A Labor Market Sorting Model of Scarring and Hysteresis*

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Abstract

We develop a theory of sorting and learning in a labor search model with firm and worker heterogeneity, aggregate risk, life-cycle dynamics, and match-dependent human capital accumulation. We estimate the model on administrative matched employer-employee data. We show that following a negative productivity shock the scarring effects on workers' careers reflect not only a worsening of workers' outside options in bad times, but also a reduction in the overall productive capacity of the economy. Recessions thus have a sullying effect. Decomposing the cumulative output loss after a negative shock, we show that worsening in worker-firm sorting and distortions in human capital accumulation explain more than 60% of the loss in aggregate output. The model is then also used to offer a new explanation for the increased length of recessions and their heterogeneous effects across the age, income and human capital distributions.

Keywords: Economic hysteresis, Business cycle fluctuations, Human capital accumulation, Labor market sorting, Labor market scarring

JEL codes: J24, J63, E24, E32

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1 Introduction

Recessions are known to impact workers' careers because of increased displacements and reduced career opportunities (Kahn 2010; Oreopoulos, Von Wachter and Heisz 2012; Huckfeldt 2022). Displacements, in particular, have strong consequences for future earnings and job quality at the worker level (Jacobson, LaLonde and Sullivan 1993; Schmieder, von Wachter and Heining 2022; Jarosch 2023). At the same time, downturns could free up resources and reallocate inputs towards more efficient uses at the aggregate level (Davis, Haltiwanger and Schuh 1996; Baley, Figueiredo and Ulbricht 2022).

It is then natural to ask whether recessions are associated with *hysteresis*, the persistence of depressed aggregate conditions *sullying* the productivity distribution (Barlevy 2002; Haltiwanger et al. 2022), or if on the contrary they generate *cleansing* effects through efficient reallocation. To answer this question, we build a directed search model that embeds an overlapping generations structure, firm and worker heterogeneity, flows in and out of employment and out of the labor force, search on the job, match-dependent human capital accumulation, and aggregate risk. Workers are risk averse and heterogeneous in their human capital level and in their ability to accumulate it on the job. Firms are heterogeneous depending on their productivity level, chosen by risk neutral entrepreneurs while opening vacancies. We estimate the model using large-scale administrative data and show that it yields a realistic characterization of life-cycle earnings profiles, job ladders, and workers' cross-sectional earnings dynamics. The joint modeling of these features provides us with a unique novel framework to understand the interaction of labor market sorting that is, the degree to which more-productive workers are assigned to more-productive firms, with output fluctuations (Lise and Robin, 2017, Herkenhoff, Phillips and Cohen-Cole, 2019, Carrillo-Tudela and Visschers, 2023).

While recessions persistently decrease the labor share through displacements and a deterioration of workers' outside options, we also find a more general sullying effect on the productivity distribution, persistently depressing firm quality and human capital levels. We show that the interaction of labor market sorting and business cycle fluctuations is a key determinant of the persistence of aggregate shocks, generating earnings and output hysteresis. At an intuitive level, this is because recessions flatten the job ladder as returns to career progression are dampened by worse labor market conditions and outside options. The flattening of the job ladder has persistent effects as it compounds the difficulty of opening of the most productive vacancies, which is highly cyclical, with the scarcity of higher human capital workers during recessions. As human capital accumulation depends on firm quality, these effects hinder workers' ability to climb the ladder and prolong the negative effects of recessions.

A growing body of literature argues that firm quality is a major determinant of workers' human capital accumulation and skill development (Herkenhoff et al. 2018; Lise and Postel-Vinay 2020; Arellano-Bover and Saltiel 2022). In our model, firms produce according to a technology that yields increasing returns in both worker and firm productivity. The value of matches thus depends not only on promised earnings but also on the implied human capital accumulation path during the worker's tenure at the firm. We assume human capital production depends on the match between workers and firms so that human capital is accumulated at a faster pace in jobs at more-productive firms, which are harder to obtain. Workers therefore, both when employed and when unemployed, direct their search toward firms that are heterogeneous in productivity, trading off the value of each vacancy with the likelihood of matching.

We model the worker-firm relationship as a fully state-contingent wage protocol that splits the match surplus. Profit-maximizing firms design a menu of contracts to attract and retain the most-productive workers. Workers' risk aversion creates a demand for insurance from fluctuations of match value, as in Holmstrom (1983), Balke and Lamadon (2022), and Souchier (2022). The resulting state-contingent contract backloads compensation to maximize retention for the firm, while partially insuring workers against idiosyncratic and aggregate shocks. Because workers accumulate human capital on the job, incentives to search on the job vary over job tenure, and so does the wage growth within the match. Given risk aversion, firms' human capital production function, and the incentives embedded in the contract, we prove that the model economy features a monotonically increasing mapping in the search strategies of workers toward firms in equilibrium depending on human capital and firm productivity. We also use the framework to show that the severity of recession is state-contingent (Dupraz, Nakamura and Steinsson, 2022), with fluidity in job flows (Engbom, 2021) and characteristics of the labor share distribution (Ai and Bhandari, 2021) being its most important determinants.

To capture the role of cyclical displacements, we incorporate a two-sided commitment friction that generates inefficient separations. A sizeable literature identifies substantial resistance to nominal or real wage cuts (Altonji and Devereux 1999; Agell and Lundborg 2003; Grigsby, Hurst and Yildirmaz 2021). Downward wage rigidity limits the ability of firms to reduce losses when they are hit with aggregate or idiosyncratic shocks, which can lead them to become insolvent and lay off workers in bad times. The model is then able to reproduce empirical separation patterns across age, worker, and firm productivity, and matches the correlation of separations with business cycles. Whenever aggregate conditions change, firms respond endogenously by adjusting their respective wage schedules and job openings while workers optimally adjust their quit and search policies. The resulting pattern of wage growth, matches, and separations is consistent with the data and reproduces a host of results from the empirical labor literature on scarring effects on workers careers (Schwandt and von Wachter 2019, Schmieder, von Wachter and Heining 2022, Bertheau et al. 2023).

The dynamics of sorting is central to our analysis of human capital accumulation, career paths and real output fluctuations. Recessions impact labor market sorting primarily by forcing the dissolution of non-viable worker-firm matches. We show that the model replicates the empirical regularity of strongly counter-cyclical separation patterns and is able to generate realistic volatility in both unemployment and output. Because of search frictions, changes in sorting can take a long time to revert, which in turn scars workers' careers and human capital accumulation opportunities. In bad times, the reallocation of workers across firms is weaker. Firstly, because job-to-job transitions are strongly pro-cyclical. Secondly, because the quality of jobs workers switch to when moving is strongly pro-cyclical as well. Workers' degree of risk aversion plays an important role in the quality adjustment of their search behavior.

Uniquely in the literature, the richness of the model allows us to evaluate the implications of the interactions between aggregate fluctuations and labor market sorting along multiple dimensions. In particular, we can quantify the forces that shape aggregate dynamics around recessions. To this end, we analyze the effects of output contractions through a decomposition of displacement, sorting and human capital channels. In the model, the amount of displacements constitutes the main driver of output dynamics at the onset of a recession and significantly correlates with the depth of a recession's trough. Sluggish employment and output recoveries - hysteresis - depend instead on a worsened worker-firm sorting, whereby workers match with firms of lower quality and thus form less productive matches. This in turn leads to lower human capital accumulation. From a decomposition of the cumulative output loss two years after a negative productivity shock, we show that approximately 48% of losses stems from a worsened sorting between workers and firms, with adjustments to workers' search accounting for approximately 35%. Human capital accumulation instead accounts for approximately 17%. Taken together, therefore, these channels explain more than 60% of the cumulative loss in aggregate output. The outcome of this exercise shows that recessions have a sullying effect on the economy (Barlevy, 2002): distortions in sorting and human capital accumulation lead to a persistent deterioration of matches' quality after a recession.

An important ingredient of the model is that workers are risk-averse. Departing from risk neutrality allows also us to address distributive concerns between labor and capital, and to make meaningful welfare considerations. We estimate the welfare cost of business cycles, that is the amount of utility agents would be willing to give away in order to eliminate aggregate risk, to be two orders of magnitude above what traditionally estimated by Lucas (1987) and broadly in line with more recent work (Barlevy, 2004). Business cycle costs are markedly heterogeneous by income and age. We find them to be roughly U-shaped in income, with the greatest costs at the extremes of the income distribution. For low–skill workers, on average, the displacement effect prevails, which explains the high costs faced by workers with low wages. For higher income workers, instead, the distortions in sorting and human capital accumulation matter more. Business cycle costs also differ by age, and workers above 40 years old suffer more than their younger counterparts. These workers have a harder time obtaining re-employment after displacement, and can be left with few options beyond early retirement. These results for welfare echo and extend the results for earnings found by Heathcote, Perri and Violante (2020).

The heterogeneity of responses across the age distribution provides additional insight into the potential presence of cleansing dynamics. In fact, for a subset of young workers recessions force a transition into unemployment, and lead towards the formation of new matches with better firms after unemployment. The wage contract in the model implies that these workers would be able to obtain large wage gains if not separated, and that after the recession their earnings increase less than their human capital growth would imply, because of a worse bargaining position. However, their search strategy does not internalize the benefit to their future employers of increased human capital accumulation after moving jobs. This highlights the main channel through which a cleansing effect might operate, and clarifies why for older workers, who are less subject to human capital accumulation externalities, the sullying effect dominates. In this vein, the model can be used to provide an explanation for the secular increase in the length of recoveries from recessions across all developed economies as in Fukui, Nakamura and Steinsson (2023). We show that the entirety of the increase in the length of recoveries from downturns, as regard the cyclical component of output, can be attributed to the greater prevalence in the labor market of higher skill workers and the progressive aging of the population, with older workers less able to reallocate themselves in the labor market after a negative shock.

Related literature. The paper is related to a large body of empirical and theoretical studies that analyze the importance of aggregate fluctuations for workers' career and earning dynamics. Our model of the labor market builds on the directed search models developed in Menzio and Shi (2010), Menzio, Telyukova and Visschers (2016). We also adopt a wage setting protocol taken from the literature on dynamic contracts as in Thomas and Worral (1988) and, more recently, Balke and Lamadon (2022). In both cases, we extend their framework to explicitly account for workers' risk aversion, on-the-job human capital accumulation and aggregate risk to study how business cycles interact with workers' decisions, firms' optimal retention policies, and influence labor market sorting.

Our focus on human capital accumulation and the centrality of worker-firm sorting in busyness cycle dynamics is closely related to Baley, Figueiredo and Ulbricht (2022). The paper develops a theory of cyclical mismatch in which workers progressively learn about their ability in specific tasks while on-the-job. In their setting, workers that are displaced find it optimal to redirect their search towards jobs that require different tasks compared to their previous employment, in which they discovered were not well versed. In our case, there is no uncertainty in workers and firms' qualities nor we impose ex-ante that the gains in human capital accrued while on the job are firm specifics. In both models, recessions increase the mismatch in the labor market, worsening worker-firm sorting. However, while in our model this is due to workers' risk-aversion and the equilibrium dynamics of sorting, in their model this is due to uncertainty about workers' abilities.

The importance of business cycles and separations for workers' career is also related to Jarosch (2023). In that study, workers progressively climb the job ladder to gain job security as jobs are characterized by different degrees of job security. Similarly to this setting, also in our case jobs are characterized by a different separation probabilities. However, in our model the separation probabilities, and hence job security, are endogenous objects that are determined by the productivity of both firms and workers and the contract that they decide to stipulate when the match is formed. In our setting, highly tenured workers in relatively unproductive firms are those endogenously more exposed to separation risks because they command an unsustainable share of the surplus.[1] In addition, we allow the human capital accumulation to depend on the match workers are in, while in his setting human capital evolves stochastically depending on workers' employment status.

We also contribute to a growing literature that investigates the relevance of firm qualities for workers' careers (as in Arellano-Bover, 2022, Arellano-Bover and Saltiel, 2022, Mion, Opromolla and Ottaviano, 2022) and the importance of different experiences for earning dynamics over the life cycle and across educational levels (see Roca and Puga, 2017, Deming, 2023). We add to these studies by developing a structural model that allows us to study the importance of on-the-job human capital accumulation for the economy's aggregate dynamics and at business cycle frequencies while matching micro-level facts on life-cycle earnings and labor market dynamics, a feature that is absent from this literature.

Outline. Section 2 develops our model; Section 3 illustrates the main theoretical properties of our framework; Section 4 presents our estimation results and evaluates the

[1] Given the model's treatment of pay dynamics within the worker-firm match, our research is also significantly related to a strand of literature in labor and finance analyzing the firms' management of liquidity and labor compensation dynamics (Xiaolan (2014); Favilukis, Lin and Zhao (2020); Acabbi and Alati (2022); Acabbi, Panetti and Sforza (2023)).

model's fit; Section 5 discusses the model's dynamics and the composition of output losses; Section 6 discusses additional results on recession lengths and the fragility of the economy. Section 7 concludes.

2 Model

2.1 Environment

Time is discrete, runs forever and is indexed by $t \in \mathbb{N}$. We denote future values in recursive expressions by adding a $'$ to them, or index elements by t in non-recursive ones.

The economy is populated by $T \geq 2$ overlapping generations of finitely lived, hand-to-mouth, risk-averse workers and a continuum of risk-neutral entrepreneurs. All agents in the economy share the same discount factor $\beta \in (0, 1)$. Each household lives for T periods, with age $\tau \in \mathcal{T} \equiv \{1, 2, 3, \ldots, T\}$. Workers are either employed, with value function W, or unemployed, with value function U.

Workers maximize lifetime flow-utility from non-durable consumption:

$$\mathbb{E}_{t_0} \left(\sum_{\tau=1}^{T} \beta^{\tau} u(c_{\tau, t_0+\tau}) \right),$$

where t_0 characterizes the time of entry into the labor market, and τ characterizes the age of the agent. $c_{\tau, t_0+\tau}$ thus refers to the consumption of workers of age τ in time $t_0 + \tau$.

Workers are characterized by heterogeneous human capital levels h, with $h \in \mathcal{H} \equiv [\underline{h}, \overline{h}]$. Workers are heterogeneous also with respect to their education level $\iota \in \mathcal{I} \equiv \{\text{g, s}\}$, which indicates college and high school education, respectively. Both types enter the labor market with a baseline level of human capital drawn from type-specific exogenous continuous distributions. Upon entry in the labor market, $\mathbb{E}[h|\text{g}] > \mathbb{E}[h|\text{hs}]$. To account for the different number of education years in the data, graduate workers entry to the labor market is delayed accordingly. Workers exit the labor force when their employment prospects deteriorate below a certain utility threshold. This allows us to distinguish long-term unemployment from non-participation.

Lise and Postel-Vinay (2020) and Xiaolan (2014) show that human capital accumulation can be heterogeneous across firms, and that the maximum level of human capital attainable at each firm is a key variable in determining workers' compensation. We model the human capital accumulation process assuming that workers' gain in human capital depends on the quality of the firm they are matched with and their own initial level of human capital, h. Firms are characterized by different levels of quality $y \in \mathcal{Y} \equiv [\underline{y}, \overline{y}]$, which is isomorphic to capital levels. Workers accumulate human capital only while employed and according to a law of motion that is match-specific: $h' = \phi(h, y, \iota, \psi) = g_\iota(h, y) + \psi$, $n : \mathcal{H} \times \mathcal{Y} \times \mathcal{I} \to \mathcal{H}$, where g_ι is the deterministic component of the human capital accumulation dynamics, and ψ constitutes the stochastic component. The function g_ι is concave in both its arguments. The deterministic component of human capital accumulation is akin to a "catching-up" of the firm's quality, up to a point when the worker will not be able to learn any more from the match.[2] The only difference between graduates and non-graduates (indexed by ι) is the speed of the "catching-up". Graduate workers are workers that can become more specialized over time, and will thus catch up faster.

