

The Persistence of Differences in Productivity, Wages, Skill Mix and Profits Across Firms*

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Abstract

This paper constructs a dynamic assignment model to explain observed persistent differences in productivity, wages, skill mix and profits across firms. Large organization capital attracts skilled workers, who can create better organization in future. This positive feedback brings about persistent differences in these variables. When organization capital is unobservable, assignment occurs between skill and a firm's reputation. It is shown that a firm's reputation and its real capability interactively influence persistence. Our theory predicts that the heterogeneity of skill and the noisiness of information increase persistence. We structurally estimate the parameters and calibrate the model. Our calibration results show that if there were no assignment problem, the relative advantages of a firm would disappear in 4-6 years.

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1 Introduction

Why are some firms persistently more productive than others? Evidence repeatedly reveals that there are substantial and persistent differences in productivity across plants and firms [e.g. Baily, Hulten and Campbell (1992) and Haltiwanger, Lane, and Spletzer (2000)]. Apparently productivity is not the only variable that exhibits persistence. Evidence shows that skill compositions and the wage payment are also persistently different across firms [e.g. Haltiwanger, Lane, and Spletzer (2000)] and that differences in firms' profits are persistent, too [e.g. Megna and Mueller (1991)].

The coexistence of persistent differences in these variables is not coincidence. It is found that productive firms employ highly skilled workers and pay high wages. [e.g. Haltiwanger, Lane, and Spletzer (1999)] and that skill and the market value of a firm are positively correlated [Abowd et al. (2004)]. These findings suggest that an interaction among productivity, human capital and profits is a key to understand the persistence of productivity.

Another well-known puzzling aspect of productivity difference exists. Table 1 depicts time series analysis of relative productivity (= labor productivity relative to industry and year average). It shows that after controlling the first lag of relative productivity, the higher order lags still influence current productivity difference. Hence, a desirable theory for productivity difference must explain not only why it persists across firms, but also why the first order lag fails to summarize an influence from its past.

This paper aims to construct a model that can provide a unified explanation for these findings. It constructs a recursive positive assortative assignment model between skill of workers and firm specific resources, which we call a firm's organization

The dependent variable is $\ln y_t^r$

constant	$\ln y_{t-1}^r$	$\ln y_{t-2}^r$	$\ln y_{t-3}^r$	$\ln y_{t-4}^r$
.004*	.572***	.219***	.049***	.050***
(.002)	(.009)	(.010)	(.010)	(.009)
# of observations	15000		Adjusted-R ²	0.696

Table 1: AR 4

The variable, $\ln y_t^r$, is labor productivity relative to industry and year average. The variables are constructed from an industry annual dataset in COMPUSTAT. The regression uses a manufacturing sector from 1975 to 2004. Data construction is explained in Appendix. The standard errors are reported in parentheses. * means significant at 10 percent level. *** means significant at 1 percent level.

capital¹, and shows how the assignment model explains the previous evidence.

A basic logic is explained as follows. The firm is characterized by its organization capital. Workers are characterized by their own skill. As skill is assumed to be complementary to organization capital, skilled workers are assigned to large organization capital. On the other hand, skill is needed for the accumulation of organization capital. Hence, the economy exhibits a positive feedback mechanism: more organization capital attracts more skilled workers, which in turn generate more organization capital. We investigate how much this positive feedback mechanism raises the persistence of a firm's organization capital. As productivity, wages, skill composition and profits of a firm are shown to be a strictly increasing function of the organization capital, the model can explain not only the persistence of these variables, but also positive correlations among productivity, skill, wages and profits.

Although this simple logic can explain why productivity persistently differs across

¹More specifically, we define organization capital by all types of intangible assets embodied in an organization. It might consist of organizational structure, daily practice, routine, information held by an organization, corporate culture, reputation and so on. A firm cannot buy these assets from a market and it can jointly produce them with output.

firms, it cannot explain why higher order lags matter. In order to explain this evidence, we need to extend the model to that with unobserved organization capital. When organization capital is not observable, assignment occurs between the quality of workers and a belief on organization capital, which is interpreted as a firm's reputation. If a firm has a good reputation, it attracts better workers who can construct a better organization in future. As a current reputation, which can influence organization capital at the next period, is constructed from past observations, the higher order lags can influence future organization capital by changing the current reputation.

The theory predicts that a rise in the heterogeneity of skill increases the persistence of organization capital. When the variance of skill is large, the top organization can receive the most advantages because it is the firm that can attract the best workers. Hence, the larger the variance of skill is, the longer the top organization enjoys the relative advantages. It is shown that when there are no idiosyncratic shocks, every firm stays the same position forever and the relative advantages (disadvantages) of a firm never disappear.

The extension to unobserved organization capital provides the model with further theoretical and empirical advantages. Theoretically, this extension allows us to analyze a dynamic interaction between a firm's real capacity and reputation. On the one hand, as assignment occurs between a firm's reputation and skill, a good reputation attracts better workers and establishes better organization in future. On the other hand, if a firm has a productive organization, its current performance is likely to be good, which generates a good reputation in future. Hence, we can analyze how this dynamic positive feedback between real capacity and reputation influences the persistence of productivity, skill mix, wage payments and profits.

In particular, we examine how the speed of learning organization capital influences persistence. When output is a noisy signal to predict the level of organization capital, managers cannot rely on the information very much and, therefore, put more

weight on their prior beliefs. Hence, as their beliefs do not change much, they are more persistent. It is shown that the persistence of organization capital positively depends not only on the ratio of the standard deviation of skill to that of idiosyncratic shocks, but also on the noisiness of information. In particular, when the output has no predictive power on organization capital, their beliefs never change. As real organization capital exhibits the reversion to a constant belief, the belief serves as a firm specific fixed effect in a regression. In other word, this paper puts an interpretation on a fixed effect that appears in data.

The extension to unobserved organization capital has another advantage. It allows us to distinguish the effect of assignment on persistence from others. As the assignment is based on reputation, which can be generated from higher than the first order lags, the influence of higher than the first order lags provide us information about the effect of assignment on persistence. Exploiting this intuition, we structurally estimate the parameters of the model and separately identify the parameters of technological persistence, the assignment effect on the persistence and the accuracy of information to predict organization capital. We investigate a manufacturing sector from 1975 to 2004 using an industry annual dataset in COMPUSTAT. Our regression analysis reveals that the observed labor productivity is quite informative to infer fundamental capacity of firms. It also shows that the initial prior on the labor productivity has long run impacts on current productivity, though the effect declines over time.

The estimated parameters are all significant and coincide with theoretical prediction. Hence, data fairly supports the prediction of the model. Using the estimated parameters, we calibrate the correlation between current relative productivity (= a firm's labor productivity relative to industry and year average) and the lagged relative productivity. The calibrated correlation fits data quite well. Using our model, we conduct a counterfactual experiment. It shows that if there were no skill difference across workers, and, therefore, if there were no assignment problem, the

relative advantages (disadvantages) of a firm would almost disappear in 4-6 years. The effect of the assignment on the persistence of productivity differences is huge.

It has been long recognized that an individual firm possesses its particular resources [e.g., Kaldor (1934), Robinson (1934) and Lucas (1978)]. As a source of specific resources, many economists consider that a firm accumulates a firm specific knowledge through experience [e.g., Penrose (1959) and Rosen (1972)]. Prescott and Visscher (1980) calls the accumulated specific knowledge a firm's organization capital. Recently, interests in organization capital reemerge probably due to the coming of information ages. Jovanovic and Rousseau (2001), Atkeson and Kehoe (2005) and Samaniego (2006) quantify the impacts of organization capital on macro economy and Faria (2004) explains observed merger wages by an assignment model between organization capital and skill. However, no paper addresses a question: why some firms succeeded to accumulate its organization capital, while others not. Without answering this question, we cannot fully explain reasons for persistent differences in productivity. This is the main theme of this paper.

Positive assortative assignment models also have a long history. Becker (1973) originally derives a condition for a positive assortative matching in a marital market and Sattinger (1979) analyzes a positive assortative assignment equilibrium between physical capital and skill. More recently, many economists rediscover the importance of the assignment models [e.g. Kremer (1993), Costell and Loury (2004) and Shimer (2005)]. However, all papers assume the exogenous distributions of two assigned variables. By endogenizing the distribution of organization capital, this paper analyzes an interaction between assignment and the dynamics of distribution.

A key assumption behind the positive assortative matching is the complementarity between organization capital and skill. Evidence for supporting this assumption also exists. Chandler (1977) reports historical evidence that the development of administrative hierarchy is essential to monitor and coordinate resources in the modern business firms. Chandler (1977) demonstrates that this organizational structure

demands skilled workers to process information. Recent evidence by Bresnahan, Brynjolfsson and Hitt (2002) shows that reorganization accommodated with IT investment demands more skilled workers. For example, the use of flexible machinery often requires that workers have greater discretion, which in turn requires data analysis skills and general problem-solving ability.

A process of learning the level of organization capital is another important feature of our model. Similar to Jovanovic (1982), a firm gradually learns its own productive capacity. However, different from Jovanovic (1982), their productive capacity itself is changing as a result of active investment and uncertainty arising from investment. This feature of the dynamics of productive capacity is similar to Ericson and Pakes (1995), though they do not have any learning process on it. Hence, our model can be seen as a hybrid of passive learning by Jovanovic (1982) and active investment in research by Ericson and Pakes (1995).

Finally, several models generate an equilibrium distribution of wages [e.g. Burdett and Mortensen (1998)] and that of productivity [e.g. Jovanovic (1998), and Eeckhout and Jovanovic (2002)]. These papers show how ex ante homogeneous agents can generate ex post heterogeneity. Different from theirs, this paper assumes the ex ante heterogeneity of skill and generates the distribution of organization capital, productivity, wage and profits. An advantage of our approach is that our stationary distribution is globally stable. Hence, our distribution is robust against any disturbances.

The paper is organized as follows. The next section sets up a recursive positive assortative assignment model when the organization capital is perfectly observable. It clarifies a mechanism why skill differences enhance the persistence in this model. Section 3 extends the model to imperfect observation. Organization capital is not observed but must be inferred from observed output. We show that the model can generate rich dynamics that is consistent with evidence. Section 4 discusses how to identify the parameters from data and Section 5 reports our empirical studies. It

shows that the data supports the model well. We also calibrate our model to examine the importance of the assignment on the persistence. Finally, section 6 concludes.

2 A Dynamic Assignment Model when Organization Capital is Observable

This section constructs a recursive positive assortative assignment equilibrium when organization capital is observable. This section provides a clear intuition for why the assignment brings about the persistence of variables. Suppose that firms at date t are characterized by its organization capital, k_t^o , and workers are characterized by their quality of skill, q_t .

A firm employs one unit of workers and produces y_t . A firm's production function is a function of its organization capital and the quality of skill:

$$y_t = A (k_t^o)^\alpha q_t^\psi, \alpha > 0, \psi > 0$$

where A, α, ψ are constant parameters. A skilled worker is needed not only for the production of outputs, but also for the production of organization capital at the next period:

$$k_{t+1}^o = B (k_t^o)^\phi q_t^\gamma e^{\varepsilon_t}, 0 \leq \phi < 1, \gamma > 0$$

where B, ϕ, γ are constant parameters and ε_t are a random variable, which is normally distributed with a mean $-\frac{\sigma_\varepsilon^2}{2}$ and a standard deviation σ_ε . The parameter ϕ measures technological persistence of organization capital. A part of organization capital might be depreciated. We assume that ϕ fraction of organization capital can be carried over to the next period. Note that we allow $\phi = 0$. It means that we allow no technological persistence. It is shown later that the persistence of organization capital can endogenously appear without any technological persistence.

