Identification and Counterfactuals in Dynamic Models of Market Entry and Exit

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Abstract

This paper deals with a fundamental identification problem in the structural estimation of dynamic oligopoly models of market entry and exit. Using the standard datasets in existing empirical applications, there are three components of a firm's profit function that are not separately identified: the fixed cost of an incumbent firm, the entry cost of a new entrant, and the scrap-value of an exiting firm. We study the implications of this result on the power of this class of models to identify the effects of different comparative static exercises and counterfactual public policies. First, we derive a closed-form relationship between the three unknown structural functions and two functions that are identified from the data. We use this relationship to provide the correct interpretation of the estimated objects that are obtained under the 'normalization assumptions' considered in most applications. Second, we characterize a class of counterfactual experiments that are identified using the estimated model, despite the non-separate identification of the three primitives. Third, we show that there is a general class of counterfactual experiments of economic relevance that are not identified. We present a numerical example that illustrates how ignoring the non-identification of these counterfactuals (i.e., making a 'normalization assumption' on some of the three primitives) generates sizable biases that can modify even the sign of the estimated effects. Finally, we discuss possible solutions to deal with these identification problems.

Keywords: Dynamic structural models; Market entry and exit; Identification; Fixed cost; Entry cost; Exit value; Counterfactual experiment; Land price.

JEL codes: L10; C01; C51; C54; C73.

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1 Introduction

Dynamic models of market entry and exit are important tools in the empirical analysis of firm competition. These models are particularly useful to study empirical questions for which it is key to take into account the endogeneity of market structure and its evolution over time. During recent years, the structural estimation of this class of models has experienced substantial developments, both methodological and empirical, and there is a growing number of empirical applications.[1] In all these applications, the answer to the empirical questions of interest is based on the implementation of counterfactual experiments using the estimated model. Sometimes the purpose of a counterfactual experiment is to evaluate the effects of a hypothetical public policy, such as a new tax or subsidy, but many times the main purpose of the experiment is to measure the effects of a parameter change, e.g., we may want to obtain the values of firms' profits and consumer welfare when we shut-down a parameter that captures the degree of learning-by-doing. In the estimation of dynamic structural models of market entry-exit, we distinguish two main components in a firm's profit function: *variable profit* and *fixed cost*. Parameters in the *variable profit* function (i.e., demand and variable cost parameters) can be identified using data on firms' quantities and prices combined with a demand system and a model of competition in prices or quantities.[2] The *fixed cost* is the part of the profit that comes from buying, selling, or renting inputs that are fixed during the whole active life of the firm.[3] It is constant with respect to the amount of output that the firm produces and sells in the market, but it depends on the amount and the prices of fixed inputs, such as land or fixed capital, and on the firm's current and past incumbent status. The parameters in the fixed cost are estimated using data on firms' choices to be active or not in the market, combined with a dynamic model of market entry-exit. The identification of this fixed cost function is based on the principle of revealed preference, i.e., if a firm chooses to be active in the market is because its expected value of being in the market is greater than its expected value of not being in the market.

This paper deals with a fundamental identification problem in the structural estimation of dynamic oligopoly models of market entry and exit. Using the standard datasets in existing empirical

[1] Examples of recent applications are: Ryan (2012) on environmental regulation of an oligopoly industry; Suzuki (forthcoming) on land use regulation and competition in retail industries; Hashmi and Van Biesebroeck (2012) and Kryukov (2010) on the relationship between market structure and innovation, Sweeting (2011) on competition in the radio industry and the effects of copyright fees; Collard-Wexler (2010) on demand uncertainty and industry dynamics; Snider (2009) on predatory pricing in the airline industry; or Aguirregabiria and Ho (2012) on airlines network structure and entry deterrence.

[2] See Berry and Haile (2010 and 2012) for recent identification results in the estimation of demand and supply models of differentiated products.

[3] There is some abuse of language in using the term "cost" to refer to this component of the firm's profit. This fixed component of the profit may include positive income/profits associated with sales of owned inputs such as land, buildings, etc. For this reason, sometimes in this paper we will use "fixed profit" instead of "fixed cost" to denote this component.

applications, there are three key components of a firm's profit function that are not separately identified: the fixed cost of an incumbent firm, the entry cost of a new entrant, and the exit-value, or scrap-value, of an exiting firm.[4] We study the implications of this result on the power of this class of models to identify the effects of different comparative static exercises and counterfactual public policies. This is an important issue because many empirical questions on market competition, as well as the evaluation of the effects of public policies in oligopoly industries, involve changes in some of these structural functions (see Ryan 2012, Dunne et al. 2011, Lin 2012, and Varela 2011 among others).

First, we derive a closed-form relationship between the three unknown structural functions and two functions that are identified from the data. We use this relationship to provide the correct interpretation of the estimated objects that are obtained under the 'normalization assumptions' considered in most applications, e.g., the exit-value function is 'normalized' to zero. Second, we characterize a class of counterfactual experiments that are identified using the estimated model, despite the non-separate identification of the three primitives. This class of identified counterfactuals consists in an additive change in the structural function(s) where the change is known to the researcher. Third, we show that there is a general class of counterfactual experiments of economic relevance that are not identified. For instance, within this class of experiments we have a change in the stochastic process of an input price that is an argument in the entry cost, fixed cost, and exit value functions, e.g., land price. We present numerical examples that illustrate how ignoring the non-identification of these counterfactuals (i.e., making a 'normalization assumption' on some of the three primitives) generates sizable biases that can modify even the sign of the estimated effects. Finally, we discuss possible solutions to deal with these identification problems. Collecting and using data on firms' scrap-values is a possible approach. Also, in industries where the trade of firms is frequent and the researcher observes transaction prices (Kalouptsidi 2011), this information can be used to solve this identification problem. In the absence of this type of data, the researcher can apply a bounds-approach in the spirit of Manski (1995). We discuss the implementation of this approach in this context.

The rest of the paper proceeds as follows. Section 2 presents the model of market entry and exit. Section 3 describes the identification problem and the relationship between structural functions and identified objects. Section 4 deals with the identification of counterfactual experiments and presents numerical examples. In section 5, we discuss different approaches to deal with this identification

[4] This identification problem is *fundamental* in the sense that it does not depend on other econometric issues that appear in this class of models, such as the stochastic structure of the unobservables, the non-idependence between observable and unobservable state variables, or the existence of multiple equilibria in the data. Of course, these issues may generate additional identification problems. However, dealing with, or solving, these other identification problems does not help to the separate identification of the three components in the fixed cost function.

problem. We summarize and conclude in section 6.

2 Dynamic model of market entry and exit

2.1 Model

We start with a single-firm version of the model or dynamic model of monopolistic competition. Later in this section we extend our framework to dynamic games of oligopoly competition.

Time is discrete and indexed by t. Every period t the firm decides to be active in the market or not. Let $a_t \in \{0,1\}$ be the binary indicator of the firm's decision at period t, such that $a_t = 1$ if the firm decides to be active in the market at period t, and $a_t = 0$ otherwise. The firm takes this action to maximize its expected and discounted flow of profits, $\mathbb{E}_t\left(\sum_{r=0}^{\infty} \beta^r \ \Pi_{t+r}\right)$, where $\beta \in (0,1)$ is the discount factor, and Π_t is the firm's profit at period t. We distinguish two main components in the firm's profit at time t: variable profits, VP_t, and fixed profits, FP_t, with $\Pi_t = VP_t + FP_t$. The variable profit is the part that varies continuously with the firm's output, and it is equal to the difference between revenue and variable costs. By definition, if the firm is not active in the market, variable profit is zero. If active in the market, the firm observes its demand curve and variable cost function and chooses its price to maximize variable profits at period t. This static price decision determines an indirect variable profit function that relates this component of profit with state variables:[5]

$$VP_t = a_t \ \ vp(\mathbf{z}_t^v) \tag{1}$$

where $vp(.)$ is a real-valued function, and $\mathbf{z}_t^v$ is a vector of exogenous state variables affecting demand and variable costs, e.g., market size, consumers' socioeconomic characteristics, and prices of variable inputs such as wages, price of materials, energy, etc.

The fixed profit is the part of the profit that comes from buying, selling, or renting inputs that are fixed during the whole active life of the firm. These fixed inputs may include land, buildings, some type of equipment, or even managerial skills. We distinguish three components in the fixed profit: the fixed cost of an active firm, FC_t, the entry cost of a new entrant, EC_t, and the scrap value of the exiting firm, SV_t. Each of these three components may depend on a vector of exogenous state variables $\mathbf{z}_t^c$ that includes prices of fixed inputs (e.g., prices of land and fixed capital inputs), and it may have elements in common with the vector $\mathbf{z}_t^v$ (e.g., market size might also affect fixed

[5] The variable profit function is $VP_t = p_t \ q_t - VC(q_t, \mathbf{z}_t^v)$, where p_t and q_t are the firm's price and output, respectively, and VC is the variable cost function that depends on output and on the vector of state variables $\mathbf{z}_t^v$. Output equals demand, $q_t = D(p_t, \mathbf{z}_t^v)$, where D is the demand function. Maximization of variable profit implies the well-known marginal condition $D(p_t, \mathbf{z}_t^v) + \ p_t \ [\partial D(p_t, \mathbf{z}_t^v)/\partial p_t] - \ MC(D(p_t, \mathbf{z}_t^v), \mathbf{z}_t^v) \ [\partial D(p_t, \mathbf{z}_t^v)/\partial p_t] = 0$, where MC represents the marginal cost function. Solving in this condition for price, we get the optimal pricing function $p_t = p^*(\mathbf{z}_t^v)$. And plugging-in this optimal price into the variable profit function, we get the indirect variable profit function $vp(\mathbf{z}_t^v) \equiv p^*(\mathbf{z}_t^v) \ D(p^*(\mathbf{z}_t^v), \mathbf{z}_t^v) - \ VC(D(p^*(\mathbf{z}_t^v), \mathbf{z}_t^v), \mathbf{z}_t^v)$.

costs):

$$
\begin{aligned}
FP_t &= -FC_t - EC_t + SV_t \\
&= -a_t \; fc\left(\mathbf{z}_t^c\right) - a_t \left(1 - i_t\right) \; ec\left(\mathbf{z}_t^c\right) \; + \left(1 - a_t\right) i_t \; sv\left(\mathbf{z}_t^c\right)
\end{aligned}
\tag{2}
$$

where $fc(.)$, $ec(.)$, and $sv(.)$ are real-valued functions, and $i_t \equiv a_{t-1}$ is the indicator of the event "the firm was active at period $t-1$". The fixed cost is paid every period that the firm is active (i.e., when $a_t = 1$). It includes the cost of renting some fixed inputs, and taxes that should be paid every period and that depend on the amount of some owned fixed inputs, e.g., property taxes. The entry cost is paid if the firm is active at the current period but it was not active at previous period (i.e., when $a_t = 1$ and $i_t = 0$), and it includes the cost of purchasing fixed inputs, and transaction costs related to the startup of the firm. The firm receives a scrap or exit value when it was active at the previous period but decides to exit at the current period (i.e., when $a_t = 0$ and $i_t = 1$). This scrap value includes earnings from selling owned fixed inputs, minus transaction costs related to closing the firm such as compensations to workers and to lessors of capital from breaking long term contracts.

EXAMPLE 1. Consider the decision of a hotel chain to operate or not a hotel in a local market or small town. To start its operation (entry in the market), the firm should purchase or lease some fixed inputs such as land, building, furniture, elevators, restaurant, kitchen, and other equipment. If this equipment is purchased at the time of entry, the cost of purchasing these inputs is part of the entry cost. Other components of the entry cost are the cost of a building permit, or for the case of franchises, franchise fees. Some fixed inputs are leased. Therefore, a hotel fixed cost includes the renting cost of leased fixed inputs. It also includes property taxes (that depend on land prices), royalties to the franchisor, and maintenance costs of owned fixed inputs. At the time of its closure, the hotel operator may recover some amount of money by selling the owned fixed inputs such as land, buildings, furniture, and other equipment. These amounts correspond to the scrap value. ■

The one-period profit function can be described as:[6]

$$
\Pi_t = \begin{cases} i_t \; sv\left(\mathbf{z}_t^c\right) & \text{if } a_t = 0 \\ \\ vp(\mathbf{z}_t^v) - fc\left(\mathbf{z}_t^c\right) - \left(1 - i_t\right) \; ec\left(\mathbf{z}_t^c\right) & \text{if } a_t = 1 \end{cases}
\tag{3}
$$

The vector of state variables of this dynamic model is $\{\mathbf{z}_t, \, i_t\}$, where $\mathbf{z}_t \equiv \{\mathbf{z}_t^v, \, \mathbf{z}_t^c\}$. The vector of state variables $\mathbf{z}_t$ follows a Markov process with transition probability function $f_z(\mathbf{z}_{t+1}|\mathbf{z}_t)$. The indicator of incumbent status follows the trivial endogenous transition rule, $i_{t+1} = a_t$.

[6] In this version of the model, there is no "time-to-build" or "time-to-exit" such that the decision of being active or not in the market is taken at period t and it is effective at the same period, without any lag. At the end of this section we discuss variations of the model that involve "time-to-build" or/and "time-to-exit". These variations do not have any incidence in our (non) identification results, though they imply some minor changes in the interpretation of the identified objects.

