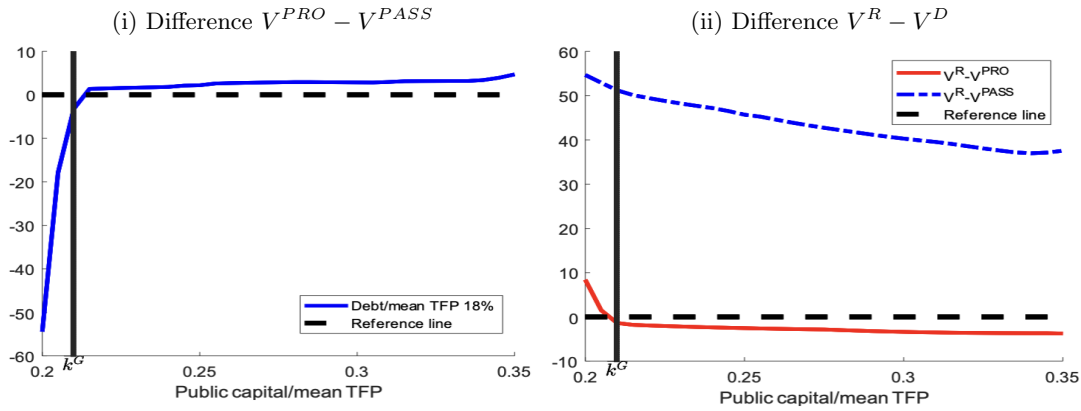
The first term on expression (44) corresponds to a difference in marginal cost of allocating one unit of resource to public capital between proposing and passing, and the second term on the expression corresponds to a difference in marginal benefit of allocating one unit of resource to public capital between proposing and passing. Therefore, the first and second terms can be interpreted as cost and benefit of debt settlement in the current period (both evaluated by one-unit investment of public capital).

When current public capital is high, output in the current period after productivity losses remains high. With enough resources, public capital in the next period remains high and in turn, output in the next period (after debt settlement) y'_{t+1} is also high. A difference in output and

public capital between good and bad credit records in the next period is high. Benefits of debt settlement in the current period are large and exceed costs of debt settlement—the renegotiation channel of public capital dominates the autarky channel of public capital.

On the contrary, when current public capital is low, output in the current period with productivity losses remains low. With limited resources, public capital in the next period remains low and in turn, output in the next period is also low. A difference in output between good and bad credit records in the next period is small. Costs of debt settlement in the current period are large and exceed benefits of debt settlement—the autarky channel of public capital dominates the renegotiation channel of public capital.

Figure 8: Value Functions at the Low TFP



To illustrate these two channels of public capital graphically, panel (i) in Figure 8 reports a difference between the two value functions of proposing (and settlement) and passing when TFP is low. When public capital is at k^G , the value of proposing (and settlement) is equal to the value of passing. When capital is high $k > k^G$, the value function of proposing (and settlement) exceeds that of passing corresponding to a positive difference between the two value functions. The renegotiation channel of public capital dominates the autarky channel of public capital. When public capital is low $k < k^G$, the value function of passing exceeds that of proposing (and settlement) corresponding to a negative difference between the two value functions. The autarky channel of public capital dominates the renegotiation channel of public capital.

Second, we focus on how the smoothing channel, the autarky channel, and the renegotiation channel of public capital play a role on the sovereign's choice between repayment and default. In order to provide insights on these three channels, we assume that the sovereign could start debt renegotiation in the current period and re-write function of defaulting as follows:

$$V^D(b_t, k_t^g, 0, a_t) = \begin{cases} V^{PRO}(b_t, k_t^g, 1, a_t) & \text{if propose \& settlement} \\ V^{PASS}(b_t, k_t^g, 1, a_t) & \text{if pass} \end{cases} \quad (13a)$$

We take a first derivative of objective function of repayment and defaulting (equations 8 and 13a) with respect to public capital (k_{t+1}^g) and evaluate two derivatives given $c_t^R < c_t^{PRO} < c_t^{PASS}$,

$g_t^P = g_t^{PRO} = g_t^{PASS}$, $k_{t+1}^{g,R} = k_{t+1}^{g,PRO} = k_{t+1}^{g,PASS}$, and $\tilde{y}_{t+1} < y'_{t+1} < y_{t+1}$ where y_{t+1} denotes output in the next period with good credit record and no default experience.

$$(1 - \pi) \frac{\partial u(c_t^R, l_t)}{\partial c_t^R} (-1) + \beta \int_A (1 - \pi) \frac{\partial u(c_{t+1}^R, l_{t+1})}{\partial c_{t+1}^R} \left(\frac{\partial y_{t+1}}{\partial k_{t+1}^g} + (1 - \delta^k) \right) d\mu(a_{t+1}|a_t)$$

$$\left\{ \begin{array}{l} (1 - \pi) \frac{\partial u(c_t^{PRO}, l_t)}{\partial c_t^{PRO}} (-1) + \beta \int_A (1 - \pi) \frac{\partial u(c_{t+1}^R, l_{t+1})}{\partial c_{t+1}^R} \left(\frac{\partial y'_{t+1}}{\partial k_{t+1}^g} + (1 - \delta^k) \right) d\mu(a_{t+1}|a_t) \text{ if propose \& settlement} \\ (1 - \pi) \frac{\partial u(c_t^{PASS}, l_t)}{\partial c_t^{PASS}} (-1) + \beta \int_A (1 - \pi) \frac{\partial u(c_{t+1}^{PASS}, l_{t+1})}{\partial c_{t+1}^{PASS}} \left(\frac{\partial \tilde{y}_{t+1}}{\partial k_{t+1}^g} + (1 - \delta^k) \right) d\mu(a_{t+1}|a_t) \text{ if pass} \end{array} \right\}$$

A difference between the two expressions (divided by $(1 - \pi)$) can be expressed as follows:

$$\left\{ \begin{array}{l} \left[\frac{\partial u(c_t^R, l_t)}{\partial c_t^R} - \frac{\partial u(c_t^{PRO}, l_t)}{\partial c_t^{PRO}} \right] + \beta \int_A \left[\frac{\partial u(c_{t+1}^R, l_{t+1})}{\partial c_{t+1}^R} \left\{ \frac{\partial y_{t+1}}{\partial k_{t+1}^g} + (1 - \delta^k) \right\} - \frac{\partial u(c_{t+1}^R, l_{t+1})}{\partial c_{t+1}^R} \left\{ \frac{\partial y'_{t+1}}{\partial k_{t+1}^g} + (1 - \delta^k) \right\} \right] d\mu(a_{t+1}|a_t) \text{ if propose \& settlement} \\ \left[\frac{\partial u(c_t^R, l_t)}{\partial c_t^R} - \frac{\partial u(c_t^{PASS}, l_t)}{\partial c_t^{PASS}} \right] + \beta \int_A \left[\frac{\partial u(c_{t+1}^R, l_{t+1})}{\partial c_{t+1}^R} \left\{ \frac{\partial y_{t+1}}{\partial k_{t+1}^g} + (1 - \delta^k) \right\} - \frac{\partial u(c_{t+1}^{PASS}, l_{t+1})}{\partial c_{t+1}^{PASS}} \left\{ \frac{\partial \tilde{y}_{t+1}}{\partial k_{t+1}^g} + (1 - \delta^k) \right\} \right] d\mu(a_{t+1}|a_t) \text{ if pass} \end{array} \right\} \quad (45)$$

The first term on the expression corresponds to a difference in marginal cost of allocating one unit of resource to public capital between repayment and defaulting, and the second term on the expression corresponds to a difference in marginal benefit of allocating one unit of resource to public capital between repayment and defaulting.

When current public capital is high, the sovereign opts to propose and settle conditional on default. Therefore, the renegotiation channel of public capital plays a larger role. In this case, the sovereign decides to repay or default comparing combined benefits of defaulting (i.e., higher current private consumption under defaulting $c_t^{PRO} > c_t^R$) and proposing (settlement), and benefits of repayment in full (i.e., higher output in the next period due to no productivity losses $y_{t+1} > y'_{t+1}$). As a result, the sovereign's choice is determined by comparing between a combination of the autarky channel and the renegotiation channel of public capital, and the smoothing channel of public capital.

On the contrary, when current public capital is low, the sovereign opts to pass conditional on default. Therefore, the autarky channel of public capital plays a larger role. In this case, the sovereign decides to repay or default comparing combined benefits of defaulting (i.e., higher current private consumption under default $c_t^{PASS} \gg c_t^R$) and passing (i.e., delaying settlement), and benefits of repayment in full (i.e., higher output in the next period due to no productivity losses $y_{t+1} \gg \tilde{y}_{t+1}$). As a result, the sovereign's choice is determined by comparing between a combination of the autarky channel and the renegotiation channel of public capital, and the smoothing channel of public capital.

To illustrate these three channels of public capital graphically, panel (ii) in Figure 8 reports a difference

between the two value functions of repayment and defaulting when TFP is low. When capital is high $k > k^G$, conditional on default, the sovereign proposes (and settles) as explained above. Taking into account the choice of proposing (and settlement) conditional on default, the sovereign opts to default. The value function of defaulting and proposing exceeds that of repayment corresponding to a negative value of difference between the two value functions (red solid line). A combination of the autarky channel and the renegotiation channel of public capital dominates the smoothing channel of public capital. When capital is low $k < k^G$, conditional on default, the sovereign passes as explained above. Taking into account the choice of passing conditional on default, the sovereign opts to repay debt. The value function of repayment exceeds that of defaulting and passing corresponding to a positive value of difference between the two value functions (blue solid line). The smoothing channel of public capital dominates a combination of the autarky channel and the renegotiation channel of public capital.

6 Comparison with Alternative Models

6.1 Endogenous Public Capital vs. Exogenous Public Capital

We contrast our baseline model of endogenous public capital with a model of exogenous public capital (Arellano and Bai 2017; Cuadra et al. 2010). In this model, restructurings are settled with recovered debt payments in cash and the sovereign fixes public investment to maintain the constant (given) level of public capital. Panel (i) in Figure 9 shows the model of exogenous public capital (green dotted line) does not replicate the dynamics of public investment as observed in the data (blue solid line) because public investment remains fixed. As a result, the sovereign settles quickly with restructuring duration of 4.8 quarters, shorter than that in our baseline mode of endogenous public capital. Panel (iii) in Figure 9 shows that short restructuring duration of 4.8 quarters is due only by productivity recovery delays with no public capital accumulation delays.

Figure F2 in Appendix F.3 shows that the model of exogenous public capital accounts for two facts such as first fact—a large number of restructurings with recovered debt payments in cash at settlement (panel (i)-a) and fourth fact—a short-lived decline and quick recovery of public consumption and transfers (panel (iii)-a). On the contrary, the model of exogenous public capital does not replicate any of second, third, and fifth facts. The second fact—higher haircuts and longer duration for restructurings with recovered debt payments in cash at settlement—is not replicated (panel (i)-a) because the model generates only short restructuring duration with average of 4.8 quarters. Neither the third fact—a severe decline and a slow recovery of public investment—nor fifth fact—an association between public investment dynamics and restructuring delays—are replicated because public investment remains unchanged.

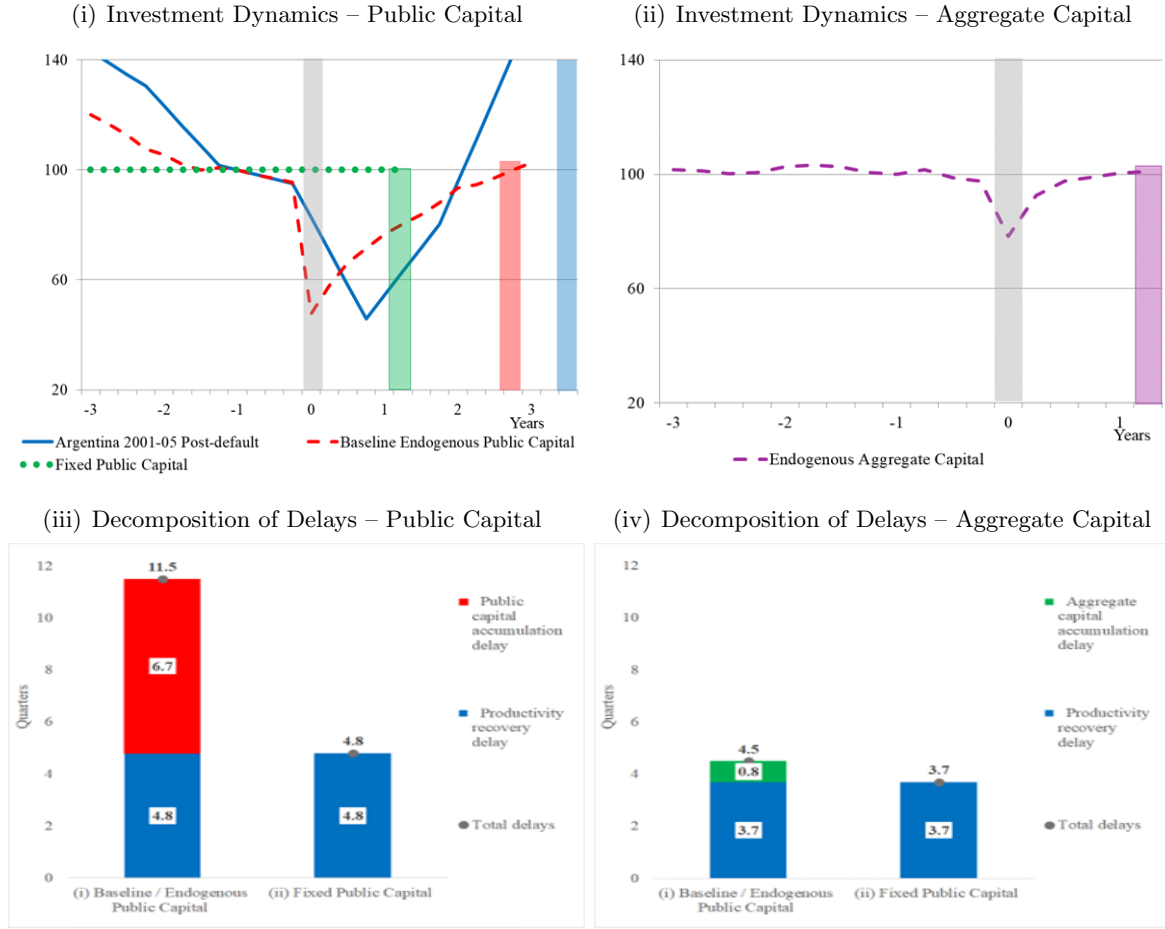
6.2 Endogenous Public Capital vs. Endogenous Aggregate Capital

We contrast our baseline model of endogenous public capital with a model of endogenous aggregate capital with no separation between private and public sectors in which a sovereign has no distortionary taxation but lump-sum taxation (e.g., Gordon and Guerron-Quintana 2018; Park 2017; Galli 2021).³⁰ In this model, restructurings are settled with recovered debt payments in cash, and there are no distinct public consumption and transfers, and only aggregate consumption is available.

