Intangible Capital, Asset Prices, and Business Cycles*

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Abstract

This paper demonstrates that a seemingly natural single innovation to an otherwise standard macroeconomic model simultaneously explains the origin of a news shock and the origin of the long-run risk. The innovation is a two-stage product cycle: i.e., when a new product is created with research and development (R&D), it is innovative and is produced monopolistically; but it is matured stochastically and after which time it is produced competitively. An important assumption is that an innovative product is a seed of new products in the sense that expert knowledge about it is essential for R&D. I interpret an innovation in the R&D sector-specific productivity shock as a news shock. The model replicates empirical responses recently found by Sims (2009) including a seemingly puzzling stock market appreciation happening with an investment slump on impact of the shock. The same shock is also the origin of the long-run risk. Equipped with a recursive utility, the model generates a large equity premium with a low and stable risk-free rate. Importantly, these results are robust to the value of intertemporal elasticity of substitution while most existing long-run risk models depend on an empirically controversial parametric assumption that the intertemporal elasticity of substitution is greater than 1. Simulated business cycle moments are in line with the data including those of R&D spending and factor shares.

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1 Introduction

Recent empirical research motivated by the IT revolution has argued that intangible capital is an important missing piece in economic analysis for business cycles and asset prices.[1] The definition of intangible capital differs among researchers, but at minimum they include knowledge created by research and development (R&D), and some include brand equity and organizational capital. The size of intangible investment is large; business R&D alone is around 2% of GDP; broadly defined intangible investment is now roughly equal to investment in tangible capital. While intangible investments are currently treated as expenses, the Bureau of Economic Analysis recognizes their importance and is planning to incorporate R&D spending as investment in 2013.[2] In this paper, I introduce R&D in a standard endogenous product variety model (Romer (1990) and Grossman and Helpman (1991)) and investigate its role in the business cycle.

A distinguishing feature of my model is a product cycle.[3] A product is innovative when it is created. It is produced monopolistically in this stage. But a product matures later in its life. After that, the product is produced competitively. In addition, I assume that a new product is created based on expert knowledge about an innovative product. Because I also assume that expert knowledge is perishable, only a firm that is currently producing an innovative-monopolistic product can conduct a successful R&D.[4] This paper demonstrates how this single innovation improves performance of an otherwise standard macroeconomic model.

[1] Baldwin, Gu, Lafrance, and Macdonald (2008), Bloom and Reenen (2007), Bond and Cummins (2000), Corrado, Hulten, and Sichel (2005), Corrado, Hulten, and Sichel (2009), Fukao, Miyagawa, Mukai, Shinoda, and Tonogi (2008), Hall (2000), Hall (2001b), Jalava, Aulin-Ahmavaara, and Alanen (2007), Lin (2009), Marrano, Haskel, and Wallis (2009), McGrattan and Prescott (2001), and van Rooijen-Horsten, van den Bergen, de Haan, Klinkers, and Tanriseven (2008).

[2] See Aizcorbe, Moylan, and Robbins (2009).

[3] The product cycle has long history in the trade literature (Vernon (1966)). But it has been paid less attention in the macroeconomics.

[4] This is consistent with recent empirical studies that find most new products are invented within an existing firm or an existing facility. Broda and Weinstein (2007) find that 92% of product creation occurs within an existing firm. Bernard, Redding, and Schott (2006) find that 94% of product addition occurs within an existing facility.

First, the model offers a structural interpretation on a news shock. A news shock is a change in expectation about future productivity. Motivated by the IT revolution, a rich literature has burgeoned on the topic. Starting point of the literature is that in a standard neoclassical macroeconomic model, good news about future productivity generates a recession today because of the wealth effect. Since this prediction is counter-intuitive for most people, researchers propose possible extensions to a standard model with which the model generates what is known as a Pigou cycle, i.e., simultaneous increases in output, consumption, investment, and labor hours in response to a favorable news shock (Beaudry and Portier (2004), Jaimovich and Rebelo (2010), and Christiano, Ilut, Motto, and Rostagno (2007)). Beaudry and Portier (2006) gave empirical support for these responses using a small-scale vector autoregression model.

But recently, Sims (2009) claim that a simple neoclassical model's predictions are more in line with data. His vector autoregression model has macroeconomic aggregates and several forward looking variables in a single system. He identifies a news shock as the one that best explains future movements in a utilization-adjusted measure of aggregate technology among all shocks that are orthogonal to technology innovations. A favorable news shock is associated with an increase in consumption and declines in output, investment, and labor hours on impact. After the impact effects, they all track the productivity improvement. These are roughly the predictions of a simple neoclassical model augmented with news shocks.

This finding seems to be good news for macroeconomists because we can continue to rely on the same workhorse model without major surgeries. But actually a puzzle remains. That is, a news shock identified by Sims (2009) is also associated with an immediate appreciation of a stock market index. Using a similar identification strategy, Barsky and Sims (2009) also find an appreciation in the stock market index. Therefore, a news shock simultaneously produces a stock market boom and an investment slump on impact. This is hardly interpretable in light of a simple neoclassical model because the q theory of investment predicts that investment is high when the stock price is high.

My model offers consistent interpretation. A "news shock" in my model economy is an innovation in the R&D sector-specific productivity shock. The model's impulse responses to this news shock are consistent with the empirical responses found by Sims (2009). In particular, a favorable news shock is associated with a stock market boom because the value of an innovative-monopolistic product increases on impact. The value increases because it reflects values of products it is going to create in the future. The assumption that products are invented based on expert knowledge about a current innovative product is crucial for this result. If R&D can be conducted by any firm, a favorable news shock decreases the value of a current monopolistic product since it simply means that more rivals will be entering into the market.

These responses to a news shock also help in generating a large equity premium. I consider a household who has a recursive utility of Epstein and Zin (1989) and prefers early resolution of uncertainty. Such a household demands an extra premium for an asset whose return has a large positive covariance with its lifetime utility value. Because a favorable news shock is associated with both a stock market boom and an increase in the lifetime utility value, a large premium is demanded.

This mechanism resembles the so-called long-run risk in the finance literature (Bansal and Yaron (2004) and Bansal (2007)). In the literature, however, persistent fluctuation in consumption and dividend growth rates are exogenously given. In my model, they emerge endogenously. Moreover, a news shock and long-run risk, two important subjects studied independently, share the same origin. Another important difference is the role of intertemporal elasticity of substitution. Most existing long-run risk models assume that the intertemporal elasticity of substitution is greater than 1. This is crucial for their results because in their models, consumption and dividend growth rates share a same stochastic component. An upward revision of consumption and dividend growth rates simultaneously increases the interest rate. The equity price rises only when the intertemporal elasticity of substitution is greater than 1; otherwise the interest rate effect dominates.

But unfortunately, the assumption that the intertemporal elasticity of substitution is greater than 1 does not have much empirical support. See Cochrane (2005) for the survey. A nice property of my model is that it does not crucially depend on this controversial parameter. The intertemporal elasticity of substitution is 1 in my benchmark calibration. I also show that the intertemporal elasticity of substitution being .5 does not change the qualitative results; i.e., an innovation in the R&D sector-specific productivity shock is associated with a stock market boom and increases the equity premium. In my model, an innovation in the R&D sector-specific productivity shock increases both consumption growth rate and an interest rate. Nonetheless the equity price rises because the number of innovative-monopolistic products grows faster than aggregate consumption. This is possible because of the monopolistic-competitive product margin.

We can think of other product cycle patterns. One possibility is a product cycle from innovative-monopolistic to innovative-competitive; that is, any firm can conduct R&D but what changes is the monopolistic position in the goods producing sector. Another possibility is a product cycle from innovative-monopolistic to matured-monopolistic; that is, a product is always monopolistic but what changes is its usefulness as a seed of future products. In both cases, asset pricing implications are grossly different from the original model.

Among the theoretical studies on intangible capital, McGrattan and Prescott (2009) is the closest to my study. An important difference is that McGrattan and Prescott (2009) assume perfect foresight and focus on trend in the 90s. My model is a dynamic stochastic general equilibrium model. My focus is on fluctuations around the trend. This paper is also related to the production economy asset pricing literature.[5] Iraola and Santos (2009) study a monopolistic competition economy with endogenous product creation. They find that exogenous price markup variations and leverage are important for asset price volatility. My model features endogenously arising long-run risk in the relation with the product creation and product life cycle.

The rest of the paper is organized as follows. In section 2, the model is shown. In section 3, model parameters are shown. In section 4, main results are shown. In section 5, I investigate mechanisms by considering alternative models. They include different assumptions for the product cycle and a model in which the intertemporal elasticity of substitution is different from 1. In section 6, I present conclusions.

[5] Jermann (1998), Tallarini (2000), Boldrin, Christiano, and Fisher (2001), Croce (2008), Kaltenbrunner and Lochstoer (2008), Pananikolaou (2008), and Guvenen (2009).

2 Model

2.1 Final good production

A representative final good firm produces final good Z_t with a CES technology

$$Z_t = \left[\int_0^{N_{t-1}} z_t\left(i\right)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \tag{1}$$

$z_t\left(i\right)$ is the amount of intermediate product i bought by the firm. N_{t-1} is the number of intermediate products in the market. $\theta > 1$ is elasticity of substitution across products. Let $p_t\left(i\right)$ denote the price of intermediate product i. Solving the cost minimization problem

$$\min_{z_i(i)} \int_0^{N_{t-1}} p_t\left(i\right) z_t\left(i\right) di$$

subject to (1), we find a demand function

$$z_t\left(i\right) = \left(\frac{p_t\left(i\right)}{P_t} \right)^{-\theta} Z_t \tag{2}$$

where $P_t \equiv \left[\int_0^{N_{t-1}} p_t\left(i\right)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$. The final good market is competitive. Competition ensures that the equilibrium price of the final good is equal to P_t.

2.2 Intermediate goods production

There are competitive intermediate products and monopolistic intermediate products. A competitive product is old and is produced competitively. A monopolistic product is new and a firm that owns its trade secret produces it exclusively. For simplicity, I assume that (i) a firm can produce at most a single intermediate product in a period, and (ii) the number of potential intermediate good firms is always larger than the number of intermediate products in the market. Therefore, it is possible to identify a monopolistic product and the firm that produces it. I call such a firm a *monopolistic intermediate good firm*, and I call a firm that is not monopolistic a *competitive intermediate good firm*.

The production function of competitive intermediate products is

$$z_{C,t} = A_t k_{C,t}^{\alpha} l_{C,t}^{1-\alpha} \tag{3}$$

A_t is goods producing sector-specific productivity shock which follows a stationary stochastic process

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_{A,t}, \quad \varepsilon_{A,t} \sim \text{i.i.d.} N\left(0, \sigma_A^2\right)$$

$k_{C,t}$ is tangible capital input whose rental price is $rental_t$. $l_{C,t}$ is labor input whose real wage is W_t. The marginal cost of production is

$$\frac{1}{A_t}\left(\frac{rental_t}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}$$

The Bertland competition ensures that

$$rental_t k_{C,t} = \alpha z_{C,t}\left[\frac{1}{A_t}\left(\frac{rental_t}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\right] \tag{4}$$

$$W_t l_{C,t} = (1-\alpha) z_{C,t}\left[\frac{1}{A_t}\left(\frac{rental_t}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\right] \tag{5}$$

$$\frac{p_{C,t}}{P_t} = \left[\frac{1}{A_t}\left(\frac{rental_t}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\right] \tag{6}$$

where $p_{C,t}$ is price of a competitive product.