Human capital accumulation is risky: at any period any employed worker is subject to the idiosyncratic human capital shock ψ, which enters additively with respect to the deterministic component.[3] The shock affects workers' ability and can amplify, shrink, or even reverse human capital accumulation. We further allow for the possibility that human capital deteriorates while workers are unemployed, according to an arbitrary process g_u.[4]

Firms are modeled as one worker–one job matches, thus abstracting from firm size. Each job match is characterized by a promised utility to the worker $V \in \mathcal{V}$. We group worker-specific characteristics in a tuple $\chi \in \mathcal{X} \equiv \{\mathcal{H} \times \mathcal{T} \times \mathcal{I}\}$. The aggregate state of the economy Ω is characterized by the productivity level $a \in \mathcal{A} \subseteq \mathbf{R}_0^+$ and by the distribution of agents across states $\mu \in \mathcal{M} : \{W, U\} \times \mathcal{Y} \times \mathcal{X} \times \mathcal{V} \to [0, 1]$. Let $\Omega = (a, \mu) \in \mathcal{A} \times \mathcal{M}$ represent the aggregate state of the economy and let $\mathcal{M}$ represent the set of distributions μ over the states of the economy. Let $\mu' = \Phi(\Omega, a')$ be the law of motion of the distribution. Aggregate productivity evolves as a stationary monotone increasing Markov process, namely $a' \sim F(a'|a) : \mathcal{A} \to \mathcal{A}$, with the Feller property.

2.2 Labor markets

Search is directed. Each labor market is organized as a continuum of submarkets indexed by the expected lifetime utility offered by firms of type y, $v_y \in \mathcal{V} \equiv [\underline{v}, \overline{v}]$. Workers are indexed by the tuple $\chi = (h, \tau, \iota)$.[5] The process of starting a firm, which amounts to posting a vacancy at a quality-specific cost $\kappa(y)$, will be described in **Section 2.4**.

The search process is characterized by a constant return to scale, twice continuously differentiable, matching function $M(u, \nu)$ for each submarket. The tightness of each

[2]Workers who match with a low-quality firm will see their ability deteriorating with the same g function.

[3]The additive nature of the shock keeps the properties of monotonicity and uniqueness of workers' search strategies unaltered, which is essential for tractability.

[4]This process might be without loss of generality deterministic or stochastic, and might or might not depend on current human capital h.

[5]As in Menzio and Shi (2010) the equilibrium will be separating. Given a menu of offers from any firm, each worker will visit only a particular submarket. For this reason submarkets can then be indexed directly by workers' current characteristics (see **Section 2.3**).

submarket in $\mathcal{X} \times \mathcal{V}$ is defined as $\theta = \nu/u$, with $\theta(\cdot) : \mathcal{X} \times \mathcal{V} \to \mathbf{R}_0^+$. Job finding rates are defined as $p(\theta(\cdot)) = M(u, \nu)/u$, where $p(\cdot) : \mathbf{R}_0^+ \to [0, 1]$ is a twice continuously differentiable, strictly increasing, and strictly concave function with $p(0) = 0$, $\lim_{\theta \to +\infty} p(\theta) = 1$ and $p'(0) < \infty$. The vacancy-filling probability is in turn defined as $q(\theta(\cdot)) = M(u, \nu)/\nu$, where $q(\cdot) : \mathbf{R}_0^+ \to [0, 1]$ is twice continuously differentiable, strictly decreasing, and strictly convex, with $q(0) = 1$, $\lim_{\theta \to +\infty} q(\theta) = 0$ and $q'(0) < 0$. Given these properties $q(\theta) = p(\theta)/\theta$, and $p(q^{-1}(\cdot))$ is concave.

Upon match, workers produce according to the twice-continuous increasing and concave production function $f(h, y; a) + x(a) : \mathcal{A} \times \mathcal{H} \times \mathcal{Y} \to \mathbf{R}_0^+$. The $x(a)$ component of the production function is a fixed cost, which can depend on the aggregate productivity realization.[6] Workers' compensation is determined by means of dynamic contracts through which firms deliver a promised utility, as described in **Section 2.5**.

Workers can search on the job with probability λ_e. Matches are destroyed at an exogenous rate λ_τ each period, with the exogenous separation rate possibly varying by age. Matches separate also if the worker is poached by another firm, if the worker voluntarily decides to quit and become unemployed (quits), or if the value of the match for the firm becomes negative (firings). Lastly, unemployed workers whose expected value of re-employment falls below a threshold $\underline{\mathrm{p}}$ are assumed to have permanently exited the labor force.

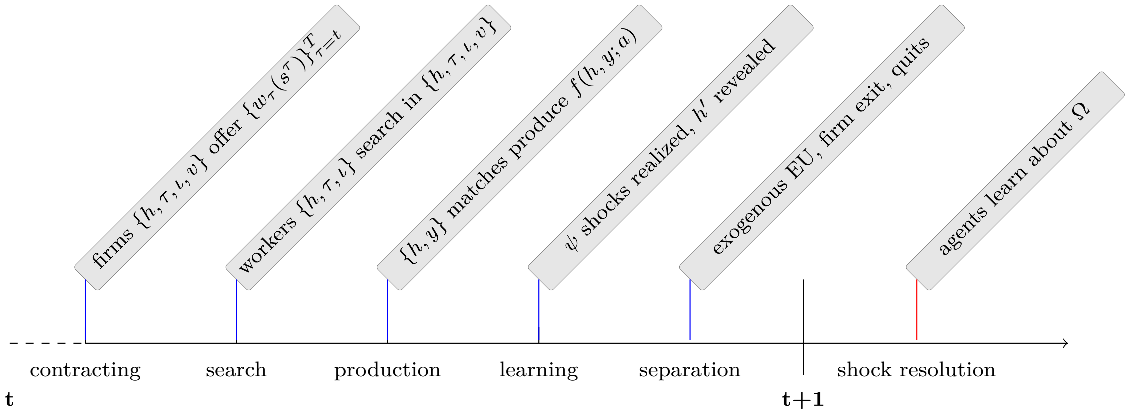
Timing is represented in Figure 1. At the beginning of each period an aggregate productivity shock is drawn; entrepreneurs open vacancies across submarkets and post their offers; workers search from unemployment or on-the-job, and move to a new job if the search is successful; production takes place; workers accumulate human capital depending on their employment status and idiosyncratic shock realization; an exogenous share of matches breaks down, while some firms endogenously exit and some workers quit.

2.3 Informational and contractual structure

Firms post fully state-contingent contracts. Each contract prescribes an action for each realization of the history of the worker–firm match. The state of a match at a generic time t is defined by $s_t = (h_t, \tau_t, \iota, a^t, \mu^t) \in \mathcal{S}^t = \mathcal{X} \times \mathcal{Y} \times \Omega^t = \mathcal{H} \times \mathcal{T} \times I \times \mathcal{Y} \times \Omega^t$, that is the worker skill, age, education, the history of aggregate productivity shocks, and workers' distributions across their employment history. A given history of realizations between t and k periods ahead is thus $s^{t+k} = (s_t, s_{t+1} ...., s_{t+k})$. The contract defines a transfer of utility from the risk-neutral firm to the risk-averse worker within the match for all future possible histories of shocks. We define τ_{t_0} as the age at which the worker is hired and

[6]This is a reduced-form way of incorporating financial frictions in the model, which make fixed costs loom larger over flow-production in downturns.

Figure 1. Timeline of Worker–Firm Match



T is the retirement age. The history of realizations between t_0, the time of hiring of the worker, and $t_0 + (T - \tau_{t_0})$, the time of maximum duration of the match with the worker before retirement, is thus $s^{t_0+(T-\tau_{t_0})}$.

Histories of workers and productivity shocks are common knowledge, and the future realizations of shocks are fully contractible. While the contract is state-contingent, markets are incomplete: workers' actions are private knowledge in the search stage, so firms are unable to directly counter outside offers. The contracts offered by firms are then defined as:

$$\mathcal{C}^{\tau_{t_0}} := (\mathbf{w}, \zeta) \text{ with } \mathbf{w} := \{w_t(s^{\tau_t - \tau_{t_0} + t_0})\}_{t=t_0}^{t_0+(T-\tau_{t_0})}, \text{ and } \zeta := \{v_t(s^{\tau_t - \tau_{t_0} + t_0})\}_{t=t_0}^{t_0+(T-\tau_{t_0})}. \tag{1}$$

Firms promise a series of state-contingent wages defined by the series of utility values v_t sought at each node of the history.[7] ζ is the action suggested by the contract, which is bound to be incentive compatible for the worker. The resulting relationship between workers and firms is characterized by a contract with forward-looking constraints. The state space of the worker problem can be expressed in terms of their current lifetime utility, as in Spear and Srivastava (1987), so as to avoid having to keep track of all past histories s^t at each period. The relevant state space is then $\mathcal{X} \times \mathcal{V}$.

2.4 Vacancy creation and free entry

The economy is populated by a continuum of risk-neutral entrepreneurs. Each entrepreneur can invest to reach the desired level of firm quality y. The start-up costs of the firm are priced in terms of the consumption good and they coincide with vacancy posting costs in the frictional labor market. The cost of each vacancy is positively

[7] Similarly to Menzio and Shi (2010), Tsuyuhara (2016), and Balke and Lamadon (2022), to guarantee that the problem is well behaved and the firm profit function is concave, the contract will require a two-point lottery, which specifies probabilities over the actions prescribed. We omit it here for conciseness.

related to the quality of the firm being created. To post a vacancy for the creation of a firm with quality y the entrepreneur must thus pay $c(y)$, a vacancy cost priced in terms of the consumption good. The vacancy cost function $c(y)$ is a strictly convex function of firm quality y.[8]

At a generic time t each entrepreneur chooses in which submarket to post the vacancy selecting a lottery over the offered utility W, which maps into the set of firms' qualities $y \in \mathcal{Y}$, and worker characteristics $(\chi, V) \in \mathcal{X} \times \mathcal{V}$.

We define $J(h, \tau, \iota, W, y; \Omega) \in \mathcal{X} \times \mathcal{V} \times \mathcal{Y} \times \Omega$ as the value function of a firm, which capitalizes all future profits from the match. As entrepreneurs choose the submarkets in which to open a vacancy, they face the following problem:

$$\Pi(h, \tau, \iota, W, y; \Omega) = \sup_{y,h,\tau,\iota,W} - c(y) + q(\theta(h, \tau, \iota, W; \Omega))[J(h, \tau, \iota, W, y; \Omega)] \tag{2}$$

Given perfect competition, free entry and the possibility for all entrepreneurs to choose *any* possible firm kind y, the expected profits from creating a vacancy are driven down to 0 in submarkets that actually open.[9] This translates into a free entry condition:

$$\Pi(h, \tau, \iota, W, y; \Omega) \leq 0 \;\text{ for } \forall\{h, \tau, \iota, W, y; \Omega\} \in \{\mathcal{X} \times \mathcal{V} \times \mathcal{Y} \times \Omega\} \tag{3}$$

Assuming that $q(\cdot)$ is invertible, the equilibrium tightness in each submarket is:

$$\theta(h, \tau, \iota, W; \Omega) = q^{-1}\left(\frac{c(y)}{J(h, \tau, \iota, W, y; \Omega)}\right). \tag{4}$$

2.5 Firm problem

Firms commit to the delivery of a utility value to workers, but exit when the present value of future profits becomes negative. Workers' limited commitment implies they will search for new jobs whenever they have the possibility to do so. Firms cannot observe poaching offers and thus cannot counteract them. The sequence of past histories s^t is common knowledge, and while the firm cannot observe any of actions of its workers, it has enough information to internalize their optimal search policy decisions.

Define $\widetilde{p}(\chi, V; \Omega)$ as the optimal retention function and $\widetilde{r}(\chi, V; \Omega)$ as the optimal utility return from workers' solution to the on-the-job search problem. The value function of an incumbent firm y in state $(h, \tau, \iota, W_y; \Omega)$ can be written recursively using

[8] We assume that entrepreneurs can borrow from risk-neutral, deep-pocketed financiers to finance the vacancy. In Herkenhoff (2019) this assumption implies the cost of credit for entrepreneurs to coincide with the risk-free rate.

[9] Notice that in this case the expectation does not refer to realizations of the aggregate state Ω or the human capital shock ψ, but to the vacancy-filling probability q.

the promised utilities as additional state variables as:

$$
\begin{aligned}
J(h,&\tau,\iota,W,y;\Omega) = \\
&\sup_{\pi_i,\{w_i,W_i'\}} \sum_{i=1,2} \pi_i \Bigg(f(y,h;a) - w_i \\
&+\beta\mathbb{E}_\psi\left[\max\left\{0,\mathbb{E}_\Omega\left[\widetilde{p}(h',\tau+1,\iota,W_i';\Omega')J(h',\tau+1,\iota,W_i',y;\Omega')\right]\right\}\right]\Bigg)
\end{aligned} \tag{5}
$$

$$
s.t.\ W = \sum_{i=1,2} \pi_i \left(u(w_i) + \beta\mathbb{E}_{\Omega,\psi}\left(\widetilde{r}(h',\tau+1,\iota,W_i';\Omega')\right)\right), \tag{6}
$$

$$
\sum_{i=1,2} \pi_i = 1, \tag{7}
$$

where **Equation** (6) is the promise keeping constraint ensuring that the current value of the contract is based on the current wage and future utility promises with $\widetilde{r}_t(\cdot)$. The firm (the principal) chooses the wage(s) to be offered in the current period w_i, the utility promises W_i' and the probability π_i in the two-point lottery. The optimization implicitly takes into account the utility of workers (the agent) and their incentive compatible best replies, through the retention probability $\widetilde{p}(\cdot)$ and the expected utility gain $\widetilde{r}(\cdot)$. The continuation value for an incumbent firms allows for the possibility of dissolving the match if its value falls below zero.

Incumbent firms make their exit decisions before the realization of aggregate productivity but after the realization idiosyncratic human capital shocks for the next period.[10] At the beginning of a period they already know whether they will exit. Exit is therefore completely determined by the current state and can be summarized by a threshold for the aggregate productivity level and can be described as follows.

Definition 2.1 (Exit policy)**.** *The following indicator takes a value of one if the firm does not decide to exit in the following period:*

$$
\eta(h,\tau,\iota,W,y;\Omega,\psi) = \begin{cases} 1 & \textit{if } a \geq \max\{0,a^*\} \\ 0 & \textit{otherwise} \end{cases}
$$

with the productivity threshold defined as

$$
a^*(h,\tau,\iota,W,y;\Omega,\psi) : \mathbb{E}_\Omega[J(h',\tau+1,\iota,W',y;\Omega')] = 0. \tag{8}
$$

[10]This amounts to having the firm making a state-contingent exit decision in advance of the idiosyncratic shock's realization as in Gomes (2001) and Xiaolan (2014).

2.6 Worker problem

Given current lifetime utility V, job seekers with characteristics χ have to decide in which submarket to direct their search. Submarkets are indexed by worker type χ and by offered utility v associated to firms' posted vacancies. As discussed in **Section 2.4**, the choice over v will also indirectly determine which kind of firm y the worker matches with, and thus the implied human capital accumulation path. For now, let us assume this (conditional) mapping exists. This amounts to assuming that the function $v(y;\chi,V)$ is an injective function $f_v : \mathcal{Y} \times \mathcal{X} \times \mathcal{V} \to \mathcal{V}$. Upon observing a job offer with utility v, a worker χ with current utility V will be able to infer which firm type y is posting the offer.