The model of endogenous aggregate capital does not generate public investment dynamics, but only aggregate investment dynamics, i.e., combined private and public investment. Panel (ii) in Figure 9 shows

³⁰In a model of “private capital”, private and public sectors are separated by distortionary taxation (e.g. Esquivel 2023).

Figure 9: Investment Dynamics and Decomposition of Delays in Alternative Models



a sharp decline in aggregate investment, i.e., a 25-percent decline from the level of default and start of a post-default restructuring similar to that in Gordon and Guerron-Quintana (2018). The sharp decline in aggregate investment is smaller than that in public investment, i.e., a 50-percent decline in our baseline model of endogenous public capital. Most importantly, however, the model of endogenous aggregate capital (purple dashed line) generates a quick recovery in aggregate investment to the pre-restructuring level during the restructuring. This is because in the model of endogenous aggregate capital, the sovereign can extract resources from the private sector without distortion and allocate enough resources to aggregate investment quickly.

Panel (iv) shows that the model of endogenous aggregate capital generates total delays of 4.5 quarters which are much shorter than our baseline model of endogenous public capital (11.5 quarters). What makes total delays shorter in the model of endogenous aggregate capital are aggregate capital accumulation delays of only 0.8 quarters. These are much shorter than public capital accumulation delays of 6.7 quarters in our baseline model of endogenous public capital.

Figure F2 in Appendix F.3 shows that the model of endogenous aggregate capital accounts for first fact—a large number of restructurings with recovered debt payments in cash at settlement (panel (i)-b). On the contrary, the model of endogenous aggregate capital does not replicate any of remaining four facts. The second fact—higher haircuts and longer duration for restructurings with recovered debt

payments in cash at settlement—is not replicated (panel (i)-b) because the model generates only short restructuring duration with average of 4.5 years. None of the third, fourth and fifth facts are replicated because the model of endogenous aggregate capital does not replicate dynamics of public investment or public consumption and transfers, but generate dynamics of only aggregate investment or consumption. Even focusing on aggregate investment, panels (ii)-b and (v) show that the model of aggregate capital generates a sharp decline of aggregate investment at the onset of the restructuring, but not slow recovery of aggregate investment, and that there is no association between aggregate investment dynamics and restructuring delays. Focusing on aggregate consumption, panel (iii)-b shows that the the model of endogenous aggregate capital (purple dashed line) replicates a mild decline and a quick recovery in aggregate consumption.

6.3 Robustness Checks

Table 7: Sensitivity Analysis—Baseline Model with Recovered Debt Payments in Cash

	Adjustment Costs			Depreciation Rate			Risk Aversion		
	10	14	20	0.025	0.04	0.075	2	3	4
Default probability (%)	4.10	3.80	3.02	3.90	3.80	3.21	4.30	3.80	2.98
Public investment (std. dev.)/output (std. dev.)	2.69	2.41	1.74	2.95	2.41	2.02	2.51	2.41	3.20
Non-target statistics									
Pre-default periods									
Average public investment/GDP ratio (%)	2.35	2.30	2.20	1.87	2.30	3.10	2.35	2.30	2.10
Average public investment/public expenditure ratio (%)	9.8	9.3	9.1	8.22	9.3	12.3	9.8	9.3	8.2
Average debt/GDP ratio (%)	53.4	55.4	59.4	55.0	55.4	59.7	64.2	55.4	55.5
Renegotiation periods									
Average public investment/GDP ratio (%)	1.83	1.85	1.95	1.37	1.85	3.80	2.21	1.85	1.67
Average public investment/expenditure ratio (%)	8.2	8.2	8.3	6.9	8.2	16.2	12.1	8.2	7.3
Average debt/GDP ratio (%)	86.0	88.7	86.1	88.1	88.7	86.1	97.1	88.7	86.4
Average duration of renegotiations (quarters)	9.0	11.5	12.2	8.8	11.5	14.3	9.9	11.5	12.3
Average recovery rate (%)	19.4	22.5	25.0	19.7	22.5	24.5	22.8	22.5	23.1
Corr.(decline in public investment, duration)	-0.05	-0.10	-0.08	-0.20	-0.10	-0.09	-0.23	-0.10	-0.11
Corr.(recovery in public investment, duration)	0.30	0.58	0.48	0.53	0.58	0.56	0.42	0.58	0.37

Source: Authors' computation

Table 7 reports how a change in these parameter values keeping other parameter values constant influences quantitatively the main moment statistics. Low adjustment costs on public capital increases both investment volatility and a difference in public investment-to-GDP ratio between the pre-default and renegotiation periods. In this case, the sovereign is more willing to cut public investment severely at the onset of the debt crises and allocate more resources to public investment during restructurings. Moreover, in the case of low capital depreciation rate, public investment is lower in both the pre-default and renegotiation periods than that in our baseline model because the sovereign needs to allocate less resources to public investment. When the household becomes less risk averse, the sovereign opts to allocate less resources to public consumption and transfers and more resources to public investment because it finds less necessary to improve current household utility. Table F4 in Appendix F.4 also confirms robustness of our baseline model without recovered debt payments in cash at settlement.

7 Conclusion

The current paper explores the role of public capital on sovereign debt crises and resolutions. We code two new comprehensive datasets on (a) public expenditure composition and (b) sovereign debt restructurings with and without recovered debt payments in cash at settlement in 1975–2020. We find five new stylized facts on post-default restructurings with and without recovered debt payments in cash at settlement, and non-debt crisis recessions in 1975–2020. To explain these facts, we embed endogenous public capital accumulation, expenditure composition and production with public capital and labor in a theoretical model of sovereign long-term debt with endogenous defaults and renegotiations. Our model quantitatively replicates these stylized facts and shows public investment dynamics delay debt settlement—“public capital accumulation delays”. Empirical evidence supports our theoretical predictions.

Our paper empirically and theoretically sheds light on how the sovereign optimally chooses components of public expenditure, i.e., consumption and transfers, and investment during sovereign debt restructurings. However, how the sovereign optimally chooses components of public revenues remains unexplored enough. Furthermore, a question on how the sovereign’s choice of different taxation methods are related with its choice of default, debt settlement and delay also remains unanswered. The literature on sovereign debt explores the interactions between a combination of single tax revenue instrument and single expenditure instrument, and the sovereign’s default choice (Cuadra et al. 2010).³¹ For future work, on the basis of our understanding on how the sovereign optimally chooses multiple expenditure instruments, we could explore how it optimally chooses multiple tax revenue instruments among consumption tax, labor income tax, and capital tax together with its choice of default, debt settlement and delay. The future work could provide insights to the ongoing policy debate on the optimal combination of multiple tax revenue and expenditure instruments and overall contributions under fiscal consolidation.

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³¹Ongoing work, Asonuma, Joo and Zhang (2023) construct a new dataset on public revenue compositions in 1975–2020 for 75 countries that have experienced sovereign debt restructurings with private external creditors.

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Appendix A Datasets

A.1 Public Capital and Investment Dataset

IMF (2015) measures public investment using gross fixed capital formation (GFCF) of the general government (i.e., central plus subnational governments). The approach allows for the use of the comparable data available for a large number of countries but ignores alternative modes by which governments support overall investment (e.g., investment grants, loan guarantees, tax concessions, the operations of public financial institutions, government-backed saving schemes).

IMF (2015) explains a methodology applied to construct public capital stocks following a conventional approach (Kamps 2006; Gupta et al. 2014). The capital stock series are computed based on the traditional inventory equation:

$$K_{i,t+1} = (1 - \delta_{i,t})K_{i,t} + (1 - \delta_{i,t}/2)I_{i,t} \quad (\text{A1})$$

where for each country i , $K_{i,t+1}$ is the stock of public capital at the beginning of period $t + 1$; $\delta_{i,t}$ is a time-varying depreciation rate; and $I_{i,t}$ is gross fixed public capital formation in period t , assuming that new investment is operational in the middle of the period.

Below summarize three main components and underlying datasets. All series (output, investment, capital stock) are expressed in constant international 2005 prices (using purchasing power parity).

(1) Investment (flow): Several databases are used to provide a comprehensive database of the public capital stock series covering the period 1860–2014.

For the Organization for Economic Cooperation and Development (OECD) countries, the OECD Analytical Database (August 2014 version) is used and covers 26 countries for the period 1960–2013. The series retrieved (in national currency and constant prices) are comprised of government GFCF, private GFCF, and real GDP, and are converted to 2005 international dollars using OECD purchasing power parities. Data are filled to the extent possible using the IMF WEO database (the April 2014 version) when there are data patches in the OECD database.

For non-OECD countries, the Penn World Tables (PWT, version 8.0) are used and cover 132 countries for the period 1960–2011. The series retrieved are comprised of GDP and total gross fixed capital formation in 2005 constant prices, and are converted to 2005 international dollars using PWT purchasing power parities. Total investment from PWT is disaggregated into private and public investment by using the WEO database. Private and public investment shares, as percent of total investment, are calculated from the WEO database, and these shares are applied to the total PWT investment series. Data are extended to 2013 using the WEO database.

(2) Capital stock at initial period: Following a conventional approach in Kamps (2006), the initial capital stock is set to 0 for all countries in 1860. A total investment series is mechanically constructed between 1860 and the first available data point under an assumption that investment grew by 4 percent a year to reach its five-year forward moving average (first available) observed level. Similarly, for private and public investment, two investment series are mechanically constructed between 1860 and the first available data point under an assumption that private and public investment grew at the same rate as total investment to reach their five-year forward moving average (first available) observed levels, respectively.

(3) Capital depreciation rates: Data on country-specific capital depreciation rates are not available. We follow the convention in the literature on discount rate assumptions used in three groups with different income levels. Following Kamps (2006), the depreciation rate for high-income countries are assumed to rise from 2.5 percent in 1960 to 4.6 percent in 2013, and from 4.25 percent to 10.4 percent for public and private capital, respectively (Table A1). Similarly, different depreciation rates are assumed for middle-income and low-income countries following Gupta et al. (2014).

Table A1: Depreciation Rates (in percent)

	1860	1960	2013
Public Capital			
Low-income	2.50	2.50	2.50
Middle-income	2.50	2.50	3.51
High-income	2.50	2.50	4.59
Private Capital			
Low-income	4.25	4.25	4.25
Middle-income	4.25	4.25	8.10
High-income	4.25	4.25	10.41

Note: Income classifications are based on the World Bank's World Development Indicators' country groupings.

A.2 Coding Public Expenditure Composition Dataset

We follow public expenditure classification and definition in US BEA (2005) for our coding (Table A2).

Table A2: Public Expenditure Classification and Definition (US BEA 2005)

Government consumption expenditure

Gross output of general government

Value added

Compensation of general government employee

Supplement to wages and salaries

(Employer contributions for government social insurance)

Consumption of general government fixed capital

Intermediate goods and Services

Durable goods

Nondurable goods

Services

Less: Own account investment

Sales to other sectors

Government (gross) investment

Structures

New

Industrial

Military facilities

Net purchases of used structures

Residuals

Equipment and software

Aircraft, missiles, ships, and vehicles

Equipment

Software (including electronics)

Government (current) transfer payments

Government social benefits

To persons

To the rest of the world

Other current transfer payments to the rest of the world (net)

Table A3: Public Consumption, Investment, Transfers, and Capital in 1975–2020

(A) 1st group—5 countries (Albania, Algeria, Argentina, Barbados, and Belize)