The production function of a monopolistic product is

$$z_{M,t} = A_t k_{M,t}^{\alpha} l_{M,t}^{1-\alpha} \tag{7}$$

A monopolistic firm's problem is

$$\max_{p_{M,t}, k_{M,t}, l_{M,t}} \left\{\left(\frac{p_{M,t}}{P_t}\right)^{1-\theta} Z_t - rental_t k_{M,t} - W_t l_{M,t}\right\}$$

subject to

$$\left(\frac{p_{M,t}}{P_t}\right)^{-\theta} Z_t \leq A_t k_{M,t}^{\alpha} l_{M,t}^{1-\alpha}$$

where $p_{M,t}$ is price of the monopolistic product. First-order conditions are

$$rental_t k_{M,t} = \alpha z_{M,t} \left[\frac{1}{A_t}\left(\frac{rental_t}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\right] \tag{8}$$

$$W_t l_{M,t} = (1-\alpha) z_{M,t} \left[\frac{1}{A_t}\left(\frac{rental_t}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\right] \tag{9}$$

$$\frac{p_{M,t}}{P_t} = \frac{\theta}{\theta-1}\left[\frac{1}{A_t}\left(\frac{rental_t}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\right] \tag{10}$$

2.3 Product creation and product cycle

New intermediate products are created by R&D activity. Here I make a central assumption: only monopolistic firms engage in the R&D. The reasoning goes as follows. I assume that a monopolistic product is an innovative product. I also assume that a new product is only possibly created based on expert knowledge about a current innovative product. Finally, I assume that expert knowledge is perishable. Therefore, only current monopolistic firms have chance to conduct successful R&D.

If a monopolistic firm spends r_t of final good in the R&D, it creates

$$n_{E,t} = S_t r_t^{\nu} \tag{11}$$

of new products. ν is between 0 and 1. S_t is R&D sector-specific productivity shock which follows a stationary stochastic process

$$\log S_t = \rho_S \log S_{t-1} + \varepsilon_{S,t}, \quad \varepsilon_{S,t} \sim \text{i.i.d.} N\left(0, \sigma_S^2\right)$$

$\varepsilon_{A,t}$ and $\varepsilon_{S,t}$ are independent. New products are always monopolistic. Because the technology does not allow a multi-product firm, a monopolistic firm that created a product sells its exclusive production right to a competitive firm. The price of an exclusive production right is $q_{N,t}$. A competitive firm finances the cost by issuing equity. The monopolistic firm's optimal R&D decision is found by solving

$$\max_{r_t \geq 0} \{q_{N,t} n_{E,t} - r_t\}$$

subject to (11). The first order condition is

$$\nu \frac{n_{E,t}}{r_t} q_{N,t} = 1 \tag{12}$$

Let R_t denote the aggregate R&D input. Because $R_t = r_t N_{M,t-1}$, the aggregate product creation is

$$n_{E,t} N_{M,t-1} = S_t R_t^{\nu} N_{M,t-1}^{1-\nu}$$

Hence, not only the aggregate R&D input but the stock of expert knowledge about innovative products contribute to create new products in this economy.

Let $N_{M,t}$ denote the number of monopolistic products at the beginning of period t. $N_{M,t}$ evolves according to

$$N_{M,t} = (1 - \delta_N)(1 - \sigma) N_{M,t-1} + n_{E,t} N_{M,t-1} \tag{13}$$

This equation says that a monopolistic product could "depreciate" for two different reasons. One is death shock. That is, a fraction δ_N of randomly chosen monopolistic products becomes unavailable forever. The trade secret of a dying monopolistic product also disappears at the same time. The other is maturing shock. That is, a fraction σ of randomly chosen monopolistic products loses its trade secret but the product itself remains as a competitive one. In both cases, the monopolistic firm which has been producing the product becomes a competitive firm.

Let $N_{C,t}$ denote the number of competitive products at the beginning of period t. $N_{C,t}$ evolves according to

$$N_{C,t} = (1 - \delta_N) N_{C,t-1} + (1 - \delta_N) \sigma N_{M,t-1} \tag{14}$$

$(1 - \delta_N) \sigma N_{M,t-1}$ is a flow of competitive products from monopolistic products. A competitive product also faces the death shock. (13) and (14) imply that N_t evolves according to

$$N_t = (1-\delta_N)\, N_{t-1} + n_{E,t} N_{M,t-1}$$

because $N_t = N_{M,t} + N_{C,t}$.

2.4 Representative household

The representative household maximizes the expected discounted utility

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\log C_{t+j} + \varphi \log\left[1 - L_{t+j}\right]\right) \tag{15}$$

C_t is consumption. L_t is labor hours. For initial physical capital stock K_{-1} and the number of monopolistic firms $N_{M,-1}$, the representative household chooses the stochastic sequence of consumption, labor hours, investment, and the number of competitive firms it finances $\{C_t, L_t, I_t, N_{E,t}\}_{t\geq 0}$ to attain the maximum utility (15) subject to the sequence of budget constraints and law of motions

$$C_t + I_t + q_{N,t} N_{E,t} = W_t L_t + rental_t K_{t-1} + d_t N_{M,t-1} \tag{16}$$

$$K_t = (1-\delta_K)\, K_{t-1} + I_t \tag{17}$$

$$N_{M,t} = (1-\delta_N)\,(1-\sigma)\, N_{M,t-1} + N_{E,t} \tag{18}$$

for a given sequence of stochastic prices $\{q_{N,t}, W_t, rental_t\}_{t\geq 0}$ and dividend from a monopolistic firm

$$d_t = \pi_{Z,t} + \pi_{N,t} \tag{19}$$

$\pi_{Z,t}$ is profit earned in the good producing sector

$$\pi_{Z,t} = \frac{1}{\theta - 1} z_{M,t} \left[\frac{1}{A_t}\left(\frac{rental_t}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\right] \tag{20}$$

$\pi_{N,t}$ is profit earned in the R&D sector

$$\pi_{N,t} = (1-\nu)\, n_{E,t} q_{N,t} \tag{21}$$

First-order conditions are

$$W_t = \varphi \frac{C_t}{1-L_t}$$

$$1 = E_t \left[M_{t+1} \left(rental_{t+1} + 1 - \delta_K \right) \right]$$

$$q_{N,t} = E_t \left[M_{t+1} \left(d_{t+1} + q_{N,t+1} \left(1-\delta_N\right) \left(1-\sigma\right) \right) \right] \tag{22}$$

$$M_{t+1} = \beta \frac{C_t}{C_{t+1}}$$

2.5 Equilibrium

The equilibrium is a sequence of quantities $\{k_{C,t}, l_{C,t}, k_{M,t}, l_{M,t}, r_t, C_t, L_t, I_t, N_{E,t}\}_{t=0}^{\infty}$ and a sequence of prices $\{p_{C,t}, p_{M,t}, q_{N,t}, W_t, rental_t\}_{t=0}^{\infty}$ such that

1. $\{k_{C,t}, l_{C,t}, , p_{C,t}\}$ satisfies (3), (4), (5), and (6) every period,
2. $\{k_{M,t}, l_{M,t}, p_{M,t}, r_t\}$ satisfies (7), (8), (9), (10), and (12) every period,
3. $\{C_t, L_t, I_t, N_{E,t}\}_{t=0}^{\infty}$ maximizes (15) subject to (16), (17), (18), and initial K_{-1} and $N_{M,-1}$
4. labor market clears $L_t = N_{M,t-1} l_{M,t} + N_{C,t-1} l_{C,t}$; rental market clears $K_{t-1} = N_{M,t-1} k_{M,t} + N_{C,t-1} k_{C,t}$; equity market clears $N_{E,t} = n_{E,t} N_{M,t-1}$; and final good market clears $Z_t = C_t + I_t + r_t N_{M,t-1}$ where

$$Z_t = \left[N_{M,t-1} \left(A_t k_{M,t}^{\alpha} l_{M,t}^{1-\alpha} \right)^{\frac{\theta-1}{\theta}} + N_{C,t-1} \left(A_t k_{C,t}^{\alpha} l_{C,t}^{1-\alpha} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

   and $N_{M,t-1}$ and $N_{C,t-1}$ are determined recursively by (13) and (14).

2.6 National income accounting, factor shares, and intangible capital value

Following the current national income accounting convention, I treat R&D spending as expense and define GDP as final good production minus aggregate R&D spending

$$Y_t \equiv Z_t - r_t N_{M,t-1}$$

From the good market clearing,

$$Y_t = C_t + I_t$$

From the household budget constraint,

$$Y_t = W_t L_t + rental_t K_{t-1} + d_t N_{M,t-1} - q_{N,t} N_{E,t}$$

This is income account of GDP. The last two terms are net aggregate profit made by intermediate good firms. Aggregate capital increase $q_{N,t} N_{E,t}$ is subtracted because aggregate dividend from current monopolistic firms $d_t N_{M,t-1}$ includes this amount.

The net aggregate profit is crucial for factor shares because wage compensation is a constant fraction of GDP minus the net aggregate profit;

$$W_t L_t = (1-\alpha)\left[Y_t - d_t N_{M,t-1} + q_{N,t} N_{E,t}\right] \tag{23}$$

The net aggregate profit is crucial for aggregate intangible capital value too. Because the market value of a monopolistic product is determined by (22), we find

$$\begin{aligned} q_{N,t} N_{M,t} &= E_t\left[M_{t+1}\left(d_{t+1} N_{M,t} + q_{N,t+1} N_{M,t}\left(1-\delta_N\right)(1-\sigma)\right)\right] \\ &= E_t\left[M_{t+1}\left(d_{t+1} N_{M,t} - q_{N,t+1} N_{E,t+1} + q_{N,t+1} N_{M,t+1}\right)\right] \end{aligned}$$

Forward substitution leads to

$$q_{N,t}N_{M,t} = E_t\left[\sum_{j=1}^{\infty}\beta\frac{C_t}{C_{t+j}}\left(d_{t+j}N_{M,t-1+j} - q_{N,t+j}N_{E,t+j}\right)\right] \tag{24}$$

Therefore, the aggregate intangible capital value is the present discount value of future net aggregate profits. The net aggregate profit can be simplified to

$$\begin{aligned} d_tN_{M,t-1} - q_{N,t}N_{E,t} &= \pi_{Z,t}N_{M,t-1} + (1-\nu)\, n_{E,t}q_{N,t}N_{M,t-1} - q_{N,t}N_{E,t} \\ &= \pi_{Z,t}N_{M,t-1} - \nu n_{E,t}q_{N,t}N_{M,t-1} \\ &= \pi_{Z,t}N_{M,t-1} - r_tN_{M,t-1} \end{aligned}$$

Because in the Appendix we show

$$\pi_{Z,t} = \frac{1}{\theta}\frac{Z_t}{N_{t-1}}\left[\frac{N_{M,t-1}}{N_{C,t-1}}+1\right]\left[\frac{N_{M,t-1}}{N_{C,t-1}}+\left(\frac{\theta}{\theta-1}\right)^{\theta-1}\right]^{-1}$$

we find

$$d_tN_{M,t-1} - q_{N,t}N_{E,t} = \frac{1}{\theta}Z_t\left[\frac{N_{M,t-1}}{N_{C,t-1}}\right]\left[\frac{N_{M,t-1}}{N_{C,t-1}}+\left(\frac{\theta}{\theta-1}\right)^{\theta-1}\right]^{-1} - r_tN_{M,t-1}$$

Therefore, the net aggregate profit is not only affected by final good production but also by the composition of products and the aggregate R&D spending. It decreases when R&D spending increases and it increases when the composition of products changes toward a larger number of monopolistic products. These observations are important for interpreting main results.

