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Epidemics and Macroeconomic Dynamics

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Epidemics and Macroeconomic Dynamics*

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Abstract

We propose a novel SIR-macro model in which virus transmission is uncertain. The model is solved with the perturbation method around a deterministic infectious steady state. Assuming a stationary infection process, a positive infection shock increases infection while reducing consumption and hours worked for susceptible individuals. Further, we estimate our model with the recent US data on the COVID-19 outbreak. Historical decomposition obtained with Bayesian techniques finds that the dis-containment rule that encourages people to work more, as well as infection shock and technology shock, play an important role in characterizing US infection and macroeconomic dynamics.

Keywords: COVID-19, SIR-macro, Perturbation method, Bayesian estimation
JEL classification: E1, I1, H0

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1 Introduction

The recent pandemic not only created a large loss in human lives but also devastated the economy on an unprecedented scale. This necessitated an analysis of the epidemic and the economy, namely, a trade-off between infection and economic activities. Importantly, the outbreak and development of epidemics are often hard to predict which obligate people to make economic decisions under uncertainty. How does "infection shock" create a trade-off between infection and economic activities? What kind of policy shocks or rules are successful at mitigating infectious disease while limiting economic losses? How do we quantify the impact of infection on economic activities? To answer these questions, we provide a macroeconomic model of epidemics with uncertainty in a full-scale DSGE model. We believe that the analytical framework we propose is general enough and can be applied to any type of infectious disease such as influenza and STD.

We model uncertainty with respect to virus transmission. Specifically, "infection shock" that triggers a new infection is defined as a random variable. There are three types of agents: susceptible, infected and recovered. New infections every period arise not only through direct contact of susceptible individuals with infected individuals but also from their economic activities, namely consumption and working. Importantly, susceptible individuals decide how much to work and to consume while facing an uncertain future infection rate. The theoretical model is calibrated with recent US data under the COVID-19 crisis. It is then solved with the standard perturbation technique around a deterministic infectious steady state. We show how infection shock, as well as other types of shock including policy shock, propagates and creates a trade-off between infection and economic activity. Finally, we estimate the infection process together with other exogenous processes with Bayesian methods for US data.

Our findings are the following. A positive infection shock increases new infections while it reduces aggregate consumption and hours worked as a result of precautious behavior of susceptible individuals. A positive technology shock providing higher income also increases infections since it encourages people to consume more. Surprise in the consumption tax reduces infections since it discourages people from consuming and working.

In estimation, we find that a policy rule that systematically decreases the consumption tax along with the development of new infections, accounts for the dynamics of infection and aggregate consumption and aggregate hours worked in the US economy. The historical decomposition based on our estimation results reveals that the development of new infection is largely due to infection shock. We also find that despite the development of infections in the US, a decrease in aggregate consumption is avoided largely due to a positive technology improvement while a fall in aggregate hours worked is avoided mainly because of the systematic dis-containment policy rule that encourages people to work more.

To the best of our knowledge, this is the first paper to incorporate the uncertainty in the macroeconomic model of epidemics. Many attempts have been made since the outbreak of COVID-19 among which a seminal work by Eichenbaum et al. (2020b) analyzes such trade-offs by adding macroeconomics to an otherwise classical SIRD model. Eichenbaum et al. (2020c) explore the impact of testing while Eichenbaum et al. (2020a) extend their framework into a New Keynesian setup. Along this line, Krüger et al. (2021) incorporate a two-sector setup in which substitutability between goods differs. Giagheddu and Papetti (2020) extend the SIR-macro model by incorporating different age groups and age-specific economic interactions. Hamano et al. (2020) introduce misperception of households about the true state of infection.

Bodenstein et al. (2020) incorporate deterministic epidemiological dynamics into a multi-sector growth model. Similarly, Acurio Vásconez et al. (2020) introduce deterministic epidemiological dynamics into a DSGE model with financial frictions. Different from the endogenous labor supply decision of susceptible individuals in Eichenbaum et al. (2020b), in their setup, the labor supply is assumed to be exogenous and tied into the epidemiological dynamics. All these papers, however, rely on a deterministic perfect foresight simulation without explicitly treating uncertainty with respect to virus transmission, as ours does.

There have been many attempts to estimate SIR-type epidemiological models. For example, Fernández-Villaverde and Jones (2020) take a standard SIRD model to the data from various cities, states, and countries by allowing for time-varying contact rates. These time variations are meant to capture changes in social distancing. The work of Arias et al. (2021) is the most closely related to our paper. They cast the SIRD dynamics to a non-linear state-space representation. They use Sequential Monte Carlo techniques and Belgian data to estimate time-varying parameters that govern the infection dynamics. While they focus on the SIRD epidemiological model, we estimate the SIR-macro model that incorporates interactions between epidemiological dynamics and economic activity. Krüger et al. (2021) estimate some of the key model parameters that are related to health status in a SIR-macro model. Their approach is based on a grid search method to minimize the root mean square error between the actual and model-implied weekly deaths from Swedish data while our estimation strategy is based on a Bayesian posterior simulation. Fujii and Nakata (2021) attempt to forecast the economic impact of the COVID-19 pandemic in Japan by establishing a reduced-form relationship between the state of the pandemic and output with a perfect foresight simulation. The advantage of our approach is that it is quite suitable for utilizing the existing computational routines.[1]

The remainder of this paper is organized as follows. In the next section, we present the SIR-macro model with uncertainty. In Section 3, we calibrate the model based on US data. Various types of impulse response functions are shown in Section 4. Section 5 provides the Bayesian estimation results and historical decomposition based on US data. The last section concludes the paper.

2 The Model

We present an SIR-macro model with uncertainty. Uncertainty arises with respect to future virus transmission as well as labor productivity, preference for current consumption, disutility in supplying labor, and consumption tax. To ease the comparison, other parts of the model explicitly follow Eichenbaum, Rebelo, and Trabandt (2020b) which incorporates

[1] They also update their simulation results. Our model, which incorporates the endogenous trade-off between containment policies and economic activity, potentially compliments their approach because our model can be easily estimated in real-time once the latest data become available by using the idea of online estimation developed by Cai et al. (2021).

economic activities in the standard SIR model.

2.1 Infection

In the baseline SIR model, there were four groups: susceptible, infected, recovered, and dead people. Those who are susceptible are a mass S_t at period t who are not infected yet but potentially will be in the future. The infected is a mass I_t. After the disease, some people recover or die and join a mass of recovered R_t. Among those who recover, a fraction again falls into a mass of susceptible S_t. The dynamics of the three groups are thus

$$S_{t+1} = S_t + \pi_s R_t - T_t,$$
$$I_{t+1} = T_t + (1 - \pi_r - \pi_d) I_t,$$
$$R_{t+1} = (1 - \pi_s) R_t + (\pi_r + \pi_d) I_t,$$

where π_r is the recovery rate and π_d is the death rate. π_s is the fraction of these people who become susceptible.[2] Following Eichenbaum, Rebelo, and Trabandt (2020b), we consider a linear process for new infection T_t. The total number of newly infected people T_t evolves as

$$T_t = \pi_1 \left(S_t C_t^s\right) \left(I_t C_t^i\right) + \pi_2 \left(S_t N_t^s\right) \left(I_t N_t^i\right) + \pi_3 S_t I_t + \pi_4 S_t V_t, \tag{1}$$

where C_t^j and N_t^j represent total consumption and hours worked of group $j = \{s, i, r\}$. Parameters π_1, π_2, π_3, and π_4 govern the infection due to consumption, working and through other types of contact with infected people, respectively. Furthermore, a proportion of the new infections, $\pi_4 S_t V_t$ comes from direct contact with "virus" V_t. Specifically, we assume that V_t follows the AR(1) process as

$$\log (V_t) = \rho_V \log (V_{t-1}) + \varepsilon_{Vt},$$

[2]Fukao and Shioji (2021) in a reduced form SIR-macro model consider similar stationary dynamics for individuals.

where ε_{Vt} is "infection shock" which is an independent and identically distributed (iid) random variable with zero mean, and ρ_v represents the persistence of the shock.[3] With the above new infection, the endogenous new infection rate is defined as

$$\tau_t \equiv \frac{T_t}{S_t}.$$

As a result of infection shock, the probability of a new infection is also stochastic.