Country	Time series Start	Periods End	Definition of Fiscal Sector	Public Consumption Yes/No	Public Investment Yes/No	Public Transfers Yes/No	Public Capital Yes/No	Debt Restructurings Number of episodes	Source
Albania	1985	2019	a. Central Government Budget Operation (1985-1987) b. Government Revenue and Expenditure (1988-1998) c. General Government Operations (1999-2020)	Yes	Yes	Yes	Yes	1 (1991-95)	i. IMF (1997) SM/97/155, ii. IMF (1994) EBS/94/39, iii. IMF (1998) SM/88/15, iv. IMF (1999) EBS/99/184, v. IMF (2003), Country Report No.03/63, vi. IMF (2004), Country Report No.04/22, vii. IMF (2008), Country Report No.08/128, viii. IMF (2010), Country Report No.10/205, ix. IMF (2014), Country Report No.14/78, x. IMF (2017), Country Report No.17/373, xi. IMF (2020), Country Report No.20/309, xii. Albania Ministry of Finance
Algeria	1975	2019	a. Central Government Operations (1975-2019)	Yes	Yes	Yes	Yes	2 (1991-95, 1993-96)	i. IMF (1992) SM/92/165, ii. IMF (1981) SM/81/192, iii. IMF (1988) EBS/88/15, iv. IMF (1991), SM/91/114, v. IMF (1994) SM/94/124 vi. IMF (1997) SM/97/155, vii. IMF (2001), Country Report No.01/163, viii. IMF (2006), Country Report No.6/102, ix. IMF (2007), Country Report No.07/95, x. IMF (2009), Country Report No.09/111, xi. IMF (2011), Country Report No.11/40, xii. IMF (2012), Country Report No.12/21, xiii. IMF (2013), Country Report No.13/49, xiv. IMF (2014), Country Report No.14/341, xv. IMF (2016), Country Report No.16/127, xvi. IMF (2018), Country Report No.18/168, xvii. IMF (2021), Country Report No.21/253.
Argentina	1975	2020	a. Central Government Operations (1975-79) b. General Government Operations (1980-83) c. Public Sector Operations (1984-90) d. Federal Government Operations (1991-2020)	Yes	Yes	Yes	Yes	5 (1982-85, 1985-87, 1988-93, 2001-05, 2019-20)	i. IMF (1979) SM/79/166, ii. IMF (1980) SM/80/185, iii. IMF (1983) SM/83/12, iv. IMF (1986), SM/86/35, v. IMF (1987) SM/87/16 vi. IMF (1990) SM/90/221, vii. IMF (1998), SM/98/20, viii. IMF (1999), SM/99/41, ix. IMF (2000), Country Report No.00/160, x. IMF (2003), Country Report No.03/392, xi. IMF (2011), Country Report No.05/236, xii. IMF (2006), SM/06/235, xiii. IMF (2016), Country Report No.16/67, xiv. IMF (2016), Country Report No.16/346, xv. IMF (2017), Country Report No.17/409, xvi. IMF (2019), Country Report No.19/232, xvii. IMF (2021), Country Report No.22/92, xviii. Argentina Ministerio de Hacienda xix. Argentina Secretaria de Hacienda
Barbados	1975	2020	a. Central Government Operations (1975-79)	Yes	Yes	Yes	Yes	1 (2018-19)	i. IMF (1980) SM/80/184, ii. IMF (1984) SM/84/186, iii. IMF (1987) SM/87/5, iv. IMF (1992), SM/92/123, v. IMF (1995) SM/95/54 vi. IMF (1998) SM/98/13, vii. IMF (2000), SM/00/237, viii. IMF (2004), SM/04/134, ix. IMF (2006), Country Report No.06/323, x. IMF (2010), Country Report No.10/363, xi. IMF (2014), Country Report No.14/52, xii. IMF (2016) Country Report No.16/279, xiii. IMF (2018), EBS/18/290, xiv. IMF (2020), EBS/20/192, xv. IMF (2021), Country Report No.21/128.
Belize	1978	2019	a. Central Government Operations (1975-79)	Yes	Yes	Yes	Yes	4 (2006-07, 2012-13, 2016-17, 2020)	i. IMF (1983) SM/83/107, ii. IMF (1986) SM/86/184, iii. IMF (1989) SM/89/97, iv. IMF (1993), SM/93/250, v. IMF (1998) SM/98/198 vi. IMF (2002) SM/02/324, vii. IMF (2004), Country Report No.04/101, viii. IMF (2004), Country Report No.06/369, ix. IMF (2008), Country Report No.08/88, x. IMF (2011), Country Report No.11/340, xi. IMF (2013), SM/13/139, xii. IMF (2017), SM/17/206, xiii. IMF (2019), SM/19/256, xiv. IMF (2020), SM/21/77. xv. Belize Ministry of Finance xiv. Central Bank of Belize

A.3 Non-debt Crisis Recession Episodes

We define non-debt crisis recession as follows. See Figure A1 for graphical illustration.

• **Definition:** A non-debt crisis recession episode satisfies the following four criteria:

- i Start year—A first year when GDP deviation from the HP-filtered trend starts a declining trend from peak;
- ii End year—A year when GDP deviation from the HP-filtered trend is at trough and negative (at least below zero) and starts an increasing trend;
- iii Magnitude and length—Change a debt from peak to trough is larger than 1 percentage point (based on US 2001 recession) and at least 1 year;
- iv No overlap with a debt restructuring event—There is an interval of at least one year between a non-debt crisis recession and a debt restructuring event.

Figure A1: Non-debt Crisis Recession

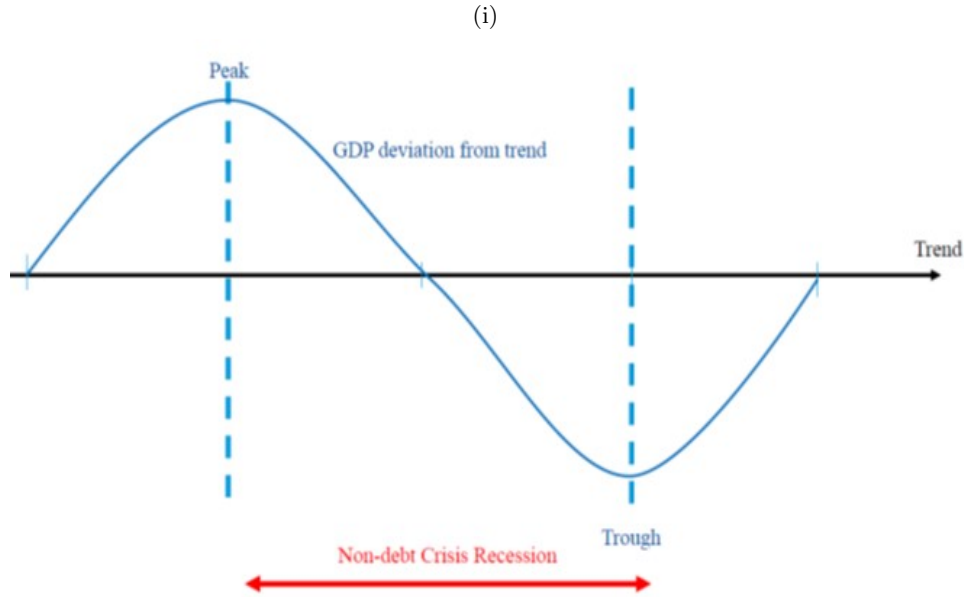


Table A4 lists some example cases of non-debt crisis recession episodes.

Table A4: Non-debt Crisis Recession Episodes in 1975–2020

Country	Business Cycles		Non-debt Crisis Recession Period		No Overlap with Restructurings (Yes/No)
	Peak	Trough	Start	End	
Albania	1976	1980	1977	1980	Yes
Albania	1981	1985	1982	1985	Yes
Albania	1996	1997	1997	1997	Yes
Albania	2001	2005	2002	2005	Yes
Albania	2008	2014	2009	2014	Yes
Algeria	1978	1980	1979	1980	Yes
Algeria	1985	1988	1986	1988	Yes
Algeria	1998	2001	1999	2001	Yes
Algeria	2005	2009	2006	2009	Yes
Argentina	1974	1976	1975	1976	Yes
Argentina	1977	1978	1978	1978	Yes
Argentina	1994	1995	1995	1995	Yes
Argentina	2007	2009	2008	2009	Yes
Argentina	2011	2014	2012	2014	Yes
Barbados	1976	1978	1977	1978	Yes
Barbados	1980	1982	1981	1982	Yes
Barbados	1989	1992	1990	1992	Yes
Barbados	2000	2004	2001	2004	Yes
Barbados	2008	2014	2009	2014	Yes
Belize	1974	1976	1975	1976	Yes
Belize	1980	1982	1981	1982	Yes
Belize	1984	1986	1985	1986	Yes
Belize	1993	1998	1994	1998	Yes
Belize	2000	2002	2001	2002	Yes
Bolivia	1998	2003	1999	2003	Yes
Bolivia	2008	2012	2009	2012	Yes
Bosnia and Herzegovina	2008	2012	2009	2012	Yes
Brazil	1976	1978	1977	1978	Yes
Brazil	1997	1999	1998	1999	Yes
Brazil	2002	2003	2003	2003	Yes

Table A5: Public Consumption, Investment, Transfers and Capital

A. Non-debt Crisis Recessions in 1975–2020 ^{1/}

	Observation	Mean	Observation	Mean	Observation	Mean
Non-debt Crisis Recession Episodes	325					
Non-debt Crisis Recession Duration	2.2					
	Pre-recession period		Recession period		Post-recession period	
			<i>Percent of GDP</i>			
Public Consumption, average ^{2/}	256	16.4	256	16.9	261	16.7
Public Investment, average ^{2/}	301	5.8	300	5.9	299	5.8
Public Transfers, average ^{2/}	249	5.8	252	5.9	259	6.2
Public Capital, average ^{2/}	321	76.6	320	80.1	319	77.8

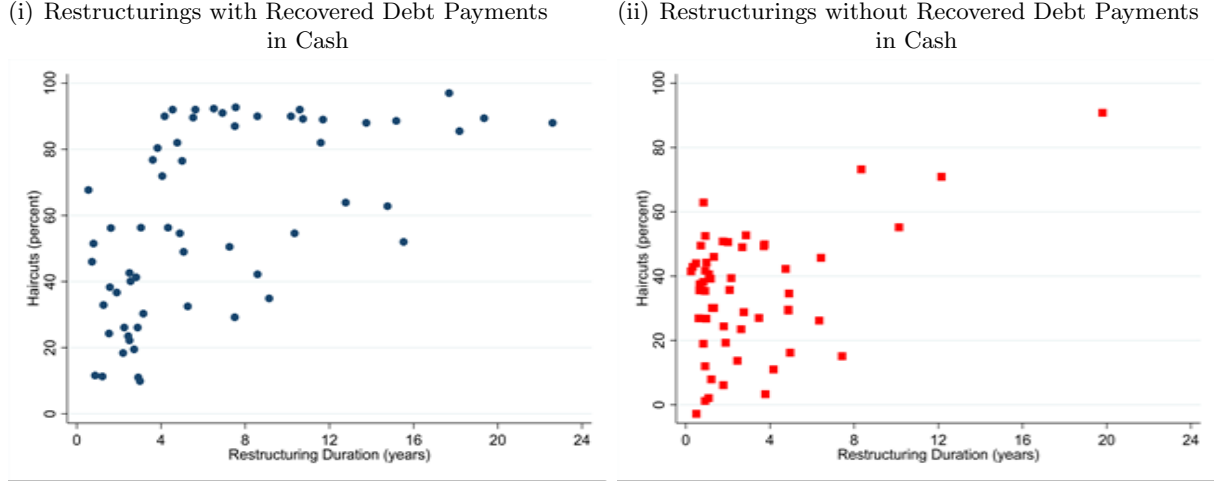
^{1/} For all components of public expenditure, our dataset covers both series in real and level (constant 2011 US dollars), and in percent of GDP.

^{2/} For each non-debt crisis recession episode, we take an average of public expenditure component series for corresponding periods: (i) pre-recession period, i.e., 3 years before the start of non-debt crisis recessions; (ii) recession period, i.e., from the start to the end of non-debt crisis recessions; (iii) post-recession period, i.e., 3 years after the end of non-debt crisis recessions. Then, we take an average of the obtained statistics across non-debt crisis recession observations.

Appendix B Further Empirical Analysis

B.1 Restructuring Duration and Haircuts

Figure B1: Restructuring Duration and Haircuts

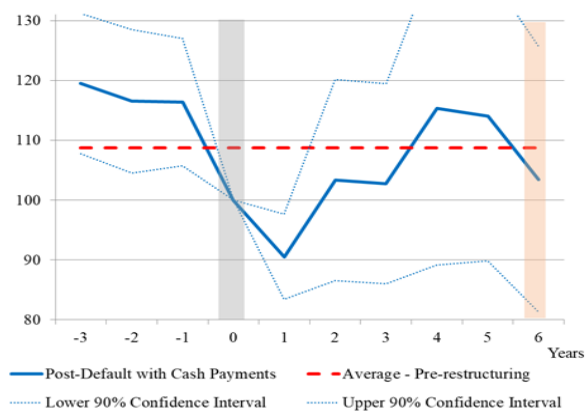


B.2 Public Investment, Consumption and Transfers

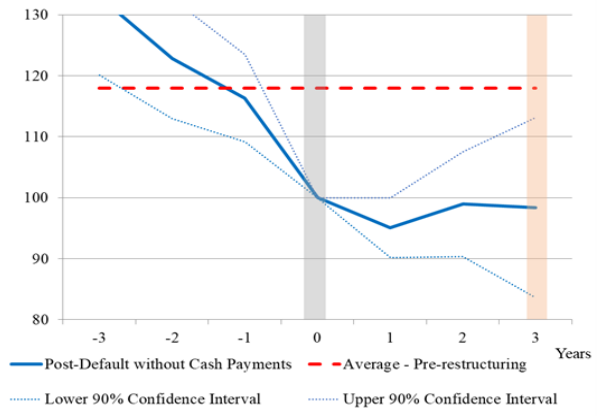
Figures B2 and B3 show the dynamics of public investment, and consumption and transfers—both as percent of GDP—around restructurings and non-debt crisis recessions. We follow the same presentation approach as in Figure 1 in terms of time horizon, timing of events, i.e., start of restructurings (recession), normalization of the series at levels at the start of restructurings (recessions), and average in the pre-restructuring (pre-recession) periods. Figures B2 and B3 show that both public investment-to-GDP and consumption and transfers-to-GDP ratios follow similar dynamics as the levels of public investment, and consumption and transfers in both restructurings and non-debt crisis recessions (Figures 1 and 2).