Each firm faces competitive wages for each quality, $w(\ln q_t; \mu_{kt}, \sigma_{kt})$, where μ_{kt} is a mean of $\ln k_t^o$ and σ_{kt} is a standard deviation of $\ln k_t^o$. The wage function implies that

the wage is a function of not only the quality of skill, but also the aggregate variables, μ_{kt} and σ_{kt} . Assume that each firm expects that economy wide state variables μ_{kt} and σ_{kt} change by $\mu_{kt+1} = f(\mu_{kt}, \sigma_{kt})$ and $\sigma_{kt+1} = g(\mu_{kt}, \sigma_{kt})$. Then a firm's profit maximization problem is written as

$$\begin{aligned} V(\ln k_t^o; \mu_{kt}, \sigma_{kt}) &= \max_{\ln q_t} \left\{ \begin{aligned} &\exp[\ln A + \alpha \ln k_t^o + \psi \ln q_t] - w(\ln q_t; \mu_{kt}, \sigma_{kt}) \\ &+ \beta \int V(\ln k_{t+1}^o; \mu_{kt+1}, \sigma_{kt+1}) d\Gamma_\varepsilon(\varepsilon_t) \end{aligned} \right\} (1) \\ \text{s.t. } \ln k_{t+1}^o &= \ln B + \phi \ln k_t^o + \gamma \ln q_t + \varepsilon_t \\ \mu_{kt+1} &= f(\mu_{kt}, \sigma_{kt}), \sigma_{kt+1} = g(\mu_{kt}, \sigma_{kt}) \end{aligned}$$

where $\beta \in (0, 1)$ is a discounting factor. We express a production function, a transition equation, a wage function and a value function as functions of $\ln k_t^o$ and $\ln q_t$ to simplify our algebra.

Suppose that $\ln k_t^o$ is normally distributed with a mean μ_{kt} and a standard deviation σ_{kt} at date t and $\ln q_t$ is normally distributed with a mean μ_q and a standard deviation σ_q at any date. For the sake of simplicity, I assume that firms and workers have reservation values of 0 and that the population of firms has the same size as the population of workers. Hence nobody chooses the outside option and every agent can find a partner. Given these assumptions, we can ignore the decisions workers and firms take when entering the market. Hence, these assumptions make it possible to focus the assignment problem.

I want to focus a positive assortative equilibrium. It means that a top x percent of $\ln k_t^o$ is assigned to a top x percent of $\ln q_t$ for any x . Let $\chi(\ln k_t^o; \mu_{kt}, \sigma_{kt})$ denote a policy function derived from equation (1) and $\Phi(\cdot)$ denote a standard normal distribution. As $\frac{\ln k_t^o - \mu_{kt}}{\sigma_{kt}}$ and $\frac{\ln q_t - \mu_q}{\sigma_q}$ are distributed with a standard normal distribution, a positive assortative equilibrium implies

$$1 - \Phi\left(\frac{\ln k_t^o - \mu_{kt}}{\sigma_{kt}}\right) = 1 - \Phi\left(\frac{\chi(k_t^o; \mu_{kt}, \sigma_{kt}^2) - \mu_q}{\sigma_q}\right), \quad \forall \ln k_t^o \quad (2)$$

It means that the policy function must satisfy

$$\chi(\ln k_t^o; \mu_{kt}, \sigma_{kt}^2) = \frac{\sigma_q}{\sigma_{kt}} [\ln k_t^o - \mu_{kt}] + \mu_q, \quad \forall \ln k_t^o$$

As you can see, the high quality of skill is assigned to large organization capital. Hence, when the dynamics of a firm's organization capital show persistence, the skill level of a worker the firm employs persists too. Given this policy function, the dynamics of organization capital can be written as

$$\ln k_{t+1}^o = \ln B + \phi \ln k_t^o + \gamma \left[\frac{\sigma_q}{\sigma_{kt}} (\ln k_t^o - \mu_{kt}) + \mu_q \right] + \varepsilon_t. \quad (3)$$

Hence, $\ln k_{t+1}$ is also normally distributed and the normality of distribution can be maintained at the next period. The dynamics of μ_{kt} and σ_{kt}^2 can be derived from equation (3):

$$\begin{aligned} \mu_{kt+1} &= \ln B + \phi \mu_{kt} + \gamma \mu_q - \frac{\sigma_\varepsilon^2}{2}, \\ \sigma_{kt+1} &= \sqrt{(\phi \sigma_{kt} + \sigma_q \gamma)^2 + \sigma_\varepsilon^2}. \end{aligned}$$

Now we can define a recursive positive assortative equilibrium as follows.

Definition 1 *A recursive positive assortative equilibrium with observed organization capital consists of $\chi(\ln k_t^o; \mu_{kt}, \sigma_{kt})$, $V(\ln k_t^o; \mu_{kt}, \sigma_{kt})$, $w(\ln q_t; \mu_{kt}, \sigma_{kt})$, $f(\mu_{kt}, \sigma_{kt})$ and $g(\mu_{kt}, \sigma_{kt})$ that satisfy the following conditions.*

1. *An individual firm solves its maximization problem (1):*
2. *A labor market is cleared:*

$$\chi(\ln k_t^o; \mu_{kt}, \sigma_{kt}^2) = \frac{\sigma_q}{\sigma_{kt}} [\ln k_t^o - \mu_{kt}] + \mu_q.$$

3. *Expectation is rational:*

$$f(\mu_{kt}, \sigma_{kt}) = \ln B + \phi \mu_{kt} + \gamma \mu_q - \frac{\sigma_\varepsilon^2}{2} \quad (4)$$

$$g(\mu_{kt}, \sigma_{kt}) = \sqrt{(\phi \sigma_{kt} + \sigma_q \gamma)^2 + \sigma_\varepsilon^2} \quad (5)$$

In order to show the existence of the equilibrium, we need to find both a value function and a wage function that are consistent with the definition of the equilibrium.

Theorem 2 *A recursive positive assortative equilibrium with observed organization capital exists and this equilibrium is supported by a strictly increasing value function and a strictly increasing wage function. (Closed form solution of the value function and the wage function is available in Appendix).*

As you can see, the firm's profits and the wage payments are positively correlated with $\ln k_t^o$. Hence, if the dynamics of $\ln k_t^o$ exhibit persistence, profits and wage payments also shows the persistence. Hence, the rest of this paper focuses the dynamics of $\ln k_t^o$.

Dynamics: The dynamics of this economy are characterized by three equations.

$$\begin{aligned}\mu_{kt+1} &= \ln B + \phi\mu_{kt} + \gamma\mu_q - \frac{\sigma_\varepsilon^2}{2} \\ \sigma_{kt+1} &= \sqrt{(\phi\sigma_{kt} + \sigma_q\gamma)^2 + \sigma_\varepsilon^2}\end{aligned}$$

and

$$\ln k_{t+1}^o - \mu_{kt+1} = \left(\phi + \gamma \frac{\sigma_q}{\sigma_{kt}} \right) (\ln k_t^o - \mu_{kt}) + \varepsilon_t^*$$

where $\varepsilon_t^* = \varepsilon_t + \frac{\sigma_\varepsilon^2}{2}$ is normally distributed with a mean 0 and a standard deviation σ_ε .

Third equation means that when $\ln k_t^o$ is larger than its industry mean, μ_{kt} , $\phi + \gamma \frac{\sigma_q}{\sigma_{kt}}$ fraction of this relative advantage is translated into the next period. The parameter, ϕ , is assumed persistence and the second term, $\gamma \frac{\sigma_q}{\sigma_{kt}}$ is a result of a positive assortative matching. When the ratio of standard deviation of skill to that of organization capital is large, organization capital is more persistent. When the ratio is larger, large organization capital brings larger advantages because it is the top organization that attracts most talented workers, who bring more organization capital. Therefore, relative advantages persist longer.

This intuition can be more strictly analyzed. First, we show that the distribution of $\ln k_t^o$ converges to a stationary distribution.

Proposition 3 *The mean, μ_{kt} , and the standard deviation, σ_{kt} , of $\ln k_t^o$ converges to stationary points, $\mu_{k\infty}$ and $\sigma_{k\infty}$, respectively. The two variables, $\mu_{k\infty}$ and $\sigma_{k\infty}$ are solved as*

$$\mu_{k\infty} = \frac{\ln B + \gamma\mu_q - \frac{\sigma_\varepsilon^2}{2}}{1 - \phi}, \quad (6)$$

$$\sigma_{k\infty} = \frac{\gamma\phi\sigma_q + \sqrt{(\gamma\sigma_q)^2 + (1 - \phi^2)\sigma_\varepsilon^2}}{(1 - \phi^2)}. \quad (7)$$

Moreover, when the distribution converges to the stationary distribution, the dynamics of organization capital can be written as a following simple AR(1) process:

$$\ln k_{t+1}^o - \mu_{k\infty} = \left(\phi + \gamma \frac{\sigma_q}{\sigma_{k\infty}} \right) (\ln k_t^o - \mu_{k\infty}) + \varepsilon_t^* \quad (8)$$

The proposition says that there is globally stable stationary points of $\mu_{k\infty}$ and $\sigma_{k\infty}$. As the stationary distribution is unique and globally stable, a real economy is expected to locate near the stationary distribution in the long run. Hence, the following discussions in this section examine the dynamics of equation (8). As information on the persistence is summarized by $\phi + \gamma \frac{\sigma_q}{\sigma_{k\infty}}$, we call it a persistence parameter below.

Persistence when $\sigma_\varepsilon = 0$: In order to have a clear intuition for persistence, we first analyze a deterministic model.

Proposition 4 *Suppose $\sigma_\varepsilon = 0$. Then the stationary distribution still exists*

$$\begin{aligned} \mu_{k\infty} &= \frac{\ln B + \gamma\mu_q}{1 - \phi}, \\ \sigma_{k\infty} &= \frac{\gamma\sigma_q}{1 - \phi}, \end{aligned}$$

and organization capital never changes its value.

$$\ln k_{t+1}^o = \ln k_t^o$$

The proposition says that if there is no shock, the ranking of organization capital never changes. When there is no idiosyncratic shocks, top organization always attract the best worker, which, in turn, creates the best organization routine at the next period. Hence, they never change its ranking and keep exactly the same organization capital in the long run.

Note that $\sigma_{k\infty}$ is not 0. Hence, different firms continue to have different organization capital, and productivity, different wages, skill mix and profits persist forever. When σ_{kt} becomes small, $\frac{\sigma_q}{\sigma_{kt}}$ becomes large. Therefore, the relative advantages of being a top organization becomes large and pushes σ_{kt} large again. In the end, $\sigma_{k\infty}$ never converges to 0.

Persistence when $\sigma_\varepsilon > 0$: When we introduce idiosyncratic shocks on the accumulation of organization capital, the reversion to the mean starts taking place. Idiosyncratic shocks introduce the possibility of changes in ranking. If a firm receives a really good shock, the firm climbs up the rank, which makes it possible to attract higher quality of workers. It means that top organization cannot be always top. There is always a possibility of moving toward the mean of distribution.