A worker of type (χ, V) that enters the search stage has lifetime utility $V + \max\{0, R(\chi, V;\Omega\}$, where the second component of the expression embeds the option value of the search, with R being the search value function. R is defined as:

$$R(\chi, V;\Omega) = \sup_{v} \Big[p(\theta(\chi, v;\Omega))\big[v - V\big]\Big]. \tag{9}$$

We denote the solution of the search problem as $v^* = v^*(\chi, V;\Omega)$, and $p^*(\chi, v^*;\Omega) = p(\theta(\chi, v^*;\Omega))$ as the associated optimal job-finding probability. The lifetime utility of an unemployed worker at the beginning of the production stage can be defined as

$$\begin{aligned} U(h,\tau,\iota;\Omega) = \ & u(b(h,\tau)) + \beta\mathbb{E}_{\Omega,\psi}\bigg(U(h',\tau+1,\iota;\Omega') \\ & + \max\{0, R(h',\tau+1,\iota, U(h',\tau+1,\iota;\Omega');\Omega')\}\bigg), \end{aligned} \tag{10}$$

where $b(h,\tau)$ is a skill and age dependent unemployment benefit. Given finite workers' lives, $U(h,\tau,\iota;\Omega) = 0 \ \forall(\chi;\Omega) \in \mathcal{X} \times \Omega$ whenever $\tau > T$. The corresponding lifetime utility of a worker employed at firm y, with human capital h, age τ, education ι and promised utility V at the beginning of production stage can be expressed as:

$$\begin{aligned} V(h,\tau,\iota;\Omega) = \ & u(w) + \beta\mathbb{E}_{\Omega,\psi}\bigg(\lambda_\tau U(h',\tau+1,\iota;\Omega') + \ (1-\lambda_\tau)\Big[V(h',\tau+1,\iota;\Omega') \\ & + \ \lambda_e \max\{0, R(h',\tau+1,\iota, V(h',\tau+1,\iota;\Omega');\Omega')\}\Big]\bigg), \end{aligned} \tag{11}$$

where w is the promised wage and $V(h',\tau+1,\iota;\Omega')$ is next period's state-contingent promised utility of remaining in the current firm, which becomes the outside option in the search problem.[11] Firms internalize incentives embedded in workers' strategies and

[11] It is here implied that, in case there is an endogenous separation, this future promised value is

post wages and utility offers to maximize profits by optimizing retention. This way, future promised utilities incorporate both future wages or option values of search.

The policy functions are uniquely defined and allow us to identify target y as long as the injective mapping between the offered utility v and y given χ exists.[12]. The solution of employed workers' on-the-job search problem defines a search policy function. In turn, this policy function leads to the definition of two equilibrium objects, which firms internalize in their optimization in order to incorporate workers' incentive compatibility.

Definition 2.2 (Optimal retention probability and utility return)**.** *The solution to the worker's problem defines a retention function* $\widetilde{p} : \mathcal{X} \times \mathcal{V} \times \Omega \to [(1-\lambda)(1-\lambda_e), 1-\lambda]$ *and a utility return* $\widetilde{r} : \mathcal{X} \times \mathcal{V} \times \Omega \to \mathcal{V}$*:*

$$\widetilde{p}(\chi, V; \Omega) \equiv (1-\lambda_\tau)(1-\lambda_e p^*(\chi, v^*; \Omega)) \tag{12}$$

$$\widetilde{r}(\chi, V; \Omega) \equiv \lambda_\tau U(\chi; \Omega) + (1-\lambda_\tau)\Big[V + \lambda_e \max\{0, R(\chi, V; \Omega)\}\Big] \tag{13}$$

2.7 Equilibrium definition

Recursive Equilibrium. Let $\Theta = \mathcal{A} \times \mathcal{M} \times \mathcal{H} \times \mathcal{T} \times \mathcal{I}$. A recursive equilibrium in this economy consists of a market tightness $\theta : \Theta \times \mathcal{V} \to \mathbb{R}_+$, a search value function $R : \Theta \times \mathcal{V} \to \mathbb{R}$, a search policy function $v^* : \Theta \times \mathcal{V} \to \mathcal{V}$, an unemployment value function $U : \Theta \to \mathbb{R}$, a firm value function, $J : \Theta \times \mathcal{V} \times \mathcal{Y} \to \mathbb{R}$, a series of contract policy functions $\{c_\tau\}_{\tau=1}^{T} : \mathcal{S}^\tau \times \mathcal{Y} \to \mathcal{C}^\tau$, an injective mapping between firm qualities and promised utilities at hiring $f_v : \mathcal{X} \times \mathcal{V} \times \mathcal{Y} \to \mathcal{V}$, an exit threshold for aggregate productivity $a^* : \mathcal{X} \times \mathcal{V} \times \mathcal{Y} \to \mathcal{A}$, a human capital accumulation process $\phi(h, y, \iota, \psi)$, $\mathcal{H} \times \mathcal{Y} \times \mathcal{I} \times \Psi \to \mathcal{H}$, and a law of motion for the aggregate state of the economy $\Phi_{\Omega,a} : \mathcal{A} \times \mathcal{M} \to \mathcal{A} \times \mathcal{M}$ such that:

1. Given the mapping f_v, market tightness satisfies **Equation** (4).

2. The unemployment value function solves **Equation** (10).

3. Search value functions solve the search problem in **Equation** (9) and v^* is the associated policy function.

4. Firm value functions and associated contract policy functions solve **Equation** (5) for each $t \leq T$.

5. The exit threshold satisfies **Equation** (8).

equivalent to the value of being unemployed.

[12]Proofs of the uniqueness of policy functions and individuals' optimal policy are provided in Online Appendix Section 3.

6. The law of motion for the aggregate state of the economy respects the search and contract policy functions and the exogenous process of aggregate productivity.

Definition 2.3 (Block Recursive Equilibrium)**.** *A Block Recursive Equilibrium (BRE) is a recursive equilibrium such that the value and policy functions depend on the aggregate state only through aggregate productivity, $a \in \mathcal{A}$ and not through the distribution of agents across states $\mu \in \mathcal{M}$.*

3 Discussion

The objective of our model of dynamic sorting is to understand the properties of firm creation and worker search in a setting with two-sided heterogeneity. The following properties guarantee a high degree of tractability.[13]

Property 3.1 (Unique Injective Mapping)**.** *Upon matching, firm quality y and utility promises in vacancy postings v are related by an injective mapping conditional on the aggregate state of the economy, Ω, and workers characteristics (χ, V).*

The previous proposition establishes that workers' directed search toward promised values is equivalent to directed search toward firms' types. We can focus on the properties of the search strategy to get a complete view of how sorting works in equilibrium.

Property 3.2 (Search Monotonicity and Uniqueness)**.** *The optimal search strategy when unemployed, conditional on age τ and the aggregate state Ω, is unique and weakly increasing in workers' characteristics (h, ι). The optimal search strategy when employed, conditional on age τ and the aggregate state Ω, is unique and weakly increasing in workers' characteristics (h, ι) and current level of lifetime utility V.*

Property 3.2 guarantees that, abstracting from idiosyncratic as well as aggregate shocks, workers sort positively with respect to their education and human capital. **Property 3.1**, in turn, guarantees that workers agree on firms' relative ranking. Firms are thus vertically differentiated, and there is a separating equilibrium whereby workers with different characteristics optimally search in distinct firms.

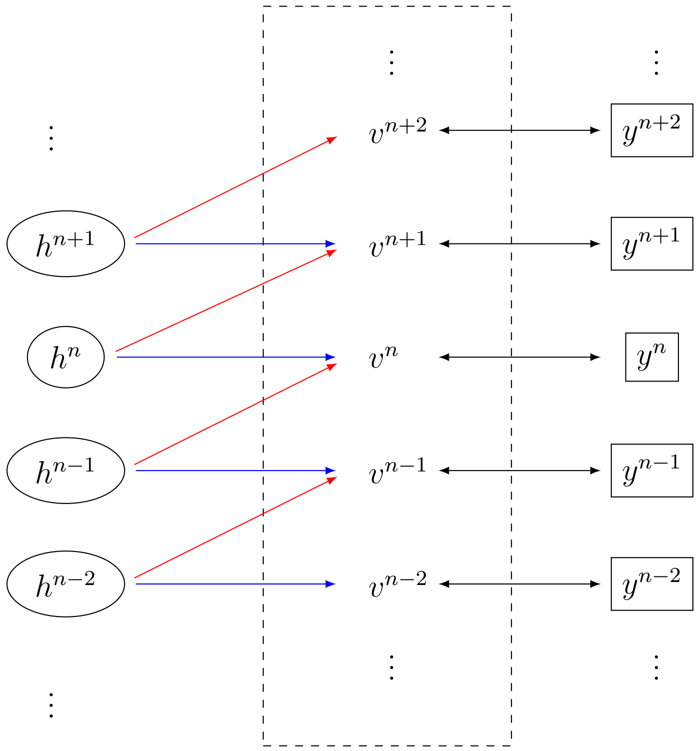
Because we are interested in how aggregate fluctuations shape the distribution of matches, we now turn to changes in search strategies across aggregate states.

Property 3.3 (Search in Good and Bad Times)**.** *The optimal search strategy is increasing in the aggregate productivity level, a.*

[13] We report the essential proofs to the propositions discussed in this section in **Appendix B** and we refer the reader to the Online Appendix for a more in-depth discussion of the theoretical properties of the model.

At this point we are able to illustrate one of the main mechanisms of the model, which is represented in **Figure 2**. The figure highlights one way in which aggregate fluctuations modify sorting in the labor market. The value of vacancies posted by each firm in equilibrium changes with the business cycle, as submarkets become less tight in bad times. Faced with a lower probability of successfully matching with the firm they would aim to match with in good times, risk-averse workers will then adjust their search downward.

Figure 2. Search Behavior.



Note: Schematic representation of labor market sorting along the business cycle. Unemployed workers, ordered by human capital levels, search in bad times and good times toward values offered by the (unique) corresponding firm type, presented as an ordered list with respect to order n.

Firms' offers will optimally respond to workers' incentives for on-the-job search.

Property 3.4 (Optimal Retention)**.** *Retention probabilities,* $\widetilde{p}(h, \tau, \iota, W; \Omega)$ *are:*

(i) increasing in the value of promised utilities, W

(ii) decreasing in aggregate productivity, a

Despite continuation values within the match being procyclical and workers searching more ambitiously in good times, firms are more likely to see workers leave in times of expansion. This is consistent with the data, as employment-to-employment transitions are strongly pro-cyclical. **Property 3.4** highlights another important aspect of the incentives that shape the contract designed by firms: retention grows in continuation values W.

To close the model, we need a rule for surplus sharing between firms and workers, that is, a wage protocol for firms to deliver lifetime utility promises to workers.

Property 3.5 (Wage Protocol)**.** *The optimal contract delivers a wage that satisfies:*

$$\frac{\partial \log \widetilde{p}(\chi', W_i'; \Omega')}{\partial W_i'} J(\chi', W_i', y; \Omega') = \frac{1}{u'(w_i')} - \frac{1}{u'(w_i)}, \tag{14}$$

with $\chi' \equiv (\phi(h, y, \iota, \psi), \tau + 1, \iota)$ *being the definition of individual characteristics and* w_i' *being the wage paid in the future state, conditional on realizations of idiosyncratic risk* ψ *and aggregate risk* a'*.*

This result extends the wage equation in Balke and Lamadon (2022) to an environment with two-sided heterogeneity. Wage growth is proportional to the residual continuation value of the match, J and the semi-elasticity of the worker's retention probability to future value promised. Limited liability provides the rationale for inefficient separations. At the same time, it also gives rise to wage rigidity, as it ensures that both elements in Equation 14 are weakly positive if the firm does not close down.[14]

Property 3.6 (Countercyclical Separations)**.** *Conditional on the existing contract and on worker and firm types, there exists an aggregate state* a^* *below which firms will not continue the contract. The threshold* a^* *is, all things being equal, increasing in the value promised to workers, and decreasing in worker and firm types.*

A clear implication of **Property 3.6** is that, at the onset of recessions, firms are significantly more likely to lay off workers. In addition, lower-skilled workers and low-productivity firms are more likely to separate in recessions. The counter-cyclicality of separations is a common feature in labor market data, together with the lower job security enjoyed by workers who are less productive, or provided by firms that are less productive.

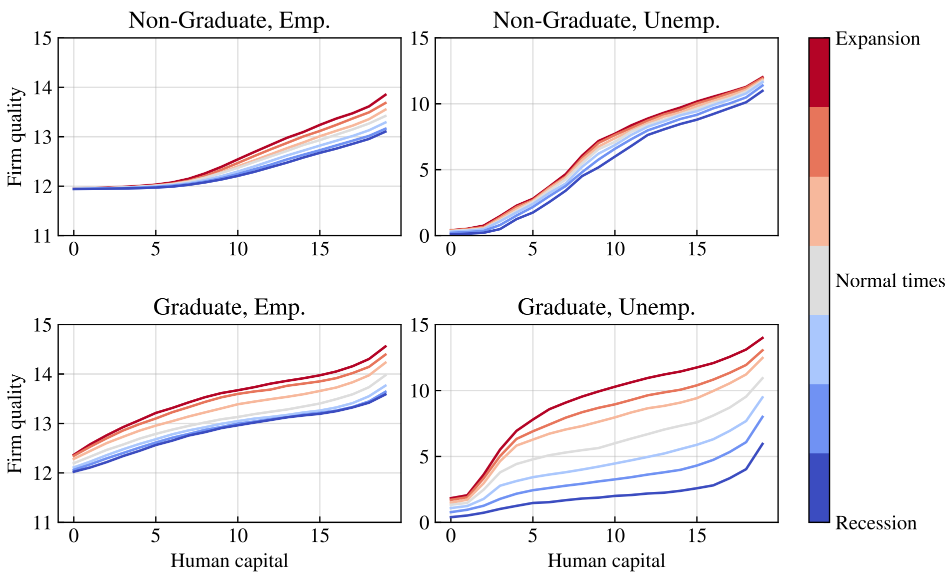
3.1 Sorting in equilibrium

The theory discussed in this section predicts that workers' search is monotonic in individual characteristics and in the aggregate state (see **Proposition 3.2**).

In **Figure 3** we plot the equilibrium mapping between workers' human capital and search behavior at each point in the aggregate state using a model simulation. Search is strongly monotonic in both dimensions. Search strategies of college-educated workers are more sensitive to shifts in aggregate conditions. Because expected human capital

[14]Notice that, in the presence of risky human capital accumulation, J will fluctuate together with the human capital levels of the worker even in the absence of aggregate fluctuations. However, because the contract provides insurance to workers, changes in their human capital will have asymmetric effects on wage growth, thus weakly increasing the labor share over time.

Figure 3. Search Behavior



Note: Search policy function by human capital level and aggregate state, averaged across labor market experience and wage promises.

accumulation in their matches is higher, the duration of corresponding firm profits is longer. This makes vacancy creation more volatile with respect to shifts in aggregate conditions. Productive jobs that would enhance human capital accumulation become scarcer, and workers respond by moderating their job search strategy, thus increasing misallocation in downturns.

4 Bringing the model to the data

The model features internally and externally calibrated parameters. To estimate the first group of parameters, we target moments from Italian administrative data, provided by the Uniemens dataset of the Italian Social Security Administration (INPS), for all years between 1996 and 2018.[15] To obtain model moments, we simulate a population of overlapping generations working for 45 years (180 quarters, from 18 to 63 years old, the legal retirement age for most years in our period of analysis). We then use a simulated method of moments (SMM) approach. This section will first present the quantitative setup of the model, present calibration choices for the parameters that are set externally, and finally present our estimation results.

[15]Details of data construction and sources are discussed in Online Appendix Section 6.

