Cautious Hiring*

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December 19, 2016

Abstract

Business cycles in the US since the mid-1980s have behaved very differently from previous cycles in two dimensions. First, employment recovers much more slowly in recent recessions than in earlier ones. Second, labor productivity is procyclical in previous recessions, but acyclical or countercyclical in recent ones. We explore the extent to which both phenomena can be accounted for by an increase in the importance of firm-specific human capital.

We develop a model in which firm-specific human capital introduces a hold-up problem between firms and workers. A senior worker with firm-specific skills has the same outside option as a comparable junior worker without these skills. Due to the low outside option of senior workers, wage bargaining results in their receiving a wage that is lower than their productivity. Since senior workers are a valuable asset to a firm, competition drives firms to pay junior workers a wage above their productivity. Firms are cautious about hiring junior workers, because they lower expected profits and increase the probability of bankruptcy. As the importance of firm-specific human capital rises, firms choose to hire more slowly, which leads to slower recovery for employment.

We estimate returns to tenure from the PSID, and use it to infer the importance of firm-specific human capital. We find that wage returns to average tenure have increased from 4.84% for the period prior to 1982 to 13.02% for the period after. We calibrate a quantitative version of the model and show that the estimated increase in the importance of firm-specific human capital can account for two-thirds of the slower employment recovery, and also reverses the cyclicality of labor productivity.

*We are indebted to V. V. Chari, Tim Kehoe, and Fabrizio Perri for valuable advice. We thank Naoki Aizawa, Manuel Amador, Kyle Herkenhoff, Larry Jones, Patrick Kehoe, Pawel Krolikowski, Jeremy Lise, Erzo G. J. Luttmer, Chris Phelan, David Rahman, Terry Roe, David Wiczer, and Kei-Mu Yi for their helpful comments. We thank Joseph Altonji and Nicolas Williams for kindly sharing their SAS codes for estimating returns to seniority from PSID. We are also grateful to participants at the SED Annual Meeting, Midwest Macro Meeting, XIX Workshop on Dynamic Macroeconomics, UW St Louis Graduate Students Conference, MEA Annual Meeting, McMaster, and Loyola seminars. Any remaining errors are our own.

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1 Introduction

It is nearly eight years since the 2008 recession, but employment continues to remain below the pre-recession level. This slow rate of recovery in employment is a phenomenon shared by the two previous recessions (1990 and 2001). However, it contrasts sharply with recessions prior to the mid-1980s, in which recoveries in employment are relatively fast.[1] This emergent pattern is directly related to a conversation commonly referred to by policymakers, journalists, as well as in the economic literature, as *jobless recovery*.[2] Jobless recoveries refer to the periods following recessions in which rebounds in aggregate output are accompanied by much slower recoveries in employment.[3]

Closely related to jobless recovery is the cyclicality of average labor productivity. When the recovery in employment lags the recovery in GDP, labor productivity rises. In fact, during the three most recent recessions, average labor productivity has been countercyclical, as opposed to being procyclical as in earlier recessions.[4]

We propose a novel mechanism that accounts for the slower rate of recovery in employment and also reverses the cyclicality of labor productivity. Our mechanism works through the increase in the importance of firm-specific human capital, namely, the relative productivity of senior to junior workers.

Central to our model is the idea that hiring workers functions as a type of investment. The marginal productivity of newly hired workers (henceforth junior workers) is low relative to the productivity of workers who have worked at the job for a period of time (henceforth senior workers). The existence of firm-specific skills, combined with a lack of commitment by firms, creates a hold-up problem between firms and workers. This hold-up problem implies that wage rates for senior and junior workers are compressed relative to their marginal productivities: senior workers are paid less than their marginal productivity and junior workers are paid more than their marginal productivity.

Intuitively, when a junior worker works at a firm for a period of time, they acquire firm-specific skills and become a senior worker. These firm-specific skills improve their marginal productivity at the firm. If the skills were general (i.e., not firm-specific), the senior worker could ask the firm for a corresponding raise in the wage by threatening to quit this firm and join another firm. However, with firm-specific skills, this threat is no longer credible. Departure from this firm results in a loss of all firm-specific skills for the senior worker, and they

[1] See Gali et al. (2012).

[2] Examples include Gordon (1993), Bernanke (2003), Gali et al. (2012), and Jaimovich and Siu (2012).

[3] This definition is taken from Jaimovich and Siu (2012). A reconciliation of this defintion with Gali et al. (2012) is included in section 2.

[4] This has previously been documented by McGrattan and Prescott (2012), Gali and van Rens (2010), and Berger (2012), among others.

would have to start over as a junior worker at another firm. Due to this low outside option of senior workers, wage bargaining implies that they are paid less than their productivity, and firms are able to reap positive profits off their senior workers. In anticipation of these positive profits once a worker becomes senior, competition in the labor market pushes the wage for junior workers above their marginal productivity. As a result, wage rates for senior and junior workers are compressed relative to their marginal productivities.

This compression of wages relative to marginal productivities leads to cautious hiring. A cost is incurred in the period a junior worker is hired, and a profit is made later when the junior worker turns senior. The stock of senior workers within a firm is valuable to the firm. Hiring junior workers lowers the profits of the firm in the period of hiring, and makes bankruptcy more likely to occur. Bankruptcy is costly to a firm, since it causes a separation of the firm from the stock of senior workers. Firms trade off a higher rate of growth with a higher probability of default. As a result, they hire cautiously and grow slowly.

The key parameter in our model is the relative productivity of senior to junior workers. As this ratio increases, firms are incurring more costs on each junior worker and earning more profits off each senior worker. This makes hiring more risky: each junior worker hired is more likely to trigger bankruptcy, and the stock of senior workers is more painful to lose if bankruptcy occurs. Therefore, increasing the relative productivity of senior to junior workers endogenously slows firm hiring. Our model focuses on the recovery following a recession. In the context of our model, a recession results in a larger than normal fraction of firms declaring bankruptcy. Recovery is driven by bankrupt firms that start over from size zero and grow back to the mature size. Slower firm hiring translates into a slower rate of employment recovery following a recession.

Next, we discuss the implication of our mechanism on average labor productivity. When the economy is recovering from a recession, there are two opposing economic forces on labor productivity. The first is related to the composition of senior to junior workers employed in the economy. Following an unexpected shock, a large number of senior workers are separated from firms. Firms gradually hire back junior workers. Therefore, in the ensuing recovery, the proportion of junior workers temporarily increases. This exerts downward pressure on average labor productivity, since junior workers are less productive than senior workers. The more quickly firms choose to hire junior workers, the stronger this composition force is.

The opposing force on labor productivity works through the decreasing returns to scale production technology. During recovery, average firm size is smaller than in the stationary equilibrium, for two reasons. First, following a recession, output and consumption are low relative to future periods. This increases the importance of present consumption relative to future growth, resulting in a smaller number of junior hires and a smaller average firm size.

Second, a recession causes a fraction of firms to go bankrupt. Bankrupt firms start over from size zero and engage in cautious hiring, and are smaller in size than an average firm in the stationary equilibrium. Due to the decreasing returns to scale production technology, a smaller average firm size drives up average labor productivity.

When firm-specific human capital is less important, firms grow fast. This large influx of junior workers into the economy strengthens the composition effect. As a result, the composition effect outweighs the decreasing returns to scale effect, resulting in a procyclical pattern for average labor productivity. When firm-specific human capital becomes more important, firms grow more slowly. This dilutes the influx of junior workers across time and weakens the composition effect, causing the decreasing returns to scale effect to dominate. As a result, a countercyclical pattern of average labor productivity emerges.

We estimate the return to seniority from the data and use it to infer the importance of firm-specific human capital.[5] Our estimation indicates that firm-specific human capital has significantly increased in importance since the mid-1980s.

This increase in the importance of firm-specific human capital leads to more cautious hiring at the firm level, which slows the speed of employment recovery at the aggregate level following a recession. In our quantitative analysis, our cautious hiring mechanism accounts for 65% of the slowdown in employment recovery. Further, our benchmark model suggests that changes in the importance of firm-specific human capital flip labor productivity from procyclical to countercyclical.

2 Related Literature

In this section, we briefly explain how our work ties into the existing empirical and theoretical literature regarding slow labor recoveries and emergent acyclical labor productivity patterns.

A number of papers including Gali et al. (2012), Gordon (1993), Groshen and Potter (2003), and Bernanke (2003) have documented the slower rate of recovery in labor through recent business cycles. Our model explains this change through an increase in the importance of firm-specific human capital since the mid-1980s.

Our theoretical explanation for the slow labor recovery overlaps with several others via the introduction of a mechanism which separates the marginal productivity of labor from the wage. For example, Hall (2014) and Midrigan et al. (2014) emphasize the role of discount rates. In their models, hiring a new worker incurs an upfront vacancy posting cost which is earned back from future differences between productivity and wage. If somehow the

[5] We follow methods used by both Altonji and Williams (2005) and Topel (1991) to estimate the returns to seniority over time.

effective discount rate of firms falls during the recession (for example, due to a tightening of borrowing constraints as in Midrigan et al. (2014)), hiring new workers becomes less profitable. Similar to their models, our model also features an upfront cost of hiring new workers. The difference is that in our model, this cost is due to the lower productivity of newly hired workers relative to their wage. In addition, our mechanism for generating the slower recoveries in recent recessions is an increase in the gap between the productivity levels of new and existing workers, instead of a reduction in the effective discount rate. An added benefit of our model is its ability to explain the emergent pattern in labor productivity over the business cycle.

Our mechanism does not fully account for the slower recovery in employment observed in the data. Complementary explanations include worker reallocation, the use of overtime hours, changes in worker effort, permanently destroyed employment at large firms, increased access to credit card borrowing, extension of countercyclical unemployment benefits, rigid real wages due to either lack of inflation or efficiency wages, changes in female labor force participation and gender-biased technological change.[6]

In addition to explaining the slow recovery, our work is related to the literature on the cyclicality of average labor productivity. Prior to the mid-1980s, labor productivity was procyclical. Explanations for this procyclical pattern include labor hoarding (for example, Oi (1962) and Fay and Medoff (1985)) and procyclical technology shocks in standard RBC models (for example, Kydland and Prescott (1982)).

Documentation and explanations for the new pattern in average labor productivity starting from the mid-1980s include Mulligan (2011), McGrattan and Prescott (2014) and McGrattan and Prescott (2014). To explain the new pattern in labor productivity, McGrattan and Prescott (2014) incorporates intangible capital into a standard RBC model, Arellano et al. (2012) shows that increased uncertainty at the firm level slows down firm hiring without dragging down labor productivity, Gali and van Rens (2010) demonstrates lower labor adjustment cost results in less labor hoarding hence less procyclical average labor productivity, and Barnichon (2010) attributes declined cyclicality of labor productivity to declined aggregate demand shocks.[7]

[6] Groshen and Potter (2003), Schreft et al. (2005), Bils et al. (2014), Luttmer (2012), Reinhart and Rogoff (2008), Herkenhoff (2014), Calvo et al. (2014), Shimer (2012), Sahin and Albanesi (2013) and Rabinovich and Mitman (2012).

[7] As discussion of new patterns in labor productivity are core to our model, a brief aside to consider the findings of Gali et al. (2012) is warranted. Gali et al. (2012) prefer to describe the emergent patterns following recent recessions as *slow recoveries* rather than *jobless recoveries* since recoveries of GDP (and several other series) have also slowed following recent recessions. We agree that this finding warrants additional recognition in the literature by corroborating this finding in the data and replicating the slower recovery in GDP through our model. However, we also argue that the relationship between GDP and labor productivity has fundamentally shifted. Gali et al. (2012) rely on aggregate statistics observed at four and eight quarters

Most related to our paper are those which attempt to provide explanations for both the jobless recoveries and the change in the cyclicality of labor productivity. Sims and Pries (2011) suggest that the nature of aggregate shocks has grown more asymmetric across sectors. This leads to a reallocation of labor across sectors and a slow recovery. Our mechanism can also be motivated through sectoral reallocation in that workers reallocate from routine sectors into non-routine sectors. Our paper is also related to Berger (2012) which attributes changes in average labor productivity to firm restructuring and the removal of less productive employees during recoveries. In both papers, hiring workers is initially costly but profitable later on. In Berger (2012), only certain new employees are good matches. As a result, hiring new employees is costly and the existing stock of workers is valuable. In our model, senior employees have acquired firm-specific skills which are not fully compensated in the wage. These workers represent a real value to the firm.

Our mechanism relies on the consideration of a firm's hiring decision as a form of investment. In this sense, our paper builds on the seminal work of Becker (1964). In this work, the costs and returns of firm-specific training are largely attributed to firms and there is a difference between the marginal product of labor and the wage. Additional empirical work on this topic includes Frazis and Loewenstein (2006), and Isen (2012). Our mechanism is also at work in Hudomiet (2015), where adaptation costs are incurred due to the lower productivity of newly hired workers. This leads to fewer jobs and a longer unemployment duration. However, our application differs from the work of Hudomiet (2015) in its exploration of how differences in adaptation costs across various occupations generate higher unemployment rates for less educated workers.

Firm-specific human capital is of critical importance to our mechanism. We use the returns to seniority as a proxy for firm-specific skills and estimate it following the methodology of Altonji and Shakotko (1985), Altonji and Williams (2005), and Topel (1991). This is related to a large literature estimating the returns to seniority, including Buchinsky et al. (2005), Abraham and Farber (1986), Dustmann and Meghir (2005), and Neal (1995).

The rest of the paper is organized as follows. In Section 3, we document two main patterns which have changed over more recent recessions. Section 4 presents the dynamic model and defines the stationary equilibrium. In Section 5, we present our stationary equilibrium results. Section 6 calibrates our parameters to the data. In Section 7, we present the business

following each recession to establish changes in recovery patterns. Our observations on average labor productivity make use of the time series as well as the correlation statistic. Our findings reveal that the pattern in average labor productivity following a recession is not monotonic which implies that discrete observations at various points could mask underlying changes. Observations of the correlation between GDP and labor productivity, which can be observed in Figure 3, provide additional support that the relationship between GDP and average labor productivity has changed.

cycle properties of our model and compare them to the data. Section 8 concludes.

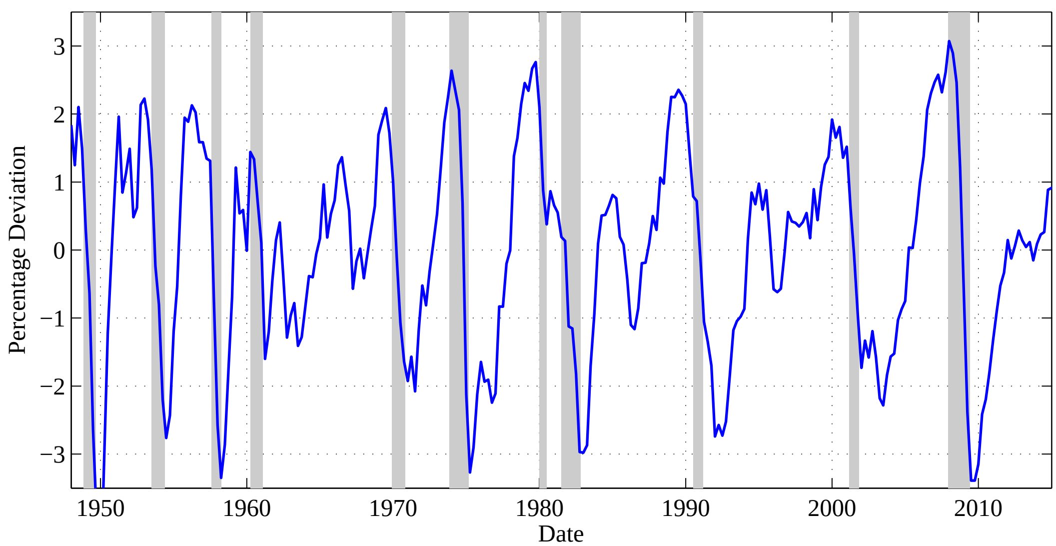
3 Data

Slower employment recoveries began with the recession of 1991. Moreover, not only have employment recoveries become slower after the mid-1980s, they have also become slower *relative* to the corresponding recoveries in GDP. This can be observed through the cyclicality of average labor productivity. In this section, we formally present these emergent patterns with a special focus on business cycles.

3.1 Employment Recovery

We measure employment as total hours worked in the nonfarm business sector from BLS. The time series of the percentage deviation of employment from its HP filtered trend is displayed in Figure 1.

Figure 1 – Employment: Total Hours



Source: Bureau of Labor Statistics, Authors' calculations

To compare the speed of employment recovery for different recessions, we calculate the number of years it takes for employment to return to its pre-recession peak. To standardize comparison across business cycles, we measure the speed of employment recovery as the length of time between the NBER end date and the date at which employment returns to its pre-recession peak. Measuring recovery from the NBER end date avoids conflating the length of recession with the length of recovery. The result is displayed in Table 1.

Table 1 – Employment Recovery Comparison

	Fast Recovery Recessions						
NBER Recession Starting Year	1953	1957	1960	1969	1973	1981	Average
Recovery Length	2	1	1.25	2.75	4	1.25	2.04

	Slow Recovery Recessions		
NBER Recession Starting Year	1990	2001	2007
Recovery Length	> 6	6.5	> 6

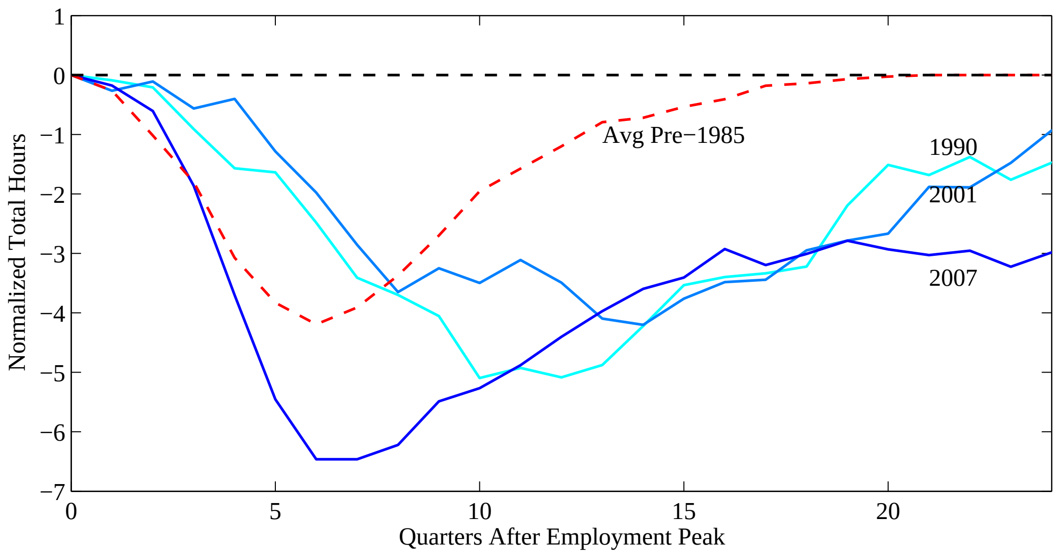
Source: Bureau of Labor Statistics, NBER recession dates, and Author's calculations

For the recessions prior to the mid-1980s (1953, 1957, 1960, 1969, 1973 and 1981 recessions[8]), employment fully recovers to pre-recession levels within four years. The average recovery length is 2.04 years. In contrast, even six years after each of the three most recent recessions (1990, 2001 and 2008 recessions), employment remains significantly below the pre-recession level.

