Monetary Policy and Inflation Dynamics in Asset Price Bubbles*

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Abstract

This paper studies optimal monetary policy in a monetary DSGE model which features both an asset price bubble and agency costs in firms' price-setting decisions. Amplified by nominal wage rigidities, an asset price bubble causes an inefficiently excessive boom. Inflation, however, remains moderate in the boom, because a loosening in financial tightness lowers the agency costs and adds downward pressure on marginal costs and inflation. In such an environment, strictly inflation targeting, which stabilizes inflation completely, makes the excessive boom even excessive in the short run. The optimal monetary policy calls for monetary tightening to curve the boom at the cost of a drop in inflation.

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1 Introduction

In academic and central bank circles a heated debate has taken place over whether monetary policy should mainly focus on inflation stabilization or react to asset price developments instead.[1] No consensus has yet been reached and views on the issue are evolving following the global financial crisis of 2007-2008. One reason for the disagreement is that little is understood about inflation dynamics in asset price booms. As documented by Adalid and Detken (2007), Bordo and Wheelock (2004, 2007) and Christiano et al. (2010), in many countries asset price booms, including possible asset price bubbles, have been periods of moderate inflation. The moderate inflation raises a concern that monetary policy focusing on inflation stabilization may fail to address inefficiencies, if there are any, in asset price booms. Why does inflation tend to be moderate in asset price booms? How should monetary policy be conducted in such an environment? What distortions could affect the framework of optimal monetary policy in asset price booms?

This paper aims to address these questions and derive policy implications. To this end I extend the business cycle model with an asset price bubble, developed by Miao, Wang and Xu (2012), to a monetary dynamic stochastic general equilibrium (DSGE) model with agency costs in firms' price setting decisions. The agency costs are modeled as credit constraints on working capital.[2] A tighter credit constraint makes it more costly to raise funds for working capital and to produce goods. Consequently, the financial tightness affects marginal costs and appears endogenously as a cost-push shock in the Phillips curve. The agency costs coexist with an asset price bubble which emerges endogenously as in Miao, Wang and Xu (2012). In sum, the model features financial tightness as an endogenous cost-push shock in the Phillips curve and an asset price bubble within the standard DSGE framework developed by Christiano, Eichenbaum and Evans (2005, CEE for short hereafter) and Smets and Wouters (2007).

I estimate the model using U.S. time series data and make five points based on the estimated model.

First, the model succeeds in generating moderate inflation in an asset price bubble. In addition to the standard upward pressure on inflation caused by an inflating asset price, the model features downward pressure on inflation arising from a loosening in financial tightness. The downward pressure has to do with the fact that firms, subject to credit constraints, have to finance the cost of working capital. An asset price bubble raises the value of firms, which makes it easier for firms to raise funds for working capital and lowers the cost of production. This adds downward pressure on marginal costs and hence on inflation. The simulation shows that the downward pressure on inflation limits a rise in

[1] See, for example, the papers and speeches in Hunter, Kaufman and Pomerleano (2003).

[2] The importance of working capital is stressed and confirmed empirically in the literature on the cost channel of monetary policy. See, for example, Barth and Ramey (2001) and Ravenna and Walsh (2006).

inflation significantly. In particular, inflation rises only by half the extent it would without the downward pressure.

Second, the optimal monetary policy calls for monetary tightening to restrain an asset price boom more than what would be warranted by price stability. The optimal monetary policy is derived as a solution to the Ramsey problem in which a benevolent planner with commitment chooses the nominal interest rate to maximize the utility of a representative household. In response to an asset price bubble the optimal monetary policy raises the real interest rate to curb an increase in real variables such as output and investment. The monetary tightening, however, is too strong to stabilize inflation as inflation falls in the short run. The fall in inflation implies that policy makers could face a difficult trade off between stabilizing the real economy and stabilizing inflation in an asset price bubble.

Third, it is nominal wage rigidities that mainly contribute to generating inefficiently amplified responses to an inflating asset price bubble. An asset price bubble allows firms to invest and hire more and increases the demand for labor. Nominal wage rigidities prevent real wages from rising sharply and stimulate the demand for labor. The increase in labor amplifies a boom in output and investment, which, in turn, raises the credit capacity and fuels the boom. The optimal monetary policy aims to restrain the inefficiently excessive boom at the cost of greater volatility in inflation. Interestingly, without nominal wage rigidities, the optimal monetary policy would be nearly consistent with price stability. The result highlights the importance of the nature of the labor market in the conduct of monetary policy in an asset price bubble.

Fourth, in an asset price bubble the inflation stabilization policy (i.e., strict inflation targeting) escalates an excessive boom in the short run, though the policy contributes to stabilize the boom in the long run. Stabilizing inflation requires stabilizing marginal costs. Because a loosening in financial tightness decreases marginal costs in the short run, strict inflation targeting calls for monetary easing at the onset of a boom. The monetary easing fuels the heated economy, making an excessive boom even excessive in the short run.

Fifth, an effective monetary policy to contain asset price bubbles is not a precondition for monetary policy to respond to asset price developments. In the model, monetary policy has little effect on the size of bubble because it is economic agents' self-fulfilling beliefs that mainly drive a bubble. The fact that monetary policy has little effect on the size of bubble, however, does not prevent a benevolent Ramsey planner from responding to an asset price boom more than what would be warranted by price stability. Although monetary policy may not be useful to contain a bubble, it is still able to serve as a useful tool to restrain inefficiently amplified responses of real variables in a bubble.

Related Literature

This paper is not the first to address moderate inflation in asset price booms in a DSGE framework. Christiano et al. (2010) build a monetary DSGE model in which expectations of improvements in technology in the future cause not only a boom but also low inflation today because of lower expected inflation in the future. This paper proposes another mechanism of moderate inflation, which is complementary to the one presented by Christiano et al. (2010).[3]

This paper is motivated by Bernanke and Gertler (1999, 2001), who provide important insights about the conduct of monetary policy in an asset price bubble. They argue that focusing on inflation stabilization remains a useful policy framework even in an asset price bubble. In contrast, this paper shows that deviating from inflation stabilization to restrain a boom is optimal in an asset price bubble. The different policy implication originates from three differences between this paper and the studies by Bernanke and Gertler (1999, 2001). First and the most importantly, the model in this paper features additional frictions, including nominal wage rigidities and working capital. Without nominal wage rigidities, inflation stabilization would perform nearly the same as the optimal monetary policy, consistent with Bernanke and Gertler (1999, 2001). Second, this paper considers optimal monetary policy formally, while they consider the effect of various policies on the volatility of variables. Third, the model features a rational asset price bubble, while Bernanke and Gertler consider a non-rational asset price bubble such that there remains opportunities for arbitrage.

Moreover, this paper is closely related to Gali (2011), which studies optimal monetary policy in rational bubbles in an overlapping generations framework. He shows, contrary to this paper, that monetary tightening in a bubble could enhance fluctuations and worsen welfare. The conflicting implications stem from a difference in a way how a bubble is sustained. While in Gali (2011) a bubble increases at the rate of real interest rate, in this paper a bubble itself generates benefits by mitigating credit constraints so that the bubble does not have to grow at the rate of real interest rate. Therefore, while in Gali (2011) a bubble is sensitive to monetary policy, which affects the real interest rate, it is not the case in the present paper. In Gali (2011), monetary tightening in an asset price bubble raises the real interest rate, which may accelerate the growth rate of the bubble and destabilize the economy.

This paper is also related to the literature on monetary policy and asset prices. Gilchrist and Leahy (2002) study shocks associated with asset prices and find that focusing on stabilizing inflation remains a useful policy framework, while Dupor (2002, 2005) considers non-rational shocks to asset prices and makes the case that incorporating asset prices in monetary policy rules improves welfare.

Broadly speaking, this paper can be placed in the literature on optimal monetary policy. The majority of the literature supports the view that stabilizing inflation is close to optimal

[3]Kimura (2012) proposes another mechanism of moderate inflation, namely one in which liquidity-abundant firms invest in market share by keeping prices down in booms. He examines and confirms the mechanism empirically using survey data for Japan.

(see Schmitt-Grohe and Uribe (2007) for a medium scale DSGE model, Faia and Monacelli (2007) for a DSGE model with financial frictions, and Carlstrom, Fuerst and Paustian (2010) for a simple model in which financial tightness appears in the Phillips curve as in this paper). In contrast Erceg, Henderson and Levin (2000) show that stabilizing inflation is not optimal in a staggered wage setting. This paper shows that nominal wage rigidities à la Erceg, Henderson and Levin (2000) play the crucial role for the conduct of monetary policy in an asset price bubble. Stabilizing inflation is far from optimal in an asset price bubble when nominal wages are not adjusted flexibly.

The rest of the paper is organized as follows. Section 2 presents the model used for the analysis. I then estimate the model using U.S. data in Section 3. Section 4 uses the estimated model and studies inflation dynamics and monetary policy in an asset price bubble. Section 5 concludes the paper.

2 Model

The model used here builds on the study by Miao, Wang and Xu (2012), who develop a business cycle model with a rational asset price bubble, extended in two ways. First, the model features a credit constraint on working capital, which will be shown to add interesting inflation dynamics to the model. Second, the model is built in a standard monetary DSGE framework developed by CEE (2005) and Smets and Wouters (2007). There are six types of agents in the model: households, wholesale goods firms, retailers, final goods firms, investment goods firms and a government. This section starts from the description of households, which is followed by the description of the most important part of the model: wholesale goods firms. The wholesale goods firms are subject to a credit constraint and a bubble could emerge in the share price of the wholesale goods firms. Next, the common building blocks of a standard monetary DSGE model are presented. Retailers and final goods firms serve as a modeling device to introduce nominal price rigidities. Investment goods firms face investment adjustment costs, which add a persistence mechanism to the model. The government consists of a fiscal and a monetary authority. Following the description of the model, the Phillips curve is derived to show that the financial tightness appears endogenously as a cost-push shock.

2.1 Households

There is a continuum of households with measure unity. The households own all firms in the economy. The households supply specialized labor, indexed by $j \in (0, 1)$, and have monopoly power over nominal wage for specialized labor. A competitive employment agency combines all households' specialized labor and transform into a homogeneous labor. Then, the employment agency provides the homogeneous labor to the wholesale goods firms.

The household problem can be divided into three: a consumption and saving problem, an employment agency's problem and a workers' wage setting problem.

Consumption and Saving Problem

The households trade both nominal bonds and the share of the wholesale goods firms. The firms differ in age τ and the average ex-dividened share price of the firms with age τ, $S_{t,\tau}$, depends on τ. The share of the newly born wholesale goods firms is assumed to be distributed equally among the households. Then, household j chooses consumption C_t, the amount of nominal bonds D_t, and the share of the wholesale goods firm with age τ, $e_{t+1,\tau+1}$, for $\tau = 0, 1, ...,$ to maximize utility

$$E_t \sum_{s=0}^{\infty} \beta^s \zeta_t \left[\log \left(C_{t+s} - hC_{t+s-1} \right) - \psi \frac{L_{t+s}(j)^{1+\nu}}{1+\nu} \right], \tag{1}$$

subject to a flow budget constraint,

$$\begin{aligned} & P_t C_t + D_t + \sum_{\tau=0}^{\infty} S_{t,\tau} e_{t+1,\tau+1} \\ \leq & W_t(j) L_t(j) + R_{t-1} D_{t-1} + \sum_{\tau=0}^{\infty} \left(\Pi_{t,\tau}^s + S_{t,\tau} \right) e_{t,\tau} + \Pi_t^p + \Theta_t(j) + T_t, \end{aligned} \tag{2}$$

where $L_t(j)$ denotes the specialized labor of the household, P_t denotes the price level, R_t denotes the nominal interest rate, Π_t^s denotes the average dividends of the wholesale goods firms with age τ, Π_t^p denotes the profits of producers and T_t denotes lump-sum taxes. The households insure against the opportunity of wage changes so that an individual household chooses the same level of consumption, nominal bonds and firm shares, keeping a representative agent framework. The net cash flow arising from insuring the opportunity of wage changes is denoted as $\Theta_t(j)$ in (2). The structural parameters satisfy $0 < \beta < 1$, $\nu > 0$ and $\psi > 0$. The preference shock, ζ_t in (1), follows an AR(1) process:

$$\log(\zeta_t) = \rho_\zeta \log(\zeta_{t-1}) + \epsilon_{\zeta,t}, \quad 0 \leq \rho_\zeta < 1,$$

with $\epsilon_{\zeta,t} \sim i.i.d.N \left(0, \sigma_\zeta^2\right)$.