3 Parameter values

Parameter values are summarized in Table 1. Time unit is a quarter of a year. The capital depreciation rate δ_K is .025. The subjective time discount rate β is .995. They are standard values in the literature. The probability of death shock δ_N is .01. The value is chosen to match an empirical finding of Broda and Weinstein (2007). That is, they find that the value of the products disappearing in a year is around 4 percent of the total expenditure. The probability of maturing shock σ is .045. $\sigma = .045$ and $\delta_N = .01$ together imply 20% annual depreciation rate of monopolistic products' knowledge capital, which is an empirically plausible value (see Corrado, Hulten, and Sichel (2009)). The elasticity of substitution across products θ is 3. This value is consistent with marketing research that directly measures the price elasticity of branded products (Tellis (1988)). $\theta = 3$ implies that the markup of a monopolistic product is 50%, which is also consistent with empirical studies measuring markup (Hall (1988) and Barsky, Bergen, Dutta, and Levy (2003)).[6] Capital share α is .31. The value is chosen to match the steady state labor share (68% of GDP). The coefficient of marginal utility of leisure φ is 3.66. The value is chosen to match the steady state labor hours (20% of the available time). The elasticity of R&D spending ν is .17. The value is chosen to match the steady state R&D spending (2% of GDP). Finally, parameters of productivity processes $(\rho_A, \sigma_A, \rho_S, \sigma_S)$ are estimated by maximum likelihood. The data are linearly detrended quarterly GDP from 1st quarter of 1950 to 2nd quarter of 2009 and annual R&D spending performed by business from 1959 to 2004. The sample length and frequencies of the two series do not match, but I integrate out missing R&D observations in computing the likelihood.

I close this section by showing some of the steady state relations implied by my calibration. First, competitive products are the majority. The number of competitive products is 4.4 times larger than the number of monopolistic products. The size of the total tangible capital stock is 252% of GDP. The size of the total intangible capital stock, i.e., the total market value of monopolistic products, is 54% of GDP. Under a reasonable assumption that the corporate sector's hard asset (plants and equipments) is 65% of GDP (Hall (2001a)), the steady state profit share is 8.9% of GDP.

[6] After Rotemberg and Woodford (1992), the elasticity of substitution $\theta = 6$ and 20% markup become relatively standard in the literature. But notice that Rotemberg and Woodford (1992) themselves state that, after reviewing empirical evidence, their choice of the markup is "extremely conservative."

4 Results

4.1 Impulse response functions

I plot impulse response functions of GDP, consumption, investment, labor hours, aggregate R&D spending, and the number of products in Figure 1. Thin lines are percentage deviations from the steady state after a one standard deviation positive innovation in good producing sector-specific productivity shock A_t. An innovation increases GDP, consumption, investment, labor hours, and aggregate R&D spending on impact. Because aggregate R&D spending increases, the number of products also increases.

As a benchmark, I also plot impulse response functions of GDP, consumption, investment, and labor hours in a standard RBC model (in dotted line). The model is easily described by a planner's problem that maximizes (15) subject to

$$A_t K_{t-1}^{\alpha} L_t^{1-\alpha} + K_t = C_t + (1-\delta_K) K_{t-1} \tag{25}$$

This model's parameter values are the same as in Table 2. Responses in the standard RBC model and our two-sector model are almost identical in the short run. However, GDP, consumption, and investment in the standard RBC model is less persistent than in our model. This is because our model has a propagation mechanism through endogenous product creation. As I derive in the Appendix, final good production Z_t is

$$Z_t = A_t N_{t-1}^{\frac{1}{\theta-1}} K_{t-1}^{\alpha} L_t^{1-\alpha} \left[\left(\frac{N_{M,t-1}}{N_{C,t-1}} \right) + 1 \right]^{\frac{-1}{\theta-1}} \left[\left(\frac{N_{M,t-1}}{N_{C,t-1}} \right) + \left(\frac{\theta}{\theta-1} \right)^{\theta} \right]^{-1} \left[\left(\frac{N_{M,t-1}}{N_{C,t-1}} \right) + \left(\frac{\theta}{\theta-1} \right)^{\theta-1} \right]^{\frac{\theta}{\theta-1}}$$

Because $N_{t-1}^{\frac{1}{\theta-1}}$ enters multiplicatively, expanding product variety has the same effect as productivity improvement. Because an increase in A_t increases N_{t-1}, responses become more persistent.

Thick lines are percentage deviations from the steady state after a one standard deviation positive innovation in the R&D sector-specific productivity shock S_t. Initial response to an innovation in S_t are much smaller than initial responses to an innovation in A_t. In addition, not all the variables respond in the same direction; consumption and aggregate R&D spending increase and output, investment, and labor hours decline on impact. For S_t improves and the R&D spending increases, the number of product varieties increases. Expanding product varieties improves productivity in the final good production, but the process is gradual. Correctly understand this gradual future productivity improvement, the household increases consumption right now. Higher consumption level decreases the marginal utility of consumption and reduces the household's incentive to work. This substitution effect causes an initial drop in the labor hours. Because the labor input declines, the final good production also declines. Higher level of R&D spending makes GDP even smaller. Investment also drops because output is small but consumption is large.

After small initial responses, GDP, consumption, and investment all start to grow. They are tracking the productivity improvement, and slow down only slightly even at 120 quarters after the innovation. In the medium-run, responses to an innovation in S_t become larger than responses to an innovation in A_t.

4.2 Business Cycle Statistics

I report business cycle statistics in Table 1. Series are detrended using the band-pass filter of Christiano and Fitzgerald (2003). The filter extracts cyclical components up to 32 quarters for quarterly data and cyclical components up to 8 years for annual data. I generate 1,000 artificial data from the model economies and report average across simulations.

The second column reports the business cycle statistics of the post-war U.S. data. The third column reports the same statistics in the model economy. I first focus on GDP, consumption, investment, and labor hours. The model successfully generates general movements of these key macroeconomic variables. The ratio of GDP standard deviation in the model economy to GDP standard deviation in the actual economy is around 75%. The investment is about three times more volatile than GDP, and consumption is about half as volatile as GDP both in the model economy and the actual economy. Labor hours are much smoother in the model economy than in the actual economy, but this is a common problem in the real business cycle models. Contemporaneous correlations between GDP and consumption, investment, and labor hours are all high both in the model economy and the actual economy.

The fourth column reports simulation results in the model where the R&D sector-specific productivity shock S_t is set to be constant. Statistics of GDP, consumption, investment, and labor hours in the third and the fourth columns are very close. Therefore, I conclude that the good producing sector-specific productivity shock A_t accounts for most of the high-frequency fluctuations of these variables.

Now I turn to the R&D spending. The R&D spending is volatile in the actual economy. The standard deviation is 1.79, which is slightly larger than that of output but smaller than that of investment. The R&D spending is also pro-cyclical in the actual economy, but is much weaker than other key macroeconomic variables. Contemporaneous correlation between GDP and the R&D spending is .47 while contemporaneous correlation between GDP and consumption, investment, and labor hours are all above .75.

Our model economy replicates the general pattern of the R&D spending very well. As the third column shows, standard deviation in the model economy is 1.86, which is almost identical with the same statistics in the actual economy. Correlation with output in the model economy is .33, which is again very close to the same statistics in the actual economy.

The fourth column shows that once the R&D sector-specific productivity shock S_t is set to be constant, the model cannot replicate general patterns of the R&D spending anymore. Standard deviation in the model is .80, less than a half of the same statistics in the actual economy. Correlation with output is .99 while the same statistics in the actual economy are .47. I conclude that the R&D sector-specific productivity shock S_t is important in accounting for high-frequency R&D fluctuations.

4.3 Labor share

Labor share is constant in most of the standard RBC models, but this is not the case in my model. In the left panel of Figure 3, I plot impulse functions of the labor share. A thick line is percentage deviation from the steady state after a one standard deviation positive innovation in the R&D sector-specific productivity shock S_t. The labor share increases by 3.3 basis points on impact.

It subsequently decreases and undershoots the steady state level at around 20 quarters after the innovation. A thin line is percentage deviation from the steady state after a one standard deviation positive innovation in the good producing sector-specific productivity shock A_t. It does not have a noticeable impact on the labor share.

From (23), we find that

$$
\begin{aligned}
W_t L_t &= (1-\alpha)\left[Y_t - \left(\pi_{Z,t} N_{M,t-1} - r_t N_{M,t-1}\right)\right] \\
&= (1-\alpha)\left[Z_t - \pi_{Z,t} N_{M,t}\right] \\
&= (1-\alpha)\, Z_t \left[1 - \frac{1}{\theta}\left[\frac{N_{M,t-1}}{N_{C,t-1}}\right]\left[\frac{N_{M,t-1}}{N_{C,t-1}} + \left(\frac{\theta}{\theta-1}\right)^{\theta-1}\right]^{-1}\right]
\end{aligned}
$$

Therefore,

$$
\frac{W_t L_t}{Y_t} = (1-\alpha)\frac{Z_t}{Z_t - r_t N_{M,t-1}}\left[1 - \frac{1}{\theta}\left[\frac{N_{M,t-1}}{N_{C,t-1}}\right]\left[\frac{N_{M,t-1}}{N_{C,t-1}} + \left(\frac{\theta}{\theta-1}\right)^{\theta-1}\right]^{-1}\right] \tag{26}
$$

The labor share in my model moves for two factors. First, it moves as the ratio of final good production to GDP, $Z_t/\left(Z_t - r_t N_{M,t-1}\right)$, moves because an increase in R&D spending depresses the corporate profit disproportionately. This mechanism explains the initial labor share increase after an innovation in S_t. Consistent with my model's prediction, the labor share is negatively correlated with business R&D spending in the actual economy; the contemporaneous correlation is .23. Second, the labor share moves as the composition of products, $N_{M,t-1}/N_{C,t-1}$, moves because a larger share of monopolistic products increases the net aggregate profit and decreases the labor share. Because a positive innovation in S_t increases the share of monopolistic products as plotted in the right panel of Figure 3, this mechanism explains the labor share undershooting. As we see in a subsequent section, we do not observe this undershooting property in a model $\sigma = 0$ because all the products are monopolistic and the composition effect is shut down.

In Table 3, I report variance and correlation of labor share at the business cycle frequency. In the actual economy the labor share is volatile and persistent. The standard deviation is .47 and the first-order autoregression coefficient is .69. In my model economy the standard deviation is

.045 and the first-order autoregression coefficient is .69. The labor share volatility in my model economy is only one-tenth of the labor share volatility in the actual economy. But this is huge improvement from the benchmark RBC model where the labor share is constant at all frequencies. In addition, the labor share in my model economy is smooth partly because I calibrate my model parameters using the business R&D, which is only 2% of GDP. But Corrado, Hulten, and Sichel (2005) include advertisement and employer-provided training as part of intangible investment and find that the size of intangible investment measured in this way is as large as tangible investment. If I adopt these broadly defined measures, the labor share will swing more widely. The persistence is well replicated.

The labor share is countercyclical both in the actual and in the model economies; the contemporaneous correlation with GDP is -.16 in both economies. The labor share is countercyclical in my model because an innovation in S_t increases the labor share but decreases GDP on impact. Both in the actual and the model economies, the labor share is negatively correlated with the Solow residual measured by

$$\log SR_t = \log Y_t - \alpha \log K_{t-1} - (1-\alpha)\log L_t$$

The contemporaneous correlation with the Solow residual is -.48 in the actual economy and it is -.16 in the model economy. The Solow residual in my model economy is

$$\begin{aligned}\log SR_t \;=\; & \log\left[\frac{Z_t - r_t N_{M,t-1}}{Z_t}\right] + \log A_t + \frac{1}{\theta-1}\log N_{t-1} \\ & + \log\left[\left[\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right)+1\right]^{\frac{-1}{\theta-1}}\left[\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right)+\left(\frac{\theta}{\theta-1}\right)^{\theta}\right]^{-1}\left[\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right)+\left(\frac{\theta}{\theta-1}\right)^{\theta-1}\right]^{\frac{\theta}{\theta-1}}\right]\end{aligned}$$

Therefore, an increase in the R&D spending decreases the Solow residual. As we discussed, an increase in the R&D spending increases the labor share. Hence fluctuations in the R&D spending generate a negative correlation between the labor share and the Solow residual.