2.2 Susceptible People

The susceptible person maximizes the following lifetime utility U_t^s under uncertainty about the future infection:

$$U_t^s = u\left(c_t^s, n_t^s\right) + \beta \mathrm{E_t}\left[\left(1-\tau_t\right) U_{t+1}^s + \tau_t U_{t+1}^i\right],$$

where c_t^s and n_t^s are consumption and hours worked for the susceptible person, U_t^i is the lifetime utility in case of infection, and β represents the discount factor. Due to infection shock, the infection rate τ_t is stochastic and defined as:

$$\tau_t = \pi_1 c_t^s \left(I_t C_t^i\right) + \pi_2 n_t^s \left(I_t N_t^i\right) + \pi_3 I_t + \pi_4 V_t. \tag{2}$$

The susceptible person maximizes her lifetime utility subject to the following budget constraint

$$\left(1+\mu_{ct}\right) c_t^s = w_t n_t^s + \Gamma_t^s, \tag{3}$$

where μ_{ct} is the tax rate on consumption, w_t is real wage, and Γ_t^s represents the lump-sum transfer from the government. In the following section, we assume that μ_{ct} has zero mean and follows an AR(1) process.

[3] Infection shock is very similar to "technology shock" in the neoclassical growth model (Solow, 1957). We don't know when the epidemic outbreaks will occur or how infectious the virus is. Just a no one observes "technology", and this is the reason why we often label it as "residual", no one can see the virus infecting susceptible individuals, and thus, it is also considered a residual transmission.

The first-order condition with respect to c_t^s yields:

$$u_1\left(c_t^s, n_t^s\right) - \left(1 + \mu_{ct}\right)\lambda_{bt}^s + \lambda_{\tau t}^s \pi_1 \left(I_t C_t^i\right) = 0.$$

where λ_{bt}^s is the Lagrange multiplier for the budget constraint (3) and $\lambda_{\tau t}^s$ is the Lagrange multiplier for the infection rate (2). The first-order condition with respect to n_t^s gives

$$u_2\left(c_t^s, n_t^s\right) + w_t \lambda_{bt}^s + \lambda_{\tau t}^s \pi_2 \left(I_t N_t^i\right) = 0.$$

The first-order condition with respect to τ_t is

$$\lambda_{\tau t}^s = \beta\left[U_{t+1}^i - U_{t+1}^s\right] + \beta \mathrm{E_t}\left[\lambda_{\tau t+1}^s \left(\pi_1 c_{t+1}^s C_{t+1}^i + \pi_2 n_{t+1}^s N_{t+1}^i + \pi_3\right) S_t\right].$$

The second term on the right-hand side of the equation arises because today's decision to prevent infection influences the number of people infected in the future.

2.3 Infected People

In the case of infection, the patient will recover with a probability π_r or stay infected. Her lifetime utility is given by:

$$U_t^i = u\left(c_t^i, n_t^i\right) + \beta \mathrm{E_t}\left[\left(1 - \pi_r - \pi_d\right) U_{t+1}^i + \pi_r U_{t+1}^r\right],$$

where c_t^i and n_t^i are consumption and hours worked of the infected person, respectively, and U_t^r is the lifetime utility of a recovered person. The infected person maximizes her lifetime utility subject to the following budget constraint:

$$\left(1 + \mu_{ct}\right) c_t^i = w_t \phi^i n_t^i + \Gamma_t^i, \tag{4}$$

where ϕ^i denotes productivity in the case of infection and Γ_t^i represents the lump-sum transfer from the government. The first-order condition with respect to c_t^i yields:

$$u_1\left(c_t^i, n_t^i\right) - \left(1 + \mu_{ct}\right)\lambda_{bt}^i = 0,$$

where λ_{bt}^i is the Lagrange multiplier for (4). The first-order condition with respect to n_t^i gives

$$u_2\left(c_t^i, n_t^i\right) + \phi^i w_t \lambda_{bt}^i = 0.$$

2.4 Recovered People

A recovered person may become susceptible again with the exogenous probability π_s. Her lifetime expected utility is:

$$U_t^r = u\left(c_t^r, n_t^r\right) + \beta \mathrm{E_t}\left[\left(1 - \pi_s\right) U_{t+1}^r + \pi_s U_{t+1}^s\right],$$

where c_t^r and n_t^r are consumption and hours worked of the recovered person, respectively. She maximizes the above utility subject to the following budget constraint:

$$\left(1 + \mu_{ct}\right) c_t^r = w_t n_t^r + \Gamma_t^r, \tag{5}$$

where Γ_t^r represents the lump-sum transfer from the government. The first-order condition with respect to c_t^r yields:

$$u_1\left(c_t^r, n_t^r\right) - \left(1 + \mu_{ct}\right)\lambda_{bt}^r = 0,$$

where λ_{bt}^r is the Lagrange multiplier for (5). The first-order condition with respect to n_t^r gives

$$u_2\left(c_t^r, n_t^r\right) + w_t \lambda_{bt}^r = 0.$$

2.5 Firms

Production in the economy is isomorphic as Eichenbaum, Rebelo, and Trabandt (2020b). There is a continuum of competitive firms of unit measure. They produce consumption goods C_t using the following linear production technology:

$$C_t = A_t N_t,$$

where N_t is the aggregate hours worked. The firm chooses N_t by maximizing its current profits. A variable A_t represents the level of technology that has mean one and follows an AR(1) process as

$$\log\left(\frac{A_t}{A}\right) = \rho_A \log\left(\frac{A_{t-1}}{A}\right) + \varepsilon_{At},$$

where ε_{At} is "technology shock" which is iid random variables with zero mean and ρ_A represents the persistence of the shock.

2.6 Government and Welfare

The budget of the government is balanced as:

$$\mu_{ct}\left(S_t C_t^s + I_t C_t^i + R_t C_t^r\right) = \Gamma_t^s S_t + \Gamma_t^i I_t + \Gamma_t^r R_t.$$

Additionally, the government implements the following simple containment policy:

$$\mu_{ct} = \rho_{\mu_c}\mu_{ct-1} + \xi \log\left(\frac{T_t}{T}\right) + \varepsilon_{\mu_c t}, \tag{6}$$

where $\varepsilon_{\mu_c t}$ is "containment shock" which is iid random variables with zero mean and ρ_{μ_c} represents the persistence of the shock. The parameter ξ governs the reaction following the development of the newly infected. A priori, we do not know whether ξ is positive or negative.

Finally, social welfare is defined as the lifetime utility at the initial date of being susceptible and infected as

$$U_0 = S_0 U_0^s + I_0 U_0^i.$$

2.7 Equilibrium

The model is completed by the following two market-clearing conditions: The goods market clears as

$$S_t C_t^s + I_t C_t^i + R_t C_t^r = C_t.$$

The labor market clears as

$$S_t N_t^s + I_t N_t^i \phi^i + R_t N_t^r \phi^r = N_t.$$

In equilibrium, we have $C_t^j = c_t^j$ and $N_t^j = n_t^j$ for $j = s, i, r$. Also, we assume the same amount of subsidy across different types of agents as $\Gamma_t \equiv \Gamma_t^s = \Gamma_t^i = \Gamma_t^r$.

