Imperfect Common Knowledge, Staggered Price Setting, and the Effects of Monetary Policy*

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Abstract

This paper studies the consequences of a lack of common knowledge in the transmission of monetary policy by integrating the Woodford (2003a) imperfect common knowledge model with Taylor-Calvo staggered price-setting models. The average price set by monopolistically competitive firms depends on their higher-order expectations about not only the current state of the economy but also about the states in the future periods in which prices are to be fixed. This integrated model provides a plausible explanation for the observed effects of monetary policy: it shows analytically how price adjustments are delayed and how the response of output to monetary disturbances is amplified.

Keywords: Imperfect common knowledge; Higher-order expectations; Public and private information; Staggered price setting.
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1 Introduction

Modern macroeconomic theory provides two main explanations for why monetary policy has real effects in the short run: imperfect information about the policy shocks and short-run rigidity in price or wage adjustment. The imperfect information approach was originally developed by Phelps (1970) and Lucas (1972) in the era in which the traditional output-inflation relationship collapsed. Their arguments, however, were criticized for their practical irrelevance: the Phelps-Lucas models imply that the real effects of monetary policy only last while the precise public information about aggregate disturbances is unavailable, which seems to contradict the observed persistence of business fluctuations despite the availability of macroeconomic data with little delay. To analyze the persistent real effects of monetary policy, many current monetary models of business fluctuations assume short-run rigidity in price or wage adjustment, typically by incorporating staggered price setting as in Taylor (1980) or Calvo (1983).

Recently, some authors have reconsidered the imperfect information approach. Mankiw and Reis (2002) consider sticky information rather than sticky prices, which means some price setters cannot choose their prices based on current information. Woodford (2003a) considers imperfect common knowledge about nominal disturbances in an environment among monopolistically competitive suppliers. These models can generate persistent real effects of monetary policy. Moreover, they can also explain the observed delay in the monetary policy effect on inflation, which implies they overcome a major problem faced by the Taylor-Calvo staggered price-setting models.

These imperfect-information models, however, still leave the original problem in the Phelps-Lucas models unsolved. The source of persistence of the real effects of monetary policy in the Mankiw-Reis model is the outdated information that influences current price setting. In their model, there are always some suppliers who set their prices based on very old information because the probability of obtaining new information in each period is constant and identical for all suppliers however recent their last updates. In the Woodford model, suppliers choose their prices solely on the basis of the history of their subjective observations that contain idiosyncratic perception errors. They never obtain, nor pay attention to, precise information about aggregate demand and even about the actual quantities they sold at their chosen prices. In both models, there would be no persistent real effects of monetary policy if the true state of the economy were revealed to all suppliers with a delay of only one period. These models do not explain why price setters fail to use widely and readily available macroeconomic data.[1]

In this paper, we develop a model that integrates Woodford's imperfect common knowledge model with Taylor-Calvo staggered price-setting models in order to overcome the problems in each of them and explain plausibly the observed effects of monetary policy. The model is based on the standard monopolistic competition framework as in Blanchard and Kiyotaki (1987). Following Woodford, we assume that price setters can only observe the state of the economy through noisy private signals. Their optimal pricing strategy depends not only on their own estimates of the aggregate demand but also on their expectations of the average estimates by other price setters. In such an environment, the overall price level is determined by a weighted sum of price setters' "higher-order expectations," that is, what others expect about what others expect ... about aggregate demand.[2] Meanwhile, we drop Woodford's unrealistic assumption by assuming that the true state of the economy is revealed to all price setters with a delay of one period. Given staggered price setting, however, the model can generate persistent real effects of monetary policy. The average price chosen in each period depends on higher-order expectations about not only the current state of the economy but also about the states in the future periods in which prices are to be fixed. For simplicity, we assume that half of the price setters in the economy set their prices fixed until the next period, namely two-period staggered price setting. Our model of

[1] Some recent studies attempt to explain how agents rationally choose to be inattentive under informational constraints: Sims (2003) considers limited capacity for processing information, while Reis (2006) considers costs of acquiring, absorbing, and processing information and provides a micro-foundation to the assumption of Mankiw and Reis (2002).

[2] Keynes (1936) described the role of higher-order expectations in an asset-pricing context by introducing the famous metaphor of financial markets as "beauty contests." Recently, higher-order beliefs have been extensively studied in the theoretical literature on "global games" (Morris and Shin, 2003) and applied to various fields.

imperfect common knowledge, however, can be integrated with more general price setting that allows for multiple-period staggered price setting[3] including the one analogized with Calvo-type price setting.[4] Despite the complexity in those dynamic and staggered higher-order expectations, the model can be solved analytically by virtue of the assumption that the true current state becomes common knowledge in the subsequent period.

The main results of the model are as follows. The noisier are the private signals, the more sluggish is the initial response of prices to a monetary disturbance. The response that operates through dynamic and staggered higher-order expectations is, in many cases, more sluggish than the one that operates through static and simultaneous higher-order expectations under flexible prices. Following this initial response, price adjustments are delayed and inflation may peak later than in the corresponding full-information staggered price-setting model. The response of output is amplified by the lack of common knowledge and continues to exceed the response in the full-information model. Even a small amount of noise in the private signals may significantly delay the adjustment of prices and amplify the response of output. The model nests the full-information staggered price-setting model as one limit case. As another limit case, it also nests the predetermined-prices model, in which all firms either have no information about the current aggregate disturbances or are simply assumed to set their prices one period in advance.[5] The case of imperfect common knowledge is between these two limit cases, and explains endogenously how price adjustments are delayed.

We extend the above baseline model by introducing a noisy public signal in addition to the private signals and study the consequences of a more general information structure following Hellwig (2002) and Amato and Shin (2003). These authors emphasize the separation of information into public and pri-

[3]Fukunaga (2006) provides some extensions to multiple-period staggered price setting.

[4]Calvo-type price setting corresponds to random-period staggered price setting given a constant probability with which price setters can change their prices in each period. As pointed out by Guerrieri (2006), allowing for a distribution of price durations makes fixed-period staggered price setting closer to Calvo-type price setting.

[5]The latter limit case is analyzed in Section 3.1 of Chapter 3 in Woodford (2003b) to demonstrate a simple and direct way of generating delayed effects of nominal disturbances on inflation.

vate signals and criticize the Woodford model for focusing only on private signals and for lacking considerations of problems involving informational interaction between decision makers. As they argue, in an economy in which decision makers' information sets are heterogeneous, public information has disproportionately large effects on their decisions. Whereas Amato and Shin assume that price setters never obtain precise information as in the Woodford model, we retain the assumption that the true state of the economy is revealed to all price setters with a delay of one period. The public signal in our extended model, then, may represent preliminary data that is to be revised or noisy information promptly provided by the media, the government, and so on. When it is interpreted as a communication tool of the monetary authority, the model has interesting implications for the conduct of monetary policy involving, for example, transparency.

We first show that provision of the public signal alleviates the effects of monetary disturbances. The noisier is the public signal, as are the private signals, the more sluggish is the initial response of prices. Compared with the baseline model without the public signal, which corresponds to the case with an infinite amount of noise in the public signal, the initial response of prices is less sluggish and the response of output is less amplified. Meanwhile, the provision of the public signal exposes firms to an additional aggregate disturbance, namely noise in the public signal itself. For example, a *negative* informational disturbance, that is, downwardly biased information about current aggregate demand, generates delayed inflation and positive response of output as does a positive monetary disturbance. Unlike the responses to monetary disturbances, the responses to informational disturbances do not have a monotonic relationship with the amount of noise in the public signal. A small improvement in the precision (i.e., reduction in the amount of noise) of the public signal may amplify, rather than reduce, the responses to informational disturbances and increase output volatility. This happens because improving precision of the public signal does not only reduce the size of informational disturbances but also makes firms rely heavily on the public signal, and therefore, indirectly generates high responsiveness to informational disturbances. This mechanism corresponds to the one proposed by Morris and

Shin (2002) who show that improving precision of public information could lower welfare in their model. We examine the total effect of the improving precision on the responses to both monetary and informational disturbances and find that the possibility of increasing output volatility is small in our model.

Recent studies on imperfect common knowledge have obtained important results for the analysis of monetary policy and social welfare. Adam (2007) determines the optimal monetary policy in an economy with idiosyncratic information errors about real demand and supply shocks. Amato and Shin (2003) consider a targeting rule in an economy in which firms can access both public and private signals about the natural rate of interest. The debate on the precision of public information, or the central-bank transparency, has received much attention since Svensson (2006) claimed that the model of Morris and Shin (2002) actually implied pro-transparency, that is, improving precision of public information does not, as they argued, lower welfare.[6] Hellwig (2005) extends the Hellwig (2002) model of nominal adjustment and finds welfare-improving effects of public information in the form of reduced price dispersion. Angeletos and Pavan (2005) provides a general analytical framework that relates the inefficiency of business cycles to the social value of information. These studies, however, are typically based on static models, or assume an unrealistic lack of awareness or attentiveness as does Woodford (2003a).

Meanwhile, the attempt to integrate imperfect common knowledge with staggered price setting has never been made until very recently.[7] Nimark (2005) considers firms' private information about their own marginal costs that includes an idiosyncratic component, and derives a Phillips curve based on

[6] In response to Svensson's claim, Morris, Shin, and Tong (2006) and Morris and Shin (2005) develop their arguments, emphasizing the endogenous nature of the precision of public information.

[7] It has been argued, however, that imperfect information and nominal rigidities are closely related to each other as plausible explanations for the real effects of monetary policy. Ball and Cecchetti (1988) develop a model in which monopolistically competitive firms gain information by observing the prices set by others and then the staggered price setting arises endogenously as the equilibrium outcome under certain conditions. Kiley (2000) develops a model that has costs of nominal price adjustment as well as costs of acquiring information in order to estimate the degree of price stickiness.

Calvo-type price setting. Morris and Shin (2006) consider higher-order expectations iterated in a forward-looking manner, which can be applied to the purely forward-looking New Keynesian Philips curve based on Calvo-type price setting. These approaches, however, are not sufficiently tractable for obtaining various analytical results.

The remainder of the paper is organized as follows. Section 2 describes the baseline model and presents the main results on the effects of monetary disturbances. Section 3 extends the baseline model by introducing a noisy public signal, in addition to private signals, and examines the effects of informational disturbances as well as monetary disturbances. Section 4 concludes.

2 The Baseline Model

In this section, we incorporate a lack of common knowledge into a simple two-period staggered price-setting model and then analytically examine the effects of monetary disturbances.

2.1 Set-up

Consider an economy in which a continuum of monopolistically competitive firms indexed by $i \in [0, 1]$ produce individual-specific goods and set their own prices. Goods are perishable and capital is not required as a factor of production. Following Woodford (2003a) and Mankiw and Reis (2002), we begin with the static optimal price-setting condition of firm i.[8]

$$p_t^*(i) = E_t^i\, p_t + \xi\, E_t^i\, y_t, \quad 0 < \xi < 1 \tag{1}$$

All variables are expressed in terms of log deviations from the full-information symmetric equilibrium. $p_t^*(i)$ is firm i's desired price in period t and would be the actual price if firms could set their prices flexibly. p_t is the overall price index and y_t is the output gap. The parameter ξ is assumed to be less

[8]This condition can be derived from a standard monopolistic competition model such as Blanchard and Kiyotaki (1987). Woodford (2003a) provides a simple explanation of the background of equation (1).

than unity so that firms' price-setting decisions are strategic complements. The higher the elasticity of substitution among the differentiated goods or the lower the elasticity of marginal cost with respect to output, the smaller is ξ and the greater is the degree of strategic complementarity.