In Table 4, I report the phase-shift of labor share with respect to output. The labor share lags output in the actual economy. The correlation between the current output and next year's labor share is .55 while the correlation with current output and last year's labor share is -.23. The model economy replicates the asymmetry very weakly. The correlation between the current output and the next year's labor share is .03 and the correlation between the current output and the last year's labor share is -.01. In fact, the labor share lags output in my model economy because an innovation in S_t generates an immediate decline in output and a following labor share undershooting. But the undershooting is too slow to vividly appear in the filtered data.

Finally, I report dynamic relation between GDP and labor share and dynamic relation between the Solow residual and labor share in bivariate vector autoregression (VAR) models. Using the actual data, Rios-Rull and Santaeulalia-Llopis (2009) find that a productivity shock, which is identified with the assumption that the innovation to the factor share is purely redistributive, produces a reduction of labor share on impact, but it also subsequently produces a long-lasting increase in labor share that peaks above mean five years later at a level larger in absolute terms than the initial drop. They call this property overshooting.

I perform the same exercise using the artificial data in my model economy. Figure 3 reports the results. Though the magnitude is very small, the model replicates the overshooting property qualitatively. The orthogonalized output shock generates an initial drop in the labor share, but the mean response subsequently overshoots its long-run average level. The orthogonalized Solow-residual shock generates an initial drop in the labor share, and it subsequently recovers to the long-run average.

4.4 News shock

Sims (2009) carefully investigates dynamic effect of a news shock. He studies a VAR model featuring several forward-looking variables and macroeconomic aggregates, and identifies the news shock as the one that explains future movements in a utilization-adjusted measure of aggregate technology among all shocks that are orthogonal to technology innovations. He finds that a favorable news shock is associated with an increase in consumption and declines in output, hours, and investment on impact. These are consistent with a neo-classical model. But he also finds that a favorable news shock is associated with an increase in the stock market index.[7] Therefore, a news shock is

[7] Barsky and Sims (2009) uses a similar identification technique and finds a favorable news shock produces the

associated with both a stock market boom and an investment slump. This prediction is puzzling because the q theory of investment predicts that investment is high when the capital price is high.

My model offers consistent interpretation for a news shock and these empirical responses. A "news shock" in my model economy is an innovation in the R&D sector-specific productivity shock. Impulse responses of GDP, consumption, investment, and labor hours in my model economy are consistent with empirical responses of these variables (see Figure 1). In this section, I show that a favorable news shock is also associated with a stock market boom in my model economy.

In the left panel of Figure 4, I plot the impulse response function of the aggregate intangible capital value $q_{N,t}N_{M,t}$ to a positive innovation in S_t. It rises on impact, and it peaks at around 70 quarters after the innovation. Assuming that the steady state corporate sector's tangible capital value is 65% of annual GDP and the ratio of the corporate tangible capital to the non-corporate tangible capital is constant over time, I plot the impulse response function of the corporate sector's total asset value in the right panel of Figure 4. It also rises on impact. As shown in Figure 1, tangible capital investment drops after an innovation in S_t. Therefore, the appreciation of the corporate sector's total asset value is accounted for by the appreciation of the aggregate intangible capital value.

From (24), we find that the aggregate intangible capital value is determined by

$$q_{N,t}N_{M,t} = E_t \left[\sum_{j=1}^{\infty} \beta \frac{C_t}{C_{t+j}} \left(\frac{1}{\theta} Z_{t+j} \left[\frac{N_{M,t-1+j}}{N_{C,t-1+j}} \right] \left[\frac{N_{M,t-1+j}}{N_{C,t-1+j}} + \left(\frac{\theta}{\theta - 1} \right)^{\theta-1} \right]^{-1} - r_{t+j} N_{M,t-1+j} \right) \right] \tag{27}$$

The equation highlights the importance of the composition effect. An innovation in S_t produces a growth trend in both final good production Z_t and consumption C_t. (27) says these two effects almost perfectly offset each other because an increase in consumption decreases the discount factor (increases interest rate). But an innovation in S_t also increases the number of monopolistic products and changes the product composition. Because of this additional margin, an increase in the growth rate of net aggregate profit exceeds an increase in the interest rate and the aggregate tangible capital value rises.

stock market boom on impact.

In the top panel of Figure 5, I plot impulse response functions of the value of a single monopolistic product and the number of monopolistic products. It is clear from them that the majority of an initial appreciation of the aggregate intangible capital is explained by an initial appreciation of an individual product's value. The value of an individual monopolistic product is understood from goods producing sector's profit $\pi_{Z,t}$ and R&D sector's profit $\pi_{N,t}$. In the lower panel of Figure 5, I plot their responses to an innovation in S_t. The goods producing sector profit plunges on impact, and subsequently decreases further. The R&D sector profit jumps up around 1.7 percentage points on impact. It subsequently declines and undershoots its steady state level at 48 quarters after the innovation. Therefore, fat R&D sector profits for relatively short periods after the innovation account for an initial appreciation of an individual monopolistic product.

The R&D sector profit is

$$\pi_{N,t} = (1 - \nu) S_t r_t^{\nu} q_{N,t}$$

A monopolistic firm is making a positive profit in the R&D sector when ν is smaller than 1. It is easy to see that a positive innovation in S_t increases current and future $\pi_{N,t}$. This direct impact from S_t is reinforced by an increase in R&D spending r_t and an increase in the value of a monopolistic product $q_{N,t}$. That is, a larger profit stream increases the value of a monopolistic product. A higher value of a monopolistic product stimulates R&D because it makes the R&D more profitable. A larger R&D spending further increases the R&D sector's profit, which then increases the value of a monopolistic firm even more. This process continues until the marginal profit of product creation decreases to one. Decreasing returns to scale in the product creating function (11) is crucial to make this system stable.

After the initial jump, the value of a monopolistic product quickly decreases and even undershoots its steady state level at 20 quarters after the innovation. This means that monopolistic products are so scarce in the early stage of product creation boom because it is an essential input for the R&D activity. But a high price of a monopolistic product generates a massive increase in the R&D spending. The number of monopolistic products quickly increases and scarcity disappears. The value of a monopolistic product even undershoots below its steady state level because S_t is higher than its steady state level.

4.5 Asset pricing

In this section, I report the model's asset pricing implications. My focus is on the equity premium and the risk-free rate. In the actual economy, the equity premium is both high and volatile (mean is 5.57 percent and standard deviation is 14.36 percent in the post-war U.S. data) and the risk-free rate is both low and stable (mean is .98 percent and standard deviation is .65 percent in the post-war U.S. data). It is well known that replicating them in a DSGE model with reasonable parameter values is very difficult.

To explore how changes in the risk attitude affect asset prices, I replace (15) to the recursive utility of Epstein and Zin (1989):

$$U_t = (\underline{u} + \log C_t + \varphi \log [1 - L_t]) + \beta \left(E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right) \tag{28}$$

$\underline{u}$ is a positive constant. I also assume that there is a lower bound for consumption $\underline{C}$ and an upper bound for labor hours $\bar{L}$ and they satisfy $\underline{u} + \log \underline{C} + \varphi \log \left[1 - \bar{L}\right] > 0$. These assumptions ensure a solution U_t exists. (15) and (28) are identical when both $\underline{u}$ and γ are equal to 0. The stochastic discount factor changes to

$$M_{t,t+1} = \beta \frac{C_t}{C_{t+1}} \left(\frac{U_{t+1}}{E_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{-\gamma} \tag{29}$$

Other equilibrium conditions are the same as before. I solve the model up to the second order using the algorithm of Swanson, Anderson, and Levin (2006).

The exact relation between γ and the coefficient of relative risk aversion, a commonly used risk-aversion measure, is not straightforward when the utility function has consumption margin and leisure margin. See Swanson (2010) for a detailed discussion. But fortunately, Swanson (2010) finds an analytical solution of the coefficient of relative risk aversion at the steady state. In Table 5, I report simulation results in my model economy at different levels of risk aversion gauged with this measure.

The gross return on tangible capital is defined as

$$r_{K,t+1} = rental_{t+1} + 1 - \delta_K$$

The gross return on intangible capital is defined as

$$r_{N,t+1} = \frac{\pi_{Z,t+1} + \pi_{N,t+1} + (1 - \delta_N)(1 - \sigma) q_{N,t+1}}{q_{N,t}}$$

The risk-free rate is defined as

$$r_{f,t} = \frac{1}{E_t [M_{t+1}]}$$

An excess return is defined as an asset return minus the risk-free rate.

When the coefficient of relative risk aversion is equal to 1, the excess return on tangible capital is .001 annually and the excess return on intangible capital is .02 annually. When the household becomes more risk averse, the excess return on intangible capital increases appreciably while the excess return on tangible capital does not change much. When the coefficient of relative risk aversion is equal to 50, the excess return on intangible capital is 1.23 annually while the excess return on tangible capital is -.004 annually. Chan, Lakonishok, and Sougiannis (2001) empirically finds that R&D intensive firms earn large equity returns, which is consistent with our model's cross-sectional prediction.

This cross-sectional difference is understood from the following equations:

$$E_t [r_{i,t+1} - r_{f,t}] = -\frac{Cov_t [M_{t+1}, r_{i,t+1} - r_{f,t}]}{E_t [M_{t+1}]} \quad \text{for } i = N \text{ or } K$$

I derive them from the household's first order conditions. They state that an asset that has a strong negative covariance with the stochastic discount factor has a large excess return. The stochastic discount factor is small when consumption is large. The stochastic discount factor is also small when the continuation utility U_{t+1} is large since γ is greater than 1 for all the cases we consider. Because the excess return on intangible capital is large only when γ is large, it has to do with covariance between the intangible capital return and the continuation utility value.

Positive covariance between the intangible capital return and the continuation utility (i.e., negative covariance between the intangible capital return and the stochastic discount factor) is caused by both the goods producing sector-specific productivity shock A_{t+1} and the R&D sector-specific productivity shock S_{t+1}. A positive innovation in A_{t+1} increases the goods producing sector profit, the R&D sector profit, and the value of a monopolistic product. All of them raise the intangible capital return. At the same time, a positive innovation in A_{t+1} raises the continuation utility. Therefore, a fluctuation in A_{t+1} contributes to a positive covariance between the intangible capital return and the continuation utility. A positive innovation in S_{t+1} decreases the goods producing sector profit but increases the R&D sector profit and the value of a monopolistic product (Figure 7). Because the latter two dominate the first, a positive innovation in S_{t+1} raises the intangible capital return. At the same time, a positive innovation in S_{t+1} raises the continuation utility because it positively changes expectations about productivity. Therefore, a fluctuation in S_{t+1} also contributes to generating a positive covariance between the intangible capital return and the continuation utility.

To compare importance of the two shocks, in the bottom panel of Table 5, I report simulation results in the model in which S_t is set to be constant. The excess return on intangible capital when the coefficient of relative risk aversion is 50 reduces to .72 from 1.23. Therefore, both productivity shocks are important in generating a large excess return on intangible capital.

The excess return on tangible capital is only -.004 percent when the coefficient of relative risk aversion is 50. This is because an innovation in S_{t+1} decreases the tangible capital return but increases the continuation utility, resulting in a negative covariance between the two. In other words, the tangible capital stock works as a good hedge against changes in the R&D sector-specific productivity shock. Investment adjustment cost helps generate a large excess return on tangible capital. However, a strong adjustment makes investment volatility and labor hour volatility counterfactually small. I will discuss these points in a subsequent section.

