Recommendation Mechanisms and Mechanism Design with Multiple Principals, and Multiple Agents

Takuro Yamashita*
Graduate School of Economics,
Hitotsubashi University
ed041009@srv.cc.hit-u.ac.jp

Abstract

It is well known that the standard revelation principle with type revelation mechanisms does not hold when there are multiple principals. We introduce a new mechanism called the *recommendation mechanism*. We show that any pure strategy equilibrium outcomes of any contract game is attained as an equilibrium outcome of the game where (1)each principal offers a recommendation mechanism and (2)the agents sincerely respond to each recommendation mechanism.

*I am grateful to Koichi Tadenuma, Hideshi Itoh, members of Contract Theory Workshop and Contract Theory Workshop East for helphul discussions and comments. I gratefully acknowledge financial support from the 21st Century COE program "Dynamics of Knowledge, Corporate System and Innovation" at Graduate School of Commerce and Management, Hitotsubashi University.

1 Introduction

Most of the literature on mechanism design considers a situation where only one principal offers a contract to agents. In such a situation, the revelation principle holds: when we arbitrarily fix a strategy space of the principal (including the set of type revelation mechanisms) and find an equilibrium, then the equilibrium outcome is attained at an equilibrium of the game in which (1) the principal offers a type revelation mechanism where each agent is required to report his type,[1] and (2) each agent reports his type truthfully. Therefore, we can restrict our attention to type revelation mechanisms when we search for an optimal contract.

However, there are many situations in real economies where multiple principals compete with each other, and strategically offer their contracts to the agents. For example, some principals may compete with each other to win a contract with an agent, because the agent can sign a contract only with one principal. Some existing literature analyzes such multiple principal models. McAfee (1993) analyzed aucion models with many sellers and buyers, and Stole (1995) analyzed competition of firms with differentiated goods. In the theory of industrial organization, Gal-Or (1991) considered duopolistic competitions between retailers(principals) who make exclusive contracts with manufacturers(agents), and compete in a market. She showed that if each agent can observe the contract offered by a principal but cannot observe that offered by the other principal, the revelation principle holds in the exclusive dealing models.

When there are multiple principals, however, the revelation principle dose not generally hold. If we allow the principals to offer contracts other than type revelation mechanisms, then some equilibrium outcomes may not be attainable at any equilibrium of the game where each principal offers a type revelation mechanism. This problem has been pointed out by Peck (1997),

[1]This kind of mechanism is usually called a *direct revelation mechanism*. In this paper we call it a *type revelation mechanism* in order to make it clear that agents are required to announce their types.

Epstein and Peters (1999), and Martimort and Stole (2002). The main reason why the standard revelation principle fails is as follows: In the case of multiple principals, asymmetric information between each principal and each agent consists of not only the type of the agent but also contracts offered by the other principals. Though the agent observes all the contracts offered by the principals, each principal cannot observe those offered by the other principals. Because each principal may increase her expected payoff by adjusting her action once she knows the contracts offered by other principals, each principal has an incentive to make a message space strictly larger than the type space and to ask each agent to reveal not only his type but also information about the other principals' offers.

To find all equilibrium outcomes of contract games, there seems to be two ways in the case of multiple principals. One is to search for the restictions on the model under which the standard revelation principle holds. The result of Gal-Or (1991) above is interpreted as an answer of this line of research. The other is to search for mechanisms other than type revelation mechanisms with which a variant of the revelation principle holds. It is the motivation of this paper.

In this paper we consider a mechanism design problem with multiple principals and multiple agents. We introduce a new mechanism called a *recommendation mechanism*. A recommendation mechanism coincides with a menu contract if only one agent participates in the mechanism. In this case, the principal offers a subset of allocations, and let the agent choose an allocation in the subset. Then the principal assigns the chosen allocation to the agent. However, when two or more agents participate in the mechanism, the recommendation mechanism differs from a menu contract. In this case, the principal asks each agent to report his type and a "recommendation table" which recommends allocation profiles to the principal for each type report. If all participants report the same recommendation table, the principal follows it, otherwise she punishes all the participants. Therefore, each agent has strong incentive to report the same table.

The agents' report which consists of their truthful types and the same recommendation tables is called a *sincere response*. To give an incentive to each agent to report his type truthfully, the recommendation table must satisfies incentive compatibility conditions of the agents' types. Therefore, methods of calculaton of optimal type revalation mechanisms is useful to calculate recommendation tables.

We show that any pure strategy equilibrium outcomes of any contract game is attained as an equilibrium outcome of the game where (1)each principal offers a recommendation mechanism and (2)the agents sincerely respond to each recommendation mechanism. Thus, when we search for each equilibrium outcome, we can restrict our attention to recommendation mechanisms together with some proper incentive compatibility constraints.

There are several important contributions in this line of research. Epstein and Peters (1999) consider competing mechanism with two principals and two agents where each agent can sign a contract with only one of the principals. They introduce the "universal message space". Each message in this space consists of the agent's type and a sequence of expected utility, which reflects information of the other principal's offer. They show that under some assumption on the utility function of each agent, any equilibrium outcome of any contract game can be attained as equilibrium outcome of the game where (1) each principal offers a contract with the universal message space, and (2) each agent sends messages sincerely, that is, each agent reports his true type and conveys correct information of the other principal's offer.

Peters (2001) and Martimort and Stole (2002) consider single agent common agency models. They define a menu contract, where each principal offers a subset of allocations, the agent chooses one of them, and the principal gives the chosen allocation to him. They show that any equilibrium outcome of any contract game can be achieved as an equilibrium outcome of the game where each principal offers a menu contract. Martimort and Stole (2002) call this result the *delegation principle*. As Peters (2001) pointed out, however, the delegation principle with menu contracts does not generally hold for the case of multiple agents.[2]

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 presents an example which shows that both the standard revelation principle and the delegation principle fail if there are mutiple principals and multiple agents. Section 4 introduces recommendation mechanisms and sincere responses, and then establish our main theorem. Section 5 discusses some remaining issues about relations of recommendation mechanisms with type revelation mechanisms and menu contracts, possible weakenings of assumptions for the main result, and an extension of our result to the case with mixed strategy equilibria. Section 6 conludes.

2 Model

There are two principals($j = 1, 2$) and two agents($i = 1, 2$). [3] We denote the set of all principals as J , and the set of all agents as I. Each agent i has a private information(type) $\theta_i \in \Theta_i$. We assume $\Theta = \Theta_1 \times \Theta_2$ is finite. After realizing $\theta \in \Theta$ with probability $f(\theta)$, each principal simultaneously offers a contract [4] to both agents. Each agent observes these offers. Then each agent chooses which contracts to participate in, and sends a message to each of them. Lastly, each principal j gives an allocation $y_{ji} \in Y$ to agent i according to her contract when i participates in her contract, where Y is given feasible allocation space, a compact subset of $\mathbb{R}^n$. Let $y_{ji} = o$ when i does not participate in j's contract. [5] Then we call $y_{JI} = (y_{ji})_{i,j=1,2}$ *outcome* of the game.

We assume utility function of each principal $j = 1, 2$ and each agent

[2]Han (2004) showed that the delegation principle can be extended to the cases of multiple agents where each principal negotiates with each agent independently so that allocations to the agent are independent of the other agents' messages to the principal.

[3]We use "she" for pronoun of a principal, and use "he" for an agent.

[4]Calzolari and Pavan (2000) analyzed a model of sequential offers, and shows that standard revelation principle holds in the model.

[5]We denote zero vector as $o \in Y$.

$i = 1, 2$ as,

$$\begin{aligned} V_j &: \Theta \times Y^4 \to \mathbb{R}, \\ U_i &: \Theta \times Y^4 \to \mathbb{R}. \end{aligned}$$

We explain each stage more precisely. At stage 0, a type profile $\theta \in \Theta$ realizes. While only agent i knows his true type θ_i, agent $h \neq i$ expects i's type is $\tilde{\theta}_i$ with conditional probability $f_h(\tilde{\theta}_i|\theta_h)$, and each principal expects both agents' type is θ with probability $f(\theta)$. At stage 1, each principal j simultaneously offers a contract γ_j to the agents. Note that γ is observable to both of the agents, while it is not observable to the other principal. We assume that for each principal j the message space in which each agent chooses his message to the principal is exogenously given and denoted as M_j. That is, choosing a contract and choosing an allocation rule are equivalent.