In the econometric or empirical version of the model we distinguish two different types of state variables: those observable to the researcher and those unobservable. Here we consider a general additive specification of the unobservables where every primitive function has an unobservable component:

$$\begin{array}{rcl} VP_t & = & a_t \ \left[vp(\mathbf{z}_t^v) + \varepsilon_t^{vp}\right] \\ FC_t & = & a_t \ \left[fc(\mathbf{z}_t^c) + \varepsilon_t^{fc}\right] \\ EC_t & = & a_t \left(1-i_t\right) \ \left[ec(\mathbf{z}_t^c) + \varepsilon_t^{ec}\right] \\ SV_t & = & \left(1-a_t\right) i_t \ \left[sv(\mathbf{z}_t^c) + \varepsilon_t^{sv}\right] \end{array} \tag{4}$$

where $\boldsymbol{\varepsilon}_t \equiv \{\varepsilon_t^{vp}, \varepsilon_t^{fc}, \varepsilon_t^{ec}, \varepsilon_t^{sv}\}$ is the vector of state variables that are observable to the firm at period t but unobservable to the researcher. These unobserved state variables have zero mean conditional on $\mathbf{z}_t$. For simplicity, we also assume that $\boldsymbol{\varepsilon}_t$ is i.i.d. over time and independent of $(\mathbf{z}_t, i_t)$ (Rust (1994)). Allowing for serially correlated unobservables does not have any substantive influence on the positive or negative identification results in this paper. Serial correlation in the unobservables creates the so called "initial conditions problem", that is an identification problem of different nature to the one we study in this paper. Whatever the way the researcher deals with the "initial conditions problem", he still faces the problem of separate identification of the three components in the fixed profit. Therefore, the specification of the one-period profit function including unobservable state variables is:

$$\Pi_t = \pi\left(a_t, i_t, z_t\right) + \varepsilon_t\left(a_t\right) = \begin{cases} i_t \ sv\left(\mathbf{z}_t\right) + \varepsilon_t(0) & \text{if } a_t = 0 \\ vp(\mathbf{z}_t) - fc\left(\mathbf{z}_t\right) - \left(1-i_t\right) \ ec\left(\mathbf{z}_t\right) + \varepsilon_t(1) & \text{if } a_t = 1 \end{cases} \tag{5}$$

where $\pi(.)$ is the component of the one-period profit that does not depend on unobservables, $\varepsilon_t(0) \equiv i_t \ \varepsilon_t^{sv}$, and $\varepsilon_t(1) \equiv \varepsilon_t^{vp} - \varepsilon_t^{fc} - (1-i_t)\, \varepsilon_t^{ec}$.

The value function of the firm, $V\left(i_t, \mathbf{z}_t, \boldsymbol{\varepsilon}_t\right)$, is the unique solution to the Bellman equation:

$$V\left(i_t, \mathbf{z}_t, \boldsymbol{\varepsilon}_t\right) = \max_{a_t \in \{0,1\}} \left\{ \pi\left(a_t, i_t, \mathbf{z}_t\right) + \varepsilon_t(a_t) + \beta \sum_{\mathbf{z}_{t+1} \in \mathbf{Z}} f_z(\mathbf{z}_{t+1}|\mathbf{z}_t) \int V\left(a_t, \mathbf{z}_{t+1}, \boldsymbol{\varepsilon}_{t+1}\right) \ dG\left(\boldsymbol{\varepsilon}_{t+1}\right) \right\} \tag{6}$$

where $\beta \in (0,1)$ is the discount factor, and $G(.)$ is the CDF of $\boldsymbol{\varepsilon}_t$. Similarly, the optimal decision rule of this dynamic programming problem is a function $\alpha\left(i_t, \mathbf{z}_t, \boldsymbol{\varepsilon}_t\right)$ from the space of state variables into the action space $\{0,1\}$ such that:

$$\alpha\left(i_t, \mathbf{z}_t, \boldsymbol{\varepsilon}_t\right) = \arg \max_{a_t \in \{0,1\}} \left\{ v\left(a_t, i_t, \mathbf{z}_t\right) + \varepsilon_t\left(a_t\right) \right\} \tag{7}$$

where

$$v\left(a_t, i_t, z_t\right) = \pi\left(a_t, i_t, \mathbf{z}_t\right) + \beta \sum_{\mathbf{z}_{t+1} \in \mathbf{Z}} f_z(\mathbf{z}_{t+1} | \mathbf{z}_t) \int V\left(a_t, \mathbf{z}_{t+1}, \boldsymbol{\varepsilon}_{t+1}\right) \ dG\left(\boldsymbol{\varepsilon}_{t+1}\right). \tag{8}$$

By the additivity and the conditional independence of the unobservable ε's, the optimal decision rule has the following threshold structure:

$$\alpha\left(i_t, \mathbf{z}_t, \boldsymbol{\varepsilon}_t\right) = 1\{\widetilde{\varepsilon}_t \leq \tilde{v}\left(i_t, \mathbf{z}_t\right)\} \tag{9}$$

where $\tilde{v}\left(i_t, \mathbf{z}_t\right) = v\left(1, i_t, \mathbf{z}_t\right) - v\left(0, i_t, \mathbf{z}_t\right)$ and $\widetilde{\varepsilon}_t \equiv \varepsilon_t(0) - \varepsilon_t(1) \equiv (\varepsilon_t^{fc} + \varepsilon_t^{ec} - \varepsilon_t^{vp}) + i_t \ (\varepsilon_t^{sv} - \varepsilon_t^{ec})$.

For our analysis, it is helpful to define also the *Conditional Choice Probability* (CCP) function $P(i_t, \mathbf{z}_t)$ that is the optimal decision rule integrated over the unobservables:

$$\begin{aligned} P(i, \mathbf{z}) &\equiv \Pr\left(\alpha\left(i_t, \mathbf{z}_t, \boldsymbol{\varepsilon}_t\right) = 1 \mid i_t = i, \mathbf{z}_t = \mathbf{z}\right) \\ &= \Pr\left(\widetilde{\varepsilon}_t \leq \tilde{v}\left(i, \mathbf{z}\right)\right) \\ &= F_{\widetilde{\varepsilon}|i}\left(\tilde{v}\left(i, \mathbf{z}\right)\right) \end{aligned} \tag{10}$$

where $F_{\widetilde{\varepsilon}|i}$ is the CDF of $\widetilde{\varepsilon}_t$ conditional on $i_t = i$.[7] Note that $P(0, \mathbf{z}_t)$ is the probability of market entry, and $[1 - P(1, \mathbf{z}_t)]$ is the probability of market exit. In our model, the only decision made is the market entry-exit. However, our non-identification results extend to more general models where incumbent firms make investments in product quality, capacity, etc.

2.2 Extensions: Dynamic Game and Time-to-Build

We also study the identification of three extensions, or variations, of the basic model described above: (a) model with no re-entry after market exit; (b) model with time-to-build and time-to-exit; and (c) dynamic oligopoly game of market entry and exit.

(a) No re-entry after market exit. Some models and empirical applications of industry dynamics assume there is no re-entry after exit from the market. This model is practically the same as the one presented above, with the only difference that the value of market exit $v\left(0, 1, \mathbf{z}_t\right) + \varepsilon_t(0)$ is simply the scrap value $sv(\mathbf{z}_t) + \varepsilon_t^{sv}$.

(b) Time-to-build and time-to-exit. In this version of the model, it takes one period to make entry and exit decisions effective, though the entry cost is paid at the period when the entry decision is made, and similarly the scrap value is received at the period when the exit decision is taken. Now, a_t is the binary indicator of the event "the firm will be active in the market at period $t+1$", and

[7] The distribution of $\widetilde{\varepsilon}_t$ depends on i_t if the entry cost and the scrap value contain unobservable components and these unobservables are different, i.e., $\varepsilon_t^{sv} - \varepsilon_t^{ec} \neq 0$.

$i_t = a_{t-1}$ is the binary indicator of the event "the firm is active in the market at period t". For this model, the one-period profit function is:

$$\Pi_t = \begin{cases} i_t[vp(\mathbf{z}_t) - fc\,(\mathbf{z}_t) + sv\,(\mathbf{z}_t)] + \varepsilon_t(0) & \text{if } a_t = 0 \\ \\ i_t[vp(\mathbf{z}_t) - fc\,(\mathbf{z}_t)] - (1 - i_t)\;\; ec\,(\mathbf{z}_t) + \varepsilon_t(1) & \text{if } a_t = 1 \end{cases} \tag{11}$$

Given this structure of the profit function, we have that the Bellman equation, optimal decision rule, and CCP function are defined exactly the same as above in equations (6), (9), and (10), respectively.

(c) Dynamic oligopoly game of entry and exit. We follow the standard structure of dynamic oligopoly models in Ericson and Pakes (1995) but including firms' private information as in Doraszelski and Satterthwaite (2010).[8] There are N firms that may operate in the market. Firms are indexed by $j \in \{1, 2, ..., N\}$. Every period t, the N firms decide simultaneously but independently whether to be active or not in the market. Let a_{jt} be the binary indicator for the event "firm j is active in the market at period t". Variable profits at period t are determined in a static Cournot or Bertrand model played between those firms who choose to be active. This static competition determines the indirect variable profit functions of the N firms:

$$VP_{jt} = a_{jt} \quad \left[vp_j(\boldsymbol{a}_{-jt}, \mathbf{z}_t^v) + \varepsilon_{jt}^{vp} \right] \tag{12}$$

where VP_{jt} is the variable profit of firm j; $\boldsymbol{a}_{-jt}$ is the $N-1$ dimensional vector with the binary indicators for the activity of all the firms except firm j; and ε_{jt}^{vp} is a private information shock in the variable profit of firm j that is unobservable to other firms and to the researcher. Note that the variable profit functions, vp_j, can vary across firms due to permanent, common knowledge differences between the firms in variable costs or in the quality of their products.

The three components of the fixed profit function have specifications similar to the case of monopolistic competition, with the only differences that the functions can vary across firms, and the unobservable $\varepsilon' s$ are private information shocks of each firm: $FC_{jt} = a_{jt}\ [fc_j(\mathbf{z}_t^c) + \varepsilon_{jt}^{fc}]$; $EC_{jt} = a_{jt}$ $(1 - i_{jt})\ [ec_j(\mathbf{z}_t^c) + \varepsilon_{jt}^{ec}]$; and $SV_{jt} = (1 - a_{jt})\ i_{jt}\ [sv_j(\mathbf{z}_t^c) + \varepsilon_{jt}^{sv}]$, where $\boldsymbol{\varepsilon}_{jt} \equiv \{\varepsilon_{jt}^{vp}, \varepsilon_{jt}^{fc}, \varepsilon_{jt}^{ec}, \varepsilon_{jt}^{sv}\}$ is a vector of state variables that are private information for firm j, they are unobservable to the researcher, and i.i.d. across firms and over time with CDF G.

Following the literature on dynamic games of oligopoly competition, we assume that the outcome of the dynamic game of entry and exit played by the N firms is a Markov Perfect Equilibrium (MPE). In a MPE, firms' strategy functions depend only on payoff relevant state variables. Let

[8] In Ericson and Pakes (1995), there is time-to-build in the timing of firms' decisions. Here we consider a version of the dynamic game without time-to-build. As described below, all our results on (non-)identification apply similarly to models with or without time-to-build.

$\alpha_j(\mathbf{i}_t, \mathbf{z}_t, \boldsymbol{\varepsilon}_{jt})$ be a strategy function for firm j, where $\mathbf{i}_t$ is the vector $\{i_{jt} : j = 1, 2, ..., N\}$ with firms' indicators of previous incumbent status, $i_{jt} \equiv a_{jt-1}$. A MPE is a N-tuple of strategy functions, $\{\alpha_j : j = 1, 2, ...N\}$ such that every firm maximizes its value given the strategies of the other firms:

$$\alpha_j(\mathbf{i}_t, \mathbf{z}_t, \boldsymbol{\varepsilon}_{jt}) = \arg \max_{a_{jt} \in \{0,1\}} \left\{ v_j^\alpha(a_{jt}, \mathbf{i}_t, \mathbf{z}_t) + \varepsilon_{jt}(a_{jt}) \right\} \tag{13}$$

where

$$v_j^\alpha(a_{jt}, \mathbf{i}_t, \mathbf{z}_t) \equiv \pi_j^\alpha(a_{jt}, \mathbf{i}_t, \mathbf{z}_t) + \beta \sum_{\mathbf{z}_{t+1} \in \mathbf{Z}} f_z(\mathbf{z}_{t+1}|\mathbf{z}_t) \int V_j^\alpha(a_{jt}, \mathbf{z}_{t+1}, \boldsymbol{\varepsilon}_{jt+1}) \ dG(\boldsymbol{\varepsilon}_{jt+1}), \tag{14}$$

and $\pi_j^\alpha(a_{jt}, \mathbf{i}_t, \mathbf{z}_t)$ is the expected profit of firm j at period t given that the other firms follow strategies $\{\alpha_i : i \neq j\}$. This expected profit is equal to $(1 - a_{jt})\ i_{jt}\ sv_j(\mathbf{z}_t^c) + a_{jt}\ [vp_j^\alpha(\mathbf{i}_t, \mathbf{z}_t) - fc_j(\mathbf{z}_t^c) - (1 - i_{jt})\ ec_j(\mathbf{z}_t^c)]$, where $vp_j^\alpha(\mathbf{i}_t, \mathbf{z}_t)$ is the expected variable profit $\int vp_j(\alpha_{-j}(\mathbf{i}_t, \mathbf{z}_t, \boldsymbol{\varepsilon}_{-jt}), \mathbf{z}_t^v)\ dG(\boldsymbol{\varepsilon}_{-jt})$.

As we did in the model for a monopolistic firm, we can represent firms' strategies using CCP functions:

$$\begin{aligned} P_j(\mathbf{i}, \mathbf{z}) &\equiv \Pr(\alpha_j(\mathbf{i}_t, \mathbf{z}_t, \boldsymbol{\varepsilon}_{jt}) = 1 \mid \mathbf{i}_t = \mathbf{i},\ \mathbf{z}_t = \mathbf{z}) \\ &= F_{\tilde{\varepsilon}|i}\left(\ \tilde{v}_j^{\mathbf{P}}(\mathbf{i}, \mathbf{z}) \ \right) \end{aligned} \tag{15}$$

where $\tilde{v}_j^{\mathbf{P}}(\mathbf{i}, \mathbf{z})$ is the differential value function $v_j^{\mathbf{P}}(1, \mathbf{i}, \mathbf{z}) - v_j^{\mathbf{P}}(0, \mathbf{i}, \mathbf{z})$, and $v_j^{\mathbf{P}}(a_j, \mathbf{i}, \mathbf{z})$ is equivalent to the conditional choice value function $v_j^\alpha(a_j, \mathbf{i}, \mathbf{z})$ but when we represent players' strategies using CCPs.