4.1 Calibration and estimation

Quantitative Setup. **Table 1** collects all the functional form choices. We assume a Cobb-Douglass production function and allow for potentially cyclical maintenance costs, captured by the parameter x. We follow Schaal (2017) and Menzio and Shi (2010) in picking a CES function in market tightness. Vacancy creation imposes increasing costs in firm's quality y, according to the convex function $c(y)$. Workers are risk-averse with constant-relative-risk-aversion (CRRA) utility. The human capital production technology is concave in the firm quality, y, which is scaled by a parameter ξ, and in the existing stock of human capital, h. Future human capital is also subject to additive i.i.d. shocks, $\psi \sim \mathcal{N}(0, \sigma_\psi)$. Home production is increasing in the stock of human capital according to the parameter ξ_b. Finally, we allow the exogenous separation rate to be age dependent to capture age-specific aspects of worker quality that are unrelated to business cycles but still empirically relevant. The model is then characterized by seven externally calibrated parameters and by 18 jointly estimated parameters.[16]

Table 1. Functional Forms

Functions	
Production function	$f(y,h) = Ay^\alpha h^{1-\alpha} - x(A-1)$
Job-finding probability	$p(\theta) = \theta(1+\theta^\gamma)^{-\frac{1}{\gamma}}$
Vacancy creation cost	$c(y) = \frac{y^\kappa}{\kappa}$
Utility function	$U(c) = \frac{c^{1-\nu}}{1-\nu}$
Human capital accumulation	$g_\iota(h,y) = (\xi y)^{\phi_\iota} h^{1-\phi_\iota} + \psi$
Home production	$b(h,\tau) = b + \xi_b h$
Exogenous exit rate	$\lambda = \frac{\lambda_b}{\lfloor \tau/4 \rfloor}$

Calibration. Preference parameters (discount factor β, and agents' risk aversion ν), and the annualized risk-free rate r_f are set in line with the literature. We calibrate the persistence and volatility of the aggregate shock, (ρ_a, σ_a) by estimating an AR(1) on the detrended series of Italian real total factor productivity (TFP). In addition, workers draw their innate ability and human capital upon entry into the market from an initial distribution. We set this initial distribution of human capital for high school–educated workers as a $Beta(\mu_L, \sigma_L)$. College-educated workers draw their initial human capital from the same distribution plus a constant spread, ϑ. We set the shape and scale of the beta distribution and internally estimate the scaling factor to match the ratio of average initial incomes between the two groups of workers. Finally, to properly account for the empirical age distribution, we weigh simulated data according to the age distribution of the Italian working-age population.[17]

[16] Online Appendix Section 7 provides more details on the model solution and estimation procedure.

[17] Age weights are constructed following the age distribution of the 2010 census from the website of the Italian National Institute of Statistics (ISTAT).

Table 2. Parameter Values

Parameter	Description	Value
	Externally Calibrated	
ν	Risk aversion	2.000
β	Discounting	0.990
r_f	Real interest rate	0.011
(μ_L, σ_L)	Shape and scale of initial human capital dist.	(2.50,10.00)
(ρ_A, σ_A)	Mean and std of TFP process	(0.95, 0.009)
	Jointly Estimated	
α	Production function elasticity to firm quality	0.556
γ	Matching function	1.092
ϕ	Human capital adjustment rate, High School	0.038
ϕ_g	Human capital adjustment rate, College	0.285
b	Unemployment benefit	1.103
λ_b	Exogenous separation prob., initial	0.119
κ	Vacancy cost	2.440
λ_e	On-the-job-search prob.	0.447
ξ	Scaling factor in human capital accumulation	0.644
ξ_b	UB dependence on human capital	0.063
l	Linear loss of humanc capital while unemployed	0.169
τ_{ee}	Human capital retention after EE	0.866
τ_{eu}	Human capital loss after EU	0.763
x	Cyclical component of cost function	-1.711
$\underline{\mathrm{p}}$	Out of labor force threshold	0.037
σ_ψ	Std of idiosyncratic human capital shock	0.698
ϑ	Initial scaling in human capital distribution	0.351
$\underline{\mathrm{y}}$	Lowest bound of firm distribution	2.831

Estimation and Identification. We estimate the remaining 18 parameters via SMM, targeting a set of standard labor market moments: labor market flows by age, as well as their correlations with aggregate output; the profile of wage growth over workers' careers; the average unemployment rate; the average inactivity rate in the Italian labor market; the average degree of sorting between workers and firms; and the distribution of firms' value-added. We define sorting as the average over time of the correlation between the firm and worker fixed effects from an AKM model yearly estimated on the Italian administrative data. In the model, sorting is the correlation between firms' and workers' qualities. **Table 2** reports the estimated parameter values.

The model fits employment flows by age, capturing labor market dynamism in the data (see Engbom, 2021). We match the cyclical properties of employment flows to account for the jump in job destruction and the drop in job creation during recessions. The elasticity of the matching function γ is primarily identified by the cyclical moves in employment-to-employment transitions, whereas the counter-cyclical role of separations mostly identifies the component of firms' cost function x. Characteristics of labor market fluidity and dynamism are further disciplined by matching the life cycle profiles of employment flows. They jointly help identifying the parameter that governs the frictional nature of on-the-job search, λ_e, the age-dependent separation probability λ_b, and the (firm-dependent) cost of vacancy opening, κ. The model also reproduces the life-cycle wage growth of Italian workers by education levels. Parameters that govern the earnings dynamics drive the estimates of human capital accumulation parameters, ϕ and ϕ_g, together with the parameters that determine the loss in human capital stemming from employment transitions, τ_{ee} and τ_{eu}.

An issue of modelling double heterogeneity is that we need a way to discipline the relative scale of workers' human capital and firm quality, as well as their cross sectional distribution - which in the model depend on ξ, and $\underline{y}$. To do so, we match the ratio between the third and fourth to the first quintiles of the firm value added distribution. Labor market sorting depends on the importance of firm quality in both production and human capital accumulation - thus it primarily identifies the production function parameter, α. Intuitively, the inactivity rate identifies the threshold that determines the exit from the labor force, $\underline{p}$. **Table 3** summarizes the comparison of model and empirical moments.

Table 3. Target Moments

Moments	Mean	
	Data	Model
A. Labor Market Flows		
Employment-to-Employment Transition Rate*	1.3%	0.6%
Employment-to-Unemployment Transition Rate*	4.1%	2.6%
Employment-to-Employment Correlation w/GDP	0.68	0.48
Employment-to-Unemployment Correlation w/GDP	-0.19	-0.17
B. Earnings		
Earnings Growth (College)*	129.7%	149.4%
Earnings Growth (High School)*	60.7%	75.9%
Entry Salary Ratio: College to High School	1.46	1.00
C. Other Statistics		
Unemployment Rate	9.7%	8.4%
Labor Market Sorting	0.40	0.57
Inactivity Rate†	21.9%	23.5%
Firm Value Added Distribution**	1.18	1.38

Note: (*): We match the life-cycle profiles with nine age bins for each profile. The table reports average values. (**): We match the ratio between the third and fourth to the first quintiles. (†) The average inactivity rate is taken from the inactivity rates by age groups from 1996 to 2019 from ISTAT.

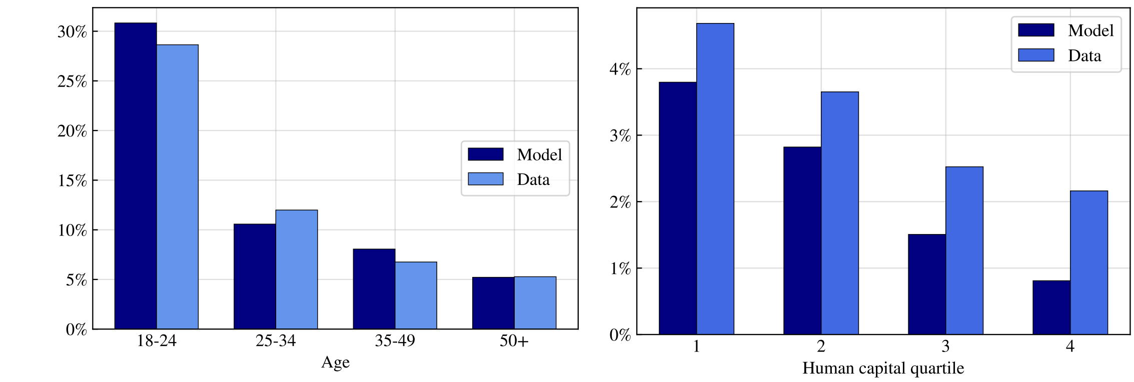
4.2 Model and data

To assess the quality our estimation we check how the model performs in matching a series of untargeted moments both in the cross section and over time as well as both at the macro and micro level.

Cross-sectional properties. While in the estimation we target the average unemployment rate, the model exhibits a very good fit also for the age profile of the unemployment rate (see **Figure 4a**).

Figure 4. Cross-Sectional Features, Model and Data

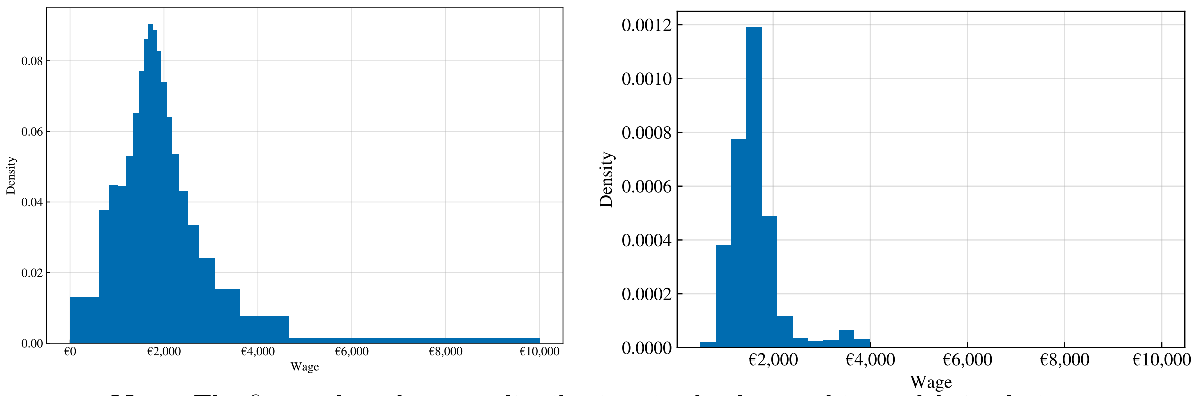
(a) Unemployment rates **(b)** Separation rates by worker qualities



Note: Panel (**a**) plots the unemployment rate by age groups in model simulations and in the data. *Sources:* unemployment rates are taken from the Italian National Statistical Agency (ISTAT). Panel (**b**) reports the average separation rates by worker quality. In the data worker quality is measured by the worker-specific AKM fixed effect. In the model simulations, worker quality is workers' human capital.

Figure 5. Wage distributions

(a) Data **(b)** Model



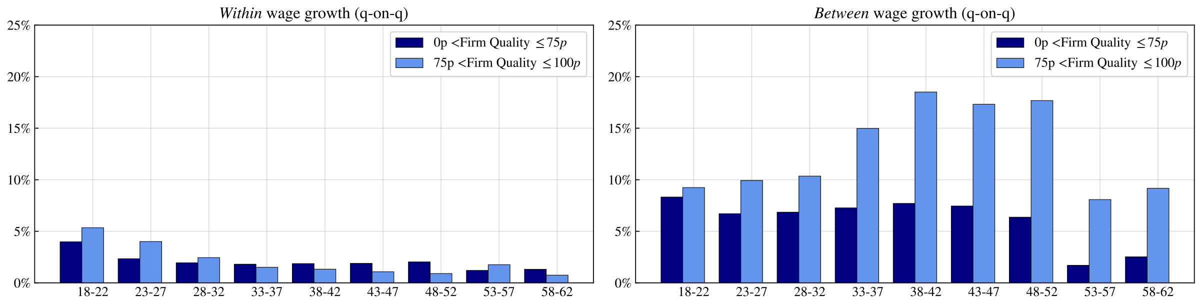
Note: The figure plots the wage distributions in the data and in model simulations.

In addition, **Figure 4b** compares the separation rates by worker types in the model and in the data, highlighting how the model is able to well capture the higher fragility of workers with low human capital, despite not directly targeting these moments in the estimation.

Our baseline model is also able to qualitatively reproduce the distribution of earnings. **Figure 5** displays the cross-sectional distribution of earnings in the data and in the model. The empirical wage distribution is centered at slightly below €2, 000 and skewed to the left, with most observations below €4, 000. What the model fails to generate is the long right tail of wages in the data, which corresponds mainly to managerial figures whose earnings command premia that our mechanism is not meant to capture.

Exploiting the rich features of the model, in **Figure 6**, we decompose the average growth in wages *within* jobs, that is, within the same job spell, and *between* jobs, that is, after a job-to-job transition. Consistent with the data, the model simulation implies that the bulk of wage growth is due to job-to-job transitions. We also observe that average *within*-wage growth is declining in age and firm quality. This is due to the fact that search frictions make it difficult for workers to reach progressively higher-productivity firms. Those firms thus enjoy relatively greater retention probabilities without the need of substantially adjusting wages (as in Gouin-Bonenfant, 2022). The resulting wage profiles for higher-productivity firms are flatter. Given that the highest skill workers tend to be older, one obtains decreasing wage growth over the life cycle and with firm quality.

Figure 6. Within vs. Between Wage Growth by Age and Firm Quality in the Model



Note: The figure plots the average wage growth, by age and firm quality, *within* employment spells and after employment-to-employment transitions (*between*). For the *betweeen* component, the firm quality quartiles are computed on the distribution of origin firms.

Table 4. Wage Growth of Movers vs. Stayers: Data and Model

		Age				
		18-27	27-36	36-45	45-54	54-62
Data	Between	7.1%	4.8%	3.7%	2.9%	2.2%
	Within	1.7%	1.2%	0.9%	0.6%	0.3%
Model	Between	8.4%	9.2%	13%	11%	6.9%
	Within	3.1%	1.9%	1.7%	1.7%	1.1%

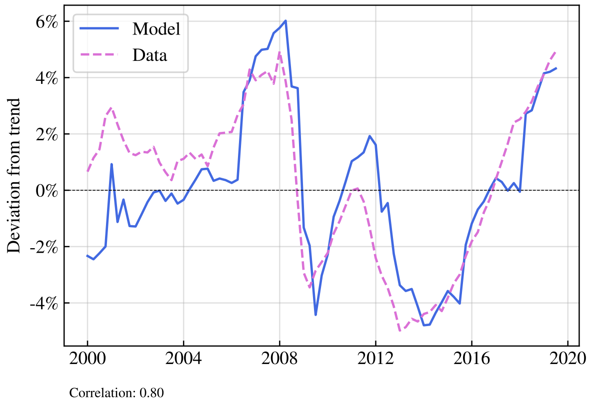
Note: The table reports the median yearly wage growth for movers (between wage growth) and stayers (within wage growth). Data: INPS, for all years between 1996 and 2018

The importance of sorting for human capital accumulation and workers' careers can be validated by measuring the correlation of workers ex-post wages with their ex-ante employer quality after an employment to unemployment to employment (EUE) transitions.[18] The correlation is increasing in previous firm qualities, indicating that workers benefit from employment in good firms even once the match is dissolved.[19]

[18]This is an adaptation of Herkenhoff et al. (2018)'s analysis to our model setting. In their paper, they rely on co-workers wages a proxy for firm quality. Given the nature of our framework, we use value-added per employee in the data to measure firm quality, and we control for workers' pre-transition wage.

[19]We report the estimated correlations in the **Table C.1**.

Figure 7. GDP: Model and Data



Note: The figure plots the cyclical components of real GDP for Italy and for a model simulation in which the TFP process is matched to the Italian TFP realizations from 2000 to 2019, both series are quadratically detrended and their correlation is robust to the choice of the filter.