2.8 Preference and Shock Process

For the preference, we assume that a group-j has the following utility function:

$$u(c_t^j, n_t^j) = \alpha_t \ln(c_t^j) - \frac{\theta_t}{2}(n_t^j)^2,$$

where α_t stands for the shift in the preference towards consumption, and θ captures the disutility from working. These exogenous variables are assumed to follow the following AR (1) processes:

$$\log(\alpha_t) = \rho_\alpha \log(\alpha_t) + \varepsilon_{\alpha t}, \qquad \log\left(\frac{\theta_t}{\theta}\right) = \rho_\theta \log\left(\frac{\theta_{t-1}}{\theta}\right) + \varepsilon_{\theta t},$$

where $\varepsilon_{\alpha t}$ and $\varepsilon_{\theta t}$ are iid random variables with zero mean and ρ_α and ρ_θ represent the persistence of the shock.

3 Calibration

In this section, we calibrate the model presented in the previous section. In calibration, we use recent US data under the COVID-19 crisis. Specifically, we define the infectious steady state and argue its characteristics.[4]

3.1 Infectious Steady State

Our purpose is to capture the infection dynamics and its trade-off with economic activity in the epidemic crisis. To this end, we assume an infectious steady state. One period in our model corresponds to one week. Specifically, we set the share of new infection at the steady state $T/(S+I+R)$ as 0.15% so that it corresponds to the average share of new infection from the first week of February 2020 to the second week of April 2021 in the United States. As of May 5, 2021, after the third wave of infection, the Centers for Disease Control and Prevention (CDC) reports that approximately 35% of the population has been infected in the United States. Given this number, we assume that 80 % of the population $S/(S+I+R)$ is susceptible implying that 20% of the population has been infected, has recovered or is dead at the infectious steady state. We swap π_4, the parameter that determines the transmission from virus V_t, for the first share and π_s for the second share and we get $\pi_4 = 0.00095304$ and $\pi_s = 0.0076267$, respectively. Specifically, the latter implies that 0.76 % of those recovered have no herd immunity. Note that by setting $\pi_4 = 0$, the model is identical to that discussed in Eichenbaum, Rebelo, and Trabandt (2020b) where they assume a zero infection steady state and discuss the pandemic with a deterministic perfect foresight simulation.

Based on influenza and activity survey data in the US, Eichenbaum, Rebelo, and Trabandt (2020b) assume that 1/6 of a new infection comes from consumption and 1/6 of a new infection is related to working. Following them, we assume that the 1/6 of transmission comes from consumption and 1/6 of a new infection is related to working, and the remaining new infections are attributed to direct contact with infected individuals

[4]All the computation and estimation in the paper are implemented by the RISE toolbox developed by Junior Maih.

or with the "virus". Specifically, at the infectious steady state, we assume that

$$\frac{\pi_1 C^s I C^i}{\pi_1 C^s I C^i + \pi_2 N^s I N^i + \pi_3 I + \pi_4 V} = \frac{1}{6},$$

$$\frac{\pi_2 N^s I N^i}{\pi_1 C^s I C^i + \pi_2 N^s I N^i + \pi_3 I + \pi_4 V} = \frac{1}{6},$$

$$\frac{\pi_3 I + \pi_4 V}{\pi_1 C^s I C^i + \pi_2 N^s I N^i + \pi_3 I + \pi_4 V} = \frac{2}{3}.$$

In calibration, we set $V = 1$ without loss of generality and obtain $\pi_1 = 1.1321 \times 10^{-7}$, $\pi_2 = 0.00014372$, and $\pi_3 = 0.10476$.[5] Note that, by construction, the model's basic reproduction number at the infectious steady state is 1.[6] This is, however, no longer the case once the epidemic starts.

Our choice of other parameter values in the preference also follows Eichenbaum, Rebelo, and Trabandt (2020b). We assume and $\phi^i = 0.8$, which implies lower productivity in the case of infection. The discount factor is $\beta = 0.96^{1/52}$. We set the level of technology $A = 39.8352$ and the weight on disutility from working $\theta = 0.0013$ so that the steady-state matches the observed 28 hours of work for recovered individuals and $\$58,000/52$ weekly income in the United States.

We provide the steady state values in Table 1. As argued, in our infectious steady state, the share of susceptible individuals S and new infections T are set at 0.8 and 0.15, respectively. As a result, there is substantial heterogeneity across agents. While recovered and infected individuals work the same hours ($N^i = N^r$), because of lower productivity due to infection, consumption by individuals who are infected is lower ($C^i < C^s$). Susceptible individuals work fewer hours to avoid the possibility of being infected at the steady state ($N^s < N^i = N^r$). Recovered individuals get the highest lifetime utility followed by that of susceptible individuals while those who are infected get the lowest value ($U^r > U^s > U^i$).

[5]These values are in line with those obtained in Eichenbaum, Rebelo, and Trabandt (2020b) which find $\pi_1 = 7.8 \times 10^{-8}$, $\pi_2 = 0.000124$, and $\pi_3 = 0.390186$ at the pre-infection steady state.

[6]With (2.1), the basic reproduction number at the steady state is $\mathcal{R} \equiv \frac{T/I}{\pi_r} = 1$.

Table 1: Infectious steady state

S	C^s	N^s	u^s	U^s	λ_b^s
0.8	1.1007×10^3	27.6320	6.5168	8.2237×10^3	8.9663×10^{-4}
I	C^i	N^i	u^i	U^i	λ_b^i
0.0038	892.3077	28	6.2938	8.1898×10^3	0.0011
R	C^r	N^r	u^r	U^r	λ_b^r
0.1962	1.1154×10^3	28	6.5170	8.2312×10^3	8.9655×10^{-4}
T	λ_τ^s	μ_c	Γ	A	V
0.0015	-41.8892	0	0	39.8352	1

4 Macroeconomic Dynamics with Epidemics

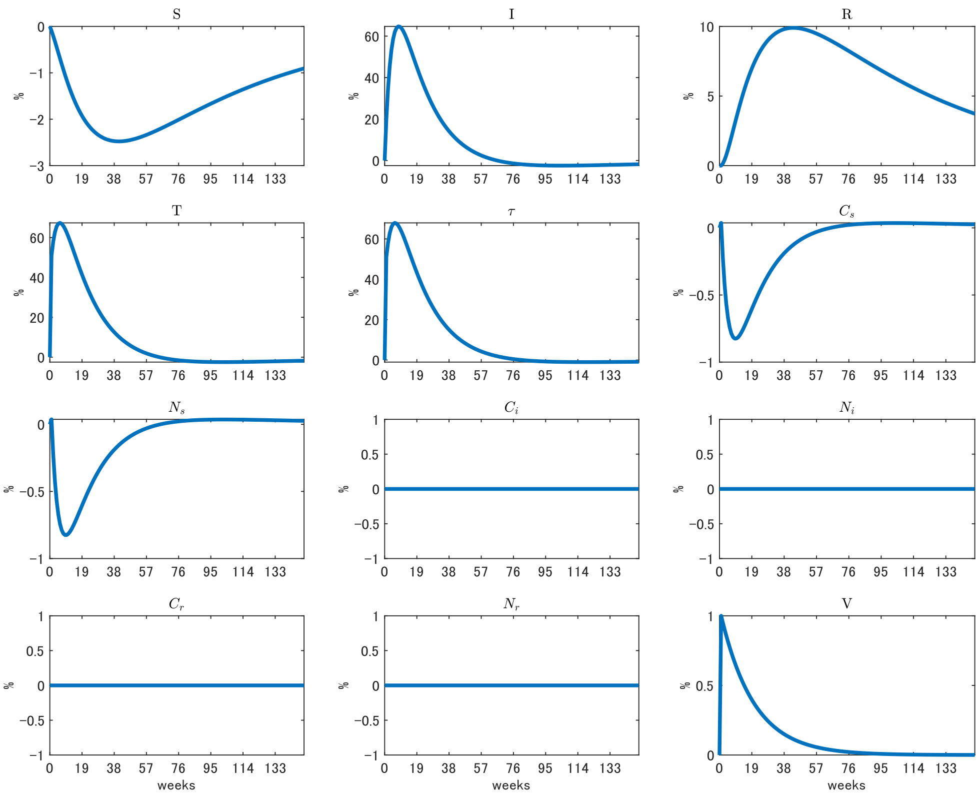
To understand the macroeconomic dynamics with epidemics, we solve the model with the perturbation (Judd and Guu, 1993 and Juillard, 2003) around the above-mentioned infectious steady state. We present the impulse response functions following infection shock, technology shock, and containment policy shock. Through the exercises, we will see a clear trade-off between infection and economic activities such as consumption and hours worked. Finally, we show that the policy rule that we specify can be a powerful stabilization device in containing the outbreak of epidemics.