Figure B2: Public Investment (percent of GDP)

(i) Restructurings with Recovered Debt Payments in Cash



(ii) Restructurings without Recovered Debt Payments in Cash



(iii) Non-debt Crisis Recessions

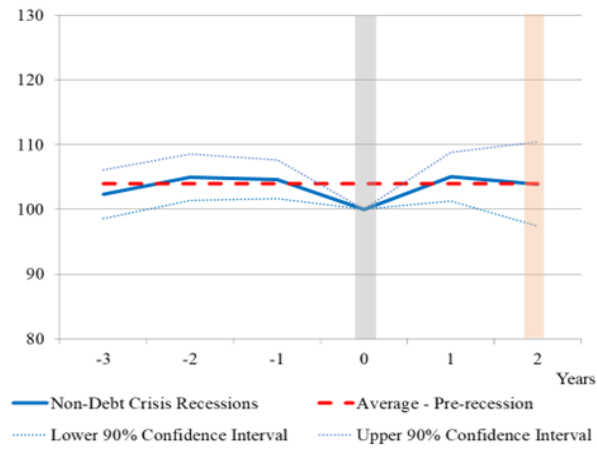
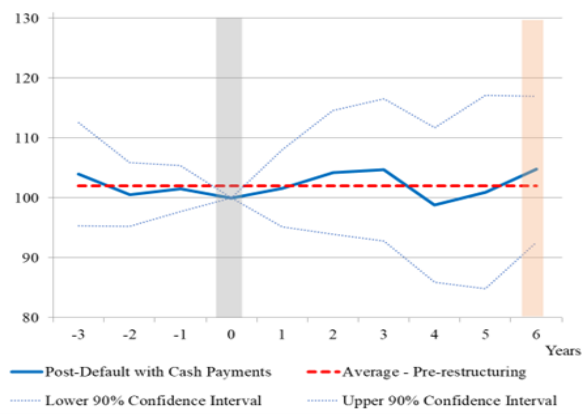
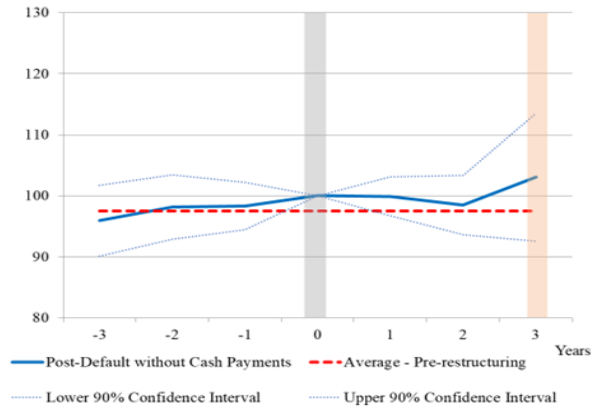


Figure B3: Public Consumption and Transfers (percent of GDP)

(i) Restructurings with Recovered Debt Payments in Cash



(ii) Restructurings without Recovered Debt Payments in Cash



(iii) Non-debt Crisis Recessions

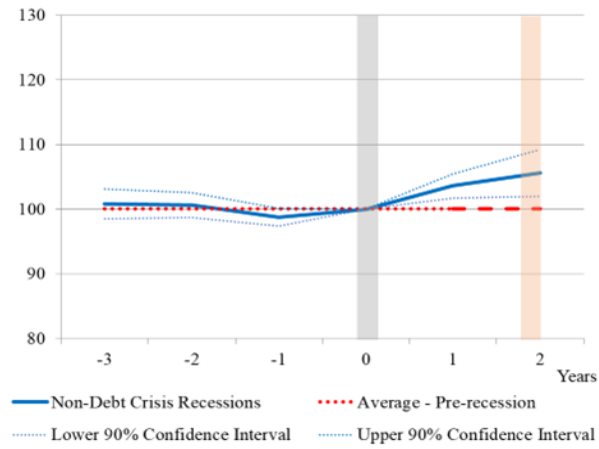


Table B1: Public Investment, Consumption and Transfers

	Public Investment			Public Consumption and Transfers		
	Restructuring with recovered debt payments in cash	Restructuring without recovered debt payments in cash	Non-debt crisis recession	Restructuring with recovered debt payments in cash	Restructuring without recovered debt payments in cash	Non-debt crisis recession
	deviation from trend, current ^{1/}	deviation from trend, current ^{1/}	deviation from trend, current ^{1/}	deviation from trend, current ^{1/}	deviation from trend, current ^{1/}	deviation from trend, current ^{1/}
	(1)	(2)	(3)	(4)	(5)	(6)
Restructuring period (current, dummy) ^{2/}	-0.12*** (0.04)	-0.15*** (0.05)	-	-0.03 (0.04)	-0.0006 (0.05)	-
Post-restructuring period (current, dummy) ^{3/}	-0.05 (0.04)	0.008 (0.05)	-	-0.02 (0.04)	-0.06 (0.06)	-
Recession period (current, dummy) ^{2/}	-	-	-0.03** (0.01)	-	-	-0.0009 (0.007)
Post-recession period (current, dummy) ^{3/}	-	-	-0.006 (0.01)	-	-	0.00004 (0.007)
PPG external debt (lagged, percent of GDP)	-0.0006** (0.0003)	-0.002*** (0.0008)	-3.4e-6** (1.8e-6)	0.00003 (0.0003)	-0.0004 (0.0008)	-3.0e-9** (9.3e-7)
GDP deviation from trend (current, percent) ^{1/}	0.03*** (0.004)	0.03*** (0.006)	0.01*** (0.002)	0.02*** (0.004)	0.02*** (0.006)	0.001 (0.001)
Constant	0.06* (0.03)	0.21*** (0.05)	0.03** (0.01)	0.002 (0.04)	0.03 (0.06)	0.002 (0.007)
Episode-specific fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Number of restructuring (recession) episodes	52	41	273	42	39	270
Number of observations	627	359	2,068	449	298	1,975
F-statistics	20.30	13.12	14.37	6.94	3.17	0.37
R ²	0.087	0.100	0.028	0.068	0.046	0.001

Notes: The table shows results from fixed effects OLS regressions. The dependent variables are public investment deviation from the trend in columns (1)–(3) and public consumption and transfers deviation from the trend in columns (4)–(6). The main explanatory variables are dummy variables for the restructuring and recession periods. Columns (1) and (4), (2) and (5), (3) and (6) report regression results for debt restructurings with recovered debt payments in cash, for debt restructurings without recovered debt payments in cash, and for non-debt crisis recessions, respectively. All regressions include episode-specific fixed effects. Significance levels are denoted by *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$, respectively. Robust standard errors clustered on the episode level are in parentheses.

^{1/} A deviation from the trend is a percentage deviation from the trend obtained by applying a Hodrick-Prescott (HP) filter to annual series with filter of 6.25.

^{2/} A dummy variable for the restructuring (recession) period is set 1 in the restructuring (recession) period and 0 in both the pre- and post-restructuring (pre- and post-recession) periods.

^{3/} A dummy variable for the post-restructuring (post-recession) period is set 1 in the post-restructuring (post-recession) period and 0 in both the pre-restructuring and restructuring (pre-recession and recession) periods.

B.3 Declines and Recoveries in Public Investment and Restructuring Duration

Table B2: Declines and Recoveries in Public Investment and Restructuring Duration

	Restructuring Duration (years)			
	Restructuring with Recovered Debt Payments in Cash		Restructuring without Recovered Debt Payments in Cash	
	Declines	Recoveries	Declines	Recoveries
	(1)	(2)	(3)	(4)
Declines in public investment-to-GDP ratio (percentage change from t-1 to the lowest, percent) ^{1/}	-0.96*** (0.32)	-	-0.59** (0.23)	-
Recoveries in public investment-to-GDP ratio (periods from the lowest to the pre-crisis average, years) ^{2/}	-	0.26** (0.11)	-	0.11 (0.07)
Constant	4.91*** (0.81)	4.83*** (0.92)	2.21*** (0.40)	2.28*** (0.52)
Number of observations	56	49	50	43
Adjusted- R^2	0.124	0.085	0.107	0.031
Root MSE	4.89	4.97	2.46	2.55

Notes: The table shows results from ordinary least square (OLS) regressions. The dependent variable is restructuring duration (years). The main explanatory variables are declines and recoveries in public investment-to-GDP ratio. Significance levels are denoted by *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$, respectively. Standard errors are in parentheses.

^{1/} Percentage change of public investment-to-GDP ratio from the level in year -1 to the level when public investment-to-GDP ratio is at the lowest level i.e., end of its downward trend.

^{2/} Length of time (years) from the time which public investment-to-GDP ratio is at the lowest level i.e., end of its downward trend to the time which it recovers to the pre-restructuring average.

Table B3: Declines and Recoveries in Public Investment and Restructuring Duration–Robustness

	Restructuring Duration (years)			
	Restructuring with Recovered Debt Payments in Cash		Restructuring without Recovered Debt Payments in Cash	
	Declines	Recoveries	Declines	Recoveries
	(1)	(2)	(3)	(4)
Declines in public investment-to-GDP ratio (percentage change from t-1 to the lowest, percent) ^{1/}	-0.62* (0.35)	-	-0.29 (0.40)	-
Recoveries in public investment-to-GDP ratio (periods from the lowest to the pre-crisis average, years) ^{2/}	-	0.32** (0.13)	-	0.11 (0.11)
PPG external debt (lagged, percent of GDP)	0.04** (0.01)	0.03** (0.01)	-0.01 (0.01)	-0.01 (0.01)
GDP deviation from trend (end, percent) ^{3/}	0.20* (0.11)	0.25** (0.12)	0.001 (0.05)	0.02 (0.06)
Export-to-debt service ratio (end)	0.13 (0.11)	0.12 (0.11)	0.41*** (0.14)	0.37** (0.16)
Constant	2.49 (1.54)	2.22 (1.62)	1.61 (1.09)	1.79 (1.18)
Number of observations	50	44	42	38
Adjusted- R^2	0.213	0.272	0.159	0.173
Root MSE	4.81	4.62	2.43	2.49

Notes: The table shows results from ordinary least square (OLS) regressions. The dependent variable is restructuring duration (years). The main explanatory variables are declines and recoveries in public investment-to-GDP ratio. Significance levels are denoted by *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$, respectively. Standard errors are in parentheses.

^{1/} Percentage change of public investment-to-GDP ratio from the level in year -1 to the level when public investment-to-GDP ratio is at the lowest level i.e., end of its downward trend.

^{2/} Length of time (years) from the time which public investment-to-GDP ratio is at the lowest level i.e., end of its downward trend to the time which it recovers to the pre-restructuring average.

^{3/} A deviation from the trend is a percentage deviation from the trend obtained by applying a Hodrick-Prescott (HP) filter to annual series with filter of 6.25.

Appendix C Implications for Key Theoretical Assumption

We explore model implications for following one key theoretical assumption: taxation method. In particular, we discuss how a change in the assumption keeping other assumptions and parameter values unchanged influences the sovereign’s choice between repayment and default, and between settlement and delay. Our main qualitative implications are robust.

Panels (i) and (ii) in Figure C1 repeat panels A-(i) and A-(ii) in Figure 5: the sovereign’s choice when the sovereign’s TFP is at the mean level. Panels (i) and (ii) report the sovereign’s choice at good credit record ($h_t = 0$) and at bad credit record ($h_t = 1$), respectively.

Figure C2 reports the sovereign’s choice between repayment and default, and between settlement and delay in two different assumptions of taxation; panel A: two-stage consumption tax; panel B: labor income tax. First, we allow the sovereign to increase consumption tax rate to raise tax revenues during debt restructurings—equivalent to additional revenue measures conditional on default. In this case, the sovereign is more willing to settle because of the improvement in repayment capacity driven by an increase in tax revenues (the enlarged “settlement” region in panel A-(ii)). Due to lower default costs—shorter periods of financial exclusion owing to high likelihood of debt settlement—, the sovereign is more willing to default ex ante (the enlarged “default” region in panel A-(i)).

Second, replacing consumption tax with labor income tax (Arellano and Bai 2017) does not influence the sovereign’s choice between repayment and default, and between settlement and delay. This is because, labor income tax and consumption tax are conceptually identical in that both affect the household’s intra-temporal substitution between consumption and labor (equation 3), but not the sovereign’s inter-temporal substitution between consumption—public consumption and transfers—and saving (i.e., public investment).