3 Identification of structural functions

3.1 Conditions on Data Generating Process

Suppose that the researcher has panel data with realizations of firms' decisions over multiple markets/locations and time periods. We use the letter m to index locations. The researcher observes a random sample of M markets with information on $\{a_{jmt}, \mathbf{z}_{mt}, i_{jmt} : j = 1, 2, ..., N, t = 1, 2, ..., T\}$, where N and T are small (they can be as small as $N = T = 1$) and M is large. For the identification results in this section we assume that M is infinite and $T = 1$. For most of the rest of the paper, we assume that the variable profit functions $vp_j(.)$ are known to the researcher, or more precisely, they have been already identified using data on firms' prices, quantities, and exogenous demand and variable cost characteristics. However, we also discuss below the case when the researcher does have data on prices, quantities, or revenue to identify in a first step the variable profit function.

We want to use this sample to estimate the structural 'parameters' or functions of the model: the three functions in the fixed profit, $fc_j(\mathbf{z}_t)$, $ec_j(\mathbf{z}_t)$, and $sv_j(\mathbf{z}_t)$; the transition probability of the state variables, f_z; and the distribution of the unobservables $F_{\tilde{\varepsilon}|i}$. Following the standard approach in dynamic decision models, we assume that the discount factor, β, is known to the researcher. Finally, note that the transition probability function $\{f_z\}$ is nonparametrically identified.[9] Therefore, we assume that $\{vp_j(.), f_z, \beta\}$ are known, and we concentrate on the identification of the functions $fc_j(.)$, $ec_j(.)$, $sv_j(.)$, $F_{\widetilde{\varepsilon}|0}$, and $F_{\widetilde{\varepsilon}|1}$.

All our identification results apply very similarly to the model of monopolistic competition and to the dynamic game of oligopoly competition. Given the identification of the variable profit function $vp_j(.)$, and given players' CCP functions, it is clear that the expected variable profit of firm j in the dynamic game is also identified.[10] From an identification point of view, a relevant difference between the monopolistic and the oligopoly models is that in the oligopoly case the CCP function of a firm depends not only on its own incumbent status i_{jt} but on the incumbent status of all the firms, as represented by the vector $\mathbf{i}_t$. In principle, given that a firm's fixed profit does not depend on the incumbent status of the other firms, one may think that this exclusion restriction might help to separately identify the three components of a firm's fixed profit. However, that is not the case. The oligopoly model provides additional over-identifying restrictions that can be tested, but these over-identifying restrictions do not help in the separate identification of the three components of the fixed cost. Therefore, for notational simplicity, we omit the firm subindex for the rest of the paper and we use the notation of the monopolistic case. When necessary, we comment some differences with the dynamic oligopoly game, and why the additional restrictions implied by the dynamic game do not help in our identification problem.

3.2 Identification of the distribution of the unobservables

We might consider a semiparametric version of our model where the conditional distributions $F_{\widetilde{\varepsilon}|0}$ and $F_{\widetilde{\varepsilon}|1}$ are parametrically specified and known to the researcher up to the scale parameters $\sigma_{\widetilde{\varepsilon}|0} \equiv \sqrt{var(\widetilde{\varepsilon}_t|i_t = 0)}$ and $\sigma_{\widetilde{\varepsilon}|1} \equiv \sqrt{var(\widetilde{\varepsilon}_t|i_t = 1)}$. In that model, the identification of the scale parameters $\sigma_{\widetilde{\varepsilon}|0}$ and $\sigma_{\widetilde{\varepsilon}|1}$ requires the following type of exclusion restriction: there is a *special state variable* y_t included in $\mathbf{z}_t$ such that this variable enters in the variable profit function $vp(.)$ (a function that has been identified using data on prices and quantities) but not in the fixed profit. It turns out that this exclusion restriction, together with a large support condition, is also sufficient for the nonparametric identification of the distribution functions $F_{\widetilde{\varepsilon}|0}$ and $F_{\widetilde{\varepsilon}|1}$. Proposition 1 establishes the nonparametric identification of $F_{\widetilde{\varepsilon}|0}$ and $F_{\widetilde{\varepsilon}|1}$.

PROPOSITION 1. Suppose that the following conditions hold: (a) the vector of unobservables

[9] Note that $f_z(\mathbf{z}'|\mathbf{z}) = \Pr(\mathbf{z}_{mt+1} = \mathbf{z}' \mid \mathbf{z}_{mt} = \mathbf{z})$. We can estimate consistently these conditional probabilities using a nonparametric method such as a kernel or a sieve method.

[10] Note that the expected variable profit $vp_j^{\mathbf{P}}(a_j, \mathbf{i}, \mathbf{z})$ is equal to $[\prod_{k \neq j} P_k(\mathbf{i}, \mathbf{z})^{a_k} (1 - P_k(\mathbf{i}, \mathbf{z}))^{1-a_k}]\ vp_j(a_{-i}, \mathbf{z}_t^v)$. Therefore, given vp_j and CCPs $\{P_k : k \neq j\}$, the expected variable profit function $vp_j^{\mathbf{P}}$ is known.

ε_t is independent of $\mathbf{z}_t$; (b) $F_{\widetilde{\varepsilon}|0}$ and $F_{\widetilde{\varepsilon}|1}$ are strictly increasing over the real line; and (c) the vector of state variables $\mathbf{z}_t$ includes an 'special' state variable, y_t, such that y_t is included in $\mathbf{z}_t^v$ but not in $\mathbf{z}_t^c$ (i.e., it enters in the variable profit function but not in the fixed profit function), the variable profit function $vp(.)$ is strictly monotonic in y_t, and conditional on i_t and on the other state variables in $\mathbf{z}_t$, the distribution of y_t has support over the whole real line. Under these conditions, the distributions $F_{\widetilde{\varepsilon}|0}$ and $F_{\widetilde{\varepsilon}|1}$ are nonparametrically identified. Furthermore, given $vp(.)$, the nonparametric estimation of these distributions can be implemented separately from the estimation of the other structural functions in the fixed profit.

Proof: The proof of this Proposition 1 is a direct application of Proposition 4 in Aguirregabiria (2010) to our model of market entry and exit. Proposition 4 in Aguirregabiria (2010) applies to a general class of binary choice dynamic structural models with finite horizon, and it builds on previous results by Matzkin (1992, 1994). Norets and Tang (2012) have extended that result to infinite horizon dynamic decision models. ■

So far, we have assumed that the researcher has data on prices and quantities and the variable profit function vp can be identified using this information and independently of the dynamic decision model. Nevertheless, in many empirical applications of models of market entry and exit there is not data on prices and quantities. In that context, the specification of the (indirect) variable profit function typically follows the approach in the seminal work by Bresnahan and Reiss (1990, 1991, and 1994). This specification has the following features: (i) variable profit is proportional to market size; (ii) market size does enter in the fixed profit; and (iii) the researcher observes market size. In the monopolistic case, this specification is $VP_t = y_t\ \alpha$, where α is a parameter and the variable y_t represents market size. In the model of oligopoly competition we have $VP_{jt} = y_t\ [\alpha_j - \sum_{k\neq j} \delta_{jk}\ a_{kt}]$, where $\alpha's$ and $\delta's$ are parameters, and y_t still represents market size. It is possible to extend Proposition 1 to this specification of the variable profit, with the only difference that the parameter α is not identified separately from the distribution functions $F_{\widetilde{\varepsilon}|0}$ and $F_{\widetilde{\varepsilon}|1}$. The only relevant implication of the non-identification of α is that the estimated values of the fixed profit (and the δ parameters) are measured in units of y_t instead of dollar amounts.

For the rest of the paper, we assume that the conditions of Proposition 1 hold and that the distributions $F_{\widetilde{\varepsilon}|0}$ and $F_{\widetilde{\varepsilon}|1}$ are identified.

3.3 Identification of functions in the fixed profit

By definition, the CCP function $P(i, \mathbf{z})$ is equal to the conditional expectation $\mathbb{E}(a_{mt} \mid i_{mt} = i, \mathbf{z}_{mt} = \mathbf{z})$ and therefore, it is nonparametrically identified using data on $\{a_{mt}, i_{mt}, \mathbf{z}_{mt}\}$. Given the CCP function $P(i, \mathbf{z})$ and the inverse distribution $F_{\widetilde{\varepsilon}|i}^{-1}$, we have that the differential value function

$\tilde{v}(i,\mathbf{z})$ is identified from the expression:

$$\tilde{v}(i,\mathbf{z}) = F^{-1}_{\widetilde{\varepsilon}|i}\left(P(i,\mathbf{z})\right). \tag{16}$$

Under the conditions of Proposition 1, function $\tilde{v}(i,\mathbf{z})$ is nonparametrically identified everywhere, such that we can treat $\tilde{v}(i,\mathbf{z})$ as a known/identified function. $\tilde{v}(i,\mathbf{z})$ is the value of being active in the market minus the value of not being active for a firm with incumbent status "i" at previous period. This differential value is equal to the inverse function of $F_{\widetilde{\varepsilon}|i}$ evaluated at $P(i,\mathbf{z})$, that is the probability of being active in the market for a firm with incumbent status "i" at previous period.

Functions $\tilde{v}(i,\mathbf{z})$, $vp(\mathbf{z})$, and $F_{\widetilde{\varepsilon}|i}$ summarize all the information in the data that is relevant for the identification of the three functions in the fixed profit. We now derive a closed-form relationship between these identified functions and the unknown structural functions fc, ec, and sv. Given that by construction $\tilde{v}(i,\mathbf{z})$ is equal to $v(1,i,z) - v(0,i,z)$, and given the definition of conditional choice value function in equation (8), we have the following system of equations: for any value of $(i,\mathbf{z})$,

$$\tilde{v}(i,\mathbf{z}) = vp(\mathbf{z}) - [fc(\mathbf{z}) + ec(\mathbf{z})] + i\ \ [ec(\mathbf{z}) - sv(\mathbf{z})] + \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\ \ \left[\bar{V}\left(1,\mathbf{z}'\right) - \bar{V}\left(0,\mathbf{z}'\right)\right] \tag{17}$$

where $\bar{V}(i,\mathbf{z})$ is the integrated value function $\int V(i,\mathbf{z},\boldsymbol{\varepsilon})\ dG(\boldsymbol{\varepsilon})$, i.e., the value function integrated over the distribution of the unobservables in $\boldsymbol{\varepsilon}$. This system summarizes all the restrictions that the model and data impose on the structural functions.

Using the definition of the integrated value function $\bar{V}(i,\mathbf{z})$, we can express it as follows:

$$\begin{aligned}\bar{V}(i,\mathbf{z}) &= \int \max_{a\in\{0,1\}} \{v(a,i,\mathbf{z}) + \varepsilon(a)\}\ dG(\boldsymbol{\varepsilon}) \\ &= v(0,i,\mathbf{z}) + \int \max\{0\ ;\ \widetilde{v}(i,\mathbf{z}) - \widetilde{\varepsilon}\}\ dF_{\widetilde{\varepsilon}|i}(\widetilde{\varepsilon}) \\ &= v(0,i,\mathbf{z}) + S(\tilde{v}(i,\mathbf{z}), F_{\widetilde{\varepsilon}|i}),\end{aligned} \tag{18}$$

where $S(\tilde{v}(i,\mathbf{z}), F_{\widetilde{\varepsilon}|i})$ represents the function $\int_{-\infty}^{\tilde{v}(i,\mathbf{z})} [\tilde{v}(i,\mathbf{z}) - \widetilde{\varepsilon}]\ dF_{\widetilde{\varepsilon}|i}(\widetilde{\varepsilon})$. Note that the arguments of function S, i.e., functions $\tilde{v}$ and $F_{\widetilde{\varepsilon}|i}$, are identified. Therefore, function S is also a known or identified function. Plugging expression $\bar{V}(i,\mathbf{z}) = v(0,i,\mathbf{z}) + S(\tilde{v}(i,\mathbf{z}), F_{\widetilde{\varepsilon}|i})$ into equation (17), and taking into account that $v(0,1,\mathbf{z}) - v(0,0,\mathbf{z}) = sv(\mathbf{z})$, we have the following system of equations that summarizes all the restrictions that the data and model impose on the unknown structural functions fc, ec, and sv.

$$\begin{aligned}\tilde{v}(i,\mathbf{z}) &= vp(\mathbf{z}) - [fc(\mathbf{z}) + ec(\mathbf{z})] + i\ \ [ec(\mathbf{z}) - sv(\mathbf{z})] + \beta \sum_{\mathbf{z}'\in\mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\ sv(\mathbf{z}') \\ &+ \beta \sum_{\mathbf{z}'\in\mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\ \ \left[S(\tilde{v}(1,\mathbf{z}'), F_{\widetilde{\varepsilon}|1}) - S(\tilde{v}(0,\mathbf{z}'), F_{\widetilde{\varepsilon}|0})\right]\end{aligned} \tag{19}$$

To study the identification of functions fc, ec, and sv, it is convenient to sum up all the identified functions in equation (19) into a single term. Define the function $Q(i,\mathbf{z}) \equiv \tilde{v}(i,\mathbf{z}) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})$ $[S(\tilde{v}(1,\mathbf{z}'), F_{\tilde{\varepsilon}|1}) - S(\tilde{v}(0,\mathbf{z}'), F_{\tilde{\varepsilon}|0})]$. It is clear that function $Q(i,\mathbf{z})$ is identified. This function has also an intuitive interpretation. It represents the difference between the firm's value under two different 'ad-hoc' strategies: the strategy of being in the market today, exiting next period, and remaining out of the market forever in the future, and the strategy of exiting from the market today and remaining out of the market forever in the future. Using this definition for function $Q(i,\mathbf{z})$, we can rewrite the system of equations (19) as follows:

$$Q(i,\mathbf{z}) \quad = \quad vp(\mathbf{z}) - [fc(\mathbf{z}) + ec(\mathbf{z})] + i \; [ec(\mathbf{z}) - sv(\mathbf{z})] + \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \; sv(\mathbf{z}') \tag{20}$$

This system of equations provides a closed form expression for the relationship between the unknown structural functions and the identified function $Q(i,\mathbf{z})$.