4.1 Infection Shock

Figure 1 shows the impulse response functions of major variables following one percentage point increases in infection shock ε_{Vt}. In the figure, we set $\rho_v = 0.95$ and $\xi = 0$ (no containment policy). Following such an infection shock, new infection T_t and infection rate τ_t increase dramatically achieving more than 60 percentage points higher in 7 weeks than their initial steady-state values. After the peak, these values start to decrease gradually. In 65 weeks, they become lower than their initial steady-state values before achieving the original steady state. Infection I_t also shows a similar pattern. In our model, in addition to the number of new infections T_t and infection I_t, the dynamics of S_t and R_t are stationary because of the possibility of being susceptible to infection after recovery ($\pi_s > 0$).

Following the development of infection, susceptible individuals reduce consumption

Figure 1: IRFs following infection shock



Each entry shows the percentage-point response of one of the model's variables to a one-percentage deviation of the infection shock.

C_t^s and hours worked N_t^s, the extent of which, however, is very limited compared to the dramatic evolution of the epidemic. Both consumption and hours worked of susceptible individuals fall the most by more than 0.8 percentage points in 9 weeks in our benchmark setting of parameter values. Interestingly, consumption and hours worked decline for the first 66 weeks and then increase slightly after achieving their initial values. These economic boom periods correspond to those with a lower rate of infection than the steady state infection. Contrary to susceptible individuals, the consumption and hours worked of those who are infected and recovered, or dead, (C_t^i, C_t^r, N_t^i and N_t^r) remain unchanged following the infection shock.

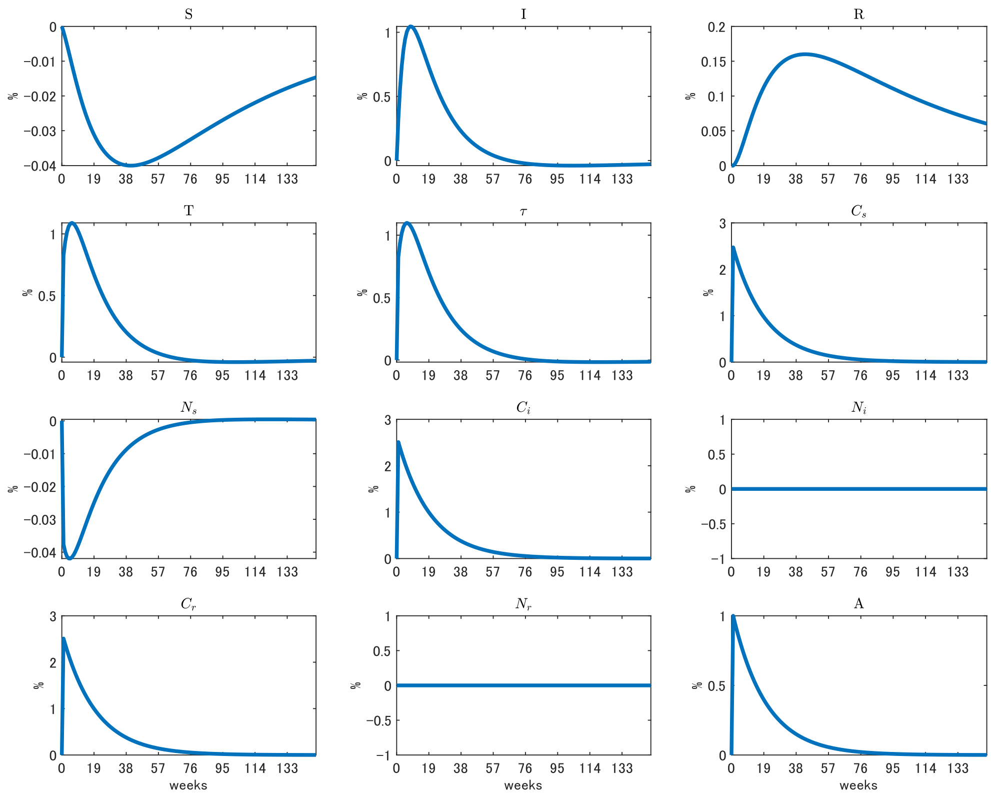
4.2 Technology Shock

Figure 2 shows the result of IRFs following one percentage point increase in labor productivity shock ε_{At}. As expected, a higher income provided by such a nice technology increases consumption of all types of agents. C_t^s, C_t^i and C_t^r increase by approximately 2.5 percentage points on impact. In our infectious economy, however, a higher level of consumption induces a new round of transmissions. New infections T_t and the number of infected individuals I_t increase by approximately 1 %. Accordingly, the number of susceptible individuals S_t gradually decreases, while those who are recovered or dead R_t gradually increases before reaching the original infectious steady state. Following such an outbreak of the epidemic, susceptible individuals cut consumption and hours worked, which is why the hours worked of susceptible individuals N_t^s declines. Consumption of the susceptible individuals C_t^s still increases since the income effect due to a nice technology dominates their precautionary consumption behavior. The hours worked of those who are infected and recovered or dead N_t^i and N_t^r remain unchanged following the technology shock with our specific preference as (2.8).

4.3 Consumption Tax Shock

As a salient feature of the SIR macro model, the epidemic can be further contained by scarifying economic activity. Figure 3 shows the IRFs following one percentage point increase in containment policy shock $\varepsilon_{\mu_c t}$. Containment policy embedded as a unilateral increase of consumption tax reduces consumption and hours worked for all agents. C_t^s, C_t^i and C_t^r and N_t^s, N_t^i and N_t^r decrease by more than 40 percentage points. Less consumption and fewer hours worked both for individuals who are susceptible and infected result in mitigating the spread of new infection T_t and infection I_t to the same extent. Accordingly, the number of susceptible individuals S_t increases gradually, while those who are recovered or dead R_t decrease before achieving the original steady state. The result shows the specificity of the perturbation around the infectious steady state. Obviously, if there were no infections at the steady state, the containment policy shock would have no impact.