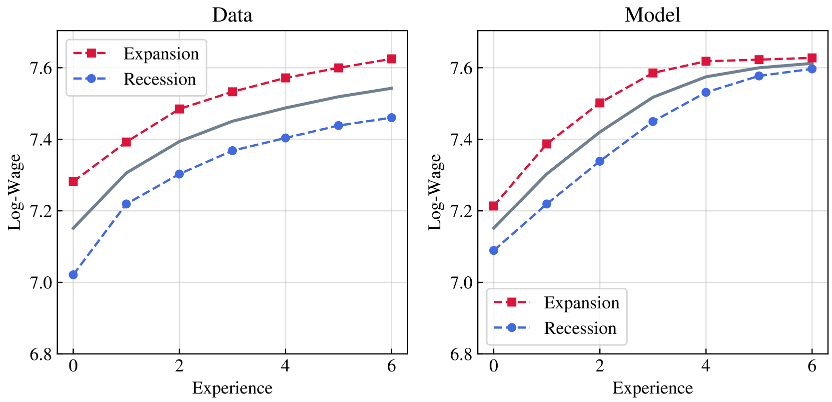
Time series properties. Replicating aggregate time-series properties of the data provides an additional validation of the channels in the model. We use the detrended quarterly series of Italian TFP, and project it on a discrete grid to simulate a series of aggregate shocks in the model. The model tracks the empirical series of GDP quite well, capturing the peaks and throughs as well as the overall behavior of the empirical series. Notably, together with matching well the volatility of output (the standard deviation of detrended log-output is approximately 3% both in the model and in the data) the model is able to generate also a volatility for the unemployment rate close to the data - 1.6% in the model versus 1.4% in the data - a result known to be hard to generate in search and matching models (Shimer, 2005).[20]

The model is also able to replicate the long-run effects of business cycles on workers' career outcomes at the micro level. In particular, we adapt the reduced-form models proposed in the literature on the effects of recessions on labor market entrants (Kahn 2010, Schwandt and von Wachter 2019) and we run it on both the Italian administrative data and on a model-simulated panel.[21] Consistently with the literature, entering the labor market in a downturn is associated with persistent losses in earnings. As shown in **Figure 8**, our baseline model is able to generate scarring effects that, on average, are approximately 25-30% of those observed in the data. The model matches reasonably well both the magnitude and the dynamics of the scarring effects of business cycles.

[20] See also Schaal (2017) for a notable exception in which a model with idiosyncratic, time-varying volatility can also generate empirically consistent volatilities of output and unemployment. To compute the volatility of unemployment we follow Shimer (2005) and we remove a slow moving trend from the time-series of unemployment, specifically we filter the quarterly series with an Hodrick-Prescott filter with smoothing equal to 10^5.

[21] In these empirical specifications we control for age, period, and cohort effects. Following the literature, we address the well-known identification issues in this class of models by proxying cohort fixed effects with the cyclical realization of GDP. We report the empirical estimates in Online Appendix Section 8.

Figure 8. Scarring Effect of Recessions



Note: The figure plots the wage profiles estimated on the data and on model simulations for cohorts of workers entering the labor market. The counterfactual profiles for expansions (recessions) are obtained considering a positive (negative) two standard deviation realization of cyclical GDP.

4.3 Alternative specifications

Our theoretical framework departs from the standard assumptions in the literature for two specific choices. One is the role of separations when the match becomes not profitable. The other, and most important, is the functional form of the human capital production function, which makes it depend on firm quality. we check the importance of these assumptions by comparing our baseline model with two alternative versions: one in which human capital accumulation is linear and independent from firm quality and one in which we allow the contract to continue even if the expected value of remaining in the match is negative, which eliminates endogenous separations in firm-worker relationships.[22] In both cases, we follow the procedure described in the previous paragraphs and re-estimate the models on the same moments summarized in **Table 3**.

Table 5 reports the correlations between the simulated GDP, Unemployment and labor share series in the baseline model and two alternative models. When human capital accumulation does not dependent on firm quality the model is not able to replicate the business cycle co-movements of unemployment and inequality in Italian economy. When we eliminate endogenous separations, the ability of the model to replicate the data remains good although its ability the business cycle correlations of unemployment and inequality are severely limited compared to our baseline economy. The table shows how both channels are important to obtain a good fit the with the data.

Panels 9a and **9b** report the unemployment rate by age and the separation patterns for the alternative models. It is clear from the figure that, although the models are re-estimated to achieve the best possible fit on the same moments used in the estimation of

[22]In the linear model, we replace the function governing human capital accumulation function (see **Table 1**) with a linear process. In this version of the model each worker type, ι, accumulates human capital following a random walk with linear trend ϕ_ι, i.e. $g(h) = \phi_\iota + \psi + h$.

Table 5. Co-movements at business cycle frequency

(a) Correlations between model generated and data time series

Corr. Model/Data	Baseline Model	Linear Human Capital Acc.	No Separations
Aggregate output	0.80	0.79	0.79
Unemployment	0.61	-0.49	0.65
Gini Index	0.57	0.60	0.60

(b) Correlations with aggregate output

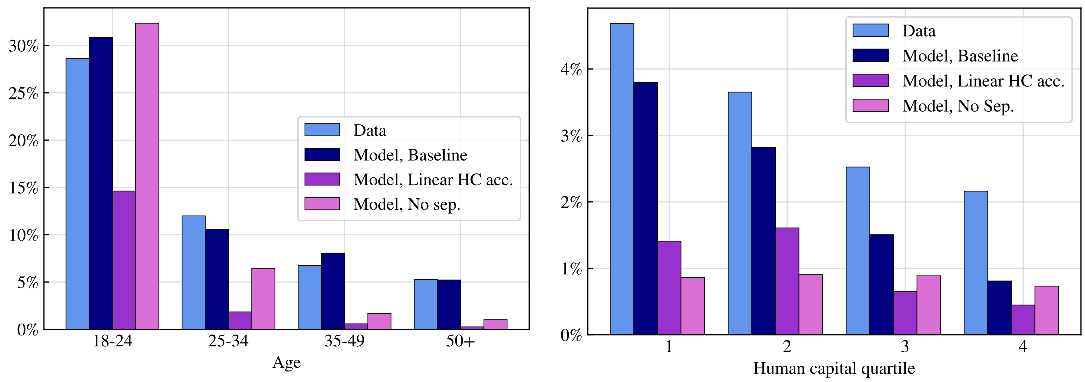
		Model		
	Data	Baseline	Linear Human Capital Acc.	No Separations
Unemployment	-0.83	-0.85	0.33	-0.69
Gini Index	-0.60	-0.78	-0.60	-0.33

Note: The table reports the correlation between model simulated series and their data counterparts (Panel 5a) and the correlations with aggregate output in the model and in the data (Panel 5b). The simulations are obtained feeding the model with a series of TFP that matches the realizations of Italian TFP from 2000Q1 to 2019Q4. All series have been quadratically detrended. We report the correlations for our baseline model and for two alternative ones. One in which human capital accumulation is linear (i.e. $h_{\iota,t+1} = \phi_\iota + h_{\iota,t} + \epsilon_{\iota,t}$) and one in which we do not allow for endogenous separations. Both alternative models have been re-estimated on the same moments used for the baseline estimation.

the our baseline model, detaching human capital accumulation makes the model unable to capture the important cross-sectional patterns for both unemployment and separations.

Figure 9. Cross-Sectional Features, Alternative Specifications

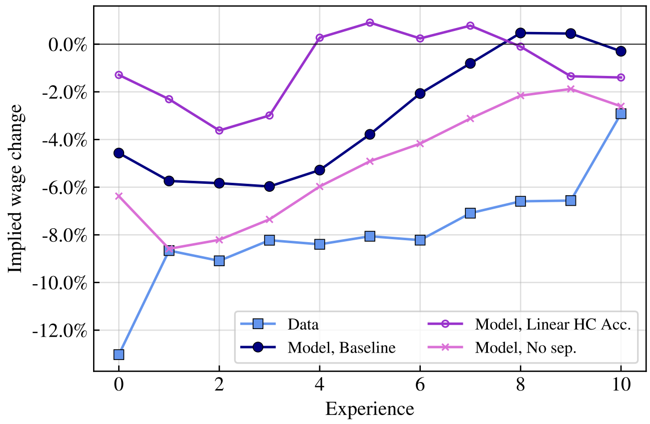
(a) Unemployment rates **(b)** Separation rates by worker qualities



Note: Panel (**a**) plots the unemployment rate by age groups for the data and for alternative models: one in which the human capital accumulation is independent from firm quality, and one where there are no endogenous separations. Panel (**b**) reports the average separation rates by worker quality for model and data. Data is as in **Figure 4**.

Eliminating endogenous separations amplifies scarring effects (see **Figure 10**). The intuition is that when separations are purely exogenous the composition of workers affected by negative output realizations is relatively more skewed towards high-human capital workers for which separations are more costly. Unsurprisingly, when firm quality does not matter for human capital accumulation the model is not able to generate any

Figure 10. Scarring Effect of Recessions



Note: The figure plots the implied loss in monthly wage from entering the labor market when cyclical GDP is two-standard deviations below trend, as in **Figure 8**.

scarring effect. On the contrary, the selection effect implies that workers that are able to find a job during a recession are positively selected compared to the mean, so they command a higher wage, leading to the very short lived positive effects in the figure for this model.

5 Anatomy of recessions

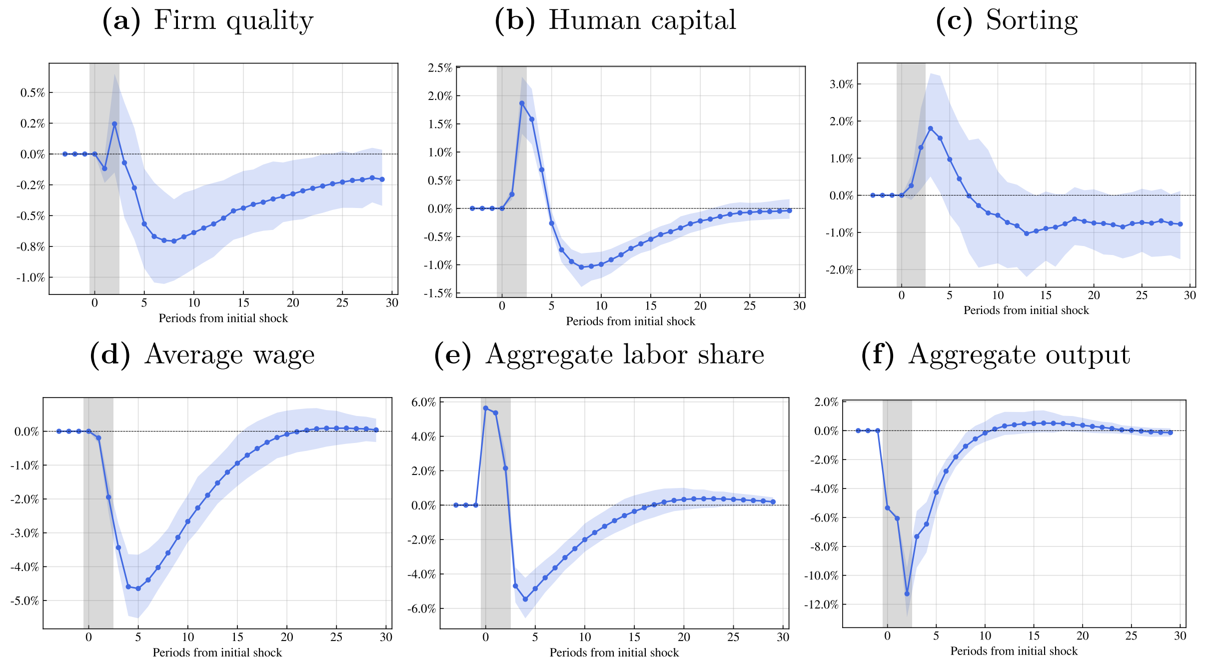
Through the lenses of firm and worker sorting, the model developed in **Section 2** can be used to analyze how aggregate shocks propagate through the economy, what channels are responsible for their persistent effects on aggregate output and how these channels differ depending on the heterogeneity of the agents in the economy.

To illustrate these points we provide a series of decomposition of the economy's impulse responses to a TFP shock. To compute the impulse response function, we compare a series of simulations of the model with aggregate shocks and their counterparts in which the economy experiences three consecutive negative realizations of the TFP process.[23] We then look at both labor market outcomes of affected cohorts and the response of aggregate GDP to the recession. We illustrate how the shock propagates in **Figure 11** and we then decompose the response of aggregate output into sorting, human capital and displacement channel in **Figure 12**.

Shock propagation. The dynamics of firm quality and average human capital, respectively in **Figure 11a** and **Figure 11b**, offer a clearer picture of the transmission of aggregate shocks in the economy. While the onset of the recession is accompanied by a sort of "Schumpeterian" response, as implied by the initial marginal increase in human capital shown in **Figure 11b**, firm quality is persistently crippled by the

[23]The cumulative drop in TFP is approximately 15% (equally split in each quarter) .

Figure 11. Recession Experiment



Note: The panels in the figure plot the ratios of the aggregate variables between an economy in which we impose a three-quarter negative TFP realization and an economy without aggregate shocks, that serves as a benchmark. The gray shaded area are the quarters in which TFP is below trend, while the blue shaded area are the 90-10 quantile ranges across one hundred model simulations.

recession, with the average quality of firms active in the economy remaining approximately 0.5% below the no-recession economy even two years after the end of the recession. The average human capital in the long run settles to a similar lower level despite the initial increase. **Figure 11c** shows the correlation between firms and workers' quality relative to the baseline simulation. After a short-lived improvement, firm-worker sorting is persitently dampened by the recession, remaining persistently below its pre-recession levels even five years after the onset of the recession. The prolonged reduction in the quality of the factors of production increases the persistence of the initial shock on output beyond the original duration of the recession. In fact, **Figure 11f** shows that even after two years, aggregate output its still approximately 1.5% below its counterfactual level.

Significantly, the recession has a significant impact on both the average wage and the average labor share in the economy, as shown in **Figure 11d** and **11e** respectively. As matches that are no longer viable are destroyed, average wages drop less than the average firm output. This allows for a brief spike in the labor share that quickly declines and remains below the counterfactual economy for a long time. The decline occurs for two reasons: the matches that form in this recovery period are still subject to the sullying in firm quality, and the human capital that favors a split in the match surplus increases the firms' share the expense of workers' compensation. This specific dynamic of the labor share also plays a central role in shaping the resilience of the economy to repeated shocks,

Table 6. Job characteristics by aggregate state at their inception

	Recession	Low-TFP	Normal	High-TFP	Boom
Tenure	1.00	1.01	1.05	1.14	1.10
Wage	1.00	1.04	1.10	1.17	1.24

Note: The table reports the average duration, wage for jobs that begin in different aggregate states. Recession values are normalized to one.

as we discuss in **Section 6**.