PROPOSITION 2. The structural functions $fc(\mathbf{z})$, $ec(\mathbf{z})$, and $sv(\mathbf{z})$ are not separately identified. However, we can identify two combinations of these structural functions which have a clear economic interpretation: (a) the sunk part of the entry cost when entry and exit occur at the same state $\mathbf{z}$, i.e., $ec(\mathbf{z}) - sv(\mathbf{z})$; and (b) the sum of fixed cost and entry cost minus the discounted expected scrap value in the next period, i.e., $fc(\mathbf{z}) + ec(\mathbf{z}) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \; sv(\mathbf{z}')$.

$$\begin{aligned} ec(\mathbf{z}) - sv(\mathbf{z}) \quad &= \quad Q(1,\mathbf{z}) - Q(0,\mathbf{z}), \\ fc(\mathbf{z}) + ec(\mathbf{z}) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \; sv(\mathbf{z}') \quad &= \quad -Q(0,\mathbf{z}) + vp(\mathbf{z}). \end{aligned} \tag{21}$$

Proof. (i) No identification. Let ec, fc, and sv be the true values of the functions in the population. Based on these true functions, define the functions: $ec^*(\mathbf{z}) = ec(\mathbf{z}) + \lambda(\mathbf{z})$; $sv^*(\mathbf{z}) = sv(\mathbf{z}) + \lambda(\mathbf{z})$, and $fc^*(\mathbf{z}) = fc(\mathbf{z}) - \lambda(\mathbf{z}) + \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \; \lambda(\mathbf{z}')$, where $\lambda(\mathbf{z}) \neq 0$ is an arbitrary function. It is clear that ec^*, sv^*, and fc^* also satisfy the system of equations (20). Therefore, ec, fc, and sv cannot be uniquely identified from the restrictions in (20). *(ii) Identification of two combinations of the three structural functions.* We can derive equations in (21) after simple operations in the system (20). ■

Proposition 2 can be easily extended to the dynamic oligopoly game. In particular, the additional structure in the dynamic game does not help to the identification of the components in the fixed profit. Equation (20) also applies to the dynamic game, only with the following modifications: (i) all the functions are firm-specific and should have the firm subindex j; (ii) function Q_j includes as an argument also the past incumbent statuses of the other firms, $\mathbf{i}_{-j}$, such that we have $Q_j(i_j, \mathbf{i}_{-j}, \mathbf{z})$; and (iii) the expected variable profit function depends on firms' CCPs and it includes as an argument the incumbent statuses of all the firms, i.e., $vp_j^{\mathbf{P}}(\mathbf{i}, \mathbf{z})$. Given this modified version of equation, (20), it is straightforward to extend the two parts to Proposition 2 to the dynamic game model. However, the dynamic game provides over-identifying restrictions. For instance, we have that for every value of $\mathbf{i}_{-j}$ the following equation should hold: $ec_j(\mathbf{z}) - sv_j(\mathbf{z}) = Q_j(1, \mathbf{i}_{-j}, \mathbf{z}) - Q(0, \mathbf{i}_{-j}, \mathbf{z})$. This implies that the value of $Q_j(1, \mathbf{i}_{-j}, \mathbf{z}) - Q(0, \mathbf{i}_{-j}, \mathbf{z})$ should be the same for any value of $\mathbf{i}_{-j}$, which is a testable over-identifying restriction.

Proposition 2 applies also to the version of the model without market re-entry, with the only difference that $Q(0, \mathbf{z})$ is now defined as $\tilde{v}(0, \mathbf{z}) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\ S(\tilde{v}(1, \mathbf{z}'), F_{\tilde{\varepsilon}|1})$ such that it does not include the term $\beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\ S(\tilde{v}(0, \mathbf{z}'), F_{\tilde{\varepsilon}|0})$. We discuss the identification of the model with time-to-build in section 3.5 below.

3.4 'Normalizations' and interpretation of estimated functions

In empirical applications, the common approach to deal with this identification problem is to restrict one of the three structural functions to be zero at any value of $\mathbf{z}$. This is often referred to as a 'normalization' assumption. The most common 'normalization' is making the scrap value equal to zero. That is the approach in applications such as Snider (2009), Collard-Wexler (2010), Dunne et al. (2011), Varela (2011), Ellickson et al. (2012), Lin (2012), Aguirregabiria and Mira (2007), or Suzuki (forthcoming), among others. In other papers, such as Pakes et al. (2007), Ryan (2012), Sweeting (2011) or Igami (2012), the normalization consists in making the fixed cost equal to zero. Though making the entry cost equal to zero is other possible normalization, this has not been common in empirical applications.

Though most of the papers in the literature admit that setting the fixed cost of the scrap value to zero is not really an assumption but a 'normalization', they do not derive the implications of this 'normalization' on the estimated parameters, and on the counterfactual experiments using the estimated model. Based on our derivation of the relationship between identified objects and unknown structural functions in the system of equations (20), or equivalently in (21), we can obtain the correct interpretation of the estimated functions under any possible normalization. Ignoring this can lead to misinterpretations of the empirical results. Table 1 reports the relationship between the estimated structural functions and the true structural functions under different normalizations. Functions $\widehat{fc}$, $\widehat{sv}$, and $\widehat{ec}$ represent the estimated vectors under a given normalization, and they should be distinguished from the true structural functions fc, sv, and ec. The expressions in Table 1 are derived as follows. First, the estimated functions $\widehat{fc}$, $\widehat{sv}$, and $\widehat{ec}$ satisfy the identifying restrictions in (20) and (21). In particular, $\widehat{ec}(\mathbf{z}) - \widehat{sv}(\mathbf{z}) = Q(1, \mathbf{z}) - Q(0, \mathbf{z})$, and $\widehat{fc}(\mathbf{z}) + \widehat{ec}(\mathbf{z}) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\ \widehat{sv}(\mathbf{z}')$. Of course, these conditions are also satisfied by the true values of these

Table 1: Interpretation of Estimated Structural Functions Under Various "Normalizations"

Normaliz.	Estimated Functions		
	$\widehat{sv}(\mathbf{z})$	$\widehat{fc}(\mathbf{z})$	$\widehat{ec}(\mathbf{z})$
$\widehat{sv}(\mathbf{z})=0$	0	$fc(z)+sv(z)$ $-\beta\ E[sv(z_{t+1})\mid z_t=z]$	$ec(z)-sv(z)$
$\widehat{fc}(\mathbf{z})=0$	$sv(z)$ $+\sum_{r=0}^{\infty}\beta^r E[fc(z_{t+r})\mid z_t=z]$	0	$ec(z)$ $+\sum_{r=0}^{\infty}\beta^r E[fc(z_{t+r})\mid z_t=z]$
$\widehat{ec}(\mathbf{z})=0$	$sv(z)-ec(z)$	$fc(z)+ec(z)$ $-\beta\ E[ec(z_{t+1})\mid z_t=z]$	0

functions. Therefore, it should be true that for any normalization we have that:

$$\begin{aligned} \widehat{ec}(\mathbf{z})-\widehat{sv}(\mathbf{z}) &= ec(\mathbf{z})-sv(\mathbf{z}), \\ \widehat{fc}(\mathbf{z})+\widehat{ec}(\mathbf{z})-\beta\sum_{\mathbf{z}'\in\mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\ \widehat{sv}(\mathbf{z}') &= fc(\mathbf{z})+ec(\mathbf{z})-\beta\sum_{\mathbf{z}'\in\mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})sv(\mathbf{z}'). \end{aligned} \tag{22}$$

These expressions, and the corresponding normalization assumption, provide a system of equations where we can solve for the estimated functions, and obtain the expression of these estimated functions in terms of the true functions. These expressions provide the correct interpretation of the estimated functions.

Suppose that the normalization is $\widehat{sv}(\mathbf{z})=0$. Including this restriction into the system (22) and solving for $\widehat{ec}(\mathbf{z})$ and $\widehat{fc}(\mathbf{z})$, we get that $\widehat{ec}(\mathbf{z})=ec(\mathbf{z})-sv(\mathbf{z})$, and $\widehat{fc}(\mathbf{z})=fc(\mathbf{z})+sv(\mathbf{z})$ $-\beta\sum_{\mathbf{z}'} f_z(\mathbf{z}'|\mathbf{z})\ sv(\mathbf{z}')$. The estimated entry cost is in fact the entry cost minus the scrap value at the same state, i.e., the 'ex-ante' sunk entry cost.[11] And the estimated fixed cost is the actual fixed cost plus the difference between the current scrap value and the expected, discounted next period scrap value. When the normalization is $\widehat{ec}(\mathbf{z})=0$, we can perform a similar operation to obtain that $\widehat{sv}(\mathbf{z})=sv(\mathbf{z})-ec(\mathbf{z})$, and $\widehat{fc}(\mathbf{z})=fc(\mathbf{z})+ec(\mathbf{z})$ $-\beta\sum_{\mathbf{z}'\in\mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\ ec(\mathbf{z}')$. That is, the estimated scrap value is equal to the 'ex-ante' sunk entry cost but with the opposite sign, and the estimated fixed cost is equal to the actual fixed cost plus the difference between current

[11] The 'ex-ante' sunk entry cost is not necessarily equal to the 'ex-post' or realized sunk cost because the value of the state variables affecting the scrap value may be different at the entry and exit periods.

entry cost and expected discounted next period entry cost. When the normalization is applied to the fixed cost, such that $\widehat{fc}(\mathbf{z}) = 0$, obtaining the solution of the estimated functions in terms of the true functions is a bit more convoluted because the solution of the system of equations is not point-wise or separate for each value of $\mathbf{z}$, but instead we need to solve recursively a system of equations that involves every possible value of $\mathbf{z}$. We have the recursive systems $[\widehat{ec}(\mathbf{z}) - ec(\mathbf{z})] = fc(\mathbf{z}) + \beta \sum_{\mathbf{z}'} f_z(\mathbf{z}'|\mathbf{z}) \, [\widehat{ec}(\mathbf{z}') - ec(\mathbf{z}')]$ and $[\widehat{sv}(\mathbf{z}) - sv(\mathbf{z})] = fc(\mathbf{z}) + \beta \sum_{\mathbf{z}'} f_z(\mathbf{z}'|\mathbf{z}) \, [\widehat{sv}(\mathbf{z}') - sv(\mathbf{z}')]$. Solving recursively these functional equations, we get that $\widehat{ec}(\mathbf{z}) = ec(\mathbf{z}) + \sum_{r=0}^{\infty} \beta^r \, \mathbb{E}[fc(\mathbf{z}_{t+r}) \mid \mathbf{z}_t = z]$, and $\widehat{sv}(\mathbf{z}) = sv(\mathbf{z}) + \sum_{r=0}^{\infty} \beta^r \, \mathbb{E}[fc(\mathbf{z}_{t+r}) \mid \mathbf{z}_t = z]$. That is, the estimated entry cost is equal to the actual entry cost plus the discounted and expected sum of the current and future fixed costs of the firm if it would be active forever in the future. A similar interpretation applies to the estimated scrap value.

EXAMPLE 2. Suppose an industry where firms need to use a particular capital equipment to operate in the market. For some reason (e.g., informational asymmetries) there is not a rental market for this equipment, or it is always more profitable to purchase the equipment than to rent it. Let z be a state variable that represents the current purchasing price of the equipment. Suppose that the entry cost is $ec(z) = ec_0 + z$, where $ec_0 > 0$ is a parameter that represents costs of entry other than those related to the purchase of capital. The fixed cost depends also on the price of capital through property taxes that firms should pay every period they are active: $fc(z) = fc_0 + \tau \, z$, where $\tau \in (0,1)$ is a parameter the captures how the property tax depends on the price of the owned capital. The scrap value function is $sv(z) = \lambda \, z$, where $\lambda \in (0,1)$ is a parameter that captures the idea that there is some capital depreciation, or a firm-specific component in the capital equipment, such that there is a wedge between the cost of purchasing capital and the revenue from selling it. Now, consider the identification of these functions. For simplicity, suppose that the real price of capital z is constant over time, though it varies across markets in our data such that we can estimate the effect of this state variable. When the normalization is $\widehat{sv}(z) = 0$, we have that $\widehat{ec}(z) = ec_0 + (1 - \lambda) \, z$. An interpretation of $\widehat{ec}(z)$ as the true entry cost, instead of the sunk entry cost, implies to underestimate the effect of the price of capital on the entry cost. The estimated fixed cost is $\widehat{fc}(z) = fc_0 + (\tau + (1 - \beta)\lambda) \, z$, such that ignoring the effects of the normalization and treating $\widehat{fc}(z)$ as the true fixed cost leads to an over-estimation of the effect of the price of capital on the fixed cost. That is, we over-estimate the impact of the property tax on the fixed cost. Similar arguments can be applied when the normalization is $\widehat{ec}(z) = 0$. In particular, $\widehat{fc}(z) = fc_0 + (\tau + (1 - \beta)) \, z$, such that the over-estimation of the incidence of the property tax on the fixed cost is even stronger than before. When we normalize the fixed cost to zero, both the scrap value and the entry cost are overestimated by $(fc_0 + \tau \, z)/(1 - \beta)$. The estimated effect of the price of capital on the cost of entry includes not only the purchasing cost but also the discounted value of the infinite stream of property taxes. ■

3.5 Model with Time-to-build and Time-to-exit

Most of the expressions for the basic model still hold for this extension, except that now the one-period payoff $\pi(a, i, \mathbf{z})$ has a different form. In particular, now $\pi(1, i, \mathbf{z}) - \pi(0, i, \mathbf{z}) = -i\ sv(\mathbf{z}) - (1-i)\ ec(\mathbf{z})$, and this implies that the expression for the differential value function $\tilde{v}(i, \mathbf{z})$ is:

$$\tilde{v}(i, \mathbf{z}) = -i\ sv(\mathbf{z}) - (1-i)\ ec(\mathbf{z}) + \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \left[\bar{V}\left(1, \mathbf{z}'\right) - \bar{V}\left(0, \mathbf{z}'\right)\right] \tag{23}$$

Also, now we have that $v(0, 1, \mathbf{z}) - v(0, 0, \mathbf{z}) = \pi(0, 1, \mathbf{z}) - \pi(0, 0, \mathbf{z}) = vp(\mathbf{z}) - fc(\mathbf{z}) + sv(\mathbf{z})$. Therefore, the system of identifying restrictions (20) becomes:

$$Q(i, \mathbf{z}) \quad = \quad -i\ sv(\mathbf{z}) - (1-i)\ ec(\mathbf{z}) + \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\ [vp(\mathbf{z}') - fc(\mathbf{z}') + sv(\mathbf{z}')] \tag{24}$$

where $Q(i, \mathbf{z})$ has exactly the same definition as before in the model without time-to-build. Given this system of equations, Proposition 2 also applies to this model with the only difference that now we have the following relationship between true functions and identified objects:

$$\begin{aligned} ec(\mathbf{z}) - sv(\mathbf{z}) \quad &= \quad Q(1, \mathbf{z}) - Q(0, \mathbf{z}) \\ ec(\mathbf{z}) + \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\ [fc(\mathbf{z}') - sv(\mathbf{z}')] \quad &= \quad -Q(0, \mathbf{z}) + \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})\ vp(\mathbf{z}') \end{aligned} \tag{25}$$

The first equation is exactly the same as in the model without time to build. The second equation is slightly different: instead of current value of variable profit minus the fixed cost, $vp(\mathbf{z}) - fc(\mathbf{z})$, now we have the discounted and expected value of this function at the next period, i.e., $\beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z})$ $[vp(\mathbf{z}') - fc(\mathbf{z}')]$. Based on these equations, we can construct the following Table 2 for the model with time-to-build with the interpretation of the estimated functions under three different normalizations.

