Inflation under a monetary policy game

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Theme

- My thesis is about applying Calvo's(1978) model as a monetary policy game. This application is largely based on Chang(1998).
- My original conrtibutions are:
  - Computing the sustainable equilibrium value set using Judd et al.(2000)'s method by introducing the public randomization.
  - Studying inflation under a monetary policy game as the purnishment for deviation.
- Today's presentation is about:
  - Explaining the time inconsistency problem and Chang's paper.
  - Introducing the public randomization into Chang's environment.
  - Computing the sustainable eq. value set using Judd et al.'s method, and show the numerical results.(If have time)

Time inconsistency problem

- What is a monetary policy game?
  - Strategic behavior between a gov't and hshlds.
  - There arises a time inconsistency problem, which was first studied by Kydland and Prescott(1977) and Calvo(1978). The time inconsistency problem *cannot* be solved recursively with the usual optimal control theory.
- Time inconsistency problem is: The inconsistency of the optimal gov't policy before and after the hshlds' action.
- The analyses of the problems are classified into two types:
  - With gov't commitment("the Ramsey problem")
  - Without gov't commitment
- We study the optimal policy *without* commitment, and its basic idea is based on Chari and Kehoe(1990). By their work, time inconsistency problem is better viewed as a dynamic game between the gov't and hshlds. Abreu et al.(1986,1990) is a natural starting point for analysis.

Time inconsistency problem(cont.)

- Recently, there is a progress: The approach using "the marginal utility of state"(Marcet and Marimon(1998), based on Kydland and Prescott(1980)). Merging this with APS, we can consider more difficult cases without commitment.
  - Chang(1998) studied the optimum monetary policy under the condition without commitment, and showed the Friedman rule is sustainable even without commitment.
    - In Friedman rule, the optimal policy is to satiate the economy with real balance, and by generating deflation that drives the net interest rate to zero.
  - Recently, Judd et al.(2000) shows the way of implementation. One of my contributions is to implement this method on Chang's environment.

The model

- We assume that the gov't can commit its future policy in time 0.
- the hshld's problem

$$\max_{\{c_t, m_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(m_t)] \tag{1}$$

$$\text{s.t. } c_t + m_t + x_t \leq y_t + m_{t-1}\frac{P_{t-1}}{P_t}, \qquad y_t = f(x_t) \tag{2}$$

$$f(0) > 0, f'(0) = 0, f''(x) < 0, f(-x) = f(x)$$

- In Calvo's model, time inconsistency problem is due to: (a) The nature of the demand for money, (b) The fact that the inconsistency implies in general the use of distortionary taxation.
- The gov't budget constraint:

$$M_t^s - M_{t-1}^s = -P_t x_t \tag{3}$$

$$\text{Using } \frac{M_t^s}{P_t} = m_t, \frac{M_{t-1}^s}{M_t^s} = h_t, m_t(h_t - 1) = x_t \tag{4}$$

The model(cont.)

- Let $E = [0, \bar{m}] \times \Pi \times X$, where Π and X are the compact sets and $E^\infty = E \times E \times ....$ We can prove that:

  Prop.1. A competitive equilibrium is completely characterized by an outcome path $\{m_t, x_t, h_t\}_{t=0}^{\infty}$ such that, for all t, $m_t \in [0, \bar{m}], h_t \in \Pi, x_t \in X$, and:

$$m_t(1 - h_t) = -x_t \tag{5}$$

$$m_t\{u'(c_t) - v'(m_t)\} \leq \beta[u'(c_{t+1})(m_{t+1} + x_{t+1})] \quad \text{with equality if } m < \bar{m} \tag{6}$$

- An element in E^∞ satisfying (5)-(6) will be called *a competitive equilibrium path*, $\{m_t, x_t, h_t\}_{t=0}^{\infty}$ and the set of all such sequences is denoted by CE.