As a matter of convenience, we assume that an agent who does not participate in principal j's contract is supposed to send a message $\bar{m} \notin M_j$ to the principal. Let $\bar{M}_j = M_j \cup \{\bar{m}\}$. Then a contract of principal j is defined as $\gamma_j = (\gamma_{j1}, \gamma_{j2})$ such that for each i,

$$\gamma_{ji} : (\bar{M}_j)^2 \to Y.$$

She gives an allocation $y_{ji} = \gamma_{ji}(m_{ji}, \bar{m})$ to agent i when only he participates in j's contract and reports $m_{ji} \in M_j$. She assigns $y_{ji} = \gamma_{ji}(m_{j1}, m_{j2})$ for agent i when both agents participate in and report $(m_{j1}, m_{j2}) \in (M_j)^2$. It means that the principal j can observe the set of the agents participating in her contract as well as their messages. Recall that the principal gives nothing to the agent if he does not participate in her contract. Therefore, for each j, i, if $j \notin p_i$, then $m_{ji} = \bar{m}$ and $y_{ji} = \gamma_{ji}(\bar{m}, m_{j,-i}) = o$ [6] for any $m_{j,-i} \in \bar{M}_j$. We denote the set of all contracts principal j can offer as Γ_j. Let $\Gamma = \Gamma_1 \times \Gamma_2$.

At stage 2, each agent i simultaneously makes *participation decision* $p_i \in P_i$, where P_i is a subset of the power set of J and exogenously given. Agent i is said to be a participant of principal j's contract, if $j \in p_i$. For example;

[6] We write comma between j and $-i$ to avoid confusion.

Intrinsic common agency As each agent can choose to participate in both principals or in neither, we have $P_1 = P_2 = \{J, \emptyset\}$.

Delegated common agency As each agent can choose to participate in either of the principals or both or neither, we have $P_i = 2^J = \{\{1\}, \{2\}, J, \emptyset\}$.

Competing mechanism As each agent can choose to participate in either of the principals or neither, we have $P_i = \{\{1\}, \{2\}, \emptyset\}$.

Exclusive dealing As each agent can choose to participate in exogenously given principal or neither, we have $P_i = \{\{i\}, \emptyset\}$.

Before the next stage, each agent observes $p = (p_1, p_2)$, while each principal j can only observe the set of her partners, $I_j(p) = \{i \in I | j \in p_i\}$. [7]

At stage 4, each agent i simultaneously sends a message to principal j if $j \in p_i$, i.e., agent i participates in principal j's contract, and sends $\bar{m}$ to principal k if $k \notin p_i$. Then, his *message choice* is defined as $m_{Ji} = (m_{ji})_{j \in J} \in M_{p_i}$, where $M_{p_i} = \prod_{j \in p_i} M_j \times \prod_{j \notin p_i} \{\bar{m}\}$. We denote $m_{jI} = (m_{j1}, m_{j2})$ and $m_{JI} = (m_{1I}, m_{2I})$.

An action profile $(\gamma, p, m_{JI}) = (\gamma_1, \gamma_2, p_1, p_2, m_{J1}, m_{J2})$ realizes an allocation from each principal to each agent, $\gamma_{JI}(m_{JI}) \in Y^4$. We call it *outcome following from* (γ, p, m_{JI}). More precisely, $\gamma_{JI}(m_{JI}) = (\gamma_{ji}(m_{jI}))_{i \in I, j \in J}$.

Remark. In some situations, we should assume that the agents simultaneously make their participation decisions and message choices, or equivalently,

[7] The same kind of conclusion holds if we assume the principal j can observe not only the set of her partners but also that of the other's partners. The same kind of conclusion holds as well even if agent i observes the participants in principal j if and only if $j \in p_i$, i.e., he observes (i)only $I_j(p)$ if $p_i = \{j\}$ and (ii)$(I_1(p), I_2(p))$ if $p_i = J$(it is equivalent to observing p).

that each agent make their message choices before he gets to know the participation decision of the other agent. We analyze those situations in a special case of this model: assume $P_1 = P_2 = \{J\}$, i.e., nothing happens in the participation stage, and that there exists a "not participate in" message, $m^o \in M_j$, such that for each j and any $\gamma_j \in \Gamma_j$,

$$m_{ji} = m^o \Rightarrow \gamma_{ji}(m_{jI}) = o.$$

A pure strategy of principal j is to choose a contract γ_j in Γ_j. A pure strategy of agent i consists of two plans: a *participation plan* π_i which specifies a participation decision $\pi_i(\theta_i, \gamma) \in P_i$ for each θ_i and γ, and a *message plan* σ_i which specifies a message choice $\sigma_{Ji}(\theta_i, \gamma, p) \in M_{p_i}$ for each θ_i, γ, and p. Note that $\sigma_{ji}(\theta_i, \gamma, p) \in M_j$ if $j \in p_i$ and $\sigma_{ji}(\theta_i, \gamma, p) = \bar{m}$ if $j \notin p_i$. For notational simplicity, let $\pi(\theta, \gamma) = (\pi_1(\theta_1, \gamma), \pi_2(\theta_2, \gamma))$, $\sigma_{jI}(\theta, \gamma, p) = (\sigma_{j1}(\theta_1, \gamma, p), \sigma_{j2}(\theta_2, \gamma, p))$, and $\sigma_{JI}(\theta, \gamma, p) = (\sigma_{J1}(\theta_1, \gamma, p), \sigma_{J2}(\theta_2, \gamma, p))$.

In this paper, we treat only pure strategies for simplification. The case of mixed strategies will be discussed in the section 6, and we suggest that the same kind of conclusions will hold with appropriate modifications on recommendation mechanisms.

Once $\theta \in \Theta$ is realized, along with the pure strategy profile (γ, π, σ), $\gamma_{JI}(\sigma_{JI}(\theta, \gamma, \pi(\theta, \gamma)))$ is realized as the outcome.

We denote this Bayesian game as G. Now we define *equilibrium* of G. As noted above, we treat only pure strategy equilibrium for simplification. We consider perfect Bayesian equilibrium(PBE) as equilibrium concept. Because both agents simultaneously act after both principals simultaneously act, an equilibrium consists of two parts: firstly we define a *rational response rule* of the agents, a rule which specify a rational response to each γ, and then define the principals' *optimal contract* profile.

A response rule of the agents consists of two parts: a strategy profile of the agents (π, σ) and a belief system $\beta = (\beta_1, \beta_2)$, where $\beta_i : \Theta_i \times \Gamma \times P \rightarrow$

$\Delta(\Theta_{-i})$. [8] $\beta_i(\theta_{-i}|\theta_i,\gamma,p)$ specifies the probability with which agent i believes that the type of the other agent is θ_{-i}, when his type is θ_i and observes γ and p.

For each γ, a response rule (π,σ,β) specifies a response to γ, $(\pi(\gamma),\sigma(\gamma),\beta(\gamma))$, where for each i,

$$\begin{aligned}
\pi_i(\gamma) &= (\pi_i(\theta_i,\gamma))_{\theta_i\in\Theta_i},\\
\sigma_i(\gamma) &= (\sigma_{Ji}(\theta_i,\gamma,p))_{\theta_i\in\Theta_i,p\in P},\\
\beta_i(\gamma) &= (\beta_i(\theta_{-i}|\theta_i,\gamma,p))_{\theta_i\in\Theta_i,p\in P}.
\end{aligned}$$

We call the response is rational to γ if this satisfies the following rationality conditions.