In order to understand this mechanism, note that equation (7) implies the persistence parameter can be written as a strictly positive function of $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$.

$$\phi + \gamma \frac{\sigma_q}{\sigma_{k\infty}} = \phi + \frac{(1 - \phi^2)}{\phi + \sqrt{1 + (1 - \phi^2) \left(\frac{\gamma\sigma_q}{\sigma_\varepsilon}\right)^{-2}}} \in (\phi, 1), \quad (9)$$

$$\frac{d\left(\phi + \gamma \frac{\sigma_q}{\sigma_{k\infty}}\right)}{d\frac{\gamma\sigma_q}{\sigma_\varepsilon}} > 0, \quad \lim_{\frac{\gamma\sigma_q}{\sigma_\varepsilon} \rightarrow \infty} \left(\phi + \gamma \frac{\sigma_q}{\sigma_{k\infty}}\right) = 1, \quad \lim_{\frac{\gamma\sigma_q}{\sigma_\varepsilon} \rightarrow 0} \left(\phi + \gamma \frac{\sigma_q}{\sigma_{k\infty}}\right) = \phi.$$

When $\sigma_\varepsilon > 0$, $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$ is finite. Therefore, the persistence parameter is always less than 1. It means that equation (8) is covariance stationary. Hence, the dynamics exhibit a reversion to the mean and eventually initial advantages die out. In this case, it is

easy to show that

$$\rho_{kj} \equiv \frac{E(\ln k_t^o - \mu_{k\infty})(\ln k_{t-j}^o - \mu_{k\infty})}{\sigma_{k\infty}^2} = \left(\phi + \gamma \frac{\sigma_q}{\sigma_{k\infty}} \right)^j.$$

That is, the larger the persistence parameter, the larger the autocorrelation. Hence, if $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$ is large, the persistence parameter is large and the autocorrelation is large.

Note that the parameter $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$ can be interpreted as the relative strength of an assignment effect to a reshuffling effect. If $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$ is large, top organization receives the relatively large benefits from better assignment compared to the fears of changes in ranking. Hence, the dynamics exhibits longer persistence. If it is small, the opposite is true. Note also that when $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$ goes infinite, the persistence parameter converges to 1. Hence, the results in the proposition 4 can be reinterpreted: when $\sigma_\varepsilon = 0$, organization capital persists forever because the relative strength of an assignment effect is infinite.

3 A Dynamic Assignment Model when Organization Capital is not Observable

In the previous section, it is shown that while large variation of skill makes organization capital persistent. Although the previous model brings about a clear intuition for how assignment influences the persistence of variables, it might be too simple to meet data. As the dynamics of productivity in the previous section is captured by AR(1) process, once we control the lagged productivity, the higher order lags should not influence current productivity.

This section examines the same dynamic assignment model when organization capital is not observable. It is shown that the higher lagged organization capital can influence current organization capital after controlling the first lag. Furthermore, this extension allows us to analyze the impact of reputation on persistence and to estimate the parameters of the model.

We assume that k_t^o cannot be directly observed, but can be inferred from the realization of output. In order to capture this idea, we add idiosyncratic shocks on the production function:

$$y_t = e^{u_t} A (k_t^o)^\alpha q_t^\psi, \quad \alpha \geq 0, \psi \geq 0$$

where u_t is normally distributed with a mean $-\frac{\sigma_u^2}{2}$ and a variance σ_u^2 . When the firm makes an employment decisions, the output is not realized. Hence, it must make a decision based on its expectation given its belief on the level of organization capital. We assume $\ln k_t$ is normally distributed with a mean μ_{kt} and σ_{kt}^2 . Then the firm's expected output is

$$E [y_t | \mu_{kt}, \sigma_{kt}^2] = \exp \left(\ln A + \alpha \mu_{kt} + \frac{\alpha^2 \sigma_{kt}^2}{2} + \psi \ln q_t \right) \quad (10)$$

We assume that all firm share the same σ_{kt} . However, μ_{kt} differs across firms. We interpret that μ_{kt} represents a firm's reputation. In this section, we consider an assignment problem between reputation and skill. After the firm employs a worker, the output realizes. From the realized output, the firm can recognize $e^{u_t} (k_t^o)^\alpha$ and infer $\ln k_t^o$. That is, a firm knows the following signal s_t to predict $\ln k_t^o$:

$$s_t \equiv \ln k_t^o + u_t^*$$

Note that $u_t^* = \frac{1}{\alpha} \left(u_t + \frac{\sigma_u^2}{2} \right)$ is normally distributed with a mean 0 and a standard deviation, $\frac{\sigma_u}{\alpha}$. As $\mu_{kt+1} = E [\ln k_{t+1}^o | s_t, \mu_{kt}, \sigma_{kt}]$ and $\sigma_{kt+1} = \sqrt{\text{Var} [\ln k_{t+1}^o | s_t, \mu_{kt}, \sigma_{kt}]}$, the dynamics of μ_{kt} and σ_{kt+1} can be written as follows:

$$\mu_{kt+1} = \ln B + \phi [(1 - h_t) \mu_{kt} + h_t s_t] + \gamma \ln q_t - \frac{\sigma_\varepsilon^2}{2} \quad (11)$$

$$\sigma_{kt+1} = \sqrt{\phi^2 (1 - h_t) \sigma_{kt}^2 + \sigma_\varepsilon^2} \quad (12)$$

$$\text{where } h_t = \frac{\left(\frac{\alpha \sigma_{kt}}{\sigma_u} \right)^2}{1 + \left(\frac{\alpha \sigma_{kt}}{\sigma_u} \right)^2}$$

As both μ_{kt} and s_t are normally distributed, $E [\ln k_t^o | s_t, \mu_{kt}, \sigma_{kt}]$ is the weighted sum of the prior belief, μ_{kt} , and new information s_t . The variable h_t is a weight on the

new information. If $\frac{\alpha\sigma_{kt}}{\sigma_u}$ is small, the variance of the noise term is relatively larger than the prior variance of $\ln k_t^o$, the new observation does not contain much additional information. Hence, h_t is small and the firm would put smaller weight on the new information. If $\frac{\alpha\sigma_{kt}}{\sigma_u}$ is large, the variance of noise term is relatively small. Hence, rational agents have large h_t and rely more on the new information. In this way, h_t measures reliability of new information².

The μ_{kt} is assumed to be normally distributed with a mean μ_{kt}^e and a standard deviation $\sigma_{\mu t}$. Similar to the previous problem, the wage function depends not only on the quality of workers, but also the aggregate state variables, μ_{kt}^e , $\sigma_{\mu t}$ and σ_{kt}^a where σ_{kt}^a is a prevailing standard deviation of $\ln k_t$. Let $\mathbf{x}_t = (\mu_{kt}^e, \sigma_{\mu t}, \sigma_{kt}^a)^T$ denote the vector of these aggregate state variables. The wage function can be written as $w(\ln q_t : \mathbf{x}_t)$. The firm's problem can be written as

$$V^*(\mu_{kt}, \sigma_{kt} : \mathbf{x}_t) = \max_{\ln q_t} \left\{ \begin{array}{l} E[y_t | \mu_{kt}, \sigma_{kt}^2] - w(\ln q_t : \mu_{kt}^e, \mathbf{x}_t) \\ + \beta \int V^*(\mu_{kt+1}, \sigma_{kt+1} : \mathbf{x}_{t+1}) d\Gamma_s(s_t | \mu_{kt}, \sigma_{kt}) \end{array} \right\} \quad (13)$$

s.t. (10), (11), (12)

$$\mu_{kt+1}^e = f(\mathbf{x}_t), \sigma_{\mu t+1} = g(\mathbf{x}_t), \sigma_{kt+1}^a = m(\mathbf{x}_t)$$

Let $\chi(\mu_{kt}, \sigma_{kt}^2 : \mathbf{x}_t)$ denote a policy function of the problem (13). Again we consider a positive assortative assignment model between a firm's reputation and skill. Similar to the previous section, $\chi(\mu_{kt}, \sigma_{kt}^2 : \mu_{kt}^e, \sigma_{\mu t})$ must satisfy

$$\chi(\mu_{kt}, \sigma_{kt} : \mathbf{x}_t) = \frac{\sigma_q}{\sigma_{\mu t}} [\mu_{kt} - \mu_{kt}^e] + \mu_q \quad (14)$$

It shows that the quality of skill is a strictly positive function of a belief on firms' organization capital. Hence, the dynamics of the belief can be translated into the

²In facts, h_t can be also rewritten as follows.

$$h_t = 1 - \frac{E[\text{Var}[\ln k_t^o | s_t, \mu_{kt}, \sigma_{kt}]]}{\sigma_{kt}^2}$$

This equation shows that h_t would be larger if the average conditional variance becomes smaller relative to the prior variance. It measures the accuracy of information, which is previously used by Takii (2003) as the measure of prediction ability.

dynamics of skill. The dynamics of the belief can be derived from equations (11) and (14):

$$\mu_{kt+1} = \ln B + \phi [(1 - h_t) \mu_{kt} + h_t \ln k_t^o] + \gamma \left[\frac{\sigma_q}{\sigma_{\mu t}} (\mu_{kt} - \mu_{kt}^e) + \mu_q \right] - \frac{\sigma_\varepsilon^2}{2} + \phi h_t u_t^*.$$

Because μ_{kt} , $\ln k_t^o$ and u_t^* are normally distributed, μ_{kt+1} is also normally distributed. Hence, the normality of the distribution is preserved. The mean and standard deviation of the belief at the next period is easily derived:

$$\begin{aligned} \mu_{kt+1}^e &= \ln B + \phi \mu_{kt}^e + \gamma \mu_q - \frac{\sigma_\varepsilon^2}{2}, \\ \sigma_{\mu t+1} &= \sqrt{(\phi \sigma_{\mu t} + \gamma \sigma_q)^2 + \phi^2 h_t \sigma_{kt}^2}. \end{aligned}$$

Definition 5 *A recursive positive assortative equilibrium with unobserved organization capital consists of $\chi(\mu_{kt}, \sigma_{kt} : \mathbf{x}_t)$, $V(\mu_{kt}, \sigma_{kt} : \mathbf{x}_t)$, $w(q_t : \mathbf{x}_t)$, $f(\mathbf{x}_t)$, $g(\mathbf{x}_t)$ and $m(\mathbf{x}_t)$ that satisfy the following conditions.*

1. *An individual firm solves its maximization problem (13):*
2. *A labor market is cleared:*

$$\chi(\mu_{kt}, \sigma_{kt} : \mathbf{x}_t) = \frac{\sigma_q}{\sigma_{\mu t}} [\mu_{kt} - \mu_{kt}^e] + \mu_q.$$

3. *Expectation is rational:*

$$f(\mathbf{x}_t) = \ln B + \phi \mu_{kt}^e + \gamma \mu_q - \frac{\sigma_\varepsilon^2}{2}, \quad (15)$$

$$g(\mathbf{x}_t) = \sqrt{(\phi \sigma_{\mu t} + \gamma \sigma_q)^2 + \phi^2 h_t (\sigma_{kt}^a)^2}, \quad (16)$$

$$m(\mathbf{x}_t) = \sqrt{\phi^2 (1 - h_t) (\sigma_{kt}^a)^2 + \sigma_\varepsilon^2}, \quad (17)$$

where

$$h_t = \frac{(\alpha \sigma_{kt}^a)^2}{\sigma_u^2 + (\alpha \sigma_{kt}^a)^2}$$

The next theorem shows the existence of equilibrium. On the equilibrium, $\sigma_{kt}^a = \sigma_{kt}$. Hence, σ_{kt} also denote the aggregate state variable later.