Recursive structure of the Ramsey problem

- the Ramsey problem: Max(1) s.t. (5)-(6) (The gov't is benevolent), wrt $\{h_t\}_{t=0}^{\infty}$.
  - the RHS of Euler eq., $\theta_{t+1} = u'[f(x_{t+1})](m_{t+1} + x_{t+1})$ can be seen as the period $t+1$ marginal utility of money "promised" by the equilibrium in period t.
  - the Ramsey problem can be represented as the usual optimal control(recursive) problem in which θ_t is the state variable.
  - This means that any "optimal" action (m_t, x_t, h_t) and next period's marginal value of money θ_{t+1} can be represented as time invariant function of θ_t.
- The following corollary makes precise a sense in which the set of competitive equilibrium path has a recursive structure with the co-state variable θ.

Corollary 1. The continuation of a competitive equiribrium itself is a competitive equilibrium. In other words, if $\{m_t, x_t, h_t\}_{t=0}^{\infty} \in CE$, then $\{m_s, x_s, h_s\}_{s=t}^{\infty} \in CE$ for all t.

Sustainable Plans

- A sustainable plan (σ, α) map the observed public history of the economy into the outcome path $\{m_t, x_t, h_t\}_{t=0}^{\infty}$. Such a strategy profile is defined for *any* histories.
- Now we drop the assumption of commitment. The followings are just informal descriptions.
  - The public history: $\{h_s\}_{s=0}^{t} = (h_0, h_1, ..., h_t), h_s \in \Pi, \forall s$.
  - *A strategy* for the gov't: $\sigma = \{\sigma_t\}_{t=0}^{\infty}$ such that $\sigma_0 \in \Pi$ and $h_t = \sigma_t(h^{t-1})$.
  - σ is *admissible* if after any history $h^{t-1} = \{h_s\}_{s=0}^{t-1}$, the continuation history $\{h_s\}_{s=t}^{\infty}$ is defined as consistent with $\{m_t, x_t, h_t\}_{t=0}^{\infty} \in CE$.
  - *An allocation rule*: $\alpha = \{\alpha_t\}_{t=0}^{\infty}$ such that $(m_t, x_t) = \alpha_t(h^t)$.
  - α is *competitive* if given after any $h^{t-1} = \{h_s\}_{s=0}^{t-1}$ and $h_t = \sigma_t(h^{t-1})$, (σ, α) induce a competitive equilibrium path, $\{m_t, x_t, h_t\}_{t=0}^{\infty} \in CE$.

DEF. A government policy and an allocation rules (σ, α) constitute *a Sustainable Plan* if
(i) σ is admissible.
(ii) α is competitive given σ.
(iii) After any $h^{t-1} = \{h_s\}_{s=0}^{t-1}$, the continuation of σ is optimal for government.

Sustainable Plans(cont.)

- Especially, the condition (iii) can be represented the following the gov't incentive constraint (Notice w is decomposed, and $x = m(h-1)$):

$$w = u(f(x)) + v(m) + \beta w'$$
$$\geq \max_{h} \min_{m,x} [u(f(x)) + v(m)] + \beta \underline{w}'$$

- There is also the property which enable us to apply recursive methods. This property can be handled with the state variable w.

Prop.3. Given any history $h^{t-1} = \{h_s\}_{s=0}^{t-1}$, the continuation of a sustainable plan is itself a sustainable plan.

- From Prop.3 and Corollary 1, we can guess that (w, θ) characterize the set of sustainable equilibrium values in a recursive manner:
  - First, because the government has a time consistency problem, any SP must provide incentives for government not to deviate. These incentive constraints can be handled by the continuation value w.
  - Second, after any history, the continuation of a SP is consistent with a CE for the infinite future. This constraint can be handled with the promised marginal utility of money θ.

The public randomization

- the set $Z \subset W \times \Omega$ has an element (w, θ).
  - The purpose of introducing the public randomization device: to make the set $Z \subset W \times \Omega$ *convex* to apply Judd et al.'s method.
- At the beginning of period t, the outcome r_t of uniform $[0, 1]$ random variable R_t is publicly observed.
  - $\{R_t\}$ are serially uncorrelated and independent of any choices made by the government or the households.
  - $\{R_t\}$ can be used as cordination devices to synchronize the gov't and the hshld's moves and beliefs in a similar fashion as sunspot equilibria.
- The hshld's problem becomes stochastic, and an outcome process solves the problem: $h_t, m_t, x_t : [0, 1]^{t+1} \rightarrow \mathbf{R}_+$. $h_t(r^t), m_t(r^t), x_t(r^t)$ are random and depend on the sequence of random outcomes $r^t = (r_0, ..., r_t)$.