Let Λ_t denote a Lagrange multiplier on the budget constraint (2). Maximizing (1) with respect to C_t, B_t and $e_{t+1,\tau+1}$ subject to (2) yields

$$P_t \Lambda_t = \frac{\zeta_t}{C_t - hC_{t-1}} + E_t \frac{\beta h \zeta_{t+1}}{C_{t+1} - hC_t}, \tag{3}$$

$$1 = E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} R_t, \tag{4}$$

$$1 = E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\Pi_{t+1,\tau+1}^s + S_{t+1,\tau+1}}{S_{t,\tau}}. \tag{5}$$

Equations (4) and (5) imply that the household equates the expected discounted return from investing in the bonds and in the firms shares so that the household chooses the portfolio optimally.

Employment Agency's Problem

A competitive employment agency combines a collection of households' specialized labor, $\{L_t(j)\}_j$, and produces homogeneous labor, L_t, using a CES aggregation technology,

$$L_t = \left(\int_0^1 L_t(j)^{\frac{1}{\lambda_{w,t}}} dj \right)^{\lambda_{w,t}}, \quad \lambda_{w,t} > 1, \tag{6}$$

where the exogenous wage markup follows an ARMA(1,1) process:

$$\log(\lambda_{w,t}/\lambda_w) = \rho_w \log(\lambda_{w,t}/\lambda_w) + \epsilon_{w,t} - \gamma_w \epsilon_{w,t-1}, \quad 0 \leq \rho_w < 1, \quad \lambda_w > 1,$$

with $\epsilon_{w,t} \sim i.i.d.N\left(0, \sigma_w^2\right)$. Following Smets and Wouters (2007), the MA(1) term aims to capture some of the high-frequency fluctuations in wages. Given the nominal wage W_t for homogeneous labor and the nominal wages for specialized labor $\{W_t(j)\}_j$, the employment agency chooses the amount of specialized labor to maximize its profits,

$$\max_{\{L_t(j)\}} W_t L_t - \int_0^1 W_t(j) L_t(j) dj,$$

subject to (28). The solution to this problem yields a demand curve for the specialized labor,

$$L_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{\frac{\lambda_{w,t}}{1-\lambda_{w,t}}} L_t, \tag{7}$$

where the nominal wage is given by,

$$W_t = \left(\int_0^1 W_t(j)^{\frac{1}{1-\lambda_{w,t}}} dj \right)^{1-\lambda_{w,t}}.$$

Wage Setting Problem

The households face nominal wage change frictions à la Erceg, Henderson and Levin (2000) in setting their wage for specialized labor. That is, the households can change their nominal wage with probability $0 < 1 - \xi_w < 1$ identically and independently over time and across households. Let $\tilde{W}_t\left(j\right)$ denote the wage set by household j in period t. Following CEE (2005), the wage, which has not been reset since period t, is assumed to follow an indexation rule:

$$W_{t+s}(j) = \begin{cases} \tilde{W}_t(j) & \text{if } s = 0 \\ \Pi_{k=1}^{s} (\pi_{t+k-1} z_{t+k-1})^{\nu_w} \left(\pi z\right)^{1-\nu_w} \tilde{W}(j) & \text{if } s = 1, 2, ... \end{cases} \tag{8}$$

where $0 \leq \nu_w \leq 1$ denotes the degree of indexation to the product of past inflation and the past growth rate of TFP. Then, household j sets $\tilde{W}_t(j)$ to maximize (1), resulting in a following problem:

$$\max_{\{\tilde{W}_t(j)\}} E_t \sum_{s=0}^{\infty} (\beta \xi_w)^s \left[\Lambda_{t+s} W_{t+s}(j) L_{t+s}(j) - \psi \frac{L_{t+s}(j)^{1+1/\nu}}{1+1/\nu} \right], \tag{9}$$

subject to the demand curve for specialized labor (7) and the indexation rule (8). The objective in (9) consists of the utility from earning the wage, the first term in the brackets, and the disutility from supplying labor, the second term in the brackets.

2.2 Wholesale Goods Firms

There is a continuum of wholesale goods firms with measure unity. The firms own capital stock. In every period, a fraction $0 < \delta_e < 1$ of firms exits the market and the same number of firms enters the market. The new firms are endowed with an initial capital stock, $K_{0,t}$. An individual wholesale goods firm, indexed by j, produces identical wholesale goods, Y_t^j, following a Cobb-Douglas production function:[4]

$$Y_t^j = \left(K_t^j\right)^{\alpha} \left(A_t L_t^j\right)^{1-\alpha}, \quad 0 < \alpha < 1, \tag{10}$$

where K_t^j denotes the capital stock held by the firm in period t, L_t^j denotes labor, and $A_t^{1-\alpha}$ denotes the total factor productivity (TFP). The growth rate of A_t, $z_t \equiv A_t/A_{t-1}$, follows an AR(1) process:

$$\log\left(z_t/z\right) = \rho_z \log\left(z_{t-1}/z\right) + \epsilon_{z,t}, \quad 0 \leq \rho_z < 1,$$

with $\epsilon_{z,t} \sim i.i.d.N\left(0, \sigma_z^2\right)$. The firm accumulates capital stock according to

$$K_{t+1}^j = (1-\delta) K_t^j + \mu_t \varepsilon_t^j I_t^j, \quad 0 < \delta < 1, \tag{11}$$

where δ denotes the capital depreciation rate, I_t^j denotes investment in final goods units, μ_t denotes a shock to the marginal efficiency of investment (MEI) and ε_t^j denotes an idiosyncratic investment shock. The MEI shock follows an AR(1) process,

$$\log\left(\mu_t\right) = \rho_\mu \log\left(\mu_{t-1}\right) + \epsilon_{\mu,t}, \quad 0 \leq \rho_\mu < 1,$$

with $\epsilon_{\mu,t} \sim i.i.d.N(0, \sigma_\mu^2)$. The idiosyncratic shock, ε_t^j, follows c.d.f $\Phi(\varepsilon)$, independently and identically across firms and over time.

The wholesale goods firms have to finance funds for investment and for working capital in advance of production. The firms' financing process, such as borrowing and repayment,

[4] The index j here is different from the index for the households.

is completed within the period. The firms can raise funds only from households who do not own them.[5] In raising funds, the firms have the enforcement problem so that they must pledge their capital stock as collateral to a lender. Let $V_t^j\left(K_t^j\right)$ denote the stock value of wholesale goods firm j as a function of capital K_t^j in period t. In case of default, the lender can seize a fraction $0 < \kappa < 1$ of the collateral and receive $V_{t+1}^j\left(\kappa K_t^j\right)$ in period $t+1$. Assuming that the firm has all the bargaining power in case of default, the lender would receive $V_{t+1}^j\left(\kappa K_t^j\right)$ in case of default. Then, it is incentive compatible for the lender to provide funds up to the discounted value of $V_{t+1}^j\left(\kappa K_t^j\right)$, resulting in the following credit constraint for the firm:

$$P_t^I I_t^j + W_t L_t^j \leq \left(1-\delta_e\right) E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^j\left(\kappa K_t^j\right), \tag{12}$$

where P_t^I denotes the price of investment goods. The RHS of (12) defines the credit limit for the firm. The value of collateral is multiplied by $1-\delta_e$ because the firm exits the market and has no value with probability δ_e in the end of period t. Within the credit limit, the firm raises funds for investment, $P_t^I I_t^j$, and for working capital, $W_t L_t^j$, the wage income paid to households in advance of production.

Investment is assumed to be irreversible at the firm level: $I_t^j \geq 0$. Let P_t^w denote the price of wholesale goods. Wholesale goods firm j chooses investment, $I_t^j \geq 0$, and labor, $L_t^j \geq 0$, to maximize its value,

$$V_t^j\left(K_t^j\right) = \max_{\left\{I_t^j \geq 0,\ L_t^j \geq 0\right\}} P_t^w Y_t^j - \left(P_t^I I_t^j + W_t L_t^j\right) + \left(1-\delta_e\right) E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^j\left(K_{t+1}^j\right), \tag{13}$$

subject to production technologies, (10), capital accumulation technologies, (11), and the credit constraint, (12). Let ξ_t^j denote the Lagrange multiplier on the credit constraint. The first-order condition with respect to L_t^j yields

$$P_t^w = \frac{W_t\left(1+\xi_t^j\right)}{\left(1-\alpha\right) Y_t^j / L_t^j} = \frac{W_t\left(1+\xi_t^j\right)}{\left(1-\alpha\right) A_t^{1-\alpha}\left(K_t^j\right)^{\alpha}\left(L_t^j\right)^{-\alpha}}. \tag{14}$$

Equation (14) shows that the price of wholesale goods depends not only on nominal unit labor costs, $W_t L_t^j / Y_t^j$, but also on the tightness of credit constraints, ξ_t^j. The tightness would disappear from (14)) without working capital in the credit constraint. Substituting Y_t^j and L_t^j using (10) and (14) respectively into problem (13) yields

$$V_t^j\left(K_t^j\right) = \max_{\left\{I_t^j \geq 0\right\}} R_t^j K_t^j - P_t^I I_t^j + \left(1-\delta_e\right) E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^j\left(K_{t+1}^j\right), \tag{15}$$

[5] If firms could borrow from owner households, there would be no frictions between the firm (borrower) and the household (lender).

subject to (10), (11) and (12). where R_t^j denotes the firm's return of capital, given by

$$R_t^j \equiv \frac{\alpha+\xi_t^j}{1+\xi_t^j}\left[\frac{1-\alpha}{\left(1+\xi_t^j\right)W_t/A_t}\right]^{\frac{1-\alpha}{\alpha}}\left(P_t^w\right)^{\frac{1}{\alpha}}.$$

Following Miao, Wang and Xu (2012) problem (15) is solved by guessing and verifying a solution. The value of wholesale goods firm j is conjectured to take the following functional form:

$$V_t^j\left(K_t^j\right) = Q_t^j K_t^j + B_t^j,$$

where Q_t^j denotes the marginal value of capital held by firm j, B_t^j represents a bubble which could be either zero or positive. Here, the solution is presented and its intuition explained, while the derivation of the solution to problem (15) is provided in the Appendix. Problem (15) is linear in investment so that the investment turns out to have a bang-bang solution:

$$I_t^j = \begin{cases} \left[Q_t\left(\kappa K_t^j\right) + \bar{B}_t^j - \frac{1-\alpha}{\alpha+\xi_t^j}R_t^j K_t^j\right]\left(P_t^I\right)^{-1} & \text{if } \varepsilon_t^j \geq \varepsilon_t^*, \\ 0 & \text{if } \varepsilon_t^j < \varepsilon_t^*, \end{cases} \tag{16}$$

where Q_t, $\bar{B}_t^j$ and ε_t^* are defined as

$$Q_t \equiv (1-\delta_e)\,E_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}Q_{t+1}^j, \tag{17}$$

$$\bar{B}_t^j \equiv (1-\delta_e)\,E_t\beta\frac{\Lambda_{t+1}}{\Lambda_t}B_{t+1}^j, \tag{18}$$

$$\varepsilon_t^* \equiv P_t^I/\left(Q_t\mu_t\right).$$

In (17) the discounted marginal value of capital, Q_t, is conjectured to depend only on aggregate states, which will be confirmed later. Equation (16) implies that only firms with idiosyncratic productivity above threshold ε_t^* raise funds up to their credit limit and make investments. Other firms with productivity below ε_t^* do not make investments. At the firm level the investment becomes lumpy, consistent with the observation reported by Cooper and Haltiwanger (2006). Solving problem (15) also yields the expression for the Lagrange multiplier on the credit constraint:

$$\xi_t^j = \frac{\varepsilon_t^j}{\varepsilon_t^*} - 1 \geq 0. \tag{19}$$

Equation (19) reflects the mirror image of the solution for investment in (16): only firms with $\varepsilon_t^j > \varepsilon_t^*$ borrow up to their credit limit and make investments so that the credit constraint is binding, i.e., $\xi_t^j > 0$.

Substituting (16) into (15) and matching the coefficients in the value function yields the solution for Q_t^j and B_t^j. Substituting these solutions into (17) and (18) yields the solution for Q_t and $\bar{B}_t^j$ (see the Appendix for the derivation):

$$\begin{aligned} Q_t = &(1-\delta_e) E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \Bigg[R_{t+1}^j + Q_{t+1} (1-\delta) \\ &+ \int_{\varepsilon \geq \varepsilon_{t+1}^*} \left(\frac{\varepsilon}{\varepsilon_{t+1}^*} - 1 \right) \left(\kappa Q_{t+1} - \frac{1-\alpha}{\alpha + \xi_{t+1}^j} R_{t+1}^j \right) d\Phi(\varepsilon) \Bigg], \end{aligned} \tag{20}$$

and

$$\bar{B}_t^j = (1-\delta_e) E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \bar{B}_{t+1}^j (1 + G_{t+1}), \tag{21}$$

where

$$G_t = \int_{\varepsilon \geq \varepsilon_t^*} \left(\frac{\varepsilon}{\varepsilon_t^*} - 1 \right) d\Phi(\varepsilon).$$

As conjectured in (17), the discounted marginal value, Q_t, depends only on aggregate states because j in R_{t+1}^j in (20) reflects only an idiosyncratic state, which is averaged out in taking the expectation.

