In Public Debt Growth-Enhancing or Growth-Reducing?

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Abstract

- Question:
  How does the accumulation of public debt affect economic growth ?
  - A well-known effect is the crowd-out effect.
  - Recent empirical studies provides mixed evidence.
    $\longrightarrow$ Probably, other effects exist.
- Result:
  We construct a theoretical model that can generate the "inversed-U" relationship.

Motivation

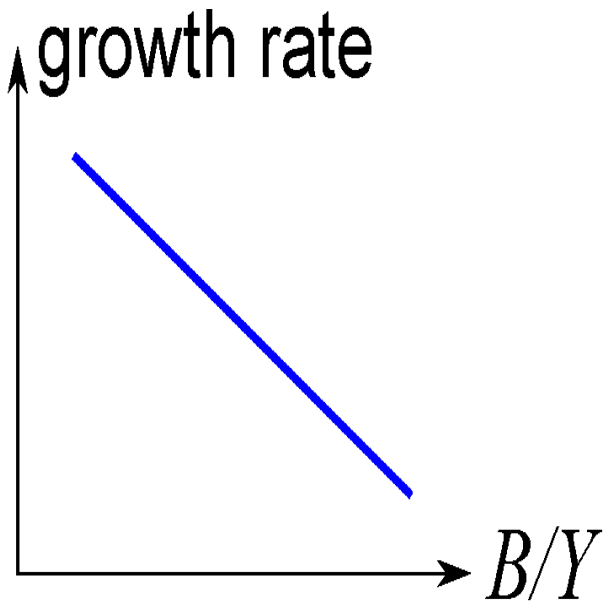
- Growth effects of the accumulation of public debt?

- Mixed evidence.
  - Negative & threshold effect of public debt.
    - ★ Reinhart and Rogoff (2010, AER)
    - ★ Kumar and Woo (2010, IMF WP)
    - ★ Balassone et al. (2013, Oxford Handbook)
  - Inverted U-shaped relationship.
    - ★ Baum et al. (2013, JIMF)
    - ★ Checherita-Westphal and Rother (2012, EER)
    - ★ Checchetti et al. (2011, BIS WP)
  - Questioning the threshold effect.
    - ★ Kourtellos et al. (2013, J Macro)
    - ★ Panizza and Presbitero (2013, SJES)

Motivation

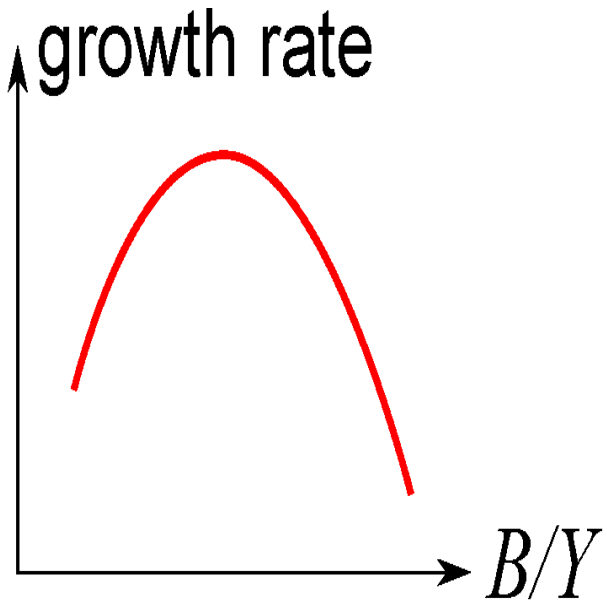
- Growth effects of the accumulation of public debt?
- Public debt $\uparrow \Rightarrow$ growth $\downarrow$ [Saint-Paul (1992, QJE)]

  because of the crowd-out effect.
  (debt $\uparrow \Rightarrow$ private investment $\downarrow$)

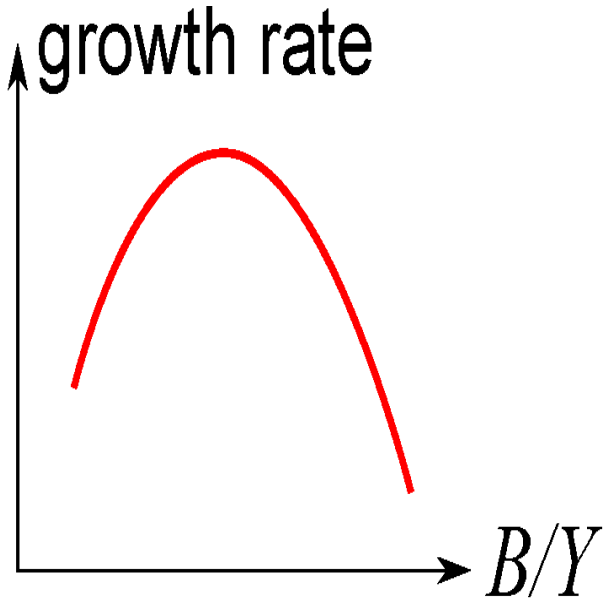


Motivation (Cont.)

- The crowd-out effect on economic growth? Empirical evidence $\longrightarrow$ Not only! We consider public debt's crowd-in effect.