Firms cannot precisely observe aggregate variables such as p_t and y_t in the current period, t. Moreover, their information sets are heterogeneous, which is the main feature of our model. Accordingly, the expectations operator conditional on i's information set at period t, E_t^i, is applied to p_t and y_t in the above equation. The details of the private information set and the signal extraction problem are explained in the next subsection.

Next, we introduce the two-period staggered price setting as in Taylor (1980). In period t, half of the firms in the economy set their prices for the current period, t, and the next period, $t+1$. Since they must set the same price for both periods, prices are not just pre-determined but fixed. The price chosen by firm i that sets its price in t is given by[9]

$$\begin{aligned} x_t(i) &= \frac{1}{2}\,(p_t^*(i) + E_t^i\, p_{t+1}^*(i)) \\ &= \frac{1}{2}\,(E_t^i\, p_t + \xi\, E_t^i\, y_t + E_t^i\, p_{t+1} + \xi\, E_t^i\, y_{t+1}). \end{aligned} \tag{2}$$

In period $t+1$, the remaining half of the firms set their prices for periods $t+1$ and $t+2$. In period $t+2$, the firms that set their prices in period t then re-set their prices for periods $t+2$ and $t+3$, and so on. The overall price index is given by

$$p_t = \frac{1}{2}\,(x_t + x_{t-1}), \tag{3}$$

where x_t is the average price chosen by the firms that set their prices in t, that is, $x_t \equiv 2\int_0^{0.5} x_t(i)\, di$ when $t = \cdots, -2, 0, 2, \cdots$, and $x_t \equiv 2\int_{0.5}^{1} x_t(i)\, di$ when $t = \cdots, -1, 1, \cdots$.

We specify the demand side of the economy by introducing an exogenous stochastic process for aggregate nominal spending as follows.

[9] For simplicity, we assume that the discount rate applied to the firm's profits in the next period is negligible.

$$m_t - m_{t-1} = \rho\,(m_{t-1} - m_{t-2}) + \sigma\,\epsilon_t, \qquad \epsilon_t \sim N(0,\,1) \tag{4}$$

where

$$m_t = p_t + y_t \tag{5}$$

and ϵ_t is Gaussian white noise. One may interpret m_t as "money" that households must hold for their spending. Following Mankiw and Reis (2002), we treat the above process as a plausible stochastic process for representing the actual money supply (M2) in the U.S.[10] Alternatively, m_t can be interpreted more broadly as a generic variable affecting aggregate demand. This simple specification for aggregate demand, however it is interpreted, allows us to concentrate on examining the consequences of alternative specifications for price-setting behavior.

2.2 Signal Extraction

Here we specify firms' information sets. As in Lucas (1972) and Woodford (2003a), each individual firm estimates the current state of the economy by using their private information. In period t, firm i has access to a noisy private signal about current aggregate demand, m_t, which is represented as follows.

$$z_t(i) = m_t + \sigma_u\, u_t(i), \qquad u_t(i) \sim N(0,\,1) \tag{6}$$

where $u_t(i)$ is Gaussian white noise, which is distributed independently of both ϵ_t and $u_t(j)$ for all $j \neq i$.

Unlike Woodford, we assume that the true value of m_t becomes common knowledge among all firms with a delay of only one period, in period $t+1$. Therefore, the information set of firm i comprises the private signal, $z_t(i)$, and the history of realized aggregate nominal spending, $\{m_{t-s}\}_{s=1}^{\infty}$. The result of firms' signal extraction for estimating m_t is given by

[10] Woodford (2003a) specifies almost the same stochastic process as (4), except that he adds a drift term that represents the long-run average growth rate of aggregate nominal spending.

$$
\begin{aligned}
E_t^i\, m_t \;&\equiv\; E[\, m_t \mid z_t(i),\, m_{t-1},\, m_{t-2}, \ldots] \\
&=\; b\, z_t(i) + (1-b)\, \{m_{t-1} + \rho\, (m_{t-1} - m_{t-2})\}
\end{aligned}
\tag{7}
$$

where

$$
b \equiv \frac{\sigma^2}{\sigma^2 + \sigma_u^2}
$$

represents firms' reliance on their private signals. Given the variance of aggregate nominal spending, this reliance is greater, the higher is the precision of the signals (the smaller is σ_u).

2.3 Higher-Order Expectations

Unlike the Lucas model, our model considers an environment among monopolistically competitive firms whose pricing strategies depend on the other firms' strategies. The prices chosen by the firms depend not only on their own estimates of current aggregate demand but also on their expectations of the average estimate among the other firms, their expectations of the average estimate of that average estimate, and so on.

Averaging (7) over i, we have

$$
\begin{aligned}
\overline{E}_t\, m_t \;&=\; b\, m_t + (1-b)\, \{m_{t-1} + \rho\, (m_{t-1} - m_{t-2})\} \\
&=\; b\, \sigma\, \epsilon_t + \{m_{t-1} + \rho\, (m_{t-1} - m_{t-2})\},
\end{aligned}
\tag{8}
$$

where $\overline{E}_t$ is the average expectations operator. The second line implies that the average estimate is not equal to the true value of m_t defined by (4) despite the assumption that the mean of the private signals is equal to the true value. The average estimate is closer to the true value when the private signals are more precise and reliable. When $\sigma_u = 0$, all firms can access homogeneous precise signals and the average expectations operator no longer needs to be defined.

The average expectations operator, defined for heterogeneous information sets, does not satisfy the law of iterative expectations. Firm i's expectation of the average estimate (8) can be calculated as follows.

$$\begin{aligned} E_t^i\,[\,\overline{E}_t\, m_t\,] &= b\,[\,b\, z_t(i) + (1-b)\ \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\}\,] \\ &\quad +(1-b)\ \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\} \end{aligned}$$

Averaging again over i, we have

$$\begin{aligned} \overline{E}_t\,[\,\overline{E}_t\, m_t\,] &= b^2\, m_t + (1-b^2)\ \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\} \\ &= b^2\,\sigma\,\epsilon_t + \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\}, \end{aligned}$$

which differs from (8). Therefore, we need to define the j-th order average expectations as follows.

$$\begin{aligned} \overline{E}_t^{(0)}\, m_t &\equiv m_t \\ \overline{E}_t^{(j+1)}\, m_t &\equiv \overline{E}_t\,[\,\overline{E}_t^{(j)}\, m_t\,] \end{aligned}$$

The higher-order average expectations can be calculated as

$$\begin{aligned} E_t^i\,[\,\overline{E}_t^{(j)}\, m_t\,] &= b^{j+1}\, z_t(i) + (1-b^{j+1})\ \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\} \\ \overline{E}_t^{(j+1)}\, m_t &= b^{j+1}\, m_t + (1-b^{j+1})\ \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\} \\ &= b^{j+1}\,\sigma\,\epsilon_t + \{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\}. \end{aligned} \tag{9}$$

Since b is less than 1, the infinite-order average expectation converges to the expectations that are conditional only on common knowledge about the history of realized aggregate nominal spending.

2.4 Solving the Model

We seek to find a rational expectations equilibrium, which is defined as a set of $\{\, p_t,\, y_t\,\}$ that satisfies the model equations (1), (2), (3), and (5) given the exogenous process for aggregate nominal spending (4) and the information structure described in the preceding subsections. The key endogenous variable in the model is the re-set price, x_t. Combining equations (1) through

(5) yields

$$
\begin{aligned}
x_t(i) &= \frac{1}{2}\,(E_t^i\,p_t + \xi\,E_t^i\,y_t + E_t^i\,p_{t+1} + \xi\,E_t^i\,y_{t+1}) \\
&= \frac{1}{2}\,\{\xi\,E_t^i\,m_t + (1-\xi)\,E_t^i\,p_t + \xi\,E_t^i\,m_{t+1} + (1-\xi)\,E_t^i\,p_{t+1}\} \\
&= \frac{1}{2}\,\{\xi\,E_t^i\,m_t + \xi\,E_t^i\,m_{t+1} + (1-\xi)\,E_t^i\,x_t + \frac{1-\xi}{2}\,E_t^i\,x_{t+1} + \frac{1-\xi}{2}\,x_{t-1}\} \\
&= \frac{1}{2}\,\{\xi\,(2+\rho)\,E_t^i\,m_t - \xi\,\rho\,m_{t-1} \\
&\quad +(1-\xi)\,E_t^i\,x_t + \frac{1-\xi}{2}\,E_t^i\,x_{t+1} + \frac{1-\xi}{2}\,x_{t-1}\}.
\end{aligned}
$$

The price chosen by firm i that sets its price in period t depends on its estimate of current aggregate demand, m_t, its estimate of the average price among the firms that set their prices in the same period, x_t, and its estimate of the future average price chosen by the other group of firms, x_{t+1}. The price also depends on the past realized value of aggregate nominal spending, m_{t-1}, and the past average price chosen by the other group of firms, x_{t-1}, which are known in period t and therefore the expectations operators need not be added to these terms.

Averaging $x_t(i)$ over the group of firms that set their prices in t, we have

$$
\begin{aligned}
x_t &= \frac{1}{2}\,\{\xi\,(2+\rho)\,\overline{E}_t\,m_t - \xi\,\rho\,m_{t-1} + \\
&\quad (1-\xi)\,\overline{E}_t\,x_t + \frac{1-\xi}{2}\,\overline{E}_t\,x_{t+1} + \frac{1-\xi}{2}\,x_{t-1}\},
\end{aligned}
\tag{10}
$$

where the average expectations operator is defined as $\overline{E}_t(\cdot) \equiv 2\int_0^{0.5} E_t^i(\cdot)\,di$ when $t = \cdots, -2, 0, 2, \cdots$, and $\overline{E}_t(\cdot) \equiv 2\int_{0.5}^{1} E_t^i(\cdot)\,di$ when $t = \cdots, -1, 1, \cdots$.