To visually observe this slower recovery in employment, we provide a plot in Figure 2. In this figure, employment is normalized to zero at the pre-recession peak. Employment drops during each recession and gradually recovers afterwards. The three blue solid lines are the recovery paths of employment following the recent three recessions. The red dashed line is the average recovery path for recessions prior to the mid-1980s. This comparison highlights the slower recovery of the three more recent recessions. Additional comparison in the speed of recovery in employment is included in Appendix A.

[8] The recovery following the 1980 recession is interrupted by the 1981 recession, resulting in an incomplete recovery path.

Figure 2 – Employment Recovery Comparison



Source: Bureau of Labor Statistics, Authors' calculations
Pre-1985 recessions include 1948, 1953, 1957, 1960, 1969, 1973, and 1981 recessions

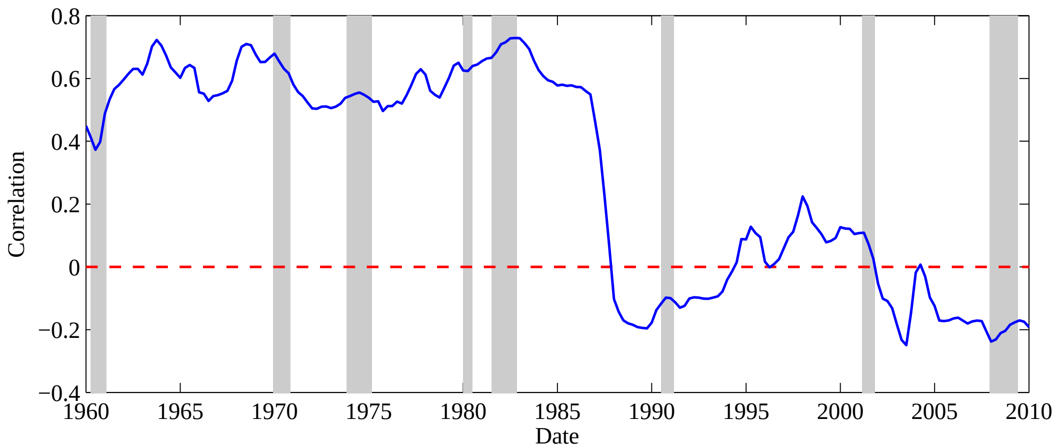
3.2 Average Labor Productivity

Not only have employment recoveries become slower for recent recessions, they have also become slower relative to the corresponding recoveries in output.[9] We define average labor productivity to be real output over total hours worked. If GDP recovers more slowly than employment following a recession, average labor productivity will stay below its stationary equilibrium level along the recovery path. Consequently, average labor productivity will be pro-cyclical. Alternatively, if GDP recovers faster than employment, average labor productivity will stay above its stationary equilibrium level along the recovery path and display a counter-cyclical pattern.

Figure 3 displays the 10-year centered moving-average correlation between labor productivity and GDP. Each point in the series is calculated as the correlation between GDP and average labor productivity for an interval of 10 years centered at the displayed date. Prior to the mid-1980s, labor productivity was procyclical. However, following the mid-1980s, labor productivity has been acyclical or even slightly counter cyclical.

[9]This fact has been previously documented. See Gali and van Rens (2010), Barnichon (2010), Berger (2012), and McGrattan and Prescott (2012) for examples.

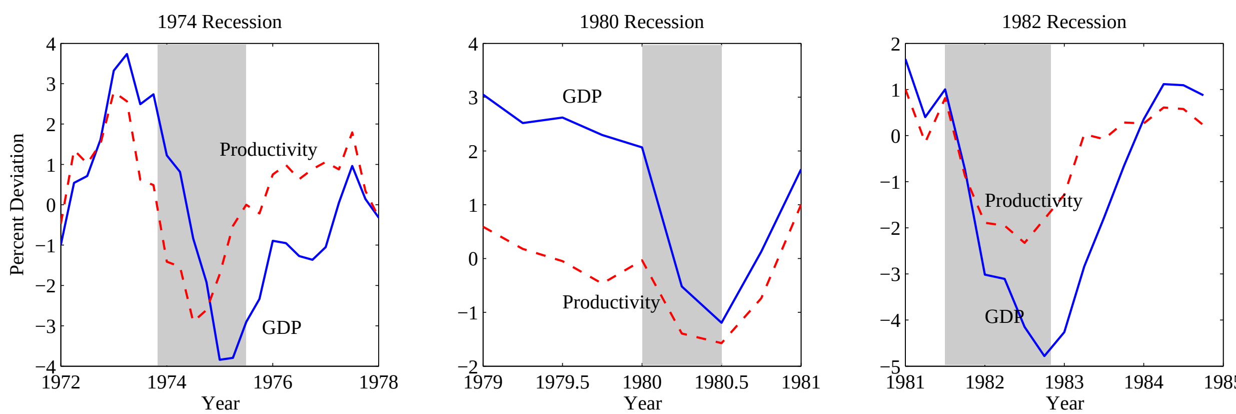
Figure 3 – Ten Year Centered MA Correlation in Labor Productivity and GDP



Source: Bureau of Labor Statistics, Bureau of Economic Analysis, Authors' calculations

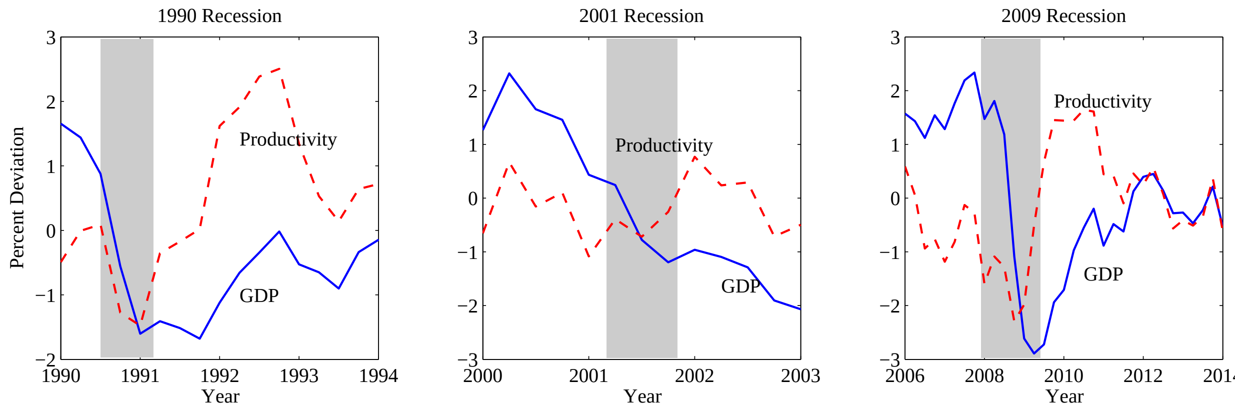
To focus on the correlation between average labor productivity and GDP over the business cycle, we plot the two data series from each of the six most recent recessions. In the three recessions immediately prior to the mid-1980s (which we will label the "Fast Recovery" recessions), average labor productivity tracks GDP closely: dropping at or just before the onset of the recession and recovering fairly quickly following the recession. However, in the three most recent recessions (the "Slow Recovery" recessions), the comovement between average labor productivity and GDP becomes weaker. Average labor productivity remains fairly flat leading into the recession, increases rapidly beginning mid-recession, and falls a few years after the recession. These features can be observed in Figures 4 and 5.

Figure 4 – The "Fast Recovery" Recessions



Source: Bureau of Labor Statistics, Authors' calculations

Figure 5 – The "Slow Recovery" Recessions



Source: Bureau of Labor Statistics, Authors' calculations

4 Environment

Our economy features a simple model of firm growth. Unlike standard models where the growth of firms occurs through the accumulation of physical capital, in our model, growth occurs through the accumulation of human capital in the form of senior workers.

Time is discrete. There are three types of agents: a continuum of representative households, a continuum of heterogeneous firms, and financial intermediaries. The problems of households and financial intermediaries are simple. The primary focus is on firms.

4.1 Households

There is a unit measure of representative households. Households supply labor to firms and consume wage and dividend income. Formally, the household's problem is given by

$$\max_{\{c_t, L_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u\left(c_t, L_t\right)$$
$$\text{s.t. } c_t \leq w_t L_t + d_t, \text{ for all } t$$

where w_t is the average wage income of the household and d_t is the dividends paid out by firms. Our workforce is composed of senior and junior workers which receive different wage rates. For simplicity, we assume that households are organized into families. Each family has a representative share of senior and junior workers. Consequently, $w_t = w_t^J s_t^J + w_t^S s_t^S$, where w_t^J and w_t^S represent the wage rates of junior and senior workers. s_t^J and s_t^S represent the share of junior and senior workers employed in the economy. Similarly, each family also owns a representative share of the firms. This family structure removes idiosyncratic shocks at households level and allows us to focus on firms. (This risk-sharing arrangement is similar to Merz (1995), Andolfatto (1996), and Midrigan et al. (2014).)

In equilibrium, the resource constraint implies that consumption for each household is equal to the total output of the economy: $c_t = Y_t$.

4.2 Firms

There is a unit measure of firms. They produce output according to the production function $y = zn^{\alpha}$. z is an idiosyncratic productivity shock that is i.i.d across firms and over time. The cumulative distribution function of z is F. n is the total effective labor units hired by the firm.

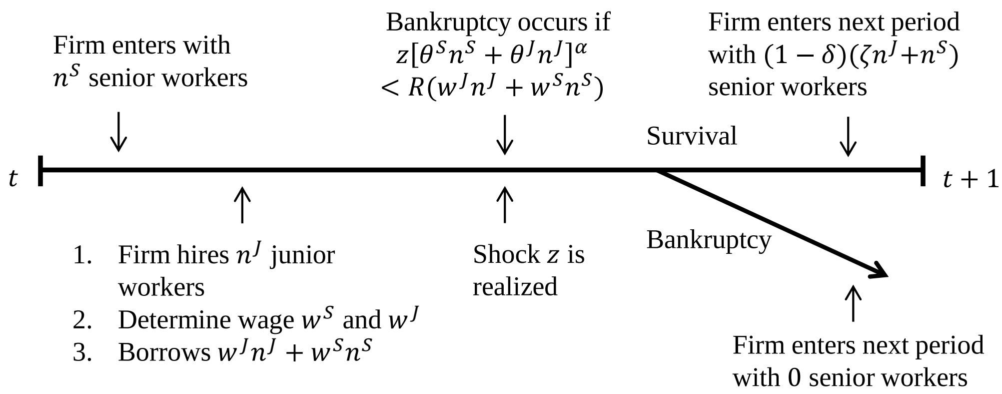
At each firm, workers differ in terms of their firm-specific skills. In the first period a worker is employed at a firm, they are junior without any firm-specific skills. Junior workers convert 1 unit of time into θ^J units of effective labor. After the initial period of employment, junior workers acquire firm-specific skills and become senior with probability ζ. Senior workers convert 1 unit of time into θ^S units of effective labor. Senior workers stay senior unless they are separated from the firm. In the case of separation, senior workers lose all firm-specific skills and start over as a junior worker in the labor market since skills are firm-specific. Firms are unable to directly hire senior workers. Instead, they must hire junior workers who become senior in subsequent periods. A firm with n^S senior workers which decides to hire n^J junior workers has total effective labor units given by $n = \theta^J n^J + \theta^S n^S$.[10]

There is a competitive market for junior workers. For a firm, junior workers hired in the past who have not become senior are no different than junior workers in the labor market who have never worked for this firm before. As a result of this observation, the only state variable we need to keep track of for each firm is the number of senior workers it has.

[10] Our model abstracts from general forms of human capital which can be transferred across firms. We assume that any forms of human capital which modify a worker's outside option are fully reflected in their wage.

One way our model implicitly includes general forms of human capital (albeit in a relatively rigid framework) is by assuming that a worker with h units of general human capital provides h units of effective labor of the category of their firm specific human capital. Firms choose how many units of effective junior labor to hire and pay w^J for each unit. A junior worker with h units of general human capital receives hw^J in wages and provides $h\theta^J$ effective units of labor to the firm. When that worker turns senior, they receive a wage of hw^S and provide $h\theta^S$ units of effective labor to the firm. Finally, we normalize the average general human capital per household to 1 so that the total supply of labor remains unchanged. In this manner, our model implicitly can account for general forms of human capital without any additional modification.

Figure 6 – Timing



4.2.1 Timing

Firms enter each period with n^S senior workers and decide how many junior workers n^J to hire. The wage bill for both types of workers has to be paid before production takes place. Firms finance their wage bill $\left(w^S n^S + w^J n^J\right)$ by taking out an intra-period working capital loan from the financial intermediaries. Next, productivity shock z is realized and production occurs. If revenue is sufficient to pay back the financial intermediaries, firms pay back their debt, distribute the remaining profits as dividends to the households, and continue into the next period with $(1-\delta)\left(n^S + \zeta n^J\right)$ senior workers, where δ is the exogenous separation rate between firms and workers. If the proceeds from output sales are insufficient to pay back financial intermediaries, firms go bankrupt. When a firm declares bankruptcy, all revenue from the period is confiscated by the financial intermediaries, and in the next period, the firm starts over with 0 senior workers. A visual depiction of the timing is displayed in Figure 6.[11]

We normalize the price of final output to be 1 and measure wage rates w^J and w^S in units of real output.

Firm's hiring problem is most easily explained in the backward order. We will start with the calculation of interest rate on working-capital loans, then move on to the determination

[11] We assume that at the end of each period, firms pay out all profits (if any) as dividends. That is to say, firms are restricted from retaining earnings. There exists a large literature in finance which argues that there are substantial costs of maintaining a large buffer stock of retained earnings. For example, Jensen (1986) argued that, in practice, if firms retain a large amount of their earnings in order to build up a buffer, managers use these funds in ways that benefit their private interests rather than shareholder interests. Since shareholders understand this, they give managers incentives to pay out funds immediately rather than retain them. We crudely model this effect by preventing firms from retaining any earnings. For brand new start-up firms, this assumption makes no difference in the first period. But it slows down firm hiring in subsequent periods. Allowing retention of earnings weakens our result quantitatively but not qualitatively.

of wage rates for senior and junior workers, and finally describe the hiring decision of firms.

4.2.2 Financial Intermediaries

Competitive financial intermediaries make intra-period loans to firms. Since productivity shocks are i.i.d. across firms, financial intermediaries are not subject to any aggregate uncertainty and behave as risk-neutral lenders. They offer an interest rate schedule $R\left(n^S, n^J\right)$ based on the number of senior workers n^S at a firm as well as the number of junior workers n^J hired by the firm. Interest rate schedule $R\left(n^S, n^J\right)$ and cutoff productivity level $z^*\left(n^S, n^J\right)$ for bankruptcy are jointly determined by:

$$z^*\left[n^S\theta^S + n^J\theta^J\right]^\alpha = \left(w^S n^S + w^J n^J\right) R \tag{1}$$

$$w^S n^S + w^J n^J = \left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right)\left(w^S n^S + w^J n^J\right) R + \int_{\underline{z}}^{z^*\left(n^J, n^S\right)} z\left[n^S\theta^S + n^J\theta^J\right]^\alpha f(z)dz \tag{2}$$

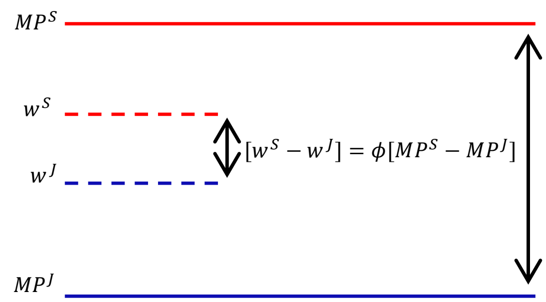
where Equation (1) defines the cutoff level in productivity draws z^* below which firms are unable to repay the loan and Equation (2) is the break-even condition for risk-neutral financial intermediaries. Intuitively, firms with higher risk of bankruptcy need to pay a higher interest rate on their working-capital loan to compensate financial intermediaries for the lower risk of getting paid back.

Implicitly assumed here is a lack of insurance for firms. Financial intermediaries are not allowed to sign long term contracts with firms to insure against their idiosyncratic productivity risk. This lack of insurance can arise when there is asymmetric information between firms and financial intermediaries. For example, if the productivity shocks are unobservable by financial intermediaries so that no contracts can be contingent upon the shock realizations. Here we assume this lack of commitment for simplicity. This is more natural for small firms which have not built up a high credit score, or established any reputation or relationship with banks.

4.2.3 Wage Bargaining

There is a competitive labor market for junior workers. The wage rate for junior workers is determined by market clearing condition. Due to firm-specific skills, there exists a joint surplus when a senior worker stays with their firm. If this match is destroyed, both the firm and the senior worker become worse off: the firm has to replace the senior worker with unskilled junior workers; and the senior worker has to start over at another firm as a junior

Figure 7 – Wage Compression



worker. We assume that the match surplus between a firm and its senior workers is shared via bargaining.

Implicitly assumed here is a lack of commitment by firms. This lack of commitment, together with firm-specific skills, create a hold-up problem between workers and firms. When a junior worker acquires firm-specific skills and turns senior, although their productivity has increased, they cannot claim an equivalent gain in wage from the firm. This is because the senior worker cannot credibly threaten to quit the firm, as they are unable to bring any of their firm-specific skills with them to another firm. As a result of their low outside option, senior workers are paid less than their marginal productivity. Competition in the labor market pushes the wage rate for junior workers above their marginal productivity.[12] In sum, the lack of commitment by firms leads to a compressed wage structure: junior workers are overpaid and senior workers underpaid relative to their marginal productivities. This idea is represented visually in Figure 7.

The wage rate for junior workers is determined in the competitive labor market, and the wage rates for senior workers are determined through bargaining between senior workers and firms. Let J be the value for junior workers and $S\left(n^S, n^J\right)$ be the value function for senior

[12] If the wage rates for both senior and junior workers were below their productivities, there would be excess demand for labor and the market would not clear.

workers at a firm which has n^S senior workers and hires n^J junior workers. Then

$$\begin{aligned}
J &= w^J\left(n^S,n^J\right) \\
&+\beta\underbrace{(1-\delta)\left(1-F\left(z^*\left(n^J,n^S\right)\right)\right)}_{\text{No Seperation from Firm}}\zeta S\left((1-\delta)\left(n^S+n^J\right)\right) \\
&+\beta\underbrace{\left[1-(1-\delta)\left(1-F\left(z^*\left(n^S,n^J\right)\right)\right)\zeta\right]}_{\text{Seperation from Firm or Not Transitioning into Senior}} J
\end{aligned} \tag{3}$$

$$\begin{aligned}
S\left(n^S,n^J\right) &= w^S\left(n^S,n^J\right) \\
&+\beta\underbrace{(1-\delta)\left(1-F\left(z^*\left(n^J,n^S\right)\right)\right)}_{\text{No Seperation from Firm}} S\left((1-\delta)\left(n^S+n^J\right)\right) \\
&+\beta\underbrace{\left[1-(1-\delta)\left(1-F\left(z^*\left(n^S,n^J\right)\right)\right)\right]}_{\text{Seperation from Firm}} J
\end{aligned} \tag{4}$$

The value J to a junior worker at a firm of size $\left(n^S,n^J\right)$ consists of the current period wage they receive $w^J\left(n^S,n^J\right)$ plus the continuation value. If there is neither bankruptcy of the firm nor exogenously separation of the worker from the firm, then the junior worker transition into a senior worker with probability ζ in the subsequent period. Otherwise, they stay junior. The value to a senior worker is similar. The only difference is that senior workers stay senior with probability 1, unless either bankruptcy or exogenous separation occurs.