Idea



- (i): (crowd-out) $>$ (crowd-in).
- (ii): (crowd-out) $<$ (crowd-in).

crowd-in effect [Woodford (1990, AER p&p)] :
Public debt $\uparrow \Rightarrow$ private investment $\uparrow$

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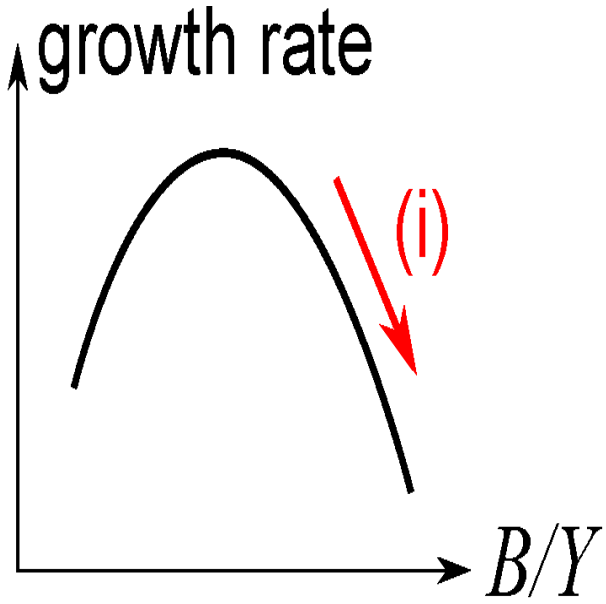

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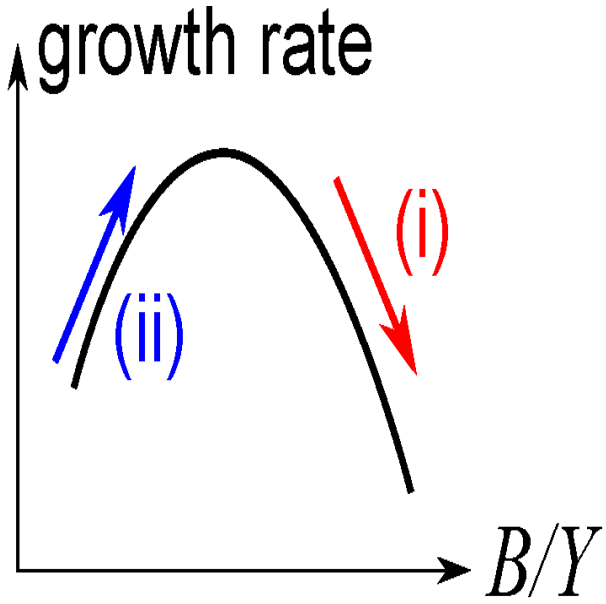

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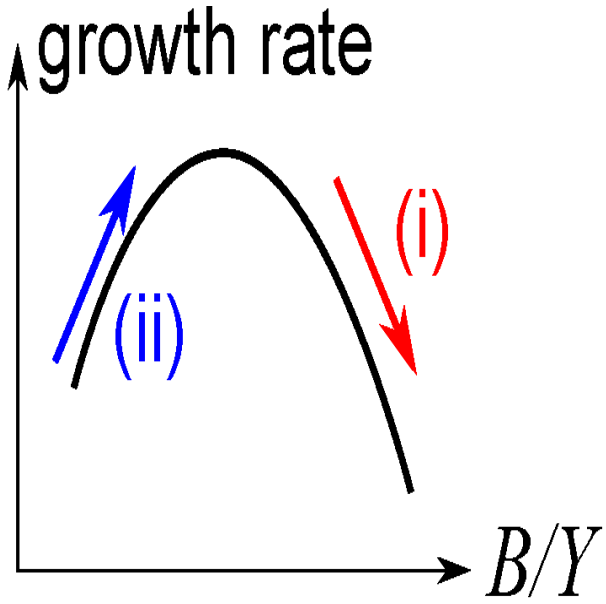

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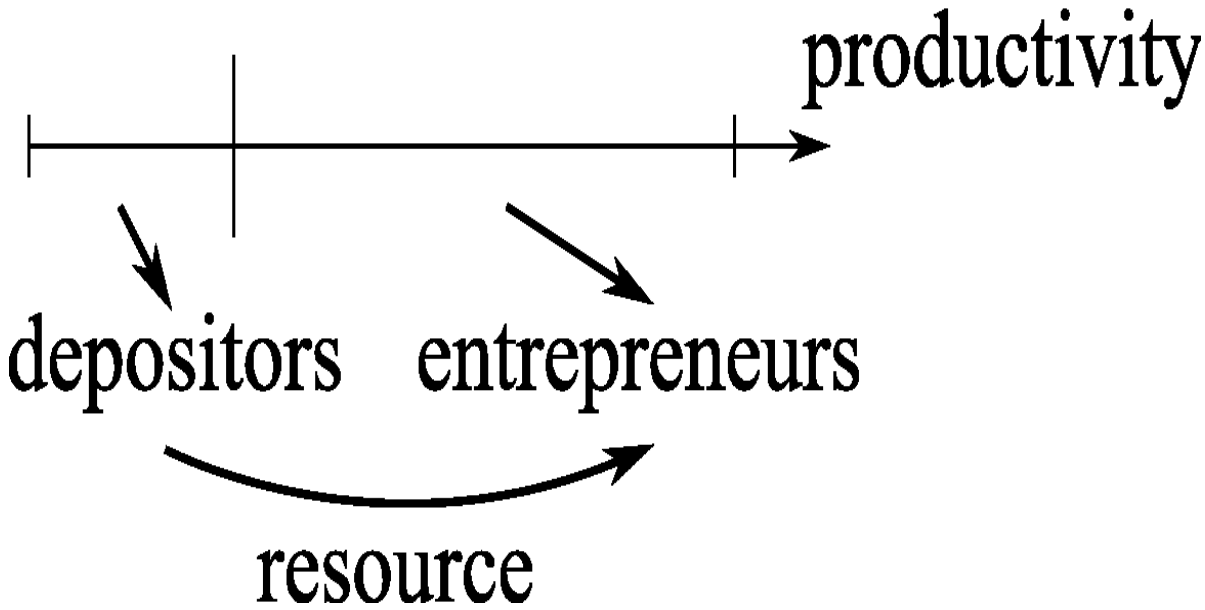
Intuition

- In our model, individuals
  - have two saving methods: investment or saving.
  - face iid productivity shock.



- Public debt $\uparrow \Rightarrow$ interest rate $\uparrow$.
  1. Less productive individuals: a investor $\Rightarrow$ a depositor.
     $\rightarrow$ The number of investors $\downarrow$.
  2. Interest rate $\uparrow \Rightarrow$ depositor's income $\uparrow$.
     $\rightarrow$ Investment $\uparrow$ in the next period by relaxed borrowing constraints.