Definition 1. Let $\gamma\in\Gamma$. $(\pi^*(\gamma),\sigma^*(\gamma),\beta^*(\gamma))$ is a *rational response to* γ if,

1. For agent $i=1,2$, for each θ_i and each p,

$$\sigma^*_{Ji}(\theta_i,\gamma,p)\in \arg\max_{m_{Ji}\in M_{p_i}}\sum_{\theta_{-i}}\beta^*_i(\theta_{-i}|\theta_i,\gamma,p)U_i(\theta,\gamma_{JI}(m_{Ji},m^*_{J,-i})),$$

where $m^*_{J,-i}=\sigma^*_{J,-i}(\theta_{-i},\gamma,p)$.

$$\text{Define}\quad U^*_i(\theta_i,\gamma,p)=\sum_{\theta_{-i}}\beta^*_i(\theta_{-i}|\theta_i,\gamma,p)U_i(\theta,\gamma_{JI}(\sigma^*_{JI}(\theta,\gamma,p))).$$

2. For agent $i=1,2$, for each θ_i,

$$\pi^*_i(\theta_i,\gamma)\in \arg\max_{p_i\in P_i}\sum_{\theta_{-i}}f_i(\theta_{-i}|\theta_i)U^*_i(\theta_i,\gamma,p_i,p^*_{-i}).$$

where $p^*_{-i}=\pi^*_{-i}(\theta_{-i},\gamma)$.

[8] $\Delta(\Theta_{-i})$ denotes the set of all probability distributions on Θ_{-i}.

3. For agent $i = 1, 2$, for each θ_i and each p,

$$\beta_i^*(\theta_{-i}|\theta_i, \gamma, p) = \begin{cases} \frac{f_i(\theta_{-i}|\theta_i)}{\sum_{\theta_{-i}} f_i(\theta_{-i}|\theta_i) 1\{\pi_{-i}^*(\theta_{-i},\gamma)=p_{-i}\}} & \text{if } \exists \theta_{-i} \text{ s.t. } f_i(\theta_{-i}|\theta_i) > 0, \\ & \text{and } \pi_{-i}^*(\theta_{-i}, \gamma) = p_{-i}, \\ f_i(\theta_{-i}|\theta_i) & \text{otherwise,} \end{cases}$$

where $1\{\cdot\}$ is the indicator function.

The first condition requires the message choice of each agent to be rational to γ, i.e., each agent sends a message to maximize his expected utility at every possible situation given his belief system. The second condition requires the participation decision of each agent to be rational to γ, i.e., each agent makes a participation decision to maximize his expected utility at every possible situation given the message plan of the agents. The third condition requires the belief system of each agent to be consistent with the strategy profile of them. [9]

If $(\pi^*, \sigma^*, \beta^*)$ makes a rational response to every γ, we call it *rational response rule*. It is not obvious whether a rational response to given γ exists, and whether it is unique even if it actually exists. For the present, we assume its existence, but don't assume its uniqueness. Let $r(\Gamma)$ denotes the set of all rational response rules.

In an equilibrium, each principal is asked to offer an optimal contract in the sense that she maximizes her expected utility given the others' equilibrium strategy.

Definition 2. $(\gamma^*, \pi^*, \sigma^*, \beta^*)$ is an *equilibrium* of G if,

1. For principal $j = 1, 2$,

$$\gamma_j^* \in \arg\max_{\gamma_j \in \Gamma_j} \sum_\theta f(\theta) V_j(\theta, \gamma_{jI}(m_{jI}^*), \gamma_{-jI}^*(m_{-jI}^*)),$$

[9] All of our results is not dependent on the definition of the belief off-the-equilibrium-path.

$$\text{where } \forall k,\ m^*_{kI} = \sigma^*_{kI}(\theta, \gamma_j, \gamma^*_{-j}, \pi^*(\theta, \gamma_j, \gamma^*_{-j})).$$

2. For the agents, $(\pi^*, \sigma^*_I, \beta^*) \in r(\Gamma)$.

In the next section, we introduce an example of mechanism design with two principals and two agents. We demonstrate that the standard revelation principle and the delegation principle fails. The logic of this result is basically the same as that of the example in Martimort and Stole (2002).

3 Example

Take an example of potential competition between firms. There are two retailers(principals) in a market who want to sell some good to consumers. The good consists of two essential components, therefore each principal can supply the good if and only if she buys both components. Component $i = 1, 2$ is produced only by manufacturer i(agent), and has three patterns of design(say model number 2, 3, and 4).

Agent i has imperfect information of the demand of the good, $\theta_i \in \{1, 2\}$, as his private information. More precisely, the consumers demand for the good, d, is one if both components have model number $\theta_1 + \theta_2 (\in \{2, 3, 4\})$, and is zero otherwise. Assume $\theta_i \in \{1, 2\}$ is independently determined as $\Pr(\theta_i = 1) = \Pr(\theta_i = 2) = 0.5$. For simplicity, the price of the good is supposed to be fixed at 10, and each component is produced with cost 1.

The allocation from principal j to agent i consists of two variables: $x_{ji} \in \{0, 2, 3, 4\}$, the model number of the component i ($x_{ji} = 0$ means "no production"), and $t_{ji} \in [0, 10]$, monetary transfer from the principal to the agent. Let $Y = \{0, 2, 3, 4\} \times [0, 10]$ be the allocation space.

If $\theta = (\theta_1, \theta_2) \in \{1, 2\}^2$ and $y_{JI} = (x_{ji}, t_{ji})_{j \in J, i \in I}$ are realized, the utility of each player is as follows.

1. For each principal $j = 1, 2$,

$$v_j = \begin{cases} 10 - (t_{j1} + t_{j2}) & \text{if} \quad x_{j1} = x_{j2} = \theta_1 + \theta_2, \\ -(t_{j1} + t_{j2}) & \text{otherwise.} \end{cases}$$

2. For each agent $i = 1, 2$,

$$u_i = t_{1i} - c(x_{1i}) + t_{2i} - c(x_{2i}),$$

   where $c(\cdot)$ denotes the production cost: $c(0) = 0$, and $c(2) = c(3) = c(4) = 0$.

In this example, we assume competing mechanism, i.e., each agent can sign a contract with only one of the principals. Therefore, competition between the pricnipals does occur not in the stage of sales in the market, but in the stage of contract with both of the agents. Note that if agent i does not participate in principal j, $x_{ji} = t_{ji} = 0$.

3.1 Benchmark: single principal case

As a benchmark, we consider the single principal case. The revelation principle holds in this case.

We denote a type revelation mechanism as $(x_i(\theta), t_i(\theta))_{i\in I,\theta\in\Theta}$. Setting $t_i(\theta) = 1$ for each θ, the principal gets all the surplus from the agents. Because this transfer rule is clearly incentive compatible, each agent always reveals their private information to the principal(say θ_1, θ_2), and therefore she always has correct information of the market demand. Then she should set $x_i(\theta) = \theta_1 + \theta_2$.

Therefore, since she always produces the preferable model for the consumers and always leaves no rent for the agents, her expected profit is

$$E_\theta v = \sum_\theta \frac{1}{4}(10 - 2) = 8.$$

Cleary, this outcome is the same as the first best outcome, i.e., symmetric information case.

3.2 Two-principal cases with type revelation mechanisms and menu contracts

When there are two principals, as long as each of them offers a type revelation mechanism, equilibrium outcomes with efficient production and no rent cannot be achieved. For example, suppose principal 1 offers a contract which leaves no rent for the agents. Then principal 2 can sign contracts with them and produce the good by giving some rent, say $\varepsilon > 0$, to both agents, which results in that outcomes with no rent are not supported in any equilibrium. The critical problem is that each agent participating in the type revelation mechanism can convey only information of his type, but the information about the contracts offered by the other principal.

Even if each principal is allowed to offer a menu contract, equilibrium outcomes with efficient production and no rent cannot be achieved. For example, think about the case where both agents participate in the contract of principal 1 and agent 1's type is $\theta_1 = 1$. To attain the same outcome, the agent should report $(2, 2)$ if $\theta_2 = 1$ and $(3, 3)$ if $\theta_2 = 2$. However, since agent 1 does not know θ_2, it is impossible for him. Due to lack of information about the other agent's type, it is also impossible to attain the equilibrium outcome by the menu contract.

3.3 Two-principal cases with exogenously given message spaces

However, if the principals is allowed to offer more complicated contracts, there exists an equilibrium outcome where one of the principals efficiently supplies the good and leaves no rent for the agents. That is, the standard revelation principle and the delegation principle fails in this multi-principal and multi-agent situation.