Theorem 6 *There exists a recursive positive assortative equilibrium with unobserved organization capital. On the equilibrium, the value function and the wage function are strictly increasing functions. (Closed form solution of the value function and the wage function is available in Appendix).*

Again, the theorem shows that the value function is a strictly increasing function of μ_{kt} and the wage function is a strictly increasing function of $\ln q_t$. As $\ln q_t$ is also an increasing function of μ_{kt} , the dynamics of the profits, wage and skill follows the dynamics of μ_{kt} . On the other hand, labor productivity, $\ln y_t$ is a strictly positive function of $\ln k_t$ and μ_{kt} . Hence, the dynamics of labor productivity can be derived from the dynamics of $\ln k_t$ and μ_{kt} . We focus the dynamics of $\ln k_t$ and μ_{kt} below.

Dynamics: Dynamics in this economy can be characterized by the following 5 equations:

$$\begin{aligned}\mu_{kt+1}^e &= \ln B + \phi\mu_{kt}^e + \gamma\mu_q - \frac{\sigma_\varepsilon^2}{2} \\ \sigma_{\mu t+1} &= \sqrt{(\phi\sigma_{\mu t} + \gamma\sigma_q)^2 + \phi^2 h_t \sigma_{kt}^2} \\ \sigma_{kt+1} &= \sqrt{\phi^2 (1 - h_t) \sigma_{kt}^2 + \sigma_\varepsilon^2}\end{aligned}$$

and

$$\begin{aligned}\ln k_{t+1}^o - \mu_{kt+1}^e &= \phi(\ln k_t^o - \mu_{kt}^e) + \gamma\frac{\sigma_q}{\sigma_{\mu t}}(\mu_{kt} - \mu_{kt}^e) + \varepsilon_t^* \quad (18) \\ \mu_{kt+1} - \mu_{kt+1}^e &= \phi h_t(\ln k_t^o - \mu_{kt}^e) + \left(\phi(1 - h_t) + \gamma\frac{\sigma_q}{\sigma_{\mu t}}\right)(\mu_{kt} - \mu_{kt}^e) + \phi h_t u_t^* \quad (19)\end{aligned}$$

where $h_t = \frac{\alpha^2 \sigma_{kt}^2}{\sigma_a^2 + \alpha^2 \sigma_{kt}^2}$ and $\varepsilon_t^* = \varepsilon_t + \frac{\sigma_\varepsilon^2}{2}$ is normally distributed with a mean 0 and a standard deviation σ_ε . Equation (18) shows the dynamics of $\ln k_t$. The first term

of equation (18) is influenced by technological persistence. That is, if organization capital is above average, this relative advantage is carried over to the next period by ϕ . On the other hand, the second term is influenced by the assignment effect. If an organization is believed to be better than average, it attracts skilled workers that are needed for accumulating more organization capital.

Equation (19) shows the dynamics of μ_{kt} . The first term capture how new information influences the dynamics of the belief. Managers know that organization capital is translated into the next organization capital by ϕ . However, current organization capital is not observable. It must be inferred from current output. When large output is realized, it can be a result of either a large temporal shock or large organization capital. As managers put a weight h_t on new information, organization capital is believed to be translated into the next belief by ϕh_t . Of course, new information is subjected to noise. Hence, the ϕh_t portion of u_t^* also influences the belief at the next period. This effect is captured by the third term, $\phi h_t u_t^*$, of equation (19).

The second term of equation (19) captures the effect of the prior belief on the posterior belief. There are two separate effects. When the firm makes an employment decision, the assignment occurs between the prior belief and the quality of workers. Therefore, the larger the prior belief, the higher quality of workers the firm can employ. As skilled workers assist accumulating organization capital, organization capital at the next period is believed to be large. This assignment effect is captured by $\frac{\gamma\sigma_q}{\sigma_{\mu t}}$ at the second term. On the other hand, when managers infer the current organization capital from its output, they cannot perfectly trust it. Hence, they put a weight, $1 - h_t$, on the prior belief. As ϕ fraction of current organization capital is translated into organization capital at the next period, $\phi(1 - h_t)$ fraction of the prior belief influences on the posterior. In total, the prior belief influences the posterior by $\phi(1 - h_t) + \gamma\frac{\sigma_q}{\sigma_{\mu t}}$.

Two equations (18) and (19) gives us some speculations on the dynamics of $\ln k_t^o$

and μ_{kt} . Firstly, equation (18) implies that $\ln k_t$ exhibits the reversion to the belief μ_{kt} by ϕ . Hence, the assignment does not influence the dynamics of $\ln k_t$ after conditioning μ_{kt} . Secondly, if h_t is smaller, equation (19) implies that μ_{kt} is less subjected any types of shocks, ε_t^* and u_t^* . Hence, the ranking of μ_{kt} is less likely to change. As an assignment takes place between μ_{kt} and q_t , less changes in ranking implies that μ_{kt} becomes more persistent. That is, the more noisy information is, the more persistent the belief would be.

In order to prove that these speculations are correct, we focus the dynamics of organization capital when the aggregate economy converges to the stationary distribution. The following proposition shows that economy converges to the stationary distribution.

Proposition 7 *The aggregate economy converges to a unique stationary distribution. On the stationary distribution*

$$\begin{aligned}\mu_\infty^e &= \frac{\ln B + \gamma\mu_q - \frac{\sigma_\varepsilon^2}{2}}{1 - \phi} \\ \sigma_{\mu\infty} &= \frac{\phi\gamma\sigma_q + \sqrt{(\gamma\sigma_q)^2 + (1 - \phi^2)\phi^2 h_\infty \sigma_{k\infty}^2}}{(1 - \phi^2)} \\ \sigma_{k\infty}^2 &= \frac{\alpha^2 \sigma_\varepsilon^2 - (1 - \phi^2)\sigma_u^2 + \sqrt{[\alpha^2 \sigma_\varepsilon^2 - (1 - \phi^2)\sigma_u^2]^2 + 4\alpha^2 \sigma_\varepsilon^2 \sigma_u^2}}{2\alpha^2} \\ h_\infty &= \frac{(\alpha\sigma_{k\infty})^2}{\sigma_u^2 + (\alpha\sigma_{k\infty})^2}\end{aligned}$$

Moreover, a firm dynamics is described as VAR model as follows:

$$\mathbf{k}_{t+1} = \mathbf{D}\mathbf{k}_t + \boldsymbol{\xi}_t \quad (20)$$

where

$$\begin{aligned}\mathbf{D} &= \begin{bmatrix} \phi & \gamma \frac{\sigma_q}{\sigma_{\mu\infty}} \\ \phi h_\infty & \left[\phi(1 - h_\infty) + \gamma \frac{\sigma_q}{\sigma_{\mu\infty}} \right] \end{bmatrix} \\ \mathbf{k}_t &= \begin{bmatrix} \ln k_t^o - \mu_{k\infty}^e \\ \mu_{kt} - \mu_{k\infty}^e \end{bmatrix}, \quad \boldsymbol{\xi}_t = \begin{bmatrix} \varepsilon_t^* \\ \phi h_\infty u_t^* \end{bmatrix}\end{aligned}$$

As the stationary distribution is unique and globally stable, a real economy is expected to locate near the stationary distribution in the long run. Hence, we focus the dynamics of organization capital when economy reaches the stationary distribution. It means that we investigate the property of equation (20) and discuss what influences the persistence of the dynamics.

Persistence when $\sigma_u = \infty$: It is instructive to start with an extreme case, $\sigma_u = \infty$. Hence, the firm cannot learn the level of organization capital at all. In this case, $h_\infty = 0$ and a firm put all weights on the prior belief. Hence, the prior belief never changes. The proposition shows that the movement of organization capital is reversing to its own belief.

Proposition 8 *Suppose that $\sigma_u^2 = \infty$. Then the dynamics of economy is described by*

$$\begin{aligned}\mu_\infty^e &= \frac{\ln B + \gamma\mu_q - \frac{\sigma_\varepsilon^2}{2}}{1 - \phi} \\ \sigma_{\mu_\infty} &= \frac{\gamma\sigma_q}{1 - \phi} \\ \sigma_{k_\infty}^2 &= \frac{\sigma_\varepsilon^2}{1 - \phi^2}\end{aligned}$$

and

$$\begin{aligned}\ln k_{t+1}^o &= \phi \ln k_t^o + (1 - \phi) \mu_{kt} + \varepsilon_t^* \\ \mu_{kt+1} &= \mu_{kt}.\end{aligned}$$

As the firm cannot learn about its own organization capital, the firm never changes its own belief. Hence, the belief is constant. As the assignment is based on this belief, the firm that has greater reputation always attracts good workers and keeps its position. Moreover, as real organization capital is subjected to the shocks, the movement of the organization capital temporally deviates from its own mean. Note

that when we could regress $\ln k_{t+1}^o$ on $\ln k_t^o$, $(1 - \phi) \mu_{kt}$ serves as a function of a firm specific fixed effect. Hence, this extreme case provides an interpretation on a fixed effect appeared in data.

Of course, no learning can be too extreme. However, the following argument shows that the results in this proposition can be considered as a limit of the results when σ_u is finite. Hence, it may be considered as a good approximation when time span of available data is short.

Persistence when $\sigma_u \in (0, \infty)$, $\sigma_\varepsilon > 0$ and $\phi > 0$: Let us examine a more general case below. First, we analyze the stability of equation (20). Equation (20) is stable if and only if the absolute values of all eigenvalues of \mathbf{D} are less than one. Examining the eigenvalues, we can derive the conditions that equation (20) is covariance stationary.

Proposition 9 *Let λ_1 and λ_2 denote the eigenvalues of the matrix \mathbf{D} . Then the equation (20) is covariance stationary if*

$$\begin{aligned}\lambda_1 &= \phi + \gamma \frac{\sigma_q}{\sigma_{\mu\infty}} < 1, \\ \lambda_2 &= \phi(1 - h_\infty) < 1.\end{aligned}$$

Note that the second condition is always satisfied. It means that the stability is guaranteed if the first condition is satisfied. The first condition is similar to that in the previous section. It implies the ratio of the standard deviation of skill to that of beliefs on organization capital $\frac{\sigma_q}{\sigma_{\mu\infty}}$ has to be less than $\frac{1-\phi}{\gamma}$. Hence, similar to the previous section, if the assignment effect, which is captured by $\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$, is not too strong, the dynamics reverse to the mean.