Figure C1: Debtor’s Choice between Repayment and Default, and between Settlement and Delay—Baseline Model

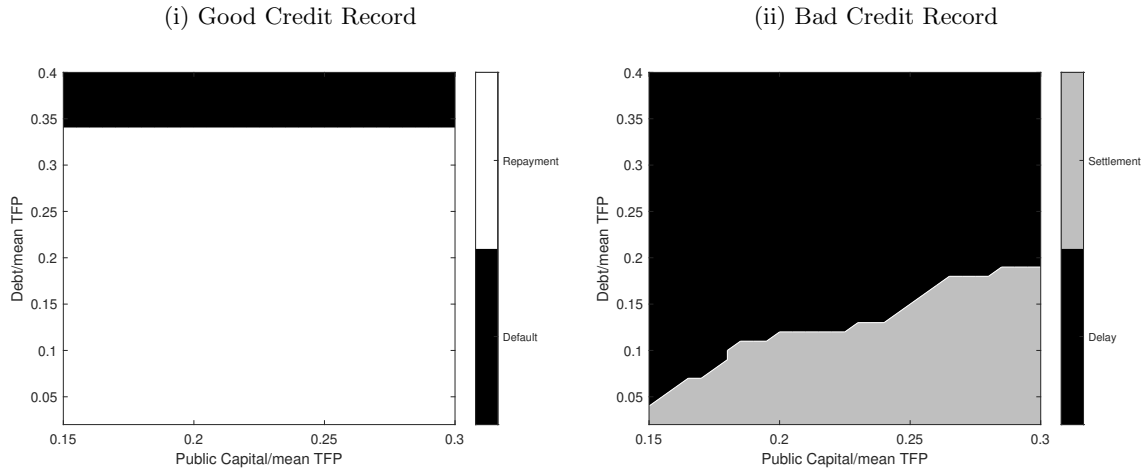
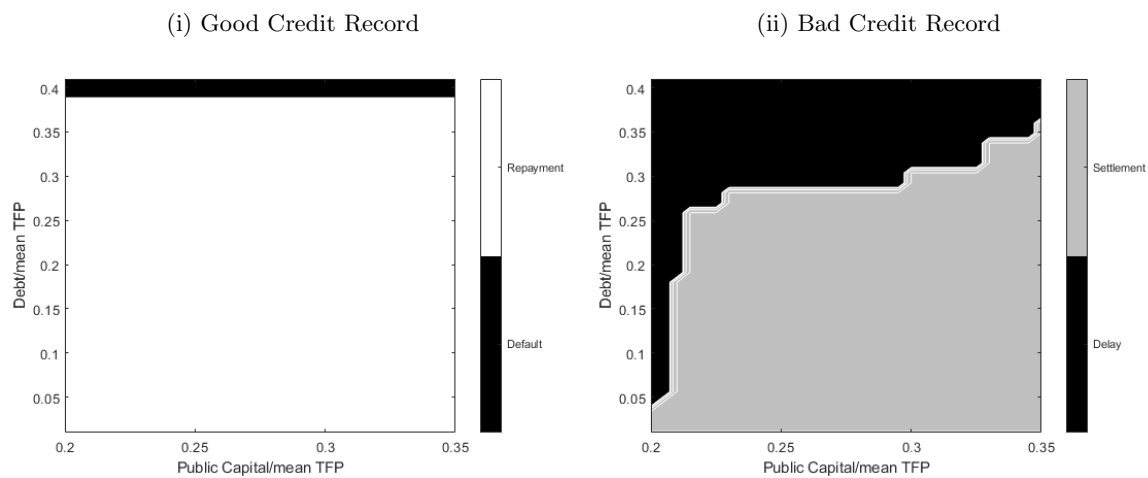
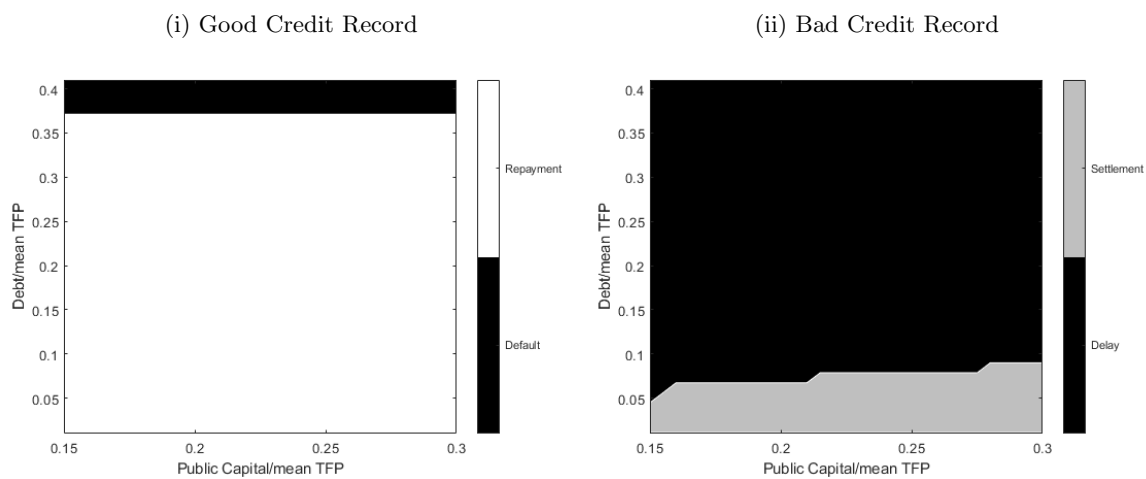


Figure C2: Debtor's Choice between Repayment and Default, and between Settlement and Delay—Taxation Methods

A: Two-stage Consumption Tax



B: Labor Income Tax



Appendix D Computation Algorithm

We use a global solution method following Chatterjee and Eyigungor (2012) and Morelli and Moretti (2023). The procedure to compute the equilibrium distribution of the model is the following:

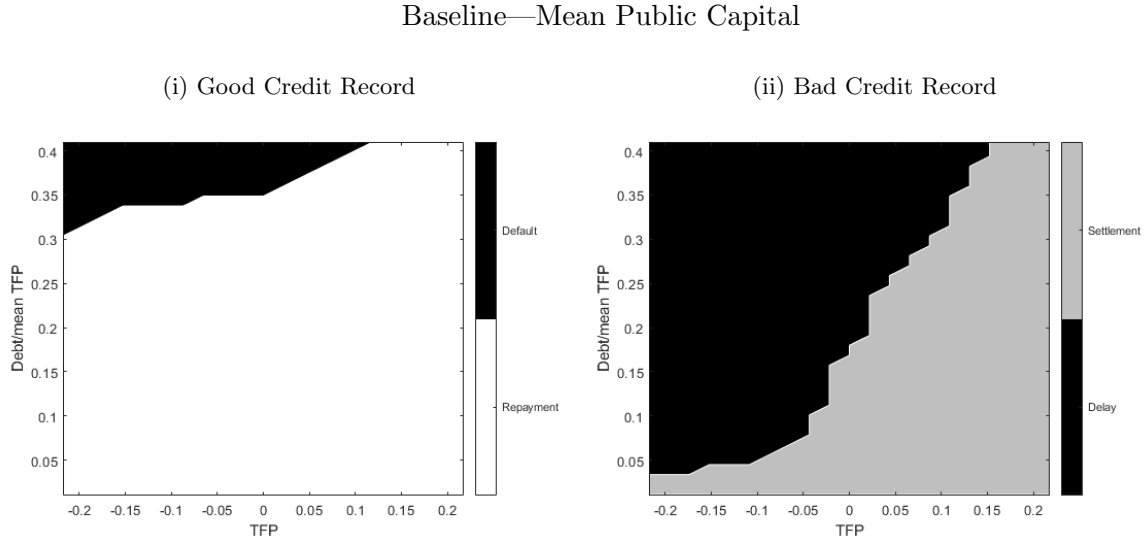
1. First, we set finite grids on the space of debt holdings, public capital and productivity as by $B = [b_{min}, b_{max}]$, $K^g = [k_{min}^g, k_{max}^g]$, and $A = [a_{min}, a_{max}]$. Limits of productivity are large enough to include large deviations from mean value of shocks. We approximate the stochastic productivity process of the sovereign shown by equation (42) using a discrete Markov chain as in Tauchen (1986). Moreover, we compute the transition matrix based on the probability distribution $\mu(a_{t+1}|a_t)$.
2. Second, we set finite grids on the space of recovery rates (δ_t). Limits of recovery rates are to ensure that they do not bind in equilibrium.
3. Third, we set the initial values for equilibrium sovereign bond price, debt renegotiation payoffs for the sovereign and the creditors, and the sovereign's value functions. We use the risk-free bond price ($q^0 = (1 + r^*)^{-1}$) for the baseline equilibrium bond price. We set payoffs for debt renegotiations for the sovereign and the creditors as $\Delta_t^{B,0} = \Delta_t^{L,0} = 0$, and the initial value functions for the sovereign as $V^0 = V^{R,0} = V^{D,0} = 0$.
4. Fourth, given the baseline equilibrium bond price, debt renegotiation payoffs, and the sovereign's value functions, we solve for the household's and the firm's maximization problems to obtain private consumption, labor supply, and labor demand.
5. Fifth, given the baseline equilibrium sovereign bond price, debt renegotiation payoffs, and the private sector's equilibrium policy functions, we solve for the sovereign's optimization problem for both good and bad credit records ($h_t = 0, 1$). Similar to Chatterjee and Eyigungor (2012) and Morelli and Moretti (2023), we solve for the default decision. This procedure finds the value functions for the sovereign ($V^1, V^{R,1}, V^{D,1}$), the optimal debt functions ($b^1, b^{R,1}, b^{D,1}$), and public capital functions ($k^{g,1}, k^{g,R,1}, k^{g,D,1}$). Furthermore, we obtain the optimal default choice. Based on the default choice, we also evaluate the default probability using the transition matrix.
6. Sixth, using the default choice in step 5, and the zero profit condition for the foreign creditors, we compute the new price of sovereign bonds (q^1).
7. Seventh, given the value functions for the sovereign, we solve the bargaining problem and compute the new payoffs for two cases either the sovereign or the creditors is the proposer ($\Delta_t^{B,1}, \Delta_t^{L,1}$).
8. We iterate steps 4, 5, 6, and 7 to have fixed optimal value functions for the sovereign, debt renegotiation payoffs, bond price and the private sector's policy functions.

Appendix E Further Equilibrium Properties

E.1 Equilibrium Properties in the Case the Sovereign Proposes

Figure E1 reports the sovereign's choice between repayment and default, and between settlement and delay when public capital is fixed at the mean level. The horizontal and vertical axes are TFP and debt-to-mean TFP ratio, respectively. On the sovereign's choice between settlement and delay in panel (ii), the sovereign is more likely to settle (delay) when debt is low (high) and TFP is high (low). This is consistent with findings in the literature on sovereign debt restructurings with multi-round renegotiations (Benjamin and Wright 2013; Bi 2008, Asonuma and Joo 2020). On the sovereign's choice between repayment and default, the sovereign is more likely to repay (default) when debt is low (high) and TFP is high (low). This is consistent with findings in the literature of sovereign default (Arellano 2008; Yue 2010).

Figure E1: Debtor's Choice between Repayment and Default, and between Settlement and Delay

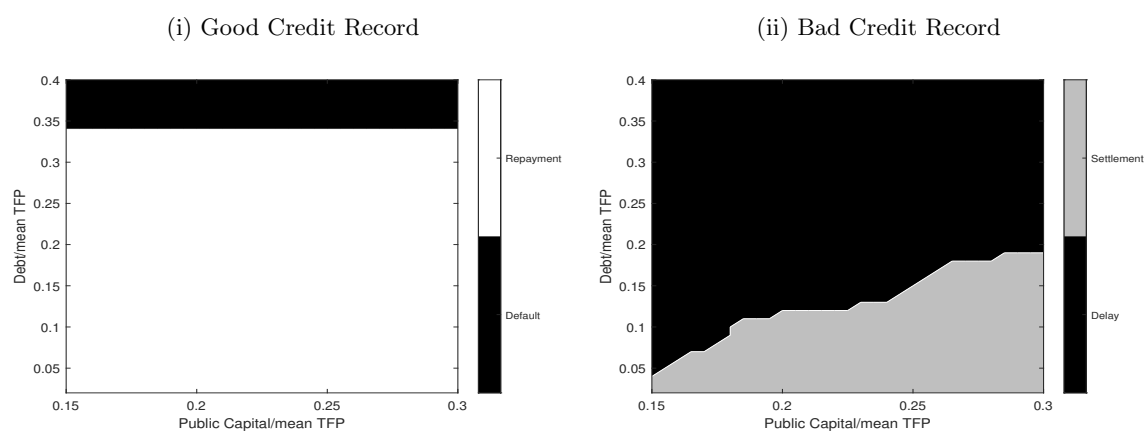


E.2 Equilibrium Properties in the Case the Creditors Propose

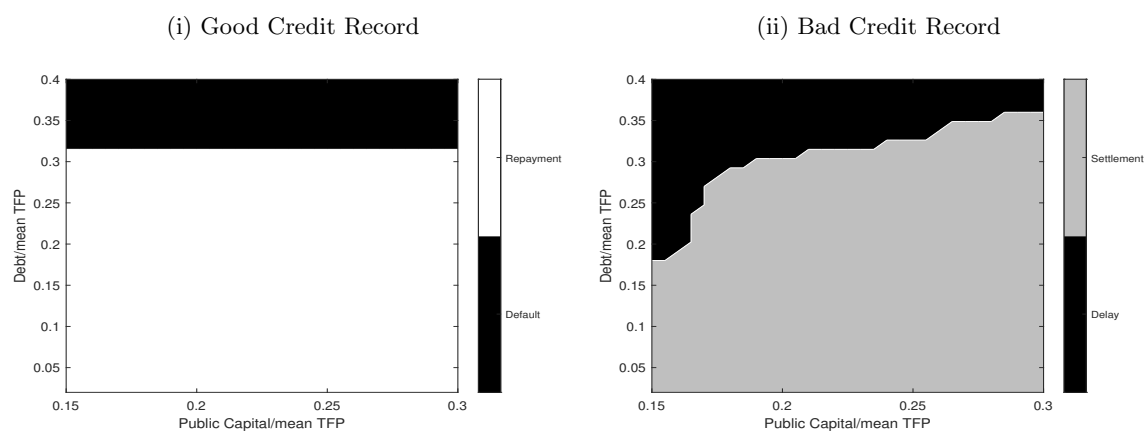
We show the sovereign's choice between repayment and default, and between settlement and delay when the creditors propose in Figure E2. We follow the same presentation approach as in Figure 5 in terms of axis, panel classifications, and regions. The sovereign's choice when the creditors propose is exactly identical to that when the sovereign proposes (Figure 5). This is the finding in the literature of multi-round renegotiations (Bi 2008; Asonuma and Joo 2020); whether both parties can reach settlement in the current period does not depend on the identity of the proposer. Intuitively, if one party proposes recovery rates that make both parties at least weakly better off by settling than postponing, this offer of recovery rates could identically be proposed by the counterpart and accepted by the original party.