The public randomization(cont.)

- Let $\tilde{E} = E \times [0,1]$, and $\tilde{E}^\infty = \tilde{E} \times \tilde{E} \times ....$ Prop.1 is rewritten as:

  Prop.1' A competitive equilibrium is completely characterized by *an outcome process* $\{m_t(r^t), x_t(r^t), h_t(r^t)\}_{t=0}^\infty$ such that, for all t, $m_t \in [0, \bar{m}], h_t \in \Pi, x_t \in X, r_t \in [0,1], r^t = (r_0, ..., r_t)$, and:

$$m_t(r^t)(1 - h_t(r^t)) = -x_t(r^t) \tag{7}$$

$$m_t(r^t)\{u'(c_t(r^t)) - v'(m_t(r^t))\}$$
$$\leq \beta E_{r_{t+1}}[u'(c_{t+1}(r^{t+1}))(m_{t+1}(r^{t+1}) + x_{t+1}(r^{t+1})) \mid r^t](= \beta E_{r_{t+1}}\theta^R(r_t)) \tag{8}$$

with equality if $m < \bar{m}$

- An element in $\tilde{E}^\infty$ satisfying (7)-(8) will be called *a competitive equilibrium process*, $\{m_t, x_t, h_t\}_{t=0}^\infty$ and the set of all such sequences is denoted by $\tilde{CE}$.

The public randomization(cont.)

- The definitions of SP are a bit modified:
  - The public history: $s^t = \{s_k\}_{k=0}^t = (s_0, s_1, ..., s_t) = (r_0, h_0, r_1, h_1, ..., r_t, h_t), r_k \in [0,1], h_k \in \Pi, \forall k$
  - a strategy and an allocation rule: $h_t = \sigma_t(s^{t-1}, r_t), (m_t, x_t) = \alpha_t(s^t)$
- ex post and ex ante value and the marginal value of money
  - given r_0, the value and the marginal utility of money:

$$w^R(r_0) = E\left[\sum_{t=0}^{\infty} \beta^t [u(c_t(r^t)) + v(m_t(r^t))] \mid r_0\right]$$
$$\theta^R(r_0) = u'[f(x_0(r_0))](m_0(r_0) + x_0(r_0))$$

  - The corresponding *expected* value and marginal value of money:

$$\tilde{w} = E_{r_0} w^R(r_0) = \int_0^1 w^R(r_0) dr_0$$
$$\tilde{\theta} = E_{r_0} \theta^R(r_0) = \int_0^1 \theta^R(r_0) dr_0$$

The public randomization(cont.)

- ex post and ex ante sustainable value set S^R, S are defined

$$\begin{aligned} S^R &= \{(w^R(r_0), \theta^R(r_0)) \mid \text{given } r_0, \text{ there is a SP } (\sigma \mid_{r_0}, \alpha \mid_{s_0}) \\ &\quad \text{whose outcome process is } \{m_t, x_t, h_t\}_{t=0}^{\infty} \in \tilde{CE} \\ &\quad \text{with value } w^R(r_0), \text{ and the marginal utility of money } \theta^R(r_0)\}. \\ S &= \{(\tilde{w}, \tilde{\theta}) \mid \text{there is a SP } (\sigma, \alpha) \\ &\quad \text{whose outcome process is } \{m_t, x_t, h_t\}_{t=0}^{\infty} \in \tilde{CE} \\ &\quad \text{with expected value } w, \text{ and expected marginal utility of money } \theta\}. \end{aligned}$$

LEMMA S is the convex hull of S^R.

- Using this LEMMA, we can define the set Z and the operator $E(Z)$ as the convex set, and apply the Judd et al.'s method.