To show that, we actually construct an equilibrium, given the message spaces $M_1 = \{1, 2\} \times \{\text{normal, deviating}\}$, and $M_2 = \{1, 2\}$. That is, principal 1 can offer a contract with more complicated message space than the type space, while principal 2 can only offer a type revelation mechanism. We consider the following contract profile: for each i,

$$\begin{aligned}
\tilde{\gamma}_{1i}(m_{1i}, \bar{m}) &= (0,0), \quad \forall m_{1i} \in M_1, \\
\tilde{\gamma}_{1i}((\theta_1, d_1), (\theta_2, d_2)) &= \begin{cases} (0,10) & \text{if} \quad d_1 = d_2 = \text{deviating}, \\ (\theta_1 + \theta_2, 1) & \text{otherwise.} \end{cases} \\
\tilde{\gamma}_{2i} &\equiv (0,0).
\end{aligned}$$

If at most only one agent participates in $\tilde{\gamma}_1$, she does nothing. If both participate in $\tilde{\gamma}_1$, unless both agents report "deviating", she sums up the information provided by the agents and follows the same production rule as in the single principal case, i.e., efficient production with no rent. If both agents report "deviating", the principal does not produce the good, and gives money to the agents. This case, however, will turn to be off the equilibrium path.

Principal 2 offers "null contract", i.e., she does nothing in any case.

As long as $\tilde{\gamma}_1$ is offered, the following response is rational for the agents: both agents always participate in $\tilde{\gamma}_1$, each agent i reports his true type θ_i and "normal" if he has observed $\tilde{\gamma}_2$ as the offer from principal 2, and reports his true type θ_i and "deviating" if he has observed principal 2's deviation from $\tilde{\gamma}_2$.

Given that agent 2 follows this response, we check agent 1's incentive of deviation to another message choice. When $\tilde{\gamma}_2$ is offered, agent 2 reports his true type and "normal", In this case agent 1 gets utility 0, regardless of his message choice, and therefore it is rational for him to report his true type and "normal". If principal 2 offers $\gamma_2 \neq \tilde{\gamma}_2$, agent 2 reports his true type and "deviating". In this case agent 1 has strong incentive to report "deviating", because he gets utility 10 by doing so. Therefore, his message choice is rational.

Since principal 1 achieves her first best outcome, she has no incentive to deviate. On the other hand, as long as principal 1 offers $\tilde{\gamma}_1$, the agents never participate in principal 2's contract, so principal 2 also has no incentive of deviation from $\tilde{\gamma}_2$.

In conclusion, the equilibrium outcome of the (benchmark) single principal model is attained in this equilibrium. Making the message space complicated, the principal can get information from the agents about the contract offered by the other principal ($\{\text{normal}, \text{deviating}\}$ plays this role in our example), and can change her action contingent on that.

The same outcome is also attained when principal 1 asks the agents to recommend what she should do. That is the key to our main theorem. Let $M_1 = \{1,2\} \times (Y^2)^4 \ni (\theta_i, a^i_{1I})$, that is, each agent i is asked to report his type θ_i and a table(mapping) $a^i_{1I} : \Theta \to Y^2$, which recommends $a^i_{1I}(\theta) = ((x_{11}(\theta), t_{11}(\theta)), (x_{12}(\theta), t_{12}(\theta))) \in Y^2$ to the principal for each θ.

Principal 1 is supposed to offer the following contract: if $a^1_{1I} = a^2_{1I}$, then she assigns $a^1_{1I}(\theta) \in Y^2$ for the agents, otherwise, she assigns $(x_{1i}, t_{1i}) = (\theta_1 + \theta_2, 0)$ for each agent i.

As long as this contract is offered by principal 1, the following response is rational for the agents: both agents always participate in this contract of principal 1, each agent i reports his true type θ_i and the following table if he has observed $\tilde{\gamma}_2$ as the offer from principal 2,

	$\theta_1 = 1$	$\theta_1 = 2$
$\theta_2 = 1$	((2,1),(2,1))	((3,1),(3,1))
$\theta_2 = 2$	((3,1),(3,1))	((4,1),(4,1))

Table 1: $\gamma_2 = \tilde{\gamma}_2$

and reports his true type θ_i and the following table if he has observed principal 2's deviation from $\tilde{\gamma}_2$.

	$\theta_1 = 1$	$\theta_1 = 2$
$\theta_2 = 1$	((0,10),(0,10))	((0,10),(0,10))
$\theta_2 = 2$	((0,10),(0,10))	((0,10),(0,10))

Table 2: $\gamma_2 \neq \tilde{\gamma}_2$

When $\tilde{\gamma}_2$ is offered by the principal 2, both agents report his true type and the same table which recommends the allocation profiles that would be assigned if the agents had reported "normal" to the principal, and allow the principal to attain her first best outcome. When a contrat other than $\tilde{\gamma}_2$ is offered, both agents report his true type and the same table which recommends the allocation profiles that would be assigned if the agents had reported "deviating" to the principal, and prevent principal 2 from deviation from $\tilde{\gamma}_2$.

We check incentives of deviation of the agents. Firstly, as long as the other agent follows the response rule above, each agent has no incentive to report a table other than the table defined above. For example, assume $\tilde{\gamma}_2$ is offered by principal 2. If agent 1 reports table 1 above, he gets utility 0, regardless of his type report. If he reports another table, however, he gets no transfer and then negative utility.

Secondly, as long as both agents report the same table defined above, each agent has no incentive to deviate from his truthful type report.

4 Recommendation Mechanism

In this section, we define *recommendation mechanism*, and show that any equilibrium outcome of game G is attainable in an equilibrium such that (1)each principal offers a recommendation mechanism and (2)the agents sincerely respond to each recommendation mechanism. Then, there is no loss of generality to restrict our attention to recommendation mechanisms in searching for equilibrium outcomes.

The motivation of recommendation mechanism is similar to that of menu contract in the sense that each agent who participates in a recommendation mechanism is asked to recommend an allocation or allocation profile to the principal. Especially, when only one agent participates in a recommendation mechanism, it is equivalent to a menu contract. That is, the principal offers a subset of the allocation space, asks the agent to choose one of them, and gives the chosen allocation to him.

If there are two agents participating in the mechanism, however, each agent i is asked to report his type θ_i and a *recommendation table* $a^i_{jI} : \Theta \to Y^2$(therefore, $a^i_{jI} \in (Y^2)^\Theta$) which recommends principal j to assign an allocation profile along with the type report. For example, assume each agent reports $(\tilde{\theta}_1, a^1_{jI}), (\tilde{\theta}_2, a^2_{jI})$ respectively such that $a^1_{jI} = a^2_{jI}$. Then principal j follows agent 1's recommendation and assign $a^1_{jI}(\tilde{\theta}) = (a^1_{j1}(\tilde{\theta}), a^1_{j2}(\tilde{\theta})) \in Y^2$. If $a^1_{jI} \neq a^2_{jI}$, the principal punishes both agents.

Since a recommendation table is a mapping from Θ to Y^2, it is similar to a type revalation mechanism. However, recommendation tables is different from type revelation mechanisms at the following two point. Firstly, recommendation tables depend on information about contracts offered by the other principal, because each agent reports the table after he observe the contracts. Secondly, the recommendation tables are not necessarily preferable for the principal, because each agent usually has different preference on allocations from the principal.

Before the definition, we introduce two assumptions.

Assumption 1. We assume the following two conditions.

A1 For each j, $M_j \supset Y$ and $M_j \supset \Theta_i \times (Y^2)^\Theta (i = 1, 2)$.

A2 There exists $\underline{y} \in Y$: for any i, any θ and any $y'_{JI} \in Y^4$, if y_{JI} satisfies either $y_{1i} = \underline{y}$ or $y_{2i} = \underline{y}$, then

$$U_i(\theta, y_{JI}) \leq U_i(\theta, y'_{JI}).$$

The former part of the first condition requires Y is a subset of M_1, M_2, i.e., each agent can report any allocation as his message to each principal. The latter of the first condition requires $\Theta_i \times (Y^2)^\Theta$ is also a subset of M_1, M_2, i.e., each agent can report both his type and a recommendation table as his message to each principal. The second condition requires existence of a *punishment allocation* $\underline{y}$.