Figure E2: Debtor's Choice between Repayment and Default, and between Settlement and Delay when the Creditors Propose

A: Restructuring with Recovered Payments in Cash (Argentina 2001–05)—Mean TFP



B: Restructuring without Recovered Debt Payments in Cash (Argentina 2019–20)—Mean TFP



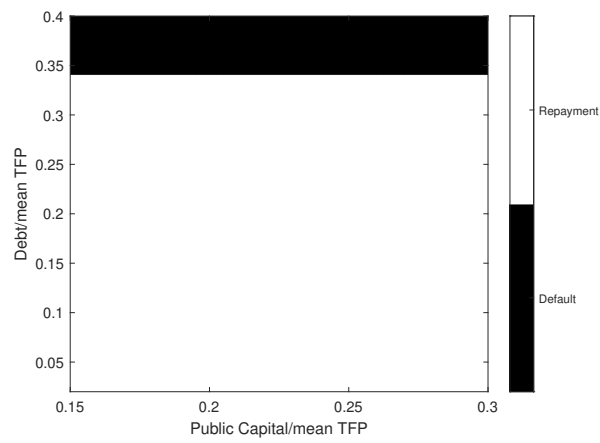
E.3 Comparison with Previous Models of Sovereign Debt Renegotiations

We contrast equilibrium properties in our baseline model of multi-round renegotiations with those in previous models of sovereign debt renegotiations. We consider two cases: (i) a model of no debt renegotiation, i.e., exogenous reentry and no recovery rates (Arellano 2008; Gordon and Guerron-Quintana 2018) and (ii) a model of a one-round negotiation (Yue 2010; Arellano and Bai 2017). To generate model features comparable to our baseline model of multi-round renegotiations, we replace our multi-round renegotiation framework in our baseline model with no debt renegotiation framework, specifically exogenous reentry and no (zero) recovery rates for the case (i), and with one-round Nash bargaining framework for the case (ii), respectively, leaving all other parameters unchanged.

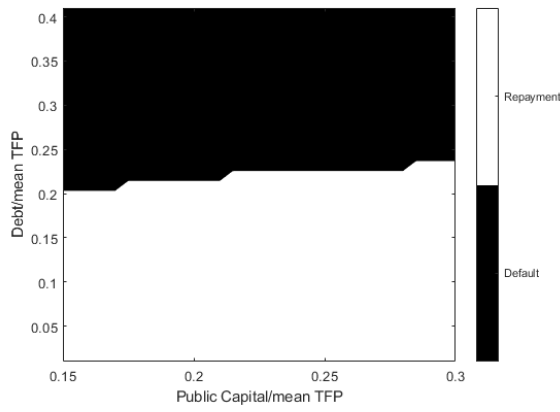
Figure E3 contrasts the sovereign’s choice between repayment and default at the mean TFP in our baseline model of multi-round renegotiations (panel (i)) with that in these two models of sovereign debt renegotiations (panels (ii) and (iii)). We follow the same presentation approach as in panel A-(i) in Figure 5 in terms of axis and regions. There are two features in these two models of sovereign debt renegotiations different from those in our baseline model of multi-round renegotiations. First, the sovereign is more willing to repay debt as public capital increases. This is shown in the enlarged (shrunk) “repayment” region when public capital is high (low) in panels (ii) and (iii). Second, the sovereign is more willing to default at low level of debt due to low default costs—fixed (i.e., exogenously determined) or short periods of financial autarky over which the sovereign suffers productivity loss—than our baseline model of multi-round renegotiations. We do not contrast the sovereign’s choice of settlement and delay in our baseline model of multi-round renegotiations with that in the model of a one-round negotiation. This is because the sovereign’s choice of settlement and delay in our baseline model of multi-round renegotiations does not correspond one-to-one with that in the model of a one-round negotiation due to a difference in the two bargaining frameworks.

Figure E3: Debtor's Choice of Repayment and Default at the Mean TFP

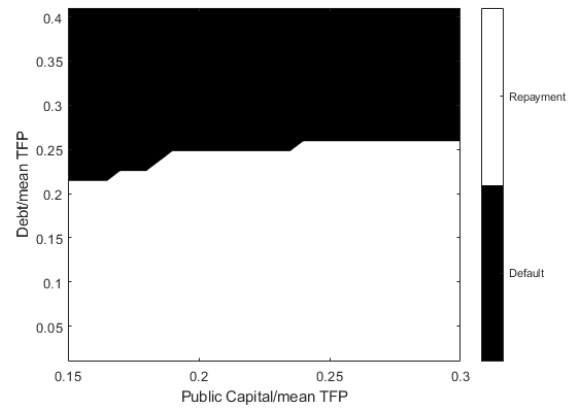
(i) Baseline Model of Multi-round Renegotiations



(ii) No Debt Renegotiation (exogenous reentry)



(iii) One-round Negotiation



E.4 Comparison with Models of Exogenous Public Capital and Endogenous Aggregate Capital

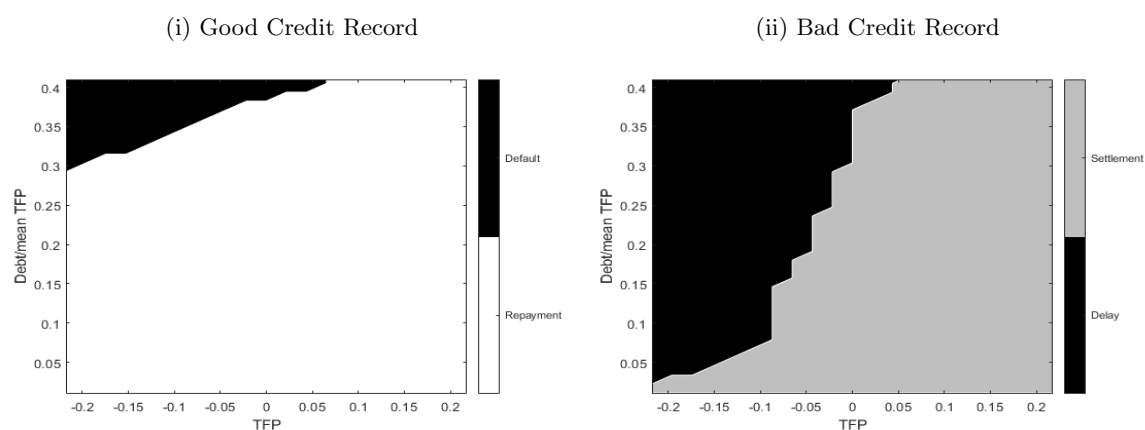
We contrast equilibrium properties in our baseline model of endogenous public capital with those in two models of exogenous public capital (Arellano and Bai 2017; Cuadra et al. 2010) and endogenous aggregate capital (Gordon and Guerron-Quintana 2018; Park 2017; Galli 2021). Figure E4 reports the sovereign’s choice between repayment and default, and between settlement and delay. Panel A follows the same presentation approach as in Figure E1 where the horizontal and vertical axes are TFP and debt-to-mean TFP ratio, respectively. Panel B follows the same presentation approach as in Figure 5 where the horizontal and vertical axes are aggregate capital-to-mean TFP ratio and debt-to-mean TFP ratio, respectively.

Panel A shows that the model of exogenous public capital shares similar qualitative and quantitative features with the baseline model of endogenous public capital (Figure E1). On the sovereign’s choice between settlement and delay, the sovereign is more likely to settle (delay) when debt is low (high) and TFP is high (low) (panel A-(ii)). On its choice between repayment and default, the sovereign is more likely to repay (default) when debt is low (high) and TFP is high (low) (panel A-(i)).

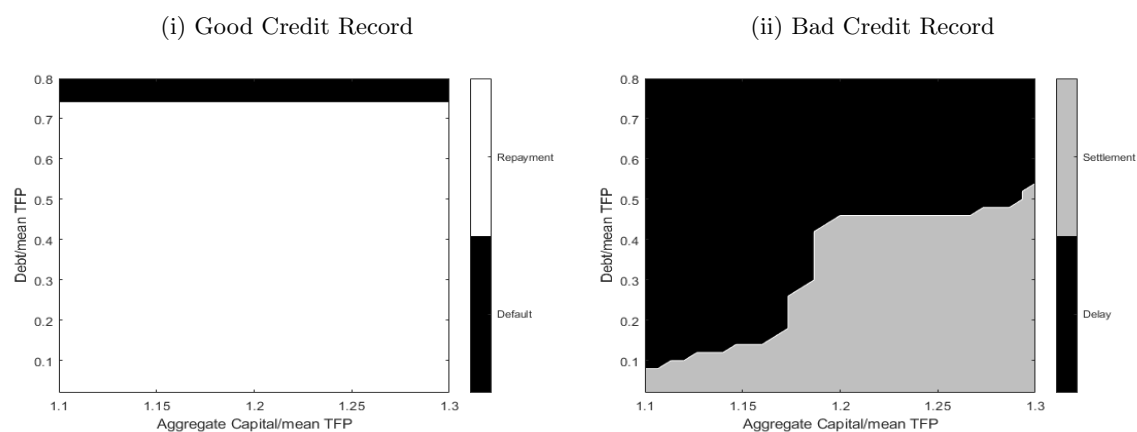
Panel B shows that the model of endogenous aggregate capital also shares similar qualitative features with the baseline model of endogenous public capital (Figure 5). On the sovereign’s choice between settlement and delay, the sovereign is more likely to settle (delay) when aggregate capital is high (low) (panel B-(ii)). Quantitatively, a boundary between “settlement” region and “delay” region is at a higher level of debt (around debt-to-mean TFP ratio of 50) in the endogenous aggregate capital model. On its choice between repayment and default, its willingness to repay remains unchanged or is weakly decreasing when aggregate capital increases—presented in unchanged or slightly enlarged “default” region in black color (panel B-(i)). Quantitatively, a boundary between “repayment” region and “default” region is also at a higher level of debt (around debt-mean TFP ratio of 75) in the endogenous aggregate capital model.

Figure E4: Debtor's Choice between Repayment and Default, and between Settlement and Delay

A: Exogenous Public Capital—Restructurings with Recovered Debt Payments in Cash



B: Endogenous Aggregate Capital—Restructurings with Recovered Debt Payments in Cash



Appendix F Further Quantitative Analysis

F.1 Further Simulation Results in Our Baseline Models

Panel (i) in Table F1 shows that business cycle statistics for private sector in our baseline models match with those in the data. Similar to previous studies (Arellano 2008; Yue 2010), our baseline models replicate both volatile consumption and trade balance-to-GDP ratio.

Table F1: Moment Statistics from Simulation Results

(i) Private Sector Business Cycle Statistics						
	Recovered Debt Payments in Cash		No Recovered Debt Payments in Cash			
	Argentine 2001-05		Argentine 2019-20			
	Data	Baseline Model	Data	Baseline Model	Cuadra et al. (2010) Recalibration ^{1/}	Gordon and Guerron-Quintana (2018) Recalibration ^{2/}
Pre-default periods ^{3/}						
Private sector						
Private consumption (std. dev.)/output (std. dev.)	1.11	1.00	1.05/1.35	1.01	1.01	1.51
Trade balance/output: std. dev. (%)	1.28	0.91	0.92/2.93	0.76	0.50	3.76
Corr.(trade balance, output)	-0.87	-0.12	-0.72/-0.03	-0.03	-0.41	-0.24
Renegotiation periods						
Private sector						
Private consumption (std. dev.)/output (std. dev.)	1.17	1.49	1.69	0.99	1.00	1.00
Trade balance/output: std. dev. (%)	0.45	0.00	2.30	0.00	0.00	0.00
Corr.(trade balance, output)	-0.97	0.00	0.47	0.00	0.00	0.00

(ii) Non-business Cycle Statistics						
	Recovered Debt Payments in Cash		No Recovered Debt Payments in Cash			
	Argentine 2001-05		Argentine 2019-20			
	Data	Baseline Model	Data	Baseline Model	Cuadra et al. (2010) Recalibration ^{1/}	Gordon and Guerron-Quintana (2018) Recalibration ^{2/}
Pre-default periods ^{3/}						
Corr.(debt/GDP, output)	-0.97	-0.23	-0.72/-0.17	-0.62	-0.28	-0.22
Renegotiation periods						
Corr.(debt/GDP, output)	-0.95	-0.63	-0.76	-0.92	-0.99	-0.33

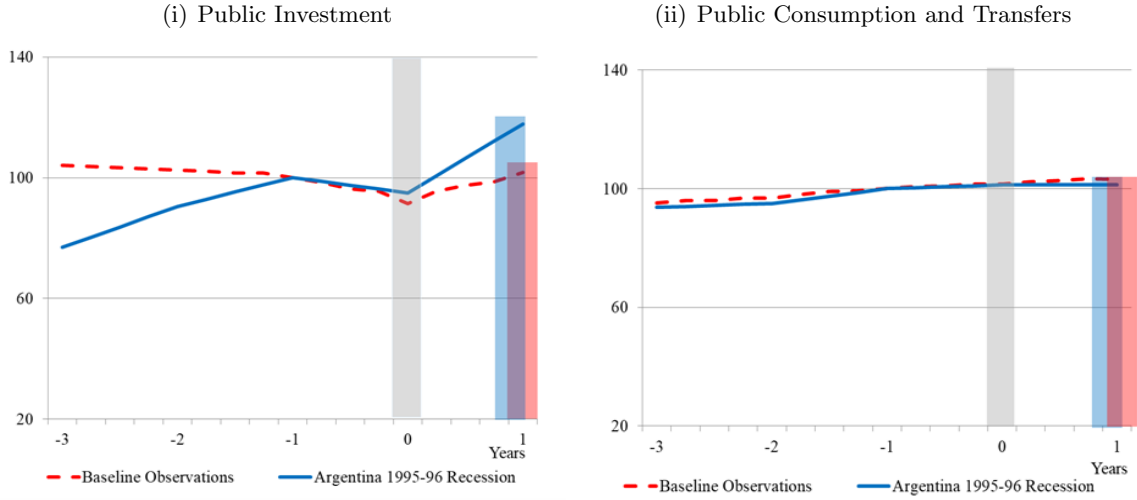
Sources: Datastream, IMF WEO, INDEC and MECON.

Notes: ^{1/} Cuadra et al. (2010) recalibration corresponds to calibration results of one-period (short-term) debt with three target statistics (i) debt service-to-GDP ratio, (ii) ratio between public consumption and transfers and private consumption, and (iii) ratio between standard deviation of public consumption and standard deviation of output.