Although the value to senior workers $S\left(n^S,n^J\right)$ depends on firm sizes $\left(n^S,n^J\right)$, the value to junior workers J does not. This is because junior workers have the freedom of choosing which firm to work for. Firms of different sizes have different probabilities of bankruptcy. Bankruptcy leads to separations of workers from firms. In this case, junior workers lose the opportunity of becoming senior and they have to start over as junior again at another firm. Therefore, firms with higher probability of bankruptcy need to offer a higher junior wage $w^J\left(n^S,n^J\right)$ to compensate junior workers for the risk. Equation (3) makes sure that firms of any size $\left(n^S,n^J\right)$ need to offer junior workers a wage $w^J\left(n^S,n^J\right)$ which yields junior workers a common value of J. This way, junior workers are indifferent about which firm to join.

Due to firm-specific skills, joint surplus is created when a senior worker stays with their firm. Wage bargaining between senior workers and their firms determines the wage of senior workers. In particular, the surplus to a senior worker of staying with their firm is $S\left(n^S,n^J\right)-J$. The surplus to a firm of being matched with a marginal senior workers is $V'\left(n^S\right)$. We assume that a fraction ϕ of the joint surplus is obtained by senior workers:

$$\frac{S\left(n^S,n^J\right)-J}{V'\left(n^S\right)} = \frac{\phi}{1-\phi}. \tag{5}$$

This pins down the gap between senior and junior values. The levels of junior and senior wage rates are determined by labor market clearing condition.

Note that wage rates w^S and w^J depend on the probability of bankruptcy $F\left(z^*\left(n^J, n^S\right)\right)$, which in turn also depends on the wage rates. An implicit assumption with regards to the timing of our model is that these four activities occur simultaneously: the firm's junior worker hiring decision, the wage determination of junior workers, wage bargaining with senior workers, and the intra-period borrowing from financial intermediaries.

Our model also features a multi-party bargaining with decreasing returns. Many potential bargaining protocols are possible: all senior workers can form a union and bargain with firms as a whole, or senior workers can bargain with firms as small groups. Here we adopt the bargaining solution of Stole and Zwiebel (1996), in which firms negotiates with each of their senior workers in turn. This bargaining protocol was first used by Cahuc and Wasmer (2001), and later on by Hawkins and Acemoglu (2014) and Elsby and Michaels (2013) among others.

There are two implications from wage bargaining Equation (5). First, the wage of senior workers relative to junior workers can be used to back out the differences in productivity levels across workers. Second, senior workers are more valuable to a firm relative to junior workers due to wage compression. In equilibrium, since firms hire junior workers in a competitive labor market, firms break even with each worker they hire. Hiring a junior worker is a costly investment which pays off when the worker becomes senior. In this sense, senior workers are really valuable to the firm. Should the firm face bankruptcy, the loss of senior workers represents a real cost. For this reason, firms are cautious in their hiring in order to balance growth against the risk of losing their stock of senior workers.

4.2.4 Firm Hiring Decision

Firms solve the following Bellman equation:

$$\begin{aligned} V(n^S) \quad = \max_{n^J} \quad & \int_{z^*(n^J,n^S)}^{\bar{z}} \Big\{ z\left[n^S\theta^S + n^J\theta^J\right]^{\alpha} \\ & - \left(w^S\left(n^S,n^J\right)n^S + w^J\left(n^S,n^J\right)n^J\right) R\left(n^J,n^S\right)\Big\} f(z)\,dz \\ & + \beta\left[F\left(z^*\left(n^S,n^J\right)\right)V(0) + \left(1-F\left(z^*\left(n^S,n^J\right)\right)\right)V\left[(1-\delta)\left(n^S+\zeta n^J\right)\right]\right] \end{aligned} \tag{6}$$

where $z^*\left(n^J, n^S\right)$ is the cutoff level of productivity for bankruptcy, $R\left(n^J, n^S\right)$ is the interest rate schedule on the working capital loan charged by financial intermediaries, and $w^S\left(n^S, n^J\right)$, $w^J\left(n^S, n^J\right)$ are the wage rates of senior and junior workers.

Lemma 1. *Combining the Bellman's equation of firms with the break-even condition of* *financial intermediaries, we can simplify the Bellman Equation (6) into:*

$$V(n^S) \quad = \max_{n^J} \quad \Big\{ E[z] \left[n^S\theta^S + n^J\theta^J \right]^\alpha - \left(w^S\left(n^S, n^J\right) n^S + w^J\left(n^S, n^J\right) n^J \right)$$
$$+\beta \left[F\left(z^*\left(n^S, n^J\right)\right) V(0) + \left(1 - F\left(z^*\left(n^S, n^J\right)\right)\right) V\left[(1-\delta)\left(n^S + \zeta n^J\right)\right]\right] \Big\}$$

Intuitively, since financial intermediaries are risk-neutral and breaking even, it is as if firms are maximizing expected profits for the current period. The only effect of bankruptcy is on the continuation value of firms. The proof of this lemma is in Appendix C.

Lemma 2. *Absent bankruptcy risk, wage bargaining Equation (5) implies:*

$$\left[S\left(n^S, n^J\right) - J \right] \quad = \quad \phi \left[\underbrace{\frac{\partial \left\{ E[z] \left[n^S\theta^S + n^J\theta^J \right]^\alpha \right\}}{\partial n^S}}_{MPL\ of\ Senior\ Workers} - \underbrace{\frac{\partial \left\{ E[z] \left[n^S\theta^S + n^J\theta^J \right]^\alpha \right\}}{\partial n^J}}_{MPL\ of\ Junior\ Workers} \right].$$

If we further assume that $\zeta = 1$ so that junior workers acquire firm-specific skills in one period with probability one, then:

$$\left[w^S - w^J \right] \quad = \quad \phi \left[\frac{\partial \left\{ E[z] \left[n^S\theta^S + n^J\theta^J \right]^\alpha \right\}}{\partial n^S} - \frac{\partial \left\{ E[z] \left[n^S\theta^S + n^J\theta^J \right]^\alpha \right\}}{\partial n^J} \right].$$

The proof of this lemma is in Appendix D. Intuitively, the value difference (wage difference) between senior and junior workers is compressed relative to the difference in their productivity difference, with the compression factor equal to the bargaining power of senior workers ϕ. Firm-specific skills introduce a hold-up problem between senior workers and firms. The lower the bargaining power of senior workers, the more severe the hold-up problem becomes, and the less the gain in value from junior to senior workers. This is visually displayed in Figure 7.

4.3 Recursive Stationary Equilibrium

Definition 1. A recursive stationary equilibrium consists of a value function $V\left(n^S\right)$ and a policy function $n^J(n^S)$ for firms, a value function $S\left(n^S, n^J\right)$ for senior workers, a value J for junior workers, wage and interest rates $\left\{ w^J\left(n^S, n^J\right), w^S\left(n^S, n^J\right), R\left(n^S, n^J\right) \right\}$, a distribution of firms sizes $g(n^S)$, output Y, and labor L such that:

1. Given $w^J\left(n^S, n^J\right)$, $w^S\left(n^S, n^J\right)$, and $R\left(n^S, n^J\right)$: $V\left(n^S\right)$ and $n^J(n^S)$ solve the firm's

Bellman equation:

$$
\begin{aligned}
V(n^S) &= \max_{n^J} \int_{z^*(n^J,n^S)}^{\bar{z}} \Big\{ z\left[n^S\theta^S + n^J\theta^J\right]^\alpha \\
&\quad - \left(w^S\left(n^S,n^J\right)n^S + w^J\left(n^S,n^J\right)n^J\right) R\left(n^J,n^S\right) \Big\} f\left(z\right)dz \\
&\quad +\beta\left[F\left(z^*\left(n^S,n^J\right)\right)V(0) + \left(1-F\left(z^*\left(n^S,n^J\right)\right)\right)V\left[(1-\delta)\left(n^S+\zeta n^J\right)\right]\right]
\end{aligned}
$$

where

$$
z^*\left[n^S\theta^S + n^J\theta^J\right]^\alpha = R\left(n^J,n^S\right)\left(w^S\left(n^S,n^J\right)n^S + w^J\left(n^S,n^J\right)n^J\right)
$$

2. Financial intermediaries break even with $R\left(n^J,n^S\right)$:

$$
\begin{aligned}
w^S\left(n^S,n^J\right)n^S + w^J\left(n^S,n^J\right)n^J &= \left(1-F\left(z^*\left(n^J,n^S\right)\right)\right)\left(w^S\left(n^S,n^J\right)n^S + w^J\left(n^S,n^J\right)n^J\right)R \\
&\quad + \int_{\underline{z}}^{z^*\left(n^J,n^S\right)} z\left[n^S\theta^S + n^J\theta^J\right]^\alpha f(z)dz
\end{aligned}
$$

3. Firms of all sizes $\left(n^S,n^J\right)$ offer the same value J to junior workers:

$$
\begin{aligned}
J &= w^J\left(n^S,n^J\right) \\
&\quad +\beta\left(1-\delta\right)\left(1-F\left(z^*\left(n^J,n^S\right)\right)\right)\zeta S\left(\left(1-\delta\right)\left(n^S+n^J\right)\right) \\
&\quad +\beta\left[F\left(z^*\left(n^J,n^S\right)\right) + \delta\left(1-F\left(z^*\left(n^J,n^S\right)\right)\right)\right]J, \forall n^S, n^J
\end{aligned}
$$

4. $S\left(n^S,n^J\right)$, the value to senior workers, is given by:

$$
\begin{aligned}
S\left(n^S,n^J\right) &= w^S\left(n^S,n^J\right) \\
&\quad +\beta\left(1-\delta\right)\left(1-F\left(z^*\left(n^J,n^S\right)\right)\right)S\left(\left(1-\delta\right)\left(n^S+n^J\right)\right) \\
&\quad +\beta\left[F\left(z^*\left(n^J,n^S\right)\right) + \delta\left(1-F\left(z^*\left(n^J,n^S\right)\right)\right)\right]J
\end{aligned}
$$

5. Joint surplus between firms and senior workers are split through bargaining:

$$
\frac{S\left(n^S,n^J\right) - J}{V'\left(n^S\right)} = \frac{\phi}{1-\phi}
$$

6. The goods market clears:

$$c = Y \quad \equiv \quad \int_z \int_{n^S} z \left[n^J(n^S)\theta^J + n^S\theta^S \right]^{\alpha} g(n^S) dn^S dF(z)$$

7. The labor market clears. Marginal disutility of labor is equal to average wage:

$$v'(L) \quad = \quad \frac{\int_{n^S} \left[n^J(n^S) w^J\left(n^S\right) + n^S w^S\left(n^S\right) \right] g(n^S) dn^S}{L}$$

where

$$L \quad \equiv \quad \int_{n^S} \left[n^J(n^S) + n^S \right] g(n^S) dn^S$$

8. The stationary distribution of firm sizes $g\left(n^S\right)$ is consistent with the policy function $n^J\left(n^S\right)$:

$$g(n) \quad = \quad \begin{cases} \int_z \int_{n^S} \left\{ \begin{array}{c} \mathbf{1}\left\{ (1-\delta)\left(n^S + \zeta n^J\left(n^S\right)\right) = n \right\} \\ \cdot \mathbf{1}\left\{ z \geq z^*\left(n^J\left(n^S\right), n^S\right) \right\} \end{array} \right\} g\left(n^S\right) f(z)\, dn^S dz & \text{if } n > 0 \\ \int_z \int_{n^S} \mathbf{1}\left\{ z < z^*\left(n^J\left(n^S\right), n^S\right) \right\} g\left(n^S\right) f(z)\, dn^S dz & \text{if } n = 0 \end{cases}$$

For a firm to reach size n, two things must occur. First, the firm has to hire just enough junior workers so that the total number of senior workers after exogenous separation (at rate δ) in the next period is n. Second, the firm must survive bankruptcy. All firms that go bankrupt start over at size 0 in the subsequent period.

5 Stationary Equilibrium

In this section, some intuition is provided about our cautious hiring mechanism. In particular, we demonstrate how the productivity gap θ^S/θ^J between senior and junior workers affects firm growth and the speed of employment recovery following a recession. In addition, we also compare the stationary equilibrium properties of our model to those in the data.

5.1 Intuitions about Cautious Hiring Mechanism

The cautious hiring behavior of firms comes from the combination of firm-specific skills and the lack of commitment by firms. Due to the existence of firm-specific skills and the lack of commitment by firms, wage rates for senior and junior workers are compressed relative to their marginal productivities. Junior workers are overpaid relative to their productivity and senior workers are underpaid. In this sense, hiring junior workers is a form of investment. Firms incur a negative expected return in the initial period of hiring and a positive expected return after junior workers transition into senior workers. Senior workers represent a real value to their firms. Hiring junior workers is risky for firms because junior workers lower the expected current period profit. This increases the probability that firms go bankrupt. Bankruptcy leads to the separation of firms from the stock of senior workers they have gradually accumulated over time, hence is costly.

Firms enter each period with a stock of senior workers who produce a positive expected cash flow. Firms use this positive cash flow to compensate for the expected losses of hiring junior workers. Junior workers that are retained have some probability of becoming senior in the subsequent period. The risky nature of hiring junior workers makes firms cautious about hiring too quickly. As firms gradually accumulate senior workers, they will eventually reach a satiation point due to the decreasing returns to scale technology. At this point, firms hire just enough junior workers each period to compensate for the exogenous separation of senior workers.

Next we analyze how a change in θ^S/θ^J affect the rate of growth for firms. Equilibrium hiring decisions are only dependent on the ratio of θ^S/θ^J rather than the level of each.[13] So without loss of generality we hold θ^J fixed at 1 and only analyze changes in θ^S throughout the remainder of this section.

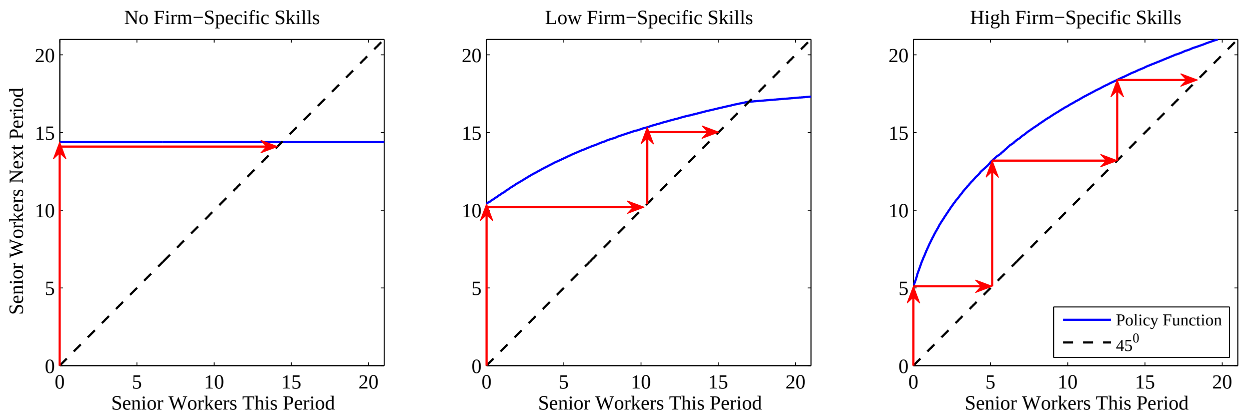
Consider a comparative statics exercise. We simulate three economies, one without firm-specific human capital $\left(\theta^S = \theta^J = 1\right)$, one with a lower importance of firms-specific human capital, θ^S_{low}, and one with a higher importance of firm-specific human capital, θ^S_{high}. Specifically $1 < \theta^S_{low} < \theta^S_{high}$. We plot and compare the growth paths of the three economies in their respective stationary equilibria.

Figure 8 is a plot of the growth path for a typical firm in each of the three economies. The horizontal axis measures the number of senior workers n^S a firm enters the period with. The vertical axis measures the number of senior workers the firm will have at the beginning of the subsequent period (specifically, it is equal to the number of senior workers, n^S, plus the number of junior workers hired, n^J, after accounting for those workers exogenously separated from the firm, $(1-\delta)\left(n^S + \zeta n^J\right)$). In the case where senior and junior workers are equally productive (left panel, $\theta^S = \theta^J = 1$), the optimal size of a firm is reached in one period no matter how many senior workers the firm starts with. This is because junior workers are as productive as senior workers so they no longer incur an initial investment upon hiring (nor

[13] Suppose θ^S and θ^J are both scaled by λ. The equilibrium wage rates will rise by λ^α. None of the firms' hiring decisions will be affected.

do senior worker offer a positive expected return in this setting). In this case, hiring decision becomes a static problem. Firms always hire the optimal number of workers in each period. When we introduce a difference in the productivity levels between senior and junior workers (middle panel, θ^S_{low}), the optimal size of a firm is gradually reached as the firm balances growth against risk of bankruptcy. A further increase in the productivity gap between θ^J and θ^S (right panel, θ^S_{high}) causes start-up firms to take even smaller steps in terms of hiring. As a result, it takes more steps (a longer time) for firms to reach the optimal size.

Figure 8 – Firm Growth Paths for Different Productivity Gaps θ^S/θ^J



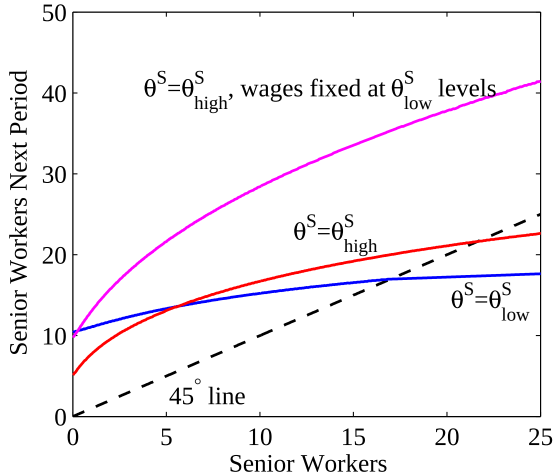
To understand how exactly an increase in θ^S/θ^J leads to slower growth of firms, we decompose the total effect into a partial equilibrium effect and a general equilibrium effect. Following an increase in θ^S, senior workers become more valuable to a firm. As a result, firms increase their hiring of junior workers who will become senior workers later on. This partial equilibrium effect tends to speed up firm growth.[14] At the same time, an increase in the demand for junior workers drives up the wage rates. This general equilibrium effect tends to discourage firm hiring and slow down firm growth.