Of course, $\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$ and h_∞ are endogenous variables. We can find more fundamental conditions for stability. When the distribution is stationary, the following relationship

is derived.

$$\begin{aligned}
h_\infty &= \frac{\left(\frac{\alpha\sigma_{k\infty}}{\sigma_u}\right)^2}{1 + \left(\frac{\alpha\sigma_{k\infty}}{\sigma_u}\right)^2} \\
\frac{\sigma_q}{\sigma_{\mu\infty}} &= \frac{(1 - \phi^2)}{\phi\gamma + \sqrt{\gamma^2 + (1 - \phi^2) \left[1 - (1 - \phi^2) \left(\frac{\alpha\sigma_{k\infty}}{\sigma_u}\right)^2 \left(\frac{\sigma_u}{\alpha\sigma_\varepsilon}\right)^2\right] \left(\frac{\sigma_q}{\sigma_\varepsilon}\right)^{-2}}} \\
\left(\frac{\alpha\sigma_{k\infty}}{\sigma_u}\right)^2 &= \frac{\left(\frac{\sigma_u}{\alpha\sigma_\varepsilon}\right)^{-2} - (1 - \phi^2) + \sqrt{\left[\left(\frac{\sigma_u}{\alpha\sigma_\varepsilon}\right)^{-2} - (1 - \phi^2)\right]^2 + 4\left(\frac{\sigma_u}{\alpha\sigma_\varepsilon}\right)^{-2}}}{2}
\end{aligned}$$

Hence, h_∞ and $\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$ can be determined by two parameters, $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$ and $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$. The following lemma shows the relationship between two endogenous variables, h_∞ and $\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$, and two exogenous variables, $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$ and $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$.

Lemma 10 *Assume that $\phi \in (0, 1)$. The measure of informativeness, h_∞ , is a strictly positive function of $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$:*

$$\begin{aligned}
h_\infty &= \eta\left(\frac{\sigma_u}{\alpha\sigma_\varepsilon}\right), \\
\eta'\left(\frac{\sigma_u}{\alpha\sigma_\varepsilon}\right) &< 0, \quad \lim_{\frac{\sigma_u}{\alpha\sigma_\varepsilon} \rightarrow 0} \eta\left(\frac{\sigma_u}{\alpha\sigma_\varepsilon}\right) = 1, \quad \lim_{\frac{\sigma_u}{\alpha\sigma_\varepsilon} \rightarrow \infty} \eta\left(\frac{\sigma_u}{\alpha\sigma_\varepsilon}\right) = 0,
\end{aligned}$$

and the ratio of standard deviation of skill to beliefs on organization capital, $\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$, is a strictly positive function of $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$ and $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$:

$$\begin{aligned}
\frac{\gamma\sigma_q}{\sigma_{\mu\infty}} &= \chi\left(\frac{\gamma\sigma_q}{\sigma_\varepsilon}, \frac{\sigma_u}{\alpha\sigma_\varepsilon}\right), \\
\chi_1\left(\frac{\gamma\sigma_q}{\sigma_\varepsilon}, \frac{\sigma_u}{\alpha\sigma_\varepsilon}\right) &> 0, \quad \lim_{\frac{\sigma_u}{\alpha\sigma_\varepsilon} \rightarrow 0} \left[\phi + \gamma\chi\left(\frac{\gamma\sigma_q}{\sigma_\varepsilon}, \frac{\sigma_u}{\alpha\sigma_\varepsilon}\right)\right] = \phi, \quad \lim_{\frac{\sigma_u}{\alpha\sigma_\varepsilon} \rightarrow \infty} \left[\phi + \gamma\chi\left(\frac{\gamma\sigma_q}{\sigma_\varepsilon}, \frac{\sigma_u}{\alpha\sigma_\varepsilon}\right)\right] = 1, \\
\chi_2\left(\frac{\gamma\sigma_q}{\sigma_\varepsilon}, \frac{\sigma_u}{\alpha\sigma_\varepsilon}\right) &> 0, \quad \lim_{\frac{\sigma_u}{\alpha\sigma_\varepsilon} \rightarrow \infty} \left[\phi + \gamma\chi\left(\frac{\gamma\sigma_q}{\sigma_\varepsilon}, \frac{\sigma_u}{\alpha\sigma_\varepsilon}\right)\right] = 1.
\end{aligned}$$

The first part of this lemma shows that the measure of informativeness of output h_∞ and $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$ have a one to one relationship. Hence, for any h_∞ we can always find a $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$ to generate the h_∞ . The parameter, $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$, is a standard deviation of a noise term

to that of shocks. If the standard deviation of a noise term is large, the firms cannot learn much, h_∞ is smaller. If the noise term has small variance, the firm can learn a lot and h_∞ is large.

The second part of lemma shows that for a given $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$, we can always find a $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$ that can support the persistence parameter $\phi + \gamma\frac{\sigma_q}{\sigma_{\mu\infty}}$ in between ϕ and 1. As explained in previous section, if $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$ is large, top organization receives the large benefits from positive assortative assignment. Hence, the dynamics exhibits longer persistence.

More interestingly, $\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$ is increasing in $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$. That is, as information becomes noisy, the assignment effect becomes larger and the persistence parameter becomes larger. When information is noisier, rational agents rely more on a prior belief to infer current organization, and, therefore, the posterior belief is less subjected to changes in ranking due to the realization of idiosyncratic shocks, ε_t^* . Without changes in ranking, large μ_{kt} always attract high q_t , which is expected to generate large organization capital in future. Hence, μ_{kt} is more persistent. Note that when $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$ goes infinite, the persistence parameter converges to 1. It means that the dynamics at $\sigma_u = \infty$ can be approximated by large $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$.

Combining proposition 9 and lemma 10, when $\phi \in (0, 1)$, as far as $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$ and $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$ are finite, the stochastic process is covariance stationary.

Proposition 11 *Suppose that $\phi \in (0, 1)$, $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$ and $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$ are finite, equation 20 is covariance stationary.*

Two eigenvalues, λ_1 and λ_2 are also important for the examination of the persistence of the stochastic process. Let ρ_{kj} denote the autocorrelation between current organization capital and organization capital j period before, and $\rho_{\mu j}$ denote the autocorrelation between belief on current organization capital and belief on organization capital j period before:

$$\rho_{kj} \equiv \frac{E(\ln k_t^o - \mu_{k\infty}^e)(\ln k_{t-j}^o - \mu_{k\infty}^e)}{\text{Var}(\ln k_t^o)}$$

$$\rho_{\mu j} \equiv \frac{E(\mu_{kt} - \mu_{k\infty}^e)(\mu_{kt-j} - \mu_{k\infty}^e)}{\sigma_{\mu\infty}^2}$$

. The following proposition derives autocorrelation of $\ln k_t$ and μ_{kt} .

Proposition 12 *The autocorrelation of $\ln k_t^o$ and μ_{kt} are functions of λ_1 and λ_2*

$$\begin{aligned}\rho_{kj} &= \left[1 - \omega \left(\frac{\gamma\sigma_q}{\sigma_\varepsilon}, \frac{\sigma_u}{\alpha\sigma_\varepsilon} \right) \right] \lambda_1^j + \omega \left(\frac{\gamma\sigma_q}{\sigma_\varepsilon}, \frac{\sigma_u}{\alpha\sigma_\varepsilon} \right) \lambda_2^j \\ \rho_{\mu j} &= \lambda_1^j \\ \text{where } \omega \left(\frac{\gamma\sigma_q}{\sigma_\varepsilon}, \frac{\sigma_u}{\alpha\sigma_\varepsilon} \right) &= \frac{\gamma \frac{\sigma_q}{\sigma_{\mu\infty}}}{\left(\phi h_\infty + \gamma \frac{\sigma_q}{\sigma_{\mu\infty}} \right) \left(1 + \left(\frac{\sigma_{\mu\infty}}{\sigma_{k\infty}} \right)^2 \right)} \\ \frac{\sigma_{\mu\infty}}{\sigma_{k\infty}} &= \frac{\phi \left(\frac{\gamma\sigma_q}{\sigma_\varepsilon} \right) + \sqrt{\left(\frac{\gamma\sigma_q}{\sigma_\varepsilon} \right)^2 + (1 - \phi^2) \left[1 - (1 - \phi^2) \left(\frac{\alpha\sigma_{k\infty}}{\sigma_u} \right)^2 \left(\frac{\sigma_u}{\alpha\sigma_\varepsilon} \right)^2 \right]}}{(1 - \phi^2) \left(\frac{\alpha\sigma_{k\infty}}{\sigma_u} \right) \left(\frac{\sigma_u}{\alpha\sigma_\varepsilon} \right)}\end{aligned}$$

The above proposition shows that the autocorrelation between current organization capital and past organization capital can be expressed as a weighted average of λ_1^j and λ_2^j ; while the correlation between current belief and past reputation is a function of λ_1^j . It means that as far as equation 20 is covariance stationary, eventually the effect of initial condition dies out. More importantly the autocorrelation will be heavily influenced by two variables, $\frac{\sigma_q}{\sigma_{\mu\infty}}$, and h_∞ . Note that $\lambda_1 > \lambda_2$. Hence, the immediate result from above proposition is as follows.

Corollary 13 *Suppose that $\frac{\gamma\sigma_q}{\sigma_\varepsilon} > 0$. The autocorrelation of a firm's reputation is always larger than its real capacity.*

$$\rho_{\mu j} > \rho_{kj}, \quad \forall j$$

The corollary says that reputation is more persistent than organization capital itself. As we discussed earlier, once we control the belief, the persistence parameter of $\ln k_t$ is only ϕ . As the main source of persistence came from the dynamics of the belief, the autocorrelation of the belief on organization capital must be larger than that of organization capital itself.

Now we want to show how $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$ and $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$ change the autocorrelation.

Proposition 14 *There exist j^* and j^{**} such that for all $j \geq j^*$*

$$\frac{d\rho_{kj}}{d\frac{\gamma\sigma_q}{\sigma_\varepsilon}} > 0$$

*and for all $j \geq j^{**}$*

$$\frac{d\rho_{kj}}{d\left(\frac{\sigma_u}{\alpha\sigma_\varepsilon}\right)} > 0.$$

For all j ,

$$\frac{d\rho_{\mu j}}{d\frac{\gamma\sigma_q}{\sigma_\varepsilon}} > 0, \frac{d\rho_{\mu j}}{d\left(\frac{\sigma_u}{\alpha\sigma_\varepsilon}\right)} > 0.$$

The proposition shows that the larger values of $\frac{\gamma\sigma_q}{\sigma_\varepsilon}$ and $\frac{\sigma_u}{\alpha\sigma_\varepsilon}$ always increase the autocorrelation of reputation, while they can definitely increase the autocorrelation of organization capital after enough period has passed. As an increase in both parameters increase the persistence due to the assignment and the assignment occurs between reputation and skill, both parameters directly increase the autocorrelation of reputation. A real organization capital moves around its reputation. Hence, after long period later, an increase in the persistence of reputation certainly increases the persistence of organization capital itself.

4 Empirical Strategy

This section examines how much the informational friction and the diversity of skill contribute to the persistence of organization. For this purpose, this section structurally identifies technological persistence, ϕ , the assignment parameter $\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$ and the measure of the accuracy of information h_∞ . First, we discuss how to identify ϕ , $\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$ and h_∞ . It gives us intuition behind our structural estimation. Later, we discuss data and our estimation procedure.