Apart from the average expectations operator, the above equation can be regarded as a second-order difference equation for x_t, similar to the ordinary two-period staggered price-setting model with full homogenous information sets. We suppose that all firms in both groups believe that the solution of the difference equation takes the following form.

$$
x_t = \lambda\,x_{t-1} + C_1\,m_{t-1} + C_2\,m_{t-2} + C_3\,\sigma\,\epsilon_t, \tag{11}
$$

where λ, C_1, C_2, and C_3 are undetermined coefficients. By substituting this solution form into (10), we eliminate the term of x_{t+1}.

$$\begin{aligned}
x_t &= \frac{1}{2}\{\xi\,(2+\rho)\,\overline{E}_t\,m_t - \xi\,\rho\,m_{t-1} + (1-\xi)\,\overline{E}_t\,x_t \\
&\quad + \frac{1-\xi}{2}\,(\lambda\,\overline{E}_t\,x_t + C_1\,\overline{E}_t\,m_t + C_2\,m_{t-1}) + \frac{1-\xi}{2}\,x_{t-1}\} \\
&= \frac{1}{4}[\,\{2\,\xi\,(2+\rho) + (1-\xi)\,C_1\}\,\overline{E}_t\,m_t + \{(1-\xi)\,C_2 - 2\,\xi\,\rho\}\,m_{t-1} \\
&\quad + (2+\lambda)\,(1-\xi)\,\overline{E}_t\,x_t + (1-\xi)\,x_{t-1}\,]
\end{aligned}$$

Note that $E^i_t\,\epsilon_{t+1} (= \overline{E}_t\,\epsilon_{t+1}) = 0$ for all i. Then, iterative substitutions for x_t yield higher-order expectations about m_t.

$$\begin{aligned}
x_t &= \frac{2\,\xi\,(2+\rho) + (1-\xi)\,C_1}{4} \sum_{j=1}^{\infty} \left\{\frac{(2+\lambda)\,(1-\xi)}{4}\right\}^{j-1} \overline{E}^{(j)}_t\,m_t \\
&\quad + \frac{(1-\xi)\,C_2 - 2\,\xi\,\rho}{4-(2+\lambda)\,(1-\xi)}\,m_{t-1} + \frac{1-\xi}{4-(2+\lambda)\,(1-\xi)}\,x_{t-1}
\end{aligned} \tag{12}$$

This implies that firms consider the weighted sum of higher-order expectations up to the infinite order when choosing their prices. Using (9) to substitute for $\overline{E}^{(j)}_t\,m_t$, we obtain

$$\begin{aligned}
x_t &= \frac{b\,\{2\,\xi\,(2+\rho) + (1-\xi)\,C_1\}}{4-(2+\lambda)\,(1-\xi)\,b}\,\sigma\,\epsilon_t \\
&\quad + \frac{2\,\xi\,(2+\rho) + (1-\xi)\,C_1}{4-(2+\lambda)\,(1-\xi)}\,\{m_{t-1} + \rho\,(m_{t-1} - m_{t-2})\} \\
&\quad + \frac{(1-\xi)\,C_2 - 2\,\xi\,\rho}{4-(2+\lambda)\,(1-\xi)}\,m_{t-1} + \frac{1-\xi}{4-(2+\lambda)\,(1-\xi)}\,x_{t-1}.
\end{aligned}$$

By matching this with the solution form (11), the values of the undetermined coefficients, which provide a unique stable solution of the difference equation for x_t, are identified as follows.

$$\lambda = \frac{1-\sqrt{\xi}}{1+\sqrt{\xi}} \quad < 1$$

$$
\begin{aligned}
C_1 &= \frac{2\sqrt{\xi}}{1+\sqrt{\xi}} + \frac{2\,\rho\sqrt{\xi}\,(1+\sqrt{\xi}+\rho\sqrt{\xi})}{(1+\sqrt{\xi})\,\{1+\sqrt{\xi}-\rho\,(1-\sqrt{\xi})\}} \\
C_2 &= -\frac{2\,\rho\sqrt{\xi}\,(1+\sqrt{\xi}+\rho\sqrt{\xi})}{(1+\sqrt{\xi})\,\{1+\sqrt{\xi}-\rho\,(1-\sqrt{\xi})\}} \\
C_3 &= \frac{2\sqrt{\xi}\,(1+\sqrt{\xi})\,(1+\sqrt{\xi}+\rho\sqrt{\xi})\,b}{\{1+\sqrt{\xi}-\rho\,(1-\sqrt{\xi})\}\,\{4-b\,(3-2\sqrt{\xi}-\xi)\}}
\end{aligned}
$$

The set of equilibrium paths $\{\, p_t,\, y_t \,\}$ can be calculated as

$$
\begin{aligned}
p_t &= \frac{1}{2}\,(\lambda\, x_{t-1} + C_1\, m_{t-1} + C_2\, m_{t-2} + C_3\,\sigma\,\epsilon_t \\
&\quad +\lambda\, x_{t-2} + C_1\, m_{t-2} + C_2\, m_{t-3} + C_3\,\sigma\,\epsilon_{t-1}) \\
&= \lambda\, p_{t-1} + \frac{C_1+C_3}{2}\, m_{t-1} + \frac{C_1+C_2-(1+\rho)\,C_3}{2}\, m_{t-2} \\
&\quad + \frac{C_2+\rho\, C_3}{2}\, m_{t-3} + \frac{C_3}{2}\,\sigma\,\epsilon_t \qquad (13) \\
y_t &= \lambda\, y_{t-1} + \left(1+\rho-\lambda-\frac{C_1+C_3}{2}\right) m_{t-1} - \left(\rho + \frac{C_1+C_2-(1+\rho)\,C_3}{2}\right) m_{t-2} \\
&\quad - \frac{C_2+\rho\, C_3}{2}\, m_{t-3} + \left(1-\frac{C_3}{2}\right)\sigma\,\epsilon_t \qquad (14)
\end{aligned}
$$

2.5 Impulse Responses

From the solution of the model obtained in the previous subsection, we examine the impulse responses of inflation and output to a monetary disturbance. We compare the responses in our model with those in the full-information two-period staggered price-setting model to study the consequences of a lack of common knowledge. Our model nests as a limit case the full-information two-period staggered price-setting model in which all firms can access homogeneous precise information about the realization of the current aggregate disturbances, $\sigma_u = 0$, so that $b = 1$. The other limit case, $\sigma_u = \infty$, so that $b = 0$, implies that all firms have no information about the current aggregate disturbances or are simply assumed to set their prices one period in advance. The case of imperfect common knowledge is between these two limit cases, and explains endogenously how price adjustments are delayed.

The impulse responses of the price level and output to a unit positive innovation in ϵ_0 are calculated as a set of equilibrium paths $\{\, \hat{p}_t, \hat{y}_t \,\}$ with

$\epsilon_0 = 1$, $\epsilon_t = 0$ for all $t \neq 0$, $p_{-1} = y_{-1} = m_{-1} = m_{-2} = m_{-3} = 0$, and $\lim_{t\to\infty} y_t = 0$ in (13) and (14). The main analytical results are summarized in the following proposition.

Proposition 1. *i) The impulse response of inflation is initially decreasing and later increasing in the amount of noise in firms' private signals, σ_u, i.e.,*

$$\begin{aligned}\frac{\partial(\hat{p}_t - \hat{p}_{t-1})}{\partial\sigma_u} &< 0, \quad t = 0,\, 1.\\ \frac{\partial(\hat{p}_t - \hat{p}_{t-1})}{\partial\sigma_u} &> 0, \quad t \geq 2.\end{aligned}$$

ii) The impulse response of output is increasing in σ_u, i.e.,

$$\frac{\partial\hat{y}_t}{\partial\sigma_u} > 0, \quad t \geq 0.$$

Proof. i) Taking the partial derivative of $(\hat{p}_t - \hat{p}_{t-1})$ with respect to σ_u sequentially, we have

$$\begin{aligned}\frac{\partial\hat{p}_0}{\partial\sigma_u} &= \frac{\sigma}{2}\,\frac{\partial C_3}{\partial b}\,\frac{\partial b}{\partial\sigma_u}\\ &= -\frac{8\sqrt{\xi}\,(1+\sqrt{\xi})\,(1+\sqrt{\xi}+\rho\sqrt{\xi})\,\sigma_u\,\sigma^3}{\{1+\sqrt{\xi}-\rho\,(1-\sqrt{\xi})\}\,\{4\,\sigma_u^2+(1+\sqrt{\xi})^2\,\sigma^2\}^2}\\ \frac{\partial(\hat{p}_1 - \hat{p}_0)}{\partial\sigma_u} &= \lambda\,\frac{\partial\hat{p}_0}{\partial\sigma_u}\\ \frac{\partial(\hat{p}_2 - \hat{p}_1)}{\partial\sigma_u} &= -(1-\lambda^2)\,\frac{\partial\hat{p}_0}{\partial\sigma_u}\\ \frac{\partial(\hat{p}_t - \hat{p}_{t-1})}{\partial\sigma_u} &= \lambda\,\frac{\partial(\hat{p}_{t-1} - \hat{p}_{t-2})}{\partial\sigma_u}, \quad t \geq 3.\end{aligned}$$

ii) Taking the partial derivative of $\hat{y}_t$ with respect to σ_u sequentially, we have

$$\begin{aligned}\frac{\partial\hat{y}_0}{\partial\sigma_u} &= -\frac{\sigma}{2}\,\frac{\partial C_3}{\partial b}\,\frac{\partial b}{\partial\sigma_u}\\ &= \frac{8\sqrt{\xi}\,(1+\sqrt{\xi})\,(1+\sqrt{\xi}+\rho\sqrt{\xi})\,\sigma_u\,\sigma^3}{\{1+\sqrt{\xi}-\rho\,(1-\sqrt{\xi})\}\,\{4\,\sigma_u^2+(1+\sqrt{\xi})^2\,\sigma^2\}^2}\end{aligned}$$

$$
\begin{aligned}
\frac{\partial \hat{y}_1}{\partial \sigma_u} &= (1+\lambda)\frac{\partial \hat{y}_0}{\partial \sigma_u} \\
\frac{\partial \hat{y}_t}{\partial \sigma_u} &= \lambda \frac{\partial \hat{y}_{t-1}}{\partial \sigma_u}, \quad t \geq 2. \quad \blacksquare
\end{aligned}
$$

The noisier are the private signals, the more sluggish is the initial response of prices. Accordingly, price adjustments are delayed and inflation may peak later than in the full-information staggered price-setting model. The response of output is amplified by the lack of common knowledge in period 0 and continues to exceed the response in the full-information model even after precise information about the disturbances becomes common knowledge in period 1. The responses in the imperfect-common-knowledge models are persistent in a sense that their half lives may be longer than those in the full-information model while their total lives are the same.[11]

Given the variance of the aggregate nominal spending, the amount of noise in the private signals, σ_u, corresponds to firms' reliance on their signals, b, one-to-one. Therefore, the above results can be applied to b instead of σ_u: the more reliable are the private signals, the less sluggish is the initial response of prices, that is, $\partial \hat{p}_0 / \partial b > 0$. Moreover, it can be shown that the extent to which a change in the reliance affects the sluggishness in the initial response of prices is greater when the private signals are more reliable, that is, $\partial^2 \hat{p}_0 / \partial b^2 > 0$. This implies that even a small amount of noise in the private signals may significantly delay the adjustment of prices and amplify the response of output.

Sample sets of impulse responses in the models in which $b = 0$, 0.25, 0.5, 0.75, and $b = 1$ (the full-information model) are shown in Figure 1. The size of the shock in the aggregate nominal spending, σ, is set to 1 in each model, which implies $\sigma_u = \infty$, $\sqrt{3}$, 1, $1/\sqrt{3}$ and $\sigma_u = 0$ in those models, respectively. In the model with $b = 0.5$, the size of the initial response of prices is about a third of that in the full-information model, and the response of output is amplified by more than 45 percent in the first four periods. The half lives of

[11] Following Chari, Kehoe, and McGrattan (2000), we define "half life" as the length of time after a disturbance before the response shrinks to half of its impact value.

the output responses are 3 periods in the models with $b = 0$, 0.25, and 0.5, while 2 periods in those with $b = 0.75$ and $b = 1$ (full information).