These two assumptions seem to be quite strong, but we can weaken, for example, the second assumption as discussed in section 6.

We define j's recommendation mechanism as one of mechanisms in Γ_j.

Definition 3. $\gamma_j \in \Gamma_j$ is principal j's *recommendation mechanism* if,

1. For all p such that $I_j(p) = \{i\}$, there exists $Y_{ji} \subset Y$;

$$\gamma_{ji}(m_{ji}, \bar{m}) = \begin{cases} m_{ji} & \text{if} \quad m_{ji} \in Y_{ji}, \\ \underline{y} & \text{if} \quad m_{ji} \notin Y_{ji}. \end{cases}$$

2. For all p such that $I_j(p) = I$,

$$\gamma_{ji}(m_{j1}, m_{j2}) = \begin{cases} a^1_{ji}(\theta_1, \theta_2) & \text{if} \quad \forall k;\ m_k = (\theta_k, a^k_{jI}) \in \Theta_k \times (Y^2)^{\Theta} \\ & \quad\quad \text{and } a^1_{jI} = a^2_{jI} \\ (\underline{y}, \underline{y}) & \quad\quad \text{otherwise} \end{cases}$$

The first condition requires that if only agent i participates in j's recommendation mechanism and he sends a message $m_{ji} \in Y_{ji}$, then she gives m_{ji}. If m_{ji} is not in menu Y_{ji}, on the other hand, the principal punishes the agent. The second condition is for the case of multiple participants. If both agents participate in j's recommendation mechanism and report their types and a recommendation table, and if both tables coincide with each other, then she assigns $a^1_{ji}(\theta_1, \theta_2)$ for agent i. That is, the principal follows agent 1's recommendation. Otherwise, both agents get punished.

A recommendation mechanism has two important characteristics. Firstly, any recommendation mechanism is one of the original mechanisms. Then, the set of all recommendation mechanisms of principal j, Γ^r_j, is a subset of Γ_j. Secondly, the principal delegates her allocation decision to the agents. Especially, if both agents participate in j's recommendation mechanism and report the same recommendation table, the principal is only to follow the agents' recommendation. It results in that choosing one recommendation mechanism is equivalent to choosing $Y_j = (Y_{j1}, Y_{j2})$. We, therefore, denote the recommendation mechanism as $\gamma^{Y_j}_j$.

Remark 1. Though the rule of recommendation mechanism for multiple participants are apparently quite different from the rule for single participant, we can interpret the rule for single participant is one of simplified versions of the following rule $\bar{\gamma}_{ji}$, which asks agent i to report his type θ_i and a kind of recommendation table $\bar{a}^i_{ji} : \Theta_i \to Y$.

$$\bar{\gamma}_{ji}(m_{ji}, \bar{m}) = \begin{cases} \bar{a}^i_{ji}(\theta_i) & \text{if} \quad m_{ji} = (\theta_i, \bar{a}^i_{ji}) \in \Theta_i \times (Y)^{\Theta_i} \\ & \quad\quad \text{and } \forall \tilde{\theta}_i,\ \bar{a}^i_{ji}(\tilde{\theta}_i) \in Y_{ji}. \\ \underline{y} & \quad\quad \text{otherwise} \end{cases}$$

This rule is essentially equivalent to the rule for single participant in the definition of recommendation mechanism: as long as the agent reports θ_i, $\bar{a}^i_{ji}(\tilde{\theta}_i)$ for every $\tilde{\theta}_i \neq \theta_i$ is *irrelevant*, and then, the agent chooses $\bar{a}^i_{ji}(\theta_i)$ to maximize his expected utility and arbitrarily chooses $\bar{a}^i_{ji}(\tilde{\theta}_i)$ for every $\tilde{\theta}_i \neq \theta_i$.

Remark 2. We note the reason why she does not restrict in the case of multiple participants, while principal j restricts the participant's message space to Y_{ji} in the cases of single participant. In the cases of single participant, principal j has to restrict the agent's recommendation, otherwise the agent recommends an allocation preferable for him (and maybe unfavorable for the principal). On the contrary, the principal needs not to do so in the case of multiple participants, because each agents' recommendation *is restricted by that of the other agents*. Therefore, for example, as long as agent 2 reports a recommendation table favorable for the principal, agent 1 has to report the same recommendation table to avoid punishment, so she does not need to restrict the message space.

We introduce a way to construct a recommendation mechanism from an arbitrary contract. It is a key to our main theorem. Let $\gamma_j \in \Gamma_j$ be an arbitrary contract of principal j. We define $\gamma_{ji}(M_j)$ for each i as the image of γ_{ji} on the allocation space Y when only agent i participates in γ_j. More precisely,

$$\gamma_{ji}(M_j) = \{y_{ji} \in Y | \exists m_{ji} \in M_j,\ y_{ji} = \gamma_{ji}(m_{ji}, \bar{m})\}.$$

We call $\gamma_j^{Y_j}$ the recommendation mechanism *constructed from* γ_j, if $Y_j = (\gamma_{j1}(M_j), \gamma_{j2}(M_j))$. Then we denote $\gamma_j^{Y_j} = c(\gamma_j)$. Since the image of $\gamma_{ji}^{Y_j}$ on Y is $Y_{ji} \cup \{\underline{y}\}$ if only agent i participates in $\gamma_j^{Y_j}$, this definition means $\gamma_{ji}^{Y_j}(M_j) = \gamma_{ji}(M_j) \cup \{\underline{y}\}$, i.e., unless he gets punished, both images coincide. Note that if γ_j is a recommendation mechanism, then the recommendation mechanism constructed from γ_j is γ_j itself, i.e., $\gamma_j = c(\gamma_j)$.

Next we consider *sincere message plans* of the agents. It is the other key of our main theorem. This asks the agents to report their true types and the same recommendation table to each recommendation mechanism in the case of multiple participants.

Definition 4. Let $\gamma_j^{Y_j} \in \Gamma_j^r$. A profile of message plan s is *sincere to* γ_j *of principal* j if for each i and all γ_{-j},

$$\begin{aligned} I_j(p) = I \Rightarrow \qquad & s_{ji}(\theta_i, \gamma, p) = (\theta_i, a_{jI}^i), \\ & \text{and } \forall\tilde{\theta},\ a_{jI}^1(\tilde{\theta}) = a_{jI}^2(\tilde{\theta}) \in Y^2. \end{aligned}$$

s is said to be sincere to principal j if s is sincere to all $\gamma_j \in \Gamma_j^r$ of principal j.

We state the main theorem of this paper. This theorem results in that we can restrict our attention to recommendation mechanisms with sincere message plans in searching for any equilibrium outcome.

Theorem 1. Let $(\gamma^*, \pi^*, \sigma^*, \beta^*)$ be an equilibrium of game G. Then there exists an equilibrium of game G, $(\bar{\gamma}, \bar{\pi}, \bar{\sigma}, \bar{\beta})$, such that

a. For each j, $\bar{\gamma}_j$ is a recommendation mechanism.

b. $\bar{\sigma}$ is sincere to both principals.

c. For each θ, equilibrium outcomes coincide with each other.

Proof. It is sufficient to show that there exists an equilibrium of game G, $(\bar{\gamma}_1, \gamma_2^*, \tilde{\pi}, \tilde{\sigma}, \tilde{\beta})$, where $\bar{\gamma}_1 \in \Gamma_1^r$ and conditions b,c are satisfied. The reason is as follows. Once $(\bar{\gamma}_1, \gamma_2^*, \tilde{\pi}, \tilde{\sigma}, \tilde{\beta})$ is shown to be an equilibrium, by applying the same methods to principal 2, it can be shown that there exists an equilibrium of game G, $(\bar{\gamma}_1, \bar{\gamma}_2, \bar{\pi}, \bar{\sigma}, \bar{\beta})$, where $\bar{\gamma}_2 \in \Gamma_2^r$ and conditions b,c are satisfied.