Another way to look at the sullying effect of recessions is to look at the different quality of jobs created along the business cycle. **Table 6** performs an exercise similar to Moscarini and Postel-Vinay (2016): we calculate the average tenure and wages for jobs created at different levels of the aggregate shock. We can see that jobs created in recessions pay on average 10% less than in "normal" times, and 24% less than in "boom" times. Job stability is also affected, with jobs created in recessions lasting 5% to 14% less than when the economy is running at full potential.[24]

5.1 Decomposing recessions

What explains the amplification and persistence of recessionary shocks? Different competing channels are at play. The first, which we call the human capital channel, captures the human capital accumulation that does not take place because of the recessionary event. The second, the sorting channel, amounts to the different joint worker-firm distributions that emerge in the periods following the shock, both because of different search strategies and because of additional unemployment spells. Finally, the standard displacement channel captures the job destruction that takes place because of the negative shock and its spillovers. We decompose the amplification channels of the model economy by shutting down each channel at a time and then comparing the resulting dynamics to the one of a baseline recession.[25]

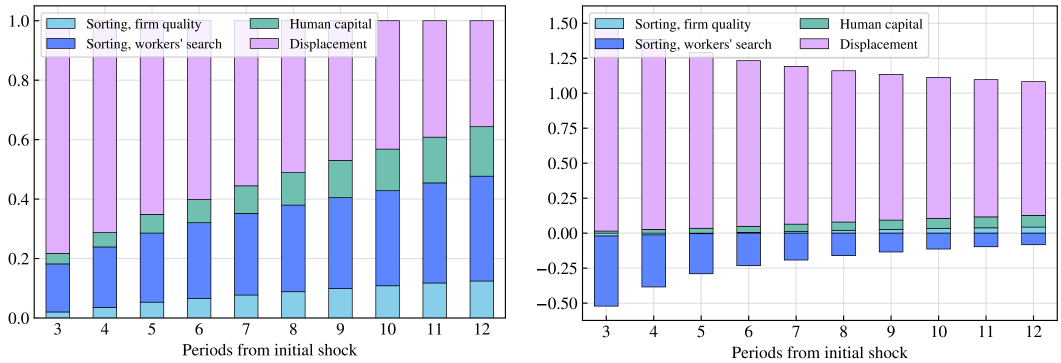
Figure 12a decomposes output dynamics after a negative shock as driven by our four channels. The displacement channel is the main driver of the recessionary dynamics on impact, explaining approximately half of the initial fall in output. The second half of the loss in output can be explained via combination of a lower search from workers as well as a lower firm quality for those that re-match during the transition. Importantly, the fact

[24] *Normal times* are periods in which TFP is at its long-run average while *Booms* are periods in which TFP is three standard deviations above average.

[25] Specifically, for the search component of the sorting channel we run model simulations in which the post-recession job-finding probabilities are those associated with search in the baseline simulation. Similarly, for the firm quality component we keep employed workers in the same firm quality they have in their baseline simulation. For the human capital channel we erase human capital losses by forcing workers' human capital in our counterfactual simulation to be the same as in the baseline one.

Figure 12. Decomposition of cumulative impulse response functions

(a) Aggregate output **(b)** Average wage



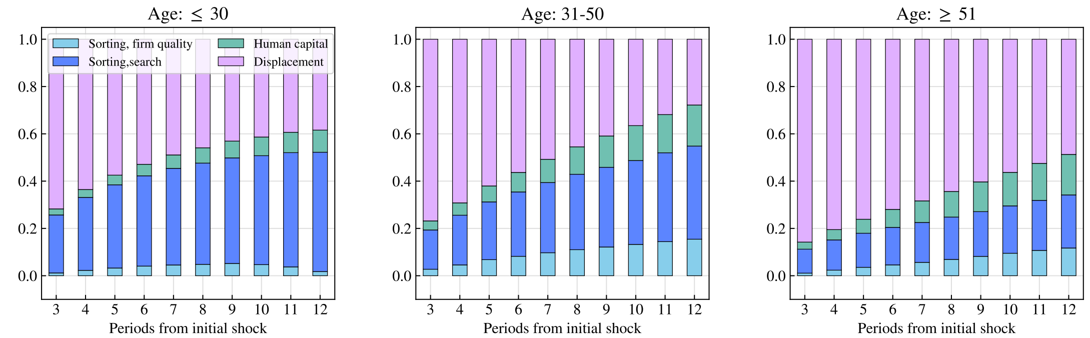
Note: The figure shows the relative importance of each transmission channel compared to the baseline recession for the cumulative response of GDP and the average wage in the two years after the onset of the recession.

that workers search towards lower quality firms in the aftermath of the recession is crucial in explaining the drop in output. This channel alone accounts for approximately 25% of output losses in the short-run. Recovery from displacement, while not immediate because search is frictional, is however relatively fast, in part because unemployed workers have lower reservation wages. We find that in the medium run, the sorting and human capital channels become more important, and contribute to the persistence of recessionary events. Contrary to the decomposition for aggregate output, the dynamics of average wage are dominated by the displacement effect, see **Figure 12b**. The large drop in wage following the onset of the recession is linked to the fact that workers move from employment to unemployment. The human capital channel gains importance over time but remains of second order importance. Interestingly, the sorting channel linked to workers' search is strongly negative at the beginning of the recovery (i.e. shutting down this channel would exacerbate the drop in average wage). This reflects the fact that when workers search facing the same job-finding probability of the baseline simulation a larger fraction of workers with relatively low human capital are pulled back into the labor market sooner than in the baseline, pushing down on the average wage.

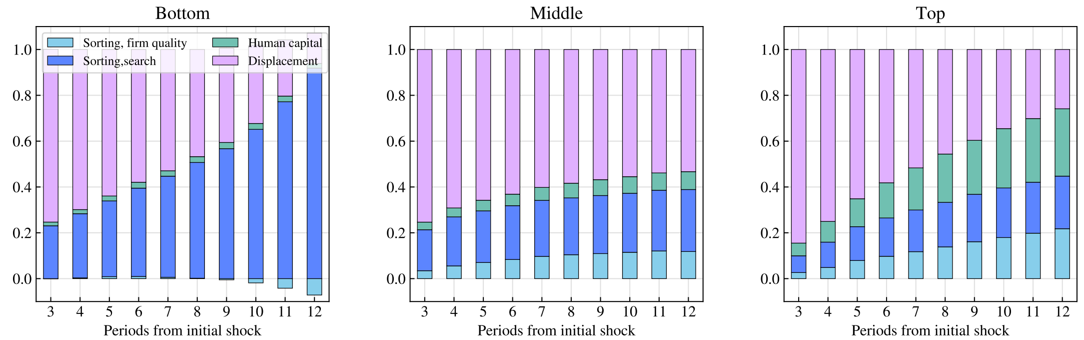
Interestingly, the three channels play a role in different parts of the workers' cross-sectional distribution (see **Figure 13**). The displacement channel depends mostly on the size of its left tail, as older and low-skilled workers are more likely to be separated. The sorting and human capital channels instead mainly relate to workers that are either young or in the right tail, for which allocative efficiency and long-run human capital accumulation matter more. This intuition is confirmed also by the progressive reduction in importance of the search component of the sorting channel as workers' age increases. This has important implications for the fragility of the economy: as the human capital distribution shifts to the right, recessions become less severe on impact, but might become

Figure 13. Decomposing aggregate output's cumulative response across the age and human capital distributions

(a) Age groups

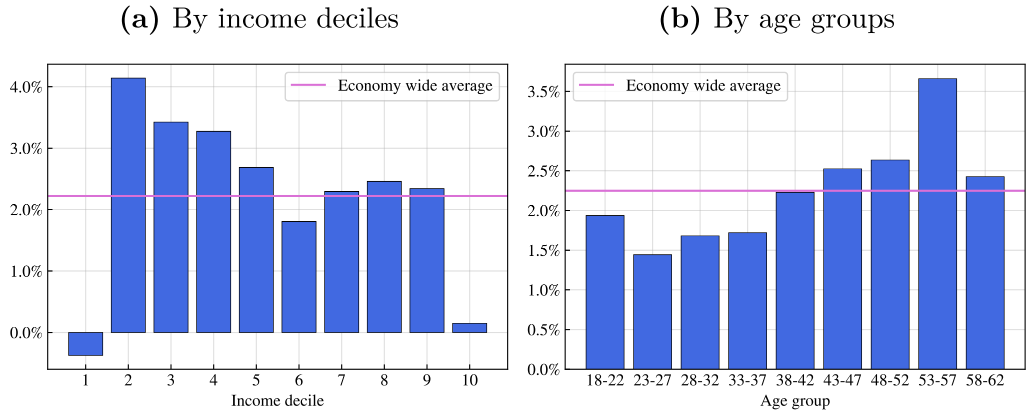


(b) Human capital terciles



Note: For each age group, Panel (**a**) shows the relative importance of each transmission channel compared to the baseline recession for the cumulative impulse response of GDP across different age groups in the two years after the onset of the shock. Panel (**b**) plots the same decomposition across human capital terciles.

Figure 14. Business Cycle Costs



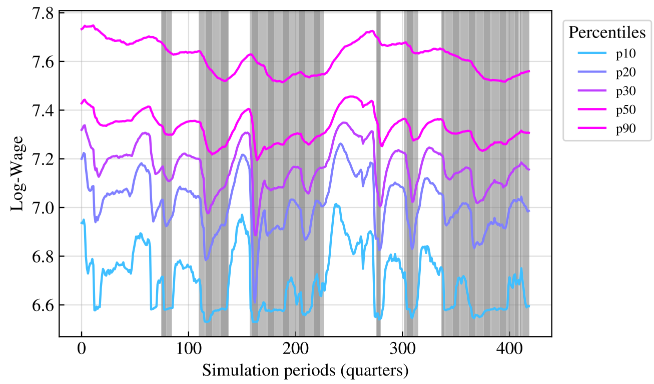
Note: Panel (**a**) and (**b**) plot the reduction in consumption-equivalent utility due to aggregate fluctuations by income deciles and age groups.

more persistent and have greater long-run effects thought human capital and firm quality match dynamics. As shown in **Table 6**, jobs that begin in expansions guarantee both longer tenure, higher wages and human capital accumulation within the spell.

5.2 The costs of recessions

Costs of Business Cycles. Influential work by Lucas (1987) argues that welfare gains from reducing business cycle volatility are negligible, and quantifies them in less than 0.1 percentage points of consumption-equivalent units. By doing an analogous calculation with our model, we estimate the cost of business cycles to be, on average, greater than 2 percentage points, more than two orders of magnitude above the Lucas estimate. Our estimate is quantitatively close to the one in Barlevy (2004), who first observed the potentially sullying effects of recessions. The richness in heterogeneity in our model though allows us to estimate how the welfare costs of business cycles vary along the income distribution. We can thus decisively show how welfare costs of business cycles crucially interact with income inequality. **Figure 14a** plots the cost of business cycles for income deciles. What emerges is that the extreme deciles, the first and the tenth, bear very little to no costs from business cycle fluctuations. For different reasons, both deciles are less affected by separation risk due to aggregate fluctuations: while the bottom decile mostly comprises unemployed workers, the top decile includes workers whose job tenure is robust to recessions. The welfare costs of business cycles are, however, much larger for all intermediate deciles, and have an interesting U-shape. Up until the seventh decile, the costs of aggregate fluctuations decrease with income: lower incomes are associated with more-fragile jobs and to a higher likelihood of unemployment in recessions. For the last three deciles, costs of business cycles are instead associated with long-run impacts on careers. For workers closer to the top of the income distribution, recessions can have

Figure 15. Inequality dynamics



Note:The figure reports the dynamics of income for different percentiles of the income distribution.

strong negative effects, as the deterioration in sorting and the ensuing lower human capital accumulation prevents them from getting to their best possible employment.

Inequality dynamics. Heathcote, Perri and Violante (2020) show that recessions impact inequality with persistence, affecting the earnings of workers in the left tail of the income distribution the most. **Figure 15** illustrates the dynamics of inequality around business cycles by displaying the pattern of losses across the earnings distribution. Recessions hit the poorest workers the hardest, and worsening job prospects push some of them out of the labor force. A prediction of the model that departs from existing literature is that the persistence of earnings losses varies across the distribution: while workers with less human capital display more volatility in earnings (mostly due to displacement, see **Figure 12b**), the impact on workers with high human capital is dampened but quite persistent.

6 Multiple shocks and recessions' length

Double dips and long expansions The model economy exhibits significant state dependence and can thus speak to the different impacts of shocks depending on the state of the economy when the shock hits. An important question is whether the depth of recessions is increasing in either built-in fragility from preceding long expansions or from the misallocating effects of a previous recession. To explore this, we perform a "double recession" experiment. The experiment works as follows: at time 0, we hit the economy with a recession identical to the one discussed in **Section 5**, then we track the evolution of human capital, firm quality, and labor share by collecting the first four moments of their distributions, absent other aggregate shocks. The three bottom panels in **Figure 16** report values for these four moments at nine different points in time following the first recession, starting one year after the return of aggregate shocks to trend and then every two years for the following eight. Then, at each of these time horizons, we hit the economy with a second shock identical to the one used to generate the first recession.

The top panel in **Figure 16** reports the difference of cumulated output responses between the first and the "second" recession. The recessions that occur within five years from the first have smaller cumulated losses. However, fragility builds up over time and recessions become more severe, peaking at the 11-year interval between the two shocks. This rather surprising result can be interpreted in light of the first decomposition of recessions, which shows the initial severity depends mostly on the impact of displacements: when the economy is recovering from a previous shock, even if unemployment has been re-absorbed, the labor share is unusually skewed to the left. This means that more workers have low wages compared to their baseline status, making the leverage problem in their match less severe. Similarly, when double dips hurt the most (for recessions number 7, 8, and 9) two facts emerge: the human capital distribution tails are less flat, but the labor share tails are fatter. This imbalance results in more separations and a more severe recession. The average length of an expansion in the data is about six years. Our results then point to the building up of fragility coming from longer-than-average expansions. In **Appendix C**, we show suggestive evidence in favor of this mechanism. In absence of more micro-data on other economies and due to the relatively low number of recessions in our sample, however, we do not have enough data points to reliably test it. While more research is definitely needed on this specific issue, we believe this result points to the usefulness of tracing the evolution of labor markets' micro-dynamics to highlight potential fragilities of the economy.

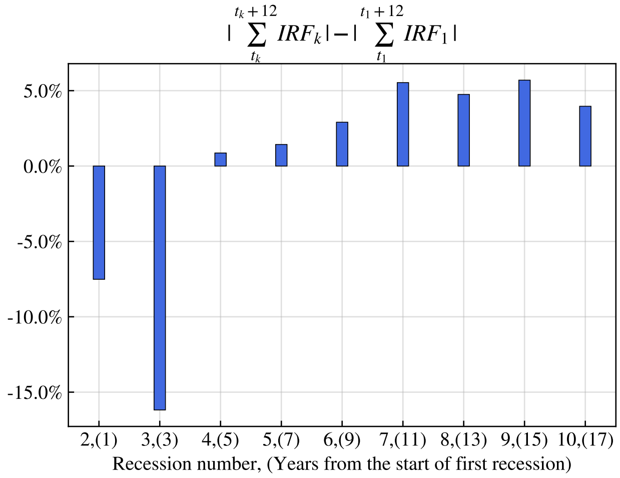
6.1 What accounts for the increased length of recessions?

The time economies take to recover from recessions has increased across developed economies in the last thirty years. **Figure 17** shows the average number of quarters aggregate GDP has been below trend during recessions for a subset of advanced economies. From the mid-1980s and early 1990s, in particular, this measure has consistently increased.[26] This is consistent with evidence presented by Fukui, Nakamura and Steinsson (2023) on the slower recovery of employment after recessions. Among other factors, an increase in job polarization and the rise in the skill premium are contemporaneous phenomena with this rise in the time economies need to recover from recessions (Goldin and Katz 2007; Goos, Manning and Salomons 2009). Our model

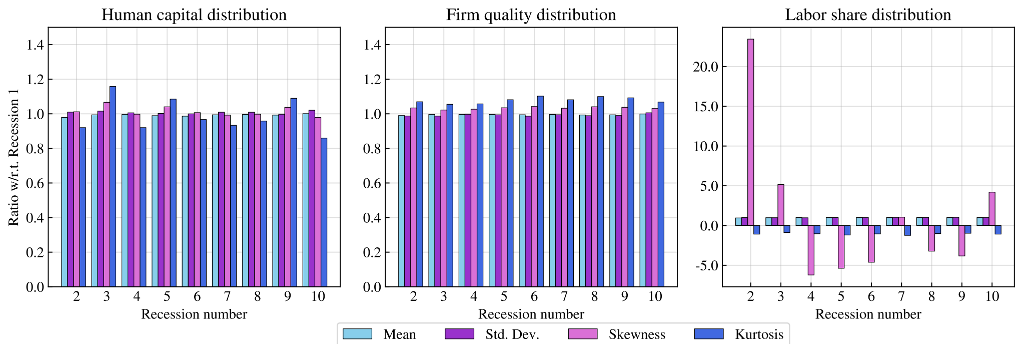
[26]Specifically, in the data, we define a recession as occurring after two consecutive quarters of negative GDP growth, and within each recession, we count the number of quarters in which GDP realizations are below trend. We obtain a similar picture if we look at alternative definitions of recession lengths, such as the number of quarters that are needed to reach pre-recession GDP levels and the number of consecutive quarters with negative GDP growth. However, these definitions cannot be transferred directly to model simulations as they rely on measures of GDP growth, which we do not explicitly model. Therefore, in our simulations, the number of consecutive quarters GDP remains below trend is our preferred measure of recession lengths.