Strategies to identify ϕ , $\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$ and h_∞ : Firstly, as we cannot observe k_t^o , we need to convert the result in the previous section to the movement of observable variables. One of such variables is output, y_t . As we assume that the number of workers is assumed to be 1, we rather measure y_t by labor productivity³. The dynamics of $\ln y_t$ and $E[\ln y_t | \mu_{kt}, \sigma_{k\infty}^2]$ on the steady state is derived from (20):

$$\ln y_t^r = b_1 y_{t-1}^r + b_2 E[\ln y_{t-1}^r | \mu_{kt-1}] + v_{t-1}, \quad (21)$$

$$E[\ln y_t^r | \mu_{kt}] = b_3 y_{t-1}^r + b_4 E[\ln y_{t-1}^r | \mu_{kt-1}] \quad (22)$$

where $\ln y_t^r = \ln y_t - E[\ln y]$, $b_1 = \phi + \frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}} \phi h_\infty$, $b_2 = \frac{\gamma\sigma_q}{\sigma_{\mu\infty}} - \frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}} \phi h_\infty$, $b_3 = \phi h_\infty + \frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}} \phi h_\infty$, $b_4 = b_1 + b_2 - b_3$ and $v_{t-1} = \alpha (\varepsilon_{t-1}^* - \phi u_{t-1}^* + u_t^*)$. Using equation (22), the expected value of labor productivity can be recursively constructed. Using this constructed expected labor productivity, we can conduct regression analysis by equation (21).

Suppose that $\frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}$ is known, which is discussed later. Then, the parameter, ϕh_∞ , ϕ and $\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$ can be derived from the following three equations:

$$\begin{aligned} \phi h_\infty &= \frac{b_3}{1 + \frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}}, \\ \phi &= b_1 + \frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}} \phi h_\infty, \\ \frac{\gamma\sigma_q}{\sigma_{\mu\infty}} &= b_2 - \frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}} \phi h_\infty. \end{aligned}$$

Hence, it is possible to identify ϕ , h_∞ and $\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$ from regression results.

³This empirical study assume that the movement of labor productivity is explained by the movement of organization capital. However, it might be influenced by the movement of physical capital too. There is no physical capital in our production function. However, as physical capital is a choice variable, it can be expressed as a function of organization capital. Hence, the movement of labor productivity is fundamentally dictated by the movement of organization capital. For example, we could assume $y_t = A (k_t^o)^\eta q_t^t k_t^t$, where k_t is capital stock per worker as a production function. Once the firm maximizes profits given a rental price, we can derive the production function in this paper. Hence adding capital stock does not change the results of our empirical study.

In order to understand intuitions behind this identification strategy, first note that equation (21) can be written as

$$\ln y_t^r = \phi y_{t-1}^r + \frac{\gamma \sigma_q}{\sigma_{\mu_\infty}} E [\ln y_{t-1}^r | \mu_{kt-1}] + \frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu_\infty}} \phi h_\infty [\ln y_{t-1}^r - E [\ln y_{t-1}^r | \mu_{kt-1}]] + v_{t-1}$$

The coefficient of $\ln y_{t-1}^r$ is the parameters on the technological persistence; the coefficient on $E [\ln y_{t-1}^r | \mu_{kt-1}]$ is the parameter of the assignment effect. As high labor productivity indicates high organization capital, it can be translated to the organization capital at the next period by ϕ . On the other hand, as assignment occurs between beliefs on organization capital and skill, the coefficient on $E [\ln y_{t-1}^r | \mu_{kt-1}]$ capture the effect of assignment on organization capital at the next period. The third term is new, which is derived from the prediction error. When the realized output is larger than the expected one, people update their belief. The updated belief attracts better workers, who can directly increase the output. Hence, if we are able to identify the effect from the prediction error, we can separately identify the assignment effect $\frac{\gamma \sigma_q}{\sigma_{\mu_\infty}}$ from the technological persistence ϕ .

The prediction error obviously is influenced by the measure of the accuracy of information, h_∞ . In our regression analysis, the parameter, b_3 provides useful information about h_∞ . Note that

$$b_1 - b_3 = \phi (1 - h_\infty).$$

Hence, the difference between b_1 and b_3 is influenced by the accuracy of information the current labor productivity provides. In order to understand the reason, note that v_t and $\ln y_t^r$ are correlated because $E [u_t^* | \ln y_t^r] \neq 0$. A high output is influenced not only by large organization capital, but also buy a current good luck. Hence, large output provides an information about the current shocks, too. A rational agent efficiently extracts this information to predict future output. That is why b_1 and b_3 are different. When the variation of u_t^* is a large component of the variation of output, the realized output is more influenced by u_t^* and less influenced by organization

capital. Hence, the observed output is useful to predict u_t^* , but not to predict $\ln k_t^o$. Therefore, h_∞ is small and the difference between b_1 and b_3 is large.

In order to identify b_1 and b_3 from data, we could apply OLS and an instrumental variable (IV) approach to the same regression. The IV estimate provides a consistent estimator of the parameter b_1 and OLS estimate provides a biased estimator of b_1 , that is b_3 . Hence, the difference between the IV estimates and the OLS tells about how much the output provides information about the error term and identify our h_∞ . More specifically, we derive the following regression equation from equation (22):

$$\ln y_t^r = b_3 \sum_{i=1}^t b_4^{i-1} \ln y_{t-i}^r + b_4^t E[\ln y_0^r | \mu_{k0}] + \varpi_t \quad (23)$$

where $\varpi_t = \ln y_t^r - E[\ln y_t^r | \mu_{kt}]$. Note that ϖ_t is not correlated with $\ln y_{t-i}^r$ and $E[\ln y_0^r | \mu_{k0}]$ because ϖ_t is a prediction error. Hence, the OLS estimate provides a consistent estimator of b_3 .

Data and Estimation Procedure for b_1 , b_2 , b_3 and b_4 : We use COMPUSTAT industry annual data set in 1970-2004 for our empirical study. We choose the manufacturing sector for this analysis. The value added per the number of workers is our proxy for labor productivity, y_t . As COMPUSTAT does not report the value added, we estimated it. Our data construction procedure is written in Appendix.

We estimate $\ln y_{jt}^r$ by $\ln y_{jt} - \frac{\sum_j^m \ln y_{jt}}{m}$ where y_{jt} is the value added over the number of workers in j th firm at year t and m is the number of firms in the same 4-digit industry. That is, $\ln y_{jt}^r$ is the deviation of labor productivity from industry and year average. We estimate the prior, $E[\ln y_0^r | \mu_{k0}]$ from the average $\ln y_{jt}^r$ over initial 5 years appeared in COMPUSTAT since 1970. Therefore, the following estimation procedure is based on data in 1975-2004.

Using equations (21) and (23), and our constructed prior, we conducted the following recursive estimation procedure. First, we picked an arbitrary value of b_4 , and construct $\sum_{i=1}^t b_4^{i-1} \ln y_{t-i}^r$ and $(b_4)^t E[\ln y_0^r | \mu_{k0}]$ from data. Second, equation (23)

is estimated by a constrained regression, where the constraint is that the coefficient of $(b_4)^t E[\ln y_0^r | \mu_{k0}]$ is 1. It identifies b_3 . Third, using these b_3 and b_4 , we estimate $E[\ln y_t^r | \mu_{kt}]$ by $b_3 \sum_{i=1}^t b_4^{i-1} \ln y_{t-i}^r + b_4^t E[\ln y_0^r | \mu_{k0}]$. Forth, using the estimated value of $E[\ln y_t^r | \mu_{kt}]$, equation (21) is estimated by the IV regression. We use $\ln y_{t-2}^r$ and $\ln w_{jt-1} - \frac{\sum_j^m \ln w_{jt-1}}{m}$ for the instruments of $\ln y_{t-i}^r$ and $E[\ln y_t^r | \mu_{kt}]$, where w_{jt} is the average wage rates in j th firm at year t ⁴. We need an additional instrument for $E[\ln y_t^r | \mu_{kt}]$ because $E[\ln y_t^r | \mu_{kt}]$ might contain a measurement error. This IV estimates produce b_1 and b_2 . Finally, as $b_4 = b_1 + b_2 - b_3$, we can replace b_4 . Using this new b_4 , we can conduct the same procedure again until the estimated b_4 converges to the assumed b_4 .

The Estimation of $\frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}$: In order to identify the parameter, we need to estimate $\frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}$. This information on $\frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}$ can be derived from the first order conditions of the firm. Integrating the first order condition, the following wage function is derived:

$$w(\chi(\mu_{kt}, \sigma_{k\infty} : \mathbf{x}_\infty) : \mathbf{x}_\infty) = b_5 E[y_t | \mu_{kt}, \sigma_{k\infty}^2] + b_6 E[V^*(\mu_{kt+1}, \sigma_{k\infty} : \mathbf{x}_\infty) | \mu_{kt}, \sigma_{k\infty}^2] \quad (24)$$

$$b_5 = \frac{\frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}}{1 + \frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}}, \quad b_6 = \frac{\beta \frac{\gamma \sigma_q}{\sigma_{\mu t}}}{\phi + \frac{\gamma \sigma_q}{\sigma_{\mu t}}}$$

Hence, in our model, the wage paid by a firm is expressed as a linear combination of the expected labor productivity and the expected market value of the firm per workers.

Note that $\frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}$ can be recovered from b_5 :

$$\frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}} = \frac{b_5}{1 - b_5}.$$

That is, $\frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}$ roughly measures the relative contribution of skill on current labor productivity.

⁴As many firms do not report labor expenses in COMPUSTAT, we need to estimate it. Our estimation of w_{jt} is discussed in Appendix.

We could directly use this equation for our estimation. However, the results turn out quite sensitive to the choice of the time period and instruments. In order to get a robust result, the following equation is derived for our regression analysis.

$$\frac{w(\chi(\mu_{kt}, \sigma_{k\infty} : \mathbf{x}_\infty) : \mathbf{x}_\infty)}{E[y_t | \mu_{kt}, \sigma_{k\infty}]} = b_5 + b_6 \frac{V^*(\mu_{kt+1}, \sigma_{k\infty} : \mathbf{x}_\infty)}{E[y_t | \mu_{kt}, \sigma_{k\infty}]} + v_t \quad (25)$$

where $v_t = b_6 \left\{ E \left[\frac{V^*(\mu_{kt+1}, \sigma_{k\infty} : \mathbf{x}_\infty)}{E[y_t | \mu_{kt}, \sigma_{k\infty}]} | \mu_{kt}, \sigma_{k\infty} \right] - \frac{V^*(\mu_{kt+1}, \sigma_{k\infty} : \mathbf{x}_\infty)}{E[y_t | \mu_{kt}, \sigma_{k\infty}]} \right\}$ and $E[y_t | \mu_{kt}, \sigma_{k\infty}^2] = E[y_t] \exp E[\ln y_t^r | \mu_{kt}]$. As $\frac{V^*(\mu_{kt+1}, \sigma_{k\infty} : \mathbf{x}_\infty)}{E[y_t | \mu_{kt}, \sigma_{k\infty}]}$ is correlated with v_t , we use the first, second and third lag of $\frac{V^*(\mu_{kt+1}, \sigma_{k\infty} : \mathbf{x}_\infty)}{E[y_t | \mu_{kt}, \sigma_{k\infty}]}$ as instruments on it. This regression derives fairly robust estimates of b_5 and b_6 .