Figure 2: IRFs following technology shock

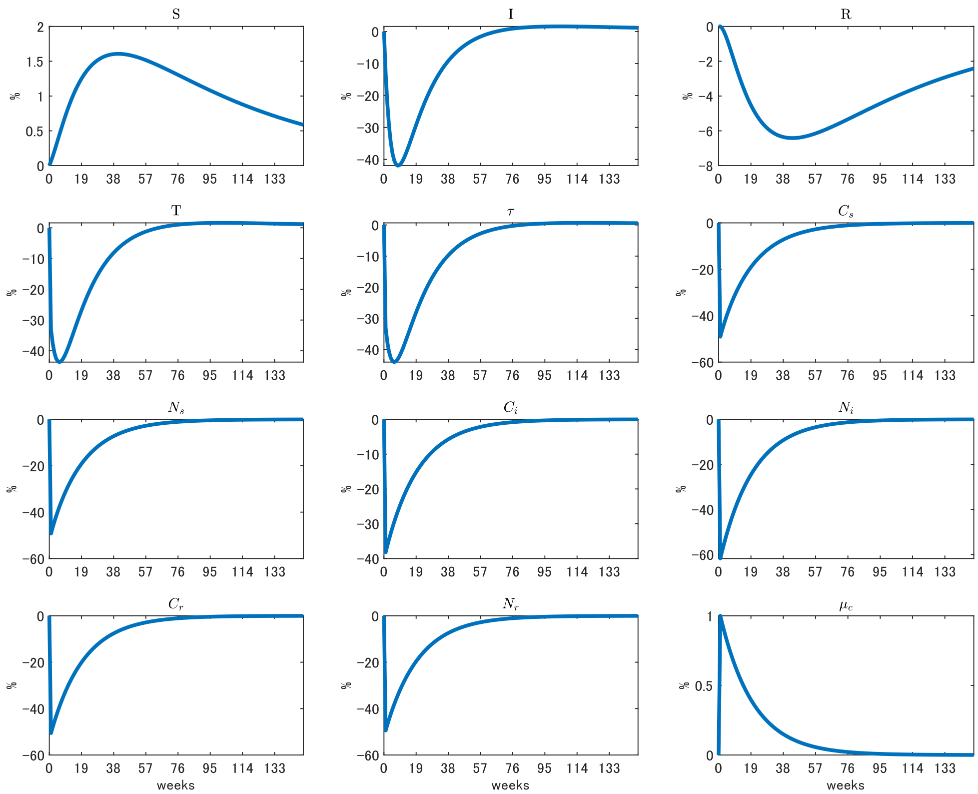


Each entry shows the percentage-point response of one of the model's variables to a one-percentage deviation of the technology shock.

4.4 Containment Policy Rule

We consider the case of the containment policy rule as specified in (6) with which the government systemically increases consumption tax following the development of new infection. Figure 4 shows the cases with different containment policies. The solid line corresponds to the case without any containment policy, $\xi = 0$ as in Figure 1. The dashed and dotted lines correspond to the case with $\xi = 0.1$ and $\xi = 0.5$, respectively. For instance, in the case of $\xi = 0.5$, by increasing the consumption tax systematically following the development of new infections, aggregate consumption and aggregate hours worked fall by approximately 40 percentage points while the infection peak becomes half (dashed lines).

Figure 3: IRFs following consumption tax shock



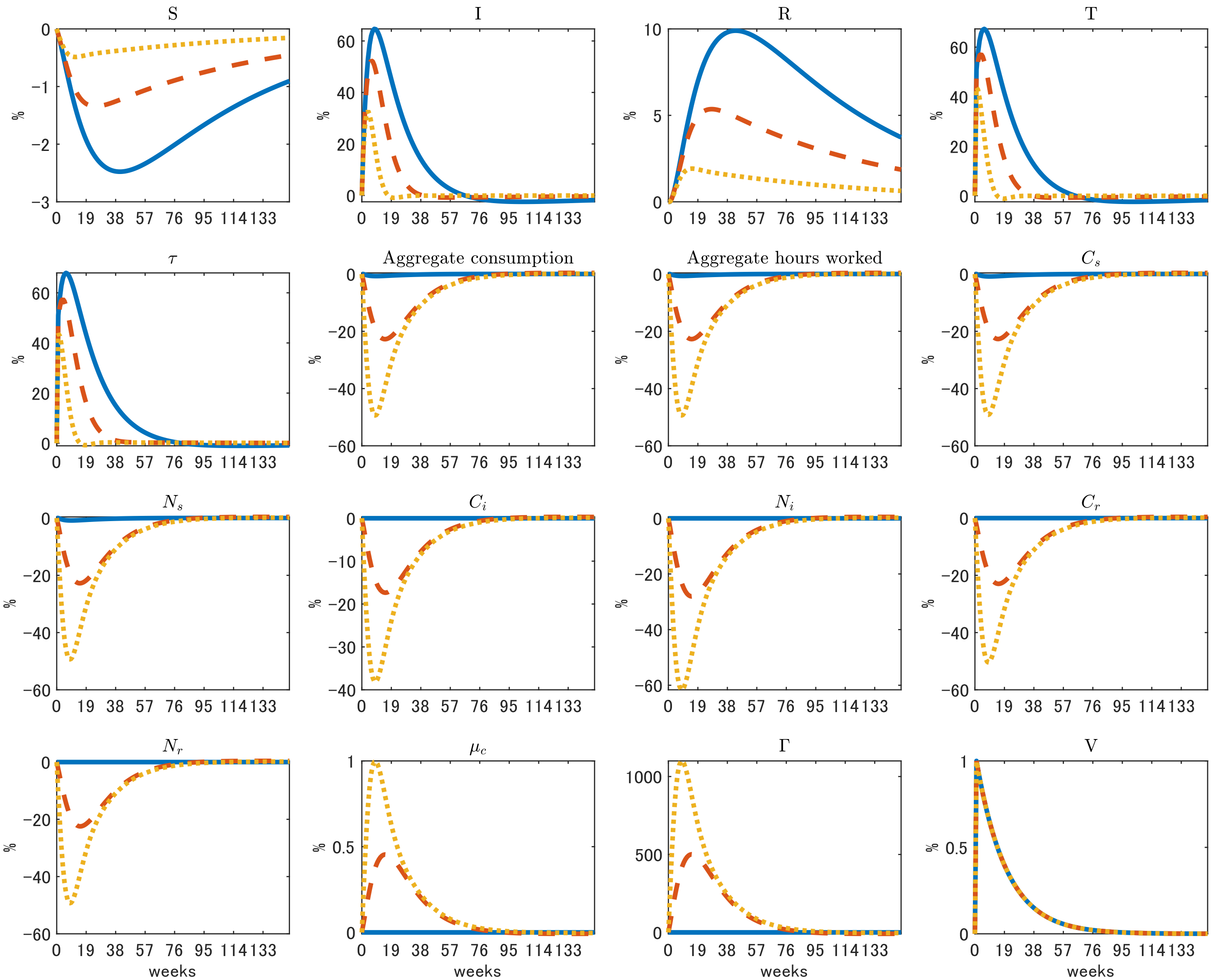
Each entry shows the percentage-point response of one of the model's variables to a one-percentage deviation of the consumption tax shock.

5 Estimation

We estimate the virus infection process, namely, the persistence and the standard deviation of the infection shock as well as other exogenous processes. Additionally, we estimate the parameter of the containment policy rule ξ. To this end, we use consumption and hours worked as well as weekly new infection for the US. The data ranges from the last week of January 2020 until the last week of March 2021, which corresponds to the first case of COVID-19 until the end of the third wave of infection. Consumption and hours worked are measured as indexes with respect to the sample average and only available at monthly frequencies. Figure 5 shows the data.