Brief Description of the Model

Based on Kunieda and Shibata's (2012) model.

- Closed economy.

- The economy consists of
  - Infinitely-lived agents:
    - ★ Consume and save (by investment and/or deposit).
    - ★ Invest 1 unit $\Rightarrow$ produce ϕ_t^i units general goods.
    - ★ Productivity ϕ_t^i is determined by an iid shock.
    - ★ Individuals can borrow when they invest but face borrowing constraints.
  - government:
    - ★ follows a certain fiscal policy rule (to be explained).
  - financial intermediary:
    - ★ Deposits $\Rightarrow$ lending/buying public debt in a competitive market.
    - ★ The financial market is competitive, so it acquires zero profits.

Agents

Agents' utility maximization problem:

$$\max\ U_t = E\left[\sum_{s=t}^{\infty} \beta^{s-t} \ln c_s(\omega^s) \middle| \Phi^t(\omega^t)\right], \tag{1}$$

subject to

$$\begin{aligned} k_s(\omega^s) + b_s(\omega^s) &= [A\Phi_{s-1}(\omega_{s-1})k_{s-1}(\omega^{s-1}) \\ &\quad + r_s b_{s-1}(\omega^{s-1})](1-\tau_s) - c_s(\omega^s), \qquad (2) \\ b_s(\omega^s) &\geq -\lambda a_s(\omega^s), \qquad (3) \\ k_s(\omega^s) &\geq 0. \qquad (4) \end{aligned}$$

- $a_s(\omega^s)$ is the net-worth $[:= k_s(\omega^s) + b_s(\omega^s)]$.
- $\mu := \lambda/(1+\lambda)$ is also the extent of financial market imperfections.

Agents (Cont.)

The Euler equation:

$$\frac{1}{c_t(\omega^t)} = \beta E\left[\tilde{R}_{t+1}(1-\tau_{t+1})\frac{1}{c_{t+1}(\omega^{t+1})}\,\middle|\,\Phi^t(\omega^t)\right], \tag{5}$$

where $\tilde{R}_s := \max\{r_s, \frac{A\Phi_{s-1}-r_s\mu}{1-\mu}\}$.

The law of motion of an agent's net worth $a_t(\omega^t)$:

$$a_{t+1}(\omega^{t+1}) = \beta\tilde{R}_{t+1}(1-\tau_{t+1})a_t(\omega^t). \tag{6}$$

Govenment

- A government follows a certain fiscal policy rule:
  - Constant government spending/GDP ratio: θ.

$$\frac{E_t}{Y_t} = \theta. \tag{7}$$

  - Income tax rate τ_t is determined by B_{t-1}/Y_{t-1}.

$$\tau_t = \tau\left(B_{t-1}/Y_{t-1}\right). \tag{8}$$

    - ★ In equilibrium, B_{t-1}/Y_{t-1} is a function of ϕ_{t-1}. $\Rightarrow \tau_t = \tau\left(\phi_{t-1}\right)$.
    - ★ $\tau(\cdot)$ is differentiable on $[G^{-1}(\mu), h]$. $\dfrac{\partial\tau(\phi_{t-1})}{\partial\phi_{t-1}} \geq 0$.

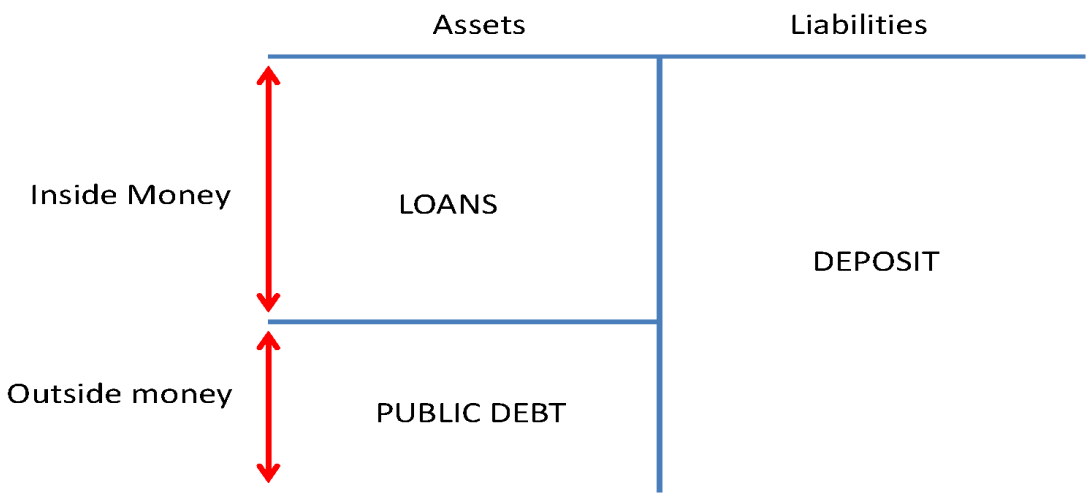
- Issuance of public debt is determined by the budget constraint.

$$B_t = r_t B_{t-1} + E_t - T_t. \tag{9}$$

Financial Intermediary (FI)

FI collects deposits and uses it to

- lend to investors, or
- purchase public debt.