At first, we construct $\bar{\gamma}_1, \tilde{\pi}, \tilde{\sigma}, \tilde{\beta}$ so that $\bar{\gamma}_1 \in \Gamma_1^r$ and $\tilde{\sigma}$ to be sincere to principal 1. Secondly, we show the outcome following from $(\bar{\gamma}_1, \gamma_2^*, \tilde{\pi}, \tilde{\sigma}, \tilde{\beta})$ is equivalent to the equilibrium outcome following from $(\gamma^*, \pi^*, \sigma^*, \beta^*)$. Lastly, we confirm $(\bar{\gamma}_1, \gamma_2^*, \tilde{\pi}, \tilde{\sigma}, \tilde{\beta})$ is actually an equilibrium of game G, but we prove this part in appendix A.

(Step 1) Define $\bar{\gamma}_1$ as $\bar{\gamma}_1 = c(\gamma_1^*)$. Also define $\tilde{\pi}, \tilde{\sigma}, \tilde{\beta}$ as follows: for each i and γ,

$$
\begin{aligned}
&(\tilde{\pi}_i(\gamma), \tilde{\sigma}_i(\gamma), \tilde{\beta}_i(\gamma)) \\
&= \begin{cases} (\pi_i^*(\gamma), \sigma_i^*(\gamma), \beta_i^*(\gamma)) & \text{if} \quad \gamma_1 \notin \Gamma_1^r, \\ (\pi_i^*(\gamma), s_i^1(\gamma), \beta_i^*(\gamma) & \text{if} \quad \gamma_1 \in \Gamma_1^r \text{ and } \gamma_1 \neq \bar{\gamma}_1, \\ (\pi_i^*(\gamma_1^*, \gamma_2), s_i^1(\gamma_1^*, \gamma_2), \beta_i^*(\gamma_1^*, \gamma_2)) & \text{if} \quad \gamma_1 = \bar{\gamma}_1, \end{cases}
\end{aligned}
$$

where for each i and θ, γ, p,

$$
\begin{aligned}
s_{2i}^1(\theta_i, \gamma, p) &= \sigma_{2i}^*(\theta_i, \gamma, p) \\
s_{1i}^1(\theta_i, \gamma, p) &= \begin{cases} \gamma_{1i}(\sigma_{1i}^*(\theta_i, \gamma, p), \bar{m}) & \text{if} \quad I_1(p) = \{i\}, \\ (\theta_i, a_{1I}) & \text{if} \quad I_1(p) = I, \end{cases}
\end{aligned}
$$

where $a_{1I}(\tilde{\theta}) = \gamma_{1I}(\sigma_{1I}^*(\tilde{\theta}, \gamma, p))$ for each $\tilde{\theta} \in \Theta$.

Each agent i follows s_{1i}^1 to each recommendation mechanism offered by principal 1. It is sincere to principal 1; if $I_1(p) = I$, he reports his true type θ_i and the same recommendation table a_{1I}.

(Step 2) We denote the equilibrium outcome following from $(\theta, \gamma^*, \pi^*, \sigma^*)$ as $y_{JI}^*(\theta)$:

$$y_{JI}^*(\theta) = \gamma_{JI}^*(\sigma_{JI}^*(\theta, \gamma^*, \pi^*(\theta^* \gamma^*))).$$

The outcome following from $(\theta, \bar{\gamma}_1, \gamma_2^*, \tilde{\pi}, \tilde{\sigma})$ is

$$y_{JI}^*(\theta) = (\bar{\gamma}_{1I}(\tilde{\sigma}_{1I}(\theta, \gamma^*, \pi^*(\theta^* \gamma^*))), \gamma_{2I}^*(\tilde{\sigma}_{1I}(\theta, \gamma^*, \pi^*(\theta^* \gamma^*)))).$$

We show that $\bar{y}_{JI}(\theta) = y_{JI}^*(\theta)$ for each θ.

Firstly, as $\tilde{\pi}(\theta, \bar{\gamma}_1, \gamma_2^*) = \pi^*(\theta, \gamma^*)$ and $\tilde{\sigma}_{2I}(\theta, \bar{\gamma}_1, \gamma_2^*, p) = \sigma_{2I}^*(\theta, \gamma^*, p)$, we have

$$\begin{aligned}
\bar{y}_{2I}(\theta) &= \gamma_{2I}^*(\tilde{\sigma}_{2I}(\theta, \bar{\gamma}_1, \gamma_2^*, \tilde{\pi}(\theta, \bar{\gamma}_1, \gamma_2^*))) \\
&= \gamma_{2I}^*(\sigma_{2I}^*(\theta, \gamma^*, \pi^*(\theta, \gamma^*))) \\
&= y_{2I}^*(\theta).
\end{aligned}$$

Secondly, we confirm the following lemma.

Lemma 1. For each θ, γ, p, if $\gamma_1^{Y_1} = c(\gamma_1)$, then

$$\gamma_{1I}^{Y_1}(s_{1I}^1(\theta, \gamma, p)) = \gamma_{1I}(\sigma_{1I}^*(\theta, \gamma, p)).$$

The proof of this lemma is as follows. If $I_1(p) = \{i\}$, then $s_{1i}^1(\theta, \gamma, p) = \gamma_{1i}(\sigma_{1i}^*(\theta_i, \gamma, p), \bar{m}) \in \gamma_{ji}(M_j)$. Since $\gamma_j^{Y_j} = c(\gamma_j)$, we have $Y_{ji} = \gamma_{ji}(M_j)$, and therefore the principal gives $s_{1i}^1(\theta, \gamma, p)$ to agent i. If $I_1(p) = I$, then $s_{1I}^1(\theta, \gamma, p) = (\theta_i, (\gamma_{1I}(\sigma_{1I}^*(\tilde{\theta}, \gamma, p)))_{\tilde{\theta} \in \Theta})$. Since the recommendation tables coincide with each other, the principal follows the agents recommendation and assigns $\gamma_{1I}(\sigma_{1I}^*(\theta, \gamma, p))$ to the agents. (The end of the proof of lemma 1.)

Since $\bar{\gamma}_1 = c(\gamma_1^*)$, we have

$$\begin{aligned}
\bar{y}_{1I}(\theta) &= \bar{\gamma}_{1I}(\tilde{\sigma}_{1I}(\theta, \bar{\gamma}_1, \gamma_2^*, \tilde{\pi}(\theta, \bar{\gamma}_1, \gamma_2^*))) \\
&= \bar{\gamma}_{1I}(s_{1I}^1(\theta, \gamma^*, \pi^*(\theta, \gamma^*))) \\
&= \gamma_{1I}^*(\sigma_{1I}^*(\theta, \gamma^*, \pi^*(\theta, \gamma^*))) \\
&= y_{1I}^*(\theta).
\end{aligned}$$

(Step 3) Lastly, we have to check that $(\bar{\gamma}_1, \gamma_2^*, \tilde{\pi}, \tilde{\sigma}, \tilde{\beta})$ is actually an equilibrium of game G. Because it is clear that $\tilde{\beta}$ is consistent to $\tilde{\pi}$, we need only to check rationality of the strategy profile. See appendix A.

□

We define a restricted game G^r such that the strategy space of each principal j is restricted to Γ_j^r and the agents decide their responses only to $\gamma \in \Gamma^r$. As we assume, Γ is larger than Γ^r. This results in that the equilibrium $(\bar{\gamma}, \bar{\pi}, \bar{\sigma}, \bar{\beta})$ of game G also constitutes an equilibrium of the game where the strategy set of each principal j is restricted to Γ_j^r, because there are less alternatives of each principal to deviate from the equilibrium strategy. On the other hand, the restricted response rule to Γ^r, say $(\bar{\pi}, \bar{\sigma}, \bar{\beta}|\Gamma^r)$, is an element in $r(\Gamma^r)$.

Corollary 1. Any equilibrium outcome of game G is attained as the equilibrium outcome of the game G^r.

4.1 Example

We investigate the example with the use of recommenation mechanism without rigorous analysis. We focus on the equilibria satisfying the following conditions: (1)both agents participate in principal 1's mechanism, (2)they report a recommendation table which recommends the best alternative for principal 1 unless it hurts the agents. Moreover, each principal j is assumed to offer the recommendation mechanism with $Y_{ji} = \{o\}$ (such recommendation mechanism uniquely exists), i.e., she does nothing unless both agents participate in her contract.