In Table 6, I report the equity premium and the risk-free rate. In the fifth column, I report results in the standard RBC model as a benchmark. The model is described by a planner's problem who maximizes (28) subject to (25). The asset side of the corporate sector's balance sheet consists of tangible capital alone. I assume that the corporate sector issues risk-free bonds. Hence, the liability side of the corporate sector's balance sheet consists of debt and equity. I also assume that the value of the debt is always the same as the value of the equity. These assumptions imply that the excess return on the corporate equity is $2\left(r_{K,t} - r_{f,t}\right)$. When the coefficient of relative risk aversion is 50, the equity premium is .03 and its volatility is .12 and the risk-free rate is 1.66 and its volatility is .15. Compared to those in the actual economy reported in the second column, the equity premium is too small and too smooth and the risk-free rate is too high in the standard RBC model.

In our model, the asset side of the corporate sector's balance sheet consists of both tangible capital and intangible capital. I assume that the corporate sector's tangible capital is a constant fraction of the economy's total tangible capital and its steady state value is 65% of annual GDP. I also assume that all the intangible capital are owned by the corporate sector. I maintain the assumption that the debt-to-capital ratio is always 1. With these assumptions, the excess return on the corporate equity is $2\left(\tilde{r}_t - r_{f,t-1}\right)$ where $\tilde{r}_t$ is the value weighted average return on the corporate sector's two capitals:

$$\tilde{r}_t = \left(\frac{(K_t/K)*(2.6*Y)}{(K_t/K)*(2.6*Y) + q_{N,t}N_{M,t}} r_{K,t} + \frac{q_{N,t}N_{M,t}}{(K_t/K)*(2.6*Y) + q_{N,t}N_{M,t}} r_{N,t}\right)$$

The coefficient of relative risk aversion is 50. The third column shows that the equity premium is 1.05 in the model economy. Although it is still smaller than 5.57 percent equity premium in the actual economy, it is significant improvement from .03 percent equity premium in the standard RBC model. In addition, 5.57 percent post-war U.S. average equity premium might be a too high bar for the model to target because there are evidence that the equity premium is declining (see Jagannathan, McGrattan, and Scherbina (2000) and Cogley and Sargent (2008)). 1.06 percent equity premium is in a reasonable range of the equity premium in light of these empirical studies while .03 percent equity premium in the standard RBC model will not be justifiable at any standard.

The fourth column shows that the equity premium in the model with constant S_t is .65. The difference between the two models, i.e., 40 basis points equity premium, reflects risk associated with a medium-term growth rate fluctuation caused by an innovation in S_t. This mechanism resembles the one argued in the long-run risk literature in finance. Most papers in the literature study the endowment economy. A few papers study a production economy in which persistent fluctuations in productivity growth rate are mechanically embedded. My model, to the best of my knowledge, is the first paper that gives a full description from the origin of the long-run risk (the R&D sector-specific productivity shock) to its mechanism for generating a high equity premium (appreciation of the intangible capital value). It is interesting that news shock and the origin of long-run risk, two subjects studied independently, are the same thing in my framework.

The risk-free rate becomes smaller as the coefficient of relative risk aversion becomes larger. In the model economy, the risk-free rate is 2.01 percents when the coefficient of relative risk aversion is 1 and 1.16 percents when the coefficient of relative risk aversion is 50. The risk-free rate becomes smaller because the risk-averse household accumulates larger tangible capital stock. The larger tangible capital stock reduces the rental price of tangible capital and reduces the risk-free rate. The risk-free rate in our model is smaller than in the standard RBC model (1.66 percent). This is because larger tangible capital investment crowds out R&D spending. The smaller steady state R&D spending results in a smaller number of intermediate products at steady state, which reduces the rental price of tangible capital and the risk-free rate.

5 Inspecting the Mechanisms

5.1 Innovative-monopolistic to innovative-competitive

In my model, only current monopolistic firms have the ability to conduct successful R&D because they have expert knowledge about an innovative product. To highlight the importance of this assumption, I consider an alternative model in which any firm can engage in the R&D with linear technology

$$n_{E,t} = S_t r_t^*$$

where r_t^* is the ready-made R&D input which is sold by the household at price $q_{R,t}$. Firm's R&D problem is

$$\max_{r_t^* \geq 0} \{q_{N,t} S_t r_t^* - q_{R,t} r_t^*\}$$

The first order condition is

$$q_{R,t} = q_{N,t} S_t$$

The household produces effective R&D input R_t^* from final good input R_t with a production function

$$R_t^* = R_t^\nu$$

An interpretation is that R_t^* is human capital and R_t is education cost. The household's problem is now

$$\max E_t \sum_{j=0}^{\infty} \beta^j \left[\log C_{t+j} + \varphi \log \left[1 - L_{t+j}\right]\right]$$

subject to

$$C_t + I_t + R_t + q_{N,t} N_{E,t} = W_t L_t + rental_t K_{t-1} + \pi_t N_{M,t-1} + q_{R,t} R_t^*$$

$$R_t^* = R_t^\nu$$

$$K_t = (1 - \delta_K) K_{t-1} + I_t$$

$$N_{M,t} = (1 - \delta_N)(1 - \sigma) N_{M,t} + N_{E,t}$$

Other components are the same as in the original model. I use the same parameter values.

In the left panel of Figure 6, I plot impulse response function of aggregate intangible capital value to a positive innovation in the R&D sector-specific productivity shock S_t. The aggregate intangible capital value does not rise but slightly drops on impact. Because the aggregate intangible capital value drops, the corporate sector's total asset value collapses on impact too (the right panel). This prediction is not consistent with empirical stock market response to a news shock found by Sims (2009).

The aggregate intangible capital value drops on impact because a monopolistic firm makes profit only in the goods producing sector. That is, the value of a monopolistic firm is

$$q_{N,t} = E_t \left[M_{t+1} \left(\pi_{Z,t+1} + (1 - \delta_N)(1 - \sigma) q_{N,t+1} \right) \right]$$

where

$$\pi_{Z,t} = \frac{1}{\theta} \frac{Z_t}{N_{M,t-1}} \left[\frac{N_{M,t-1}}{N_{C,t-1}} \right] \left[\frac{N_{M,t-1}}{N_{C,t-1}} + \left(\frac{\theta}{\theta - 1} \right)^{\theta - 1} \right]^{-1}$$

Without the dynamic link between current and future products in the R&D sector, an innovation in S_t decreases the value of a current monopolistic product since it simply means that more rivals will be entering into the market. This is essentially the same mechanism as in Greenwood and Jovanovic (1999).

In Table 7, I report asset pricing implications. The excess return on intangible capital is much smaller in the current model. When the coefficient of relative risk aversion is 50, the excess return on intangible capital is .47 in the current model while the excess return on intangible capital is 1.23 in the original model. This is because intangible capital return becomes less sensitive to long-run risk without an appreciation associated with an innovation in S_t. In the lower panel of Table 7, I report asset returns in the current model economy in which S_t is constant. Asset returns in this constant-S model are almost identical with those in the two-shock model. Actually, the excess return on intangible capital is slightly higher in the constant-S model. When the coefficient of relative risk aversion is 50, the excess return on intangible capital is .49 in the constant-S model while it is .48 in the two-shock model. This is because an initial drop in aggregate intangible capital value causes a slight negative covariance between the intangible capital return and the continuation utility.

5.2 Innovative-monopolistic only

In my model, a product is monopolistic when it is born and it turns competitive later in its life. To highlight the importance of this product life cycle, I consider an alternative model in which a product is always monopolistic. More precisely, I set the value of the maturing shock σ at $\sigma = 0$.

All the varieties are monopolistic

$$N_t = N_{M,t}$$

They depreciate only with the death shock

$$N_{M,t} = (1 - \delta_N)\, N_{M,t-1} + N_{E,t}$$

In Figure 7, I plot labor share response to a positive innovation in the R&D sector-specific productivity shock S_t. The labor share jumps up on impact and it monotonically decreases to the steady state level. This response is very different from the one in the original model. In the original model, the labor share jumps up on impact and subsequently undershoots the steady state level (Figure 3). Remember the labor share in the original model is

$$\frac{W_t L_t}{Y_t} = (1 - \alpha)\, \frac{Z_t}{Z_t - r_t N_{M,t-1}} \left[1 - \frac{1}{\theta} \left[\frac{N_{M,t-1}}{N_{C,t-1}} \right] \left[\frac{N_{M,t-1}}{N_{C,t-1}} + \left(\frac{\theta}{\theta - 1} \right)^{\theta - 1} \right]^{-1} \right]$$

The labor share undershooting is caused by an increase in the number of monopolistic products relative to competitive products. In the current model economy, the labor share is

$$\frac{W_t L_t}{Y_t} = (1 - \alpha)\, \frac{\theta - 1}{\theta} \frac{Z_t}{Z_t - r_t N_{M,t-1}}$$

Therefore, the composition effect is missing and labor share monotonically returns to the steady state.

In the left panel of Figure 8, I plot response of the aggregate intangible capital value to a positive innovation in the R&D sector-specific productivity shock S_t. A positive innovation in S_t generates a crash of the aggregate intangible capital value on impact. Because the aggregate intangible capital value crashes, the corporate sector's total asset value also crashes (the right panel of Figure 8). This prediction is inconsistent with empirical stock market response to a news shock found by Sims (2009). In the current model, the total intangible capital value is

$$q_{N,t}N_{M,t} = E_t \left[\sum_{j=1}^{\infty} \beta \frac{C_t}{C_{t+j}} \left(\frac{1}{\theta} Z_{t+j} - r_{t+j} N_{M,t-1+j} \right) \right] \tag{30}$$

An innovation in S_t produces a persistent growth trend in the productivity, consumption, and final good production. (30) says that a positive effect through growing aggregate demand (a larger Z_t) is almost perfectly offset by a negative effect through the discount rate (higher interest rate). This is the reason we do not observe an appreciation of the aggregate intangible capital value. In the original model, the same interest rate effect is working but the total intangible capital value still increases because of the composition effect helps.

I report asset pricing implications in Table 8. The excess return on intangible capital is .45 in the current model economy while it is 1.28 in the original model economy when the coefficient of relative risk aversion is equal to 50. The reason for this smaller excess return is exactly the same as in the previous section; without an appreciation associated with an innovation in S_t, intangible capital return becomes less sensitive to long-run risk. In the lower panel of Table 8, I report asset returns in the current model economy in which S_t is constant. Asset returns in the constant-S model are almost the same as those in the two-shock model. The excess return on intangible capital is slightly higher in the constant-S model than in the two-shock model. This is because an initial drop in aggregate intangible capital value causes a slight negative covariance between the intangible capital return and the continuation utility.

5.3 Innovative-monopolistic to matured-monopolistic

To be added.

5.4 Intertemporal elasticity of substitution

Reasonable value of intertemporal elasticity of substitution is hotly debated in the finance literature. More precisely, whether the parameter is greater than 1 or smaller than 1 is the focal point of the discussion. To see why this is crucial, consider a perfect-foresight endowment economy in which the household's utility is

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}}$$

ψ is the intertemporal elasticity of substitution. The price of an asset whose divided stream is $\{d_t\}_{t=0}^{\infty}$ is

$$p_0 = \sum_{t=1}^{\infty} \beta^t \left(\frac{C_t}{C_0}\right)^{-\frac{1}{\psi}} d_t$$

Assuming that d_t and C_t grow at the same pace g, we find that p_0 is

$$p_0 = d_0 \left[\sum_{t=1}^{\infty} \beta^t g^{\left(1-\frac{1}{\psi}\right)t}\right]$$

Now consider an unexpected upward revision of growth rate from g to $g' > g$. If ψ is greater than 1, p_0 increases; if ψ is smaller than 1, p_0 decreases; and if ψ is equal to 1, p_0 is unaffected by the change in the growth forecast. These differences arise because ψ determines how sensitively the interest rate is affected by changes in consumption growth. Proponents of long-run risk assume ψ greater 1 because that is how the model gets an asset price appreciation in response to an upward revision of consumption and dividend growth rates.