To separate these two effects, we conduct a counterfactual experiment. As we transit from one stationary equilibrium with θ^S_{low} to another with θ^S_{high}, we decompose the change in the policy functions into partial and general equilibrium effects. In Figure 9, we plot the respective policy functions. The blue curve is the policy function for the stationary equilibrium with θ^S_{low}, and the red curve is the policy function for the stationary equilibrium with θ^S_{high}. The pink curve is a counterfactual policy function for an economy with θ^S_{high} but with wage rates for both senior and junior workers fixed at the levels in the equilibrium with θ^S_{low}.[15]

[14]Note that there are multiple distinct forces which together comprise the complete partial equilibrium effect. This is explored further below.

[15]Since wages aren't permitted to adjust in this counterfactual experiment, the labor market does not clear for the green line.

Figure 9 – Partial and General Equilibrium Effects

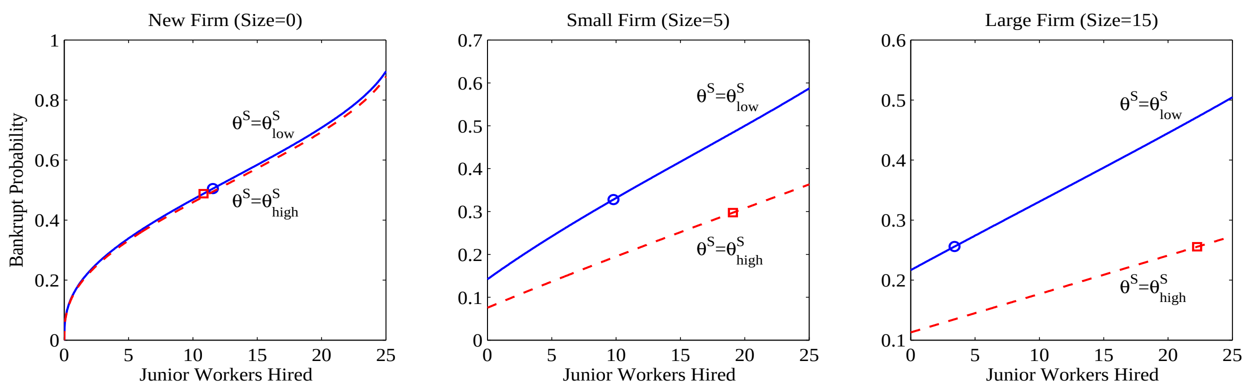


The partial equilibrium effect is represented by the change from the blue curve to the pink curve. Following an increase in θ^S (holding wage rates fixed), senior workers become more valuable to firms. As a result, firms increase their hiring of junior workers due to an increase in the future value of a worker. This partial equilibrium effect is heterogeneous across firms. Specifically it is weaker (or even negative) for small firms relative to large firms. As θ^S increases, the positive cash flow each senior worker brings into the firm increases. This lowers the probability of bankruptcy. Compared with large firms, small firms have fewer senior workers. Consequently, the bankruptcy probability for small firms does not decline as much. As a result, small firms do not increase their hiring of junior workers as much as large firms.

To illustrate the heterogeneity of the partial equilibrium effect, Figure 10 plots the bankruptcy probabilities as a function of the number of junior workers hired. The left, middle, and right panels correspond to firms of sizes 0, 5, and 15 respectively (i.e., having 0, 5, and 15 senior workers). The solid blue curves plot the bankruptcy probabilities in an economy with $\theta^S = \theta^S_{low}$. The circles on each solid curve indicate the optimal hiring choices in the θ^S_{low} economy. Regardless of firm size, the more junior workers a firm hires, the higher the probability of bankruptcy. Next, we increase the productivity level of senior workers from θ^S_{low} to θ^S_{high} while holding wage rates fixed at the equilibrium levels of θ^S_{low}. Following this increase, bankruptcy probability curves shift down from the solid blue curves to the dashed red curves. There are significant reductions in the bankruptcy probability for firms of sizes 5 and 15. Intuitively, an increase in the productivity of senior workers mitigates the bankruptcy risk and makes junior workers less risky to hire. This allows firms of sizes 5 and

15 to hire junior workers more quickly while maintaining a similar level of bankruptcy risk.[16] On the other hand, for firms of size 0, the bankruptcy probability curve under $\theta^S = \theta^S_{high}$ coincides with the case $\theta^S = \theta^S_{low}$. Since these firms have no senior workers, there is no effect on their default probabilities when θ^S increases. Consequently, these start-up firms do not significantly alter their hiring of junior workers.[17]

Figure 10 – Default Probability Comparison



Circle: optimal junior hiring when $\theta^S = \theta^S_{low}$.
Square: optimal junior hiring when $\theta^S = \theta^S_{high}$.

The general equilibrium effect is represented in Figure 9 by the change from the pink curve to the red curve. An increase in the wage rates is needed to clear the labor market as we move from the θ^S_{low} economy to the θ^S_{high} economy. This wage increase discourages firms

[16]This can be seen in the middle and right panels. The optimal hiring under θ^S_{high} is higher than that under θ^S_{low} (squares are to the right of the circles), while the bankruptcy probabilities do not increase much (squares and circles are roughly of equal heights).

[17]In fact, firms of size 0 reduce the hiring of junior workers. The Bellman equation for a firm of size 0 is:

$$V(0) = \max_{n^J} E(z)\left[n^J\theta^J\right]^\alpha - w^J n^J + \beta V\left((1-\delta)\zeta n^J\right) - \beta F\left(z^*\left(0,n^J\right)\right)\left[V\left((1-\delta)\zeta n^J\right) - V(0)\right]$$

As θ^S increases, the value function shifts upwards and becomes steeper. This has two opposing forces on the hiring decision. On the one hand, increased continuation value $V\left((1-\delta)\zeta n^J\right)$ encourages firms to hire more junior workers who will become senior workers in the following period, given that firms survive. On the other hand, increased continuation value $V\left((1-\delta)\zeta n^J\right)$ makes bankruptcy more painful (this is represented by a higher difference $\left[V\left((1-\delta)\zeta n^J\right) - V(0)\right]$). Since hiring junior workers increases the probability of bankruptcy $F\left(z^*\left(0,n^J\right)\right)$, this second force discourages firms from hiring junior workers.

For firms with a positive stock of senior workers, the Bellman equation is:

$$\begin{aligned} V\left(n^S\right) &= \max_{n^J} E(z)\left[n^S\theta^S + n^J\theta^J\right]^\alpha - \left(w^S n^S + w^J n^J\right) + \beta V\left[(1-\delta)\left(n^S + \zeta n^J\right)\right] \\ &- \beta F\left(z^*\left(n^S,n^J\right)\right)\left\{V\left[(1-\delta)\left(n^S+\zeta n^J\right)\right] - V(0)\right\} \end{aligned}$$

The two forces above are still present. However, the second force (bankruptcy concern) is mitigated by an increase in θ^S for large firms ($F\left(z^*\left(n^S,n^J\right)\right)$ shifts down as θ^S increases). Therefore, the first force dominates and it encourages firms to hire more junior workers.

from hiring junior workers. As a result, the red curve lies below the pink one. This general equilibrium effect reduces the hiring of firms across all sizes.

Combining the partial and general equilibrium effects (from the blue curve to the red curve), smaller firms with size close to 0 reduce their hiring, while larger firms increase their hiring. As θ^S increases, a start-up firm of size 0 will grow more slowly initially, followed by more rapid growth later on. If we measure firm sizes as a fraction of the size of a mature firm, then growth slows down across all firm sizes. This can be seen on the left panel of Figure 11.

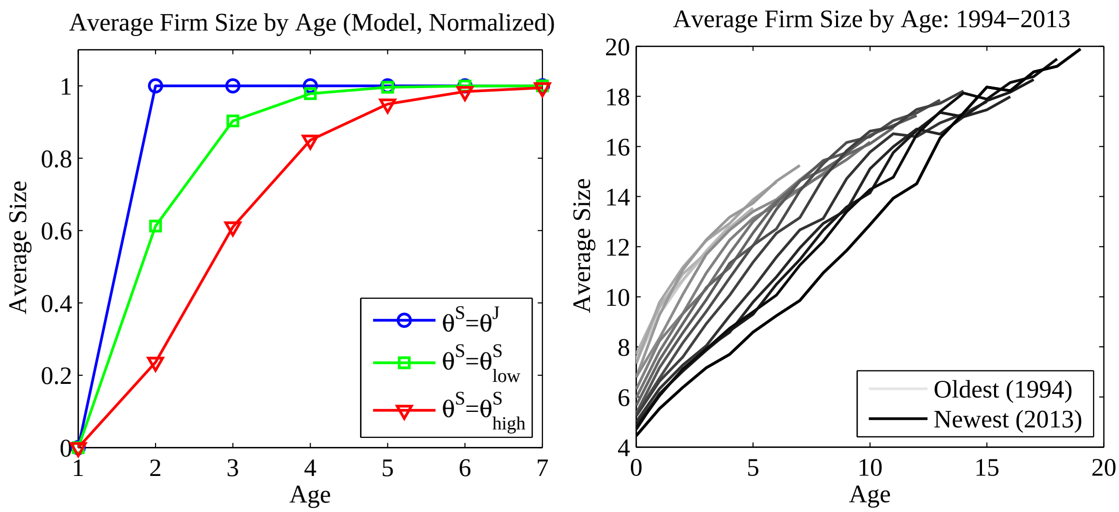
In our simulation of a recession, a fraction of firms in the economy go bankrupt following some exogenous shock. The recovery phase is mainly driven by these bankrupt firms starting over from size 0 and growing back to the mature size. A higher level of θ^S/θ^J slows down the growth rate of start-up firms and leads to a slower recovery in employment.

5.2 Comparing Stationary Equilibrium Predictions to Data

In our comparative statics exercise above, an increase in the productivity gap between junior and senior workers leads to more cautious firm hiring. As a result, the starting size of firms becomes smaller and the growth rate of firms becomes slower. It is important to check if this model prediction is consistent with the data. The Business Employment Dynamic (BED) program at the Bureau of Labor Statistics provides the age-size profiles of firms born in different years. Unfortunately, their data only range from 1994 to 2013. Information on firms born prior to 1994 is not available, making it difficult to conduct a direct comparison of the age-size profiles of firms before and after the mid-1980s. Still, the evolution of age-size profiles of firms from 1994 to 2013 is suggestive of a smaller starting size and a slower growth rate.

Figure 11 plots the average sizes of firms across ages. Data for the left panel are generated by our model. Three economies with no firm-specific skills (blue line), a lower importance of firm-specific skills (green line), and a higher importance of firm-specific skills (red line) are compared. Average firm sizes are normalized and measured as a percentage of the employment level of mature firms in the economy. The higher the importance of firm-specific skills, the smaller the starting size of firms and the longer it takes to reach the mature size.

Figure 11 – Age-Size Profile of Firms



Source: Business Employment Dynamics, Authors' calculations

The right panel in Figure 11 comes from the data. In this panel, lines of different shades correspond to different years. The line on the top with the lightest color corresponds to firms in 1994. The line at the bottom with the darkest color corresponds to firms in 2013. As can be observed in this panel, firms born in more recent years start smaller and grow more slowly than those born in earlier years. This observation has also been documented by Choi and Spletzer (2012).

6 Quantitative Analysis

Of crucial importance to our model is the contribution of firm-specific human capital to the productivity of labor. In this section, we provide an estimate of this parameter and analyze how it evolves over time.

In practice, disentangling the marginal contribution of a specific worker is quite difficult as production from most firms involves the coordinated effort of multiple individuals completing various tasks both separately and in groups. Further, hiring decisions by firms are endogenous with output.[18]

[18]Still, there do exist some attempts in the literature to identify the gains in worker productivity from experience without relying on the wage data. One attempt at quantifying the gains to experience is Shaw and Lazear (2008). They study a firm which installs windshields where output can be quantified and directly linked to individual workers. A main finding is that there is a very steep learning curve over the first 8 months on the job (53%). Further, their data show that these output gains with tenure are not reflected in equal percentage pay gains: pay profiles are much flatter than output profiles in the first year and a half on the job. Installing windshields has a relatively easy learning curve. Still, there are substantial gains to tenure which are not associated with an equivalent gain in wage providing support that workers are a form

Although output of individual workers is difficult to measure, wage rates for workers of various seniorities are observable. In Section 4, we hypothesize that the difference between the wage rates of senior and junior workers are determined through bargaining between senior workers and firms:

$$\frac{S\left(n^S, n^J\right) - J}{V'\left(n^S\right)} = \frac{\phi}{1-\phi}.$$

Only a fraction of the gains in productivity is reflected in the wage. This gives rise to our calibration strategy for θ^S/θ^J: we estimate the wage return to seniority from the data, and calibrate θ^S/θ^J so that the average wage difference between senior and junior workers in our model matches that in the data.

Existing estimations of the return to seniority include Topel (1991), Altonji and Shakotko (1985), and Altonji and Williams (2005), among others. We extend the methodology of Topel (1991) and Altonji and Williams (2005) to more recent samples of CPS and PSID in order to compare the return to seniority before and after the mid-1980s.

6.1 Determining the Return to Seniority from CPS

We estimate the return to seniority following the methodology of Topel (1991). We employ data from the Current Population Survey using the Displaced Workers, Occupational Mobility and Job Tenure supplements. In this survey, we are able to identify workers who were displaced from jobs as a result of economic reasons (layoffs and plant closings). We use the loss in wages for workers who were displaced to discipline our estimation of the return to seniority.

Using data from the Displaced Worker supplement of the CPS data is advantageous for a number of reasons. First, experience and tenure tend to move together, making it difficult to identify the contribution of each. Since separation does not alter a worker's experience but does change tenure, we attribute changes in the wage prior to displacement and post displacement to the return to tenure.[19] Second, the CPS data permits us to limit the selection bias of separation. Workers were displaced from their jobs for exogenous reasons instead of endogenous ones such as incapability of workers or bad match quality between workers and jobs.

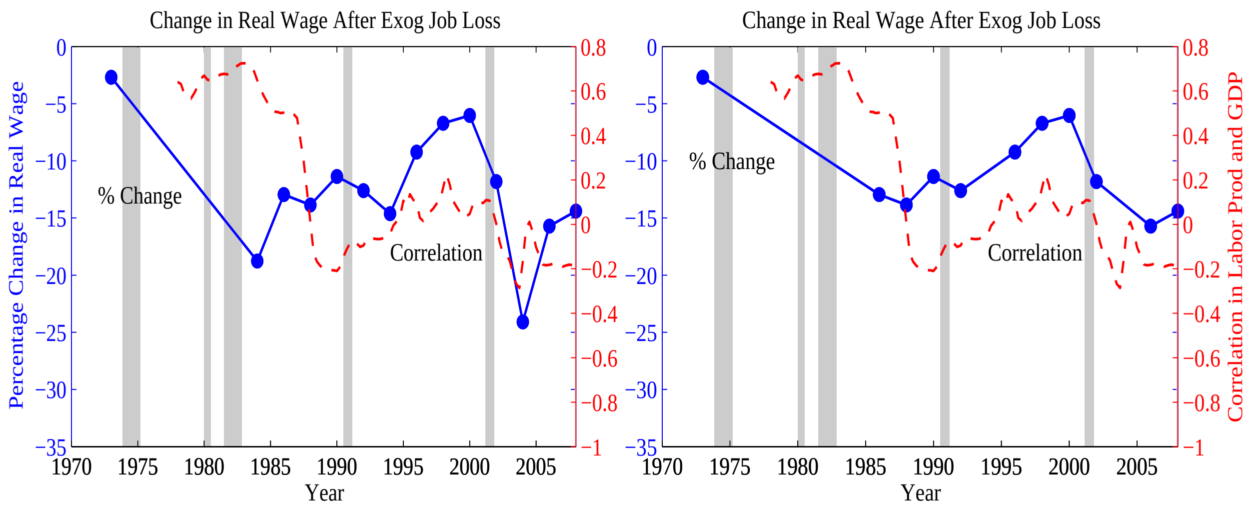
The average change in real wages after an exogenous job loss across all employees is

of investment.

[19]Depending on the year of the survey, those interviewed are asked if they were displaced in the preceding 1 to 5 years. Consequently, we are unaware of the exact year of displacement. The fact that general experience tends to increase wages causes our estimate of θ^S/θ^J to be biased downwards both for older and more recent recessions.

displayed in Figure 12. The drop in wages is affected by the business cycle. Specifically, the survey conducted immediately following each recession yields a larger drop in wages than the average drop experienced in surrounding observations.[20] Panel A of Figure 12 includes all data points whereas Panel B of Figure 12 excludes the first point following each NBER recession. The series are overlaid with the average labor productivity data from Figure 3. The figure demonstrates a strong comovement between changes in the real wage and patterns in labor productivity over the business cycle. This provides suggestive evidence linking our mechanism and result.

Figure 12 – Evidence for Changes in θ^S/θ^J



The blue line with circle markers is the percentage wage loss following a displacement. The red line is the correlation between average labor productivity and GDP.
Source: Displaced Worker's Supplement (CPS), Authors' calculations

The observed fall in wages for the single data point in 1973 was 2.75%. The average fall in wages across all subsequent points is 17.25%. If the points immediately following each recession are dropped, the fall in wages is 12.95%.

Using cross-sectional U.S. data, Christofides and Oswald (1992), Blanchflower et al. (1996), and Hildreth and Oswald (1997) estimate that a 1% increase in a firm's profitability leads to an increase in wages between 0.01% and 0.08%. Therefore, we choose a bargaining weight of $\phi = 5\%$ for senior workers.

From the average drop in wage and the bargaining weight above, we obtain an estimate for the relative productivity between senior and junior workers of $\theta^S/\theta^J = 1.55$ for periods prior to 1983, and $\theta^S/\theta^J = 4.45$ for periods after 1983 ($\theta^S/\theta^J = 3.59$ if points immediately following recessions are excluded).

Unfortunately, use of the Displaced Worker supplement of CPS to estimate the return

[20] This has also been documented by Davis and Wachter (2011).

to tenure raises several potential problems. First, the loss in wage following a displacement may be due to reasons other than the loss in seniority (e.g. scarring). This would lead to an upward bias in our estimation of returns to tenure. Second, since the loss in wage can only be calculated for workers who found a new job after displacement, we are missing the wage loss for workers who fail to find a new job. This selection may lead to a downward bias in our estimation of the percentage loss in wage. Third, the CPS Displaced Workers supplement contains relevant wage data only for years 1973, 1984, 1986, ..., up to 2010. As a result, 1973 is the only data point that can discipline our choice of θ^S/θ^J for periods prior to 1983.[21]

6.2 Determining the Return to Seniority from the PSID

As a robustness check, we also estimate the return to seniority using the methods of Altonji and Williams (2005). We update their estimations using more recent waves from the PSID.

Altonji and Williams (2005) use an instrumental variable approach to estimate the returns to seniority from 1975 to 1991. Unlike the CPS Displaced Worker supplement, job changes in PSID were not entirely due to exogenous reasons. This introduces some endogeneity problems when estimating the return to seniority. Calculating the increase in wage following a one-year increase in tenure is potentially an upward biased measure of the return to seniority due to selection. For example, a good worker, or a good match between worker and job leads to both longer tenure and higher wage payment.