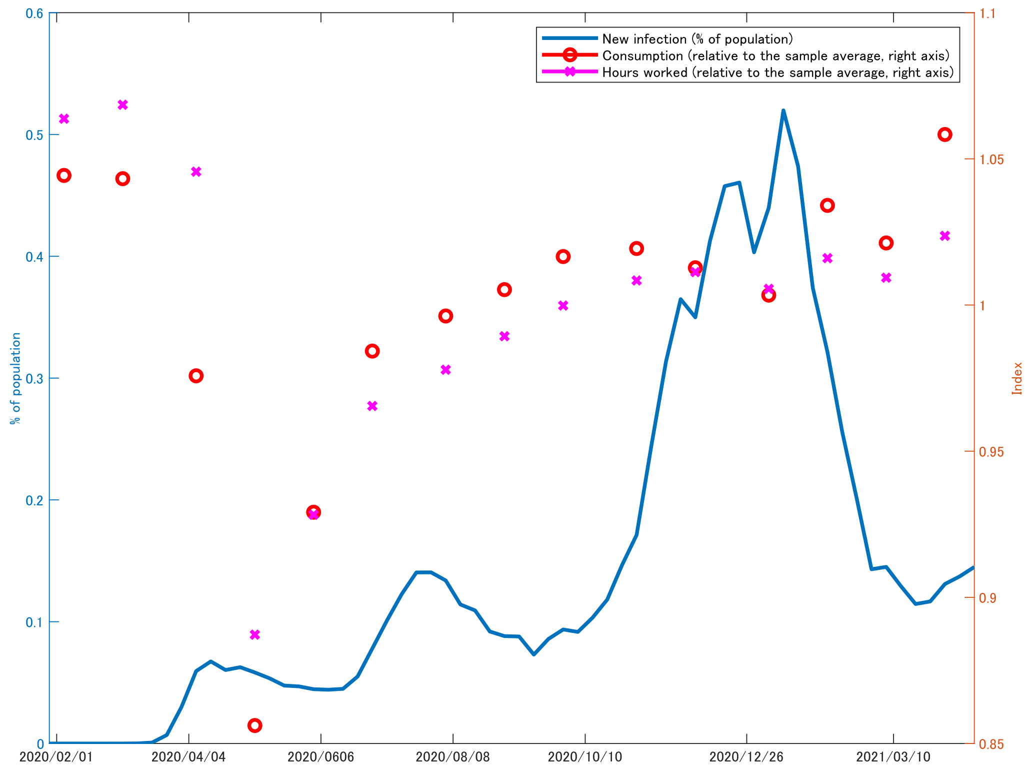
For the estimation, we rely on Bayesian techniques. Table 2 summarizes priors and

Figure 4: IRFs following infection shock with containment policy rule



Each entry shows the percentage-point response of one of the model's variables to a one-percentage deviation of the infection shock for the benchmark economy (solid line, $\xi = 0$), the economy with a higher level of containment rule (dashed line, $\xi = 0.1$), and the economy with the highest level of containment rule (dotted line, $\xi = 0.5$).

Figure 5: US data



The figure shows weekly new infection (solid line), monthly aggregate consumption (circles) and monthly aggregate hours worked (asterisks) for the US from the last week of January 2020 until the last week of March 2021. The new infection is shown as a percentage of the total population and measured on the left axis. Consumption and hours worked are defined as indexes relative to the sample average and measured on the right axis.

the result of the posterior simulation.[7] Priors in the table are shown with distribution and its upper and lower quantiles. These are standard as in the literature (Smets and Wouters, 2003 and Levin et al., 2006). We use uninformative priors for the parameter of the containment policy rule. Other priors are standard in the literature. Posterior modes and their 90% credible sets are reported.

The posterior mode of the standard deviation of infection shock is found to be significantly high ($\sigma_V = 50.20$), and it is persistent ($\rho_V = 0.93$).[8] The posterior modes of

[7]MCMC is conducted with 2500000 draws of posterior simulation in which the first 500000 draws are removed.

[8]Despite its high standard deviation and persistence, the contribution of the fourth term $\pi_4 V_t$ in generating new infections in equation (1) boils down and is comparable to other terms, all other things

standard deviation of technology shock σ_A and persistence ρ_A are well in the range found in the literature. On the other hand, persistence parameters of both preference shock ρ_α and labor disutility shock ρ_θ are only weakly identified with relatively wide 90% credible sets while playing a limited role in propagation with a small standard deviation close to zero.

Compared to the posterior mode of the standard deviation of the technology, preference, and labor disutility shocks, the posterior mode of the standard deviation of the containment policy shock is larger ($\sigma_{\mu_c} = 0.17$) and well-identified. Containment policy is sufficiently persistent as $\rho_{\mu_c} = 0.70$. Finally, the parameter of the containment policy rule is found to be slightly negative ($\xi = -0.0064$), suggesting a *dis*-containment policy rule to account for the observed US infection and macroeconomic dynamics.

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being equal. However, the other terms are endogenous. As we will see with the historical decomposition the contribution of infection shock is found to be substantially high for new infection dynamics.

Table 2: Estimation

		Prior distribution	Lower quantile	Upper quantile	Posterior mode	90% Interval
σ_V	Std D. of infection	Invgamma	0.0001	2.0000	50.1990	44.5751 59.0969
σ_A	Std D. of technology	Invgamma	0.0001	2.0000	0.0304	0.0257 0.0509
σ_α	Std D. of preference	Invgamma	0.0001	2.0000	0.0001	0.0001 0.0114
σ_θ	Std D. of labor disutility	Invgamma	0.0001	2.0000	0.0001	0.0001 0.0286
σ_{μ_c}	Std D. of containment	Invgamma	0.0001	2.0000	0.1654	0.1289 0.2652
ρ_V	Persistence of infection	Beta	0.0256	0.7761	0.9317	0.8911 0.9620
ρ_A	Persistence of technology	Beta	0.0256	0.7761	0.9598	0.8931 0.9817
ρ_α	Persistence of preference	Beta	0.0256	0.7761	0.0042	0.0000 0.7442
ρ_θ	Persistence of labor disutility	Beta	0.0256	0.7761	0.0042	0.0000 0.7421
ρ_{μ_c}	Persistence of containment	Beta	0.0256	0.7761	0.6967	0.6543 0.7291
ξ	Containement policy reaction	Uniform	-10	10	-0.0064	-0.0072 -0.0057