Operator E

- After defined SP consistent with CE, we can compute S which satisfy the particular conditions required by SP.
- Let $Z \subset W \times \Omega$ from which tomorrow's pairs $(\tilde{w}', \tilde{\theta}')$ can be chosen. Define a new set $E(Z) \subset W \times \Omega$ as follows

$$E(Z) = \{(\tilde{w}, \tilde{\theta}) \mid \text{there is } (m, x, h, \tilde{w}', \tilde{\theta}') \in \tilde{E} \times Z \text{ and}$$

$$\tilde{w} = u(f(x)) + v(m) + \beta \tilde{w}' \tag{9}$$

$$\tilde{\theta} = u'(f(x))(m + x) \tag{10}$$

$$\tilde{w} \geq \max_{h} \min_{m,x} [u(f(x)) + v(m)] + \beta \underline{\tilde{w}}' \tag{11}$$

$$- x = m(1 - h) \tag{12}$$

$$m\{u'[f(x)] - v'(m)\} \leq \beta \tilde{\theta}' \text{ with equality if } m < \bar{m}\} \tag{13}$$

- From these conditions, we can see $E(Z)$ has the recursive structure about $\tilde{w}, \tilde{\theta}$.

Operator E(cont.)

- E satisfies the following properties(Prop.4; we conjecture them):
  - (i) Self generation: If $Z \subset E(Z)$, then $E(Z) \subset S$.
  - (ii) Factorization: $S = E(S)$.
  - (iii) Monotonicity: $Z \subset Z'$ implies $E(Z) \subset E(Z')$.
- Algorithm(Prop.6): Let $Z_0 = W \times \Omega$ and $Z_n = E(Z_{n-1}), n = 1, 2, ...$ Then $Z_\infty = S$.
- Computation: Judd et al.'s outer approximation. The actions of the gov't and hshlds are discretized and it is a linear programming of (w, θ). (For details, see Appendix in the paper)

Numerical Example

- Specific functional form:

$$u(c) = 10000 \log c, \qquad f(x) = 64 - (0.2x)^2, \qquad v(m) = m^f m - \frac{m^2}{2},$$

$$u'(c) = 10000 c^{-1}, \qquad v'(m) = m^f - m,$$

$$m \in [0, m^f], \quad h \in \Pi = [\underline{\pi}, \bar{\pi}] = [0.25, 1.25], \quad x \in X = [-22.5, 7.5]$$

$$m^f = 30, \qquad \beta = \{0.9, 0.95\}$$

- For the reference,

$$w^{min} = (1-\beta)^{-1}(u[\min\{f(x)\}] + v(0))$$

$$w^{max} = (1-\beta)^{-1}(u[f(0)] + v(m^f))$$

$$w^{worst} = (1-\beta)^{-1}(u[f(0)] + v(0))$$: non monetary eq.

$$w^{C} = (1-\beta)^{-1}(u[f(0)] + v(m^{ss})),$$: m^{ss} is the SS value from Euler eq.

$$w^{R} = (1-\beta)^{-1}(u[f(x)] + v(m^f)),$$: $x = m^f(\frac{1}{\beta} - 1)$, Friedman rule

Numerical Example(cont.)

	w^{min}	w^{worst}	w^{C}	w^{R}	w^{max}
w	377849	415888	419168	419691	420388
x	-22.50	0	0	3.3333	0
h	—	0	1	1.1111	1
m	0	0	14.375	30	30

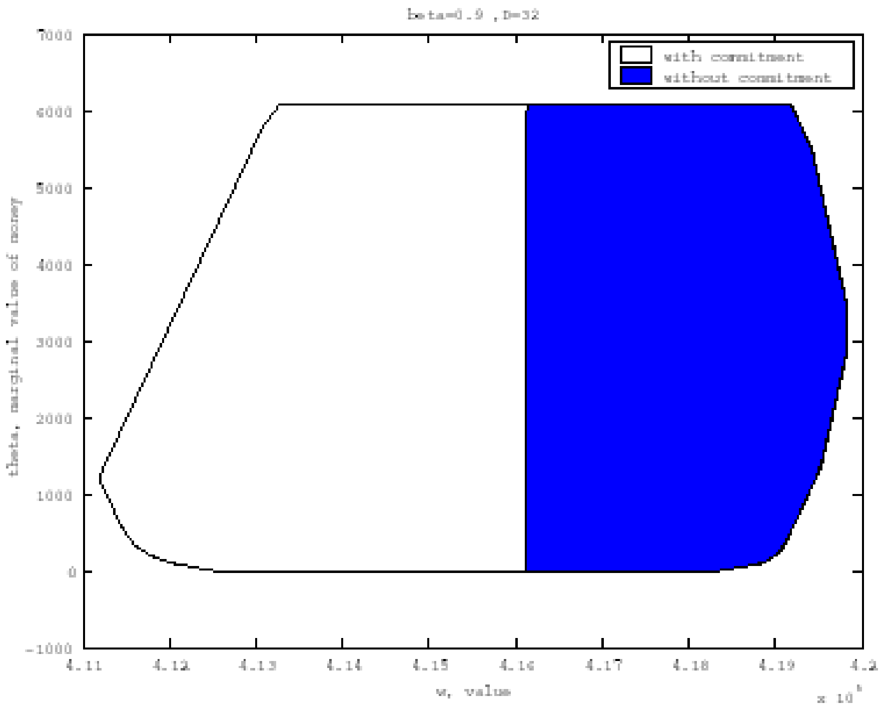
	w^{min}	w^{worst}	w^{C}	w^{R}	w^{max}
w	755698	831777	840166	840464	840777
x	-22.50	0	0	1.5789	0
h	—	0	1	1.0526	1
m	0	0	22.187	30	30