Altonji and Williams (2005) use the demeaned tenure over time for each job match as the instrument for tenure.[22] Their IV1 estimator reported a 10-year return to seniority of 5.42% for the period from 1975 to 1982, and 13.91% for the period from 1983 to 1991. We extend the methodology of Altonji and Williams (2005) to include more recent waves of the PSID. Additionally, instead of using the 10-year return, we calculate the return to tenure using the average tenure of the sample period. Using our updated data and methodology, average return to tenure is 4.84% from 1972 to 1982 and 13.02% from 1983 to 2013. Combining these numbers with a bargaining weight of 5% implies a $\theta^S/\theta^J = 1.97$ for older recessions and $\theta^S/\theta^J = 3.60$ for newer recessions.

For our subsequent simulations, we select $\theta^S/\theta^J = 2$ for recessions prior to the mid-1980s

[21] Additional information is delegated to Appendix G.

[22] Details of their regression method are delegated to Section H of the Appendix.

and $\theta^S/\theta^J = 3.5$ for recessions after.[23][24]

6.3 Parametrization

Our model includes nine parameters in the stationary equilibrium and an additional parameter to determine the magnitude of the shock. The parameters in our model are selected to levels standard in the literature or calibrated to match certain moments in the data. Following Greenwood et al. (1988), we remove the wealth effect on labor supply by adopting the GHH preference of the form

$$u(c, L) = \frac{\left[c - \frac{L^{1+\nu}}{1+\nu}\right]^{1-\gamma} - 1}{1-\gamma}.$$

Each period in our model represents a year and we select a household discount rate of $\beta = 0.96$ to match an annual interest rate of 4%. We choose a household relative risk aversion of $\gamma = 8$ in line with work in the asset pricing literature. The model is sensitive to our selection of ν which is the inverse of labor elasticity. A labor elasticity consistent with the micro literature generates too little fluctuation in employment or too much fluctuation in the wage relative to what we observe in the data. In order to simultaneously align with these two facts, a larger labor elasticity is required. Here, we set $\nu = 0.2$ which implies an elasticity of labor supply equal to 5. Alternatively, we can introduce wage rigidity which allows us to match these facts even with a perfectly inelastic labor supply. Further, an introduction of sticky wages allows us to lower our selection of household relative risk aversion. We select $\zeta = 0.5$ so it takes 2 years on average for a junior worker to transition into senior. This is consistent with Altonji and Williams (2005) which mentions that "all the estimation methods suggest that the return to seniority declines sharply after the first year or two". Labor share in the production function is chosen to be $\alpha = 0.65$.[25] The firm-specific productivity z is i.i.d. over time and across firms following the distribution $f(z) \sim U[\underline{z}, \bar{z}]$. The mean of the distribution is calibrated to match an average firm size of 16.[26]

[23]Our calibration of the ratio $\theta^S/\theta^J = 2$ for pre-1985 recessions is consistent with findings of Frazis and Loewenstein (2006) who use 1982 EOPP data to measure growth in productivity over the first two years of employment. They find that productivity of workers increases by 80% over the first two years, and that productivity growth is only partially reflected in wage growth.

[24]In our model, all firm-specific human capital is lost upon separation. In reality, some firm-specific skills may be retained across job tranistions (i.e. in the case of recall). Our identification strategy estimates only that portion of firm-specific human capital which is lost because any retained portion of firm-specific skills will not lead to a drop in wages.

[25]In our model we abstract from physical capital. Over the business cycle capital stock is much less cyclical than employment. Using a decreasing returns to scale production function simulates a larger quantity of capital per employee during recessions.

[26]Note, we assume z follows a uniform distribution for analytic tractability and computational simplicity.

Table 2 – Externally Calibrated Parameters

Parameter	Description	Value	Source
β	Discount Rate	0.96	Annual Interest Rate 4%
γ	CRRA Parameter	8	Literature†
α	Labor Share	0.65	65% Labor Share
ϕ	Bargaining Weight for Workers	0.05	Literature‡
ζ	Probability Junior Turns Senior	0.5	Altonji and Williams (2005)
M	Total Measure of Firms	1	Normalization
ν	Inverse of Labor Elasticity	0.2	Elasticity of Labor Supply 5

†Asset Pricing Literature.
‡Christofides and Oswald (1992), Blanchflower et al. (1996) and Hildreth and Oswald (1997).

The exogenous separation rate between firms and workers is chosen to be $\delta = 10\%$ to match the average job destruction rate of continuing firms from 1977 to 2013 from the Business Dynamics Statistics data. This also results in an average employment spell of approximately 2.5 years across all employees which is consistent with Shimer (2005).[27] Selection of θ^S/θ^J and the bargaining weight ϕ were discussed in the previous section.

Table 3 – Internally Calibrated Parameters

Parameter	Description	Value	Target	Model
δ	Exogenous Separation Rate	10%	Avg. Employment Spell 2.5 Years	$2.56, 2.90$
θ^J	Junior Productivity	1	Normalization	
θ^S_{old}	Senior Productivity (pre-1985)	2	Return to Tenure 1972-1982 4.84%	4.86%
θ^S_{new}	Senior Productivity (post-1985)	3.5	Return to Tenure 1983-2013 13.02%	13.00%

Our model focuses on the recovery following a recession. We remain agnostic about what is causing recessions. For simplicity, we model a recession by exogenously making a fraction of the firms go bankrupt. Recovery path is potentially dependent on whether large or small firms are forced to go bankrupt. We have explored two alternatives: forcing firms of all sizes to go bankrupt with equal probability, or introducing a one time unexpected variance shock

The exact distribution of z does not affect our results qualitatively. Since the wage bargaining in our model involves the derivative of the value functions, using a discretized normal distribution for productivity shocks introduces discontinuities into the policy function and the value functions, making convergence hard. For uniformly distributed shocks, default probabilities can be calculated analytically.

[27] To be more exact, our selection of δ results in an average employment spell of 2.56 years for older recessions and 2.90 years for more recent recessions.

to the idiosyncratic productivity level so that smaller firms are more prone to bankruptcy. The results are quite similar. For each recession, we select the magnitude of the shock to match the observed drop in output from the data.

7 Business Cycle Properties

Using the parameters from Section 6, we now compare the business cycle properties of output, employment, and average labor productivity generated by our model to those in the data. Specifically, we compare our model to the data for the largest recession in our sample period before and after the mid-1980s: the 1973 recession and the 2008 recession.

Our model accounts for the slower employment recovery between the 1973 and the 2008 recessions. It also qualitatively matches the procyclical pattern of average labor productivity for the 1973 recession and the countercyclical pattern for the 2008 recession.

7.1 Model Transition Problem

Our model focuses on the recovery following a recession. For simplicity, we model a recession by exogenously making a fraction of the firms go bankrupt.[28] These bankrupt firms are separated from all their senior workers. Recovery is driven by these firms growing from size 0 back to the mature size. We compare the recovery paths of output, employment and labor productivity of our model with those in the data.

Let the shock happen in period $t = 0$. Assume that it takes T periods for the economy to return to the stationary equilibrium. The Bellman equation for firms is given by:

$$\begin{aligned} V_t\left(n^S\right) = \max_{n^J} \; & \int_{z^*(n^J,n^S)}^{\bar{z}} \Bigg\{ z\left[n^J\theta^J + n^S\theta^S\right]^\alpha \\ & - \left[w^S n^S + w^J n^J\right] R\left(n^J, n^S\right) \Bigg\} f(z)\, dz \\ & + \Lambda_{t,t+1} F\left(z^*\left(n^J, n^S\right)\right) V_{t+1}(0) \\ & + \Lambda_{t,t+1}\left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right) V_{t+1}\left((1-\delta)\left(n^S + \zeta n^J\right)\right) \end{aligned}$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor for the household between period t and period $(t+1)$.

Value functions during the transition are indexed by time t. This is because total output/consumption is changing over the transition which affects the stochastic discount factors

[28] Our paper is primarily focused on recovery and we remain agnostic regarding the cause of the recession.

$\Lambda_{t,t+1}$.

The algorithm for solving the transition dynamics is relegated to Appendix F.

7.2 Transition Intuition

7.2.1 Slower Recovery in Employment

Following a recession, a larger fraction of firms go bankrupt relative to the stationary equilibrium. Consequently, there are more start-up firms and less mature firms. When the economy is recovering, small firms hire junior workers and gradually grow back to the mature size. Employment recovery follows.

When θ^S/θ^J is low, the difference between the productivity levels of senior and junior workers is low. Therefore, start-up firms grow fast, leading to a fast employment recovery. When θ^S/θ^J is high, start-up firms grow slowly, which leads to a slow employment recovery. The intuition and policy function comparisons are discussed in Section 5.

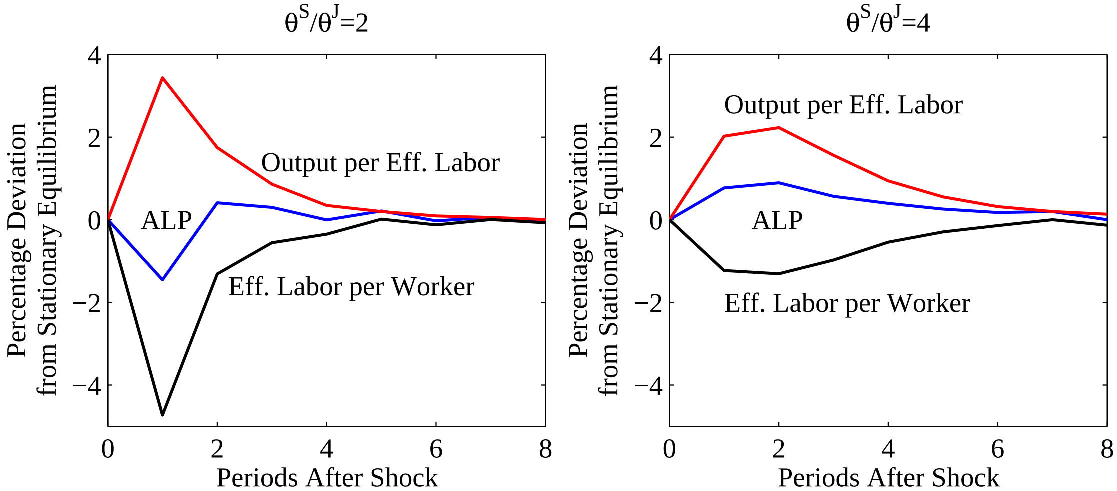
7.2.2 Change in the Cyclicality of Average Labor Productivity

Average labor productivity in our model is calculated as output per worker. Since senior and junior workers have different productivity levels, we can decompose the average labor productivity into two parts: output per effective unit of labor supplied, and the average effective labor per worker:

$$\begin{aligned}
& \text{Average Labor Productivity} \\
= \; & \frac{Y}{\text{Total Labor}} \qquad (7) \\
= \; & \frac{Y}{\text{Effective Labor Units}} \times \frac{\text{Effective Labor Units}}{\text{Total Labor}} \\
= \; & \underbrace{\frac{\int_{n^S} \left[\theta^S n^S + \theta^J n^J\right]^\alpha g\left(n^S\right) dn^S}{\int_{n^S} \left[\theta^S n^S + \theta^J n^J\right] g\left(n^S\right) dn^S}}_{\text{Decreasing Returns to Scale Factor}} \times \underbrace{\frac{\int_{n^S} \left[\theta^S n^S + \theta^J n^J\right] g\left(n^S\right) dn^S}{\int_{n^S} \left[n^S + n^J\right] g\left(n^S\right) dn^S}}_{\text{Worker Composition Factor}} \qquad (8)
\end{aligned}$$

The first term in Equation (8), Y/Effective Labor Units, (henceforth, decreasing returns to scale factor), measures how efficiently output can be produced with effective labor units. Due to the decreasing returns to scale technology, this efficiency is higher when total effective labor is smaller. The second term, Effective Labor Units/Total Labor, (henceforth, worker composition factor), measures the average amount of effective labor units provided per worker. It is a weighted average of θ^S and θ^J, with weights equal to the shares of senior and junior workers in the economy. The more senior workers there are, the larger this term will be.

Figure 13 – Decomposition of Average Labor Productivity (ALP)



During the transition, the decreasing returns to scale factor and the worker composition factor exert opposing forces on average labor productivity. On the one hand, after a recession, start-up firms desire to grow which results in a larger influx of junior workers. These junior workers provide a smaller amount of effective labor per worker than senior workers. This tends to drive down average labor productivity through the worker composition factor.

On the other hand, during the recovery, the average size of firms in the economy is smaller than in the stationary equilibrium for two reasons. First, output is small relative to future periods. This increases the importance of present consumption relative to future growth and results in a smaller average firm size. Second, the shock causes a fraction of firms to go bankrupt. Bankrupt firms start over at size 0 and engage in cautious hiring. They are smaller than their fully-grown counterparts. Because of the decreasing returns to scale production technology, a smaller average firm size increases average labor productivity.[29]

When θ^S/θ^J is low, start-up firms grow fast by hiring a lot of junior workers. The composition effect from the lower productivity of junior workers outweighs the decreasing returns to scale effect. This results in a procyclical pattern of average labor productivity. When θ^S/θ^J is high, recovery is slower. Start-up firms are more cautious in their hiring of junior workers. The downward pressure of less productive junior workers is diluted across time. This causes the decreasing returns to scale force to dominate. As a result, a countercyclical pattern of average labor productivity emerges.

Figure 13 plots this decomposition of average labor productivity. The left panel corre-

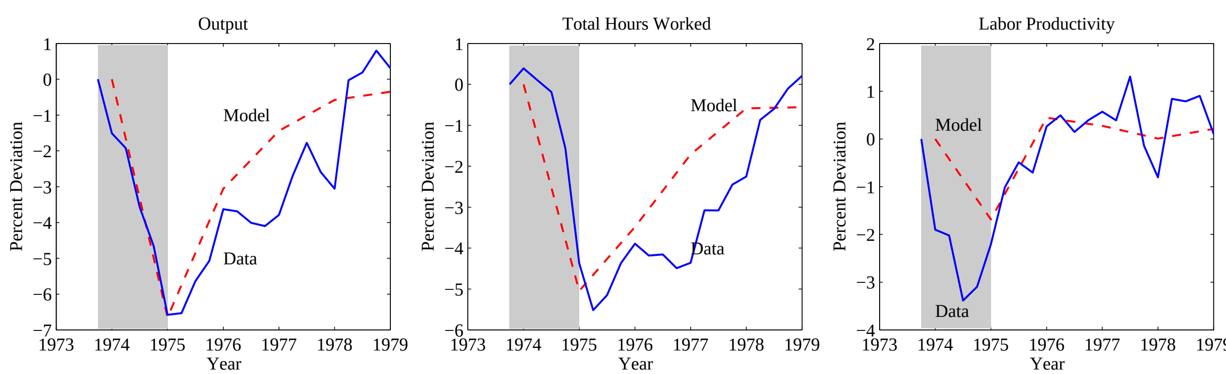
[29] Our model abstracts from capital. Empirically, over the business cycle, the fluctuations in employment are not fully matched by changes in capital. Consequently there is more capital available per worker during recessions. This serves as our motivation for modeling production using a decreasing returns to scale technology in labor.

sponds to the fast recovery economy ($\theta^S/\theta^J = 2$), and the right panel corresponds to the slow recovery economy ($\theta^S/\theta^J = 3.5$). In each panel, the red line plots average labor productivity during the transition. It is decomposed into output per effective labor unit (decreasing returns to scale effect, blue line) and effective labor per worker (worker composition effect, green line). Comparing the worker composition effect of the slow recovery economy (right panel) with the fast recovery one (left panel), we observe that the effective labor per worker (green line) falls by significantly less. As outlined above, this is because firms are hiring junior workers more cautiously in the slow recovery economy. This slower hiring also results in a slower recovery of GDP. A more steady consumption path translates into less disruption of the stochastic discount factor and average firm size. All of this contributes to a weaker decreasing returns to scale effect. In aggregate, we observe that the worker composition effect outweighs the decreasing returns to scale effect in the fast recovery economy and vice versa in the slow recovery economy.

7.3 Comparing Model Predictions to the Data

Using the parameter θ^S calibrated in Section 6, we compute the recovery path of output, employment, and average labor productivity. For the 1973 recession we use $\theta^S = 2$, and for the 2008 recession we use $\theta^S = 3.5$.[30] The magnitude of the shock η is selected to match the percentage drop in output observed from the data.

Figure 14 – 1973 Recession: Output, Employment and Labor Productivity



Source: Bureau of Labor Statistics, Authors' calculations

In Figure 14, we compare the time series of output, employment, and average labor productivity between our model and the data throughout the recession of 1973. Note that our model only uses a single parameter, η, to match the data to this particular recession.

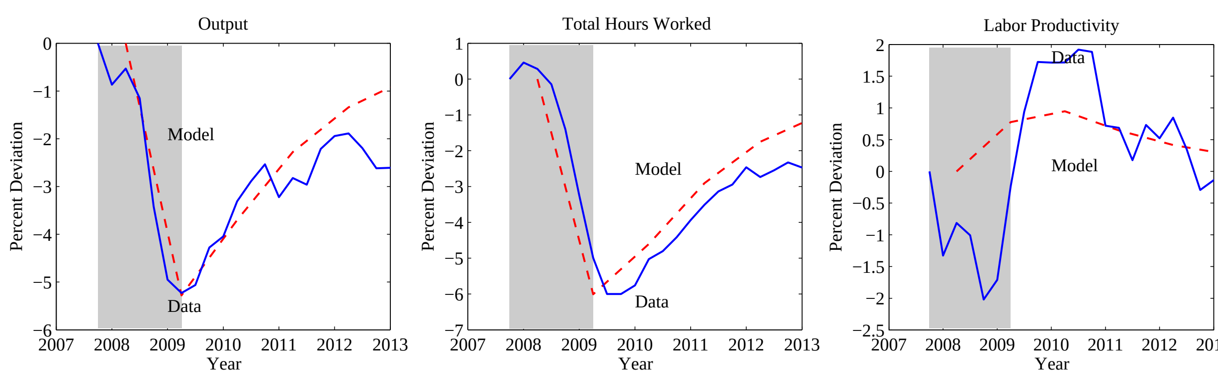
[30]Here we use the1973 recession and the 2008 recession as representatives for the older and newer recessions, respectively. For simulation results regarding other recessions, please refer to Section J of the Appendix for details.

No parameter is used to target the speed of output and employment recovery. Instead, the speed of output and employment recovery is endogenously determined by the speed of firm hiring.

Our model does a reasonable job approximating the speed of recovery in output and employment, as observed in the left and middle panels of Figure 14, given that the only moment we target is the initial drop. In the middle panel of Figure 14, employment recovers by 1.3% (from -5.5% to -4.2%) in the data over the first year of recovery. Our model delivers 1.5% employment recovery (from 5.0% to 3.5%) over this window. In addition, in our model, employment recovers to the pre-recession level in approximately three years, which is similar to the data. A more detailed comparison of the recovery of employment between our model and the data is included in Table 4.

From the right panel of Figure 14, we see that following the 1973 recession, average labor productivity dropped with output in the data. Our model delivers this procyclicality pattern of labor productivity. The intuition for this is as follows. In the 1973 recession, the difference between the productivity levels of senior and junior workers was relatively small. Therefore, start-up firms grew quickly. This fast growth in employment results in firms spending little time on the more productive portion of the decreasing returns to scale technology. However, our model suggests that average labor productivity in the economy was reduced following the recessions by the large influx of junior workers with lower productivity.