In Figure 1, the parameter value for the strategic complementarity, ξ, is set to 0.15 following Woodford (2003a), and the AR(1) coefficient on the process for quarterly aggregate nominal spending, ρ, is set to 0.5 following Mankiw and Reis (2002). A smaller ξ, that is, a higher degree of strategic complementarity implies a larger λ, which indicates more persistent responses, and also implies a smaller C_3, which indicates more sluggishness in the initial response of prices. A smaller ρ, that is, a less persistent shock process implies a smaller C_3 but has no implication for λ.

Another interesting comparison can be made between the responses in our baseline model obtained above and those in a flexible-prices model with the same information structure.[12] In the flexible-prices model, prices are set by all firms in each period based on simultaneous higher-order expectations only about the current aggregate nominal spending. The average price in the whole economy is expressed as follows.

$$
\begin{aligned}
p_t &= \overline{E}\, p_t + \xi\, \overline{E}\, y_t \\
&= \xi\, \overline{E}\, m_t + (1-\xi)\, \overline{E}\, p_t \\
&= \xi \sum_{j=1}^{\infty} (1-\xi)^{j-1}\, \overline{E}^{(j)}\, m_t,
\end{aligned}
$$

where the average expectations operator is now defined as $\overline{E}(\cdot) \equiv \int_0^1 E^i(\cdot)\, di$. Using (9) to substitute for $\overline{E}_t^{(j)}\, m_t$, we have the initial response of prices with $\epsilon_0 = 1$ and $m_{-1} = m_{-2} = 0$.

$$
\hat{p}_0^F = \frac{\xi\, b\, \sigma}{1 - b\,(1-\xi)}. \tag{15}
$$

From period 1 onward, all firms know precise information about the shock in period 0 so that they can adjust their prices flexibly to the process of the

[12] It is also interesting to make a comparison between the time-dependent staggered price-setting model (our baseline model) and a state-dependent price-setting model with the same information structure. In the latter model, the initial response of prices is likely to be sluggish due to menu costs, but the adjustment after the precise information about the initial shock becomes common knowledge might be rapid as in the flexible-prices model.

aggregate nominal spending, that is, $\hat{p}_t^F = m_t$ and $\hat{y}_t^F = 0$ for all $t \geq 1$.

Sample sets of impulse responses in the flexible-prices models in which $b = 0$, 0.25, 0.5, 0.75, and $b = 1$ are shown in Figure 2. Compared with Figure 1, it clearly shows that the responses of inflation as well as output in the flexible-prices model is less persistent than in the baseline model. Moreover, concerning the initial response, $\hat{p}_0^F < \hat{p}_0$ in the models with $b = 0.25$ and $b = 0.5$, $\hat{p}_0^F > \hat{p}_0$ in the models with $b = 0.75$ and $b = 1$, while $\hat{p}_0 = \hat{p}_0^F = 0$ in the models with $b = 0$. These imply that the initial response of prices that operates through static and simultaneous higher-order expectations under flexible prices is less sluggish than the one that operates through dynamic and staggered higher-order expectations about the future states of the economy as well as the current state when the private signals are sufficiently reliable (b is close to 1).[13] If we assume $\rho = 0$ instead of $\rho = 0.5$, the initial response of prices in the baseline model is

$$\hat{p}_0 = \frac{\sqrt{\xi}\,(1 + \sqrt{\xi})\, b\, \sigma}{4 - b\,(3 - 2\sqrt{\xi} - \xi)}. \tag{16}$$

From the comparison between (15) and (16), we can conclude that $\hat{p}_0^F > \hat{p}_0$ for most range of the parameter values.

3 Public Information

In this section, we introduce a noisy public signal in addition to private signals into the baseline model developed in the previous section and study the consequences of a more general information structure following Hellwig (2002) and Amato and Shin (2003). As they argue, in an economy in which decision makers' information sets are heterogeneous, public information has disproportionately large effects on their decisions. The public signal in our extended model may represent preliminary data that is to be revised or noisy information promptly provided by the media, the government, and so on.

[13]It can be shown that the ratio of $\hat{p}_0^F$ to $\hat{p}_0$ is increasing in b, that is, $\frac{\partial(\hat{p}_0^F/\hat{p}_0)}{\partial b} > 0$.

3.1 Private and Public Signals

First we re-specify the firms' information set. In period t, firm i has access to not only private signals (6) but also the following public signal, which is not necessarily precise.

$$z_t^P = m_t + \sigma_v \, v_t, \quad v_t \sim N(0,\, 1), \tag{17}$$

where v_t is Gaussian white noise distributed independently of both ϵ_t and $u_t(i)$ for all i. Whereas Amato and Shin (2003) assume that price setters never obtain precise information about aggregate disturbances as in the Woodford model, we retain the assumption that the true value of m_t is revealed to all firms with a delay of only one period, in $t+1$. Therefore, the information set of firm i comprises the private and public signals and the history of realized aggregate nominal spending, in which noisy information, z_t^P, as well as precise information, $\{m_{t-s}\}_{s=1}^{\infty}$, is common knowledge. Following Hellwig (2002), firms' signal extraction for estimating m_t can be calculated as

$$\begin{aligned} E_t^i \, m_t &\equiv E[\, m_t \mid z_t(i),\, z_t^P,\, m_{t-1},\, m_{t-2}, ...] \\ &= \alpha \, \Delta \, z_t(i) + (1-\alpha) \, \Delta \, z_t^P \\ &\quad +(1-\Delta) \, \{m_{t-1} + \rho \, (m_{t-1} - m_{t-2})\}, \end{aligned} \tag{18}$$

where

$$\alpha \equiv \frac{\sigma_v^2}{\sigma_v^2 + \sigma_u^2}$$

represents firms' reliance on their private signals *relative to* the public signal. Given the precision of the public signal, this *relative* reliance is greater, the higher is the precision of the private signals (the smaller is σ_u). In addition,

$$\Delta \equiv \frac{\sigma^2}{\sigma^2 + \frac{\sigma_u^2 \, \sigma_v^2}{\sigma_u^2 + \sigma_v^2}}$$

represents firms' reliance on the private and public signals. Given the variance of aggregate nominal spending, this reliance is greater, the higher is the precision of the *composite* signal.

As in the baseline model, we calculate the higher-order expectations about m_t as follows.

$$\begin{aligned} E_t^i\,[\,\overline{E}_t^{(j)}\,m_t\,] \;&=\; (\alpha\,\Delta)^{j+1}\,z_t(i) + \frac{1-(\alpha\,\Delta)^{j+1}}{1-\alpha\,\Delta}\,(1-\alpha)\,\Delta\,z_t^P \\ &\quad + \left\{1-(\alpha\,\Delta)^{j+1} - \frac{1-(\alpha\,\Delta)^{j+1}}{1-\alpha\,\Delta}\,(1-\alpha)\,\Delta\right\} \\ &\quad \{m_{t-1}+\rho\,(m_{t-1}-m_{t-2})\} \end{aligned} \tag{19}$$

$$\begin{aligned} \overline{E}_t^{(j+1)}\,m_t \;&=\; (\alpha\,\Delta)^{j+1}\,m_t + \frac{1-(\alpha\,\Delta)^{j+1}}{1-\alpha\,\Delta}\,(1-\alpha)\,\Delta\,z_t^P \\ &\quad + \left\{1-(\alpha\,\Delta)^{j+1} - \frac{1-(\alpha\,\Delta)^{j+1}}{1-\alpha\,\Delta}\,(1-\alpha)\,\Delta\right\} \\ &\quad \{m_{t-1}+\rho\,(m_{t-1}-m_{t-2})\} \\ &= \left\{(\alpha\,\Delta)^{j+1} + \frac{1-(\alpha\,\Delta)^{j+1}}{1-\alpha\,\Delta}\,(1-\alpha)\,\Delta\right\}\sigma\,\epsilon_t \\ &\quad + \frac{1-(\alpha\,\Delta)^{j+1}}{1-\alpha\,\Delta}\,(1-\alpha)\,\Delta\,\sigma_v\,v_t \\ &\quad + \{m_{t-1}+\rho\,(m_{t-1}-m_{t-2})\} \end{aligned} \tag{20}$$

Compared with firm i's own estimate of m_t, (18), its expectation of the higher-order average expectations, (19), is more responsive to public information including the public signal and the history of realized aggregate nominal spending, and less responsive to private information. The infinite-order average expectation converges to the expectations that are conditional only on common knowledge.

3.2 Effects of Monetary Disturbances

Substituting (20) into (12) in the baseline model, we obtain a unique stable solution of the following difference equation: $x_t = \lambda\,x_{t-1} + C_1\,m_{t-1} + C_2\,m_{t-2} + C_3^P\,\sigma\,\epsilon_t + C_4^P\,\sigma_v\,v_t$, where λ, C_1, and C_2 are the same as in the baseline model and

$$
\begin{aligned}
C_3^P &= \frac{2\sqrt{\xi}\,(1+\sqrt{\xi}+\rho\sqrt{\xi})}{(1+\sqrt{\xi})\,\{1+\sqrt{\xi}-\rho\,(1-\sqrt{\xi})\}}\,\frac{\{4-\alpha\,(3-2\sqrt{\xi}-\xi)\}\,\Delta}{\{4-\alpha\,\Delta\,(3-2\sqrt{\xi}-\xi)\}}, \\
C_4^P &= \frac{8\sqrt{\xi}\,(1+\sqrt{\xi}+\rho\sqrt{\xi})}{(1+\sqrt{\xi})\,\{1+\sqrt{\xi}-\rho\,(1-\sqrt{\xi})\}}\,\frac{(1-\alpha)\,\Delta}{\{4-\alpha\,\Delta\,(3-2\sqrt{\xi}-\xi)\}}.
\end{aligned}
$$

As before, we examine the impulse responses of inflation and output to a monetary disturbance, that is, a unit positive innovation in ϵ_0, and compare these responses with those in the baseline model as well as those in the full-information staggered price-setting model. The baseline model without the public signal corresponds to the case of $\sigma_v = \infty$, so that $\alpha = 1$ and $\Delta = b$, and the full-information staggered price-setting model corresponds to the case in which either σ_u or σ_v is 0, so that α is 1 or 0 and $\Delta = 1$. The responses of the price level and output in this extended model are calculated as a set of equilibrium paths $\{\,\hat{p}_t^P, \hat{y}_t^P\,\}$ with $\epsilon_0 = 1$, $\epsilon_t = 0$ for all $t \neq 0$, $v_t = 0$ for all t, $p_{-1} = y_{-1} = m_{-1} = m_{-2} = m_{-3} = 0$, and $\lim_{t\to\infty} y_t = 0$. The main analytical results are summarized in the following proposition.