5.1 Historical Decomposition

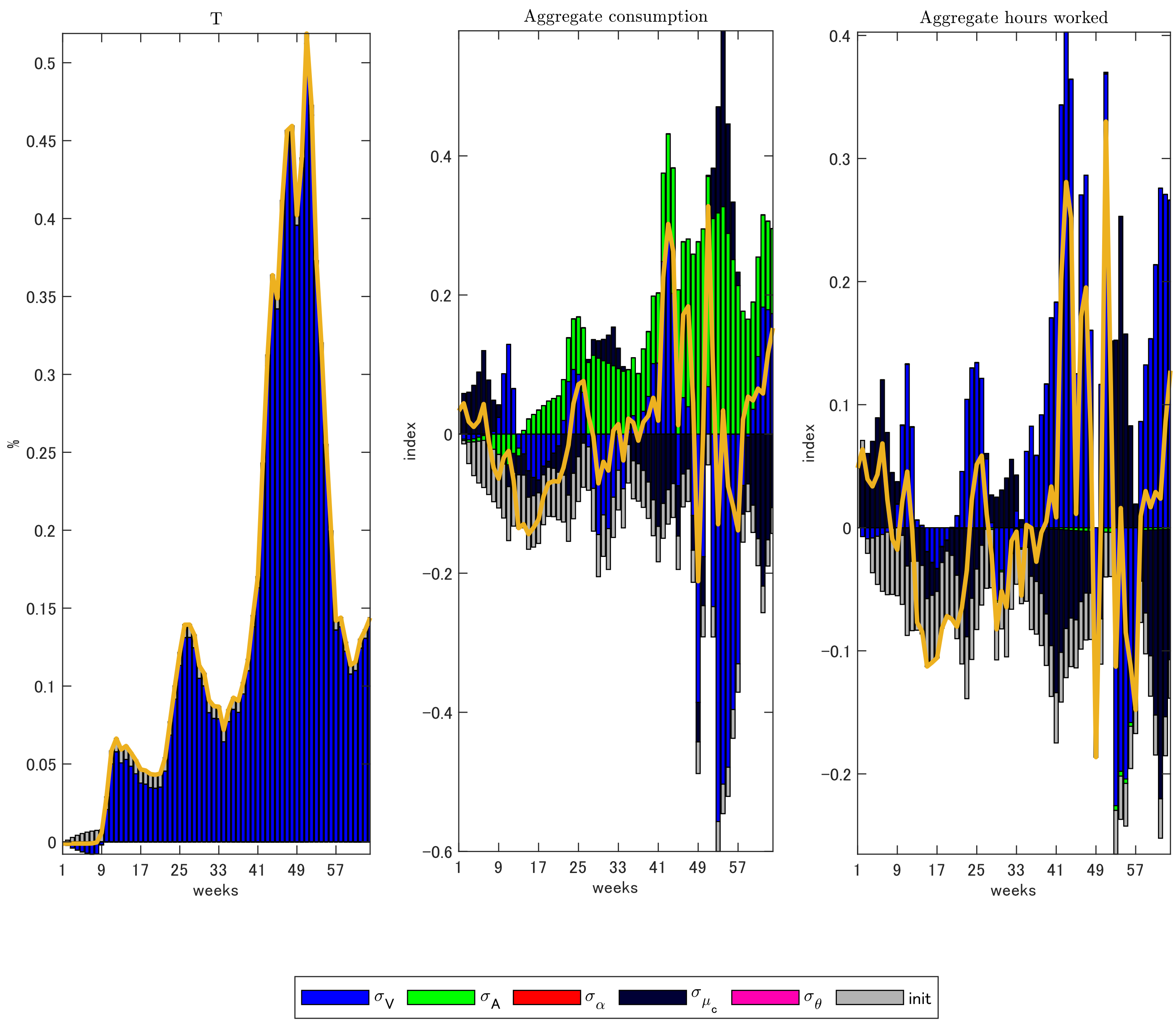
Figure 6 provides the historical decomposition obtained with our estimation results.[9] As the IRFs in the previous section indicate, the outbreak of epidemics leads to lower consumption and fewer hours worked for susceptible individuals. On the other hand, a positive technology shock ε_{At} increases consumption of all types of agents including those who are susceptible.

As the parameter of the containment policy rule is estimated with $\xi = -0.0064$, following the rise in the number of newly infected individuals T_t, consumption tax μ_{ct} decreases encouraging consumption and hours worked for both infected and recovered individuals. Such increased economic activity of infected individuals amplifies further the development of new infections T_t.

The dynamics of aggregate consumption and aggregate hours worked materialize as net dynamics of all types of agents discussed above. In Figure 6, the historical decomposition of aggregate consumption and hours worked are shown in the second and the third column panels. It can observed that aggregate consumption does not fall much along with the development of the epidemic because of technological improvements which encourage individuals to consume more (shown with the light green part in the panel of "Aggregate consumption"). Also, our historical decomposition reveals that aggregate hours worked do not fall sufficiently because of the *dis*-containment policy rule that encourages infected and recovered to work more despite the development of the epidemics (shown with the blue part in the panel of "Aggregate hours worked"). Containment policy shock $\varepsilon_{\mu_c t}$ also plays a role. Specifically, it contributes positively at each peak of the infection wave (shown with the black part in the panels of "Aggregate consumption" and "Aggregate hours worked). For new infections T_t, however, almost all its variation during the sample period is due to infection shock ε_{Vt} (shown with the blue part in the panel of T_t).

[9] The standard deviations of new infection, aggregate consumption, and hours worked in the data are 14.0147, 5.0891, and 4.7528, respectively. With the estimated shock processes and ξ, the standard deviations of these variables in the theoretical model are found to be 13.046, 22.689, and 18.84, respectively. Consumption and hours worked are four times higher than the data. Our estimation results replicate relatively well the standard deviation of new infection although the standard deviations are not targeted in estimation.

Figure 6: Historical Decomposition



Each entry shows the historical decomposition of one of the model's variables. Shocks considered are infection shock ε_{Vt}, technology shock ε_{At}, preference shock $\varepsilon_{\alpha t}$, consumption tax shock $\varepsilon_{\mu_c t}$ and labor disutility shock $\varepsilon_{\theta t}$.

6 Conclusion

We propose a novel SIR-macro model in which virus transmission is uncertain. The model is solved with the perturbation method around an infectious steady state. Assuming a stationary infection process, a positive infection shock increases the number of newly infected individuals while reducing consumption and hours worked for susceptible individuals. Furthermore, we estimate our model with the recent US data on the COVID-19 outbreak. Historical decomposition obtained with Bayesian techniques finds that containment policy shock and/or rule, as well as infection shock, play important roles in characterizing US infection and macroeconomic dynamics.

References

Acurio Vásconez, V., O. Damette, and D. W. Shanafelt (2020): "Macroepidemics and unconventional monetary policy: Coupling macroeconomics and epidemiology in a financial DSGE-SIR framework," *Covid Economics, Vetted and Real Time Papers*, 67, 199–253.

Arias, J. E., J. Fernández-Villaverde, J. F. Rubio-Ramírez, and M. Shin (2021): "Bayesian Estimation of Epidemiological Models: Methods, Causality, and Policy Trade-Offs," Working Paper No. 28617, National Bureau of Economic Research.

Bodenstein, M., G. Corsetti, and L. Guerrieri (2020): "Social distancing and supply disruptions in a pandemic," *Covid Economics, Vetted and Real Time Papers*, 19, 1–52.

Cai, M., M. Del Negro, E. Herbst, E. Matlin, R. Sarfati, and F. Schorfheide (2021): "Online estimation of DSGE models," *The Econometrics Journal*, 24, C33–C58.

Eichenbaum, M. S., S. Rebelo, and M. Trabandt (2020a): "Epidemics in the Neoclassical and New Keynesian Models," NBER Working Papers 27430, National Bureau of Economic Research, Inc.

——— (2020b): "The Macroeconomics of Epidemics," Working Paper No. 26882, National Bureau of Economic Research.

——— (2020c): "The Macroeconomics of Testing and Quarantining," Working Paper 27104, National Bureau of Economic Research.

Fernández-Villaverde, J. and C. Jones (2020): "Estimating and Simulating a SIRD Model of COVID-19 for Many Countries, States, and Cities," Working Paper No. 27128, National Bureau of Economic Research.

Fujii, D. and T. Nakata (2021): "Covid-19 and Output in Japan," Discussion Paper 21-E-004, RIETI.

Fukao, M. and E. Shioji (2021): "Is there a Tradeoff Between COVID-19 Control and Economic Activity? Implications from the Phillips Curve Debate," Tech. rep., Hitotsubashi University.

Giagheddu, M. and A. Papetti (2020): "The macroeconomics of ageâvarying epidemics," *Covid Economics, Vetted and Real Time Papers*, 58, 22–56.

Hamano, M., M. Katayama, and S. Kubota (2020): "COVID-19 Misperception and Macroeconomy," WINPEC Working Paper Series No. E2016, Waseda University.