Since each agent has no alternative if he is the only participant of each principal, we have only to consider their responses in the cases where both agents choose the same principal. The agents are supposed to report their true types and the same recommendation table. We denote the table for principal j as $a_{jI} = (x_{jI}, t_{jI})$. For simplicity, we assume $x_{j1}(\theta) = x_{j2}(\theta) = \theta_1 + \theta_2$(efficient production), and $t_{j1}(\theta) + t_{j2}(\theta) \leq 10$, for each $j$and θ.

Then both tables have to satisfy incentive compatibility conditions about the agents' types, and additionally, a_{1I} must satisfies the agents' participation constraint. Therefore, firstly, a_{2I} have to satisfies the following condition for each i and θ_i, θ_i':

$$\sum_{\theta_{-i}}(t_{2I}(\theta_i,\theta_{-i})-1)=\sum_{\theta_{-i}}(t_{2I}(\theta_i',\theta_{-i})-1),$$

and a_{1I} gives a solution to the following maximization problem:

$$\max_{\tilde{t}_{1I}}\sum_{\theta\in\Theta}\frac{1}{4}(10-(\tilde{t}_{11}(\theta)+\tilde{t}_{12}(\theta)))$$

$$\begin{aligned}\text{sub.to}\quad (PC)\quad & \sum_{\theta_{-i}}(\tilde{t}_{1i}(\theta)-1)=\max\left\{0,\sum_{\theta_{-i}}(t_{2i}(\theta)-1)\right\},\ \forall i,\forall\theta_i,\\ (IC)\quad & \sum_{\theta_{-i}}(\tilde{t}_{1i}(\theta)-1)\geq\sum_{\theta_{-i}}(\tilde{t}_{1i}(\theta_i',\theta_{-i})-1),\ \forall i,\forall\theta_i,\theta_i'.\end{aligned}$$

The condition (PC) is the participation conditions of each agent, and the condition (IC) is the incentive compatibility conditions of each agent with respect to his type. Note that (PC) depends on a_{2I}, which is not observable to principal 1, but observable to the agents.

By solving this problem, we have,

$$\begin{aligned}\frac{t_{11}(1,1)+t_{11}(1,2)}{2}-1&=\frac{t_{11}(1,1)+t_{11}(1,2)}{2}-1=\underline{u}_1,\\ \frac{t_{12}(1,1)+t_{12}(2,1)}{2}-1&=\frac{t_{12}(1,2)+t_{12}(2,2)}{2}-1=\underline{u}_2,\end{aligned}$$

where $\underline{u}_i=\max\{0,\sum_{\theta_{-i}}(t_{2i}(\theta)-1)\}$ is i's opportunity cost to participate in principal 1's contract. Condition (IC) implies that this value does not depend on θ_i. Then the expected utility of principal 1 is $8-(\underline{u}_1+\underline{u}_2)$.

There are many equilibrium outcome, but the following two extreme outcomes are interesting: (1)$\underline{u}_1=\underline{u}_2=0$(no rent outcome), and (2)$\underline{u}_1+\underline{u}_2=8$(full rent outcome). Especially, the former equilibrium outcome is the same as the equilibrium outcome with $M_1=\{1,2\}\times\{\text{normal},\text{deviating}\}$, and $M_2=\{1,2\}$ in section 3.2.

5 Discussion

In this section, we argue some remaining issues. Firstly, we consider some situations where recommendation mechanisms shrink to type revelation mechanisms or menu contracts.

Secondly, we try to weaken the assumptions. Especially, we show the assumption A2, which is the assumption about punishment allocation, can be substituted by some weaker conditions.

Lastly, we investigate mixed strategy equilibria, which is isolated through this paper. In the model with finite allocation space, we suggest that the same kind of result follows with some modification of the recommendation mechanism.

5.1 Restriction

We consider some situations where recommendation mechanisms shrink to type revelation mechanisms or menu contracts.

Firstly, when only agent i participates in a recommendation mechanism of principal j, this recommendation mechanism is the same as a menu contract. We, therefore, can restrict our attention to menu contracts in searching for any equilibrium outcome when there is only one agent in the game. It is the results of Peters (2001) and Martimort and Stole (2002), while they showed that with more complicated settings, especially mixed strategies.

Corollary 2. If there is only one agent in game G, any equilibrium outcome of the game is attained as an equilibrium outcome where each principal offers a menu contract.

Let $P_i = \{\{i\}, \emptyset\}$(exclusive dealing). In this setting, each principal has at most one participant at any situation. Then a recommendation mechanism is again the same as a menu contract.

Corollary 3. In exclusive dealing, any equilibrium outcome of game G is attained as an equilibrium outcome where each principal offers a menu contract.

Note that exclusive dealing in this paper is a little different from those in many studies analyzing exclusive dealing, because they assume each agent i can observe the offer from principal i, but cannot observe the offer from the other principal. As Gal-Or (1991) showed, in those cases, we can restrict strategy sets of principals to type revelation mechanisms in searching for any equilibrium outcome.

Next, we consider situations where standard revelation mechanisms hold in multi-principal models. If there is only one principal, the standard revelation principle holds, because asymmetric information between the principal and each agent is only the agent's type. This logic can be applied to multi-principal situations, for example situations where the message choice of the agents to each principal depends only on the contract offered by her.

We briefly illustrate the logic. Assume that agent i participates in principal 1's contract, and that he follows message plan σ_{1i} for principal 1, which does not depend on contracts offered by principal 2. In addition, assume information about participation $p = (p_1, p_2)$ is common knowledge. Then, the agent's message depends only on his type θ_i and γ_1. Since γ_1 is common knowledge between the principal and the agent, the only remaining asymmetric information is his type. Therefore, the principal have only to ask the agent to report his type, which is the same as a type revelation mechanism.

Though it is in general too restrictive to assume the conditions that p is common knowledge and σ_{1i} does not depend on γ_2, it may seem natural in some situations. For example, in single agent competing mechanism models, the agent has to choose one principal as his partner; principal j is able to know that $p_1 = \{j\}$ if the agent participates in her contract, and σ_{j1} may not depend on γ_{-j} because he actually has no contact with principal $-j$.

5.2 Punishment allocation

Existense of punishment allocation $\underline{y}$ may be too heavy to assume in some situations. In this subsection, we investigate possibility that it is substituted by more realistic or not so restrictive assumption. Because the main role of the punishment allocation is to prevent each agent from deviating from sincere responses, we can substitute it by another punishment allocation, say $\bar{y}$, if outcomes satisfying $y_{ji} = (\bar{y})$ for some j are never prefered to any equilibrium outcome by agent i.

First of all, we assume that the utility function of each agent is independent of allocation profiles for the other agent(no externality). More precisely,

A2'(a) For each i, θ, and y_{JI},

$$U_i(\theta, y_{JI}) = u_i(\theta, y_{Ji}).$$

In addition, let $Y = \prod_{h=1}^{n} X^h$ such that $X^h \subset \mathbb{R}(h = 1, \dots, n)$ is a closed interval. Then there exists $\min(Y) \in Y$. Assume that the utility function satisfies the following monotonicity condition: [10]

A2'(b) For any i, θ, y_{Ji} and y'_{Ji},

$$y_{Ji} \geq y'_{Ji} \Rightarrow u_i(\theta, y_{Ji}) \geq u_i(\theta, y'_{Ji}).$$

Under assumption A1, A2'(a), and A2'(b), we can show that any equilibrium outcome of game G is attained in the game where (1)each principal offers a recommendation mechanism with $\bar{y} = \min(Y)$ as the punishment allocation, and (2)each agent sincerely responds to each recommendation mechanism.

Because its proof is completely the same as the proof of theorem 1 (except that $\underline{y}$ is substituted by $\bar{y}$), we omit it.

[10] For any $y_{Ji} = (x^h_{Ji})^n_{h=1}$ and $y'_{Ji} = (x^{h\prime}_{Ji})^n_{h=1}$, we denote $y_{Ji} \geq y'_{Ji}$ if $x^h_{ji} \geq x^{h\prime}_{ji}$ for each $j \in J$ and $h = 1, \dots, n$.