Figure 16. State dependence and Business Cycles

(a) Cumulative losses

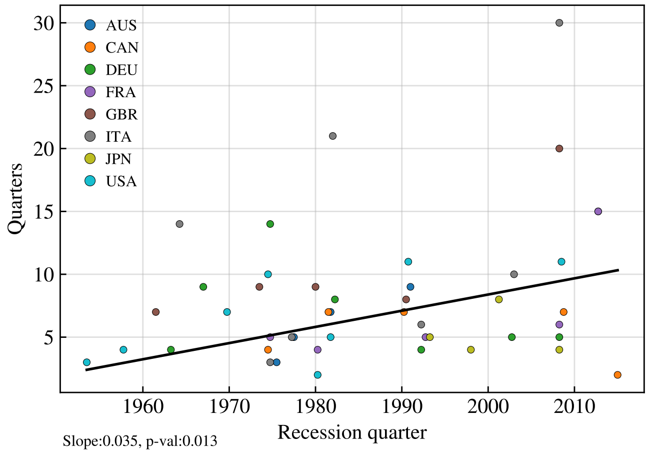


(b) Match-related distributions



Note: The figure plots a comparison between a series of simulations with two recessionary shocks of the same magnitude, one successive the other but at different horizons. The top panel shows the cumulative difference between the two recessions. The bottom panels report the ratios of the first four moments of selected variables in the quarter before each recession hit the economy. For skewness and kurtosis the denominator is the absolute value of the first recession moment.

Figure 17. Length of Recessions Over Time



Note: The figure reports the average duration of recessions for a set of OECD countries. Specifically, for each recession, we compute the number of quarters each economy's GDP remains below trend.

provides a useful structure to check whether human capital accumulation and the sorting dynamics in the labor market contribute to these aggregate developments. Our model is calibrated to match the differences in career paths and human capital accumulation of graduates and non-graduates in Italy in the last ten years. Our model shows that college-educated workers' human capital accumulates faster than other workers'. The widening of this gap through a higher weight of firm quality in the human capital production function (human capital deepening) proxies the development of a skill premium over time. We conduct the following experiment to analyze whether the rise in human capital's deepening and the lengthening of job ladders have contributed to the increase in length of recoveries. We consider two simulations of the model: our baseline and a counterfactual simulation in which the accumulation of human capital is the same across education levels. In the counterfactual economy the average earnings' growth for graduates is only 3% higher than non-graduates.[27] In our baseline economy graduates enjoy a faster human capital accumulation, which translates to approximately 30% higher earnings for graduates than non-graduates, on average.

The presence of a high-skill premium leads to recessions that are approximately 21% longer than those occurring without skill premium. In relative terms, the change in the length of recessions is remarkably close to what is observed for the subset of advanced economies in **Figure 17**. For these countries, in fact, the average duration of below-trend GDP realizations in recessions increases by approximately 29% comparing the periods before and after 1990.[28]

[27]Notice that graduates, while being older at labor market entry, draw their initial human capital from a distribution that dominates stochastically to a first order of the distribution of non-graduates. Therefore, starting from higher initial earnings, the model predicts some small further divergence in income.

[28]For Italy, the increase in this measure has been slightly more pronounced. Recession lengths went from 11 to 15 quarters, approximately a 36% increase.

We also test competing hypotheses, related to concurrent long-term trends, to show how they relate to recoveries' lengths. Increasing the firm-quality elasticity in the production function by approximately 6%, similar to the change in the share of labor compensation over GDP in Italy after 1990, would not lead to any change in the length of recessions. Jaimovich and Siu (2009) argue that population aging has had a significant impact on the business cycle properties advanced economies. To see how this has affected recession lengths, we simulate the model without a skill premium using demographic weights from the 1970 and 2011 Italian censuses, and we calculate the average recession length in these counterfactual scenarios. We find that accounting only for these demographic changes results in a 14% increase in the average recession length, about half of what observed in the data.[29]

7 Conclusions

In this paper we develop a new and tractable model of on-the-job search and human capital accumulation that features heterogeneity both on the worker side and the firm side. Ex-ante heterogeneous workers accumulate on-the-job experience which augments their skills and moves them up in the job ladder.

Consistent with the data, rigidity in labor costs amplifies negative shocks to firms, and generates inefficient separations.

We establish that workers that look for employment in bad economic times direct their search toward less-productive firms. Search frictions and aggregate uncertainty prevent an efficient allocation of workers to firms and expose different cohorts of workers to different human capital accumulation paths depending on the aggregate state at the time of entry to the labor force. Limits on workers' ability to accumulate human capital impose a drag on the overall labor productivity of the economy after recessions that tend to persist for a long time. Alterations to the sorting induced by recessions are slow to reverse and contribute not only to slow recoveries but also to long-run changes in the structure of the economy.

We show that aggregate fluctuations interact with shifts in the cross-sectional composition of labor markets, in ways consistent with the existence of hysteretic effects of recessions. We use the model to shed light on two open questions in business cycle literature. We show that increases in the importance of firms for workers' human capital accumulation, together with aging, can explain the increased length of recessions in recent decades. We also find that distributional channels can explain the reason why

[29]If we were to consider both channels simultaneously it would be sufficient to increase the skill premium so that earnings for graduates are approximately 24% higher than non-graduates to obtain a relative increase in recession lengths that matches exactly what is observed in the data.

subsequent recessions are more or less severe when the economy is hit by shocks within years after a first recession.

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Appendices

A Existence of a Block Recursive Equilibrium

In order to show that a Block Recursive Equilibrium (BRE) exists in our model we need to show that the equilibrium contracts, the workers' and the entrepreneurs value and policy functions do not depend on the distribution of employed and unemployed workers. This implies that the only element of the aggregate state that matters for a firm when making an hiring decision is the state of aggregate productivity but not the distribution of worker types (e.g. employed vs unemployed).

Proposition A.1. *A Block Recursive Equilibrium as defined in Definition 2.3 exists.*

Proof. We follow the approach in Menzio, Telyukova and Visschers (2016), Herkenhoff, Phillips and Cohen-Cole (2019) and prove the existence of a BRE using backward induction.

Consider the lifetime values of an unemployed and an employed worker before the production stage in the last period of households lives with $\tau = T$:

$$U(h, T, \iota; \Omega) = u(b(h, T)) \tag{A.1}$$

$$V(h, T, \iota; \Omega) = u(w(a)), \tag{A.2}$$

their values trivially do not depend on the distribution of types as both valuations are 0 from $T+1$ onward. Hence, $U(h, T, \iota; \Omega) = U(h, T, \iota; a)$ and $V(h, T, \iota; \Omega) = V(h, T, \iota; a)$.

The optimal contract for agents aged $\tau = T$, instead, solves the following problem

$$J(h, T, W, y; \Omega) = \sup_{w} [f(y, h; a) - w] \quad s.t.\ W = u(w),$$

that clearly does not depend on the distribution of worker types due to the directed search protocol and where the aggregate state only affects the promised utility and the optimal wage through realization of the aggregate productivity processes. Therefore, $J(h, T, \iota, W, y; \Omega) = J(h, T, \iota, W, y; a)$.

This also implies that the equilibrium market tightness

$$\theta(h, T, \iota, W; \Omega) = q^{-1}\left(\frac{c(y)}{J(h, T, \iota, W, y; a)}\right)$$

is independent from the distribution of worker types and it is only affected by realization of aggregate productivity, so $\theta(h, T, W; a)$.

This in turn implies that the search problem workers face at the beginning of the last period of their lives depends on the aggregate state only through aggregate productivity a:

$$R(h,T,\iota,V;a) = \sup_{v} \Big[p(\theta(h,T,\iota,v;a))\big[v - V\big]\Big],$$

does not depend on the distribution of worker types.

Stepping back at $\tau = T-1$, the value functions for the unemployed and the employed agents are solutions to the following dynamic programs

$$\sup_{v} u(b(h,T-1)) + \beta\mathbb{E}_{\Omega,\psi}\Big(U(h',T,\iota;a') + p(\theta(h,T,\iota,v;a'))\big[v - U(h',T,\iota,;a')\big]\Big)$$

$$u(w) + \beta\mathbb{E}_{\Omega,\psi}\left(\begin{array}{c}\lambda U(h',T,\iota;a') + \beta(1-\lambda)W+ \\ +\beta(1-\lambda)\lambda_e \max(0, R(h',T,\iota,,W);a')]]\Big]\end{array}\right),$$

where both do not depend on the distribution of worker types.

The optimal contract at this step is a solution to

$$J_t(h,T-1,\iota,V,y;a) = \sup_{\{\pi_i,w_i,W_i\}} \sum_{i=1,2} \pi_i\Big(f(y,h;a) - w_i$$
$$+\mathbb{E}_{\Omega,\psi}\left[\widetilde{p}(h',T,W_{i,\Omega'};a')(J(h',T,y,W_i;a')\right]\Big)$$

$$s.t.\ V = \sum_{i=1,2} \pi_i\left(u(w_i) + \mathbb{E}_{\Omega,\psi}\widetilde{r}(h',T,W_i;a')\right),\ h' = \phi(h,y,\iota,\psi)$$
$$\mathbb{E}_{\Omega,\psi}\sum_{i=1,2}\pi_i\left(\mathbb{E}_{\Omega,\psi}J(h',T,\iota,W_i,y;a')\right) \geq 0 \text{ and } t \leq T$$

which does not depend on types distribution.

Therefore, also the equilibrium tightness and the search gain at $T-1$ are independent from types' distributions, as

$$\theta(h,T-1,\iota,W;a) = q^{-1}\left(\frac{c(y)}{J(h,T-1,\iota,W,y;a)}\right)$$
$$R(h,T-1,\iota,V;a) = \sup_{v}\Big[p(\theta(h,T-1,\iota,v;a))\big[v - V\big]\Big].$$

Stepping back from $\tau = T-1,...,1$ and repeating the arguments above completes the proof. $\square$

B Proofs of model properties

For compactness of notation, we omit the dependence on education level, which is a fixed characteristic, and the idiosyncratic human capital shock, which is additive, from the proofs in Appendices. The logic of the proofs follows without loss of generality. We reserve a more detailed discussion of the model theoretical properties in the Online Appendix.

B.1 Proof of Property 3.1

Proof. Note: throughout the proof we drop the dependence of the functions to the state (h, τ, Ω) to ease readability.

If the function f_v is an injective function then it defines a one-to-one mapping between $\mathcal{Y}$ and $\mathcal{V}$ so that for $(y_1, y_2) \in \mathcal{Y}$, and $f_v(y_1) = W_1$ and $f_v(y_2) = W_2$, $(W_1, W_2) \in \mathcal{V}$, $f_v(y_1) = f_v(y_2) \Rightarrow y_1 = y_2$.[30] We proceed by contradiction. To begin, assume that $f_v(y_1) = f_v(y_2)$ and $y_1 \neq y_2$.

As the optimal contract is a concave function in firm quality, we know that the tangents at each point are above the graph of the function. Thus, we can define the tangents at the two points y_1, y_2 as

$$T_1(y) \equiv J(y_1) + \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_1} (y - y_1) \quad \text{and} \quad T_2(y) \equiv J(y_2) + \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_2} (y - y_2).$$

Without loss of generality, consider the case in which $y_2 > y_1$. Knowing that $T_i(y) \geq J(y)$ for $i = 1, 2$ due to the concavity of $J(\cdot)$, we can define the following inequalities:

$$T_1(y_2) - J(y_2) \geq 0 \quad \text{and} \quad T_2(y_1) - J(y_1) \geq 0.$$

Using the definitions for the tangents at y_1 and y_2 they imply that

$$\frac{J(y_2) - J(y_1)}{y_2 - y_1} \leq \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_1} \quad \text{and} \quad \frac{J(y_2) - J(y_1)}{y_2 - y_1} \geq \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_2},$$

hence combining the inequalities we get that

$$\left.\frac{\partial J(y)}{\partial y}\right|_{y=y_2} \leq \frac{J(y_2) - J(y_1)}{y_2 - y_1} \leq \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_1}. \tag{B.1}$$

However, the free-entry condition in vacancy posting implies that in the submarket (h, τ, W) both firms must be respecting $c(y_i) = q(\theta)\beta J(y_i)$ for $i = 1, 2$. As $c(y_i)$ is a linear function of firm quality $\frac{\partial c(y_i)}{\partial y_i} = c$ for $i = 1, 2$ and therefore from the free-entry condition:

$$c = q(\theta)\beta \left.\frac{\partial J(y)}{\partial y}\right|_{y=y_i}$$

which is a contradiction of the slopes of the two tangents being decreasing as shown in

[30] As the contrapositive of Definition 2.2 in Rudin (1976), that defines a one-to-one mapping for $(x_1, x_2) \in A$ as $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

Equation (B.1). Note that if $c(y)$ is convex and twice differentiable, then the derivatives of $c(y)$ are increasing in y while the derivatives of $J(\cdot)$ are decreasing leading again to a contradiction. The proof for the case in which $y_1 > y_2$ follows the same arguments and leads to a similar contradiction on the implied slopes of the optimal contract and those implied by the free entry condition. $\square$

B.2 Proof of Properties 3.2 and 3.4

The following propositions characterize the properties of workers' optimal search strategies that solve the search problem in (9). We start proving a useful lemma and then provide the proofs for the two properties.

Lemma B.1. *The composite function $p(\theta(h, \tau, v; \Omega))$ is strictly decreasing and strictly concave in v.*

Proof. For this proof we follow closely Menzio and Shi (2010), Lemma 4.1 (ii). From the properties of the matching function we know that $p(\theta)$ is increasing and concave in θ, while $q(\theta)$ is decreasing and convex. Consider that the equilibrium definition of $\theta(\cdot)$ is

$$\theta(h, \tau, v; \Omega) = q^{-1}\left(\frac{c(y)}{J(h, \tau, v, y; \Omega)}\right),$$

and that the first order condition for the wage and the envelope condition on V of the optimal contract problem in (5) implies

$$\frac{\partial J(h, \tau, v, y; \Omega)}{\partial v} = -\frac{1}{u'(w)}.$$

so that as $u'(\cdot) > 0$, $J(\cdot)$ is decreasing in v.