Proposition 2. *i) The impulse response of inflation is initially decreasing and later increasing in the amount of noise in firms' private signals, σ_u, and also in the amount of noise in the public signal, σ_v, i.e.,*

$$
\begin{aligned}
&\frac{\partial(\hat{p}_t^P - \hat{p}_{t-1}^P)}{\partial\sigma_u} < 0, \quad \frac{\partial(\hat{p}_t^P - \hat{p}_{t-1}^P)}{\partial\sigma_v} < 0, \quad t = 0,\,1. \\
&\frac{\partial(\hat{p}_t^P - \hat{p}_{t-1}^P)}{\partial\sigma_u} > 0, \quad \frac{\partial(\hat{p}_t^P - \hat{p}_{t-1}^P)}{\partial\sigma_v} > 0, \quad t \geq 2.
\end{aligned}
$$

ii) The impulse response of output is increasing in σ_u and σ_v, i.e.,

$$
\frac{\partial\hat{y}_t^P}{\partial\sigma_u} > 0, \quad \frac{\partial\hat{y}_t^P}{\partial\sigma_v} > 0, \quad t \geq 0.
$$

Proof. See Appendix A. ■

The noisier is the public signal, as are the private signals, the more sluggish is the initial response of prices. Compared with the baseline model without the public signal, the initial response of prices is less sluggish and the response of output is less amplified. Provision of the public signal alleviates the effects of monetary disturbances because it allows firms to gain common knowledge and, hence, calculate the higher-order average expectations more precisely.

Sample sets of impulse responses in the full-information model, the baseline model ($b = 0.5$), and the extended models ($\alpha = 0.1$ and 0.5 with $\Delta = 0.5$) are shown in Figure 3. As in Figure 1, the size of the shock in the aggregate nominal spending, σ, is set to 1 in each model, which implies $(\sigma_u, \sigma_v) = (\sqrt{10}, \sqrt{10}/3)$ and $(\sqrt{2}, \sqrt{2})$ in the models of $(\alpha, \Delta) = (0.1, 0.5)$ and $(0.5, 0.5)$, respectively. The other parameter values, for ξ and ρ, are the same as in Figure 1. In the extended model of $\alpha = 0.1$, the size of the initial response of prices is about half of that in the full-information model. The sluggishness is alleviated compared with the baseline model in which the ratio is about one third. The response of output is amplified by more than 35 percent in the first four periods, while the response in the baseline model is amplified by more than 45 percent.

As before, we compare the responses in the extended model with those in a flexible-prices model with the same information structure that incorporates the public signal as well as the private signals. The response in the flexible-prices model is

$$\hat{p}_0^{FP} = \frac{\{1 - \alpha\,(1 - \xi)\}\,\Delta\,\sigma}{\{1 - \alpha\,\Delta\,(1 - \xi)\}}.$$

The corresponding result in the extended model with $\rho = 0$ is

$$\hat{p}_0^{P} = \frac{\sqrt{\xi}}{1 + \sqrt{\xi}}\,\frac{\{4 - \alpha\,(3 - 2\,\sqrt{\xi} - \xi)\}\,\Delta\,\sigma}{\{4 - \alpha\,\Delta\,(3 - 2\,\sqrt{\xi} - \xi)\}}.$$

Again, the response that operates through static and simultaneous higher-order expectations under flexible prices is less sluggish than the one that operates through dynamic and staggered higher-order expectations, that is, $\hat{p}_0^{FP} > \hat{p}_0^{P}$, for most range of the parameter values.

3.3 Effects of Informational Disturbances

While the public signal reduces uncertainty in firms' higher-order expectations about aggregate nominal spending, the noise in the public signal itself adds to aggregate uncertainty. Firms with heterogeneous information sets might "over-react" to the noisy public signal to such an extent that the economy could be destabilized. We consider this side effect of public information by examining the impulse responses to an informational disturbance in the public signal.

The responses of the price level and output to a unit positive innovation in v_0 are calculated as a set of equilibrium paths $\{\,\tilde{p}_t^P,\, \tilde{y}_t^P\,\}$ with $v_0 = 1$, $v_t = 0$ for all $t \neq 0$, $m_t = \epsilon_t = 0$ for all t, $p_{-1} = y_{-1} = 0$, and $\lim_{t\to\infty} y_t = \lim_{t\to\infty} p_t = 0$. The responses of inflation and output are given by

$$
\begin{aligned}
\tilde{p}_0^P &= \frac{C_4^P}{2}\,\sigma_v \\
\tilde{p}_1^P - \tilde{p}_0^P &= \lambda\,\tilde{p}_0^P \\
\tilde{p}_2^P - \tilde{p}_1^P &= -(1-\lambda^2)\,\tilde{p}_0^P \\
\tilde{p}_t^P - \tilde{p}_{t-1}^P &= \lambda\,(\tilde{p}_{t-1}^P - \tilde{p}_{t-2}^P), \qquad t \geq 3.
\end{aligned}
$$

and

$$
\begin{aligned}
\tilde{y}_0^P &= -\frac{C_4^P}{2}\,\sigma_v \\
\tilde{y}_1^P &= (1+\lambda)\,\tilde{y}_0^P \\
\tilde{y}_t^P &= \lambda\,\tilde{y}_{t-1}^P, \quad t \geq 2
\end{aligned}
$$

Firms raise their prices when they receive a public signal biased upward from the true value of $m_0 (= 0)$, believing it to be unbiased. Since the exogenous process for m_t is not affected by the informational disturbance, the increase in prices leads to a corresponding decrease in output.

Sample sets of impulse responses of inflation and output to a *negative* informational disturbance $\{\,-\tilde{p}_t^P,\, -\tilde{y}_t^P\,\}$ are shown in Figure 4 (upper panels), in which $(\alpha,\, \Delta) = (0.5,\, 0.5)$, $(\sigma_u,\, \sigma) = (\sqrt{2},\, 1)$, the size of the shock is $\sigma_v = \sqrt{2}$, and the other parameter values are the same as in Figure 3.

Firms react to the downward-biased public signal by reducing their prices, and output increases accordingly. When the output gap begins to shrink, prices start increasing and then inflation peaks later than output. Combining this pattern of responses to a negative informational disturbance with the pattern of responses to a positive monetary disturbance examined in the previous subsection further delays the response of inflation and further amplifies the response of output.[14] Conversely, a positive (or negative) informational disturbance accompanied by a positive (negative) monetary disturbance has an effect of further alleviating the sluggishness in the initial response of prices. Figure 5 shows the responses to a negative informational disturbance, $\sigma_v \, v_0 = -\sqrt{2}$, accompanied by a positive monetary disturbance, $\sigma \, \epsilon_0 = 1$: the combination of the responses in Figure 3, $\{ \hat{p}_t^P, \, \hat{y}_t^P \}$, and those in Figure 4, $\{ -\tilde{p}_t^P, \, -\tilde{y}_t^P \}$. In this case, the sluggishness in the initial response of prices is magnified and the response of output is amplified significantly by the negative informational disturbance.

Improving precision of the public signal, that is, lowering σ_v makes the initial response of prices to *monetary* disturbances less sluggish and the response of output less amplified, as shown in Proposition 2. Meanwhile, the effects of lowering σ_v on the responses of prices and output to *informational* disturbances are ambiguous. While a small σ_v implies a small informational disturbance and directly generates small responses, it also makes firms rely heavily on the public signal, and therefore, indirectly generates high responsiveness to informational disturbances. If the latter indirect effect dominates, improving precision of the public signal induces firms to over-react to the signal and amplifies the responses. The partial derivative of the initial response of prices to a positive informational disturbance, or equivalently the initial response of output to a negative informational disturbance, with respect to the amount of noise in the public signal is

$$\frac{\partial \tilde{p}_0^P}{\partial \sigma_v} = -\frac{\partial \tilde{y}_0^P}{\partial \sigma_v}$$

[14] A negative informational disturbance accompanied by a positive monetary disturbance implies that the public signal (17) does not reflect the current monetary disturbance.

$$
\begin{aligned}
&= \frac{1}{2}\left(C_4^P + \sigma_v \frac{\partial C_4^P}{\partial \sigma_v}\right) \\
&= \frac{4\sqrt{\xi}\,(1+\sqrt{\xi}+\rho\sqrt{\xi})}{(1+\sqrt{\xi})\,\{1+\sqrt{\xi}-\rho\,(1-\sqrt{\xi})\}}\,\frac{\sigma_u^2\,\sigma^2\,\{\,4\,\sigma_u^2\,(\sigma^2-\sigma_v^2)-(1+\sqrt{\xi})^2\,\sigma_v^2\,\sigma^2\}}{\{4\,\sigma_u^2\,(\sigma^2+\sigma_v^2)+(1+\sqrt{\xi})^2\,\sigma_v^2\,\sigma^2\}^2}.
\end{aligned}
$$

The first term in the second line represents the direct effect of the disturbance ($C_4^P > 0$) and the second term represents the indirect effect caused by changes in the responsiveness ($\frac{\partial C_4^P}{\partial \sigma_v} < 0$). The sign of the partial derivative as a whole is ambiguous.

In the lower panel of Figure 4, $-\tilde{y}_0^P$ is plotted as a function of the amount of noise in the public signal, σ_v. For the amount of noise in the private signals, σ_u, and the standard deviation of aggregate nominal spending, σ, the following three sets of parameter values are chosen: $(\sigma_u, \sigma) = (\sqrt{2}, 1)$, $(2\sqrt{2}, 1)$, and $(\sqrt{2}, \sqrt{2})$. When $\sigma_v = \sqrt{2}$, these sets correspond to $(\alpha, \Delta) = (0.5, 0.5)$, $(0.2, 0.385)$, and $(0.5, 0.667)$, respectively. The relationship between the precision of the public signal and the amplitude of the responses of output to informational disturbances is non-monotonic: improving precision reduces the amplitude when precision is high (σ_v is small) but raises the amplitude when precision is low (σ_v is large). Therefore, a small improvement in the precision of a relatively noisy public signal could increase output volatility and destabilize the economy.