The combination of the two conditions, (20) and (21), implies the arbitrage condition for the share price of the firm, (5). The average ex-dividened share price of the firms with age τ, appeared in (5), is defined as $S_{t,\tau} \equiv E_j \left(S_t^j | \text{age}_t = \tau \right)$ where

$$S_t^j = (1-\delta_e) E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^j \left(K_{t+1}^j \right).$$

In this definition the operator E_j calculates a mean with respect to individual j and age_t denotes a firm's age in period t. Because the solution for $V_{t+1}^j \left(K_{t+1}^j \right)$ is already known, the share price is expressed as

$$S_{t,\tau} = Q_t K_{t+1,\tau} + B_{t,\tau},$$

where $K_{t+1,\tau} \equiv E_j \left(K_{t+1}^j | \text{age}_t = \tau \right)$ denotes the average capital stock in the beginning of period $t+1$ for firms with age τ in period t and $B_{t,\tau} \equiv E_j \left(\bar{B}_t^j | \text{age}_t = \tau \right)$ denotes the average bubble of firms with age τ in period t. Multiplying $K_{t+1,\tau}$ to (20) and adding $B_{t,\tau}$ using (21) yields

$$\begin{aligned} S_{t,\tau} = &E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \{ (1-\delta_e) E_j R_{t+1}^j K_{t+1,\tau} + Q_{t+1} (1-\delta_e) \left[(1-\delta) K_{t+1,\tau} + I_{t+1,\tau+1} \right] \\ &+ B_{t+1,\tau+1} \}, \end{aligned} \tag{22}$$

where E_j is added using the law of iterated expectation and $I_{t,\tau} \equiv E_j \left(I_t^j | \text{age}_t = \tau \right)$ is obtained from (16). The first term in the brackets in (22) corresponds to $\Pi_{t+1,\tau+1}^s$, the average dividend paid by firms with age $\tau+1$ in period $t+1$ and the second term in the brackets (22) is reduced to the fundamental component of firms share, $Q_{t+1} K_{t+2,\tau+1}$ where $K_{t+2,\tau+1} = (1-\delta_e) \left[(1-\delta) K_{t+1,\tau} + I_{t+1,\tau+1} \right]$. The third term in the brackets, the bubble component of firms share, is obtained because $E_j \left(\bar{B}_{t+1}^j | \text{age}_t = \tau \right) = B_{t+1,\tau+1} / (1-\delta_e)$. Therefore, the arbitrage condition (5) holds in this model even when there is a bubble.

The derivation of equation (22) clarifies why the household is willing to hold the share price of the firms even when the price is inflated by a bubble. In equation (21), the bubble, $\bar{B}_t^j$, itself does not yield any interest, but it generates return, G_{t+1}, by mitigating the credit constraint and allowing the firm to invest more and earn more profits when the firm is hit by a greater idiosyncratic shock than ε_{t+1}^* in the next period. The increase in investment, made it possible by the bubble, appears a part of $I_{t+1,\tau+1}$ in (22). Without the benefits generated by the increase in investment, the bubble would not be sustained in this model. In other words, the bubble can be sustained because of the credit constraint which provides the bubble to play a role. As shown by Miao and Wang (2012) analytically in a similar but much simpler framework, a bubble exists when the credit constraint is tight or when κ is small enough. As will be shown in the next section, a bubble exists under an empirically reasonable set of values for κ in this model.

This completes the description of wholesale goods firms. The following subsections describe the common building blocks of a standard DSGE model.

2.3 Retailers and Final Goods Firms

There is a continuum of retailers with measure unity, which are owned by households. Retailer, indexed by i, purchase wholesale goods from wholesale goods firms at price P_t^w and transform one unit of wholesale goods into one unit of specialized retail goods, $Y_t(i)$. Then the retailers sell the retail goods to final goods producers at price $P_t(i)$. Competitive final goods firms combine retail goods to produce final goods, Y_t, according to

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{\lambda_{p,t}}} di \right]^{\lambda_{p,t}},$$

where the exogenous price markup follows an ARMA(1,1) process:

$$\log(\lambda_{p,t}/\lambda_p) = \rho_p \log(\lambda_{p,t}/\lambda_p) + \epsilon_{p,t} - \gamma_p \epsilon_{p,t-1}, \quad 0 \le \rho_p < 1, \quad \lambda_p > 1,$$

with $\epsilon_{p,t} \sim i.i.d.N\left(0, \sigma_p^2\right)$. Following Smets and Wouters (2007), the MA(1) term aims to capture the high-frequency fluctuations in inflation. Profit maximization by final goods firms yields a demand curve for retail goods:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{\frac{\lambda_{p,t}}{1-\lambda_{p,t}}} Y_t, \tag{23}$$

for $i \in (0, 1)$, where P_t denotes the price of final goods.

Retailers face price change frictions à la Calvo (1983) in each period. That is, retailers can change prices with probability $0 < 1 - \xi_p < 1$ identically and independently over time and across retailers. When retailers have an opportunity to change the price, they set the price, $\tilde{P}_t(i)$, to maximize expected profits:

$$\max_{\{\tilde{P}_t(i)\}} E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[P_{t+s}(i) Y_{t+s}(i) - P^w_{t+s} Y_{t+s}(i) \right],$$

subject to the demand for $Y_t(i)$, (23). Following CEE (2005), the price, which has not been reset since period t, is assumed to follow an indexation rule:

$$P_{t+s}(i) = \begin{cases} \tilde{P}_t(i) & \text{if } s = 0 \\ \Pi_{k=1}^{s} (\pi_{t+k-1})^{\nu_p} (\pi)^{1-\nu_p} \tilde{P}_t(i) & \text{if } s = 1, 2, ... \end{cases}$$

where π_t denotes the inflation rate in period t, π denotes the inflation rate in the steady state and $0 \leq \nu_p \leq 1$ denotes the degree of indexation to past inflation.

2.4 Investment Goods Firms

There are competitive investment goods firms which are owned by households. They transform one unit of final goods into one unit of investment goods subject to CEE (2005) investment adjustment costs and sell the investment goods at price P^I_t to wholesale goods firms. They choose the amount of investment goods, I_t, to maximize expected profits:

$$\max_{\{I_t\}} E_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left\{ P^I_{t+s} I_{t+s} - \left[1 + \frac{S''}{2} \left(\frac{I_{t+s}}{I_{t+s-1}} - z \right)^2 \right] P_{t+s} I_{t+s} \right\}, \quad S'' > 0.$$

The CEE (2005) adjustment costs, captured by the squared term, provide a persistence mechanism and help the model generate hump-shaped responses of investment and output to various shocks, consistent with the VAR-based evidence. The growth rate in the steady state, z, appeared in the squared term, ensures that the adjustment costs are zero in the steady state.

2.5 Government

The government consists of a monetary and a fiscal authority. The monetary authority sets a nominal interest rate, R_t, according to

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[\phi_\pi \log\left(\frac{\pi_t}{\pi}\right) + \phi_{dy} \log\left(\frac{Y_t}{Y_{t-1}} \frac{1}{z}\right)\right] + \epsilon_{mp,t},$$

with $\epsilon_{mp,t} \sim i.i.d.N\left(0, \sigma^2_{mp}\right)$. The disturbance, $\epsilon_{mp,t}$, denotes a monetary policy shock. The monetary policy rule responds to inflation and the growth rate of output relative to its steady state value with lagged parameter $0 \leq \rho_R < 1$.

The fiscal authority sets the amount of government spending, which is proportional to the output, given by $g_t Y_t$, where

$$\log\left(\frac{g_t}{g}\right) = \rho_g \log\left(\frac{g_{t-1}}{g}\right) + \epsilon_{g,t}, \quad 0 \leq \rho_g < 1, \;\; 0 < g < 1,$$

with $\epsilon_{g,t} \sim i.i.d.N\left(0, \sigma_g^2\right)$. The disturbance, $\epsilon_{g,t}$, denotes a government spending shock. The fiscal authority finances spending by lump-sum taxes on households. It does not issue government bonds so that the supply of nominal bonds is fixed at zero.

This completes the description of economic agents in the model.

2.6 Aggregation

We are now in a position to aggregate over individual variables and derive aggregate relationships. The aggregation starts from bubbles and proceeds to standard variables such as output and labor.

Bubbles

Let $b_{t,\tau}$ denote the real value of the average bubble of firms with age τ in period t: $b_{t,\tau} \equiv B_{t,\tau}/P_t$. Following Miao, Wang and Xu (2012), a *sentiment shock*, θ_t, is introduced, which affects the size of the current bubbles $\{b_{t,\tau}\}_{\tau>0}$ relative to that of newly-born bubble $b_{t,0}$. Specifically, the households are assumed to believe that the relative size of bubbles for any two firms born in period t and $t+1$ evolves according to

$$\frac{b_{t+\tau,\tau}}{b_{t+\tau,\tau-1}} = \theta_t, \quad \tau \geq 1, \tag{24}$$

where θ_t follows an AR(1) process:

$$\log\left(\theta_t\right) = \rho_\theta \log\left(\theta_{t-1}\right) + \epsilon_{\theta,t}, \quad 0 \leq \rho_\theta < 1,$$

with $\epsilon_{\theta,t} \sim i.i.d.N\left(0, \sigma_\theta^2\right)$. The relative relationship, (24), implies that

$$b_{t,0} \equiv b_t^*, \;\; b_{t,1} = \theta_{t-1} b_t^*, \;\; b_{t,2} = \theta_{t-1}\theta_{t-2} b_t^*, \quad ..., \quad b_{t,\tau} = \prod_{s=1}^{\tau} \theta_{t-s} b_t^*. \tag{25}$$

In equation (25) a sentiment shock, θ_{t-1}, affects existing bubbles, $b_{t,\tau}, \tau \geq 1$, relative to a newly-born bubble, b_t^*. In the same manner, the current sentiment shock, θ_t, affects current bubbles relative to a newly-born bubble in the next period.

Let b_t denote the aggregate bubble in real terms in period t. Because the firms that have a bubble in its share price exit the market with probability δ_e in each period, b_t is expressed as

$$
\begin{aligned}
b_t &= \sum_{\tau=0}^{\infty} (1-\delta_e)^{\tau} \delta_e b_{t,\tau} = \sum_{\tau=0}^{\infty} (1-\delta_e)^{\tau} \delta_e \left(\prod_{k=1}^{\tau} \theta_{t-k} \right) b_t^* \\
&= \delta_e b_t^* + (1-\delta_e) \delta_e \theta_{t-1} b_t^* + (1-\delta_e)^2 \delta_e \theta_{t-1} \theta_{t-2} b_t^* + ... \\
&= m_t b_t^*,
\end{aligned}
\tag{26}
$$

where m_t follows

$$
m_t = m_{t-1} (1-\delta_e) \theta_{t-1} + \delta_e.
$$

Taking the average over j in equation (21) and expressing the bubble in real terms yields

$$
b_{t,\tau} = (1-\delta_e) E_t \beta \frac{\Lambda_{t+1} P_{t+1}}{\Lambda_t P_t} b_{t+1,\tau+1} (1+G_{t+1}).
$$

From (25), $b_{t,\tau} = \theta_{t-1} ... \theta_{t-\tau} b_t^*$ and $b_{t+1,\tau+1} = \theta_t ... \theta_{t-\tau} b_{t+1}^*$. Substituting these into the above expression yields

$$
b_t^* = (1-\delta_e) E_t \beta \frac{\Lambda_{t+1} P_{t+1}}{\Lambda_t P_t} \theta_t b_{t+1}^* (1+G_{t+1}).
$$

Substituting out b_t^* using equation (26) yields the arbitrage condition which governs the aggregate bubble:

$$
b_t = (1-\delta_e) E_t \beta \frac{\Lambda_{t+1} P_{t+1}}{\Lambda_t P_t} \frac{m_t}{m_{t+1}} \theta_t b_{t+1} (1+G_{t+1}).
\tag{27}
$$

Equation (27) makes clear why a bubble can be sustained in the economy. A bubble, by definition, is fundamentally useless in itself. Yet, if everyone believes that an even worthless stock does have value, the inflating stock price yields additional benefits by loosening the credit constraint, (12). A firm whose stock price has been inflated by a bubble is able to borrow more than firms whose stock price is not inflated. The additional borrowing allows the firm to take advantage of the high return of investment available and to make more profits if it is hit by a greater idiosyncratic shock than ε_{t+1}^* in the next period. This additional benefits are summarized by G_{t+1} in equation (27) because of which the belief that a bubble does have value is fulfilled and a bubble is sustained in the equilibrium.