But in my model, the equity value increases even when the intertemporal elasticity of substitution is smaller than 1. For that purpose, I replace the utility function (15) with

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{\left[C_{t+j}\left(1-L_{t+j}\right)^{\varphi}\right]^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} \tag{31}$$

(15) and (31) are identical when ψ approaches to 1. I set ψ at $\psi = .5$, and I plot impulse response function of the aggregate intangible capital value to a positive innovation in S_t in the left panel of Figure 9. The aggregate intangible capital value rises on impact. The right panel is impulse response function of the corporate sector's total asset value. It also rises on impact because of the appreciation in the aggregate intangible capital value.

The composition effect is the key for this result. The stochastic discount factor is

$$M_{t+j} = \beta \left(\frac{C_{t+j}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{1 - L_{t+j}}{1 - L_t} \right)^{\varphi\left(1-\frac{1}{\psi}\right)}$$

and the total intangible capital value is

$$q_{N,t} N_{M,t} = E_t \left[\sum_{j=1}^{\infty} M_{t+j} \left(\frac{1}{\theta} Z_{t+j} \left[\frac{N_{M,t-1+j}}{N_{C,t-1+j}} \right] \left[\frac{N_{M,t-1+j}}{N_{C,t-1+j}} + \left(\frac{\theta}{\theta - 1} \right)^{\theta - 1} \right]^{-1} - r_{t+j} N_{M,t-1+j} \right) \right]$$

An innovation in S_t increases consumption growth rate. When ψ is smaller than 1, an upward revision of consumption growth rate produces a large increase in the interest rate. However, the number of monopolistic products also increases, which has positive effect on the net aggregate profit. With this additional margin, the aggregate intangible capital value increases on impact because monopolistic products expand faster than the aggregate consumption.

Because the corporate sector's total asset value increases in response to an innovation in S_t, there is an equity premium gain from the long-run risk too. I use the recursive utility

$$U_t = \frac{\left[C_t \left(1 - L_t \right)^{\varphi} \right]^{1-\frac{1}{\psi}}}{1 - \frac{1}{\psi}} - \beta E_t \left[\left(-U_{t+1} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \tag{32}$$

Stochastic discount factor is

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{1 - L_{t+1}}{1 - L_t} \right)^{\varphi\left(1-\frac{1}{\psi}\right)} \left(\frac{-U_{t+1}}{E_t \left[\left(-U_{t+1} \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{-\gamma}$$

I report asset pricing implications in Table 9. When the coefficient of relative risk aversion is 50, the excess return on intangible capital is .87. In the model in which S_t is constant, the excess return on intangible capital at the same level of risk aversion is .67. Hence, 20 basis point equity premium is associated with the long-run risk caused by S_t.

Long-run risk implications are strengthened if I use the intertemporal elasticity of substitution greater than 1. I set ψ at $\psi = 1.5$ and plot impulse response functions of total intangible capital value and the corporate sector's total asset value to a positive innovation in S_t in Figure 10. Responses are larger than those in the original model (Figure 4). In Table 10, I report asset pricing implications. The excess return on intangible capital is 1.64 when the coefficient of relative risk aversion is 50. In a model in which S_t is constant, the excess return on intangible capital reduces to .88. Hence, 76 basis point equity premium is associated with the long-run risk caused by S_t.

6 Conclusion

To be added

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A Equilibrium relations

In this section, I derive some equilibrium properties. (10) and (6) imply

$$\frac{p_{M,t}}{p_{C,t}} = \frac{\theta}{\theta - 1}$$

Therefore, a monopolistic product is sold at a higher price than a competitive product. Because both a monopolistic product and a competitive product face the same demand functions (2),

$$\frac{z_{M,t}}{z_{C,t}} = \left(\frac{\theta}{\theta - 1}\right)^{-\theta}$$

Therefore a monopolistic product is produced less than a competitive product. Because both a monopolistic product and a competitive product are produced with the same Cobb-Douglas production function (3) and (7),

$$\frac{k_{M,t}}{k_{C,t}} = \frac{l_{M,t}}{l_{C,t}} = \left(\frac{\theta}{\theta - 1}\right)^{-\theta} \tag{33}$$

Therefore, a monopolistic product's factor demands are less than a competitive product's factor demands.

From the rental market clearing

$$\begin{aligned} K_{t-1} &= N_{M,t-1}k_{M,t} + N_{C,t}k_{C,t} \\ &= N_{t-1}k_{M,t}\left[\frac{N_{M,t-1}}{N_{t-1}} + \frac{N_{C,t-1}}{N_{t-1}}\left(\frac{\theta}{\theta - 1}\right)^{\theta}\right] \\ &= N_{t-1}k_{M,t}\left[\frac{\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right)}{\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right) + 1} + \frac{1}{\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right) + 1}\left(\frac{\theta}{\theta - 1}\right)^{\theta}\right] \end{aligned}$$

We find

$$k_{M,t} = \frac{K_{t-1}}{N_{t-1}}\left[\frac{\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right)}{\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right) + 1} + \frac{1}{\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right) + 1}\left(\frac{\theta}{\theta - 1}\right)^{\theta}\right]^{-1}$$

Doing similar calculations for the labor market,

$$l_{M,t} = \frac{L_t}{N_{t-1}} \left[\frac{\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right)}{\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right)+1} + \frac{1}{\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right)+1} \left(\frac{\theta}{\theta-1}\right)^{\theta} \right]^{-1}$$

Therefore, both N_{t-1} and $N_{M,t-1}/N_{C,t-1}$ affect a monopolistic product's factor demands. N_{t-1} appears in the denominator, meaning that N_{t-1} stretches the resources. An increase in $N_{M,t-1}/N_{C,t-1}$ increases the factor demands. This is because a monopolistic product uses less factor inputs and hence does not stretch the resources as much as a competitive product.

The final good production is

$$\begin{aligned}
Z_t &= \left[N_{M,t-1} z_{M,t}^{\frac{\theta-1}{\theta}} + N_{C,t-1} z_{C,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \\
&= N_{t-1}^{\frac{\theta}{\theta-1}} z_{M,t} \left[\frac{N_{M,t-1}}{N_{t-1}} + \frac{N_{C,t-1}}{N_{t-1}} \left(\frac{\theta}{\theta-1}\right)^{\theta-1} \right]^{\frac{\theta}{\theta-1}} \\
&= A_t N_{t-1}^{\frac{1}{\theta-1}} K_{t-1}^{\alpha} L_t^{1-\alpha} \left[\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right) + 1 \right]^{\frac{-1}{\theta-1}} \left[\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right) + \left(\frac{\theta}{\theta-1}\right)^{\theta} \right]^{-1} \left[\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right) + \left(\frac{\theta}{\theta-1}\right)^{\theta-1} \right]^{\frac{\theta}{\theta-1}}
\end{aligned}$$

Therefore an increase in N_{t-1} increases the final good production. Importantly, $N_{t-1}^{\frac{1}{\theta-1}}$ is multiplicative, meaning that an expansion of total available varieties has the same effect as an improvement of the productivity shock A_t. The composition effect is more subtle. But around the steady state N_M/N_C, an increase in $N_{M,t-1}/N_{C,t-1}$ decreases Z_t.

Because the final good price indicator implies

$$\begin{aligned}
1 &= N_{M,t-1} \left(\frac{p_{M,t}}{P_t}\right)^{1-\theta} + N_{C,t-1} \left(\frac{p_{C,t}}{P_t}\right)^{1-\theta} \\
&= N_{t-1} \left(\frac{p_{M,t}}{P_t}\right)^{1-\theta} \left[\frac{N_{M,t-1}}{N_{t-1}} + \frac{N_{C,t-1}}{N_{t-1}} \left(\frac{\theta}{\theta-1}\right)^{\theta-1} \right]
\end{aligned}$$

we find

$$\frac{p_{M,t}}{P_t} = N_{t-1}^{\frac{1}{\theta-1}} \left[\frac{\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right)}{\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right)+1} + \frac{1}{\left(\frac{N_{M,t-1}}{N_{C,t-1}}\right)+1} \left(\frac{\theta}{\theta-1}\right)^{\theta-1} \right]^{\frac{1}{\theta-1}}$$

The monopoly rent in the goods producing sector $\pi_{Z,t}$ is

$$\begin{aligned}\pi_{Z,t} &= \left[\frac{p_{M,t}}{P_t} - \frac{\theta-1}{\theta}\frac{p_{M,t}}{P_t}\right] z_{M,t} \\ &= \frac{1}{\theta}\frac{p_{M,t}}{P_t} A_t k_{M,t}^{\alpha} l_{M,t}^{1-\alpha} \\ &= \frac{1}{\theta}\frac{Z_t}{N_{t-1}}\left[\frac{N_{M,t-1}}{N_{C,t-1}}+1\right]\left[\frac{N_{M,t-1}}{N_{C,t-1}}+\left(\frac{\theta}{\theta-1}\right)^{\theta-1}\right]^{-1}. \end{aligned} \tag{34}$$

If $N_{M,t-1}/N_{C,t-1} = 0$, we have $\pi_{Z,t} = Z_t/\theta N_{t-1}$, which is a familiar expression in the standard monopolistic competition model. An increase in $N_{M,t-1}/N_{C,t-1}$ increases $\pi_{Z,t}$ because it is better for each monopolistic firm to have other intermediate products produced by monopolistic firms than by competitive firms. Finally, an increase in N_{t-1} decreases $\pi_{Z,t}$ because expanding available varieties dilutes per-variety demand.

Table 1—Parameter Values

Parameter	*Description*	value	source of information
β	Time discounting	.995	standard value
δ_K	Capital depreciation rate	.025	standard value
δ_N	Death shock	.01	product exit
σ	Maturing shock	.045	R&D depreciation
θ	Elasticity of substitution	3	empirical research
α	Capital share	.31	s.s. labor share
φ	Marginal leisure utility	3.66	s.s. labor hours
ν	Elasticity of R&D input	.17	s.s. R&D share
σ_A	Volatility of innovation in A_t	.0069	GDP and R&D
ρ_A	Persistence of A_t	.9903	GDP and R&D
σ_S	Volatility of innovation in S_t	.0097	GDP and R&D
ρ_S	Persistence of S_t	.9722	GDP and R&D

Table 2—Business Cycle Statistics

	2-32 quarters			33-160 quarters		
	U.S. data	model	constant-S	U.S. data	model	constant-S
GDP volatility	1.65	1.23	1.20	3.12	2.26	2.22
Consumption volatility	.85	.58	.56	2.32	1.56	1.55
Investment volatility	4.73	3.20	3.09	7.24	4.94	4.76
Hours volatility	1.75	.54	.52	2.71	.79	.77
R&D volatility	1.79	1.86	.80	9.78	4.21	2.34
Consumption comovement	.77	.97	.98	.96	.92	.93
Investment comovement	.79	.99	.99	.69	.94	.94
Hours comovement	.87	.98	.99	.75	.82	.81
R&D comovement	.47	.33	.99	.45	.37	.96

Table 3—Variance and Correlation of Labor Share

	U.S. Data	model
LS volatility	.47	.045
LS persistence	.73	.69
LS correlation with y_t	-.16	-.16
LS correlation with SR	-.48	-.16

Table 4—Phase Shift of Labor Share

	Correlation of y_t with								
	ls_{t-4}	ls_{t-3}	ls_{t-2}	ls_{t-1}	ls_t	ls_{t+1}	ls_{t+2}	ls_{t+3}	ls_{t+4}
U.S. Data	-.23	-.26	-.28	-.26	-.16	.11	.33	.48	.55
model	-.01	-.04	-.07	-.11	-.16	-.10	-.05	.00	.03