This non-monotonic relationship can hold in the responses to a combination of monetary and informational disturbances. In the lower panel of Figure 5, $\hat{y}_0^P - \tilde{y}_0^P$ is plotted as a function of σ_v. Although the slop is much flatter than that in Figure 4, improving precision of the public signal reduces the amplitude of the initial response of output when σ_v is large. More generally, the variance of output is expressed as

$$
\begin{aligned}
\mathrm{Var}(y_t) = &\left[\left(1-\frac{C_3^P}{2}\right)^2 + \left(\rho + \frac{C_1}{2} + C_2 - \frac{C_3^P}{2}\right)^2\right. \\
&+ 2\,\lambda\left(1-\frac{C_3^P}{2}\right)\left(\rho + \frac{C_1}{2} + C_2 - \frac{C_3^P}{2}\right) \\
&\left. -\frac{2\,\lambda\,(1+\lambda)}{1-\lambda\,\rho}\left(\rho^2 + \frac{\rho\,C_1 + (2\,\rho+1)\,C_2}{2}\right)\frac{C_3^P}{2}\right]\frac{\sigma^2}{1-\lambda^2}
\end{aligned}
$$

$$+ \left[\frac{(C_4^P)^2}{2} + 2\,\lambda \left(\rho^2 + \frac{\rho\, C_1 + (2\,\rho + 1)\, C_2}{2} \right) \frac{C_4^P}{2} \right] \frac{\sigma_v^2}{1 - \lambda^2} + \cdots,$$

where the remaining terms are independent of σ_v. The sign of the partial derivative of $\mathrm{Var}(y_t)$ with respect to σ_v is ambiguous. It is negative, however, in a very limited region of parameter values, in which ξ and ρ are close to 1 and σ_u is very small compared with σ_v.[15] While a small σ_v generates high responsiveness to informational disturbances and increases output volatility, it also generates high responsiveness of *prices* to monetary disturbances (i.e., alleviates the sluggishness of price adjustment) and decreases output volatility. The latter effect combined with the direct effect of reducing the size of informational disturbance brings the partial derivative into positive. As a result, the possibility that improving precision of the public signal destabilizes the economy is small in our model.[16]

4 Concluding Remarks

In this paper we have studied the consequences of a lack of common knowledge in the transmission of monetary policy by integrating the Woodford (2003a) imperfect common knowledge model with Taylor-Calvo staggered price-setting models. The average price set by monopolistically competitive firms that can only observe the state of the economy through noisy private signals depends on their higher-order expectations about not only the current state of the economy but also about the states in the future periods in which prices are to be fixed. This integrated model provides a plausible explanation for the observed effects of monetary policy: it shows analytically how price adjustments are delayed and how the response of output to a monetary disturbance is amplified. We have also considered a more general information

[15] See Appendix B for the derivation of $\mathrm{Var}(y_t)$ and the partial derivative of $\mathrm{Var}(y_t)$ with respect to σ_v.

[16] This result is consistent with Svensson (2006) who points out that improving precision of the public signal is welfare-improving unless the public signal is much noisier than the private signals in the Morris and Shin (2002) model. While we consider only output volatility, Hellwig (2005) shows in his model that better public information is always welfare-improving when price dispersion as well as output volatility is taken into account.

structure in which a noisy public signal, in addition to the private signals, is introduced.

Based on the models developed in this paper, at least two directions for future research can be pursued. One is policy research. The model of Section 3 could be further extended to obtain richer implications for the central bank's communication strategy. Another direction is empirical research. For example, deep parameters such as price setters' reliance on their private information could be estimated by matching impulse responses obtained from a structural model with those from an estimated VAR model. Although the models in this paper may be too simple for practical use, they are tractable, flexible, and based on plausible assumptions about information structure. We hope these models serve as a useful building block for future research in those directions.

Appendix

A Proof of Proposition 2

i) Taking the partial derivatives of $\hat{p}_0^P$ with respect to σ_u and σ_v, we have

$$
\begin{aligned}
\frac{\partial \hat{p}_0^P}{\partial \sigma_u} &= \frac{\sigma}{2}\left(\frac{\partial C_3^P}{\partial \alpha}\frac{\partial \alpha}{\partial \sigma_u} + \frac{\partial C_3^P}{\partial \Delta}\frac{\partial \Delta}{\partial \sigma_u}\right) \\
&= -\frac{8\sqrt{\xi}\,(1+\sqrt{\xi})\,(1+\sqrt{\xi}+\rho\sqrt{\xi})\,\sigma_u\,\sigma_v^4\,\sigma^3}{\{1+\sqrt{\xi}-\rho\,(1-\sqrt{\xi})\}\,\{4\,\sigma_u^2\,(\sigma_v^2+\sigma^2)+(1+\sqrt{\xi})^2\,\sigma_v^2\,\sigma^2\}^2} \\
\frac{\partial \hat{p}_0^P}{\partial \sigma_v} &= \frac{\sigma}{2}\left(\frac{\partial C_3^P}{\partial \alpha}\frac{\partial \alpha}{\partial \sigma_v} + \frac{\partial C_3^P}{\partial \Delta}\frac{\partial \Delta}{\partial \sigma_v}\right) \\
&= -\frac{32\sqrt{\xi}\,(1+\sqrt{\xi}+\rho\sqrt{\xi})\,\sigma_u^4\,\sigma_v\,\sigma^3}{(1+\sqrt{\xi})\,\{1+\sqrt{\xi}-\rho\,(1-\sqrt{\xi})\}\,\{4\,\sigma_u^2\,(\sigma_v^2+\sigma^2)+(1+\sqrt{\xi})^2\,\sigma_v^2\,\sigma^2\}^2}
\end{aligned}
$$

From $\hat{p}_1^P$ onward,

$$
\begin{aligned}
\frac{\partial(\hat{p}_1^P-\hat{p}_0^P)}{\partial \sigma_u} &= \lambda\,\frac{\partial \hat{p}_0^P}{\partial \sigma_u}, & \frac{\partial(\hat{p}_1^P-\hat{p}_0^P)}{\partial \sigma_v} &= \lambda\,\frac{\partial \hat{p}_0^P}{\partial \sigma_v}. \\
\frac{\partial(\hat{p}_2^P-\hat{p}_1^P)}{\partial \sigma_u} &= -(1-\lambda^2)\,\frac{\partial \hat{p}_0^P}{\partial \sigma_u}, & \frac{\partial(\hat{p}_2^P-\hat{p}_1^P)}{\partial \sigma_v} &= -(1-\lambda^2)\,\frac{\partial \hat{p}_0^P}{\partial \sigma_v}. \\
\frac{\partial(\hat{p}_t^P-\hat{p}_{t-1}^P)}{\partial \sigma_u} &= \lambda\,\frac{\partial(\hat{p}_{t-1}^P-\hat{p}_{t-2}^P)}{\partial \sigma_u}, & \frac{\partial(\hat{p}_t^P-\hat{p}_{t-1}^P)}{\partial \sigma_v} &= \lambda\,\frac{\partial(\hat{p}_{t-1}^P-\hat{p}_{t-2}^P)}{\partial \sigma_v}, \quad t \geq 3.
\end{aligned}
$$

Therefore, we have proved

$$
\begin{aligned}
\frac{\partial(\hat{p}_t^P-\hat{p}_{t-1}^P)}{\partial \sigma_u} < 0, \quad \frac{\partial(\hat{p}_t^P-\hat{p}_{t-1}^P)}{\partial \sigma_v} &< 0, \quad t = 0,\,1. \\
\frac{\partial(\hat{p}_t^P-\hat{p}_{t-1}^P)}{\partial \sigma_u} > 0, \quad \frac{\partial(\hat{p}_t^P-\hat{p}_{t-1}^P)}{\partial \sigma_v} &> 0, \quad t \geq 2.
\end{aligned}
$$

ii) Taking the partial derivatives of $\hat{y}_0^P$ with respect to σ_u and σ_v, we have

$$
\begin{aligned}
\frac{\partial \hat{y}_0^P}{\partial \sigma_u} &= -\frac{\sigma}{2}\left(\frac{\partial C_3^P}{\partial \alpha}\frac{\partial \alpha}{\partial \sigma_u} + \frac{\partial C_3^P}{\partial \Delta}\frac{\partial \Delta}{\partial \sigma_u}\right) \\
&= \frac{8\sqrt{\xi}\,(1+\sqrt{\xi})\,(1+\sqrt{\xi}+\rho\sqrt{\xi})\,\sigma_u\,\sigma_v^4\,\sigma^3}{\{1+\sqrt{\xi}-\rho\,(1-\sqrt{\xi})\}\,\{4\,\sigma_u^2\,(\sigma_v^2+\sigma^2)+(1+\sqrt{\xi})^2\,\sigma_v^2\,\sigma^2\}^2}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial \hat{y}_0^P}{\partial \sigma_v} &= -\frac{\sigma}{2}\left(\frac{\partial C_3^P}{\partial \alpha}\,\frac{\partial \alpha}{\partial \sigma_v} + \frac{\partial C_3^P}{\partial \Delta}\,\frac{\partial \Delta}{\partial \sigma_v}\right) \\
&= \frac{32\sqrt{\xi}\,(1+\sqrt{\xi}+\rho\sqrt{\xi})\,\sigma_u^4\,\sigma_v\,\sigma^3}{(1+\sqrt{\xi})\,\{1+\sqrt{\xi}-\rho\,(1-\sqrt{\xi})\}\,\{4\,\sigma_u^2\,(\sigma_v^2+\sigma^2)+(1+\sqrt{\xi})^2\,\sigma_v^2\,\sigma^2\}^2}
\end{aligned}
$$

From $\hat{y}_1^P$ onward,

$$
\frac{\partial \hat{y}_1^P}{\partial \sigma_u} = (1+\lambda)\,\frac{\partial \hat{y}_0^P}{\partial \sigma_u}, \quad \frac{\partial \hat{y}_1^P}{\partial \sigma_v} = (1+\lambda)\,\frac{\partial \hat{y}_0^P}{\partial \sigma_v}.
$$

$$
\frac{\partial \hat{y}_t^P}{\partial \sigma_u} = \lambda\,\frac{\partial \hat{y}_{t-1}^P}{\partial \sigma_u}, \quad \frac{\partial \hat{y}_t^P}{\partial \sigma_v} = \lambda\,\frac{\partial \hat{y}_{t-1}^P}{\partial \sigma_v}, \quad t \geq 2.
$$

Therefore, we have proved

$$
\frac{\partial \hat{y}_t^P}{\partial \sigma_u} > 0, \quad \frac{\partial \hat{y}_t^P}{\partial \sigma_v} > 0, \quad t \geq 0.
$$

B Variance of output

From the equations (3), (4) and (5) and the solution of the difference equation: $x_t = \lambda\,x_{t-1} + C_1\,m_{t-1} + C_2\,m_{t-2} + C_3^P\,\sigma\,\epsilon_t + C_4^P\,\sigma_v\,v_t$, we have

$$
\begin{aligned}
y_t - \lambda\,y_{t-1} &= \left(\rho + \frac{C_1}{2} + C_2\right)(m_{t-1} - m_{t-2}) + \frac{C_2}{2}\,(m_{t-2} - m_{t-3}) \\
&\quad + \left(1 - \frac{C_3^P}{2}\right)\sigma\,\epsilon_t - \frac{C_3^P}{2}\,\sigma\,\epsilon_{t-1} - \frac{C_4^P}{2}\,\sigma_v\,(v_t + v_{t-1}) \\
&= \left(1 - \frac{C_3^P}{2}\right)\sigma\,\epsilon_t + \left(\rho + \frac{C_1}{2} + C_2 - \frac{C_3^P}{2}\right)\sigma\,\epsilon_{t-1} \\
&\quad + \left(\rho^2 + \frac{\rho\,C_1 + (2\,\rho+1)\,C_2}{2}\right)\sum_{i=0}^{\infty}\rho^i\,\sigma\,\epsilon_{t-2-i} \\
&\quad - \frac{C_4^P}{2}\,\sigma_v\,(v_t + v_{t-1}). \qquad (21)
\end{aligned}
$$