In order to implement this regression, we need proxies for $w(\chi(\mu_{kt}, \sigma_{k\infty} : \mathbf{x}_\infty) : \mathbf{x}_\infty)$, $V^*(\mu_{kt+1}, \sigma_{k\infty} : \mathbf{x}_\infty)$ and $E[y_t | \mu_{kt}, \sigma_{k\infty}]$. The estimated wage rate is proxies for $w(\chi(\mu_{kt}, \sigma_{k\infty} : \mathbf{x}_\infty) : \mathbf{x}_\infty)$ and the market value of firms per the number of workers are proxies for $V^*(\mu_{kt+1}, \sigma_{k\infty} : \mathbf{x}_\infty)$. Both are constructed from COMPUSTAT. More details are written in Appendix. Note that we can estimate $E[\ln y_t^r | \mu_{kt}]$ by $b_3 \sum_{i=1}^t b_4^{i-1} y_{t-i}^r + b_4^t E[y_0^r | \mu_{k0}]$ and $E[y_t]$ by the industry average of y_t . This provides sufficient information for the proxies of $E[y_t | \mu_{kt}, \sigma_{k\infty}^2]$.

5 Empirical Results and Calibration

This section provides empirical results. Using the estimated parameters, we calibrate our model and examine how much the assignment influences the persistence of productivity.

The first two tables 2 and 3 show the results of our iterated regressions. The tables report the number when the estimated b_4 meets the assumed b_4 . The table 2 reports the results of regression equation (23) and the table 3 reports the results of regression equation (21).

As only small few companies report labor and related expense in COMPUSTAT,

The dependent variable is $\ln y_t^r$

	Small Sample	Small Sample	Large Sample	Large Sample
	Constrained	Unconstrained	Constrained	Unconstrained
$\sum_{i=1}^t b_4^{i-1} \ln y_{t-i}^r$	0.657***	0.658***	0.574***	0.577***
	(0.008)	(0.008)	(0.003)	(0.003)
$b_4^t E [\ln y_0^r \mu_{k0}]$	1	0.977***	1	0.899***
		(0.092)		(0.030)
# of observations	2425	2425	19542	19542

Table 2: Regression 1

The dependent variable is $\ln y_t^r$

	Small Sample	Large Sample
y_{t-1}^r	0.676***	0.603***
	(0.052)	(0.014)
$E [\ln y_{t-1}^r \mu_{kt-1}]$	0.264***	0.326***
	(0.054)	(0.015)
# of observations	2131	16690

Table 3: Regression 2

Two tables show regression results when the estimated b_4 converges to the assumed b_4 . The variable $\ln y_t^r$ is labor productivity relative to industry and year average. The prior $E [\ln y_0^r | \mu_{k0}]$ is constructed by the average of initial five years appeared in Compustat after 1970. The prior $E [\ln y_t^r | \mu_{kt}]$ is constructed by $b_3 \sum_{i=1}^t b_4^{i-1} \ln y_{t-i}^r + b_4^t E [\ln y_0^r | \mu_{k0}]$. Small sample contains only the companies that report labor and related expense and large sample expand companies using estimated labor cost. Constrained regression assume that the coefficient of $b_4^t E [\ln y_0^r | \mu_{k0}]$ is 1. Table 2 reports the results of simple OLS, and Table 3 reports the results of IV estimate. The Standard errors are reported in parentheses.

*** means significant at 1 percent level.

we estimate labor cost when the companies do not report it. The estimation procedure is written in Appendix. In order to investigate the bias due to this estimation, we also report the regression results for the companies that report labor and related expense. Small sample contains only the companies that report labor and related expense and large sample expand companies using estimated labor cost.

The table 2 shows that b_3 (the coefficient on $\sum_{i=1}^t b_4^{i-1} \ln y_{t-i}^r$) is 0.657 in the small sample and 0.574 in the large sample. The small sample produces slightly bigger values for b_3 . In order to check the bias due to constrained regression, we also run unconstrained regressions. It shows that b_3 does not change much between the constrained regression and the unconstrained regression. It backs up the reliability of our estimates.

Unconstrained regression also reveals an interesting feature of data: the weighted initial prior influences labor productivity for long time. It means that the effect of initial values declines over time, but does not fade out. The theory predicts the coefficient on the weighted initial prior is 1. The results from unconstrained regression is smaller than 1, but fairly close. In particular, the results by small sample shows 0.977 and F-test can not reject the hypothesis the coefficient is 1. This result also supports the prediction of our model.

The table 3 shows that b_1 (the coefficient on y_{t-1}^r) is 0.676 in the small sample and 0.603 in the large sample. They are slightly bigger than b_3 . It means that predicting the error terms from output does not provide additional information very much. In other words, the most of variation of labor productivity can be explained by persistent capacity of the firm, which we call organization capital.

Table 3 shows after controlling labor productivity, the belief on labor productivity still influences the labor productivity at the next period. It is consistent with the prediction by the model with unobserved organization capital.

In order to identify parameters, we need to estimate $\frac{\psi}{\alpha} \frac{\sigma_g}{\sigma_{\mu\infty}}$. Table 4 reports the results of regression equation (25). The constant term contains the information

The dependent variable is $\frac{w_t}{E[y_t|\mu_{kt},\sigma_{k\infty}]}$

	Small Sample	Large Sample
constant	0.108***	0.121***
	(0.003)	(0.001)
$\frac{V_{t+1}}{E[y_t \mu_{kt},\sigma_{k\infty}]_t}$	0.008*	0.010***
	(0.003)	0.001
# of observations	1457	10608

Table 4: Regression3

The variable w is the wage, V is the market value of a firm per the number of workers, where the market value is evaluated at the end of year. The expected labor productivity $E[y_t|\mu_{kt},\sigma_{k\infty}]$ is estimated by $E[y_t] \exp E[\ln y_t^r|\mu_{kt}]$, where $E[y_t]$ is industry average labor productivity and $E[\ln y_t^r|\mu_{kt}]$ is taken from the results in Table 2. IV regression is conducted by using the first, second and third lag of

$\frac{V}{E[y_t|\mu_{kt},\sigma_{k\infty}]}$ as instruments. Small sample contains only the companies that report labor and related expense and large sample expand companies using estimated labor cost. The Standard errors are reported in parentheses. *means significant at 10 percent level. *** means significant at 1 percent level.

about $\frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}$. More interestingly, the expected labor share $\frac{w}{E[y_t|\mu_{kt},\sigma_{k\infty}]}$ is positively correlated with the expected market to output ratio, $\frac{V}{E[y_t|\mu_{kt},\sigma_{k\infty}]}$. This is consistent with our theory that employing able workers plays a role of investment.

Using the results from our regression analysis produces the parameters of our interests; ϕ , h_∞ and $\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$. Table 5 report the results. Note that h_∞ is quite high; 0.97 in small sample and 0.95 in large sample. It means that the realized labor productivity reveals fairly accurate estimates of the firm's capacity. This result is mainly derived from a small difference between IV estimation and OLS estimation. The parameter of technological persistence is 0.61 and the measure of assignment on persistence is 0.33. Note that the persistence parameter is $\lambda_1 = \phi + \frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$. Hence,

The Estimated Parameters.

	Small Sample	Large Sample
$\frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}$	0.121	0.137
ϕ	0.605	0.534
h_∞	0.969	0.945
$\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$	0.334	0.395

Table 5: Estimated Parameters

$\frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}$ the relative contribution of skill on current labor productivity. ϕ measures technological persistence. h_∞ measures the accuracy of information held by realized labor productivity to predict the level of organization capital. $\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}$ measures the importance of the assignment for the persistence. Small sample contains only the companies that report labor and related expense and large sample expand companies using estimated labor cost.

the assignment explains about one third of the persistence parameter.

In order to understand the impacts of assignment on the persistence. We estimate the correlation between current labor productivity relative to industry and year average and the lagged relative labor productivity. Let us define the correlation as follows:

$$\rho_{ys} \equiv \frac{E[(\ln y_t - E[\ln y_t])(\ln y_{t-s} - E[\ln y_{t-s}])]}{Var(\ln y_t)}$$

The following proposition derives the theoretical prediction.

Proposition 15 *The correlation between current relative labor productivity and the relative labor productivity s period before is*

$$\rho_{ys} = \frac{\left(1 + \frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}\right) \lambda_1^{s-1} \left[\phi + \lambda_1 \left(1 + \frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}\right) \left(\frac{\sigma_{\mu\infty}}{\sigma_{k\infty}}\right)^2\right]}{1 + \left(1 + \frac{\psi}{\alpha} \frac{\sigma_q}{\sigma_{\mu\infty}}\right)^2 \left(\frac{\sigma_{\mu\infty}}{\sigma_{k\infty}}\right)^2 + \left(\frac{\sigma_u}{\alpha\sigma_{k\infty}}\right)^2}$$

The correlation between current labor productivity relative to industry and year average and the lagged relative labor productivity.

		ρ_{y2}	ρ_{y4}	ρ_{y6}	ρ_{y8}	ρ_{y10}
Small sample	data	0.81	0.69	0.59	0.51	0.42
	model	0.78	0.69	0.61	0.54	0.47
	no assignment	0.36	0.13	0.05	0.02	0.01
Large sample	data	0.75	0.66	0.62	0.55	0.51
	model	0.71	0.61	0.53	0.46	0.39
	no assignment	0.27	0.08	0.02	0.01	0.002

Table 6: Correlation between current productivity differences and lagged productivity differences

ρ_{ys} means the correlation between current relative labor productivity and the relative labor productivity s years before. Small sample contains only the companies that report labor and related expense and large sample expand companies using estimated labor cost.

where $\lambda_1 = \phi + \gamma \frac{\sigma_q}{\sigma_{\mu\infty}}$, $\left(\frac{\sigma_u}{\alpha\sigma_{k\infty}}\right)^2 = \frac{1-h_\infty}{h_\infty}$, and

$$\frac{\sigma_{\mu\infty}}{\sigma_{k\infty}} = \frac{\phi \left(\frac{\gamma\sigma_q}{\sigma_\varepsilon}\right) + \sqrt{\left(\frac{\gamma\sigma_q}{\sigma_\varepsilon}\right)^2 + (1-\phi^2) \left[1 - (1-\phi^2) \left(\frac{\alpha\sigma_{k\infty}}{\sigma_u}\right)^2 \left(\frac{\sigma_u}{\alpha\sigma_\varepsilon}\right)^2\right]}}{(1-\phi^2) \left(\frac{\alpha\sigma_{k\infty}}{\sigma_u}\right) \left(\frac{\sigma_u}{\alpha\sigma_\varepsilon}\right)}$$

$$\left(\frac{\alpha\sigma_{k\infty}}{\sigma_u}\right)^2 \left(\frac{\sigma_u}{\alpha\sigma_\varepsilon}\right)^2 = \frac{1}{h_\infty + (1-\phi^2)(1-h_\infty)}$$

$$\frac{\gamma\sigma_q}{\sigma_\varepsilon} = \phi \sqrt{\frac{h_\infty}{\left[(1-\phi) \left(\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}\right)^{-1} - 1\right] \left[(1+\phi) \left(\frac{\gamma\sigma_q}{\sigma_{\mu\infty}}\right)^{-1} + 1\right] \left[h_\infty + (1-\phi^2)(1-h_\infty)\right]}}$$

Table 6 compares the persistence of labor productivity derived from the theory with that in data. It shows that the calibration fits data well in small sample. Although the theory slightly underestimates the persistence in large sample, it still

captures the overall picture of data.