Balance Sheet of the financial intermediary

Cutoff ϕ_t and Public-debt-to-GDP Ratio

- From the market clearing condition on public debt,

$$\frac{B_t}{Y_t} = \frac{\beta(1-\theta)[G(\phi_t) - \mu]}{1 - \mu - \beta[G(\phi_t) - \mu]}. \tag{10}$$

Proposition

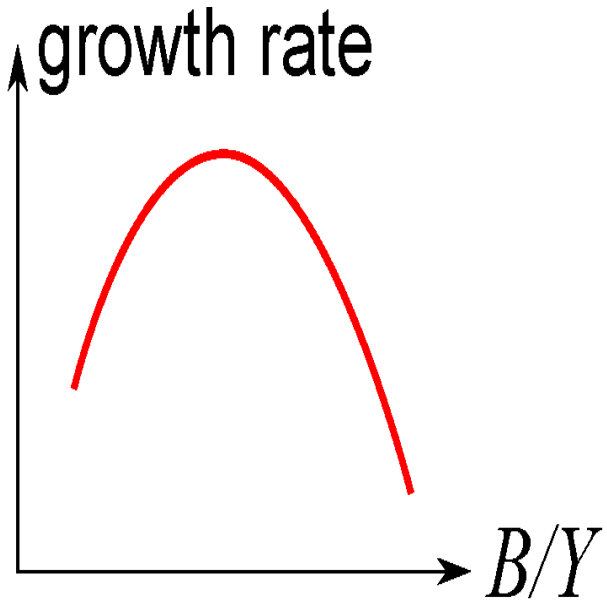
The PDG ratio, B_t / Y_t, is an increasing function of the cutoff ϕ_t.

Economic Growth Rate

Assumption

$\beta F(G^{-1}(\mu)) > G^{-1}(\mu)(1-\mu)$.

- Under the assumption, "inversed-U relationship" can be derived. [Prop. 2, 3].



Economic Growth Rate (Cont.)

- From the definition of aggregate output and the aggregate investment,

$$\frac{Y_{t+1}}{Y_t} = \frac{\beta A(1-\theta)F(\phi_t)}{1-\mu-\beta(G(\phi_t)-\mu)}, \tag{11}$$

where $F(\phi_t) = \int_{\phi_t}^{h} \Phi_t(\omega_t) \mathrm{d}G(\Phi_t)$.

- Two effects: Public debt $\uparrow \Rightarrow$ interest rate $\uparrow \Rightarrow$ ...
  - cutoff $\phi_t \uparrow \Rightarrow F(\phi_t) \downarrow$.
    The number of investors $\downarrow$ = crowd-out effect.
  - Interest rate $\uparrow \Rightarrow$ Public debt is a beneficial asset to store value. $\Rightarrow$ Less productive agents are given better saving opportunity. $\Rightarrow$ Many of them will become productive agents. Investment $\uparrow$ = crowd-in effect.

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Public Debt and the Growth Process

- From the market clearing condition on public debt,

$$\underbrace{\frac{\beta(1-\theta)(G(\phi_t)-\mu)}{1-\mu-\beta(G(\phi_t)-\mu)}}_{\Psi(\phi_t)} = \underbrace{\frac{(1-\tau(\phi_{t-1}))\phi_{t-1}(G(\phi_{t-1})-\mu)}{F(\phi_{t-1})}+\theta-\tau(\phi_{t-1})}_{\Lambda(\phi_{t-1})}. \quad (12)$$

  - $\Psi(\phi_t)$: Aggregate demand of public debt per GDP (determined in the financial market).
  - $\Lambda(\phi_{t-1})$: Aggregate supply of public debt per GDP (determined by fiscal policy).

Public Debt and Growth Process (Cont.)

We assume that the government enacts the following fiscal policy rule:

$$\tau(\phi_{t-1}) = 1 - \alpha(1 - (\phi_{t-1})^2), \tag{13}$$

where $\alpha > 0$ is a policy parameter.

- The government conducts public debt management with Eq. (13) so that the PDG ratio does not diverge.
- Given the value of ϕ_{t-1}, a greater (smaller) value of α yields a smaller (greater) value of $\tau(\phi_{t-1})$.
  - In the case of a greater value of α, the issuance of public debt plays a central role to finance government spending.
  - In the case of a smaller value of α, the issuance of public debt plays a minor role to finance it.

Public Debt and Growth Process (Cont.)

Assuming $G(\Phi) = \Phi$, $\Lambda(\phi_{t-1})$ (the RHS of (12)) becomes:

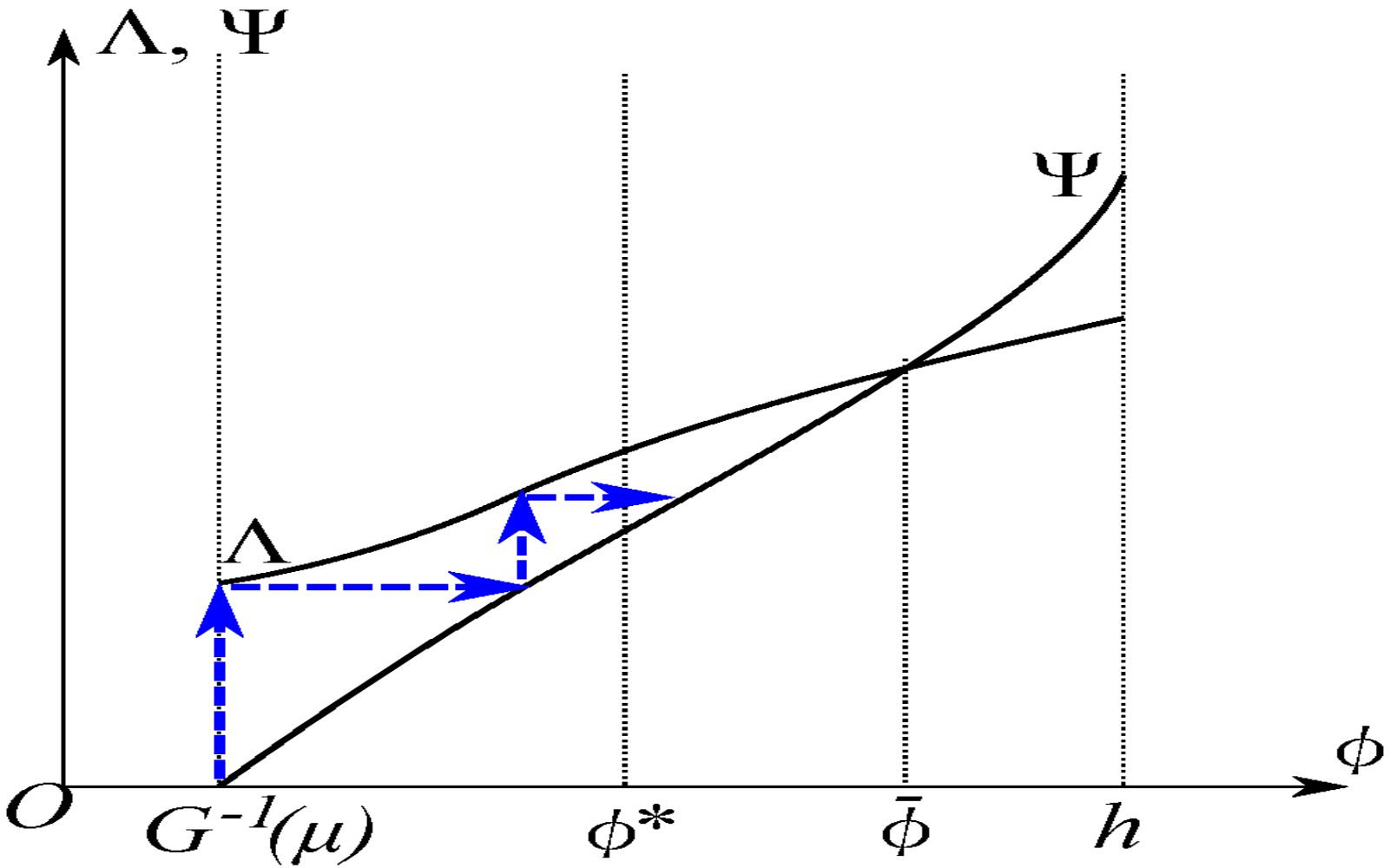
$$\Lambda(\phi_{t-1}) = \alpha(\phi_{t-1} - \mu)^2 + \alpha(1 - \mu^2) + \theta - 1. \tag{14}$$