4 Counterfactual experiments

4.1 Definition and identification of counterfactual experiments

Suppose that the researcher is interested in using the estimated structural model to obtain an estimate of the effect on firms' entry-exit behavior of a change in some of the structural functions such that the environment is partly different from the one generating the data. Let $\boldsymbol{\theta}^0 = \{vp^0, fc^0,$

Table 2: Interpretation of Estimated Structural Functions in the Model with Time-to-Build

Normaliz.	Estimated Functions		
	$\widehat{sv}\,(\mathbf{z})$	$\beta\ \mathbb{E}(\widehat{fc}\,(\mathbf{z}_{t+1}) \mid \mathbf{z}_t = \mathbf{z})$	$\widehat{ec}\,(\mathbf{z})$
$\widehat{sv}\,(\mathbf{z}) = 0$	0	$\beta\ E[fc(z_{t+1}) \mid z_t = z]$ $+sv(z) - \beta\ E[sv(z_{t+1}) \mid z_t = z]$	$ec(z) - sv(z)$
$\widehat{fc}\,(\mathbf{z}) = 0$	$sv(z)$ $+\sum_{r=1}^{\infty} \beta^r E[fc(z_{t+r}) \mid z_t = z]$	0	$ec(z)$ $+\sum_{r=1}^{\infty} \beta^r E[fc(z_{t+r}) \mid z_t = z]$
$\widehat{ec}\,(\mathbf{z}) = 0$	$sv(z) - ec(z)$	$\beta\ E[fc(z_{t+1}) \mid z_t = z]$ $+ec(z) - \beta\ E[ec(z_{t+1}) \mid z_t = z]$	0

ec^0, sv^0, β^0, $f_z^0\}$ represent the structural functions that have generated the data. And let $\boldsymbol{\theta}^* = \{vp^*$, fc^*, ec^*, sv^*, β^*, $f_z^*\}$ be the structural functions in the hypothetical or counterfactual scenario. Define $\Delta_\theta \equiv \{\Delta_{vp}, \Delta_{fc}, \Delta_{ec}, \Delta_{sv}, \Delta_\beta, \Delta_{f_z}\} \equiv \boldsymbol{\theta}^* - \boldsymbol{\theta}^0$. We refer to Δ_θ as the *perturbation* in the primitives of the model defined by the counterfactual experiment. The goal of this counterfactual experiment is to obtain how the perturbation Δ_θ affects firms' behavior as measured by the CCP function. In other words, we want to identify $\Delta_P(i, \mathbf{z})$ associated to Δ_θ, where

$$\Delta_P(i, \mathbf{z}) \equiv P(i, \mathbf{z}\ ;\ \boldsymbol{\theta}^0 + \Delta_\theta) - P(i, \mathbf{z}\ ;\ \boldsymbol{\theta}^0). \tag{26}$$

The parameter perturbations that have been considered in the counterfactual experiments in recent empirical applications include regulations that increase entry costs (Ryan 2012, Suzuki forthcoming), entry subsidies (Das et al. 2007, Dunne et al. 2011, Lin 2012), subsidies on R&D investment (Igami 2012), tax on revenue (Sweeting 2011), market size (Bollinger 2012, Igami 2012), economies of scale and scope (Aguirregabiria and Ho 2012, Varela 2011), a ban on some products (Bollinger 2012, Lin 2012), regulation on firms' predatory conduct (Snider 2009), demand fluctuation (Collard-Wexler 2010), time to build (Kalouptsidi 2011), or exchange rates (Das et al. 2007).

In the derivation of our identification results on counterfactual experiments, we exploit some properties of the mapping that relates functions Q and P. Lemma 1 below describes the mapping and its properties.[12]

[12] Lemma 1 is related but quite different to Proposition 1 in Hotz and Miller (1993). That Proposition 1 establishes

LEMMA 1: Define $\widetilde{Q} \equiv \{Q(i,\mathbf{z}) : \textit{for all } (i,\mathbf{z})\}$ *and* $\widetilde{P} \equiv \{P(i,\mathbf{z}) : \textit{for all } (i,\mathbf{z})\}$. *For given* (β, f_z), *there is a mapping* $\widetilde{q}(\widetilde{P};\beta,f_z)$ *from the space of* $\widetilde{P}$ *into the space of* $\widetilde{Q}$ *such that* $\widetilde{Q} = \widetilde{q}(\widetilde{P};\beta,f_z)$. *The definition of this mapping is,* $\widetilde{q}(\widetilde{P};\beta,f_z) \equiv \{q(i,\mathbf{z},\widetilde{P};\beta,f_z) : \textit{for all } (i,\mathbf{z})\}$ *with:*

$$\begin{aligned} q(i,\mathbf{z},\widetilde{P};\beta,f_z) \quad \equiv \quad & F^{-1}_{\widetilde{\varepsilon}|i}\left(P(i,\mathbf{z})\right) - \beta \sum_{\mathbf{z}'\in\mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \left(\int_{-\infty}^{F^{-1}_{\widetilde{\varepsilon}|1}(P(1,\mathbf{z}'))} [F^{-1}_{\widetilde{\varepsilon}|1}\left(P(1,\mathbf{z}')\right) - \widetilde{\varepsilon}] \; dF_{\widetilde{\varepsilon}|1}\left(\widetilde{\varepsilon}\right) \right) \\ + \quad & \beta \sum_{\mathbf{z}'\in\mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \left(\int_{-\infty}^{F^{-1}_{\widetilde{\varepsilon}|0}(P(0,\mathbf{z}'))} [F^{-1}_{\widetilde{\varepsilon}|0}\left(P(0,\mathbf{z}')\right) - \widetilde{\varepsilon}] \; dF_{\widetilde{\varepsilon}|0}\left(\widetilde{\varepsilon}\right) \right) \end{aligned} \tag{27}$$

The mapping $\widetilde{q}(\widetilde{P};\beta,f_z)$ *is one-to-one (invertible).*

Proof. See Appendix.

For some counterfactual experiments, we need to distinguish two components in the vector of state variables: $\mathbf{z} \equiv (\mathbf{z}^{nosv}, \mathbf{z}^{sv})$, where $\mathbf{z}^{sv}$ is the subvector of the state variables that affect the scrap value, and $\mathbf{z}^{nosv}$ represents the rest of the state variables. We can represent the transition probability of the observable state variables as $f_z(\mathbf{z}_{t+1}|\mathbf{z}_t) = f_{z,sv}(\mathbf{z}^{sv}_{t+1}|\mathbf{z}_t)\; f_{z,nosv}(\mathbf{z}^{nosv}_{t+1}|\mathbf{z}_t, \mathbf{z}^{sv}_{t+1})$.

Let $\Delta_Q(i,\mathbf{z})$ be then a function that represents the change in function Q associated to the change in parameters Δ_θ, i.e., $\Delta_Q(i,\mathbf{z}) \equiv Q(i,\mathbf{z}\,;\boldsymbol{\theta}^0 + \Delta_\theta) - Q(i,\mathbf{z}\,;\boldsymbol{\theta}^0)$. Given Lemma 1, it should be clear that $\Delta_P(i,\mathbf{z})$ is identified if and only if $\Delta_Q(i,\mathbf{z})$ is identified. For some of our results below on the (non) identification $\Delta_P(i,\mathbf{z})$, it is useful to prove them by showing the (non) identification of $\Delta_Q(i,\mathbf{z})$. Using the definition of $\Delta_Q(i,\mathbf{z})$, and taking into account equation (20) relating Q with the structural functions, we have the following equation that relates $\Delta_Q(i,\mathbf{z})$ with the structural functions and the parameter perturbation Δ_θ:

$$\begin{aligned} \Delta_Q(i,\mathbf{z}) \quad = \quad & \Delta_{vp}(\mathbf{z}) - [\Delta_{fc}\left(\mathbf{z}\right) + \Delta_{ec}\left(\mathbf{z}\right)] + i\;\; [\Delta_{ec}\left(\mathbf{z}\right) - \Delta_{sv}\left(\mathbf{z}\right)] \\ + \quad & \beta^0 \sum_{\mathbf{z}^{sv\prime}\in\mathbf{Z}} f^0_{z,sv}(\mathbf{z}^{sv\prime}|\mathbf{z}) \; \Delta_{sv}(\mathbf{z}^{sv\prime}) \\ + \quad & \beta^0 \sum_{\mathbf{z}^{sv\prime}\in\mathbf{Z}} \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z}) \; [sv^0(\mathbf{z}^{sv\prime}) + \Delta_{sv}(\mathbf{z}^{sv\prime})] \\ + \quad & \Delta_\beta \sum_{\mathbf{z}^{sv\prime}\in\mathbf{Z}} [f^0_{z,sv}(\mathbf{z}^{sv\prime}|\mathbf{z}) + \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z})] \; [sv^0(\mathbf{z}^{sv\prime}) + \Delta_{sv}(\mathbf{z}^{sv\prime})] \end{aligned} \tag{28}$$

PROPOSITION 3: Suppose that Δ_θ *implies changes only in functions* vp, fc, ec, sv, *and* $f_{z,nosv}$ *such that* $\beta^* = \beta^0$ *and* $f^*_{z,sv} = f^0_{z,sv}$. *And suppose that the researcher knows the perturbation* $\Delta_\theta =$

that for every value of the state variables $(i,\mathbf{z})$, there is a one-to-one mapping between CCPs P and differential values $\widetilde{v}$. In our binary choice model, Hotz-Miller Proposition simply establishes that function $P(i,\mathbf{z}) = F_{\widetilde{\varepsilon}|i}(\widetilde{v}(i,\mathbf{z}))$ is invertible. In contrast, Lemma 1 establishes the invertibility of the mapping between vector $\widetilde{Q}$ and vector $\widetilde{P}$. Note that every value $Q(i,\mathbf{z})$ depends on the whole vector $\widetilde{P}$.

$\{\Delta_{vp}(\mathbf{z}),\ \Delta_{fc}(\mathbf{z}),\ \Delta_{ec}(\mathbf{z}),\ \Delta_{sv}(\mathbf{z}),\ \Delta_{f_{z,nosv}}(\mathbf{z}'|\mathbf{z})\}$ *(though he does not know neither* $\boldsymbol{\theta}^0$ *nor* $\boldsymbol{\theta}^*$*). Then,* $\Delta_Q(i,\mathbf{z})$, $\Delta_P(i,\mathbf{z})$, *and* $P(i,\mathbf{z}\ ;\ \theta^0+\Delta_\theta)$ *are identified.*

Proof. Under the conditions of Proposition 3, we have that equation (28) becomes:

$$\begin{aligned}\Delta_Q(i,\mathbf{z}) &= \Delta_{vp}(\mathbf{z}) - [\Delta_{fc}(\mathbf{z}) + \Delta_{ec}(\mathbf{z})] + i\ [\Delta_{ec}(\mathbf{z}) - \Delta_{sv}(\mathbf{z})] \\ &+ \beta^0 \sum_{\mathbf{z}^{sv\prime}\in\mathbf{Z}} f^0_{z,sv}(\mathbf{z}^{sv\prime}|\mathbf{z})\ \Delta_{sv}(\mathbf{z}^{sv\prime})\end{aligned} \tag{29}$$

The researcher knows all the elements in the right-hand-side of this equation, and therefore $\Delta_Q(i,\mathbf{z})$ is identified. Given $\Delta_Q(i,\mathbf{z})$ and $Q(i,\mathbf{z}\ ;\ \boldsymbol{\theta}^0)$, we obtain $Q(i,\mathbf{z}\ ;\ \boldsymbol{\theta}^0+\Delta_\theta)$, and using the inverse mapping $\widetilde{q}^{-1}$, we get $\widetilde{P}(\boldsymbol{\theta}^0+\Delta_\theta) = \widetilde{q}^{-1}(\widetilde{Q}(\boldsymbol{\theta}^0+\Delta_\theta);\beta^0,f^*_z)$ and $\Delta_P(i,\mathbf{z})$. ■

PROPOSITION 4: Suppose that Δ_θ *implies changes in the discount factor or in the transition probability of the state variables, such that* $\Delta_\beta \equiv \beta^* - \beta^0 \neq 0$ *or/and* $\Delta_{f_{z,sv}} \equiv f^*_{z,sv} - f^0_{z,sv} \neq 0$, *where the researcher knows both* (β^0, f^0_z) *and* (β^*, f^*_z)*. Despite the knowledge of these primitives under the factual and the counterfactual scenarios, the effect of these counterfactuals on firms' behavior, as represented by* $\Delta_Q(i,\mathbf{z})$, $\Delta_P(i,\mathbf{z})$, *and* $P(i,\mathbf{z}\ ;\ \theta^0+\Delta_\theta)$, *is NOT identified.*

Proof. Under the conditions of Proposition 4 (and making $\Delta_{vp} = \Delta_{fc} = \Delta_{ec} = \Delta_{sv} = \Delta_{f_{z,nosv}} = 0$, for simplicity but without loss of generality), equation (28) becomes:

$$\begin{aligned}\Delta_Q(i,\mathbf{z}) &= \beta^0 \sum_{\mathbf{z}^{sv\prime}\in\mathbf{Z}} \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z})\ sv^0(\mathbf{z}^{sv\prime}) \\ &+ \Delta_\beta \sum_{\mathbf{z}^{sv\prime}\in\mathbf{Z}} [f^0_{z,sv}(\mathbf{z}^{sv\prime}|\mathbf{z}) + \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z})]\ sv^0(\mathbf{z}^{sv\prime})\end{aligned} \tag{30}$$

Since the scrap value sv^0 is not identified, none of the terms that form $\Delta_Q(i,\mathbf{z})$ are identified. ■

4.2 Bias induced by normalizations

Suppose that a researcher has estimated the structural parameters of the model under one of the 'normalization' assumptions that we have described in Section 3.4. And suppose that given the estimated model this researcher implements counterfactual experiments applying the same 'normalization' assumption that has been used in the estimation. For instance, the model has been estimated under the condition that the scrap value is zero, and this condition is also maintained when calculating the counterfactual equilibrium. In this section, we study whether and when this approach introduces a bias in the estimation of the counterfactual effects. We find that this approach does not introduce any bias for the class of (identified) counterfactuals described in Proposition 3. For the class of counterfactuals in Proposition 4, this approach provides a biased estimation. Of course, this is not surprising because, as shown in that Proposition, that class of counterfactual experiments are not identified. More interestingly, we show that the magnitude of this bias can be economically very significant.