Let us conduct an experiment. What would happen if there is no assignment in an economy. This experiment can be done when $\gamma \frac{\sigma_q}{\sigma_{\mu\infty}} = 0$. It means that if there is no difference in skill for creating organization capital, how much this effect reduces the persistence of productivity differences? The fourth and seventh row of Table 6 report the results of this experiment. It shows that after 4-6 years, the correlation is almost 0. It means that the relative advantages (disadvantages) of a firm persist only in 4-6 years. This striking results reveal the importance of assignment on the persistence of productivity differences.

6 Conclusion

This paper constructs a dynamic assignment model to explain the observed persistent differences in productivity, wages, skill mix and profits across plants and firms. When the organization capital and skill are complement, large organization capital attracts skilled workers, who can create better organization in future. This positive feedback brings the persistent differences in these variables. Furthermore, when organization capital is not observable, it is shown that the real organization capital exhibits a reversion to the belief on the organization capital and the belief itself is persistent. This dynamics is consistent with the evidence on productivity differences. The theory predicts that the relative diversity of skill and the lack of ability to accurately evaluate organization capital increase the persistence of variables. After structurally estimating the parameters of the model using COMPUSTAT data set, we calibrate the model. Our calibration results show that if there is no diversity in skill, the relative advantages of a firm disappear in 4-6 years.

7 Appendix

The Complete Statement of Theorem 2: A unique recursive positive assortative equilibrium with observed organization capital exists and this equilibrium is supported by

$$\begin{aligned}
V(\ln k_t^o; \mu_{kt}, \sigma_{kt}) &= \sum_{i=0}^{\infty} \Pi_{\tau=1}^i \frac{\beta\phi}{\phi + \frac{\gamma\sigma_q}{\sigma_{kt+\tau}}} \frac{\alpha E_t [y(\ln k_{t+i}^o, \mu_{kt+i}, \sigma_{kt+i})]}{\alpha + \frac{\psi\sigma_q}{\sigma_{kt+i}}} \\
w(\ln q_t; \mu_{kt}, \sigma_{kt}) &= \frac{\psi \frac{\sigma_q}{\sigma_{kt}} y(\ln k_t^o, \mu_{kt}, \sigma_{kt})}{\alpha + \psi \frac{\sigma_q}{\sigma_{kt}}} \Big|_{\ln k_t^o = \frac{\sigma_{kt}}{\sigma_q} [\ln q_t - \mu_q] + \mu_{kt}} \\
&\quad + \frac{\beta\gamma \frac{\sigma_q}{\sigma_{kt}}}{\phi + \gamma \frac{\sigma_q}{\sigma_{kt}}} \int V(\ln k_{t+1}^o; \mu_{kt+1}, \sigma_{kt+1}) dQ(\varepsilon_t) \Big|_{\ln k_t^o = \frac{\sigma_{kt}}{\sigma_q} [\ln q_t - \mu_q] + \mu_{kt}}
\end{aligned}$$

where $\Pi_{\tau=1}^0 \frac{\beta\phi}{\phi + \frac{\gamma\sigma_q}{\sigma_{kt+\tau}}} = 1$,

$$\begin{aligned}
E_t [y(\ln k_{t+i}^o, \mu_{kt+i}, \sigma_{kt+i})] &= \int \dots \int y(\ln k_{t+i}^o, \mu_{kt+i}, \sigma_{kt+i}) dQ(\varepsilon_{t+i-1}) \dots dQ(\varepsilon_t) \\
y(\ln k_{t+i}^o, \mu_{kt+i}, \sigma_{kt+i}) &= \exp \left[\ln A + \psi \left(\mu_q - \frac{\sigma_q}{\sigma_{kt+i}} \mu_{kt+i} \right) + \left(\alpha + \psi \frac{\sigma_q}{\sigma_{kt+i}} \right) \ln k_{t+i}^o \right]
\end{aligned}$$

and

$$\begin{aligned}
\ln k_{t+i}^o &= \Pi_{\tau=1}^i \left(\phi + \gamma \frac{\sigma_q}{\sigma_{kt+i-\tau}} \right) \ln k_t^o \\
&\quad + \sum_{\tau=1}^i \Pi_{x=1}^{\tau-1} \left(\phi + \gamma \frac{\sigma_q}{\sigma_{kt+i-x}} \right) \left[\ln B + \gamma \left(\mu_q - \frac{\sigma_q}{\sigma_{kt+i-\tau}} \mu_{kt+i-\tau} \right) + \varepsilon_{t+i-\tau} \right] \\
\mu_{kt+i} &= \frac{1 - \phi^i}{1 - \phi} \left(\ln B + \gamma \mu_q - \frac{\sigma_q^2}{2} \right) + \phi^i \mu_{kt} \\
\sigma_{kt+i} &= g^{*i}(\sigma_{kt}) = \overbrace{g^*(g^*(g^*(\sigma_{kt})))}^i, \quad g^*(\sigma_{kt}) = \sqrt{(\phi\sigma_{kt} + \sigma_q\gamma)^2 + \sigma_\varepsilon^2}
\end{aligned}$$

The Complete Statement of Theorem 6: There exists a unique recursive positive assortative equilibrium with unobserved organization capital. On the equilibrium,

the value function and the wage function is

$$\begin{aligned}
V^*(\mu_{kt}, \sigma_{kt} : \mathbf{x}_t) &= \sum_{i=0}^{\infty} \prod_{s=1}^i \frac{\beta \phi}{\phi + \frac{\gamma \sigma_q}{\sigma_{\mu t+s-1}}} \frac{\alpha E[y(\mu_{kt+i}, \sigma_{kt+i} : \mathbf{x}_{t+i}) | \mu_{kt}, \sigma_{kt}]}{\alpha + \frac{\psi \sigma_q}{\sigma_{\mu t+i}}} \\
w(\ln q_t : \mu_{kt}^e, \mathbf{x}_t) &= \frac{\frac{\psi \sigma_q}{\sigma_{\mu t}} E[y(\mu_{kt}, \sigma_{kt}, \mathbf{x}_t) | \mu_{kt}, \sigma_{kt}]}{\alpha + \frac{\psi \sigma_q}{\sigma_{\mu t}}} \Big|_{\mu_{kt} = \frac{\sigma_{\mu t}}{\sigma_q} (\ln q_t - \mu_q) + \mu_{kt}^e} \\
&\quad + \frac{\beta \frac{\gamma \sigma_q}{\sigma_{\mu t}}}{\phi + \frac{\gamma \sigma_q}{\sigma_{\mu t}}} \int V^*(\mu_{kt+1}, \sigma_{kt+1} : \mathbf{x}_{t+1}) d\Gamma_s(s_t | \mu_{kt}, \sigma_{kt}) \Big|_{\mu_{kt} = \frac{\sigma_{\mu t}}{\sigma_q} (\ln q_t - \mu_q) + \mu_{kt}^e}
\end{aligned}$$

where $\prod_{s=1}^0 \frac{\beta \phi}{\phi + \frac{\gamma \sigma_q}{\sigma_{\mu t+s-1}}} = 1$ and

$$\begin{aligned}
&E[y(\mu_{kt+i}, \sigma_{kt+i}, \mathbf{x}_{t+i}) | \mu_{kt}, \sigma_{kt}] \\
&= \exp \left[\begin{aligned} &\log A + \psi \left(\mu_q - \frac{\sigma_q}{\sigma_{\mu t+i}} \mu_{kt+i}^e \right) + \frac{\alpha^2 \sigma_{kt+i}^2}{2} + \\ &\left(\alpha + \frac{\psi \sigma_q}{\sigma_{\mu t+i}} \right) E[\mu_{kt+i} | \mu_{kt}, \sigma_{kt}] + \frac{\left(\alpha + \frac{\psi \sigma_q}{\sigma_{\mu t+i}} \right)^2}{2} \text{Var}[\mu_{kt+i} | \mu_{kt}, \sigma_{kt}] \end{aligned} \right] \\
E[\mu_{kt+i} | \mu_{kt}, \sigma_{kt}^2] &= \mu_{kt+i}^e + \prod_{\tau=1}^i \left(\phi + \frac{\gamma \sigma_q}{\sigma_{\mu t+i-\tau}} \right) (\mu_{kt} - \mu_{kt}^e) \\
\text{Var}[\mu_{kt+i} | \mu_{kt}, \sigma_{kt}^2] &= \sum_{\tau=1}^i \prod_{s=1}^{\tau-1} \left(\phi + \frac{\gamma \sigma_q}{\sigma_{\mu t+i-s}} \right)^2 \phi^2 h_{t+i-\tau} \sigma_{kt+i-\tau}^2
\end{aligned}$$

and

$$\begin{aligned}
\mu_{kt+i}^e &= \frac{1 - \phi^i}{1 - \phi} \left(\ln B + \gamma \mu_q - \frac{\sigma_\varepsilon^2}{2} \right) + \phi^{i-1} \mu_{kt}^e \\
\sigma_{\mu t+i} &= g^i(\mathbf{x}_t) \equiv g \left((\mu_{kt+i-1}^e, g^{i-1}(\mathbf{x}_t), m^{i-1}(\mathbf{x}_t))^T \right) \\
\sigma_{kt+i}^2 &= m^i(\mathbf{x}_t) \equiv m \left((\mu_{kt+i-1}^e, g^{i-1}(\mathbf{x}_t), m^{i-1}(\mathbf{x}_t))^T \right)
\end{aligned}$$

and

$$\begin{aligned}
g^1(\mathbf{x}_t) &= g(\mathbf{x}_t) = \sqrt{(\phi \sigma_{\mu t} + \gamma \sigma_q)^2 + \phi^2 h_t (\sigma_{kt}^a)^2} \\
m^1(\mathbf{x}_t) &= m(\mathbf{x}_t) = \sqrt{\phi^2 (1 - h_t^a) (\sigma_{kt}^a)^2 + \sigma_\varepsilon^2}
\end{aligned}$$

Data Appendix: ($(\#X)_t$ implies COMPUSTAT number X in year t .)

- Labor expense...If a firm reports labor and related expense $(\#42)_t$, we use it as labor expense. If not, we estimate it as follow. We estimate the proportion of the labor and related expense $(\#42)_t$ to cost of goods directly sold $(\#41)_t$ by the firm that report $(\#42)_t$. We estimate the average of the proportion by industry and year. We multiply this industry average proportion in year t to the cost of goods directly sold in year t $(\#41)_t$ if the firm does not report $(\#42)_t$. This is our estimate of labor expense in a large sample. Small sample simply covers firms that report $(\#42)_t$.
- y_t ...the value added over the number of employees $(\#29)_t$. The value added is constructed by sales $(\#12)_t$ minus material, where material is constructed by total expense minus labor expense and total expense is defined as operating income $(\#13)_t$ minus sales $(\#12)_t$.
- w_t ...the wage is estimated by labor cost over the number of employees $(\#29)_t$.
- V_{t+1} ... the market value of a firm at the end of year over the number of employees $(\#29)_t$, where the market value of a firm at the end of year equals the book value of assets, $(\#6)_t$, plus the market value of common equity, $(\#25)_t \times (\#199)_t$, less the book value of common equity, $(\#60)_t$ and balance sheet deferred taxes $(\#74)_t$.

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