- Our interest is in investigating the mechanism that creates the inverted U-shaped relationship between the accumulation of public debt and economic growth.
  - We focus on the simplest case in which $\Lambda(\phi)$ is monotonically increasing and there is a unique steady state $\bar{\phi}$.
  - We impose the following parameter assumption:

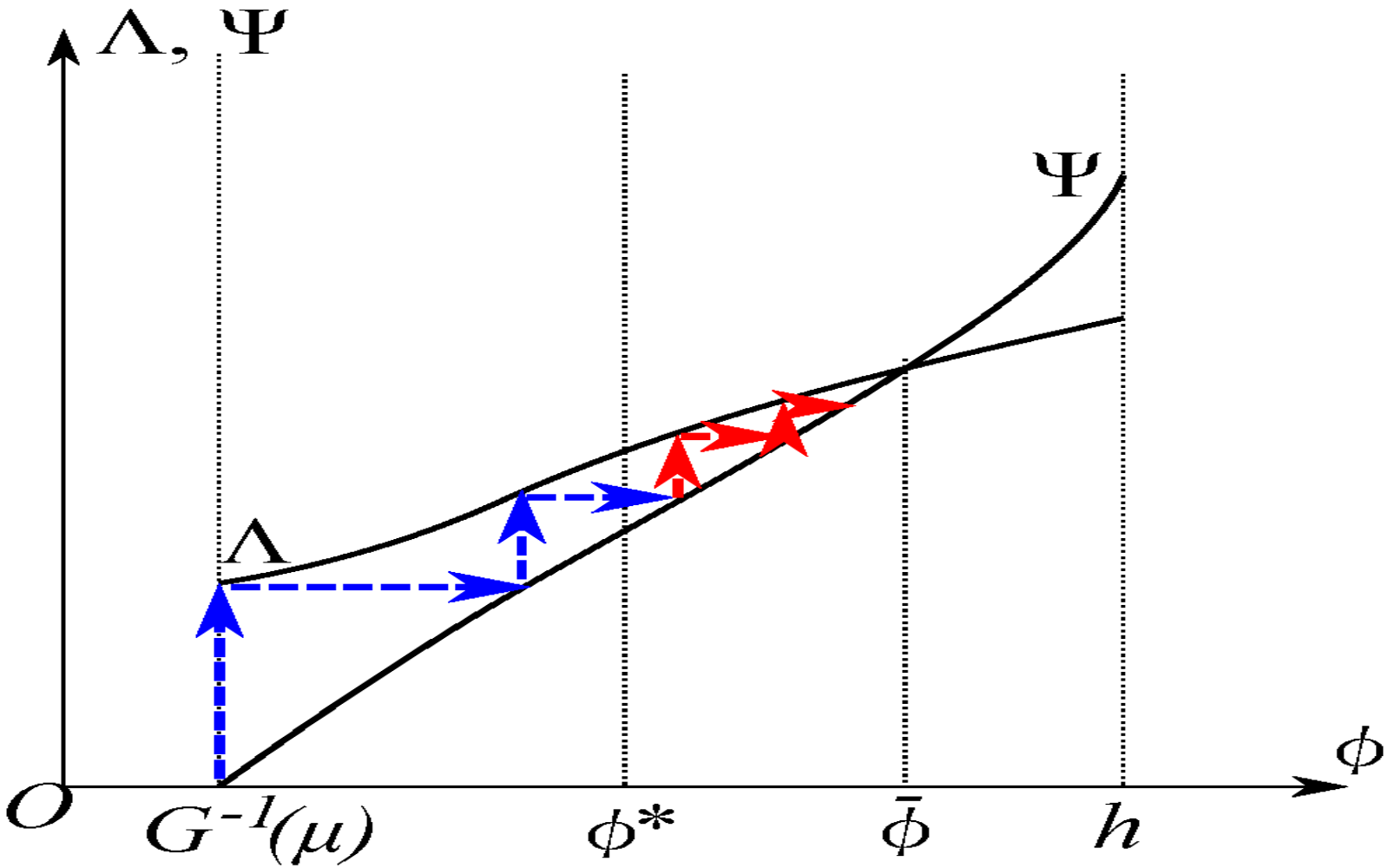
Assumption

$\frac{1-\theta}{1-\mu^2} < \alpha < \frac{1-\theta}{2(1-\beta)(1-\mu)}$.