Figure 15 – 2008 Recession: Output



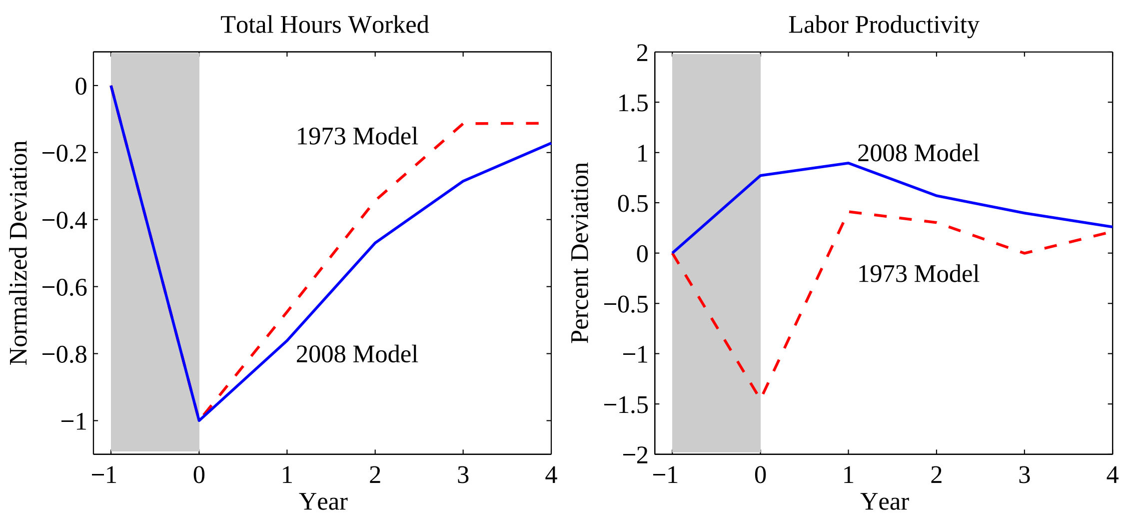
Source: Bureau of Labor Statistics, Authors' calculations

Figure 15 compares our model to the data for the 2008 recession. The only difference in the parametrization between the 1973 recession and the 2008 recession is a change in θ^S and the magnitude of the shock η. Again the model does a fairly good job at predicting the rate of recovery in output. It also generates a slower rate of recovery in employment due to the cautious hiring mechanism. Four years after the ending date of the recession, employment has not recovered to the pre-recession level, in both the data and our model.

In the 2008 recession, average labor productivity went up while output dropped. Our model also delivers this counter cyclical pattern in labor productivity (right panel of Figure 15). Intuitively, in 2008 the difference between the productivity levels of senior and junior workers is relatively large. Therefore, start-up firms grow slowly for fear of losing their senior workers upon bankruptcy. This slow growth in employment dilutes the influx of junior workers over time and weakens the downward pressure of junior workers on average labor productivity. Slower hiring also allows firms to operate at a smaller average size and take advantage of the highly productive portion of the decreasing returns to scale technology. As a result, average labor productivity rises over the recovery path.

Figure 16 compares the model output from the two different recessions. Since the magnitude of the recessions vary slightly, the employment series have been normalized for easier comparison. Specifically, the deviation of employment from the stationary equilibrium has been set to -100% for the first period of recovery. Normalizing in this manner allows us to compare the speed of recovery in employment across recessions. As can be observed in Figure 16, an increase in θ^S/θ^J causes a slower speed of recovery in employment. It also reverses the cyclicality of labor productivity. The causes for this are outlined in Section 7.2.

Figure 16 – Recession Comparison: Employment and Avg. Labor Prod.



Source: Authors' calculations

By normalizing actual employment data from the two recessions using the same method, we are able to calculate how much of the slower employment recovery can be explained by our mechanism. This comparison is displayed in Table 4. In this table, we calculate how much employment has recovered relative to its trough after one, two and three years. For example, employment reaches its trough at 5.5% below its pre-recession level in the 1973 recession. One year after the employment trough, employment is still 4.2% below its pre-recession level. We calculate the percentage recovery in employment over the first year as $(5.5\% - 4.2\%) / 5.5\% = 24\%$. Calculations are included for both the data and our model.

Table 4 – Comparison of Employment Recovery

	Years After	Percentage Recovery	
	Emp. Trough	Model	Data
1973 Recession	1	31%	24%
	2	66%	44%
	3	88%	84%
2008 Recession	1	23%	20%
	2	52%	48%
	3	71%	58%

Employment recovery for the 1973 recession features a non-monotone path in the data. Specifically, employment drops to its trough in 1975 and recovers by 24% from 1975 to 1976. However, it suffers another drop in 1977 (observable in the middle panel of Figure 14). Our model is relatively simple and there is no mechanism in our model that delivers this "double-dip" feature. Still, our model provides a relatively close match to the three-year employment recovery statistic. Our model predicts an 88% recovery in three years which is close to 84% recovery observed in the data.

For the 2008 recession, our model predicts 23% recovery in employment in the first year and a cumulative 52% over the first two years. This is also close to what we observe in the data (20% and 48%, respectively). However, over a three-year horizon, our model is only able to account for a portion of the slower employment recovery for the 2008 recession. In the data, employment has recovered by only 58% in the three years following the employment trough, while our model delivers 71%.

Table 5 – Comparison of Employment Recovery

	Years After	Percentage Recovery		
	Emp. Trough	Model	Data	Portion Explained
1973 Recession	3	88%	84%	
2008 Recession	3	71%	58%	
Difference		17%	26%	65%

By measuring the change in the speed of recovery from 1973 to 2008 as the difference in the three-year percentage recovery rate, our change in θ^S/θ^J accounts for roughly two-thirds ($17\%/26\% = 65\%$) of the change in the speed of recovery in employment between 1973 and 2008.

8 Conclusion

In this paper, we document two changes in the pattern of business cycles starting from the mid-1980s. First, the speed of recovery in employment has become significantly slower. Second, labor productivity has switched from being procyclical to acyclical or even countercyclical. We present a model that contributes to the explanation of both facts through the variation of a single parameter, namely the relative productivity of senior to junior workers.

In our model, when a recession occurs, many firms go bankrupt and need to start up again. This causes two opposing forces on average labor productivity. First, start-up firms employ a disproportionately large number of junior workers, resulting in a decrease of average labor productivity. The second and countering force is that, during the recovery, the average size of firms in the economy is smaller than in the stationary equilibrium. This effect comes through two channels. First, during a recession, output is small relative to future periods. This increases the importance of present consumption relative to future growth and results in reduced hiring. Second, the shock causes a fraction of firms to go bankrupt. These firms engage in cautious hiring and are smaller than their fully-grown counterparts. Due to the decreasing returns to scale production technology, a smaller average firm size tends to increase average labor productivity. Whether average labor productivity is procyclical or countercyclical depends on the relative strength of these two opposing forces.

In older recessions, the difference between the productivity of senior and junior workers is low so new firms tend to hire junior workers quickly. This concentrated influx of junior workers during periods immediately following a recession causes the downward force in average labor productivity to dominate. Further, since firms grow quickly, employment also recovers quickly.

As the relative productivity of senior to junior workers increases, senior workers become more valuable to a firm. Consequently, firms are more cautious (slower) in hiring in order to avoid losing their stock of senior workers. This results in a slower recovery in employment. In addition to effects on employment, this slow hiring diffuses the downward effect on labor productivity across additional periods and results in firms spending more time on the relatively productive portion of their decreasing returns to scale technology. In this case, the upward force on average labor productivity dominates the downward force and results in the countercyclicality of labor productivity observed in more recent recessions.

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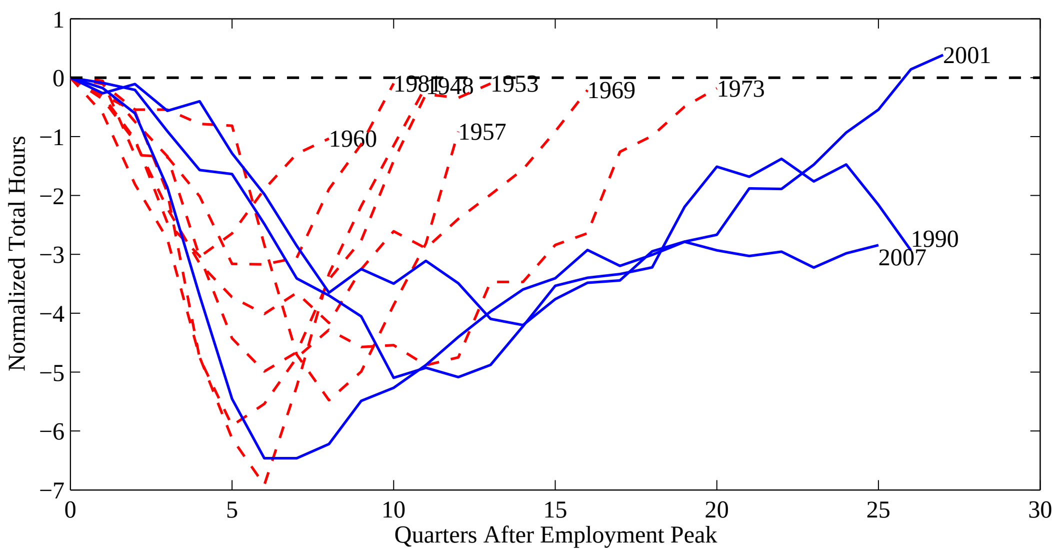
Appendix

A Comparing Recovery in Employment Across Recessions

In this section, additional comparison of the speed of employment recovery is provided. In Section 3, employment is measured as total hours of all persons in the nonfarm business sector. Total hours worked is detrended using an HP filter. In Figure 2 of that section, we compare the recovery paths of total hours for recessions before and after the mid-1980s. For clarity, only the average recovery path for recessions prior to the mid-1980s is displayed. In Figure 17, we include the recovery paths for all recessions. If a subsequent recession occurs before employment has fully recovered, the recovery path is truncated.

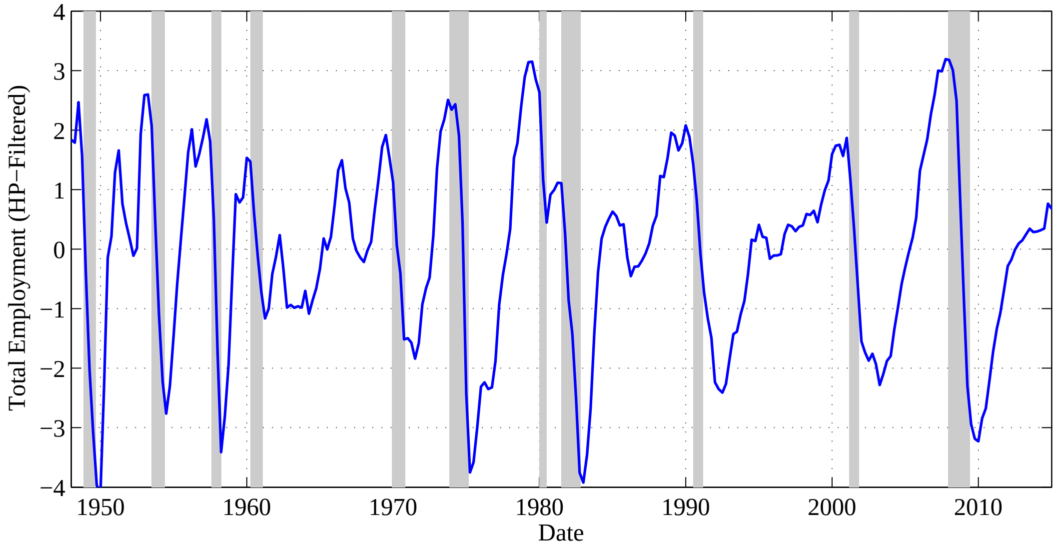
Figure 17 – Employment Recovery Comparison



Source: Bureau of Labor Statistics, Authors' calculations

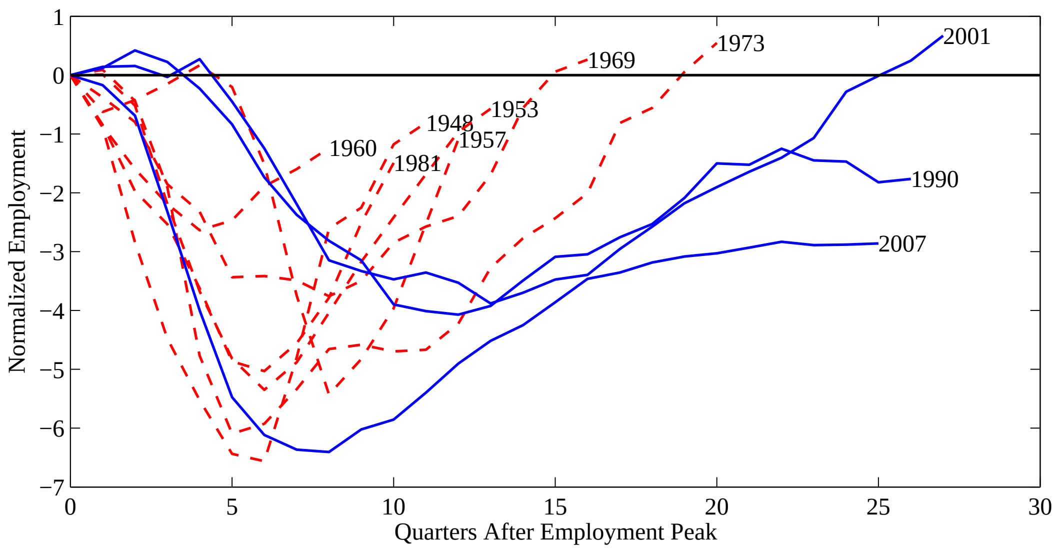
Besides total hours, another commonly used measure for employment is the total employment level in the nonfarm business sector. The time series of HP filtered employment as well as its recovery paths for different recessions are compared below:

Figure 18 – Employment



Source: Bureau of Labor Statistics, Authors' calculations

Figure 19 – Employment Recovery Comparison



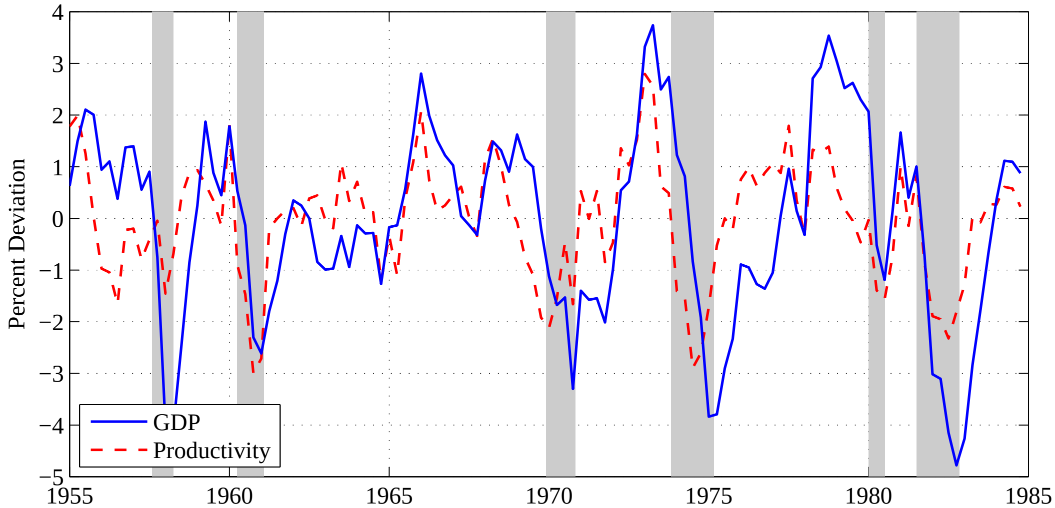
Source: Bureau of Labor Statistics, Authors' calculations

Employment exhibits a similar pattern to total hours. For the recessions prior to the mid-1980s, employment recovers to pre-recession peaks within four years after the NBER recession end date. In contrast, for the three recessions after the mid-1980s, employment is significantly below the pre-recession peak even six years after NBER recession end date.

B Average Labor Productivity and GDP

In this section, time series plots of HP filtered average labor productivity and GDP are compared for the pre-1985 and the post-1985 periods. Labor productivity is calculated as real output per hour of all persons from the non-farm business sector. We apply an HP filter and plot the residual.

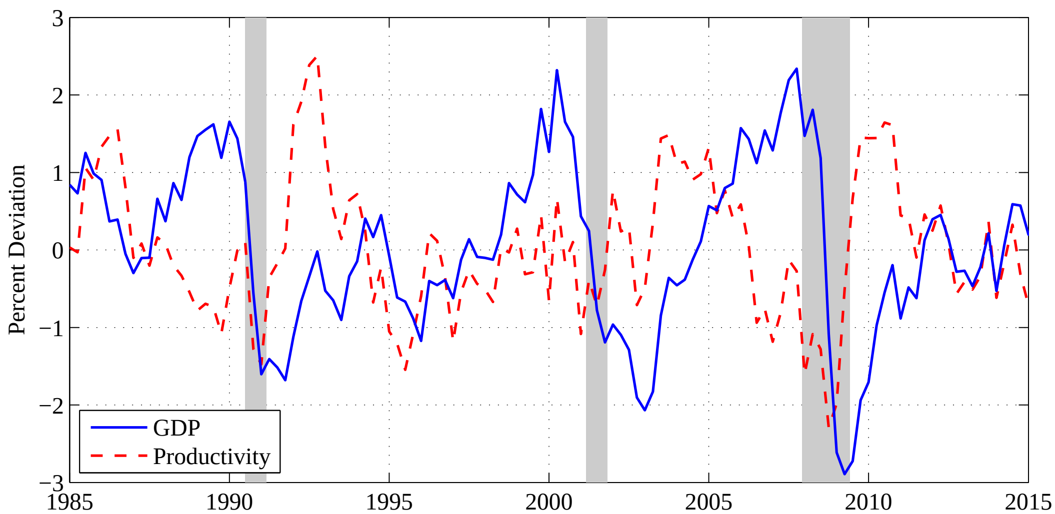
Table 6 – Before 1985: Average Labor Productivity and GDP (HP Filtered)



Source: Current Population Survey and NBER recession dates. Author's calculations

Prior to the mid-1980s, the pattern of labor productivity tracks that of GDP closely during business cycles. Labor productivity falls during recessions and recovers at approximately the same speed as GDP. This results in a procyclical pattern of labor productivity.

Table 7 – After 1985: Average Labor Productivity and GDP (HP Filtered)



Source: Current Population Survey and NBER recession dates. Author's calculations

After the mid-1980s, the correlation between GDP and labor productivity becomes weaker. In a recession, labor productivity drops with GDP. However, during the recovery, labor productivity recovers much faster than GDP, resulting in no correlation or even a negative correlation between the two series.