Table 5—Asset Pricing Implication

coefficient of RRA	$E[r_N - f_f]$	$SD[r_N]$	$E[r_K - f_f]$	$SD[r_K]$	$E[r_f]$	$SD[r_f]$
			model			
1	.02	1.68	.001	.17	2.01	.16
10	.23	1.67	.005	.17	1.84	.16
25	.62	1.67	.007	.17	1.60	.16
50	1.23	1.67	-.004	.17	1.16	.16
			constant-S			
1	.01	1.36	.001	.15	1.99	.14
10	.14	1.36	.006	.15	1.88	.14
25	.36	1.36	.011	.15	1.68	.14
50	.72	1.36	.012	.15	1.35	.14

Table 6—Equity Premium and Risk-free Rate

	U.S. data	model	constant-S	standard RBC
Equity premium	5.57	1.05	.65	.03
Equity return volatility	14.36	1.46	1.26	.12
Risk-free rate mean	.98	1.16	1.35	.1.66
Risk-free rate volatility	.65	.16	.14	.15

Table 7—Asset Pricing Implication (free entry in R&D)

coefficient of RRA	$E[r_N - f_f]$	$SD[r_N]$	$E[r_K - f_f]$	$SD[r_K]$	$E[r_f]$	$SD[r_f]$
			model			
1	.003	1.42	.001	.06	2.00	.16
10	.06	1.42	.003	.06	1.92	.15
25	.22	1.41	.008	.06	1.81	.15
50	.47	1.42	.013	.06	1.61	.15
			constant-S			
1	.005	1.41	.006	.06	2.00	.15
10	.10	1.41	.004	.06	1.93	.15
25	.24	1.42	.009	.06	1.81	.15
50	.49	1.42	.014	.06	1.61	.15

Table 8—Asset Pricing Implication (monopolistic alone)

model						
coefficient of RRA	$E[r_N - f_f]$	$SD[r_N]$	$E[r_K - f_f]$	$SD[r_K]$	$E[r_f]$	$SD[r_f]$
1	.02	1.09	.001	.15	2.00	.14
10	.09	1.09	.005	.15	1.90	.14
25	.22	1.09	.009	.15	1.74	.14
50	.45	1.09	.012	.15	1.47	.14
constant-S						
coefficient of RRA	$E[r_N - f_f]$	$SD[r_N]$	$E[r_K - f_f]$	$SD[r_K]$	$E[r_f]$	$SD[r_f]$
1	.02	1.09	.001	.15	2.00	.14
10	.09	1.09	.005	.15	1.90	.14
25	.23	1.09	.009	.15	1.74	.14
50	.46	1.08	.014	.15	1.48	.14

Table 9—Asset Pricing Implication (IES=.5)

model						
coefficient of RRA	$E[r_N - f_f]$	$SD[r_N]$	$E[r_K - f_f]$	$SD[r_K]$	$E[r_f]$	$SD[r_f]$
1	.02	1.30	.001	.19	2.01	.18
10	.18	1.31	.005	.19	1.84	.18
25	.41	1.29	.008	.19	1.60	.18
50	.87	1.29	.011	.18	1.24	.17
constant-S						
coefficient of RRA	$E[r_N - f_f]$	$SD[r_N]$	$E[r_K - f_f]$	$SD[r_K]$	$E[r_f]$	$SD[r_f]$
1	.02	1.20	.001	.15	1.99	.14
10	.13	1.21	.005	.15	1.85	.14
25	.33	1.20	.010	.14	1.61	.14
50	.67	1.20	.013	.14	1.22	.14

Table 10—Asset Pricing Implication (IES=1.5)

model						
coefficient of RRA	$E[r_N - f_f]$	$SD[r_N]$	$E[r_K - f_f]$	$SD[r_K]$	$E[r_f]$	$SD[r_f]$
1	.02	2.06	.002	.19	2.00	.16
10	.30	2.05	.008	.18	1.89	.16
25	.78	2.05	.015	.18	1.71	.16
50	1.64	2.03	.010	.19	1.39	.16
constant-S						
coefficient of RRA	$E[r_N - f_f]$	$SD[r_N]$	$E[r_K - f_f]$	$SD[r_K]$	$E[r_f]$	$SD[r_f]$
1	.02	1.65	.002	.17	1.99	.15
10	.17	1.64	.01	.17	1.91	.15
25	.43	1.64	.02	.17	1.76	.15
50	.88	1.62	.02	.17	1.49	.14

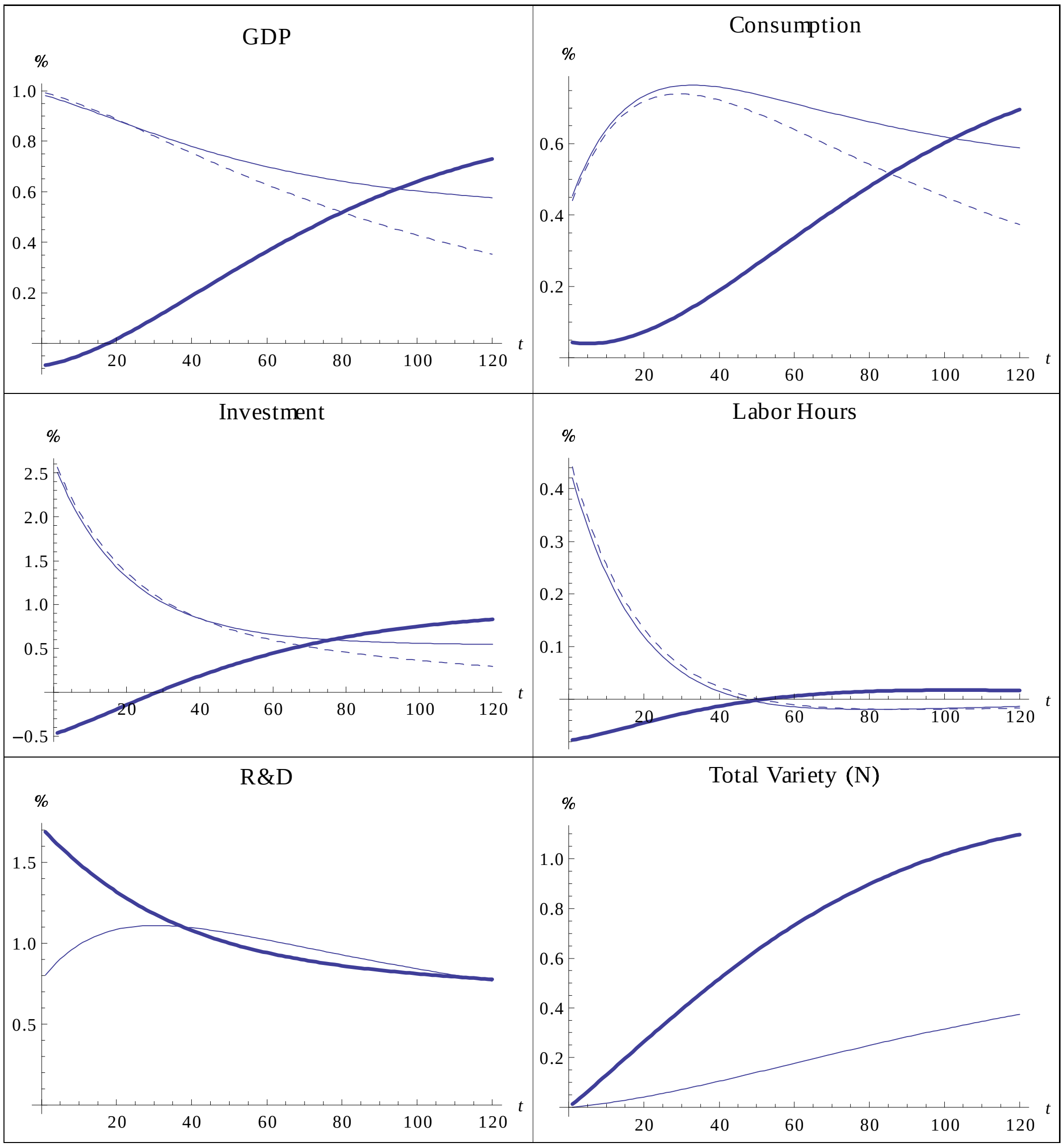

Figure 1: Percentage deviation from the steady state value. Thick lines are reponses to a one-standard deviation positive innovation in the R&D sector specific productivity shock. Thin lines are responses to a one-standard deviation positive innovation in the goods producing sector specific productivity shock. Dotted lines are responses to a one-standard deviation positive innovation in the goods producing sector specific productivity shock in the standard RBC model.

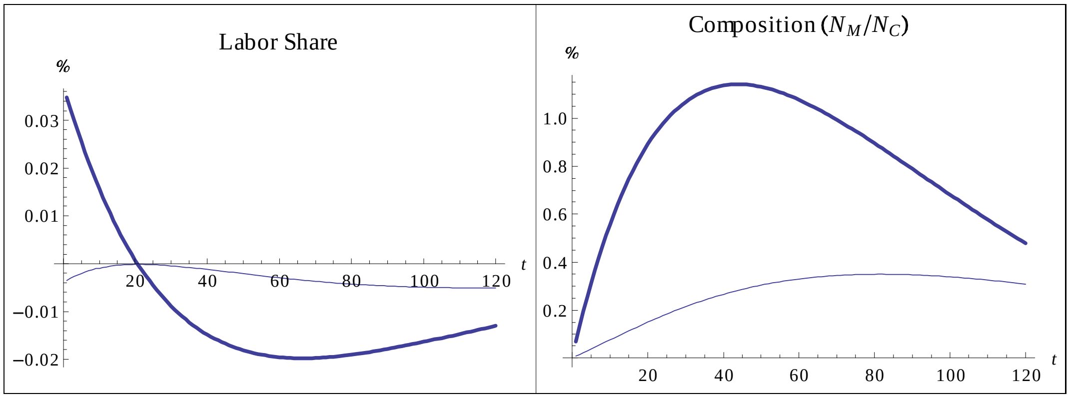

Figure 2: Percentage deviation from the steady state value. Thick lines are reponses to a one-standard deviation positive innovation in the R&D sector specific productivity shock. Thin lines are responses to a one-standard deviation positive innovation in the goods producing sector specific productivity shock.

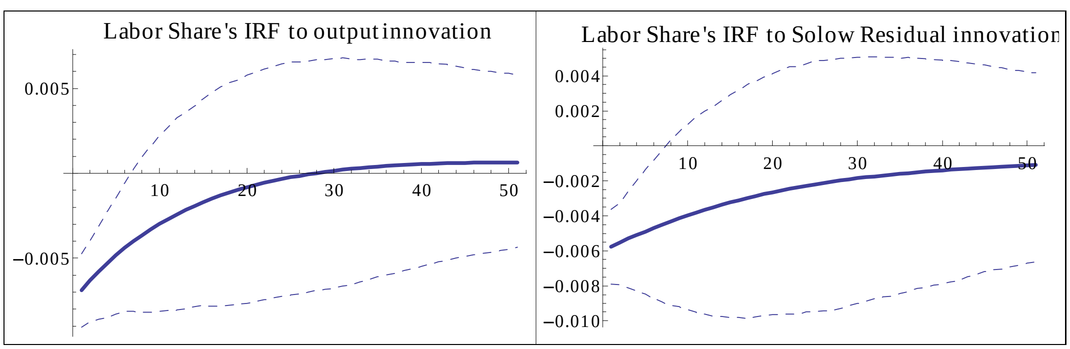

Figure 3: Impulse response functions to a one-standard deviation orthogonal output/productivity shock in a bi-variate VAR system applied to artificial data from the model. Thick lines are average among simulations. Dotted lines represent 84% and 16% quantiles.