Standard Variables

Substituting (19) into (14) and aggregating over j yields the demand for labor:

$$
L_t = (1-\alpha)^{\frac{1}{\alpha}} \left(\frac{P_t^w A_t^{1-\alpha}}{W_t} \right)^{\frac{1}{\alpha}} \left[\Phi(\varepsilon_t^*) + \int_{\varepsilon \geq \varepsilon_t^*} \left(\frac{\varepsilon_t^*}{\varepsilon} \right)^{\frac{1}{\alpha}} d\Phi(\varepsilon) \right] K_t,
\tag{28}
$$

where L_t and K_t denote aggregate labor and capital respectively. The term in square brackets, which is greater than unity, captures the inefficiency arising from the need for wholesale goods firms to finance working capital subject to credit constraints. Without working capital, the term would reduce to unity. Let L_t^* denote the average supply of labor: $L_t^* = \int_0^1 L_t(j)\,dj$. When nominal wages are dispersed, the supply of labor is given by

$$L_t = (w_t^*)^{\frac{\lambda_{w,t}}{\lambda_{w,t}-1}} L_t^*, \quad w_t^* = \left[\int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{\frac{\lambda_{w,t}}{1-\lambda_{w,t}}} dj\right]^{\frac{1-\lambda_{w,t}}{\lambda_{w,t}}},$$

where $w_t^* \leq 1$ captures the inefficiency caused by nominal wage dispersion.

Now that aggregate labor is in hands, aggregate output is ready to be derived. After aggregating over the idiosyncratic shock, ε_t^j, wholesale goods firms all have the same capital-labor ratio, because they are price takers in factor markets. Keeping this in mind, aggregating an individual output, Y_t^j, using (10), (14) and (28), yields the average supply of wholesale goods, denoted by Y_t^*:

$$Y_t^* = \frac{\Phi(\varepsilon_t^*) + \int_{\varepsilon \geq \varepsilon_t^*} \left(\frac{\varepsilon_t^*}{\varepsilon}\right)^{\frac{1-\alpha}{\alpha}} d\Phi(\varepsilon)}{\left[\Phi(\varepsilon_t^*) + \int_{\varepsilon \geq \varepsilon_t^*} \left(\frac{\varepsilon_t^*}{\varepsilon}\right)^{\frac{1}{\alpha}} d\Phi(\varepsilon)\right]^{1-\alpha}} K_t^{\alpha} (A_t L_t)^{1-\alpha}. \tag{29}$$

As in (28), the terms involving ε_t^* in (29) capture the inefficiency arising from the need to finance working capital subject to credit constraints. When prices are dispersed, aggregate output, Y_t, is given by

$$Y_t = (p_t^*)^{\frac{\lambda_{p,t}}{\lambda_{p,t}-1}} Y_t^*, \quad p_t^* = \left[\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{\frac{\lambda_{p,t}}{1-\lambda_{p,t}}} dj\right]^{\frac{1-\lambda_{p,t}}{\lambda_{p,t}}},$$

where $p_t^* \leq 1$ captures the inefficiency caused by price dispersion.

It is straight-forward to derive aggregate investment, I_t. Aggregating an individual investment, I_t^j, using (16), yields

$$I_t = \frac{(1-\Phi(\varepsilon_t^*))(Q_t \kappa K_t + B_t) - \int_{\varepsilon \geq \varepsilon_t^*} \left(\frac{\varepsilon_t^*}{\varepsilon}\right)^{\frac{1}{\alpha}} d\Phi(\varepsilon) (1-\alpha)^{\frac{1}{\alpha}} \left(\frac{W_t}{A_t}\right)^{-\frac{1-\alpha}{\alpha}} (P_t^w)^{\frac{1}{\alpha}} K_t}{P_t^I}, \tag{30}$$

where $B_t = P_t b_t$ denotes the aggregate nominal bubble. The first term in the numerator in (30) describes the amount of borrowing of wholesale goods firms conducting investment. The second term in the numerator in (30) describes the amount of borrowing assigned to working capital for firms conducting investment. Borrowing minus the funds for working capital represents the funds available for investment. Dividing the funds for investment by the price of investment, P_t^I, yields the amount of investment in final goods units.

Let K_{t+1}' denote the aggregate capital stock in the end of period t just before a fraction, δ_e, of wholesale goods firms exit the market. After they exit, the same number of new firms enter the market, each with start-up capital $K_{0,t+1}$. The start-up capital is assumed to be a fraction, $0 < \varphi < 1$, of the average capital held by firms which have exited the market: $K_{0,t+1} = \varphi K'_{t+1}$[6]. Then, the capital stock at the beginning of period $t+1$ is given by

$$K_{t+1} = (1 - \delta_e + \delta_e \varphi) K'_{t+1}.$$

Aggregating individual capital stock using (11) and (16) yields

$$\begin{aligned} K'_{t+1} &= (1-\delta) K + \mu_t \Bigg[\int_{\varepsilon \geq \varepsilon_t^*} \varepsilon d\Phi(\varepsilon) \frac{(1 - \Phi(\varepsilon_t^*))(Q_t \kappa K_t + B_t)}{P_t^I} \\ &\quad - \frac{\int_{\varepsilon \geq \varepsilon_t^*} \varepsilon^{1-\frac{1}{\alpha}} d\Phi(\varepsilon) (\varepsilon_t^*)^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1}{\alpha}} \left(\frac{W_t}{A_t}\right)^{-\frac{1-\alpha}{\alpha}} (P_t^w)^{\frac{1}{\alpha}} K_t}{P_t^I} \Bigg]. \end{aligned}$$

The above two equations constitute the law of motion for capital, K_t. This completes the description of the model.

2.7 Phillips Curve

The New Keynesian Phillips curve reveals how a loosening in financial tightness affects inflation dynamics. In deriving the Phillips curve the idiosyncratic shock, ε_t^j, is assumed to follow a Pareto distribution, $\Phi : [1, \infty) \to [0, 1]$:

$$\Phi(\varepsilon) = 1 - \varepsilon^{-\eta}, \quad \eta > 0.$$

In addition, for the sake of exposition, no price indexation and no price markup shocks are assumed, i.e., $\nu_p = 0$ and $\lambda_{p,t} = \lambda_p$, in this subsection.

Under these simplified assumptions, log-linearizing the solution to retailers' problem yields the standard Phillips curve in the marginal costs version:

$$\hat{\pi}_t = \kappa_p \hat{p}_t^w + \beta E_t \hat{\pi}_{t+1}, \quad \kappa_p = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}, \tag{31}$$

where variables with a hat denote the deviation from the steady state and $p_t^w = P_t^w / P_t$ denotes real marginal costs. Current inflation depends on real marginal costs and expected inflation given by (31).

Unlike the standard model, real marginal costs differ from real unit labor costs. Using the aggregate expression for L_t and Y_t^*, (28) and (29) respectively, marginal costs are expressed as follows:

$$\hat{p}_t^w = \widehat{ulc}_t + \frac{\chi}{\kappa_p} \hat{\xi}_t, \tag{32}$$

[6] The interpretation of the initial capital stock, $K_{0,t+1}$, is that new firms collect a fraction, φ, of capital held by the firms which have exited the market. When φ is less than one, the remaining capital stock, $(1-\varphi)\delta_e K'_{t+1}$, becomes no use.

where $ulc_t = W_t L_t / (P_t Y_t)$ denotes real unit labor costs, $\xi_t = E_j \xi_t^j$ denotes the average tightness of credit constraints, or "financial tightness" for short. Parameter χ is given by

$$\chi \equiv \frac{\alpha^2 \eta \left(\varepsilon^*\right)^{-\eta} \kappa_p}{\left[\alpha\eta + 1 - \left(\varepsilon^*\right)^{-\eta}\right] \left[\alpha\left(\eta - 1\right) + 1 - \left(1 - \alpha\right)\left(\varepsilon^*\right)^{-\eta}\right]} > 0.$$

Substituting (2) into (31) yields the Phillips curve in a unit labor cost version:

$$\hat{\pi}_t = \kappa_p \widehat{ulc}_t + \beta E_t \hat{\pi}_{t+1} + \chi \hat{\xi}_t. \tag{33}$$

In the Phillips curve, (33), financial tightness, $\hat{\xi}_t$, appears endogenously as a cost-push shock[7]. In an asset price boom financial tightness loosens so that $\hat{\xi}_t$ in (33) decreases, adding downward pressure on inflation, $\hat{\pi}_t$. To understand the mechanism, consider a wholesale goods firm that has already borrowed up to its credit limit. When a positive shock hits the economy and the firm finds it profitable to increase output, it has to finance the cost of additional working capital. Because the firm's credit constraint is binding, the only way to finance the cost of working capital is to give up some investment and use the saved funds for working capital. But by doing so the firm incurs the opportunity cost of forgoing profitable investment opportunities. The opportunity cost depends on the tightness of the credit constraint, so that marginal costs consist not only of unit labor costs but also of the financial tightness. Therefore, when the credit constraint loosens, for example due to an asset price bubble, the opportunity cost drops and adds downward pressure on marginal costs and hence on inflation.

Without working capital in the credit constraint, (12), marginal costs, p_t^w, would coincide with real unit labor costs, ulc_t, and financial tightness, ξ_t, would vanish from the Phillips curve, (33). Therefore, (33) would reduce to the standard New Keynesian Phillips curve without working capital in the credit constraint.

3 Estimation

Before proceeding to the main analysis of this paper, the model is applied to the U.S. economy and estimated using Bayesian techniques. This section includes a description of the data and the priors used in the estimation as well as a brief outline of the results of the estimation. The estimated model will be used to study monetary policy and inflation dynamics in an asset price bubbles in the next section.

[7]If the working capital in the credit constraint included the nominal interest rates so that firms should raise $R_t W_t L_t^j$ in advance, the nominal interest rates, R_t, would appear as a cost-push shock, as in the model by Ravenna and Walsh (2006). I choose not to include R_t in the credit constraint to focus on the effect of financial tightness, ξ_t.

3.1 Data

The model is estimated using time series data from 1985Q1 to 2011Q4 for the U.S. economy for the following eight variables: real per capita GDP, real per capita consumption, real per capita investment, hours worked, real wages, inflation rates, nominal interest rates and real per capita stock prices, or

$$\left[\triangle \log\left(Y_t\right), \triangle \log\left(C_t\right), \triangle \log\left(I_t\right), \log\left(L_t\right), \triangle \log\left(\frac{W_t}{P_t}\right), \pi_t, R_t, \triangle \log\left(\frac{S_t}{P_t}\right)\right],$$

where $\triangle$ denotes the first difference operator[8]. The first seven variables are standard and are constructed as in Justiniano, Primiceri and Tambalotti (2010). The nominal per capita value of the stock market, S_t, is constructed as the Dow Jones Wilshire 5000 stock price index divided by the civilian non-institutional population aged 16 and over. Following Christiano, Motto and Rostagno (2012) who use the same value of the stock market constructed in the same manner in their estimation, the variables in a form of growth rates are demeaned, because their average growth rates, which serve as proxies for the steady state growth rates in the model, differ substantially across variables. For example, the growth rate of the value of the stock market is more than four times as large as that of GDP. While demeaning may drop important information from the data, not doing so could distort the estimation, if the differences of the growth rates are due to something missing in the model. In light of the divergence in the average growth rates in the data, it appears preferable to demean the variables.

3.2 Priors

The value of several parameters which are difficult to identify or whose values are relatively uncontroversial are fixed for the analysis below. The capital income share and the capital depreciation rate are fixed at $\alpha = 0.36$ and $\delta = 0.025$ respectively. Price and wage markups are fixed at $\lambda_p = \lambda_w = 1.15$. The share of government expenditure to output in the steady state is fixed at $g = 0.2$. As for variables pertaining to financial frictions, the parameter of start-up capital is fixed at $\varphi = 0.2$, following Miao, Wang and Xu (2012). The exit rate of firms is fixed at $\delta_e = 0.01$, which is close to the value of default rates used in the financial frictions literature, such as in the studies by Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999).