Given a normalization used for the estimation of the model (not necessarily $\widehat{sv}^0(\mathbf{z}) = 0$), let $\widehat{sv}^0(\mathbf{z})$ be the estimated scrap value function. And let $\widehat{\Delta_Q}(i, \mathbf{z})$ be the estimate of $\Delta_Q(i, \mathbf{z})$ when we use $\widehat{sv}^0$ instead of the true value sv^0. Using the general expression for $\Delta_Q(i, \mathbf{z})$ in equation (28), the bias induced by the normalization is:

$$\begin{aligned}\widehat{\Delta_Q}(i,\mathbf{z}) - \Delta_Q(i,\mathbf{z}) &= \beta^0 \sum_{\mathbf{z}^{sv\prime}\in\mathbf{Z}} \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z}) \left[\widehat{sv}^0(\mathbf{z}^{sv\prime}) - sv^0(\mathbf{z}^{sv\prime})\right] \\ &+ \Delta_\beta \sum_{\mathbf{z}^{sv\prime}\in\mathbf{Z}} [f^0_{z,sv}(\mathbf{z}^{sv\prime}|\mathbf{z}) + \Delta_{f_{z,sv}}(\mathbf{z}^{sv\prime}|\mathbf{z})] \left[\widehat{sv}^0(\mathbf{z}^{sv\prime}) - sv^0(\mathbf{z}^{sv\prime})\right]\end{aligned} \tag{31}$$

PROPOSITION 5: If the counterfactual experiment is such that $\Delta_\beta = 0$ *and* $\Delta_{f_{z,sv}} = 0$*, then the bias* $\widehat{\Delta_Q}(i,\mathbf{z}) - \Delta_Q(i,\mathbf{z})$ *is zero, and the 'normalization' assumption is innocuous for this class of experiments. Otherwise, the bias* $\widehat{\Delta_Q}(i,\mathbf{z}) - \Delta_Q(i,\mathbf{z})$ *is not zero and the 'normalization' assumption introduces a bias in the estimated effect of the counterfactual experiment.*

Proof. It follows simply from equation (31). ■

Proposition 5 defines the class of counterfactual experiments for which the 'normalization' assumptions introduces a bias. This class consists of those experiments involving a change in the transition probability of a state variable that enters into the scrap value function, or a change in the discount factor. While most of the counterfactual experiments in recent applications look at the impacts of the change in structural functions other than transition functions, several studies have examined the change in firms' behavior under different transition functions. For instance, Collard-Wexler (2010) examines the effect of demand uncertainty on industry dynamics and market structure by implementing a counterfactual experiment that changes the transition probability function of state variables affecting demand. The normalization used consists of setting the scrap value to zero. This normalization is innocuous and the predictions of the counterfactual experiment are unbiased if the state variables affecting demand do not have any impact on scrap value. In another example, Das et al. (2007) examines the change in firms' decisions to enter foreign markets when the currency of their home country is devalued by 20 percent. In this case, their normalization (zero scrap value) is innocuous only if all scarp values come from selling domestic assets. If a part of scrap value comes from selling foreign assets (e.g., subsidiaries), their normalization is not innocuous any more as its value depends on the exchange rate.

4.3 Numerical Example

In this section we present a simple example that illustrates how the bias induced by the normalization assumption can be sizeable and economically significant. Consider a retail industry in which market entry requires land ownership. Examples include big-box stores and hotels. Let z_t represent land price. The form of the entry cost, fixed cost , the scrap value functions are $ec(z) = ec_0 + ec_1 \; z$, $fc(z) = fc_0 + fc_1 \; z$, and $sv(z) = sv_0 + sv_1 \; z$, where ec_0, ec_1, fc_0, fc_1, sv_0, and sv_1 are parameters. Variable profits do not depend on land price: $vp(z) = vp_0$, where vp_0 is a parameter. The stochastic process of land price is described by the following AR(1) process: $z_{t+1} = \alpha_0 + \alpha_1 \; z_t + \sigma_z \; u_{t+1}$, where u_{t+1} is an i.i.d. shock with standard normal distribution.

There are two groups of markets in the data, type H and type L. For instance, each group may represent a metropolitan area, or a region. Each group consists of a large number of local markets. Suppose that the only difference between these two groups of markets is the stochastic processes of land price:

$$
\begin{aligned}
\text{Type } H &: \quad z_t = \alpha_{0H} + \alpha_{1H} \; z_{t-1} + \sigma_H \; u_t \\
\text{Type } L &: \quad z_t = \alpha_{0L} + \alpha_{1L} \; z_{t-1} + \sigma_L \; u_t \text{ where } u \sim iid \; N(0,1)
\end{aligned}
$$

All structural cost functions are the same in all local markets, whether they belong to type H or type L. However, this is not known to the researcher. The researcher observes that land prices are, on average, different between the two groups, but he does not know whether this is the only structural difference between the two groups.

Given a data set generated from this model, the researcher observes, or estimates consistently, the CCP functions for each group of markets. For every land-price z, he knows the probabilities of entry of a potential entrant in a market type H and in market group L, i.e., $P_H(0,z)$ and $P_L(0,z)$, respectively, and the probabilities of exit of an incumbent in these two types of markets, i.e., $P_H(0,z)$ and $P_L(0,z)$. The researcher also knows the variable profit vp_0, the discount factor β, and the parameters $\{\alpha_{0j}, \alpha_{1j}, \sigma_j\}_{j\in\{H,L\}}$ in the stochastic process of land price. Suppose that the main interest of this researcher is to understand the contribution of different structural factors to the differences in the CCP functions in the two groups of markets. More specifically, he is interested in estimating what part of this difference CCPs can be attributed purely to the difference in the stochastic process of land prices, rather than to the differences in cost functions. Unfortunately, as shown in Proposition 5, the researcher cannot obtain an unbiased/consistent estimator of this effect. Performing this experiment requires the knowledge of scrap value function $sv(z)$ but functions $ec(z)$, $sv(z)$, and $fc(z)$ are not separately identified from data. Suppose that the researcher makes a normalization assumption on these functions, and uses the same normalization to implement the

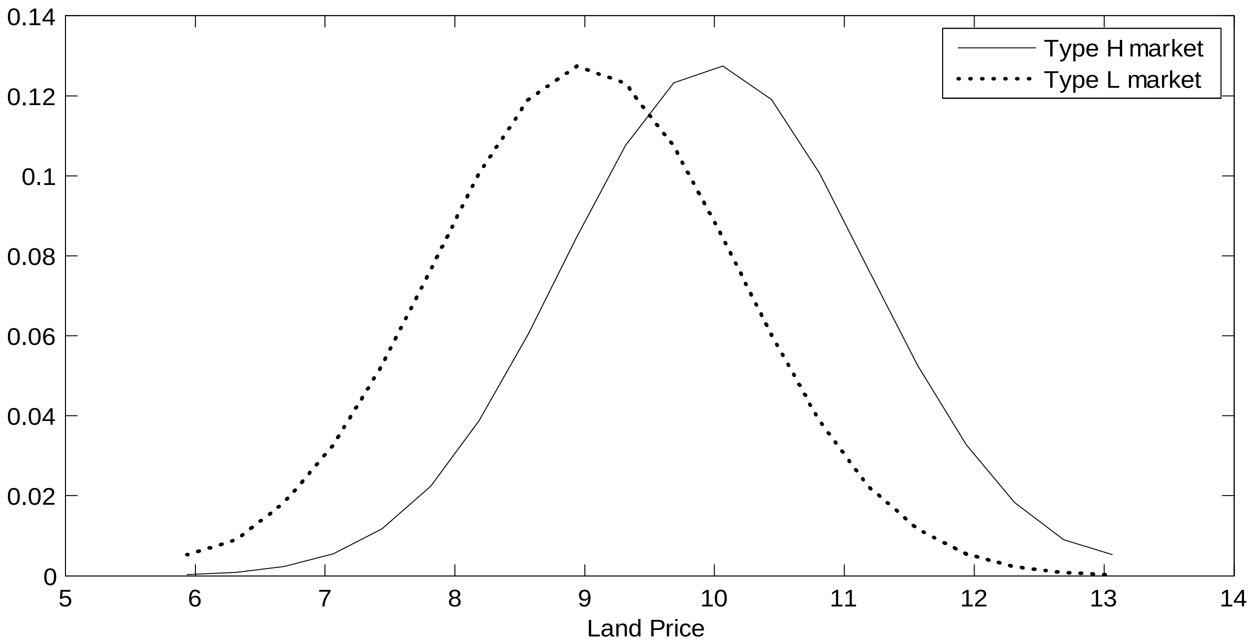

Figure 1: Stationary Distribution of Land Price: Type H vs Type L

counterfactual experiment. The goal of this numerical exercise is to quantify the extent to which this approach introduces a bias in the estimated effect of the counterfactual experiment.

For our numerical example we consider the following values for the parameters in the Data Generating Process: $ec_0 = 6.5$, $ec_1 = 1$, $sv_0 = 0.9$, $sv_1 = 0.96$, $vp_0 = 1.1$, $fc_0 = 0.1$, $fc_1 = 0.03$, $\beta = 0.95$, $\alpha_{0H} = 1.0$, $\alpha_{0L} = 0.9$, $\alpha_{1H} = \alpha_{1L} = 0.9$, and $\sigma_H = \sigma_L = 0.5$. Under this setting, the average land price of group H is ten-percent higher than that of group L (10.0 vs 9.0), while the standard deviation of land price is the same (1.3). Figure 1 illustrates the distributions of these two groups.

Figure 2 shows the CCPs of both a potential entrant and an incumbent in each group of markets. Note that the observed difference in the CCPs of these two groups is entirely due to the difference in the stochastic processes of the land price (though the researcher does not know it). For every land price, the probability of entry and the probability of staying in the market is higher for group H than for group L. This is because the lower average land price in group L implies that, at any level of current land price, a firm's expectation on future land price decreases. As a result, a new entrant is more likely to postpone its entry as future entry cost is likely to be lower. An incumbent is more likely to exit today as scrap value is likely to decrease in the future. Of course, on average, there is more entry and less exit in type L markets than in type H. For a potential entrant, the unconditional probability of being in the market is 7.1 percent in group H and 10.4 percent in group L, and these probabilities are 94.7 percent and 96.5 percent, respectively, for an incumbent.

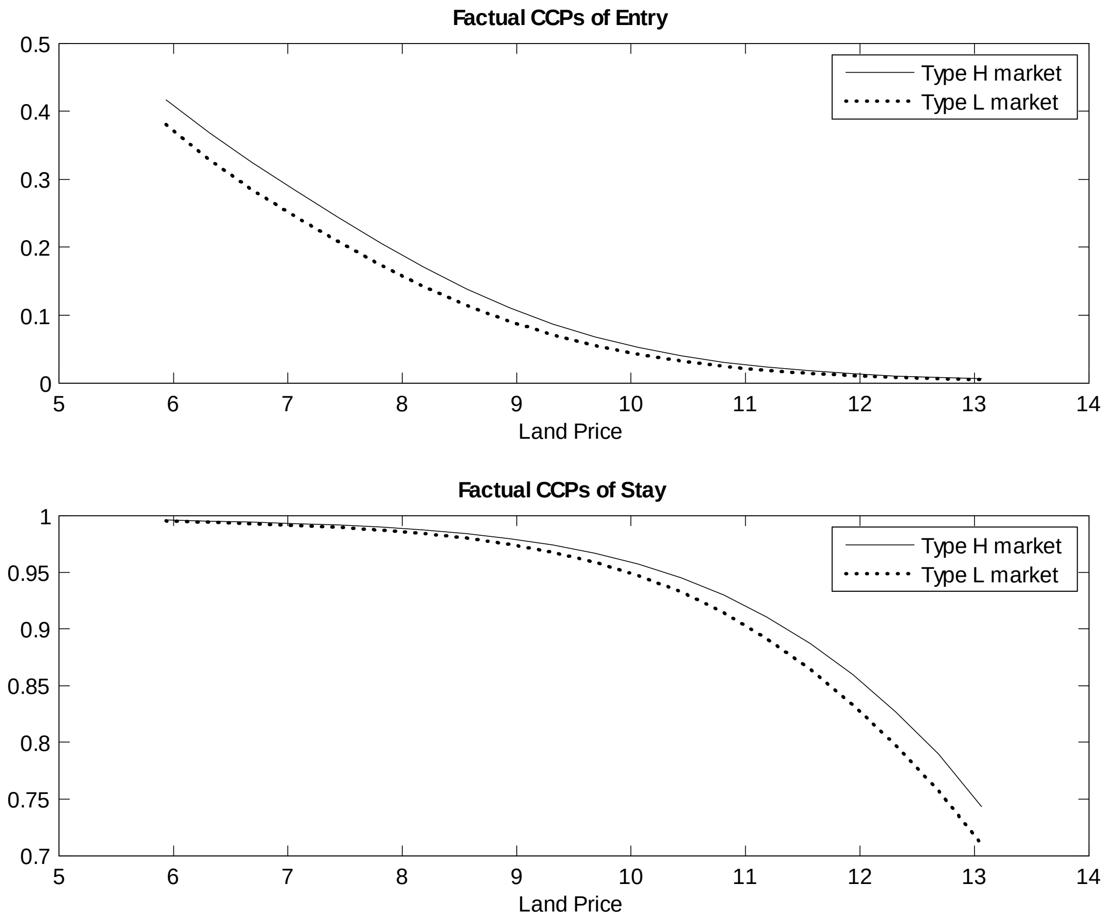

Figure 2: Estimated CCPs

Note that having a probability of staying in the market that declines with land price is not by itself evidence that there is a scrap value that depends on land price. This correlation can be also generated by a model where the scrap value is constant and the fixed cost of an incumbent firm increases with land price (e.g., property taxes, or leasing price that depends on the price of land). In this example, both effects play a role in generating this dependence.