Note that the terms including $\sum_{i=0}^{\infty}\rho^i\,\sigma\,\epsilon_{t-2-i}$ are independent of both C_3^P and C_4^P and thus independent of σ_v. Squaring both sides of (21) and taking expectations gives

$$
\begin{aligned}
& (1+\lambda^2)\,\mathrm{Var}(y_t) - 2\,\lambda\, E[\, y_t\, y_{t-1}\,] \\
= \; & \left\{ \left(1 - \frac{C_3^P}{2}\right)^2 + \left(\rho + \frac{C_1}{2} + C_2 - \frac{C_3^P}{2}\right)^2 + \left(\rho^2 + \frac{\rho\, C_1 + (2\,\rho + 1)\, C_2}{2}\right) \sum_{i=0}^{\infty} \rho^{2i} \right\} \sigma^2 \\
& + (C_4^P)^2\, \sigma_v^2. \qquad (22)
\end{aligned}
$$

Multipling (21) by y_{t-1} and taking expectations gives

$$
\begin{aligned}
& E[\, y_t\, y_{t-1}\,] - \lambda\, \mathrm{Var}(y_t) \\
= \; & E\left[\left(1 - \frac{C_3^P}{2}\right) \sigma\, \epsilon_t\, y_{t-1} + \left(\rho + \frac{C_1}{2} + C_2 - \frac{C_3^P}{2}\right) \sigma\, \epsilon_{t-1}\, y_{t-1} \right. \\
& + \left(\rho^2 + \frac{\rho\, C_1 + (2\,\rho + 1)\, C_2}{2}\right) \sum_{i=0}^{\infty} \rho^i\, \sigma\, \epsilon_{t-2-i}\, y_{t-1} \\
& \left. - \frac{C_4^P}{2}\, \sigma_v\, (v_t + v_{t-1})\, y_{t-1} \right]. \qquad (23)
\end{aligned}
$$

Combining (22) and (23), we obtain

$$
\begin{aligned}
\mathrm{Var}(y_t) \;=\; & \left[\left(1 - \frac{C_3^P}{2}\right)^2 + \left(\rho + \frac{C_1}{2} + C_2 - \frac{C_3^P}{2}\right)^2 \right. \\
& + 2\,\lambda \left(1 - \frac{C_3^P}{2}\right) \left(\rho + \frac{C_1}{2} + C_2 - \frac{C_3^P}{2}\right) \\
& \left. - \frac{2\,\lambda\,(1+\lambda)}{1 - \lambda\,\rho} \left(\rho^2 + \frac{\rho\, C_1 + (2\,\rho + 1)\, C_2}{2}\right) \frac{C_3^P}{2} \right] \frac{\sigma^2}{1-\lambda^2} \\
& + \left[\frac{(C_4^P)^2}{2} + 2\,\lambda \left(\rho^2 + \frac{\rho\, C_1 + (2\,\rho + 1)\, C_2}{2}\right) \frac{C_4^P}{2} \right] \frac{\sigma_v^2}{1-\lambda^2} + \cdots,
\end{aligned}
$$

where the remaining terms are independent of σ_v. The partial derivative of $\mathrm{Var}(y_t)$ with respect to σ_v is

$$
\begin{aligned}
& \frac{\partial \mathrm{Var}(y_t)}{\partial \sigma_v} \\
= \; & \frac{1 + \sqrt{\xi} + \rho\,\sqrt{\xi}}{(1+\sqrt{\xi})\, \left\{1 + \sqrt{\xi} - \rho\,(1 - \sqrt{\xi})\right\}^3} \, \frac{16\, \sigma_u^4\, \sigma_v\, \sigma^4}{\left\{4\, \sigma_u^2\, \sigma_v^2 + 4\, \sigma_u^2\, \sigma^2 + \left(1+\sqrt{\xi}\right)^2\, \sigma_v^2\, \sigma^2\right\}^3}
\end{aligned}
$$

$$
\begin{aligned}
&\times \Bigg[4\,\sigma_u^2\,\sigma_v^2\,\Bigg\{ \rho^4\,\left(1-\sqrt{\xi}\right)^2 - 2\,\rho^3\,\left(1-\sqrt{\xi}\right) + \rho^2\,\left(1-4\,\sqrt{\xi}-4\,\xi+4\,\sqrt{\xi}^3-\xi^2\right) \\
&+\rho\,\left(-2-3\,\sqrt{\xi}+6\,\xi+5\,\sqrt{\xi}^3-2\,\xi^2\right)+\left(2+7\,\sqrt{\xi}+7\,\xi+\sqrt{\xi}^3-\xi^2\right)\Bigg\} \\
&+4\,\sigma_u^2\,\sigma^2\,\Bigg\{ \rho^4\,\left(1-\sqrt{\xi}\right)^2 - 2\,\rho^3\,\left(1-\sqrt{\xi}\right) + \rho^2\,\left(1-4\,\sqrt{\xi}-2\xi+\xi^2\right) \\
&+\rho\,\left(-2-\sqrt{\xi}+2\,\xi+3\,\sqrt{\xi}^3+2\,\xi^2\right)+\left(1+\sqrt{\xi}\right)^2\left(2+\sqrt{\xi}+\xi\right)\Bigg\} \\
&+\left(1+\sqrt{\xi}\right)^2\sigma_v^2\,\sigma^2\,\Bigg\{ \rho^4\,\left(1-\sqrt{\xi}\right)^2 - 2\,\rho^3\,\left(1-\sqrt{\xi}\right) + \rho^2\,\left(1-4\,\sqrt{\xi}-\xi^2\right) \\
&+\rho\,\left(-2+\sqrt{\xi}+2\,\xi-3\,\sqrt{\xi}^3-2\,\xi^2\right)+\left(1+\sqrt{\xi}\right)^2\left(2-\sqrt{\xi}-\xi\right)\Bigg\}\Bigg]
\end{aligned}
$$

The sign is ambiguous, but it is negative in a very limited region of parameter values, in which ξ and ρ are close to 1 and σ_u is very small compared with σ_v.

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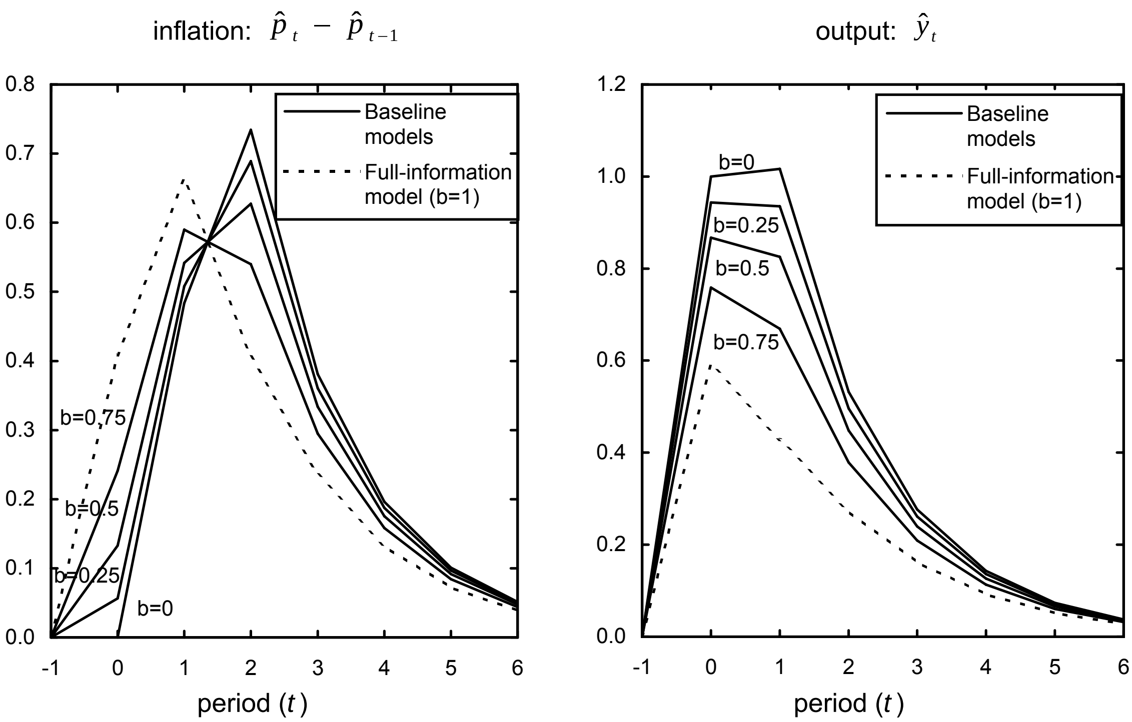

Fig. 1. Impulse responses to a positive monetary disturbance (Baseline model)

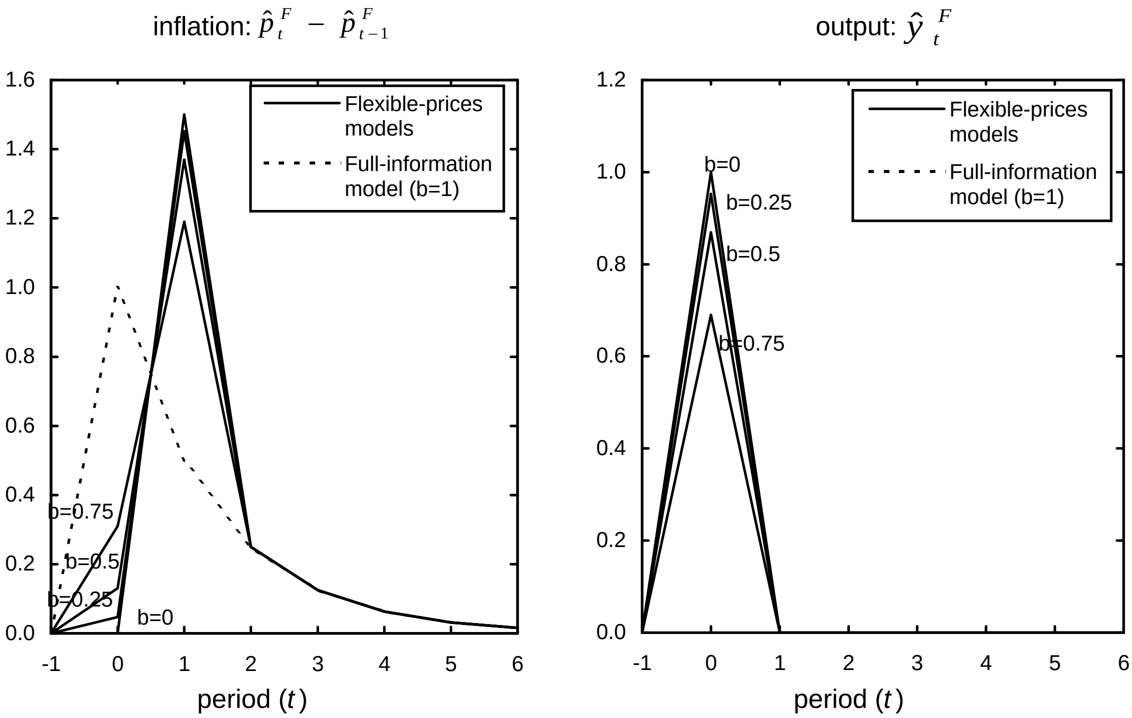

Fig. 2. Impulse responses to a positive monetary disturbance (Flexible-prices model)