For standard parameters, priors are set in line with Justiniano, Primiceri and Tambalotti (2010) and Christiano, Motto and Rostagno (2012) and are summarized in Table 1. As for parameters pertaining to credit frictions, the mean of the credit constraint parameter,

[8]The number of variables coincides with the number of shocks in the model: TFP ($\epsilon_{z,t}$), MEI ($\epsilon_{\mu,t}$), wage markup ($\epsilon_{w,t}$), price markup ($\epsilon_{p,t}$), preference ($\epsilon_{\zeta,t}$), monetary policy ($\epsilon_{mp,t}$), government spending ($\epsilon_{g,t}$) and sentiment (ϵ_{θ}).

κ, is set to 0.15 and the standard deviation to 0.05. This prior for κ falls within the range reported by Covas and den Hann (2011), who find that κ ranges from 0.1 to 0.3 for various sizes of firms. As for the parameter of the distribution of idiosyncratic shocks, η, instead of setting a prior on η directly, a prior is set for the fraction of firms which conduct investment in the steady state, $(\varepsilon^*)^{-\eta}$, which has a clear economic interpretation[9]. Cooper and Haltiwanger (2006), using data on large manufacturing plants, find that each year about 18.5 percent of plants do not have positive net investment. Given the idiosyncratic nature of investment in the model, a rough calculation implies that about 34 percent of firms ($1 - (0.185)^{1/4} = 0.34$) carry out investment in each period in the steady state. If firms were allowed to make negative investments, the value would be lower. Because firms are not allowed to make negative investments in the model, a slightly lower value, 0.25, is attached for the prior mean of $(\varepsilon^*)^{-\eta}$.

3.3 Estimation Results

Parameter Estimates

Table A in the appendix reports the posterior median and the 90 percent posterior intervals for the estimated model parameters. As for the standard parameters in the DSGE literature, the parameter estimates are in line with Justiniano, Primiceri and Tambalotti (2010) and Christiano, Motto and Rostagno (2012). As for the parameters pertaining to credit frictions, the median of the credit constraint parameter, κ, is 0.11, which is lower than the prior mean, 0.15. The median of $(\varepsilon^*)^{-\eta}$, a fraction of firms conducting investment in the steady state, is 0.17, which is also lower than the prior mean, 0.25. The median value implies that in the steady state only 17 percent of firms borrow up to their credit limit and conduct investment in every period.

Variance Decomposition

How important is the sentiment shock, which causes a wild swing in the asset price bubble, in business cycles? Table 1 answers the question by reporting the variance decomposition of the observed variables in business cycle frequencies (6 to 32 months) evaluated at the posterior medians. The sentiment shock ($\epsilon_{\theta,t}$) accounts for 16 percent of the volatility of output, 10 percent of that of consumption, 13 percent of that of investment and 19 percent of that of hours worked. It is not surprising that the shock accounts for almost all of the volatility of the value of the stock market (95 percent), because as is well-known, the value of the stock market does not fluctuate very much in standard business cycle models. The

[9]In the steady state, the fraction is expressed as follows:

$$(\varepsilon^*)^{-\eta} = \frac{[1 - (1 - \delta_e)\beta](\eta - 1)}{(1 - \delta_e)\beta}.$$

Then, setting a prior for $(\varepsilon^*)^{-\eta}$ substitutes for setting a prior for η.

Table 1: Variance Decomposition

	TFP	MEI	Wage markup	Price markup	Preference	Monetary	Government	Sentiment
Output growth	0.28	0.22	0.10	0.08	0.03	0.08	0.05	0.16
Consumption growth	0.14	0.03	0.08	0.02	0.58	0.03	0.01	0.10
Investment growth	0.11	0.45	0.07	0.08	0.07	0.08	0.00	0.13
Hours	0.04	0.29	0.18	0.12	0.03	0.11	0.03	0.19
Real wage growth	0.34	0.02	0.51	0.06	0.00	0.00	0.00	0.07
Federal funds rate	0.03	0.42	0.08	0.16	0.07	0.15	0.01	0.07
Inflation rate	0.04	0.57	0.02	0.05	0.06	0.09	0.01	0.14
Stock price growth	0.01	0.02	0.00	0.00	0.00	0.01	0.00	0.95

Note: Decomposition of the variance corresponding to periodic components with cycles of between 6 and 32 quarters, obtained using the spectrum of the DSGE model evaulated at posterior medians.

contribution of the sentiment shock is lower than that reported by Miao, Wang and Xu (2012), who estimate a similar model with a shock to κ (i.e., a financial shock) but without nominal rigidities, working capital and some of the shock considered here, such as the shock to the marginal efficiency of investment (MEI).

For the other shocks, the result is consistent with Justiniano, Primiceri and Tambalotti (2010): the technology shocks, i.e., the sum of the TFP ($\epsilon_{z,t}$) and the MEI ($\epsilon_{\mu,t}$) shocks, are the driving force of business cycles. A slight difference appears in the magnitude of the MEI shock. While the MEI shock is the dominant source of business cycles in Justiniano, Primiceri and Tambalotti (2010), accounting for 50 percent and 86 percent of the volatility of GDP and investment respectively, in the model here, the contribution is only about half as large, as can be seen in Table 1. The difference is partly attributable to the newly-added shock, a sentiment shock, and the additional observable variable, a stock price, in this study.[10]

In sum, a sentiment shock serves as the main driving force of asset price developments and as an important, if not the main, source of business cycles. Hence, it is worth using the estimated model and then exploring the role of monetary policy in an asset price bubble triggered by a sentiment shock.

4 Monetary Policy and Inflation Dynamics

This section addresses four questions using the estimated model. The first question concerns inflation dynamics, while the other three concern monetary policy: (1) Why does inflation

[10]Christiano, Motto and Rostagno (2012) argue that the positive MEI shock decreases the price of capital by shifting the supply curve of capital outward, which weakens the importance of the MEI shock in business cycles when the value of stock market is included as an observable variable.

tend to be moderate in an asset price boom, as observed historically? (2) How should monetary policy be conducted in an asset price bubble? (3) What contributes to generating inefficiencies, if there are any, which the optimal monetary policy aims to fix in an asset price bubble? (4) Does responding to asset prices or other financial variables help improve welfare in an asset price bubble? What about targeting inflation?

In the following simulations the model's parameter values and the magnitude of the shock are fixed at their posterior median reported in Table A in the appendix.

4.1 Moderate Inflation

Inflation dynamics in an asset price bubble have important implications for monetary policy. If inflation rises in line with an increase in inefficiencies caused by an asset price bubble, an inflation targeting policy will address the inefficiencies effectively. On the other hand, if inflation does not rise, an inflation targeting policy may fail to respond to an increase in the inefficiencies and disrupt the economy. Historically, asset price booms have been periods of moderate inflation in many countries, as documented by Adalid and Detlen (2007), Bordo and Wheelock (2004, 2007) and Christiano, et al. (2010). Thus, discussing monetary policy in an asset price bubble requires that the model should feature moderate inflation.

Qualitatively, the model presented here offers a hypothesis why inflation remains relatively moderate in asset price booms. In addition to the conventional mechanism in which asset price inflation caused by a bubble accelerates inflation, the model here features another mechanism in which a loosening in financial tightness decelerates inflation. In particular, inflating asset prices mitigate credit constraints and lower the shadow cost of financing working capital, adding downward pressure on marginal costs and inflation. The Phillips curve, (33), makes clear the two conflicting pressures on inflation. In conventional models, a bubble creates inflationary pressure by pushing up real unit labor costs, $\widehat{ulc}_t$, in (33). In the model here a bubble creates downward pressure on inflation as well by lowering financial tightness, $\hat{\xi}_t$, in (33).

To quantify the downward pressure on inflation, the baseline generated by the model proposed here is compared to a model without this working capital *channel*, in which the final term involving $\hat{\xi}_t$ in the Phillips curve, (33), is eliminated.[11]

[11] Here, the working capital channel is removed by replacing the credit constraint, (12), with the following:

$$P_t^I I_t^j + \omega K_t^j \leq (1 - \delta_e) E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}^j \left(\kappa K_t^j \right),$$

where parameter ω is chosen to mimic the steady state of the baseline. Another way to eliminate the working capital channel would be to remove the working capital completely from the credit constraint, (33). In this case, the model, however, would be entirely different from the baseline in terms of not only the dynamics but also the steady state. A change in the steady state affects the dynamics and makes it difficult to compare inflation dynamics in the baseline to those in a model without working capital.

Figure 1: Impulse Responses to Sentiment Shock: Working Capital Channel

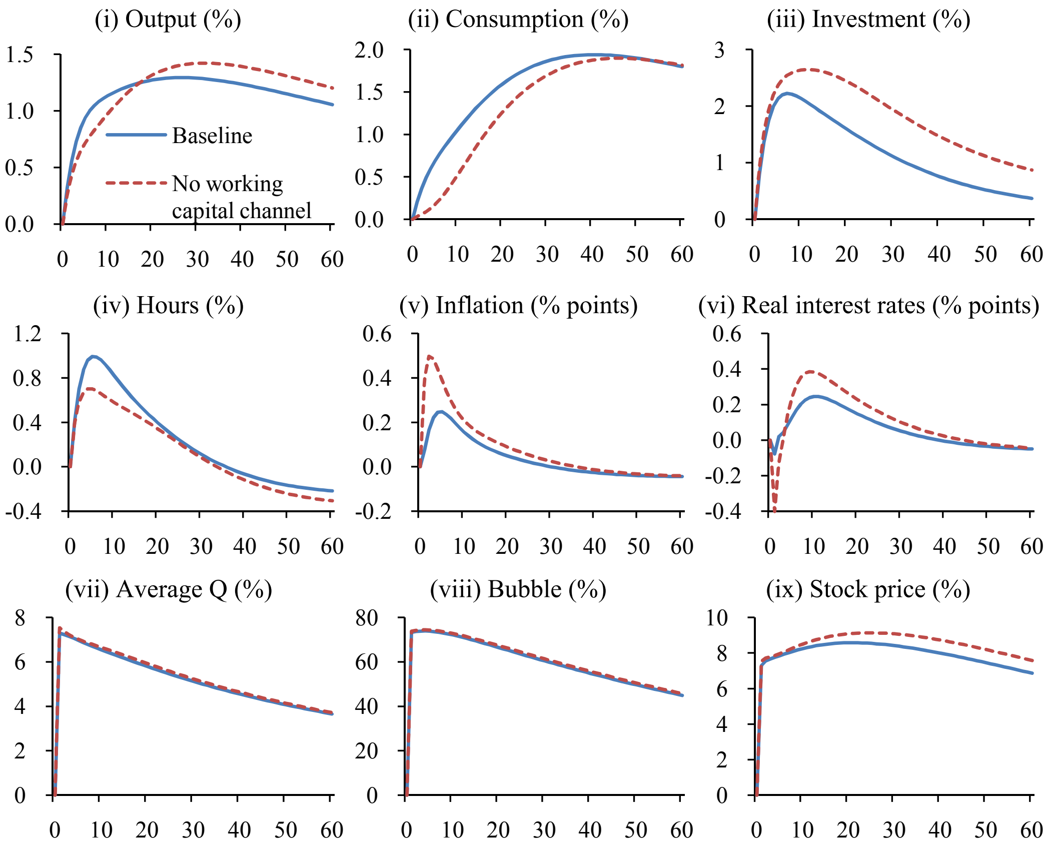
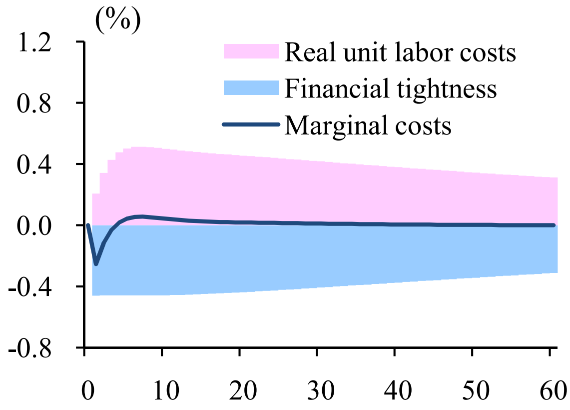


Figure 1 plots, for the baseline and for a model without the working capital channel, the impulse responses to a positive sentiment shock. In the baseline, the bubble increases more than 60 percent in response to a sentiment shock (Panel (viii)). The increase in the bubble translates into an increase in the value of the stock market (Panel (ix)) and an increase in average q, given by $(q_t K_{t+1} + b_t) / K_{t+1}$ (Panel (vii)). The increase in the value of the stock market raises firms' credit capacity and accelerates investment and production. Output, consumption, investment and hours worked all increase (Panels (i)-(iv)). Inflation rises slightly (Panel (v)). In sum, the bubble causes an asset price boom in the baseline.