Figure 3 presents the estimated entry cost and fixed cost functions in group H with zero scrap value normalization. With this normalization, the estimated entry cost function is smaller and less sensitive to a change in land price than the true function, i.e., $\widehat{ec}(z) = ec(z) - sv(z) = [5.6 + 0.4z] < [6.5 + z] = ec(z)$. In contrast, the estimated fixed cost function is larger for most land price level and more sensitive to a change in land price than the true function. Under the normalization of the scrap value to zero the researcher may interpret that the effect of land price on the probability of exit comes only the fixed operating cost, but the truth is that it may also come from the dependence of the scrap value with respect to the land price, as in this example.

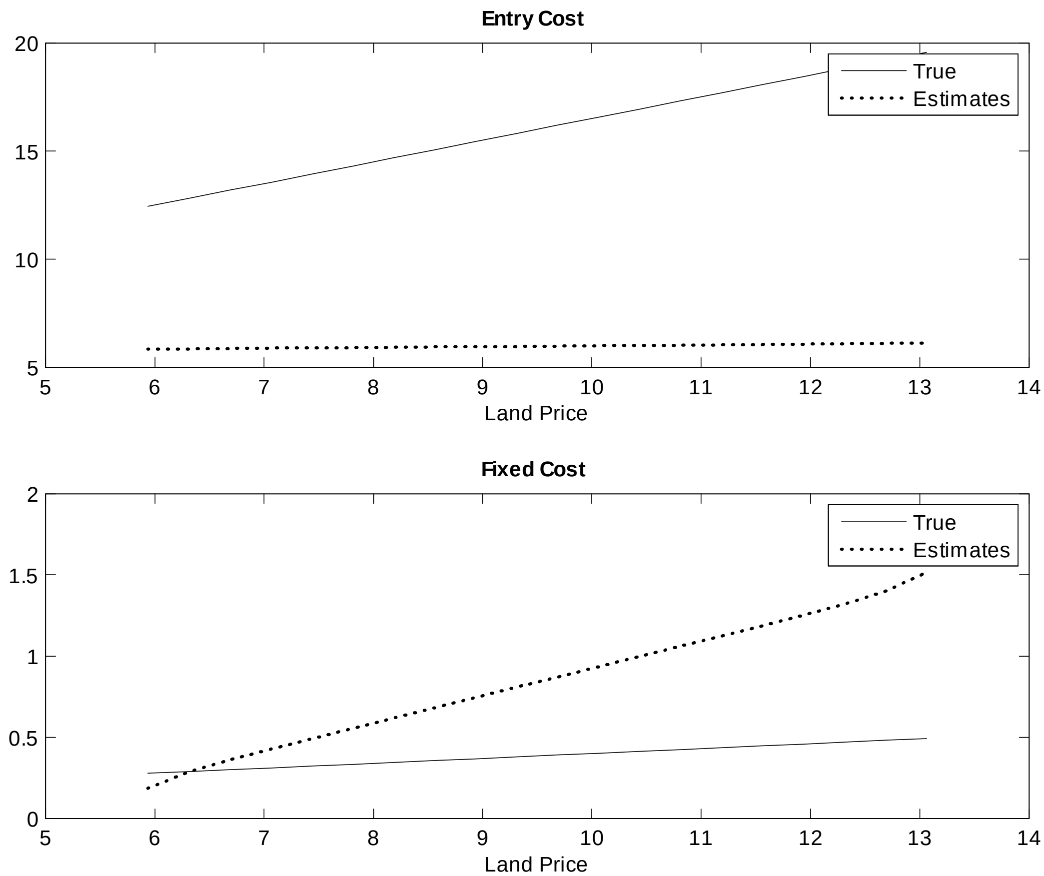

Figure 3: Estimated Structural Cost Functions with $sv(z) = 0$ Normalization

We do not present a figure with the estimated entry cost and fixed cost functions in group L. In this example and under this normalization, the estimated entry cost is the same for the two groups of markets. This is because the stochastic process of land price does not play any role in the estimated entry cost under the normalization of zero scrap value. The estimated fixed cost functions are different: $\widehat{fc}_H(z) = -0.767 + 0.1692\ z$ for group H, and $\widehat{fc}_L(z) = -0.675 + 0.1692\ z$ for group L. Given these results, the researcher will conclude that the sunk entry cost is the same in the two groups of markets. And he might be even willing to conjecture that the two groups have the same entry cost function and the same scrap value function. However, this correct conjecture is still not enough to identify the effect of a change in the stochastic process of land price. Also given the difference in the estimated fixed cost function, the researcher cannot identify how much of this difference can be attributed to the difference in the level of land prices and how much to possible actual differences in the fixed cost functions of the two groups.

Figure 4 presents the main results of a counterfactual experiment that measures to what extent the difference in average land prices can explain the different firm turnover rates between type H

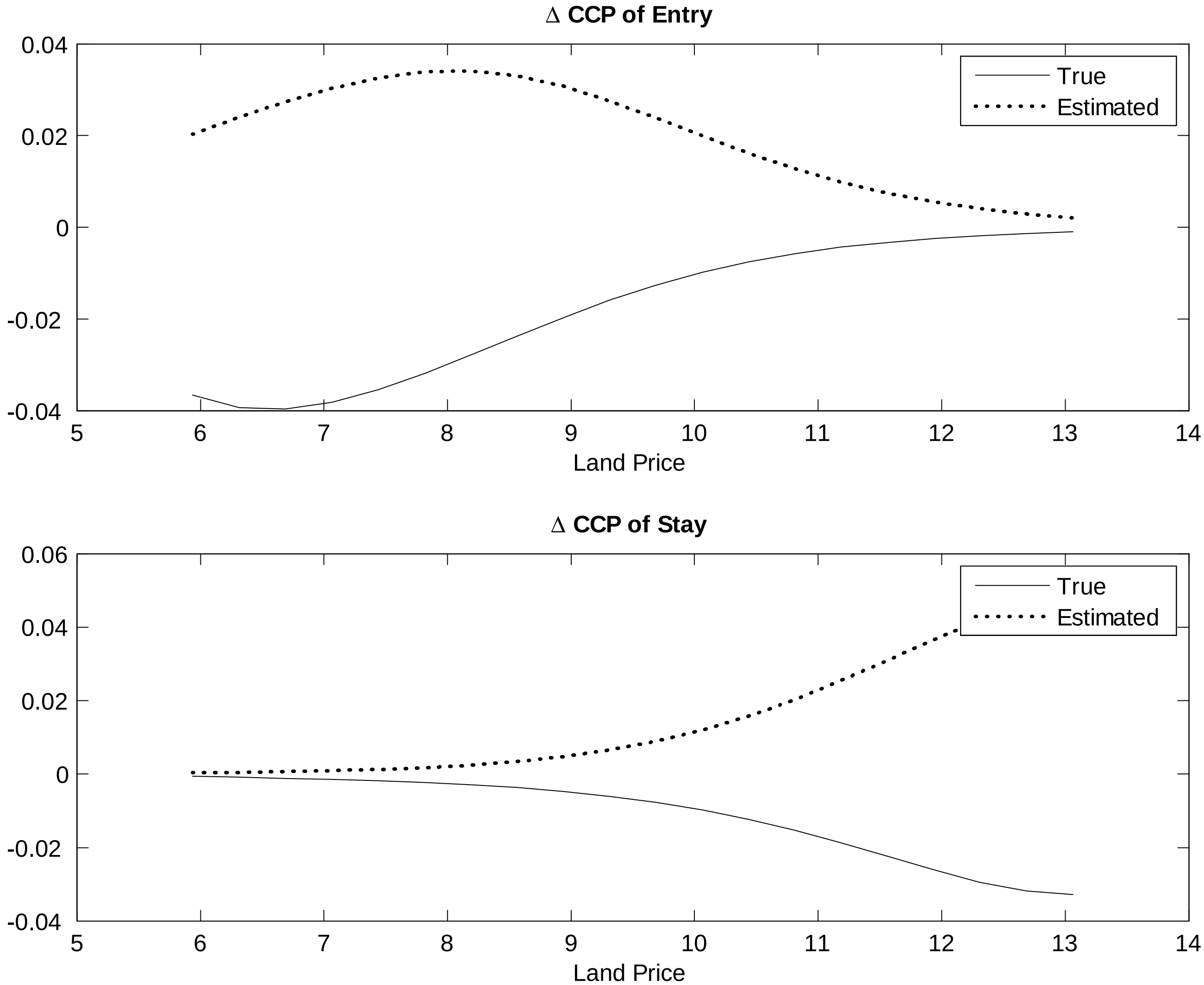

Figure 4: The Estimated Change in CCPs

and type L markets.[13] This experiment consists of solving for the equilibrium CCPs in type H market when we keep the structural cost functions at the estimated values for this group H but we replace the stochastic process of land price with that of type L. The upper panel shows the true and the estimated changes in the probability of entry (i.e., the difference between counterfactual and factual probabilities) where the estimated values are based on zero-scrap value normalization. Similarly, the lower panel shows the change in the probability of staying in the market for an incumbent.

The true effect of this counterfactual experiment, that consists of reducing average land price by ten percent, is that both new entrants and incumbents are less likely to be in the market at

[13] Land price is discretized into 20 equally spaced distinct points between one percentile and ninty-nine percentile.

every land price, i.e., the schedules that represent the probability of entry and stay, as functions of z, move downward. The reduction in average land price implies that, at any level of current land price, a firm's expectation on future land price decreases. As a result, a new entrant is more likely to postpone its entry as future entry cost is likely to be lower. An incumbent is more likely to exit today as scrap value is likely to decrease in the future. Of course, despite this shift downward in these probability schedules, we have that there is more entry and less exit on average with the lower mean land price, i.e., the counterfactual probability function of entry (stay) evaluated at the low mean land price is greater than the factual probability of entry (stay) evaluated at the high mean land price. More specifically, the counterfactual change increases the unconditional probability of being in the market from 7.1 percent to 10.4 percent for a potential entrant and from 94.7 percent to 96.5 percent for an incumbent.

The predictions are considerably different when we perform the same counterfactual experiment using the estimated structural cost functions that are 'identified' from data under zero-scrap value restriction. In contrast to the true effect, the estimated effect shows an upward shift in the schedules that represent the probability of entry and the probability of stay at every possible land price. For instance, at the mean land price in the original distribution ($z = 10$), the probability of being in the market increases by 2.0 percentage points (from 5.3 to 7.3) for a new entrant and by 1.2 percentage points (from 95.7 to 96.9) for an incumbent. This experiment also overshoots the unconditional probability of being in a market in the next period by 4.7 percentage points for a potential entrant (15.1 vs. 10.4) and by 1.4 percentage points for an incumbent (97.9 vs. 96.5).

This bias is generated by the difference in the structural cost functions. As shown in the first row of Table 1, imposing zero scrap value restriction leads overestimation of fixed cost and underestimation of entry cost. In addition, fixed cost estimates under this restriction depend on land price, while both entry cost and scrap value do not. Under the original distribution of land price, a firm's equilibrium policy under this (incorrect) cost structure is exactly equal to the one under the true cost structure. When the distribution shifts to the left, however, a firm is more likely to choose to be in the market for every land price level, as a firm expects lower fixed cost in the future. In this environment, firms do not consider the change in entry cost and scrap value as they do not depend on land price any more.

5 Brief Discussion of Possible Solutions

In this section we present a brief discussion of two natural approaches to deal with the non identification of the three structural functions in the fixed cost and some of the counterfactual experiments: (a) partial (interval) identification; and (b) using data on firms' value or scrap values.

5.1 Partial identification

There are plausible restrictions on the fixed profits functions that provide bounds of our estimates of these functions. For instance, suppose that the researcher is willing to assume that the three components of the fixed profit are always positive, i.e., $ec(\mathbf{z}) \geq 0$, $fc(\mathbf{z}) \geq 0$, and $sv(\mathbf{z}) \geq 0$. These restrictions, together with equation (21), imply sharper lower-bounds for the entry cost and the scrap value. More specifically, we have that $ec(\mathbf{z}) \geq \underline{ec}(\mathbf{z}) \equiv \max\{0, Q(1,\mathbf{z}) - Q(0,\mathbf{z})\}$, and $sv(\mathbf{z}) \geq \underline{sv}(\mathbf{z}) = -\underline{ec}(\mathbf{z})$. That is, if the 'ex ante' sunk cost is positive (i.e., $Q(1,\mathbf{z}) - Q(0,\mathbf{z}) \geq 0$), then the entry cost should be at least as large as the sunk cost, and if the sunk cost is negative (i.e., $Q(1,\mathbf{z}) - Q(0,\mathbf{z}) < 0$), then the scrap value should be at least as large as the negative sunk cost. Other restriction that seems quite plausible is $sv(\mathbf{z}) \geq \beta\ \mathbb{E}(sv(\mathbf{z}_{t+1}) \mid \mathbf{z}_t = \mathbf{z})$. Combining this restriction with equation (21), we have the following upper bound on the fixed cost function: $fc(\mathbf{z}) \leq \overline{fc}(\mathbf{z}) \equiv -Q(1,\mathbf{z}) + vp(\mathbf{z})$.