C Proof of Lemma 1

Proof. Break-even conditions of financial intermediaries are:

$$\begin{aligned}
w^S n^S + w^J n^J &= \left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right)\left(w^S n^S + w^J n^J\right) R \\
&\quad + \int_0^{z^*\left(n^J, n^S\right)} z\left[n^S \theta^S + n^J \theta^J\right]^\alpha f(z) dz \\
&= \int_0^1 \min\left\{z\left[n^S \theta^S + n^J \theta^J\right]^\alpha, \left(w^S n^S + w^J n^J\right) R\right\} f(z) dz \\
&= \int_0^1 z\left[n^S \theta^S + n^J \theta^J\right]^\alpha f(z)\, dz \\
&\quad - \int_0^1 \max\left\{0, z\left[n^S \theta^S + n^J \theta^J\right]^\alpha - \left(w^S n^S + w^J n^J\right) R\right\} f(z) dz.
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\int_0^1 \max\left\{0, z\left[n^S \theta^S + n^J \theta^J\right]^\alpha - R\left(w^S n^S + w^J n^J\right)\right\} f(z) dz \\
&= \int_0^1 z\left[n^S \theta^S + n^J \theta^J\right]^\alpha f(z)\, dz - \left(w^S n^S + w^J n^J\right).
\end{aligned}$$

Plugging this into the Bellman's equation of firms, we get:

$$\begin{aligned}
V(n^S) &= \max_{n^J}\Bigg\{\int_{z^*(n^J, n^S)}^1 \left[z\left[n^S \theta^S + n^J \theta^J\right]^\alpha - \left(w^S n^S + w^J n^J\right) R\left(n^J, n^S\right)\right] f(z) dz \\
&\quad + \beta\left[F\left(z^*\left(n^J, n^S\right)\right) V(0) + \left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right) V\left[(1-\delta)\left(n^S + \zeta n^J\right)\right]\right]\Bigg\} \\
&= \max_{n^J}\Bigg\{\int_0^1 z\left[n^S \theta^S + n^J \theta^J\right]^\alpha f(z)\, dz - \left(w^S n^S + w^J n^J\right) \\
&\quad + \beta\left[F\left(z^*\left(n^J, n^S\right)\right) V(0) + \left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right) V\left[(1-\delta)\left(n^S + \zeta n^J\right)\right]\right]\Bigg\}
\end{aligned}$$

as desired. □

D Proof of Lemma 2

Proof. With $\zeta = 1$ and without bankrputcy risk, firm's Bellman's equation can be simplified into:

$$V(n^S) = \max_{n^J} E[z]\left[n^S\theta^S + n^J\theta^J\right]^\alpha - \left(w^S n^S + w^J n^J\right) + \beta V\left[(1-\delta)\left(n^S + n^J\right)\right]$$

First order condition with respect to junior hiring and envelope condition imply:

$$\begin{aligned} 0 &= \frac{\partial\left\{E[z]\left[n^S\theta^S + n^J\theta^J\right]^\alpha\right\}}{\partial n^J} - w^J + \beta(1-\delta)V'\left((1-\delta)\left(n^S+n^J\right)\right) \\ V'\left(n^S\right) &= \frac{\partial\left\{E[z]\left[n^S\theta^S + n^J\theta^J\right]^\alpha\right\}}{\partial n^S} - w^S + \beta(1-\delta)V'\left((1-\delta)\left(n^S+n^J\right)\right) \end{aligned}$$

Combining the two conditions:

$$V'\left(n^S\right) = \left[\frac{\partial\left\{E[z]\left[n^S\theta^S + n^J\theta^J\right]^\alpha\right\}}{\partial n^S} - w^S\right] - \left[\frac{\partial\left\{E[z]\left[n^S\theta^S + n^J\theta^J\right]^\alpha\right\}}{\partial n^J} - w^J\right].$$

□

E Algorithm for Solving Stationary Equilibrium

1. Guess a value for junior workers J.
2. Guess a value function for firms $V\left(n^S\right)$, a value function for senior workers $S\left(n^S\right)$, and a default cutoff function $z^*\left(n^S, n^J\right)$.
3. Using J, $S\left(n^S\right)$, together with the junior worker wage equation to solve for junior wage: $w^J\left(n^S, n^J\right)$.
4. Using the wage bargaining equation for senior workers to solve for senior wage: $w^S\left(n^S, n^J\right)$.
5. Using the wage rates to calculate the actual default cutoff function $\hat{z}^*\left(n^S, n^J\right)$, as well as to solve for firm's optimal hiring decision. Using the optimal hiring rule of firms to update the value functions of firms and senior workers $\hat{V}\left(n^S\right)$ and $\hat{S}\left(n^S\right)$.
6. Replace the initial guesses of value functions and default cutoff function with the updated value functions and default cutoff function. Iterate until value functions $V\left(n^S\right)$, $S\left(n^S\right)$ and default cutoff function $z^*\left(n^S, n^J\right)$ converge.
7. After value functions and default cutoff converge, using the corresponding policy function to calculate the stationary distribution $g\left(n^S\right)$ of firm sizes, output Y, and labor demand L. Check if household first order condition of labor supply is satisfied. If not, adjust the value of J accordingly.
8. Iterate until J converges.

For detailed algorithm, please refer to Appendix F.

F Algorithm for Solving Transition Dynamics

Our transition problem features heterogeneous firms. Aggregation does not hold because of the decreasing returns to scale production technology. In each period of the transition, we must keep track of the entire distribution of firms. This is because the distribution affects the total output in the economy, which in turn affects the stochastic discount factor used by firms to evaluate payoffs at different dates. In addition, the distribution is also needed to check labor market clearing condition in each period.

The determination of junior and senior wage rates makes the calculation of transition path difficult. For senior workers, wage bargaining involves the derivative of firm's value function. If policy function is not solved accurately, discontinuities in the policy function lead to kinks in the value function, resulting in inaccuracies in the wage rates, which further feeds back into the discontinuities in policy functions. For junior workers, since the equation that determines junior wage is forward-looking, labor market clearing condition in each period t depends on the value of junior workers for both period t and period $t+1$: J_t and J_{t+1}. This makes it difficult to calculate the sequence of $\{J_t\}$ during the transition that clears the labor market.

We solve for the transition dynamics using the following algorithm. There are 3 nested loops: output on the most outside (which determines stochastic discount factors), value to junior workers in the middle (which clears labor market in each period), and value functions for firms and senior workers on the inside. Details are as follows:

1. Choose grids of size N_{n^S} for n^S (state variable) and N_{n^J} for n^J (control variable). Assume transition is complete in T periods. In our code, we choose $N_{n^S} = 300$ n^S, $N_{n^J} = 300$ grids for n^J, and $T = 20$ periods for transition.

2. Guess a sequence of $\{X_t : t = 1, ..., T\}$ during the transition, where $X_t \equiv C_t - v(L_t)$ determines the stochastic discount factors.

3. Guess a sequence of values to junior workers $\{J_t : t = 1, ..., T\}$ during the transition.

4. Guess a sequence of value functions to firms $\left\{V_t\left(n^S\right)\right\}$, value functions to senior workers $\left\{S_t\left(n^S\right)\right\}$, and default cutoff functions $\left\{z_t^*\left(n^S, n^J\right)\right\}$.

5. Calculate the wage rates for junior and senior workers:

$$\begin{aligned}
J_t &= w_t^J\left(n^S, n^J\right) \\
&+\beta\frac{u'\left(X_{t+1}\right)}{u'\left(X_t\right)}\left(1-\delta\right)\left(1-F\left(z_t^*\left(n^J, n^S\right)\right)\right)\zeta S_{t+1}\left(\left(1-\delta\right)\left(n^S+\zeta n^J\right)\right) \\
&+\beta\frac{u'\left(X_{t+1}\right)}{u'\left(X_t\right)}\left[1-\left(1-\delta\right)\left(1-F\left(z_t^*\left(n^J, n^S\right)\right)\right)\zeta\right] J_{t+1} \\
S_t &= w_t^S\left(n^S, n^J\right) \\
&+\beta\frac{u'\left(X_{t+1}\right)}{u'\left(X_t\right)}\left(1-\delta\right)\left(1-F\left(z_t^*\left(n^J, n^S\right)\right)\right) S_{t+1}\left(\left(1-\delta\right)\left(n^S+\zeta n^J\right)\right) \\
&+\beta\frac{u'\left(X_{t+1}\right)}{u'\left(X_t\right)}\left[1-\left(1-\delta\right)\left(1-F\left(z_t^*\left(n^J, n^S\right)\right)\right)\right] J_{t+1} \\
\frac{1-\phi}{\phi} &= \frac{V_t'\left(n^S\right)}{S_t\left(n^S, n^J\right)-J_t}
\end{aligned}$$

6. Calculate the actual default cutoff function $\left\{\hat{z}_t^*\left(n^S, n^J\right)\right\}$ based on the wage rates. Also calculate firm's optimal hiring rule based on default cutoff function and wage rates:

$$\begin{aligned}
\hat{V}_t \quad \left(n^S\right) &= \max_{n^J}\Bigg\{E\left[z\right]\left[n^S\theta^S+n^J\theta^J\right]^\alpha-\left(w_t^S n^S+w_t^J n^J\right) \\
&+\beta\frac{u'\left(X_{t+1}\right)}{u'\left(X_t\right)}F\left(\hat{z}_t^*\left(n^J, n^S\right)\right) V_{t+1}(0) \\
&+\beta\frac{u'\left(X_{t+1}\right)}{u'\left(X_t\right)}\left(1-F\left(\hat{z}_t^*\left(n^J, n^S\right)\right)\right) V_{t+1}\left[\left(1-\delta\right)\left(n^S+\zeta n^J\right)\right]\Bigg\}
\end{aligned}$$

This implies an optimal hiring rule $n_t^J\left(n^S\right)$ as well as an updated value function $\hat{V}_t\left(n^S\right)$ for firms. Using the policy function, we can calculate the updated value function for senior workers:

$$\begin{aligned}
\hat{S}_t\left(n^S, n^J\right) &= w_t^S\left(n^S, n^J\right) \\
&+\beta\frac{u'\left(X_{t+1}\right)}{u'\left(X_t\right)}\left(1-\delta\right)\left(1-F\left(\hat{z}_t^*\left(n^J, n^S\right)\right)\right) S_{t+1}\left(\left(1-\delta\right)\left(n^S+\zeta n^J\right)\right) \\
&+\beta\frac{u'\left(X_{t+1}\right)}{u'\left(X_t\right)}\left[1-\left(1-\delta\right)\left(1-F\left(\hat{z}_t^*\left(n^J, n^S\right)\right)\right)\right] J_{t+1}
\end{aligned}$$

7. Iterate until:

$$\begin{aligned}
\max_{t=1,\ldots,T}\max_{n^S}\left|V_t\left(n^S\right)-\hat{V}_t\left(n^S\right)\right| &< \text{tolerance} \\
\max_{t=1,\ldots,T}\max_{n^S}\left|S_t\left(n^S\right)-\hat{S}_t\left(n^S\right)\right| &< \text{tolerance} \\
\max_{t=1,\ldots,T}\max_{n^S}\left|z_t^*\left(n^S, n^J\right)-\hat{z}_t^*\left(n^S, n^J\right)\right| &< \text{tolerance}
\end{aligned}$$

8. Let $n_t^J\left(n^S\right)$ and $z_t^*\left(n^S, n^J\right)$ be the policy function and default cutoff function associated with the convergent value functions and default function. Using these to calculate the stationary distribution of firm sizes during the transition from forward induction:

   $g_1\left(n^S\right)$ is taken as given from the shock

$$g_{t+1}(n) = \begin{cases} \int_z \int_{n^S} \left\{ \begin{array}{c} \mathbf{1}\left\{(1-\delta)\left(n^S + \zeta n_t^J\left(n^S\right)\right) = n\right\} \\ \cdot \mathbf{1}\left\{z \geq z_t^*\left(n^J\left(n^S\right), n^S\right)\right\} \end{array} \right\} g_t\left(n^S\right) f(z)\, dn^S dz & \text{if } n > 0 \\ \int_z \int_{n^S} \mathbf{1}\left\{z < z^*\left(n_t^J\left(n^S\right), n^S\right)\right\} g_t\left(n^S\right) f(z)\, dn^S dz & \text{if } n = 0 \end{cases}$$

   Based on the distribution of firm sizes during the transition, calculate total labor demand by firms in each period:

$$L_t = \int_{n^S} \left(n^S + n_t^J\left(n^S\right)\right) g_t\left(n^S\right) dn^S$$

9. Check if labor demand and output is consistent with household's first order condition for labor supply:

$$v'(L_t) = \bar{w}_t$$

   where $\bar{w}_t$ is the average wage per hour worked given by:

$$\bar{w}_t \equiv \frac{\int_{n^S} \left(w_t^S\left(n^S\right) n^S + w_t^J\left(n^S\right) n_t^J\left(n^S\right)\right) g_t\left(n^S\right) dn^S}{L_t}$$

   If this first order condition is not satisfied, adjust the value of J_t according to the following method:

$$\begin{aligned} \hat{J}_t = & \left\{ v'(L_t) L_t - \int \frac{\phi}{1-\phi} n^S V_t'\left(n^S\right) g_t\left(n^S\right) dn^S \right. \\ & \left. + \beta \frac{u'(X_{t+1})}{u'(X_t)} \int \left[A_t\left(n^S\right) n_t^J\left(n^S\right) + B_t\left(n^S\right) n^S\right] g_t\left(n^S\right) dn^S \right\} / L_t \end{aligned}$$

   where

$$\begin{aligned} A_t\left(n^S\right) \equiv & \left[(1-\delta)\left(1 - F\left(z_t^*\left(n^J, n^S\right)\right)\right)\zeta\right] S_{t+1}\left((1-\delta)\left(n^S + \zeta n_t^J\left(n^S\right)\right)\right) \\ & + \left[1 - (1-\delta)\left(1 - F\left(z_t^*\left(n^J, n^S\right)\right)\right)\zeta\right] J_{t+1} \\ B_t\left(n^S\right) \equiv & \left[(1-\delta)\left(1 - F\left(\hat{z}_t^*\left(n^J, n^S\right)\right)\right)\right] S_{t+1}\left((1-\delta)\left(n^S + \zeta n_t^J\left(n^S\right)\right)\right) \\ & + \left[1 - (1-\delta)\left(1 - F\left(\hat{z}_t^*\left(n^J, n^S\right)\right)\right)\right] J_{t+1} \end{aligned}$$

   This update of J_t makes sure that the labor market for period t clears, taken J_{t+1} and the distribution g_t of firms in period t as given.

10. Update the value of J_t:

$$J_t = \lambda J_t + (1-\lambda)\hat{J}_t$$

Iterate until labor market clears for each period during the transition.

11. Calculate the actual output sequence:

$$\hat{Y}_t \equiv \int_z \int_{n^S} z\left[n_t^J(n^S)\theta^J + n^S\theta^S\right]^\alpha g_t(n^S)dn^S dF(z)$$

Update the initial guess of the sequence of X_t:

$$\hat{X}_t = \hat{Y}_t - v(L_t)$$

Iterate until the time series of output converge.

G Determining θ^S/θ^J Using the CPS

As outlined in Section 6, we use data from the Displayed Workers, Occupational Mobility and Job Tenure supplements of the CPS to infer the relative productivity of senior to junior workers. This supplemental survey has not been administered every year, and the survey questions have varied somewhat over time. The first time these supplemental questions were asked was in 1973. Unfortunately, questions inquiring about displacement as well as prior and current wages were not asked again until 1984. Since then, these questions have continued to be included bi-annually to present.

Following the method used in Topel (1991), we restrict attention to male respondents between the ages of 20 and 60 whose jobs end exogenously. We then deflate nominal wages by the GNP price deflator for consumption expenditure. For these workers we calculate the average change in log weekly wages for the prior and current jobs, and use Equation (5) to calculate the implied ratio of θ^S/θ^J.

The percentage drop in wages following an exogenous separation is affected by the business cycle. Specifically, the survey conducted immediately following each recession yields a larger drop in wages than the average drop experienced in surrounding observations. We therefore calculate the implied θ^S/θ^J by both including and excluding these points. For the supplemental survey administered prior to the double-dip recession, we find an average change in log weekly wages of -2.7%, which implies a value of θ^S/θ^J of 1.55.[31] For those surveys administered after the double-dip recession, we find an average change in log weekly wages of -14.7%, which implies a value of θ^S/θ^J of 4.45. When the post-recession data points are excluded, we observe an average change in log weekly wages of -11.5%, which implies a value of θ^S/θ^J of 3.59.

[31] Supplemental survey questions which include data regarding wages of the prior and current job were only included one time before the double dip recession (in 1973). Therefore, the value of θ^S/θ^J for the "fast recoveries" is based on the data from this single survey.

H Determining θ^S/θ^J Using the PSID

Altonji and Shakotko (1985) and Altonji and Williams (2005) use the following specification for estimating returns to seniority:

$$W_{ijt} = \beta_0 t + \beta_1 X_{ijt} + \beta_2 T_{ijt} + \epsilon_{ijt} \tag{H9}$$

where W_{ijt} is the log earning of person i in job j in period t. X is total labor market experience and T is tenure with the employer. The error term ϵ is potentially dependent on individual specific component μ_i and match specific component ϕ_{ij}:

$$\epsilon_{ijt} = \mu_i + \phi_{ij}$$

These individual specific and match specific components are likely to be correlated with both tenure and earning: an individual with higher ability or a good match between worker and employer are likely to lead to both longer tenure and higher earning.

To deal with this endogeneity problem, Altonji and Shakotko (1985) and Altonji and Williams (2005) propose an instrumental variable DT_{ij} for tenure. This instrument is defined to be the deviation of tenure T_{ijt} from the mean $\bar{T}_{ij}$ of the sample observations on job match ij. Effectively, bringing in this instrument is equivalent to demeaning Equation (H9) from its mean over time. Since individual and match specific components in the error term are assumed to be invariant over time, they will be eliminated after this demeaning procedure. Intuitively, although individual and match specific components are likely to affect the earning of a worker, they stay constant within a worker-job match. Therefore, if we look within each worker-job match, the increase in earning over time cannot be attributed to those two components. A similar instrument is also used for year.

Altonji and Williams (2005) extend the methodology of Altonji and Shakotko (1985) to newer PSID waves of 1983 to 1991. We further extend their estimations to 2013. Between 1968 and 1997, PSID interviews were conducted annually. Since then, interviews have been biennial. This change in the frequency of interviews does not affect the validity of the regression, because returns to tenure are estimated from earning gains within worker-job matches.

Summary statistics are included in table 8.

The results of our regression analysis are included in table 9. We report the estimated returns to tenure at 3, 5, 10 as well as the mean tenure for workers.

Table 8 – Summary Statistics, PSID

Variable	Panel 1: 1975-1982	Panel 2: 1983-2013
ln(Earnings)	2.827	2.883
	(0.458)	(0.573)
Experience	16.540	16.850
	(10.820)	(9.710)
Tenure	8.795	7.952
	(9.083)	(7.934)
Education	12.811	13.540
	(2.430)	(2.123)
Married Now	0.876	0.815
	(0.330)	(0.389)
Union	0.302	0.161
	(0.473)	(0.367)
Disability Affecting Work	0.063	0.073
	(0.244)	(0.261)
Age	36.115	37.018
	(11.240)	(9.973)
North Central Region	0.344	0.327
North Eastern Region	0.224	0.358
Southern Region	0.271	0.279
Western Region	0.160	0.182
Observations	6882	23125

Notes: Standard deviations are in parentheses. Samples are created from the Panel Study of Income Dynamics, survey years 1975-1996, and every other year 1999-2013.