A stark difference between the two models appears in the response of inflation (Panel (v)). In the baseline, the inflation rises less than half the extent it would in the model without the working capital channel. Obviously, the moderate inflation in the baseline stems from the working capital channel: a loosening in financial tightness lowers the cost of financing additional working capital, adding downward pressure on marginal costs and inflation. The two models show more or less the same responses for the other variables. Because firms do not have to finance the cost of additional labor in the model without the working capital channel, the hours worked rises less substantially (Panel (iv)), while the

Figure 2: Decomposition of Marginal Costs



investment increases more than in the baseline (Panel (iii)).

Figure 2 decomposes marginal costs, $\hat{p}_t^w$, into real unit labor costs, $\widehat{ulc}_t$, and financial tightness, $\hat{\xi}_t$, following equation (2) in the baseline. While real unit labor costs add upward pressure on marginal costs, the effect is offset by the downward pressure of the loosening in financial tightness. As a result, marginal costs drop in the initial period when the value of stock market rises, and stay just above zero in later periods when the economy is booming.

In sum, the model features financial tightness as a cost-push shock in the Phillips curve, which contributes to generating relatively moderate inflation in an asset price bubble. The following subsections explore how this feature affects monetary policy during the bubble.

4.2 The Optimal Monetary Policy

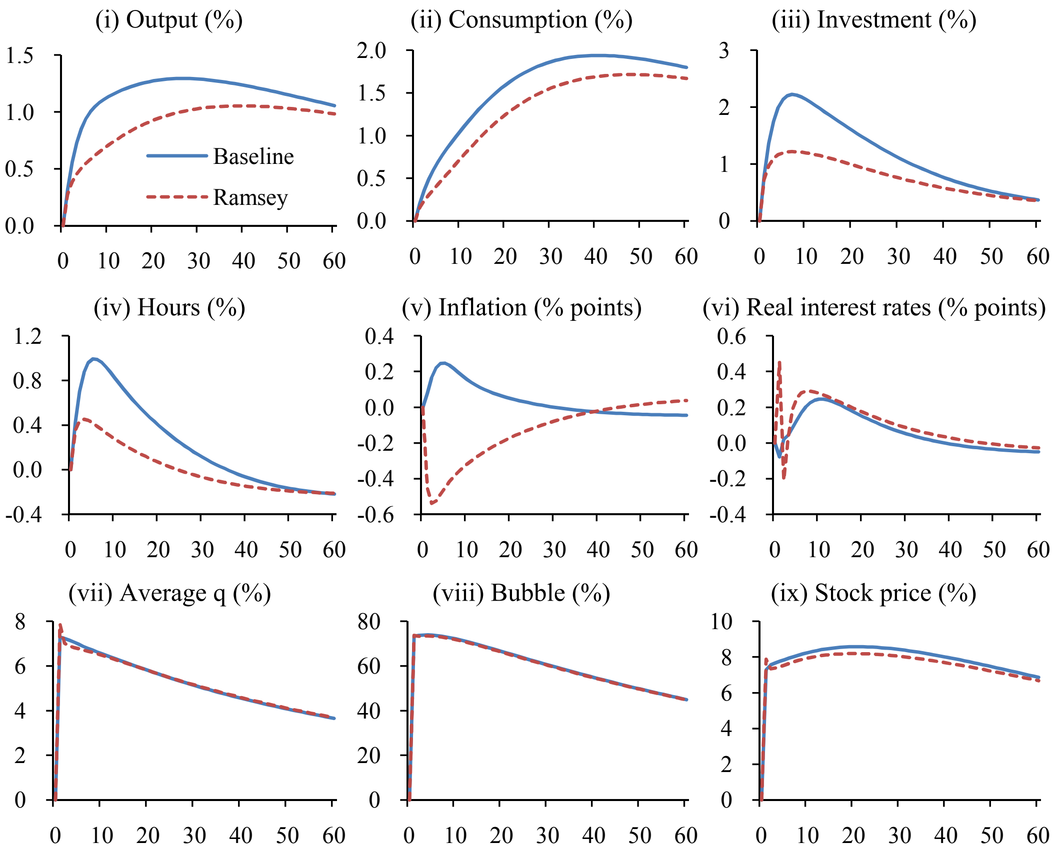
How should monetary policy be conducted in an asset price bubble? One direct way to answer this question is to compute a Ramsey optimal monetary policy. A Ramsey planner chooses an allocation and nominal interest rates among those satisfying equilibrium conditions (without a monetary policy rule) to maximize the average household's utility,

$$E_t \sum_{s=0}^{\infty} \beta^s \zeta_t \left[\log \left(C_{t+s} - h C_{t+s-1} \right) - \psi \frac{\int_0^1 L_{t+s}(j)^{1+\nu} dj}{1+\nu} \right].$$

The planner is assumed to have full commitment and to honor commitments made in the past. This is a so called timeless perspective monetary policy (Woodford, 2003). The Ramsey planner's problem is solved using the method proposed by Schmitt-Grohe and Uribe (2005).

Figure 3 plots the impulse responses to a sentiment shock for the model with the estimated monetary policy rule (baseline) and for the model with the optimal monetary policy (Ramsey economy). The optimal monetary policy performs differently from the estimated monetary policy rule in two respects. First, the increase in real variables such as

Figure 3: Ramsey Optimal Monetary Policy



output and investment is limited under the optimal monetary policy, relative to the baseline. Put differently, the asset price boom caused by the bubble is inefficiently excessive in the baseline. The investment and the hours worked in the baseline increase twice as great as in the Ramsey economy (Panel (iii) and (iv)). The output in the baseline increases by about 50 percent greater than in the Ramsey economy in the medium run (Panel (i)).

Second, inflation falls into a negative range under the optimal monetary policy (Panel (v)). The Ramsey planner aims to restrain the boom and stabilize the real economy by raising the real interest rate (Panel (vi)) more than what appears to be warranted by price stability. While the planner succeeds in limiting the inefficiently amplified responses of real variables, he or she has to sacrifice short run price stability (Panel (v)). Thus, the planner faces the difficult trade-off between stabilizing the real economy and stabilizing inflation. This observation shows that stabilizing inflation is not necessarily a precondition for the optimal monetary policy in an asset price bubble.

Interestingly, the optimal monetary policy has little effect on the size of the bubble as shown in Panel (viii). In this model, it is the sentiment shock that mainly drives a change in the bubble so that the small increase in the real interest rate in Panel (vi) has little effect on the bubble. To confirm this clearly, solving the equation governing the aggregate bubble, (27), forward recursively starting from the steady state yields the impulse response of the bubble as

$$\frac{b_t}{b} = \prod_{j=1}^{\infty} \theta_{t+j-1} \frac{(1-\delta_e)(1+G_{t+j})}{1+r_{t+j}}, \tag{34}$$

where $r_{t+1} \equiv R_t/\pi_{t+1}$ denotes the real interest rate and the expectation operator E_t is omitted because equation (34) shows an impulse response, a deterministic path after a shock hits in period t. According to the estimation results shown in Table A in the appendix, the median of one standard deviation of the sentiment shock θ_t is about 26 percent while the rise in the real interest rate in Panel (vi) is at most 0.5 annual percentage points. Hence, the fluctuation of the bubble is dominated by the sentiment shock and the optimal monetary policy, which increases the real interest rate by a small amount, has little impact on the bubble as shown in Panel (viii).

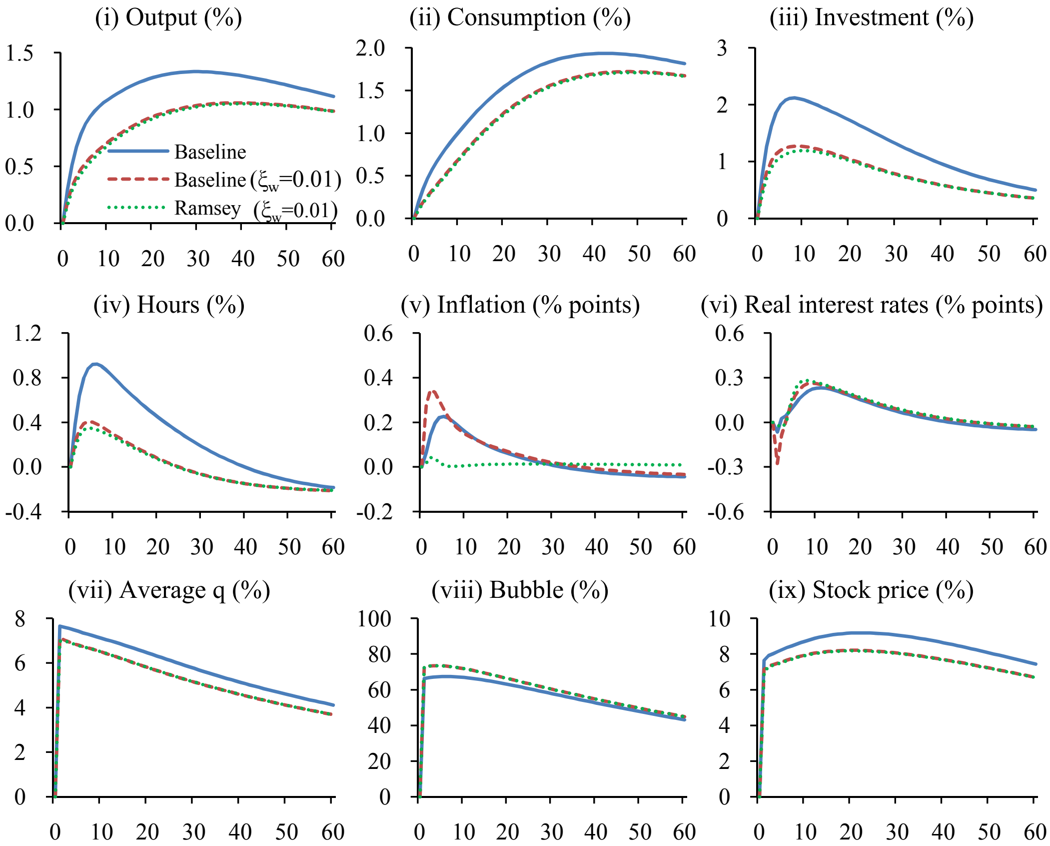
This observation seems to be a counterexample to one of the three conditions set out by Kohn (2006) under which somewhat tighter monetary policy is rationalized in the face of an inflating asset market. Specifically, he states that "..., there must be a fairly high probability that a modestly tighter policy will help to check the further expansion of speculative activity"(Kohn, 2006). He argues that the condition does not hold in practice so that a tighter monetary policy does not pay. In contrast, the optimal monetary policy in the model here calls for somewhat tighter monetary policy, even though the policy fails to check the bubble significantly as in Panel (viii). The rationale of the monetary tightening is that the real variables respond to the bubble in an inefficiently volatile manner, which monetary policy is able to fix through the traditional interest rate channel. The effectiveness of monetary policy in pricking a bubble may matter in practice, but what matters most theoretically is a rise in inefficiencies, likely to be appeared in amplified responses of real variables, which monetary policy can address. Because the inefficiencies increase in the bubble, somewhat tighter monetary policy becomes the optimal response in this model. The next subsection will investigate what frictions contribute to generating the inefficiencies.

Another interesting observation is that the Ramsey planner does not completely stabilize the boom caused by an asset price bubble. This is partly because the positive bubble works to improve welfare by mitigating credit constraints and by allowing firms to raise more funds. In the model, dampening the boom completely does harm the economy, even if the policy maker recognizes the bubble.

4.3 What Drives an Excessive Boom?

What frictions contribute to generating an excessive boom, because of which the optimal monetary policy calls for a tighter monetary policy stance? The model features four main

Figure 4: The Role of Nominal Wage Rigidities



distortions: nominal price rigidities, nominal wage rigidities, working capital and credit frictions. It turns out that it is mainly nominal wage rigidities that amplify the effect of bubble inefficiently.

To isolate the effect of nominal wage rigidities, nominal wage rigidities are shut down nearly completely by setting $\xi_w = 0.01$. Figure 4 plots the impulse responses to a sentiment shock for the original baseline ($\xi_w = 0.82$), the baseline with $\xi_w = 0.01$ and the Ramsey economy with $\xi_w = 0.01$. The figure yields three interesting observations. First, when the distortion of nominal wage rigidities is shut down nearly completely, the responses in the baseline match closely those in the Ramsey economy. The result implies that without nominal wage rigidities the estimated monetary policy rule would perform a similar function to the optimal monetary policy. Second, the optimal monetary policy stabilizes inflation, implying that focusing on inflation stabilization is nearly optimal without nominal wage rigidities. Third, the responses of real and financial variables in the original baseline are much greater than those in the baseline with $\xi_w = 0.01$. This observation implies that it is nominal wage rigidities that amplify the effect of a sentiment shock and make the boom excessive.