Public Debt and Growth Process (Cont.)



Public Debt and Growth Process (Cont.)



Public Debt and Growth Process (Cont.)

- On the dynamic transition path,
  - When B/Y is low, $B/Y \uparrow \Rightarrow$ growth rate $\uparrow$,
  - When B/Y is high, $B/Y \uparrow \Rightarrow$ growth rate $\downarrow$,

Growth Patterns subject to Fiscal Policy Rules

- How are economic growth patterns subject to fiscal policy rules?
- We compare two types of fiscal policy rules.

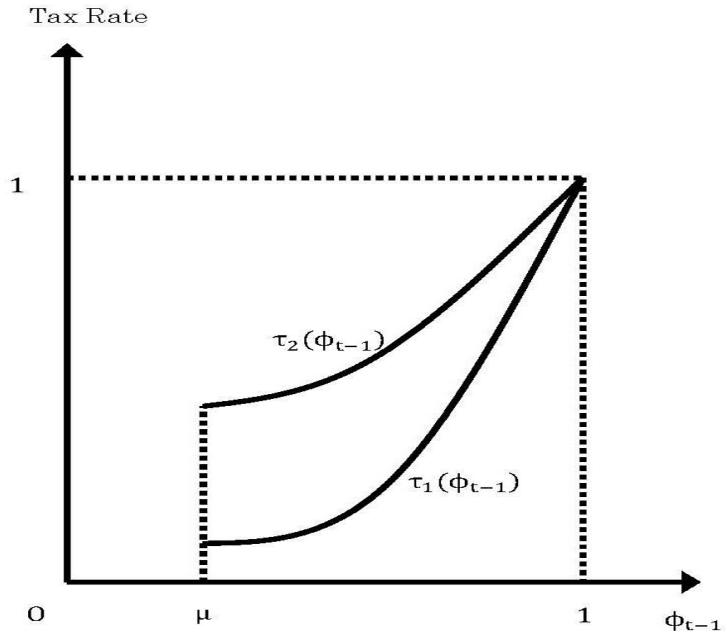
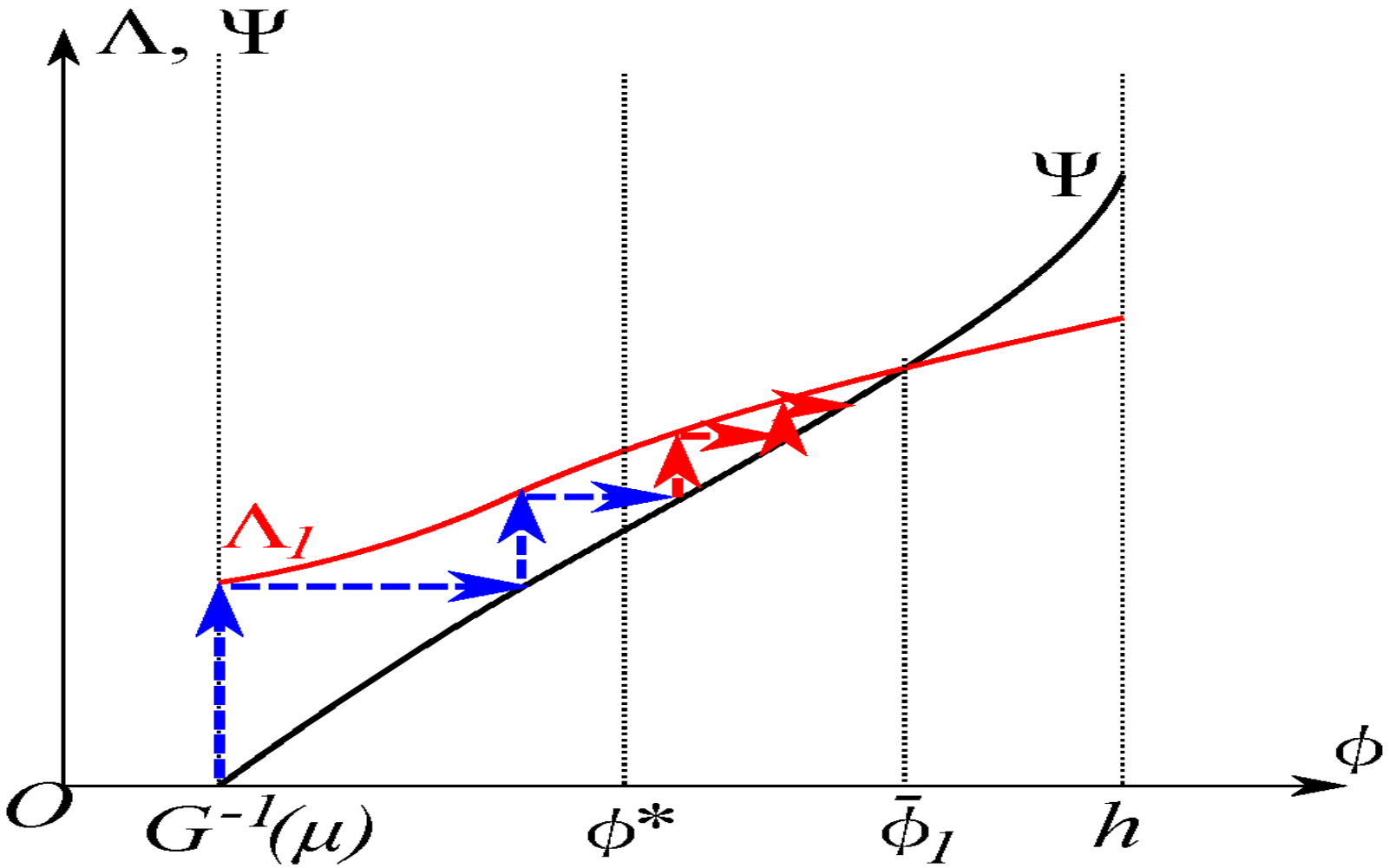


Fig. 2: Two different fiscal policy rules

- $\tau_1(\phi_t)$: a "more expansionary" policy. α is bigger
- $\tau_2(\phi_t)$: a "less expansionary" policy. α is smaller.