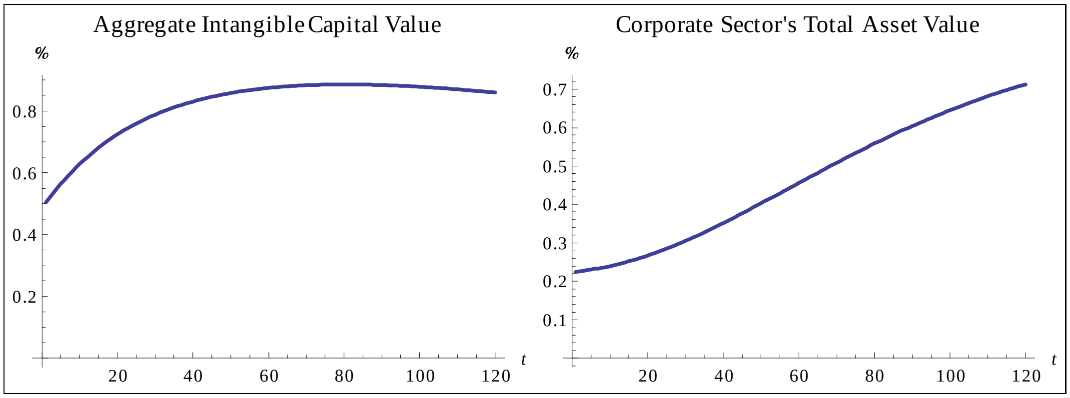

Figure 4: Percentage deviation from the steady state value. Thick lines are reponses to a one-standard deviation positive innovation in the R&D sector specific productivity shock.

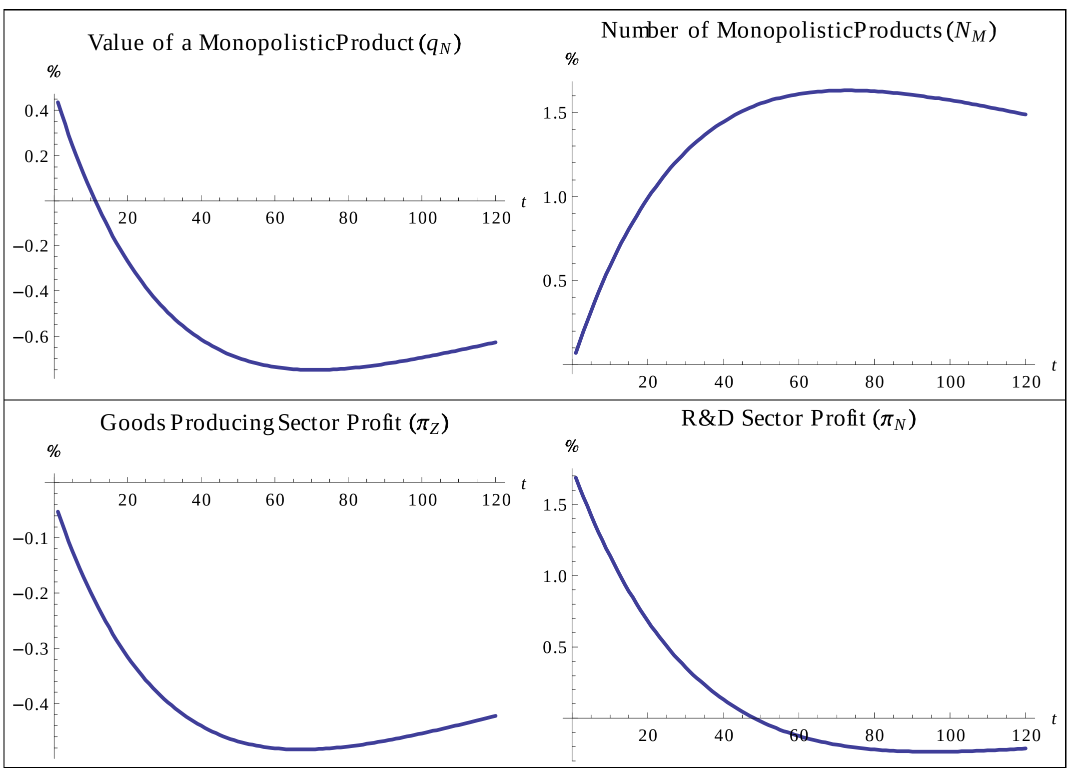

Figure 5: Percentage deviation from the steady state value. Thick lines are reponses to a one-standard deviation positive innovation in the R&D sector specific productivity shock.

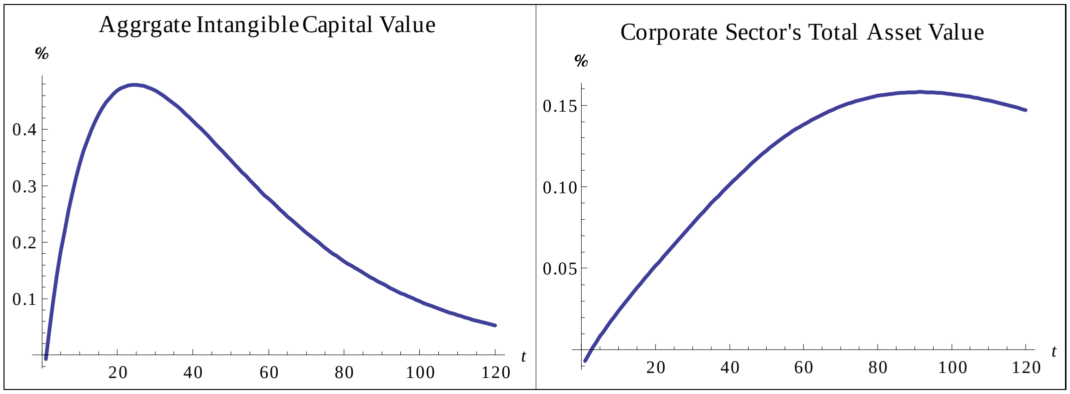

Figure 6: Percentage deviation from the steady state value in a model in which any firm can conduct R&D. Thick lines are reponses to a one-standard deviation positive innovation in the R&D sector specific productivity shock.

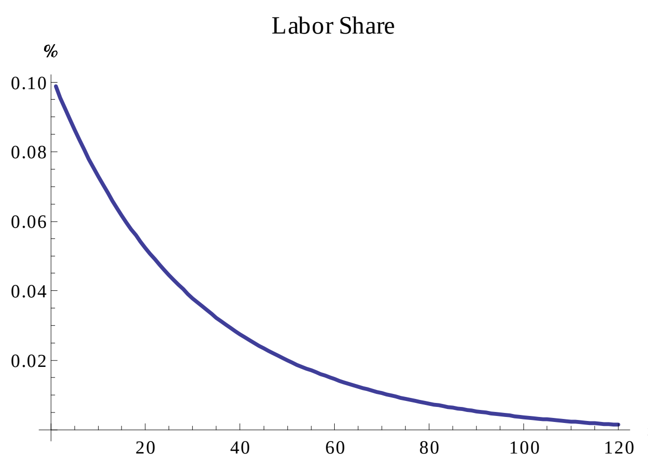

Figure 7: Percentage deviation from the steady state value in a model in which any firm can conduct R&D. Thick lines are reponses to a one-standard deviation positive innovation in the R&D sector specific productivity shock.

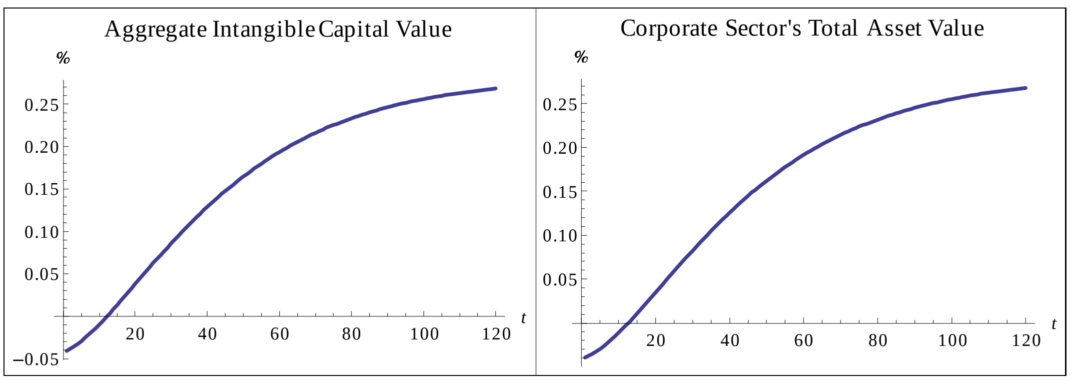

Figure 8: Percentage deviation from the steady state value in a model in which all the firms are innovative-monopolistic. Thick lines are reponses to a one-standard deviation positive innovation in the R&D sector specific productivity shock.

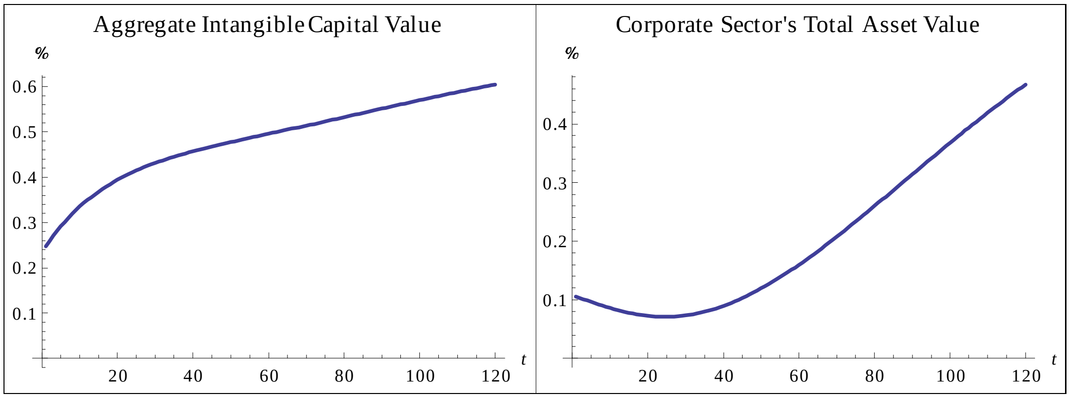

Figure 9: Percentage deviation from the steady state value in a model in which the intertemporal elasticity of substitution is .5. Thick lines are reponses to a one-standard deviation positive innovation in the R&D sector specific productivity shock.

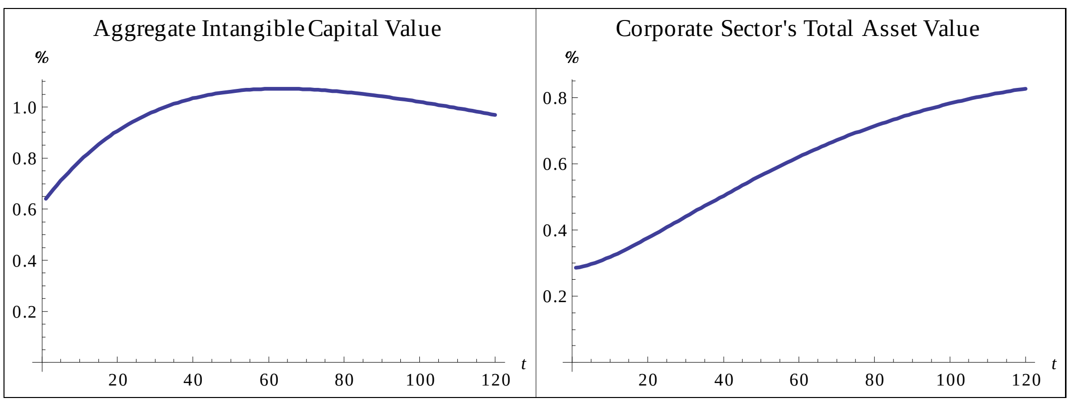

Figure 10: Percentage deviation from the steady state value in a model in which the intertemporal elasticity of substitution is 1.5. Thick lines are reponses to a one-standard deviation positive innovation in the R&D sector specific productivity shock.