How do nominal wage rigidities amplify the responses of real variables inefficiently? A positive sentiment shock increases the size of the bubble and raises the credit capacity of firms. Firms now can hire and invest more, which increases the demand for labor and adds upward pressure on real wages. When nominal wages are sticky, the upward pressure on real wages is muted and real wages stay relatively low, which translates into a large increase in labor input. The increase in labor boosts investment by raising the marginal product of capital. The increase in investment raises the value of firms and credit capacity, which, in turn, stimulates labor demand and investment. The increase in investment translates into an increase in output. Consumption also increases, amplified by a counter-cyclical markup in wages generated by nominal wage rigidities. Thus, nominal wage rigidities contribute to generating an inefficiently excessive boom in the model.

According to the literature on optimal monetary policy in a DSGE framework, financial frictions per se do not undermine the importance of focusing on inflation (Faia and Monacelli, 2007). Also, nominal wage rigidities make inflation targeting sub-optimal, but not far from the optimal policy (Schmitt-Grohe and Uribe, 2007). The observations obtained from Figure 4 imply that nominal wage rigidities play an important role when there is strong upward pressure on real wages as in an asset price bubble in the model. Nominal wage rigidities suppress the upward pressure, keeping real wages relatively low, and amplify the increase in labor input inefficiently and contribute to generating an excessive boom.

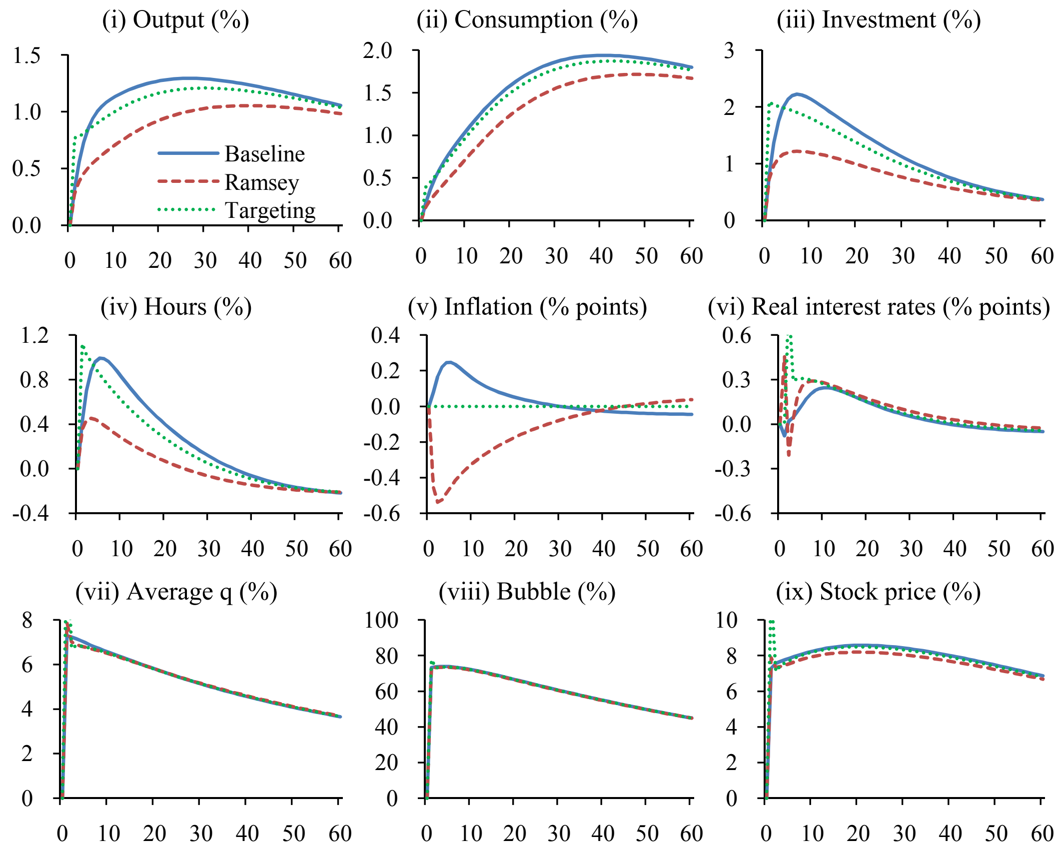
4.4 Inflation Targeting and the Role of Financial Variables

The previous analysis on the optimal monetary policy suggests that there is a room for monetary policy rules to improve welfare in an asset price bubble. In light of the debate whether monetary policy should focus on inflation or react to asset price developments, this section considers a policy stabilizing inflation completely (strict inflation targeting) and a policy responding to a financial variable.

Strict Inflation Targeting

Figure 5 plots the responses to a positive sentiment shock for a model with strict inflation targeting along with the baseline and the Ramsey economy. As is clear from Panel (v), inflation is completely stabilized under strict inflation targeting. By focusing on the inflationary pressure generated by asset price developments, strict inflation targeting is supposed to respond effectively to the toxic side effects of an asset price bubble. Actually, in the medium and long run, strict inflation targeting works to restrain an increase in real variables such as output and investment, making the responses move toward those in the Ramsey economy. There remains, however, a significant distance between the two responses. More importantly, in the short run, strict inflation targeting makes the responses of real variables more volatile than those in the baseline (Panels (i)-(iv)), performing the opposite function to the optimal monetary policy.

Figure 5: The Effect of Strict Inflation Targeting



Why does strict inflation targeting fail to address inefficiencies caused by the bubble in the short run? The decrease in marginal costs in the initial periods shown in Figure 2 lies behind the volatile fluctuations under strict inflation targeting in the short run. It is important to note that stabilizing inflation calls for stabilizing marginal costs completely for all periods. Stabilizing marginal costs, however, requires monetary easing because marginal costs are negative. This is why the responses become more volatile under strict inflation targeting in the short run, as shown in Figure 5.

Admittedly, the negative marginal costs in the short run in Figure 2 have to do with the working capital channel. Without the working capital channel inflation would be twice as great as that in the baseline and marginal costs would be positive for all periods. Thus, strict inflation targeting would not generate volatile responses in the short run without the working capital channel. In other words, relatively low inflation, caused by the working capital channel, undermines the effectiveness of a monetary policy stance focusing on inflation in an asset price bubble.

Monetary Policy Rules Augmented with a Financial Variable

A monetary policy rule augmented with an additional variable, x_t, is assumed to take a following form:

$$\log\left(\frac{R_t}{R}\right) = 0.83\log\left(\frac{R_{t-1}}{R}\right) + (1-0.83)\left[1.97\log\left(\frac{\pi_t}{\pi}\right) + 0.30\log\left(\frac{Y_t}{Y_{t-1}}\frac{1}{z}\right) + \phi_x \log\left(\frac{x_t}{x}\right)\right], \quad (35)$$

where estimated parameter values are fixed at their posterior median.[12] As an additional variable, the growth rate of asset prices and the growth rate of credit are considered. The coefficient, ϕ_x, is set such that the value of ϕ_x minimizes welfare costs, defined as the percentage of consumption the representative household is willing to give up in every period in order not to change to the monetary policy rule, (35), from the Ramsey optimal monetary policy.[13] In calculating welfare costs, only a sentiment shock is considered in order to focus on the role of monetary policy in an asset price bubble.

Figure 6 plots the responses to a positive sentiment shock under two alternative monetary policy rules, one augmented with asset price growth ($\phi_{\text{asset price}} = 0.33$) and the other with credit growth ($\phi_{\text{credit}} = 2.41$) respectively. Both monetary policy rules do restrain an asset price boom by raising real interest rates sharply (Panel (vi)). The responses of real variables such as output and consumption are restrained and become close to those under the optimal monetary policy (Panels (i)-(iv)). The policy rule augmented with asset price growth, however, is less successful in restraining the responses than that with credit growth. This observation suggests that the rule with asset price growth is less effective in addressing inefficiencies caused by the bubble than that with credit growth.

Welfare Analysis

So far, impulse responses under various monetary policy rules have been compared. The purpose here is to explore the welfare implications of different monetary policy rules quantitatively and summarize the argument on monetary policy rules.[14]

Table 2 reports welfare costs for five monetary policy rules (shown in the columns) in two different models (shown in the rows), the baseline and a model without nominal wage

[12] This monetary policy rule is in line with the studies by Christiano, et al. (2010) and Gilchrist and Zakrajsek (2011), who consider a similar rule which adds a financial variable to a standard monetary policy rule. In this paper, asset prices are given by S_t/P_t while credit is given by $p_t^I I_t + w_t L_t$.

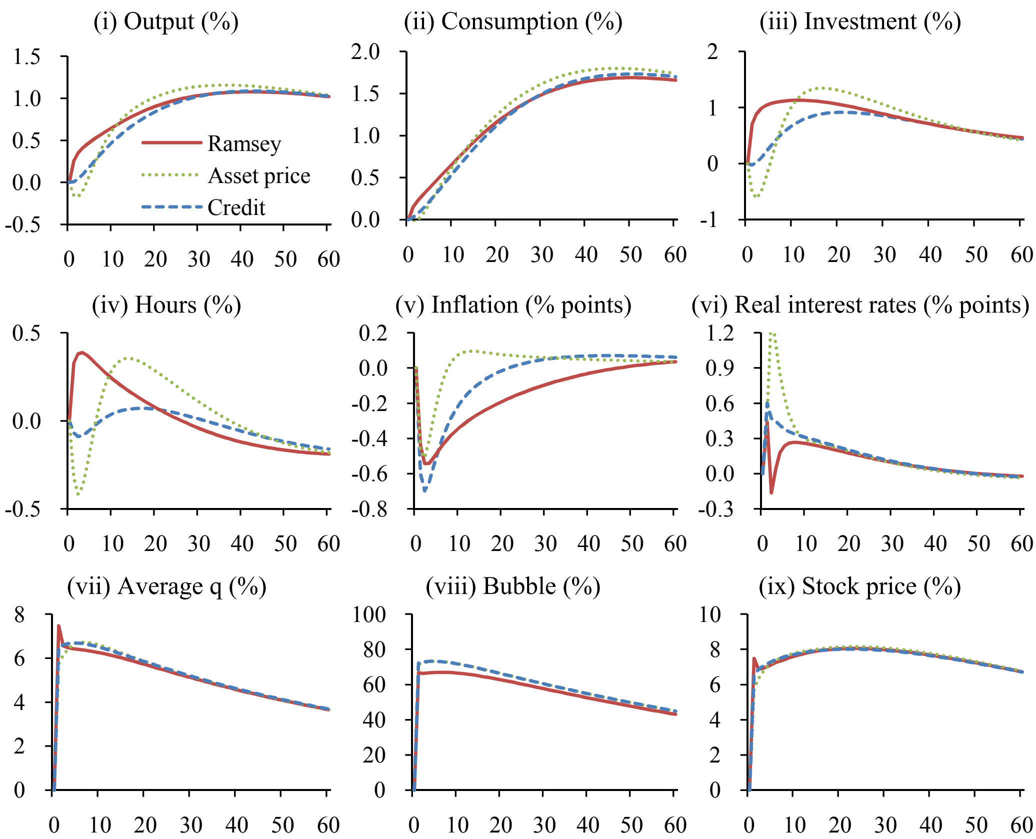
[13] Let $W_0^r(\lambda)$ be the welfare of the representative household in the Ramsey economy with parameter λ:

$$W_0^r(\lambda) = E_0 \sum_{t=0}^{\infty} \beta^t \zeta_t \left[\log\left((1-\lambda)C_t - hC_{t-1}\right) - \psi \frac{\int_0^1 L_t(j)^{1+\nu} dj}{1+\nu}\right].$$

Similarly, define W_0 as the welfare in the model with the modified monetary policy rule but with $\lambda = 0$. By definition $W_0^r(0) \geq W_0$. The welfare costs are defined as $100 \times \lambda$ such that $W_0^r(\lambda) = W_0$. A λ is approximated using second order approximation, following Schmit-Grohe and Uribe (2005, 2007).

[14] The welfare implications do not change significantly even if the estimated technology and preference shocks, which jointly account for more than half of the volatility of output, consumption and investment, are added to the model.

Figure 6: The Effect of Augmented Monetary Policy Rules



rigidities in which the wage stickiness parameter value is set to $\xi_w = 0.01$. In the baseline, welfare costs vary significantly depending on a monetary policy rule, as presented in the first row in Table 2. Under the estimated monetary policy rule the welfare costs are 1.37 percent of consumption. Strict inflation targeting improves welfare by reducing welfare costs to 0.64 percent of consumption, which is about half as large as the costs under the estimated monetary policy rule. This improvement can be seen in Figure 5, where strict inflation targeting restrains the responses of real variables in the medium and long run. The usefulness of stabilizing inflation has to do with the fact that an increase in asset prices mitigates credit constraints and raises firms' credit capacity, allowing them to invest and hire more, which translates into a surge in the demand for final goods. Because of the inflationary pressure, inflation serves as a useful, if not perfect, indicator for detecting inefficiencies caused by the asset price bubble.