Growth Patterns subject to Fiscal Policy Rules (Cont.)



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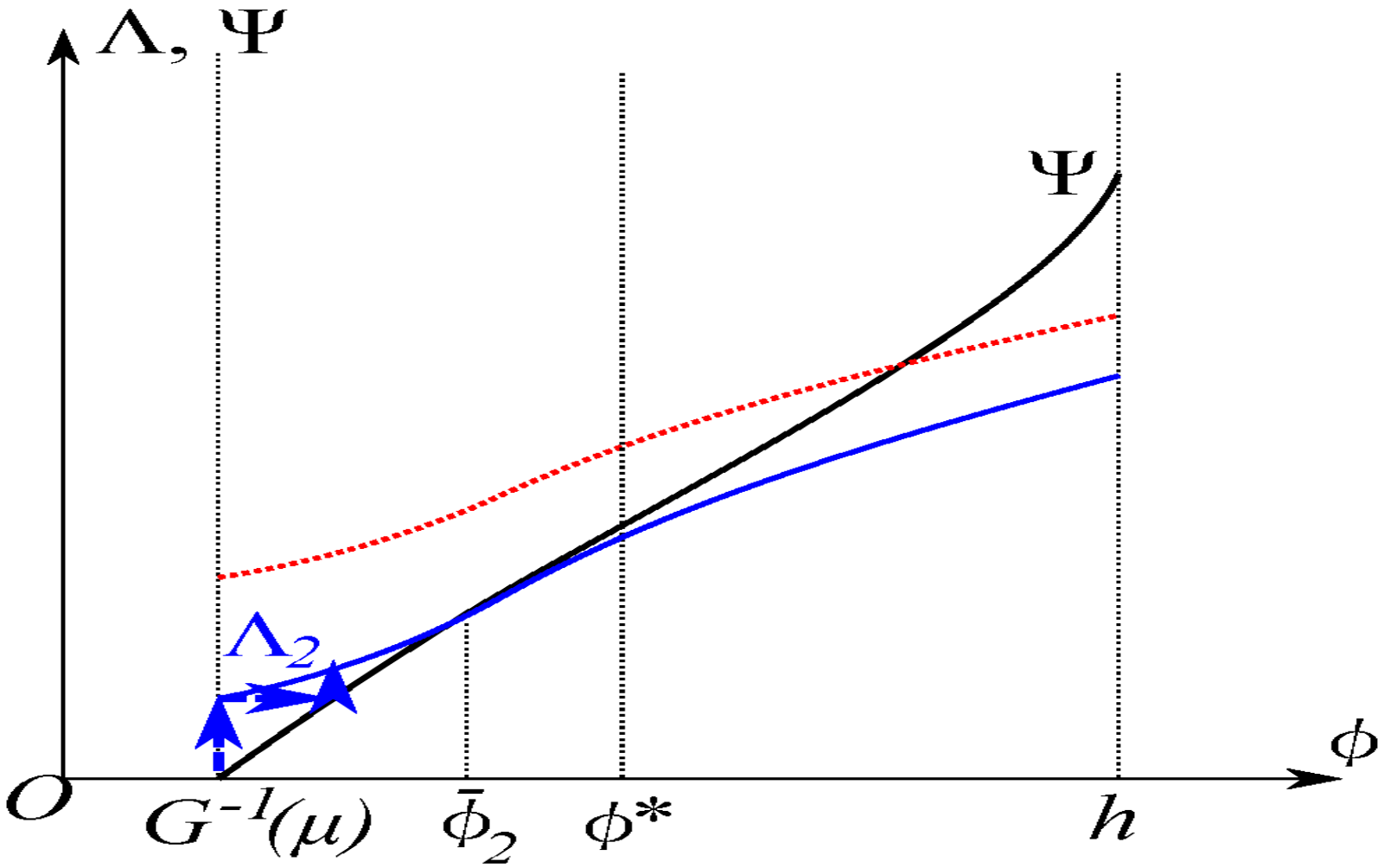


Quantitative Analysis

In any cases in equilibrium including the dynamic growth process and the steady state, the following two equation determine both PDG ratio and growth rate through the cutoff ϕ_t.

$$\frac{B_t}{Y_t} = \frac{\beta(1-\theta)[G(\phi_t)-\mu]}{1-\mu-\beta[G(\phi_t)-\mu]}. \tag{15}$$

$$\frac{Y_{t+1}}{Y_t} = \frac{\beta A(1-\theta)F(\phi_t)}{1-\mu-\beta(G(\phi_t)-\mu)}. \tag{16}$$