^{2/} Gordon and Guerron-Quintana (2018) recalibration corresponds to calibration results with four target statistics (i) average bond spreads, (ii) standard deviation of bond spreads, (iii) ratio between standard deviation of total investment and standard deviation of output, and (iv) excess consumption volatility.

^{3/} Over 1993Q1–2000Q4 and 2006Q1–18Q4. We report (i) 2006Q1–13Q3 and (ii) 2013Q4–18Q4 due to the MECON's discontinuation of the private sector business cycle data series.

Figure F1: Public Investment, Consumption and Transfers in Non-debt Crisis Recession



F.2 Comparison with Recalibrations of Previous Studies

Moment Statistics Table F2 reports non-target statistics in recalibration results of previous studies on (c) sovereign debt and fiscal policy (Arellano and Bai 2017) and (d) debt renegotiations (Benjamin and Wright 2013) in columns 4 and 5, respectively. We recalibrate these two models to match their targeted statistics leaving the remaining parameter values unchanged with our baseline model of endogenous public capital. We also report calibrated moment statistics in Benjamin and Wright (2013) in column 6. We contrast those with data (column 1) and moment statistics in calibration results of our baseline model of endogenous public capital (columns 2–3).

First of all, neither the recalibrations of Arellano and Bai (2017) nor Benjamin and Wright (2013) replicate lower average public investment in the renegotiation periods than that in the pre-default periods as observed in the data. Neither the recalibration of Arellano and Bai (2017) nor Benjamin and Wright (2013) generate any dynamics of public investment.

Second, neither the recalibrations of Arellano and Bai (2017) nor Benjamin and Wright (2013) replicate a negative relationship between a decline in public investment and duration, or a positive relationship between a recovery in public investment and duration as observed in the data. Again, neither the recalibration of Arellano and Bai (2017) nor Benjamin and Wright (2013) generate any dynamics of public investment.

Third, both recalibrations of Arellano and Bai (2017) and Benjamin and Wright (2013) replicate only short restructuring duration (2.0 and 6.0 quarters). The recalibration of Arellano and Bai (2017) generates only restructuring delays of 2.0 quarters because the model assumes one-round debt negotiation—note that the minimum restructuring delay is 2 quarters. The recalibration of Benjamin and Wright (2013) generates only restructuring delays of 6.0 quarters much shorter than calibrated restructuring delays of 33.2 quarters in Benjamin and Wright (2013). This is because Benjamin and Wright (2013) introduce stochastic process of bargaining power that depends on both the party who proposes in the current round and the current debtor income but our recalibration of Benjamin and Wright (2013) assumes constant (unchanged) bargaining power.

As in conventional models in the literature, both recalibrations of Arellano and Bai (2017) and Benjamin and Wright (2013) replicate private sector business cycle moments such as procyclical and

volatile private consumption, and countercyclical trade-balance. The recalibration of Benjamin and Wright (2013) replicates high average debt-to-GDP ratio in pre-default and restructuring periods due to multi-round debt renegotiations. While both recalibrations of Arellano and Bai (2017) and Benjamin and Wright (2013) assume short-term (one-period) debt, both generate and countercyclical bond spreads as observed in the data.

Table F2: Moment Statistics from Recalibration Results of Previous Studies

(i) Business Cycle Statistics

	Data Recovered Debt Payment in Cash	Baseline Model Recovered Debt Payment in Cash	Baseline Model Recovered Debt Payment in Cash	Arellano and Bai (2017) Recalibration ^{1/}	Benjamin and Wright (2013) Recalibration ^{2/}	Benjamin and Wright (2013) Statistics ^{3/}
Target statistics						
Pre-default periods						
Average public consumption & transfers/GDP ratio (%)	22.5	22.5	24.7	-	-	-
Public consumption & transfers (std. dev.)/output (std. dev.)	1.26	1.10	1.01	-	-	-
Public investment (std. dev.)/output (std. dev.)	2.7	2.41	2.90	-	-	-
Non-target statistics						
Pre-default periods						
Public sector						
Public consumption & transfers (std. dev.)/output (std. dev.)	1.26	1.10	1.01	1.42	-	-
Corr.(public consumption & transfers, output)	0.77	0.88	0.78	0.87	-	-
Average public consumption & transfers/GDP ratio (%)	22.5	-	-	24.6	-	-
Average public investment/GDP ratio (%)	1.31	2.30	2.72	-	-	-
Average public investment/public expenditure ratio (%)	6.2	9.3	12.2	-	-	-
Renegotiation periods						
Public sector						
Public consumption & transfers (std. dev.)/output (std. dev.)	0.99	1.43	2.80	-	-	-
Corr.(public consumption & transfers, output)	0.97	0.79	0.10	-	-	-
Average public investment/GDP ratio (%)	1.19	1.85	1.33	-	-	-
Average public investment/public expenditure ratio (%)	5.7	8.2	5.7	-	-	-

(ii) Private Sector Business Cycle Statistics

	Data Recovered Debt Payment in Cash	Baseline Model Recovered Debt Payment in Cash	Baseline Model Recovered Debt Payment in Cash	Arellano and Bai (2017) Recalibration ^{1/}	Benjamin and Wright (2013) Recalibration ^{2/}	Benjamin and Wright (2013) Statistics ^{3/}
Target statistics						
Pre-default periods						
Private consumption (std. dev.)/output (std. dev.)	1.11	-	-	-	-	-
Non-target statistics						
Pre-default periods						
Private sector						
Private consumption (std. dev.)/output (std. dev.)	1.11	1.00	1.01	1.06	1.02	1.02
Trade balance/output: std. dev. (%)	1.28	0.91	0.76	0.38	0.89	-
Corr.(trade balance, output)	-0.87	-0.19	-0.03	-0.42	-0.40	-0.10
Renegotiation periods						
Private sector						
Private consumption (std. dev.)/output (std. dev.)	1.17	1.49	0.99	-	1.00	-
Trade balance/output: std. dev. (%)	0.45	0.00	0.00	-	0.00	-
Corr.(trade balance, output)	-0.97	0.00	0.00	-	0.00	-

(iii) Non-business Cycle Statistics

	Data Recovered Debt Payment in Cash	Baseline Model Recovered Debt Payment in Cash	Baseline Model Recovered Debt Payment in Cash	Arellano and Bai (2017) Recalibration ^{1/}	Benjamin and Wright (2013) Recalibration ^{2/}	Benjamin and Wright (2013) Statistics ^{3/}
Target statistics						
Pre-default periods						
Default probability (%)	3.50	3.80	3.20	-	3.01	5.2
Average recovery rate (%)	25.0	-	-	24.5	25.6	50.0
Average debt service/GDP ratio (%)	8.0	-	-	9.7	-	-
Bond spreads: average (%)	7.2	-	7.9	7.3	-	-
Bond spreads: std. dev. (%)	3.40	3.20	4.7	-	-	-
Pre-default periods						
Default probability (%)	3.50	-	-	3.73	-	-
Average debt/GDP ratio (%)	32.6	55.4	38.0	-	37.2	76.0
Bond spreads: average (%)	7.2	-	-	-	1.17	-
Bond spreads: std. dev. (%)	3.40	-	-	13.5	1.42	-
Corr.(output, spreads)	-0.88	-0.20	-0.35	-0.43	-0.24	-0.12
Corr.(debt/GDP, spreads)	0.92	0.20	0.27	0.05	0.37	-
Renegotiation periods						
Average debt/GDP ratio (%)	109.6	88.7	75.8	-	43.2	84.0
Average duration of renegotiations (quarters)	14.6	11.5	5.0	2.00	6.0	33.2
Average recovery rate (%)	25.0	22.5	51.0	-	-	-
Corr.(decline in public investment, duration)	-0.25	-0.10	-0.52	-	-	-
Corr.(recovery in public investment, duration)	0.22	0.58	0.14	-	-	-

Sources: Datastream, IMF WEO, INDEC and MECON.

^{1/} Arellano and Bai (2017) recalibration corresponds to calibration results with three target statistics (i) average bond spreads, (ii) debt service-to-GDP ratio, and (iii) average recovery rate.

^{2/} Benjamin and Wright (2013) recalibration corresponds to calibration results with three target statistics (i) default frequency, (ii) average recovery rate, and (iii) average debtor output deviation during renegotiations.

^{3/} Benjamin and Wright (2013) statistics correspond to their moment statistics in calibration results using average emerging market income process and stochastic bargaining power.

F.3 Comparison with Models of Exogenous Public Capital and Endogenous Aggregate Capital

Moment Statistics Table F3 reports non-target statistics in quantitative analysis results of (a) a model of exogenous public capital (Arellano and Bai 2017; Cuadra et al. 2010) and (b) a model of endogenous aggregate capital (Gordon and Guerron-Quintana 2018; Park 2017; Galli 2021) in columns 4–5 and 6–7, respectively. We introduce additional specific features and parameter values, in our baseline model of endogenous public capital. We contrast those with data (column 1) and moment statistics in calibration results of our baseline model of endogenous public capital (columns 2–3).

First of all, neither the models of exogenous public capital nor endogenous aggregate capital replicate lower average public investment in the renegotiation periods than that in the pre-default periods as observed in the data. The model of exogenous public capital generates both higher average public investment and higher investment share in public expenditure in the renegotiation periods than that in the pre-default periods (4.36 vs. 2.16 percent of GDP for model with recovered debt payments in cash at settlement; 4.09 vs. 2.07 percent of GDP for model without recovered debt payments in cash at settlement). This is because of fixed (constant) public investment level and endogenous output dynamics. The model of endogenous aggregate capital does not generate any dynamics of public investment, only those of aggregate investment.

Second, neither the models of exogenous public capital nor endogenous aggregate capital replicate a negative relationship between a decline in public investment and duration, or a positive relationship between a recovery in public investment and duration as observed in the data. This is because in the model of exogenous public capital, public investment remains fixed (unchanged), and as a result, there is no relationship between public investment dynamics and restructuring duration. Again, the model of endogenous aggregate capital does not generate any dynamics of public investment, only those of aggregate investment.

Third, both models of exogenous public capital and endogenous aggregate capital replicate only short restructuring duration (4.8 and 4.5 quarters). Sections 6.1 and 6.2 provide decomposition of restructuring delays in both models. In the model of exogenous public capital, there is no “public capital accumulation delay” and only “productivity recovery delay” contributes to total restructuring delays of 4.8 quarters. In the model of endogenous aggregate capital, aggregate capital accumulation delays account for only 0.8 quarters, resulting in shorter restructuring delays of 4.5 quarters in total.

Fourth, we generate lower average recovery rate (higher average haircut) for the model of exogenous public capital with recovered debt payments in cash at settlement than for those without recovered debt payments in cash at settlement (24.5 vs. 42.0 percent). In the models of endogenous and exogenous aggregate capital (columns 6–7), we generate lower average recovery rate (higher average haircut) of 24.1 and 15.0 percent.

As in conventional models in the literature, both models of exogenous public capital and endogenous aggregate capital replicate private sector business cycle moments such as both volatile private consumption and trade balance. Furthermore, both models replicate high average debt-to-GDP ratio in both pre-default and restructuring periods, average and standard deviation of bond spreads, and countercyclical bond spreads due to long-term debt (Hatchondo and Martinez 2009; Chatterjee and Eyigungor 2012) as observed in the data.

Simulated Stylized Facts Figure F2 shows simulation results on restructuring duration, haircuts, public investment, public consumption and transfers for models of exogenous public capital and endogenous aggregate capital. For both models, post-default restructurings are settled with recovered

debt payments in cash at settlements.

First, the model of exogenous public capital in which restructurings are settled with recovered debt payments in cash account for two facts such as first fact—a large number of restructurings with recovered debt payments in cash at settlement (panel (i)-a) and fourth fact—a short-lived decline and a quick recovery of public consumption and transfers (panel (iii)-a). On the contrary, the model does not replicate any of second, third, and fifth facts. The second fact—higher haircuts and longer duration for restructurings with recovered debt payments in cash at settlement—is not replicated (panel (i)-a) because the model generates only short restructuring duration with average of 4.8 quarters. Neither the third fact—a severe decline and a slow recovery of public investment—nor fifth fact—an association between public investment dynamics and restructuring delays—are replicated because public investment remains unchanged.

Second, the model of endogenous aggregate capital in which restructurings are settled with recovered debt payments in cash account for first fact—a large number of restructurings with recovered debt payments in cash at settlement (panel (i)-b). On the contrary, the model of endogenous aggregate capital does not replicate any of remaining four facts. The second fact—higher haircuts and longer duration for restructurings with recovered debt payments in cash at settlement—is not replicated (panel (i)-b) because the model generates only short restructuring duration with average of 4.5 quarters. None of the third, fourth and fifth facts are replicated because the model of endogenous aggregate capital does not replicate dynamics of public investment or public consumption and transfers, but generates dynamics of only aggregate investment or consumption. Even focusing on aggregate investment, panel (ii)-b and panel (v) show that the model (purple dashed line) generates a sharp decline of aggregate investment at the onset of the restructuring, but not a slow recovery of aggregate investment, and that there is no association between aggregate investment dynamics and restructuring delay. Focusing on aggregate consumption, panel (iii)-b shows that the model (purple dashed line) replicates a small decline and a quick recovery of aggregate consumption.