Table 9 – IV Estimation of Returns to Tenure

	Panel 1: 1975-1982	Panel 2: 1983-2013
3 Years of Tenure	0.0444	0.0778
	(0.0169)	(0.0106)
5 Years of Tenure	0.0548	0.1143
	(0.0235)	(0.0143)
10 Years of Tenure	0.0422	0.1286
	(0.0353)	(0.0188)
Mean Tenure	0.0484	0.1302
	(0.0324)	(0.0173)

Notes: White standard errors in parentheses. Specification includes a fourth degree polynomial for estimation of returns to experience and tenure. Controls are included.

I Cross-Sectional Intuition for How θ^S/θ^J Affects Average Labor Productivity

In Section 7, intuition is provided regarding how θ^S/θ^J affects average labor productivity by looking at forces across time. In this section, we conduct a similar analysis on the cross-section. During the recovery, there are two opposing forces on average labor productivity. First, there is a composition effect. The recession leads to the bankruptcy of a fraction of firms. These firms start over with no senior workers and are less productive than the average firm in the stationary equilibrium. This is due to their high proportion of junior workers. The composition effect tends to drag down the average labor productivity. Second, there is a decreasing returns to scale effect. During a recession, output is small relative to future periods. This increases the importance of present consumption relative to future growth and results in reduced hiring. Reduced firm size translates into higher average labor productivity within each firm due to the decreasing returns to scale technology.

Whether average labor productivity will be procyclical or countercyclical depends on the relative strength of the composition effect across firms and the decreasing returns to scale effect within firms. When θ^S/θ^J is low, firm-specific skills are less important. Start-up firms grow fast. This leads to a strong composition effect. Further, when start-up firms grow fast, they employ a larger portion of the labor force which further increases the strength of the composition effect.[32] As a result, average labor productivity is procyclical. When θ^S/θ^J is high, firm-specific skills are more important. Start-up firms grow slowly. This leads to a weaker composition effect. Hence average labor productivity is countercyclical.

In Figure 20, we provide two cross-sectional snapshots (one in the steady state and one during the transition) for a fast recovery economy (θ^S_{low}/θ^J). In the left panel, we compare the size distribution of firms in the stationary equilibrium (blue bars) and during the first period of recovery (red bars). As can be observed in the figure, there are more start-up firms and less firms at the mature size in the recovery relative to the stationary equilibrium. Over time, these bankrupt firms grow back to their mature size. The right panel displays the average labor productivity by firm size. As can be observed in this figure, smaller firms are substantially less productive than larger firms as a result of their higher ratio of junior workers. Additionally, all firms of a specific size are slightly more productive in recovery relative to their stationary counterpart. This is because the stochastic discount factor causes firms to emphasize present profits over future growth during the recovery. In total, the composition effect from the increase in the proportion of less productive start-up firms outweighs the decreasing returns to scale effect caused by the smaller average firm size.

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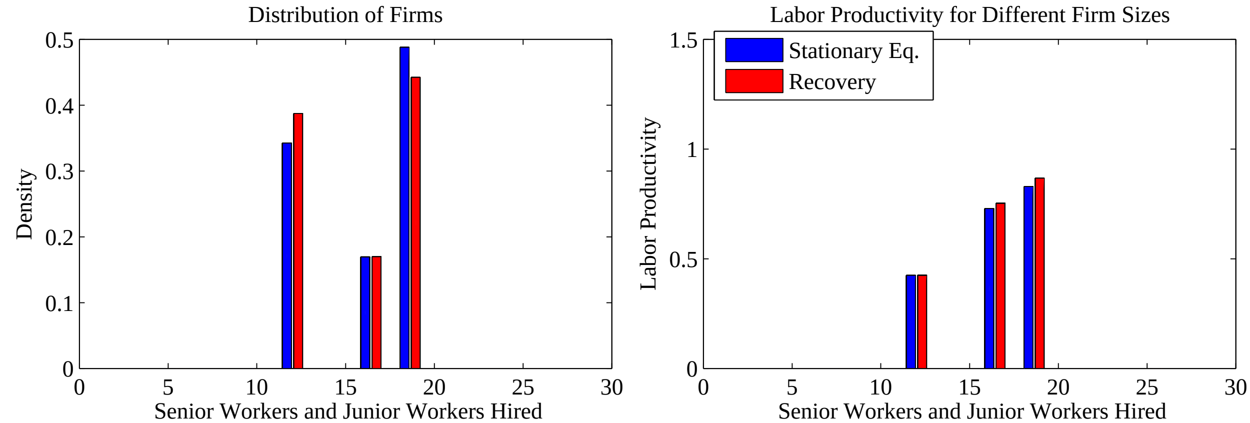
$$\text{Average Labor Productivity} \equiv \frac{Y}{L} = \sum_{n^S} \frac{g\left(n^S\right) L\left(n^S\right)}{L} \frac{Y\left(n^S\right)}{L\left(n^S\right)}$$

where $g\left(n^S\right)$ is the measure of firms of size n^S, $L\left(n^S\right)$ is the total labor used in a firm of size n^S, and $Y\left(n^S\right)$ is the expected output of a firm of size n^S.

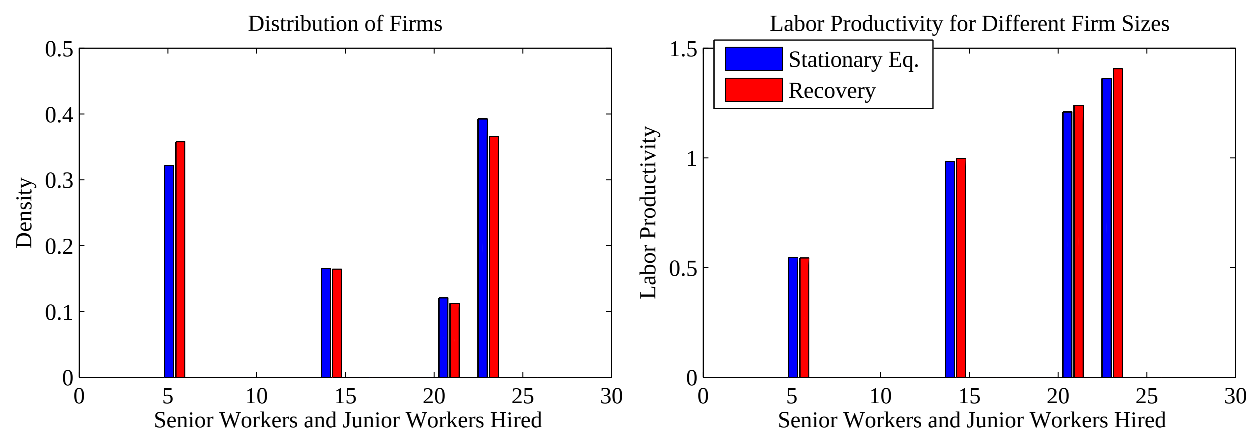
Figure 20 – Distribution of Firm Sizes and Labor Productivities by Firm Sizes ($\theta^S = \theta^S_{low}$)



Note: In this figure, although it looks like the x-axis values for blue and red bars are slightly misaligned, the densities and labor productivities are actually calculated on the same set of bins for the total number of workers.

In Figure 21, we present the cross-sectional snapshots for a slow recovery economy (θ^S_{high}/θ^J). As before, the left panel compares the size distribution of firms in the stationary equilibrium (blue bars) and during the first period of recovery (red bars). As can be observed in the figure, there are more start-up firms and less firms at the mature size in the recovery relative to the stationary equilibrium. However, small firms are much smaller than they were in the fast recovery economy. As a result, they employ a smaller portion of the labor force and their contribution to average labor productivity is significantly reduced. The graph of average labor productivity is similar to the fast recovery figure for the same reasons as before. In this case, the composition effect of an increase in start-up firms is dampened by the smaller average size of start-up firms, and the decreasing returns to scale effect dominates.

Figure 21 – Distribution of Firm Sizes and Labor Productivities by Firm Sizes ($\theta^S = \theta^S_{high}$)



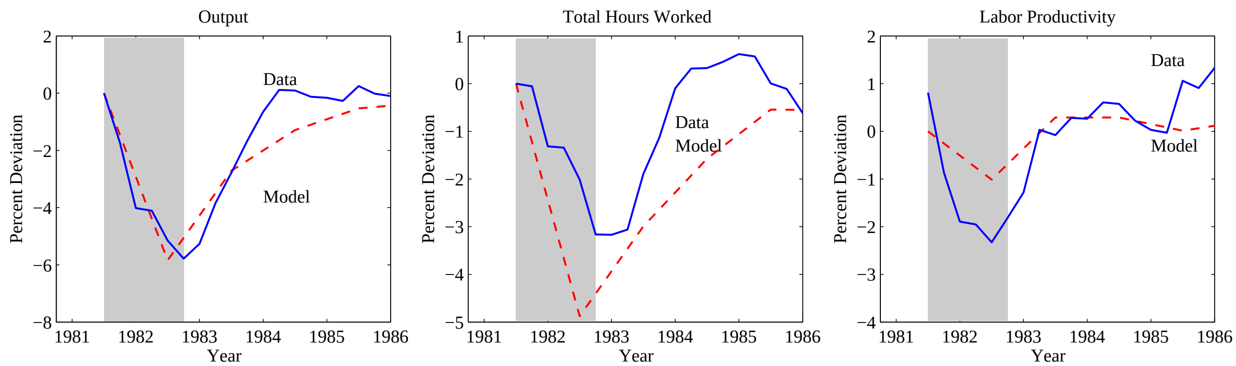
Note: In this figure, although it looks like the x-axis values for blue and red bars are slightly misaligned, the densities and labor productivities are actually calculated on the same set of bins for the total number of workers.

J Simulation Results for Other Recessions

In Section 7, we compare the simulation results of our model with the data for the 1973 recession and the 2008 recession. In this section, additional comparisons for the recessions of 1982, 1990, and 2001 are presented.

The three recessions prior to the mid-1980s include the 1973 recession, the 1980 recession, and the 1982 recession. The results for the 1973 recession are presented in Section 7. The 1980 recession is followed shortly by the 1982 recession so the recovery path is not complete. Below is the comparison of output, unemployment, and labor productivity between data and model for the 1982 recession:

Figure 22 – 1982 Recession



The three recessions after the mid-1980s are the 1900 recession, the 2001 recession, and the 2008 recession. The results for the 2008 recession are presented in Section 7. The 1990 and 2001 recessions are plotted below:

Figure 23 – 1990 Recession

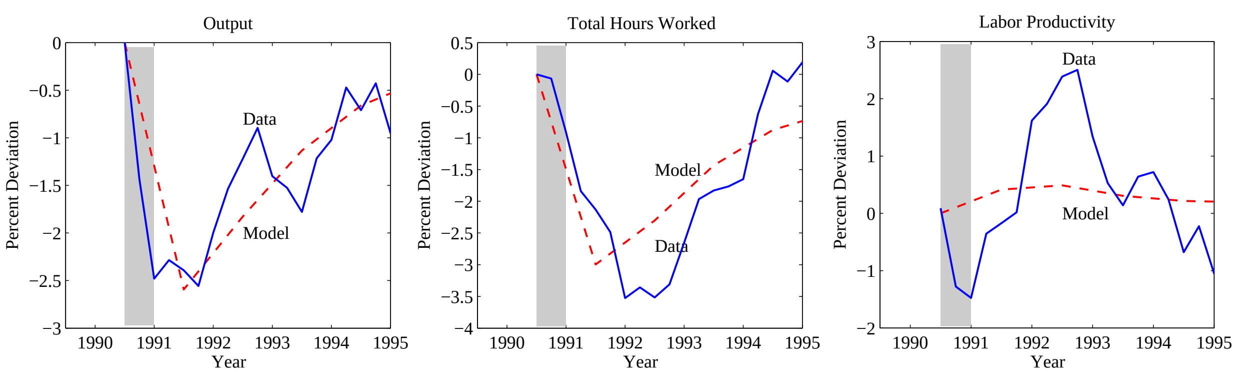
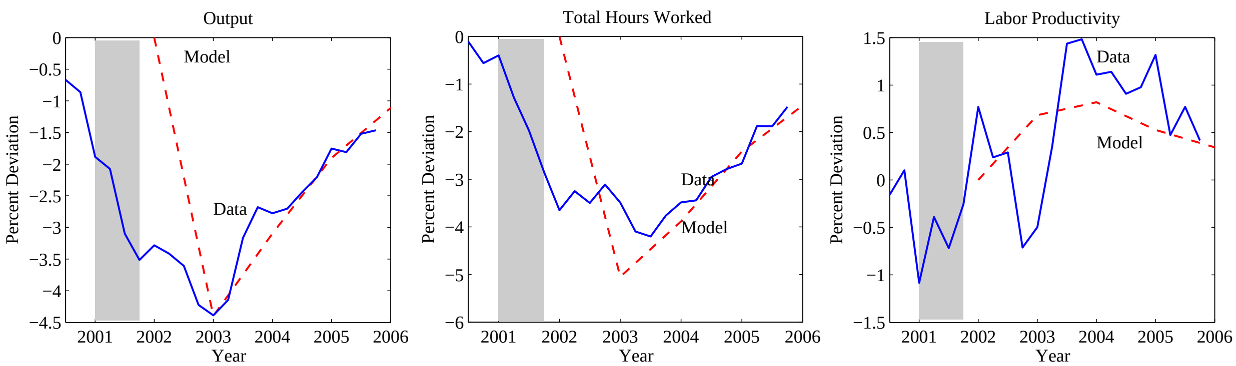


Figure 24 – 2001 Recession



K Compare Model and Data Using Employment Level

In Section 7, we compare the path of employment recovery between our model and the data. Employment is measured as the percentage deviation of total hours in nonfarm business sector from its trend. Another popular measure of employment is the employment level. In this section, we compare the path of employment recovery in our model with the employment population ratio, as a robustness check.

To remove the trend in employment level, we employ an HP filter. Figure 25 compare the employment recovery between our model and data, using two different measures of employment. Figure 26 is the comparison for the 2008 recession.

Figure 25 – 1973 Recession

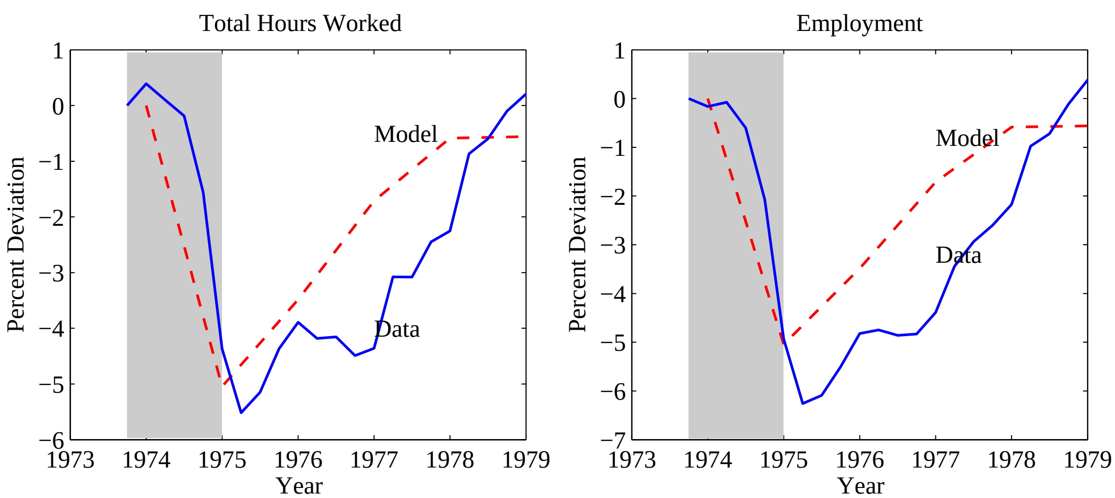
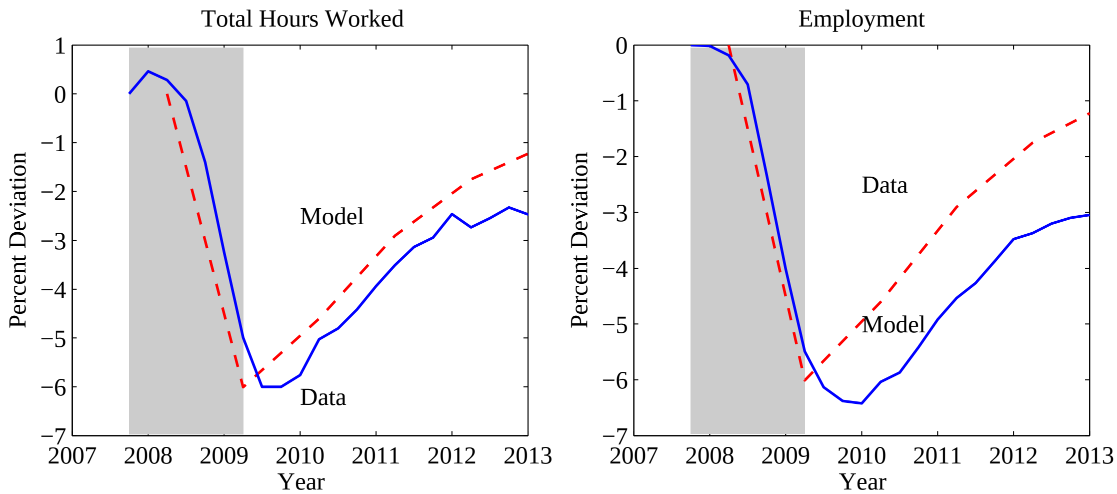


Figure 26 – 2008 Recession



L General CES Production Function

Throughout our previous analysis, we have been assuming the effective labor inputs of junior and senior workers are perfect substitutes. In this subsection, we consider a more general production function of the constant elasticity of substitution (CES) form:

$$y = \left\{ \Upsilon \left(\theta^J n^J\right)^{\varepsilon} + (1-\Upsilon)\left(\theta^S n^S\right)^{\varepsilon} \right\}^{\alpha/\varepsilon}.$$

With complementarity between the effective labor input of junior and senior workers, small firms grow even slower. This is because the labor input of senior workers complement that of junior workers. Small firms have few senior workers so their junior workers are even less productive. A similar result to the benchmark model can be achieved through changes either to the relative productivity of senior to junior workers (as before) or to the junior worker share parameter, Υ. In this setup, senior workers and junior workers are complementary. A change in their relative importance continues to affect the cyclicality of labor productivity over the business cycle in the same manner as our benchmark model. For example, suppose some of the work previously completed by junior workers is automated. In this case, if the share parameter, Υ, for junior workers drops from 40% to 20%, then recovery is slower and the cyclicality of average labor productivity is flipped. A graphical example is provided in Figure 27.[33] This provides an additional potential explanation for emergent patterns in labor productivity and unemployment recovery, namely an increased share of tasks completed by senior workers.[34]

[33] The graph was generated using $\varepsilon = 0.7$ (elasticity of substitution of 3.33), $\Upsilon = 0.4$ for older recession and $\Upsilon = 0.2$ for newer recession. $\theta^S = 2$ for both cases.

[34] It is important to note that this production function has an unrealistic implication. Namely, if a firm has a large ratio of senior employees, it is possible that the marginal productivity of junior employees is larger than their senior counterparts.

Figure 27 – CES Production Function