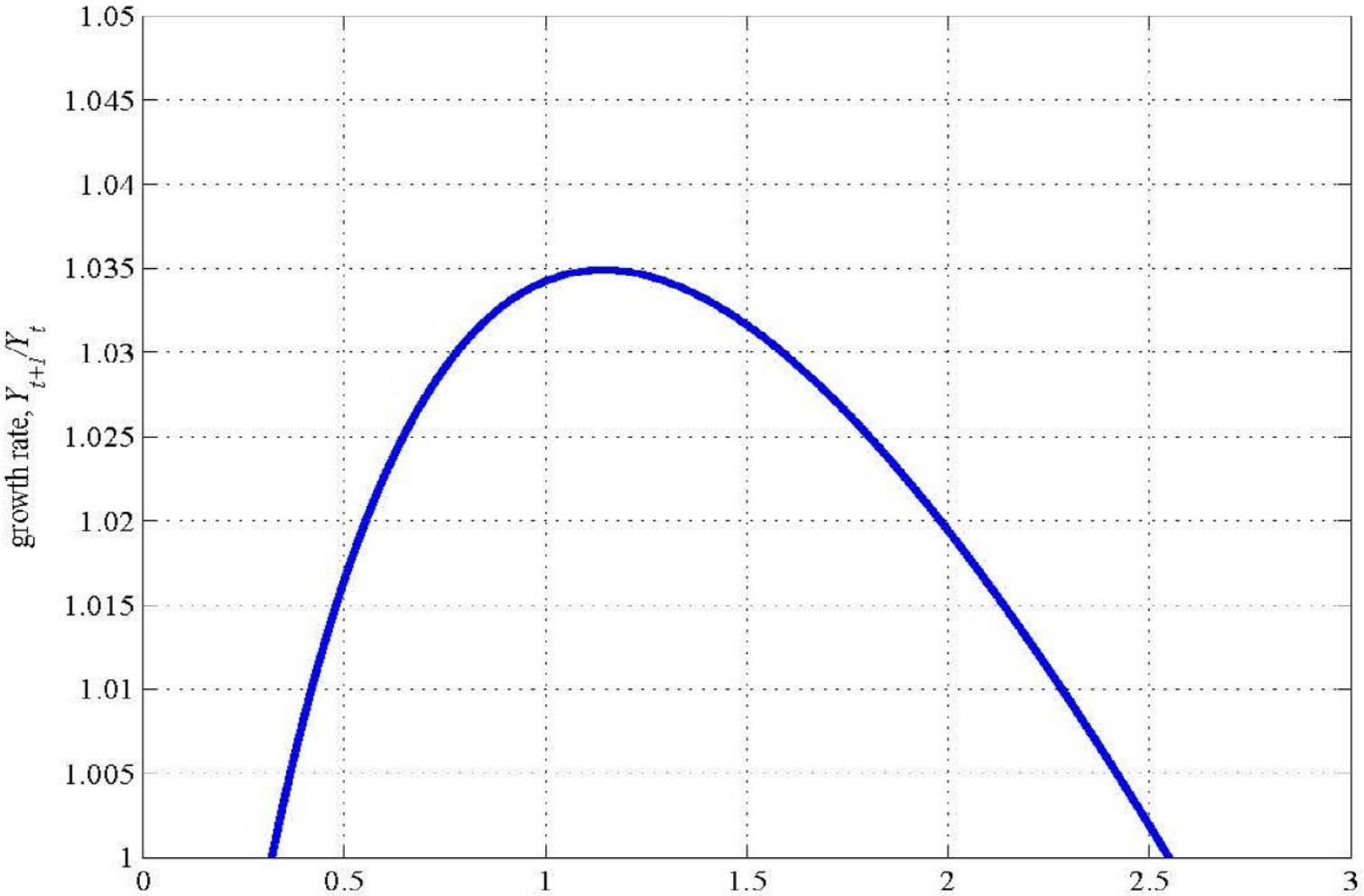
We calibrate the model and derive the growth-maximizing PDG ratio from these two equations

Quantitative Analysis (Cont.)

- $G(\Phi) = \Phi$.
- The subjective discount factor (a year): $\beta = 0.96$
- The severity of credit constraints: $\mu = 0.57$ (benchmark case).
  - Producers' net worth-to-investment ratio $a_t(\omega^t)/k_t(\omega^t) = 1 - \mu$ in our model can be regarded as owners' equity-to-total assets ratio in the actual balance sheet.
  - According to the Quarterly Financial Report created by the United States Census Bureau in 2006, the average of owners' equity-to-total assets ratio for all corporations in the NAICS manufacturing sector is around 0.43 in 2006.
- The government-spending-to-GDP ratio: $\theta = 0.157$.
  - We follow the procedure employed by Hayashi and Prescott (2002).
- The scaling parameter: $A = 1.48$.
  - We use the average growth rate and the average PDG ratio over the period 1996-2006 in the US with eqs. (14) and (15).

Quantitative Analysis (Cont.)

The inverted U-shaped relationship between the accumulation of public debt and economic growth.



Quantitative Analysis (Cont.)

Table 1: Growth-maximizing PDG ratio and net growth rate

Parameters: $A = 1.52$, $\beta = 0.96$, $\mu = 0.50$, $\theta = 0.157$

$(B/Y)^*$	129%
$(Y_{t+1}/Y_t)^* - 1$	4.32%

Parameters: $A = 1.48$, $\beta = 0.96$, $\mu = 0.57$, $\theta = 0.157$

$(B/Y)^*$	114%
$(Y_{t+1}/Y_t)^* - 1$	3.49%

Parameters: $A = 1.42$, $\beta = 0.96$, $\mu = 0.70$, $\theta = 0.157$

$(B/Y)^*$	82%
$(Y_{t+1}/Y_t)^* - 1$	2.46%

Quantitative Analysis (Cont.)

Table 2: Robustness checks with $G(\Phi) = \Phi^2$

Parameters: $A = 1.40$, $\beta = 0.96$, $\mu = 0.50$, $\theta = 0.157$

$(B/Y)^*$	74.5%
$(Y_{t+1}/Y_t)^* - 1$	2.38%

Parameters: $A = 1.39$, $\beta = 0.96$, $\mu = 0.57$, $\theta = 0.157$

$(B/Y)^*$	62.5%
$(Y_{t+1}/Y_t)^* - 1$	2.30%

Parameters: $A = 1.36$, $\beta = 0.96$, $\mu = 0.70$, $\theta = 0.157$

$(B/Y)^*$	37.2%
$(Y_{t+1}/Y_t)^* - 1$	2.50%

Concluding Remarks

- Question: How does the accumulation of public debt affect economic growth ?
  - Recent empirical evidence is mixed.
- Result: We construct a theoretical model that can generate the "inversed-U" relationship.
- Intuition: $B_t / Y_t \uparrow \Rightarrow$ interest rate $\uparrow \Rightarrow$ ...
  - The number of investors $\downarrow$ = crowd-out effect.
  - $\Rightarrow$ Public debt is a beneficial asset to store value. $\Rightarrow$ Less productive agents are given better saving opportunity. $\Rightarrow$ Many of them will become productive agents. Investment $\uparrow$ = crowd-in effect.