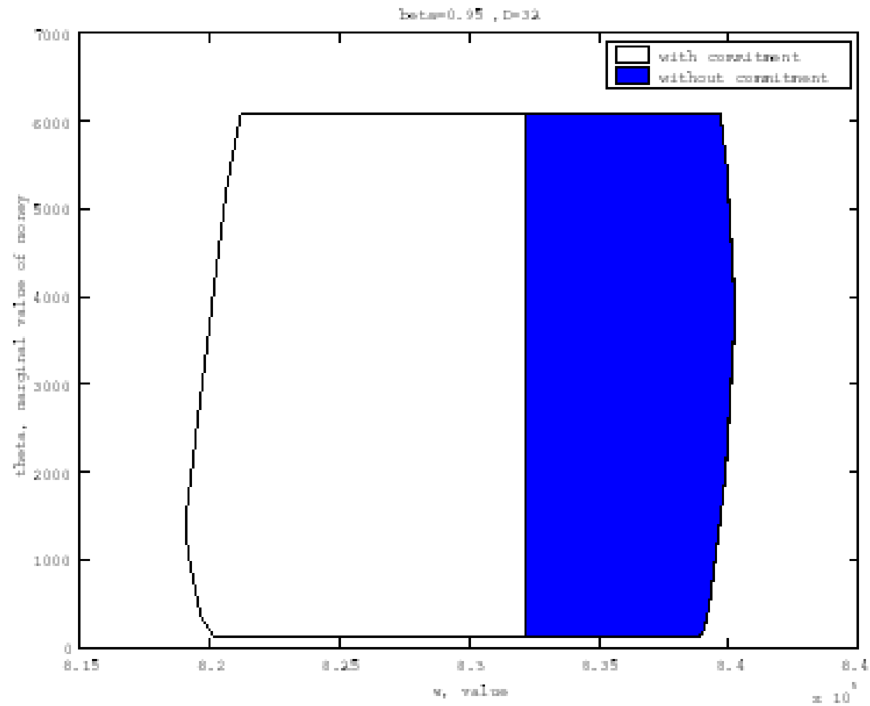
Table 1: The values and outcomes

From above, the value of $\beta = \{0.9, 0.95\}$.

Notice that $x = m(h-1)$. In non monetary equilibrium, the inflation rate is infinite, so $h = 0$.

Numerical Example(cont.)





The value of $\{\underline{w}, \bar{w}\}$(without commitment) is: $\{416105, 419825\}, \{832161, 840255\}$. The left is for $\beta = 0.9$, the right is for $\beta = 0.95$.

Concluding Remarks

- After introducing the public randomization into Chang's environment, The fundamental characters of the economy are preserved, although some proofs remain to be done.
  - As Chang(1998), the gov't can archive the Friedman rule by threat of punishment by hshlds.
  - The value of worst equilibrium(slightly higher than non monetary equilibrium) means inflation, and it is never realized. Deriving the path is to be done using Judd et al.'s ray inner approx. method.
- Inflation never happens in this economy due to the public information: To study the case of private information is one of the interesting topics for future research.