Table F3: Moment Statistics from Simulation Results in Models of Public Capital and Aggregate Capital

(i) Public Sector Business Cycle Statistics							
	Data	Endogenous Public Capital		Exogenous Public Capital ^{1/}		Endogenous Aggregate Capital ^{2/}	Exogenous Aggregate Capital ^{3/}
	Recovered Debt Payment in Cash	Recovered Debt Payment in Cash	No Recovered Debt Payments in Cash	Recovered Debt Payment in Cash	No Recovered Debt Payments in Cash	Recovered Debt Payments in Cash	Recovered Debt Payments in Cash
Target statistics							
Pre-default periods							
Average public consumption & transfers/GDP ratio (%)	22.5	22.5	24.7	22.9	23.2	-	-
Public investment (std. dev.)/output (std. dev.)	2.7	2.41	2.90	-	-	-	-
Non-target statistics							
Pre-default periods							
Public sector							
Public consumption & transfers (std. dev.)/output (std. dev.)	1.26	1.10	1.01	1.21	2.60	-	-
Corr.(public consumption & transfers, output)	0.77	0.88	0.78	0.36	0.77	-	-
Average public investment/GDP ratio (%)	1.31	2.30	2.72	2.16	2.07	-	-
Average public investment/public expenditure ratio (%)	6.2	9.3	12.2	7.4	7.1	-	-
Renegotiation periods							
Public sector							
Public consumption & transfers (std. dev.)/output (std. dev.)	0.99	1.43	2.80	0.80	4.81	-	-
Corr.(public consumption & transfers, output)	0.97	0.79	0.10	0.77	0.74	-	-
Average public consumption & transfers/GDP ratio (%)	20.2	22.5	23.2	20.6	20.6	-	-
Average public investment/GDP ratio (%)	1.19	1.85	1.33	4.36	4.09	-	-
Average public investment/public expenditure ratio (%)	5.7	8.2	5.7	16.9	15.9	-	-

(ii) Private Sector Business Cycle Statistics							
	Data	Endogenous Public Capital		Exogenous Public Capital ^{1/}		Endogenous Aggregate Capital ^{2/}	Exogenous Aggregate Capital ^{3/}
	Recovered Debt Payment in Cash	Recovered Debt Payment in Cash	No Recovered Debt Payments in Cash	Recovered Debt Payment in Cash	No Recovered Debt Payments in Cash	Recovered Debt Payments in Cash	Recovered Debt Payments in Cash
Pre-default periods							
Private sector							
Private consumption (std. dev.)/output (std. dev.)	1.11	1.00	1.01	1.00	1.00	1.22	1.01
Trade balance/output: std. dev. (%)	1.28	0.91	0.76	1.20	0.70	1.21	1.10
Corr.(trade balance, output)	-0.87	-0.12	-0.03	-0.05	-0.44	-0.07	-0.10
Renegotiation periods							
Private sector							
Private consumption (std. dev.)/output (std. dev.)	1.17	1.49	0.99	1.00	1.01	1.50	1.00
Trade balance/output: std. dev. (%)	0.45	0.00	0.00	0.00	0.00	0.00	0.00
Corr.(trade balance, output)	-0.97	0.00	0.00	0.00	0.00	0.00	0.00

(iii) Non-business Cycle Statistics							
	Data	Endogenous Public Capital		Exogenous Public Capital ^{1/}		Endogenous Aggregate Capital ^{2/}	Exogenous Aggregate Capital ^{3/}
	Recovered Debt Payment in Cash	Recovered Debt Payment in Cash	No Recovered Debt Payments in Cash	Recovered Debt Payment in Cash	No Recovered Debt Payments in Cash	Recovered Debt Payments in Cash	Recovered Debt Payments in Cash
Target statistics							
Default probability (%)	3.50	3.80	3.20	3.05	3.98	3.01	3.05
Bond spreads: average (%)	7.2	8.2	7.9	8.5	9.8	8.9	8.6
Bond spreads: std. dev. (%)	3.40	3.20	4.70	4.50	7.6	4.9	5.5
Pre-default periods							
Average debt/GDP ratio (%)	32.6	55.4	38.0	43.3	46.7	86.5	35.1
Corr.(output, spread)	-0.88	-0.20	-0.35	-0.12	-0.69	-0.19	-0.11
Corr.(debt/GDP, spreads)	0.92	0.20	0.27	0.22	0.73	0.32	0.86
Renegotiation periods							
Average debt/GDP ratio (%)	109.6	88.7	75.8	80.4	104.4	90.0	96.5
Average duration of renegotiations (quarters)	14.6	11.5	5.0	4.8	2.9	4.5	3.7
Average recovery rate (%)	25.0	22.2	51.0	24.5	42.0	24.1	15.0
Corr.(decline in public investment, duration) ^{4/}	-0.25	-0.10	-0.52	-	-	-	-
Corr.(recovery in public investment, duration) ^{5/}	0.22	0.58	0.14	-	-	-	-

Sources: Datastream, IMF WEO, INDEC and MECON.

Notes: 1/ Model with exogenous public capital corresponds to our baseline model of public capital (with the same parameter values) in which public capital is exogenously fixed at the mean level.

2/ Model with endogenous aggregate capital corresponds to our baseline model (with the same parameter values) in which the aggregate capital income share is assigned and there is no distortionary taxation (and lump-sum taxation).

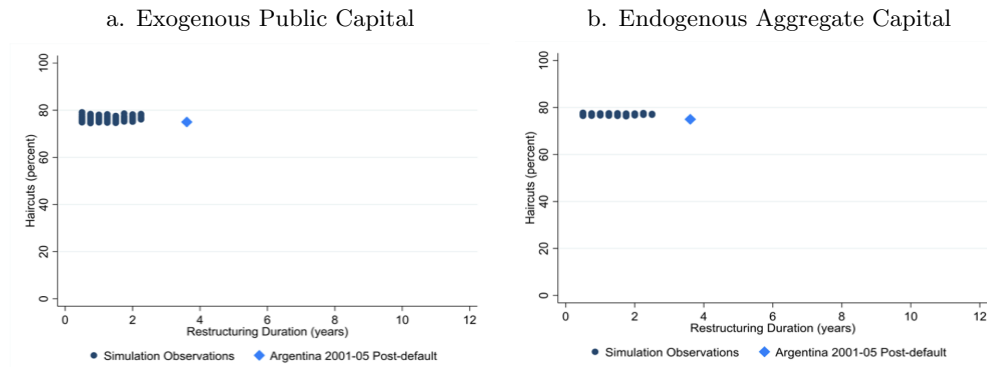
3/ Model with exogenous aggregate capital corresponds to our baseline model (with the same parameter values) in which the aggregate capital income share is assigned, aggregate capital is exogenously fixed at the mean level, and there is no distortionary taxation (and lump-sum taxation).

4/ Decline in public investment is measured in percentage change of public investment from level in -4 (quarter) to the lowest level, i.e., the level at end of declining trend.

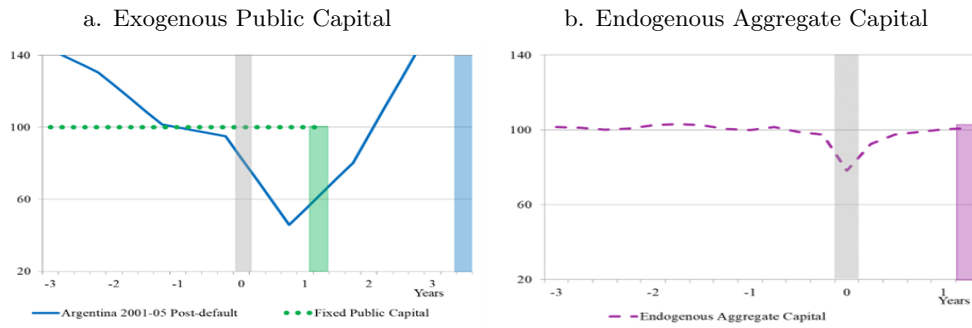
5/ Recovery in public investment is measured in length of time (years) from the time which public investment is at the lowest level to the time which it recovers to the pre-crisis average.

Figure F2: Restructuring Duration, Haircuts, Public Investment, Consumption and Transfers in Models of Public Capital and Aggregate Capital

(i) Restructuring Duration and Haircuts



(ii) Public Investment and Aggregate Investment



(iii) Public Consumption and Transfers and Aggregate Consumption

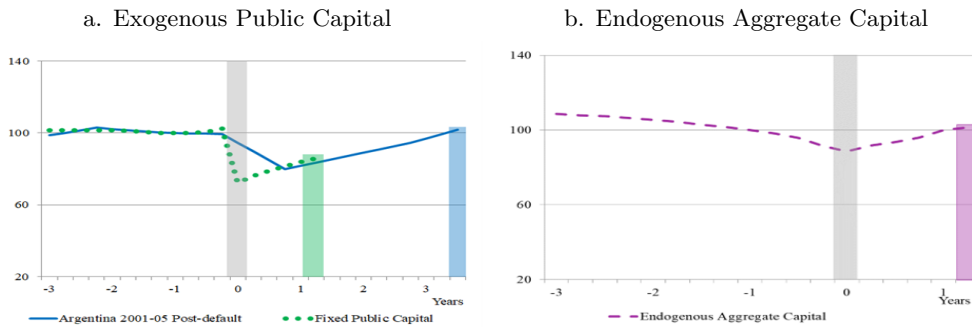
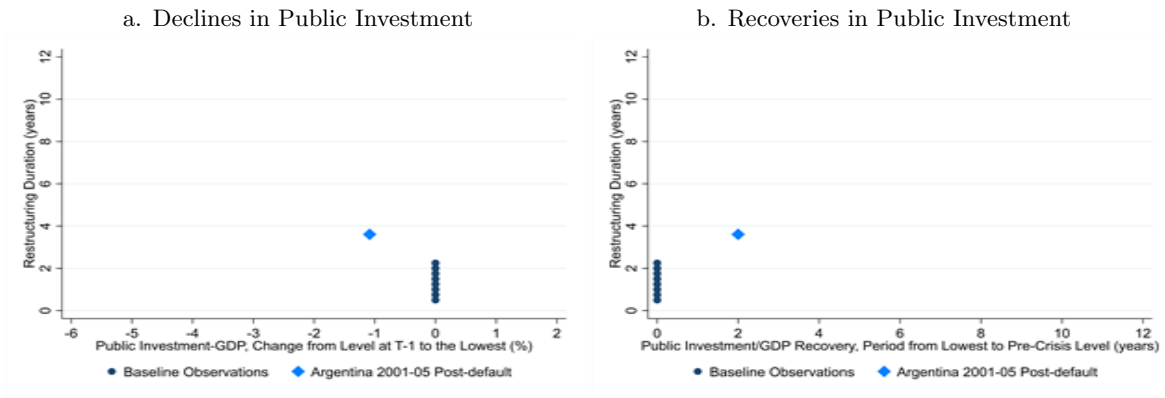
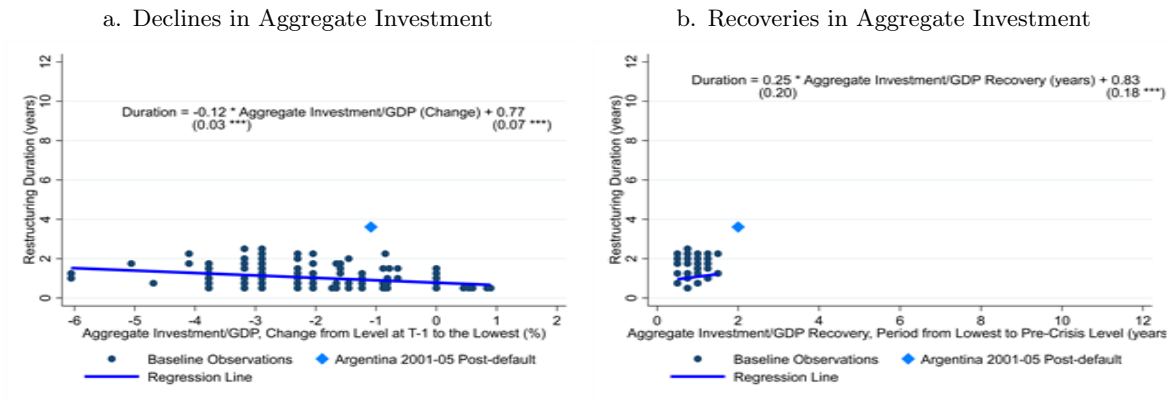


Figure F2: Restructuring Duration, Haircuts, Public Investment, Consumption and Transfers in Models of Public Capital and Aggregate Capital (cont.)

(iv) Exogenous Public Capital: Restructuring Duration and Public Investment



(v) Endogenous Aggregate Capital: Restructuring Duration and Aggregate Investment



F.4 Sensitivity Analysis

Table F4: Sensitivity Analysis—Baseline Model without Recovered Debt Payments in Cash

	Adjustment Costs			Depreciation Rate			Risk Aversion		
	10	14	20	0.025	0.04	0.075	2	3	4
Default probability (%)	4.50	3.20	3.30	3.80	3.20	3.25	3.45	3.20	3.77
Public investment (std. dev.)/output (std. dev.)	5.9	2.90	2.55	3.58	2.90	3.10	2.76	2.90	4.01
Non-target statistics									
Pre-default periods									
Average public investment/GDP ratio (%)	3.80	2.72	2.56	2.00	2.72	4.05	4.30	2.72	1.96
Average public investment/public expenditure ratio (%)	18.5	12.2	11.8	10.1	12.2	15.5	19.8	12.2	9.7
Average debt/GDP ratio (%)	58.0	38.0	35.6	35.4	38.0	35.5	35.6	38.0	35.7
Renegotiation periods									
Average public investment/GDP ratio (%)	0.90	1.33	1.30	1.05	1.33	3.95	3.50	1.33	0.95
Average public investment/expenditure ratio (%)	4.80	5.7	5.6	4.45	5.7	14.5	11.1	5.7	4.65
Average debt/GDP ratio (%)	81.4	75.8	70.5	65.9	75.8	90.4	65.4	75.8	68.7
Average duration of renegotiations (quarters)	3.90	5.0	5.7	2.2	5.0	4.7	2.6	5.0	2.2
Average recovery rate (%)	40.3	51.0	50.4	51.6	51.0	50.2	53.3	51.0	45.6
Corr.(decline in public investment, duration)	-0.34	-0.52	-0.40	-0.02	-0.52	-0.07	-0.07	-0.52	-0.03
Corr.(recovery in public investment, duration)	0.05	0.14	0.17	0.03	0.14	0.08	0.06	0.14	0.04

Source: Authors' computation