From the equilibrium definition of $\theta(\cdot)$ and noting that $q^{-1}(\cdot)$ is also decreasing due to the properties of the matching function we have that

$$\frac{\partial \theta(h, \tau, v; \Omega)}{\partial v} = \left.\frac{\partial q^{-1}(\xi)}{\partial \xi}\right|_{\xi=\frac{c(y)}{J(h,\tau,v,y;\Omega)}} \cdot \left(-\frac{\partial J(h, \tau, v, y; \Omega)}{\partial v}\right) \cdot \frac{c(y)}{(J(h, \tau, v, y; \Omega))^2} < 0,$$

which, in turn, implies that

$$\frac{\partial p(\theta(h, \tau, v; \Omega))}{\partial v} = \left.\frac{\partial p(\theta)}{\partial \theta}\right|_{\theta=\theta(h,\tau,v;\Omega)} \cdot \frac{\partial \theta(h, \tau, v; \Omega)}{\partial v} < 0.$$

Suppressing dependence on the states (h, τ, y, Ω) for readability, to prove that $p(\theta(v))$ is concave, consider that $J(v)$ is concave[31] and a generic function $\frac{c}{v}$ is strictly convex in

[31] $J(\cdot)$ concave give the two-point lottery in the structure of the contract. See Menzio and Shi (2010)

v. This implies that with $\alpha \in [0,1]$ and $v_1, v_2 \in \mathcal{V}$, $v_1 \neq v_2$:

$$\frac{c}{J(\alpha v_1 + (1-\alpha)v_2)} \leq \frac{c}{\alpha J(v_1) + (1-\alpha)J(v_2)} < \alpha \frac{c}{J(v_1)} + (1-\alpha)\frac{c}{J(v_2)}.$$

As $p(q^{-1}(\cdot))$ is strictly decreasing the inequality implies that

$$\begin{aligned} p\left(q^{-1}\left(\tfrac{c}{J(\alpha v_1+(1-\alpha)v_2)}\right)\right) &\geq p\left(q^{-1}\left(\tfrac{c}{\alpha J(v_1)+(1-\alpha)J(v_2)}\right)\right) \\ &> \alpha p\left(q^{-1}\left(\tfrac{c}{J(v_1)}\right)\right) + (1-\alpha)p\left(q^{-1}\left(\tfrac{c}{J(v_2)}\right)\right), \end{aligned}$$

and as $\theta(v) = q^{-1}(\frac{c}{J(v)})$:

$$p(\theta(\alpha v_1 + (1-\alpha)v_2)) > \alpha p(\theta(v_1)) + (1-\alpha)p(\theta(v_2))$$

so that $p(\theta(v))$ is strictly concave in v. □

The proofs for the two properties are as follows:

Proof. The proofs follow closely Shi (2009), Lemma 3.1 and Menzio and Shi (2010), Corollary 4.4. More formally, for each triplet (h, τ, Ω) given at each search stage, we can re-define the search objective function as $K(v, V) = p(\theta(v))(v - V)$ and $v^*(V) \in \arg\max_v K(v, V)$ as the function that maximises the search returns (i.e. the optimal search strategy of the worker) and prove the following.

We first show that $K(v, V)$ is strictly concave in v. Consider two values for v, $v_1, v_2 \in \mathcal{V}$ such that $v_2 > v_1$ and define $v_\alpha = \alpha v_1 + (1-\alpha)v_2$ for $\alpha \in [0,1]$. Then by definition:

$$\begin{aligned} K(v_\alpha, V) &= p(\theta(v_\alpha))(v_\alpha - V) \\ &\geq [\alpha p(\theta(v_1)) + (1-\alpha)p(\theta(v_2))][\alpha(v_1 - V) + (1-\alpha)(v_2 - V)] \\ &= \alpha K(v_1, V) + (1-\alpha)K(v_2, V) + \alpha(1-\alpha)[(p(\theta(v_1)) - p(\theta(v_2))](v_2 - v_1) \\ &> \alpha K(v_1, V) + (1-\alpha)K(v_2, V) \end{aligned}$$

where the first inequality follows from the concavity of $p(\theta(\cdot))$ (this is true if $J(\cdot)$ concave with respect to V) and the second inequality stems from the fact that $p(\theta(\cdot))$ is strictly decreasing hence $\alpha(1-\alpha)[(p(\theta(v_1)) - p(\theta(v_2))](v_2 - v_1) > 0$.

Weakly increasing in promised utility. Consider a worker employed in a job that gives lifetime utility V. Given that $v \in [\underline{v}, \overline{v}]$, and that submarkets are going to open depending on realizations of the aggregate productivity, a, there is only one region in the set of promised utilities where the search gain is positive. This set is $[V, v(a)]$ with $v(a)$ being the highest possible offer that a firm makes in the submarket for the worker

Lemma F.1.

(h, τ). Any submarket that promises higher than $v(a)$ is going to have zero tightness. Therefore, the optimal search strategy for $V \geq v(a)$ is $v^*(V) = V$, as $K(V, v(a)) = K(V, V) = K(\overline{v}, V) = 0$ (the search gain is null given the current lifetime utility V). For $V \in [V, v(a)]$, instead, as $K(v, V)$ is bounded and continuous, the solution $v^*(V)$ has to be interior and therefore respect the following first order condition

$$V = v^*(V) + \frac{p(\theta(v^*(V)))}{p'(\theta(v^*(V)) \cdot \theta'(v^*(V))}. \tag{B.2}$$

Now consider two arbitrary values V_1 and V_2, $V_1 < V_2 < \overline{v}$ and their associated solutions $W_i = v^*(V_i)$ for $i = 1, 2$. Then, V_1 and V_2 have to generate two different values for the right-hand side of (B.2). Hence, $v^*(V_1) \cap v^*(V_2) = \emptyset$ when $V_1 \neq V_2$. This also implies that the search gain evaluated at the optimal search strategy is higher than the gain at any other arbitrary strategy so that $K(W_i, V_i) > K(W_j, V_i)$ for $i \neq j$. This implies that

$$\begin{aligned} 0 >& [K(W_2, V_1) - K(W_1, V_1)] + [K(W_1, V_2) - K(W_2, V_2)] \\ &= (p(\theta(W_2)) - p(\theta(W_1)))(V_2 - V_1), \end{aligned}$$

thus, $p(\theta(W_2)) < p(\theta(W_1))$. As $p(\theta(\cdot))$ is strictly decreasing (see Lemma B.1), then $v^*(V_1) < v^*(V_2)$. Uniqueness follows directly from strict concavity.

Decreasing in a. We are now interested in the sign of:[32]

$$\left.\frac{\partial \bar{J}}{\partial a}\right|_{W_y, \pi, w, \{W'_y\}} = \frac{\partial f(\cdot)}{\partial a} + \beta \mathbb{E}_\Omega \left[\left.\frac{\partial \tilde{p}(\cdot)}{\partial a}\right|_{W_y, \pi_i, w, \{W'_y\}} \bar{J}' \right].$$

Now notice that, in equilibrium,

$$\frac{\partial \widetilde{p}(\theta)}{\partial a} \propto -\frac{\partial p(\theta)}{\partial a} = \underbrace{\frac{\partial p(\theta)}{\partial \theta}}_{>0} \cdot \underbrace{\frac{\partial \theta}{\partial J(\cdot)}}_{>0} \cdot \frac{\partial J(\cdot)}{\partial a}$$

where the sign of the second derivative on the right hand side comes from the free entry condition and the properties of vacancy filling probability function $q(\cdot)$. Given this, it has to be that $\frac{\partial p(\theta)}{\partial a}$ and $\frac{\partial J(\cdot)}{\partial a}$ have the same sign in equilibrium. Now, if both are strictly positive, both statements of our proposition are immediately true. Let's now assume they are both negative or zero. If this is the case, then $\frac{\partial \tilde{p}(\cdot)}{\partial a} \geq 0$. But this implies $\frac{\partial \bar{J}}{\partial a} > 0$, which is a contradiction. $\square$

[32]We assume that $J' = f(h', y; a') - w'$ is constant with respect to a. It is possible to prove, by backward induction, that this assumption is without loos of generality for the sake of the proof.

B.3 Proof of Property 3.3

The proof follows from the characteristic of workers optimal behaviour (see 3.2) and the following proposition.

Proposition B.1. *The Pareto frontier $J(h, \tau, \iota, W, y; a, \mu)$ is increasing in the aggregate productivity shock a.*

Proof. We proceed by backward induction.[33] For workers in period T, given that the firm increases its production while keeping the worker at least indifferent, J is at least weakly increasing in a. However, the firm can also feasibly increase the worker's wage by ε, with $\varepsilon < \frac{\partial f(\cdot)}{\partial a}$. J is thus strictly increasing in y. Consider now a worker who is $T-1$ periods old. A firm matched to a worker in submarket $\{h, T-1, y, W_{y,\Omega}\}$ will face the following Pareto frontier

$$J(h, T-1, y, W_y; a, \mu) = \sup_{w, W_y'} \Big(f(h, y; a) - w \\ + \mathbb{E}_\Omega \left[\widetilde{p}(h', T, W_y'; a', \mu')(f(h', y; a') - w') \right] \Big).$$

Assume that aggregate productivity increases from $\bar{a}$ to $\bar{a} + \varepsilon$ and that the firm keeps its policies constant. We aim at proving that, even in such a case, firm value increases while keeping the worker at least indifferent. If this is the case, it is *a fortiori* true that J increases in a after reoptimizing firms' policies. □

B.4 Proof of Property 3.5

Proof. Consider the firm problem in Equation (5). For $i = 1, 2$, the first order conditions with respect to the wage and the promised utilities are:

$$[w_i]: \ \lambda = \frac{1}{u'(w_i)} \tag{B.3}$$

$$[W_i]: \ \frac{\partial \widetilde{p}(\cdot)}{\partial W_i} J(\cdot) + \widetilde{p}(\cdot) \frac{\partial J(\cdot)}{\partial W_i} + \lambda \frac{\partial \widetilde{r}(\cdot)}{\partial W_i} = 0. \tag{B.4}$$

Note that by definition,

$$\widetilde{r}(h, \tau, V; \Omega) \equiv \lambda U(h, \tau; \Omega) + (1-\lambda) \Big[W + \lambda_e \max\{0, R(h, \tau, V; \Omega)\} \Big]$$

therefore we can use the envelope theorem as in Benveniste and Scheinkman (1979), Theorem 1 and the definition in Equation (12) to derive an expression for the derivative

[33] For compactness of notation, we omit without loss of generality the two-point lottery in the equations in the proof.

of the employment value in $t+1$ as the period ahead of the following:

$$\frac{\partial \widetilde{r}(h,\tau,W;\Omega)}{\partial W} = \widetilde{p}(h,\tau,W;\Omega).$$

Similarly, using the envelope condition on the firm problem and the first order condition for the wage, we can establish that

$$\frac{\partial J(h,\tau,y,W;\Omega))}{\partial W} = -\lambda \;\therefore\; \frac{\partial J(h,\tau,W,y;\Omega))}{\partial W} = -\frac{1}{u'(w)}. \tag{B.5}$$

Moving these two expressions one period ahead, substituting them in (B.4), focusing on $\widetilde{p(\cdot)} > 0$ and $\pi_i > 0$ and rearranging we have that:

$$\frac{\partial \widetilde{p}(\Theta)}{\partial W_i}\frac{J(\Theta)}{\widetilde{p}(\Theta)} = \frac{1}{u'(w_i')} - \frac{1}{u'(w)},$$

with $\Theta \equiv (\phi(h,y),\tau+1,W;\Omega')$ and where w' is the wage next period in state Ω'. □

B.5 Proof of Property 3.6

We discuss this property as a corollary of the following proposition.

Proposition B.2. *The Pareto frontier $J(h,\tau,\iota,W,y;a,\mu)$ is increasing in the aggregate productivity shock a, while retention probabilities, $\widetilde{p}(h,\tau,\iota,W;a,\mu)$ decrease in aggregate productivity.*

Proof. We proceed by backward induction.[34] Given the nature of the aggregate productivity process, the proposition is trivially true for workers T periods old. Given that the firm increases its production while keeping the worker at least indifferent, J is at least weakly increasing in a. However, the firm can also feasibly increase the worker's wage by ε, with $\varepsilon < \frac{\partial f(\cdot)}{\partial a}$. J is thus strictly increasing in y.

Consider now a worker who is $T-1$ periods old. A firm matched to a worker in submarket $\{h, T-1, y, W_{y,\Omega}\}$ will face the following Pareto frontier,

$$\begin{aligned} J(h,T-1,y,W_y;a,\mu) = \sup_{w,W_y'} \Big(& f(h,y;a) - w \\ & + \mathbb{E}_\Omega\left[\widetilde{p}(h',T,W_y';a',\mu')(f(h',y;a') - w')\right]\Big). \end{aligned}$$

Let's assume that aggregate productivity increases from $\bar{a}$ to $\bar{a}+\varepsilon$ and that the firm keeps its policies constant once again. We aim at proving that, even in such a case, firm value

[34] For compactness of notation, we omit without loss of generality the two-point lottery in the equations in the proof.

increases while keeping the worker at least indifferent. If this is the case, it is *a fortiori* true that J increases in a after reoptimizing firms' policies. $\square$

Corollary B.1. *There exists a productivity threshold $a^*(h, \tau, W_y, y)$ below which firms will not continue the operate.*

Proof. The proof follows immediately from **Proposition B.2** and the timing of the shock. Given the timing of the shock, exit is fully determined by the current productivity shock and incumbent firms know in advance whether they are willing to produce in the next period.

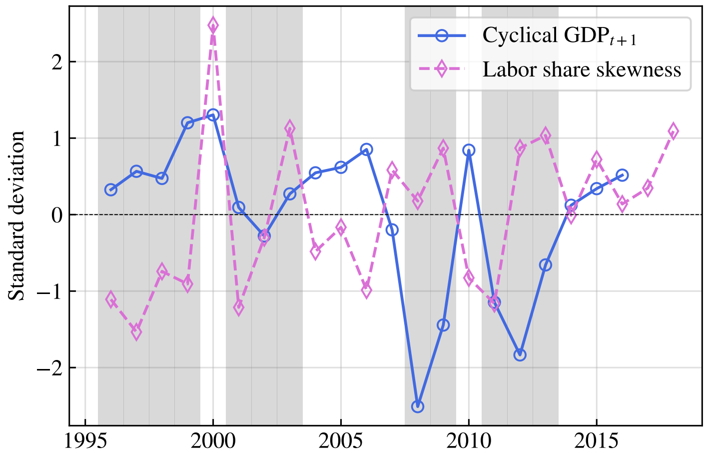
Therefore, as the Pareto frontier is strictly increasing in a, firms are willing to continue the contract if $\mathbb{E}_\Omega[J(h', \tau+1, W'_{y,}, y; a', \mu')|h, \tau, W_y, y, a, \mu] \geq 0$, so that the threshold that determines exit is

$$a^*(h, \tau, W_y, y) : \mathbb{E}_\Omega[J(h', \tau + 1, W'_y, y; a', \mu')|h, \tau, W_y, y, a, \mu] = 0.$$

$\square$

C Additional Tables and Figures

Figure C.1. Skewness of Labor Share and GDP



Note: The figure plots the cyclical component of real GDP (Hamilton filter) and the skewness of labor share, both standardized. In the data, we compute the labor share as the ratio of labor costs to value added. Shaded areas are OECD based recessions. *Data source*: ISTAT and INPS-Uniemens.

Table C.1. E-U-E transitions

(a) Data

Dep.Variable: Log-wage after E-U-E transition	(1)	(2)
Quality of origin firm (FQ):		
2^{nd} quint.	0.100	0.128
	(0.002)	(0.004)
3^{rd} quint.	0.143	0.210
	(0.002)	(0.004)
4^{th} quint.	0.153	0.182
	(0.002)	(0.004)
5^{th} quint.	0.230	0.260
	(0.002)	(0.004)
Log-wage at origin	0.669	0.609
	(0.001)	(0.002)
Experience controls	✓	✓
Sex FE	✓	✓
Year FE	✓	✓
Contract type FE	✓	✓
Full- & Part-Time FE	✓	✓
Justified dismissals	✓	
R^2	0.48	0.36
N	955,602	338,975

(b) Model

Dep. Variable: Log-wage after E-U-E transition	(1)
Quality of origin firm (FQ):	
$p0 \leq FQ < 75p$	0.743
	(0.007)
$p75 \leq FQ < 100p$	0.783
	(0.008)
Log-wage at origin	0.117
	(0.009)
Controls	✓
R^2	0.069
N	18,669

Note: Standard errors in parentheses. The tables report a specification on datasets based on workers that experience experience an Employment to Unemployment to Employment transitions (E-U-E). In Panel **(a)**, column (2) excludes separations that are justified in the Italian labor law (*giusta causa*). In the model, *Controls* include the pre-transition human capital and a polynomial in labor market experience. Referenced on page(s) [22] .