In the baseline, the optimized monetary policy rule with respect to asset price growth ($\phi_{\text{asset price}} = 0.33$) reduces the welfare costs to 0.68 percent, about half of the amount under the estimated monetary policy rule. The optimized monetary policy rule with respect to credit growth ($\phi_{\text{credit}} = 2.41$) improves welfare more and reduces the costs to 0.23 percent, i.e., about a fifth of the amount under the estimated monetary policy rule. The difference

Table 2: Welfare Costs

(% of consumption the representative household willing to pay)

	Taylor rule (estimated)	Strict inflation targeting	Augmented with asset prices	Augmented with credit	Augmented with wages
Baseline	1.37 (1.00)	0.64 (0.47)	0.68 (0.50)	0.23 (0.17)	0.09 (0.07)
No nominal wage rigidities	0.019 (1.00)	0.008 (0.45)	0.015 (0.79)	0.009 (0.46)	0.005 (0.26)

Note: Numbers in parenthesis denote welfare costs relative to those under Taylor rule (estimated).

is reflected in the volatile responses under the monetary policy rule augmented with asset prices in Figure 6. In contrast with credit growth, it appears that asset price growth is too volatile to be a good indicator for detecting inefficiencies.

The final column of Table 2 shows the optimized monetary policy rule augmented with real wage growth ($\phi_{\text{wage}} = 2.02$). As can be seen, this is the rule among the five rules that minimizes the costs, which are only 0.09 percent. This is not surprising, because stabilizing nominal wages (real wage growth + inflation) undoes inefficiencies arising from nominal wage rigidities, which are the source of the inefficiently excessive boom.

Finally, the results for the model without nominal wage rigidities under various monetary policy rules are presented in the second row in Table 2. Under the estimated monetary policy rule, the welfare costs are only 0.019 percent of consumption. This highlights the role of nominal wage rigidities in amplifying fluctuations inefficiently, as argued in Section 4.3. The strict inflation targeting policy improves welfare, as in the baseline, reducing the welfare costs to 0.008 percent. The monetary policy rule augmented with asset prices ($\phi_{\text{asset price}} = 0.04$) and credit ($\phi_{\text{credit}} = 0.43$), however, improve welfare less than in the baseline, reducing the costs to 0.015 percent and 0.009 percent, respectively.

5 Conclusion

This study explored the role of monetary policy and inflation dynamics in an asset price bubble in a DSGE framework. For this purpose, the business cycle model with an asset price bubbles, developed by Miao, Wang and Xu (2012) was extended to a monetary DSGE model and estimated using U.S. time series data. The model features a new mechanism in which financial tightness appears endogenously as a cost-push shock in the Phillips curve. The mechanism contributes to generating relatively moderate inflation in an asset price boom, which conforms with the fact that, as highlighted by Bordo and Wheelock (2004, 2007) and others, asset price booms are often periods of moderate inflation. In the model, a bubble inflates asset prices, mitigates credit constraints and loosens financial tightness, adding downward pressure on marginal costs and hence on inflation.

According to the estimated model, an asset price bubble causes an inefficiently excessive boom. While inflation serves as a useful indicator for detecting inefficiencies caused by the bubble, focusing only on inflation fails to restrain the boom effectively. In particular, strict inflation targeting makes an excessive boom even excessive and exacerbates the problem caused by the boom in the short run. The optimal monetary policy calls for monetary tightening to restrain the boom at the cost of greater volatility in inflation. A reason why such monetary tightening is called for lies in nominal wage rigidities which contribute to generating an inefficiently excessive boom. Without nominal wage rigidities, stabilizing inflation performs nearly the same function as the optimal monetary policy.

Of course, numerous challenges remain in understanding inflation dynamics in asset price bubbles and what the optimal policy response should be. Until recently, there were relatively few studies addressing monetary policy in (rational) asset price bubbles in a dynamic general equilibrium framework, partly because of a lack of models incorporating bubbles. The situation has changed considerably since the global financial crisis of 2007-2008, and there is a growing list of studies seeking to develop models of bubbles, including Aoki and Nikolov (2011), Kocherlakota (2009), Farhi and Tirole (2012), Hirano and Yanagawa (2010), Carvalho, Martin and Ventura (2012), Martin and Ventura (2012), Miao and Wang (2012) and Miao, Wang and Xu (2012). Because bubbles can take different forms and occur in different sectors, one of the challenges is to build models that take this into account and make it possible to examine the role of monetary policy in response to a bubble. This paper is an attempt to contribute in this direction.

Another, albeit ambitious, challenge, would be to develop models to examine the role of macro-prudential regulation alongside monetary policy. This would represent a large step in the direction of providing an integrated analysis of macroeconomic policy in a bubble.

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Appendix A

This appendix describes a solution to problem (15). Using the conjecture for $V_t^j(K_t^j)$ and the new notations, Q_t and $\bar{B}_t^j$, defined by (17) and (18) respectively, problem (15) is expressed as follows:

$$Q_t^j K_t^j + B_{t,\tau}^j = \max_{\{I_t^j \geq 0\}} R_t^j K_t^j + \left(Q_t \mu_t \varepsilon_t^j - P_t^I\right) I_t^j + Q_t (1-\delta) K_t^j + \bar{B}_t^j, \tag{36}$$

subject to the credit constraint,

$$P_t^I I_t^j + \frac{1-\alpha}{\alpha + \xi_t^j} R_t^j K_t^j \leq Q_t \left(\kappa K_t^j\right) + \bar{B}_t^j,$$

where K_{t+1}^j is substituted out using (11). Because of linearity in I_t^j it is straightforward to obtain a solution for I_t^j:

$$I_t^j = \begin{cases} \left[Q_t \left(\kappa K_t^j\right) + \bar{B}_t^j - \frac{1-\alpha}{\alpha+\xi_t^j} R_t^j K_t^j\right] \left(P_t^I\right)^{-1} & \text{if } \varepsilon_t^j \geq \varepsilon_t^*, \\ 0 & \text{if } \varepsilon_t^j < \varepsilon_t^*, \end{cases}$$

where $\varepsilon_t^* = P_t^I/(Q_t \mu_t)$. The first-order condition with respect to I_t^j yields the Lagrange multiplier on the credit constraint as

$$\xi_t^j = \frac{\varepsilon_t^j}{\varepsilon_t^*} - 1 \geq 0.$$

A substitution of I_t^j using (16) into problem (36) yields

$$\begin{aligned} Q_t^j K_t^j + B_t^j &= R_t^j K_t^j + Q_t (1-\delta) K_t^j + \bar{B}_t^j \\ &\quad + \max\left(\frac{\varepsilon_t^j}{\varepsilon_t^*} - 1, 0\right) \left[Q_t \left(\kappa K_t^j\right) + \bar{B}_t^j - \frac{1-\alpha}{\alpha + \xi_t^j} R_t^j K_t^j\right]. \end{aligned}$$

Matching the coefficient of K_t^j and a constant term yields

$$Q_t^j = \begin{cases} R_t^j + Q_t (1-\delta) + \left(\frac{\varepsilon_t^j}{\varepsilon_t^*} - 1\right) \left(\kappa Q_t - \frac{1-\alpha}{\alpha+\xi_t^j} R_t^j\right) & \text{if } \varepsilon_t^j \geq \varepsilon_t^*, \\ R_t^j + Q_t (1-\delta) & \text{if } \varepsilon_t^j < \varepsilon_t^*, \end{cases}$$

and

$$B_t^j = \begin{cases} \bar{B}_t^j + \left(\frac{\varepsilon_t^j}{\varepsilon_t^*} - 1\right) \bar{B}_t^j & \text{if } \varepsilon_t^j \geq \varepsilon_t^*, \\ \bar{B}_t^j & \text{if } \varepsilon_t^j < \varepsilon_t^* \end{cases}$$

respectively. A substitution of the above expression for Q_t^j and B_t^j into (17) and (18) yields (20) and (21) respectively. This completes the derivation of a solution to problem (15).

Appendix B: Prior Densities and Posterior Estimates

Table A: Prior Densities and Posterior Estimates

Coefficient	Description	Prior			Posterior[2]	
		Density[1]	Mean	Std	Median	90% interval
$(\varepsilon^*)^{-\eta}$	Fraction of firms investing in SS	G	0.25	0.03	0.17	[0.14 , 0.20]
κ	Credit friction, recovery rate	B	0.15	0.05	0.11	[0.07 , 0.14]
$100z$	SS TFP growth rate	N	0.40	0.02	0.39	[0.36 , 0.42]
$100(\beta^{-1}-1)$	Preference disount rate	G	0.250	0.05	0.24	[0.17 , 0.32]
ν	Inverse Frisch elasticity	G	1.00	0.50	0.77	[0.19 , 1.51]
h	Consumption habit	B	0.50	0.10	0.77	[0.71 , 0.84]
$\log(L)$	log hours in SS	N	0.00	0.50	-0.02	[-0.06 , 0.02]
S''	Investment adjustment costs	G	2.00	1.00	2.30	[1.42 , 3.27]
ξ_p	Calvo prices	B	0.75	0.10	0.79	[0.75 , 0.83]
ξ_w	Calvo wages	B	0.75	0.10	0.82	[0.74 , 0.88]
ι_p	Price indexation	B	0.50	0.15	0.33	[0.16 , 0.50]
ι_w	Wage indexation	B	0.50	0.15	0.35	[0.22 , 0.49]
$100(\pi-1)$	SS quarterly inflation	N	0.50	0.05	0.54	[0.46 , 0.62]
ρ_R	Taylor rule smoothing	B	0.50	0.20	0.83	[0.79 , 0.86]
ϕ_π	Taylor rule, inflation	N	1.70	0.15	1.97	[1.77 , 2.16]
ϕ_y	Taylor rule, output growth	N	0.20	0.05	0.30	[0.22 , 0.38]
ρ_z	TFP growth shock, AR	B	0.40	0.20	0.34	[0.21 , 0.48]
ρ_μ	MEI shock, AR	B	0.60	0.20	0.92	[0.86 , 0.96]
ρ_p	Price mark-up shock, AR	B	0.60	0.20	0.95	[0.91 , 0.99]
ρ_w	Wage mark-up shock, AR	B	0.60	0.20	0.90	[0.85 , 0.96]
ρ_υ	Preference shock, AR	B	0.60	0.20	0.89	[0.81 , 0.95]
ρ_{mp}	Monetary policy shock, AR	B	0.40	0.20	0.52	[0.43 , 0.62]
ρ_g	Gov. spending shock, AR	B	0.60	0.20	0.94	[0.91 , 0.97]
ρ_θ	Sentiment shock, AR	B	0.60	0.20	0.82	[0.64 , 0.98]
γ_p	Price mark-up MA	B	0.50	0.20	0.68	[0.53 , 0.81]
γ_w	Wage mark-up MA	B	0.50	0.20	0.94	[0.86 , 0.98]

(continued on the next page)

(continued)

Table A: Prior Densities and Posterior Estimates (Continued)

Coefficient	Description	Prior			Posterior[2]	
		Density[1]	Mean	Std	Median	90% interval
$100\sigma_z$	TFP growth shock, Std	I	0.50	1.00	0.83	[0.74 , 0.93]
$100\sigma_\mu$	MEI shock, Std	IG	0.50	1.00	2.62	[2.03 , 3.27]
$100\sigma_p$ [3]	Price mark-up shock, Std	IG	0.10	0.10	0.39	[0.33 , 0.46]
$100\sigma_w$ [3]	Wage mark-up shock, Std	IG	0.10	0.10	0.12	[0.10 , 0.14]
$100\sigma_\upsilon$	Preference shock, Std	IG	0.50	1.00	2.80	[2.13 , 3.59]
$100\sigma_{mp}$	Monetary policy shock, Std	IG	0.10	1.00	0.10	[0.09 , 0.11]
$100\sigma_g$	Gov. spending shock, Std	IG	0.50	1.00	1.96	[1.73 , 2.20]
$100\sigma_\theta$	Sentiment shock, Std	IG	0.50	1.00	26.08	[3.36 , 51.20]
	Marginal data density			-728.9		

(1) N stands for Normal, B Beta, G Gamma and IG Inverted-Gamma1 distribution.

(2) Median and posterior percentiles from 3 chains of 200,000 draws generated using a Random walk Metropolis algorithm, where the first 20 percent draws were discarded.

(3) The standard deviation is transformed from its original counterpart.