5.2 Using data on transaction prices from the acquisition of firms

In some industries, a common form of firm exit (and entry) is that the owner of an incumbent firm sells all the firm's assets to a new entrant. For instance, this is very frequent in the bulk shipping industry, as shown in Kalouptsidi (2011), and it is also common in some retail industries intensive in land input such as the hotel industry. Sometimes, the researcher has data on firm acquisition prices. Under some assumptions, these additional data can be used to deal with the identification problem that we study in this paper. We now illustrate this approach in a simple framework.

For simplicity, suppose that the industry is such that the only form of entry is by acquiring an incumbent firm, and similarly the only form of exit is by selling your assets to a new entrant. Then, the entry cost has two components: $ec(\mathbf{z}_t) = r(\mathbf{z}_t) + \tau_{en}(\mathbf{z}_t)$, where $r(\mathbf{z}_t)$ represents the acquisition price that the new entrant should pay to the exiting incumbent, and $\tau_{en}(\mathbf{z}_t)$ represents costs of entry other than the acquisition price. Similarly, the exit value of a firm has also two components: $sv(\mathbf{z}_t) = r(\mathbf{z}_t) - \tau_{ex}(\mathbf{z}_t)$, where $\tau_{ex}(\mathbf{z}_t)$ represents costs associated with market exit. We assume that the acquisition price $r(\mathbf{z}_t)$ is the solution of a Nash bargaining problem between the seller and the buyer. Taking into account that $\bar{V}(1,\mathbf{z}) - \bar{V}(0,\mathbf{z})$ is the value of being an incumbent minus the value of being a potential entrant, we have that the surplus of the buyer is $\bar{V}(1,\mathbf{z}) - \bar{V}(0,\mathbf{z}) - ec(\mathbf{z})$, and the surplus of the seller is $sv(\mathbf{z}) - \bar{V}(1,\mathbf{z}) + \bar{V}(0,\mathbf{z})$. The Nash bargaining solution implies that:

$$r(\mathbf{z}) = \bar{V}(1,\mathbf{z}) - \bar{V}(0,\mathbf{z}) + \alpha\ \tau_{ex}(\mathbf{z}) - (1-\alpha)\ \tau_{en}(\mathbf{z}) \tag{32}$$

where $\alpha \in (0,1)$ is a parameter that represents the seller bargaining power.

Let R_t be the selling price of a firm, and suppose that the researcher observes this price when a transaction actually occurs. For simplicity, to abstract from selection problems, consider that R_t is a deterministic function of the observable state variables $\mathbf{z}_t$ plus a measurement error ξ_t that is not a state variable of the model, has zero mean, and is independent of the observed state variables $\mathbf{z}_t$ (i.e., classical measurement error): $R_t = r(\mathbf{z}_t) + \xi_t$, with $\mathbb{E}(\xi_t|\mathbf{z}_t) = 0$.[14] Under these conditions, the pricing function $r(\mathbf{z})$ is identified from the data. Now, the researcher has three sets of restrictions to identify the three unknown functions $fc(\mathbf{z})$, $\tau_{en}(\mathbf{z})$, and $\tau_{ex}(\mathbf{z})$:

$$\begin{aligned}
\tau_{en}(\mathbf{z}) + \tau_{ex}(\mathbf{z}) &= Q(1,\mathbf{z}) - Q(0,\mathbf{z}), \\
fc(\mathbf{z}) + r(\mathbf{z}) + \tau_{en}(\mathbf{z}) - \beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \; [r(\mathbf{z}') - \tau_{ex}(\mathbf{z}')] &= -Q(0,\mathbf{z}) + vp(\mathbf{z}). \\
\beta \sum_{\mathbf{z}' \in \mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \; [r(\mathbf{z}') - \alpha \; \tau_{ex}(\mathbf{z}') + (1-\alpha) \; \tau_{en}(\mathbf{z}')] &= \widetilde{v}(i,\mathbf{z}) - Q(i,\mathbf{z})
\end{aligned} \tag{33}$$

where the third set of restrictions comes from combining the price equation in (32) with the definition of $Q(i,\mathbf{z})$ as $Q(i,\mathbf{z}) \equiv \widetilde{v}(i,\mathbf{z}) - \beta \sum_{\mathbf{z}'} f_z(\mathbf{z}'|\mathbf{z}) \; [\bar{V}(1,\mathbf{z}') - \bar{V}(0,\mathbf{z}')]$.

PROPOSITION 6: Given a value of the parameter that represents seller bargaining power, α, the set of restrictions in (33) identify the functions $\tau_{en}(.)$, $\tau_{ex}(.)$, and $fc(.)$.

6 Conclusions

The solution and estimation of dynamic structural models of oligopoly competition is a useful tool in Industrial Organization and in other fields in empirical microeconomics such as trade, health economics, or public economics, among others. Empirical applications of these models typically use panel data on firms' entry and exit decisions in multiple local markets, together with information on exogenous market and firm characteristics, and sometimes information of firms' prices and quantities. In this class of models, the functions that represent fixed operating cost, entry cost, and exit value play an important role in firms' entry and exit behavior. Furthermore, many public policies and managerial policies can be described in terms of changes in some of these functions. This paper is motivated by a fundamental identification problem: these three functions cannot be separately identified using these data. The conventional approach to deal with this identification problem has been to 'normalize' some of structural parameters to zero. We study the implications of

[14] This assumption is testable: it implies that the residual price $\xi \equiv [R - \mathbb{E}(R|\mathbf{z})]$ should be independent of firms' entry and exit decisions. In general, unless the dataset is rich enough to include in $\mathbf{z}$ all the relevant variables affecting the price of a firm, we should expect that this assumption will be rejected by the data. That is, we expect $R = r(\mathbf{z}, \boldsymbol{\varepsilon}) + \xi$, where $\boldsymbol{\varepsilon}$ is the vector of state variables observable to firms but unobservable to the researcher. Allowing for this type of unobservables as determinants of the transaction price implies that we should deal with a potential selection problem. We only observe the transaction price for those firm-market-period observations when a firm is sold, but those firms that are sold can be systematically different in terms of unobserved state variables $\boldsymbol{\varepsilon}$ from those firms that are not sold, i.e., $\mathbb{E}(R|\mathbf{z}, \text{firm is sold}) \neq \mathbb{E}(R|\mathbf{z})$.

this ‘normalization’ approach. First, we obtained closed-form expressions that provide the correct interpretation of the estimated objects that are obtained under the ‘normalization assumptions’ that have been considered in applications. Second, we show that there is a class of counterfactual experiments that are identified and for which the normalization assumptions are innocuous. We also show that there is a class of experiments for which the normalization assumptions introduce a bias. Using a simple numerical experiment, we show that this bias can be very significant both quantitatively and economically. We also discuss alternative approaches to deal with this identification problem.

APPENDIX. Proof of Lemma 1

By definition $Q(i,\mathbf{z}) \equiv \tilde{v}(i,\mathbf{z}) - \beta \sum_{\mathbf{z}'} f_z(\mathbf{z}'|\mathbf{z})\ [S(\tilde{v}\,(1,\mathbf{z})\,,F_{\widetilde{\varepsilon}|1}) -\ S(\tilde{v}\,(0,\mathbf{z})\,,F_{\widetilde{\varepsilon}|0})]$, where $\tilde{v}\,(i,\mathbf{z}) = F_{\widetilde{\varepsilon}|i}^{-1}\,(P(i,\mathbf{z}))$ and $S(\tilde{v}\,(i,\mathbf{z})\,,F_{\widetilde{\varepsilon}|i}) \equiv \int_{-\infty}^{\tilde{v}(i,\mathbf{z})}[\tilde{v}(i,\mathbf{z}){-}\widetilde{\varepsilon}]\ dF_{\widetilde{\varepsilon}|i}\,(\widetilde{\varepsilon})$. These expressions imply the mapping in equation (27). We can describe mapping $\widetilde{q}(\widetilde{P};\beta,f_z)$ as the composition of two mappings: (1) the mapping from CCPs to differential values, i.e., $\tilde{v}\,(i,\mathbf{z}) = F_{\widetilde{\varepsilon}|i}^{-1}\,(P(i,\mathbf{z}))$; and (2) the mapping from differential values to $Q's$, i.e., $Q(i,\mathbf{z}) = \tilde{v}(i,\mathbf{z}) - \beta \sum_{\mathbf{z}'} f_z(\mathbf{z}'|\mathbf{z})\ [S(\tilde{v}\,(1,\mathbf{z})\,,F_{\widetilde{\varepsilon}|1}) -\ S(\tilde{v}\,(0,\mathbf{z})\,,F_{\widetilde{\varepsilon}|0})]$. The first mapping is point-wise for every value of $(i,\mathbf{z})$, and in our binary choice model it is obviously invertible. Proposition 1 in (Hotz and Miller 1993) establishes the invertibility of that mapping for multinomial choice models. Therefore, we should prove the invertibility of the second mapping, between the vector $\widetilde{v} \equiv \{\widetilde{v}(i,\mathbf{z}) : \text{for all } (i,\mathbf{z})\}$ and the vector $\widetilde{Q} \equiv \{Q(i,\mathbf{z}) : \text{for all } (i,\mathbf{z})\}$. Define this mapping as $\widetilde{Q} = \widetilde{g}(\widetilde{v})$ where $\widetilde{g}(\widetilde{v}) \equiv \{g(i,\mathbf{z},\widetilde{v}) : \text{for all } (i,\mathbf{z})\}$ and:

$$\begin{aligned} g(i,\mathbf{z},\widetilde{v}) \;&\equiv\; \widetilde{v}(i,\mathbf{z}) - \beta \sum_{\mathbf{z}'\in\mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \left(\int_{-\infty}^{\widetilde{v}(1,\mathbf{z}')} [\widetilde{v}(1,\mathbf{z}') - \widetilde{\varepsilon}]\ dF_{\widetilde{\varepsilon}|1}\,(\widetilde{\varepsilon}) \right) \\ &+\; \beta \sum_{\mathbf{z}'\in\mathbf{Z}} f_z(\mathbf{z}'|\mathbf{z}) \left(\int_{-\infty}^{\widetilde{v}(0,\mathbf{z}')} [\widetilde{v}(0,\mathbf{z}') - \widetilde{\varepsilon}]\ dF_{\widetilde{\varepsilon}|0}\,(\widetilde{\varepsilon}) \right) \end{aligned}$$

In vector form, we can express this mapping as:

$$\widetilde{g}(\widetilde{v}) \equiv \begin{bmatrix} \widetilde{g}(0,\widetilde{v}) \\ \widetilde{g}(1,\widetilde{v}) \end{bmatrix} = \begin{bmatrix} \widetilde{v}(0,.) - \beta\ \mathbf{F}_z\ \widetilde{e}(\widetilde{v}) \\ \widetilde{v}(1,.) - \beta\ \mathbf{F}_z\ \widetilde{e}(\widetilde{v}) \end{bmatrix}$$

where $\mathbf{F}_z$ is the transition probability matrix with elements $f_z(\mathbf{z}'|\mathbf{z})$, and $\widetilde{e}(\widetilde{v}) \equiv \{\widetilde{e}(\mathbf{z};\widetilde{v}) : \text{for all } \mathbf{z}\}$ with $\widetilde{e}(\mathbf{z};\widetilde{v}) = \displaystyle\int_{-\infty}^{\widetilde{v}(1,\mathbf{z})} [\widetilde{v}(1,\mathbf{z}) - \widetilde{\varepsilon}]\ dF_{\widetilde{\varepsilon}|1}\,(\widetilde{\varepsilon}) - \int_{-\infty}^{\widetilde{v}(0,\mathbf{z})} [\widetilde{v}(0,\mathbf{z}) - \widetilde{\varepsilon}]\ dF_{\widetilde{\varepsilon}|0}\,(\widetilde{\varepsilon})$. The mapping $\widetilde{g}(\widetilde{v})$ is globally invertible if and only if its Jacobian matrix $\mathbf{J}(\widetilde{v}) \equiv \partial\widetilde{g}(\widetilde{v})/\partial\widetilde{v}'$ is non-singular for every value of $\widetilde{v}$. It is simple to show that this Jacobian matrix has the following form:

$$\mathbf{J}(\widetilde{v}) = \mathbf{I} - \beta\ \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} \otimes \left(\mathbf{F}_z\ \frac{\partial\widetilde{e}(\widetilde{v})}{\partial\widetilde{v}'} \right) \right]$$

where $\mathbf{I}$ is the identity matrix. Given the form of function $\widetilde{e}(\mathbf{z};\widetilde{v})$, it is straightforward to show that $\partial\widetilde{e}(\mathbf{z};\widetilde{v})/\partial\widetilde{v}(0,\mathbf{z}) = F_{\widetilde{\varepsilon}|0}(\widetilde{v}(0,\mathbf{z})) = -P(0,\mathbf{z})$, and $\partial\widetilde{e}(\mathbf{z};\widetilde{v})/\partial\widetilde{v}(1,\mathbf{z}) = F_{\widetilde{\varepsilon}|1}(\widetilde{v}(1,\mathbf{z})) = P(1,\mathbf{z})$. Therefore,

$$\frac{\partial\widetilde{e}(\widetilde{v})}{\partial\widetilde{v}'} = [-diag\{P(0,.)\}\ ;\ diag\{P(1,.)\}]$$

where $diag\{P(i,.)\}$ is diagonal matrix with elements $\{P(i,\mathbf{z})\}$ for every value of $\mathbf{z}$. The Jacobian matrix $\mathbf{J}(\widetilde{v})$ is invertible for every value of $\widetilde{v}$.

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