Judd, K. L. and S.-M. Guu (1993): *Perturbation Solution Methods for Economic-Growth Models*, Springer New York, New York, NY. 4.

Juillard, M. (2003): “What is the contribution of a k order approximation,” Computing in Economics and Finance 2003 286, Society for Computational Economics.

Krüger, D., H. Uhlig, and T. Xie (2021): “Macroeconomic Dynamics and Reallocation in an Epidemic: Evaluating the“Swedish Solution”,” Working Paper 2020-43, Becker Friedman Institute.

Levin, A., A. Onatski, J. Williams, and N. Williams (2006): “Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models,” in *NBER Macroeconomics Annual 2005, Volume 20*, National Bureau of Economic Research, Inc, 229–312.

Smets, F. and R. Wouters (2003): “An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area,” *Journal of the European Economic Association*, 1, 1123–1175.

Solow, R. M. (1957): “Technical Change and the Aggregate Production Function,” *The Review of Economics and Statistics*, 39, 312–320.

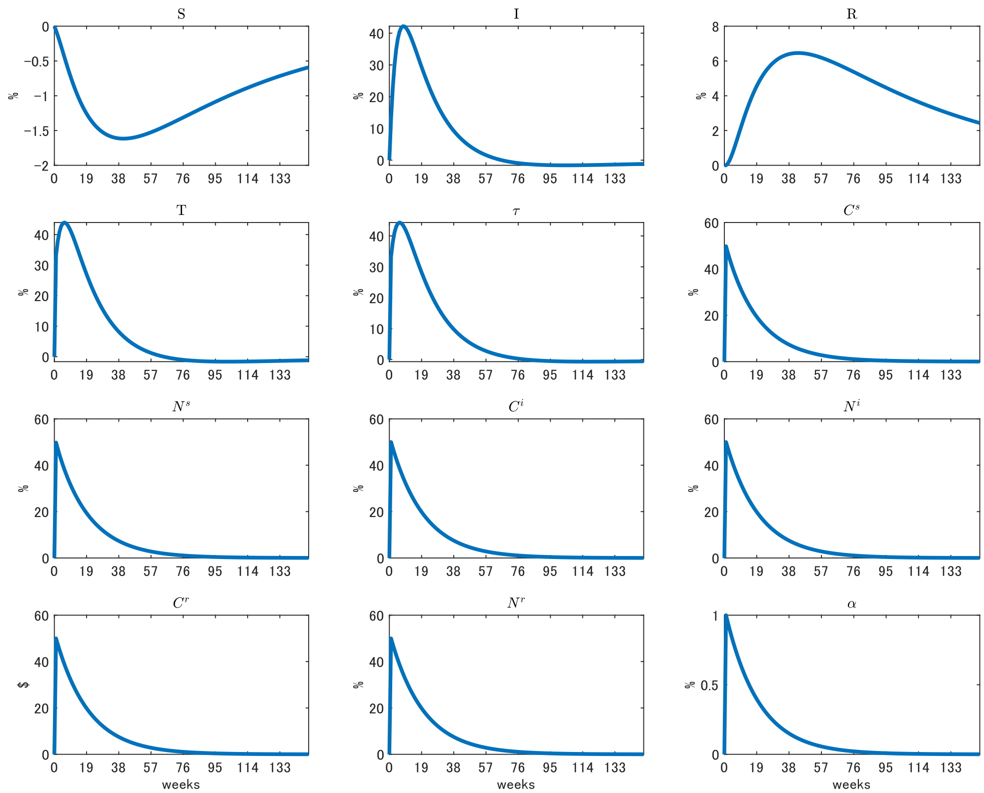
Appendix

A Preference and Labor Disutility Shock

A higher marginal utility in consumption brought by one percentage point increase in $\varepsilon_{\alpha t}$ also induces new infections. As Figure 7 shows the number of newly infected and infected individuals increase by approximately 40 percentage points. This is a direct consequence of a higher level of consumption and thus hours worked to finance consumption for both susceptible and infected individuals. Both consumption and hours worked of these agents (C_t^s, C_t^i, N_t^s and N_t^i) increase more than 40 percentage points. Thus the desire to consume prevails more the precautionary behavior of susceptible individuals.

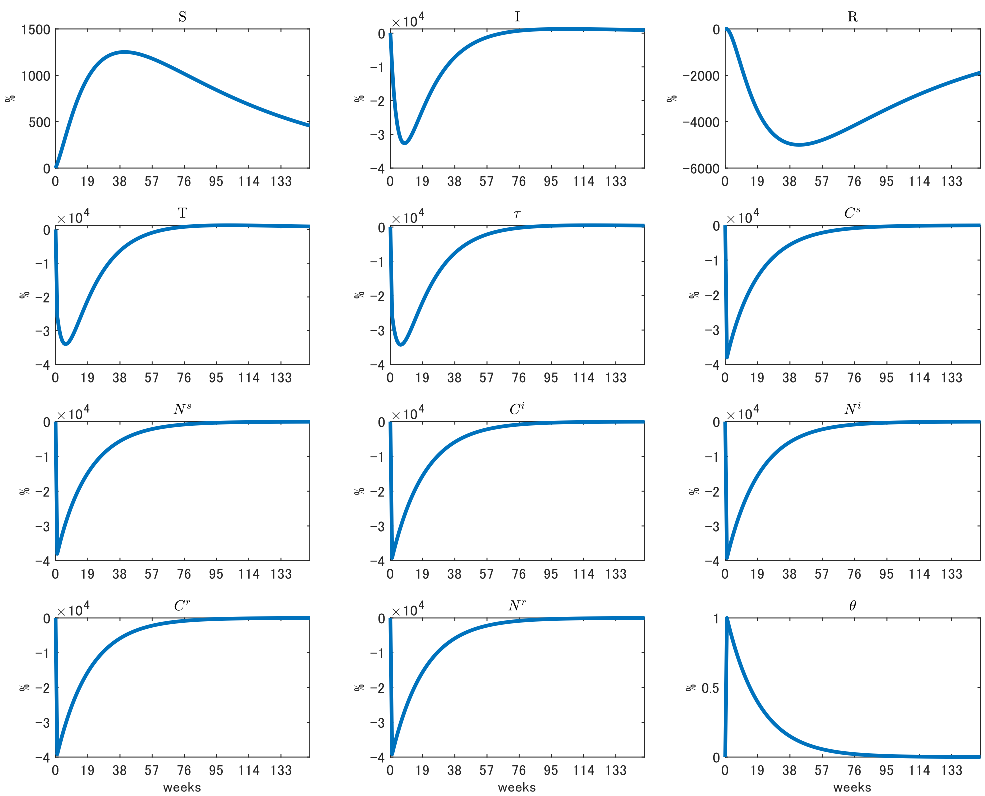
We somehow see the opposite pattern as the case of consumption preference shock argued previously in case of a rise in labor disutility. Figure 8 shows the IRFs following one percentage point increase in $\varepsilon_{\theta t}$. Consumption and hours worked for both susceptible (C_t^s and C_t^i) and infected individuals (N_t^s and N_t^i) drop substantially by approximately 35000 percentage points followed by a sharp decline in the number of newly infected T_t and infected individuals I_t to the same extent. This is, again, a direct consequence of fewer hours worked and, thus, less consumption for both susceptible and infected individuals.

Figure 7: IRFs following consumption preference shock



Each entry shows the percentage-point response of one of the model's variables to a one-percentage deviation of the consumption preference shock.

Figure 8: IRFs following labor disutility shock



Each entry shows the percentage-point response of one of the model's variables to a one-percentage deviation of the labor disutility shock.