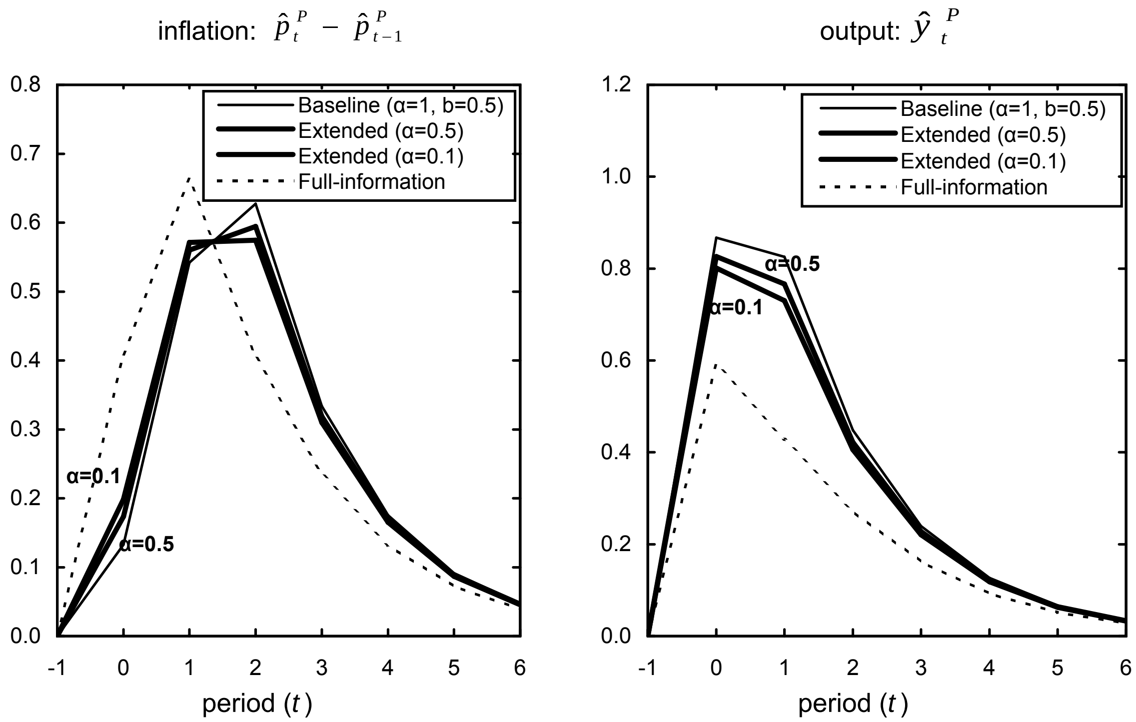

Fig. 3. Impulse responses to a positive monetary disturbance (Extended model)

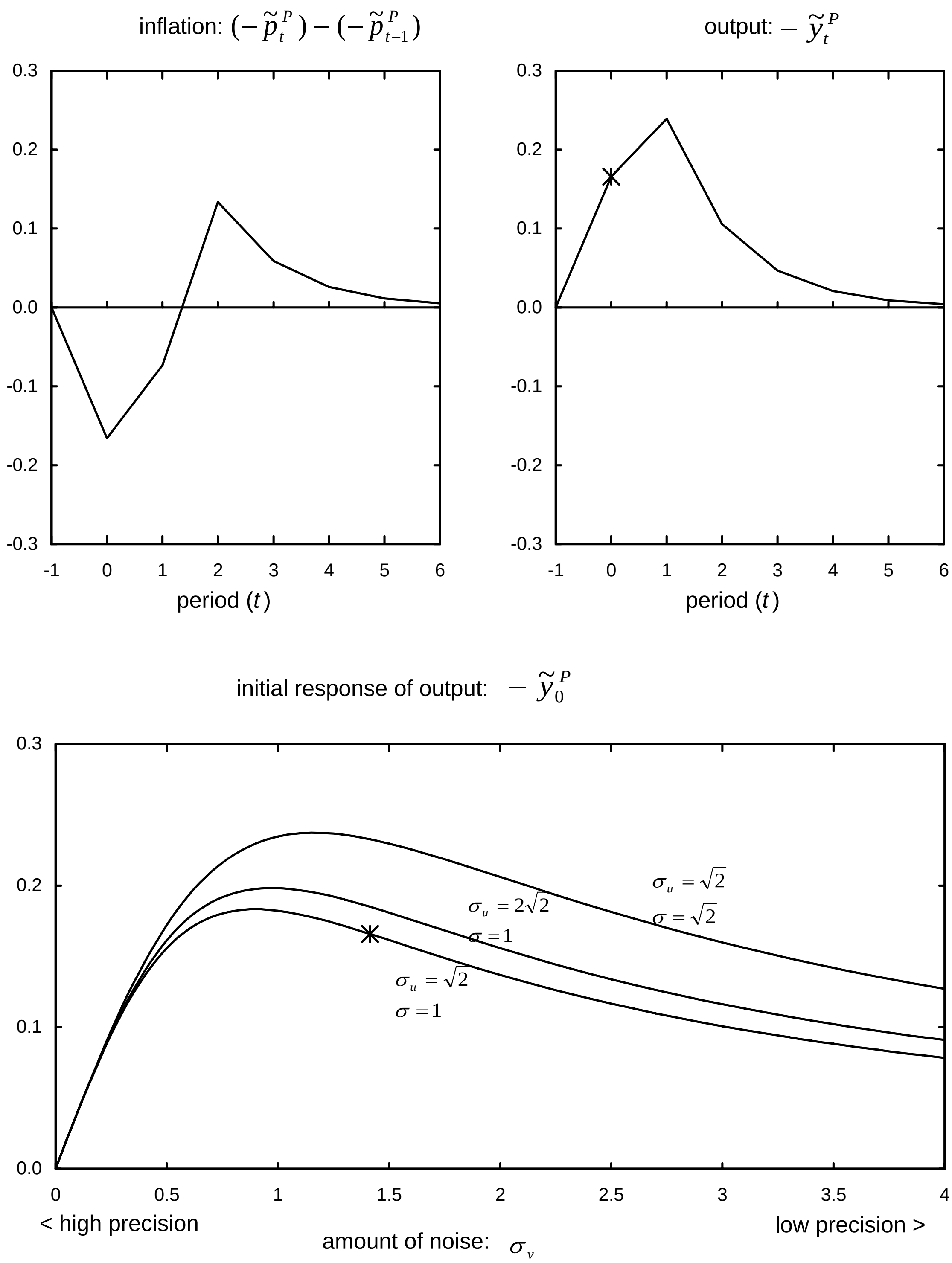

Fig. 4. Impulse responses to a negative informational disturbance

(Note: The asterisk on the $(\sigma_u, \sigma) = (\sqrt{2},\ 1)$ line in the lower panel corresponds to the response of output at t=0 in the upper-right panel.)

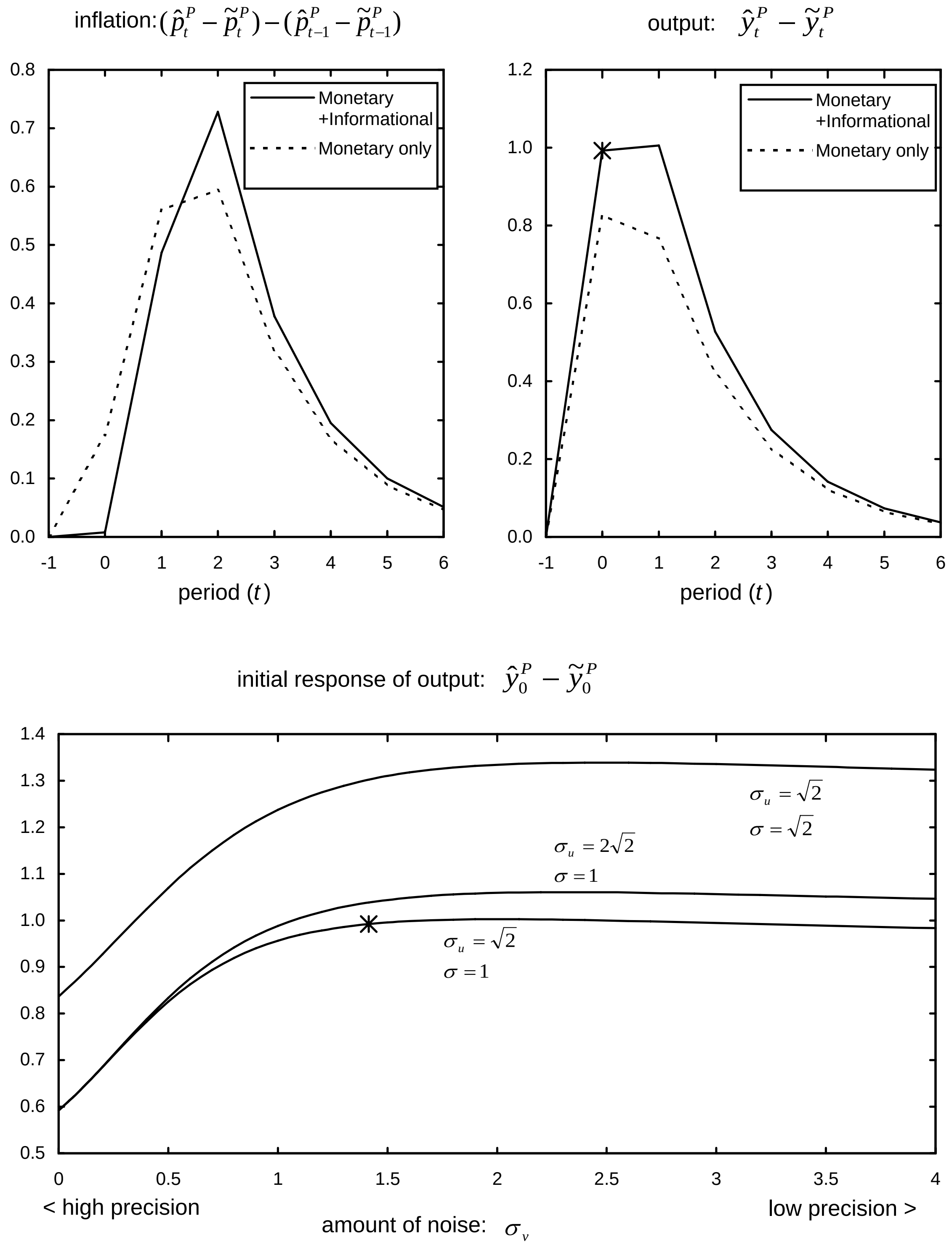

Fig. 5. Impulse responses to a negative informational disturbance accpanied by a positive monetary disturbance

(Note: The asterisk on the $(\sigma_u, \sigma) = (\sqrt{2},\ 1)$ line in the lower panel corresponds to the response of output at t=0 in the upper-right panel.)