5.3 Mixed strategy equilibria

We have not considered mixed strategy equilibria to make our analysis simple, especially to avoid discussing measure theoretical issues. However, even if we consider mixed strategy equilibria, main results of this paper may remain to hold with some modification on the definition of recommendation mechanisms. In this subsection, we briefly discuss mixed strategy equilibrium outcomes under the assumption that Y, M_1, M_2 are finite sets. [11] Note that an outcome for each $\theta \in \Theta$ is defined as one of probability distributions on Y^4.

Without any modification, recommendation mechanism defined above cannot realize some equilibrium outcome. We demonstrate it by the following example.

Example 2 Let $P_1 = P_2 = \{\{1\}, \{2\}, \emptyset\}$, $M_1 = M_2 = \{1, 2\}$, $Y = \{0, 1\}$ and $U_1 \equiv U_2 \equiv V_1 \equiv V_2 \equiv 0$. We define $\hat{\gamma} = (\hat{\gamma}_i)_{i \in I}$ and $\tilde{\gamma}(\tilde{\gamma}_i)_{i \in I}$ as follows:

$$\begin{aligned}
\forall m_1, m_2, \quad & \hat{\gamma}_1(m_1, \bar{m}) = \hat{\gamma}_1(\bar{m}, m_2) = \tilde{\gamma}_1(m_1, \bar{m}) = \tilde{\gamma}_2(\bar{m}, m_2) = 1, \\
\forall i, \forall m \neq m', \quad & \hat{\gamma}_i(m, m) = \tilde{\gamma}_i(m, m') = 1, \\
\forall i, \forall m \neq m', \quad & \hat{\gamma}_i(m, m') = \tilde{\gamma}_i(m, m) = 0.
\end{aligned}$$

Now consider the following equilibrium: principal 1 offers $\hat{\gamma}$ with probability 1; principal 2 offers $\hat{\gamma}$ with probability 0.4 and $\tilde{\gamma}$ with probability 0.6; both agents participate in principal 1's contract and report a message profile $(1, 1)$ if they are offered $\hat{\gamma}$ by principal 2, and both participate in none of the contracts if they are offered $\tilde{\gamma}$ by principal 2. Note that the contract offered by principal 2 allows the agents to correlate their mixing. The equilibrium outcome for each θ, $\mu(y|\theta) \in \Delta(Y^4)$, is as follows:

$$\begin{aligned}
\mu(y|\theta) &= 0.4 \quad \text{if } y = (y_{11}, y_{12}, y_{21}, y_{22}) = (1, 1, 0, 0), \\
\mu(y|\theta) &= 0.6 \quad \text{if } y = (0, 0, 0, 0), \\
\mu(y|\theta) &= 0 \quad \text{otherwise}.
\end{aligned}$$

[11] Because game G becomes a finite game, there exists at least one PBE.

This equilibrium outcome cannot be attained in any equilibrium where each principal mixes the recommendation mechanism constructed from his equilibrium contracts, because $\hat{\gamma}$ and $\tilde{\gamma}$ construct the same recommendation mechanism. More precisely, as principal 2 offers $c(\hat{\gamma})(=c(\tilde{\gamma}))$ with probability 1, the agents cannot correlate their decision. Therefore, if each agent independently mixes his participation decision as $p_i = \{1\}$ with $\Pr = 0.4$ and $p_i = \emptyset$ with $\Pr = 0.6$, the outcome is as follows:

$$\begin{aligned}
\mu(y|\theta) &= 0.16 \quad \text{if } y = (0,0,0,0),\\
\mu(y|\theta) &= 0.24 \quad \text{if } y = (1,0,0,0),(0,1,0,0),\\
\mu(y|\theta) &= 0.36 \quad \text{if } y = (1,1,0,0),\\
\mu(y|\theta) &= 0 \quad \text{otherwise.}
\end{aligned}$$

We define *recommendation mechanism with public signal* as a pair of a recommendation mechanism and public signal.

Definition 5. (Y_j, k) such that $Y_j \subset Y^2$, and $k \in \mathbb{N}$ is principal j's *recommendation mechanism with public signal* if,

1. $\gamma_j^{Y_j} \in \Gamma_j^r$,
2. $k \leq \sharp\{\gamma_j | c(\gamma_j) = \gamma_j^{Y_j}\}$.

The public signal asks the agents to interpret $\gamma_j^{Y_j}$ as one of the contract γ_j such that $c(\gamma_j) = \gamma_j^{Y_j}$ and to respond to it as if γ_j was offered. Due to the public signal, the agents can correlate their actions. For instance, consider example 2 again. As the mixed strategy of principal 2, we think of the following:

$$\delta_2(Y_2, k) = \begin{cases} 0.4 & \text{if} \quad Y_2 = (1,1) \quad \text{and} \quad k = 1,\\ 0.6 & \text{if} \quad Y_2 = (1,1) \quad \text{and} \quad k = 2,\\ 0 & \text{otherwise.}\end{cases}$$

Both agents participate in principal 1's contract if they observe $k = 1$ and participate in none of the contract if $k = 2$. Then the original equilibrium outcome is attained.

Though our paper focused on the pure equilibria of the game with large allocation space, it may be possible to show that any mixed strategy equilibrium outcome is attained as an mixed strategy equilibrium outcome where (1)each principal mixes some recommendation mechanisms with public signal and (2)the agents use sincere(though we do not define exactly) response to each of them.

6 Conclusion

Through the paper, we investigated mechanism design with two principals and two agents. The main purpose of this paper is to show that any pure strategy equilibrium outcomes are attainable in an equilibrium where (1)each principal offers a recommendation mechanism and (2)the agents sincerely respond to each recommendation mechanism.

One of characteristics of recommendation mechanism is similarity with type revelation mechanism and menu contract. If there is only one agent participating in the recommendation mechanisms, it is equivalent to a menu contract. If multiple agents participate in the recommendation mechanism, each agent is asked to report his true type and the same recommendation table.

We also investigated possibility to find some situations where recommendation mechanisms shrink to type revelation mechanisms or menu contracts.

In addition, we suggested the possibility to weaken the assumptions and to enlarge our result to mixed strategy equilibrium outcomes, though we only discussed it with finite allocation space.

There are many remaining tasks on mechanism design with multiple principals and multiple agents. First one is to investigate forms of optimal contracts. This may be far more complex than that of single principal case, not only because of interaction of principals, but also because complexity of the agents' decision, especially their participation decision. Secondly, we should analyze actual economic situations. In that, similarity of recommendation mechanism to type revelation mechanism and menu contract is helpful for us.

References

Calzolari, G., and A. Pavan (2000): "Sequential Contracts and Information Transmission Between Principals," *Econometric Society World Congress 2000 Contributed Papers*, (1606).

Epstein, L., and M. Peters (1999): "A Revelation Principle For Competing Mechanisms," *Journal of Economic Theory*, 88, 119–160.

Gal-Or, E. (1991): "A common agency with incomplete information," *Rand Journal of Economics*, 22(2), 274–286.

Han, S. (2004): "Menu Theorems for Bilateral Contracting," mimeo.

Martimort, D., and L. Stole (2002): "The Revelation and Delegation Principles in Common Agency Games," *Econometrica*, 70, 1659–1673.

McAfee, R. P. (1993): "Mechanism Design by Competing Sellers," *Econometrica*, 61, 1281–1312.

Peck, J. (1997): "A Note on Competing Mechanisms and the Revelation Principle," mimeo, Ohio State University.

Peters, M. (2001): "Common Agency and the Revelation Principle," *Econometrica*, 69, 1349–1372.

Stole, L. (1995): "Nonlinear Pricing and Oligopoly," *Journal of Economics and Management Strategy*, 4, 529–562.

A The proof of (Step 3)

Firstly, we check rationality of the starategy profile of the agents. If $\gamma_1 \notin \Gamma_1^r$, then $(\tilde{\pi}(\gamma), \tilde{\sigma}(\gamma), \tilde{\beta}(\gamma)) = (\pi^*(\gamma), \sigma^*(\gamma), \beta^*(\gamma))$ for each γ_2. Because $(\pi^*(\gamma), \sigma^*(\gamma), \beta^*(\gamma))$ is rational to γ, $(\tilde{\pi}(\gamma), \tilde{\sigma}(\gamma), \tilde{\beta}(\gamma))$ is also rational to γ.

If $\gamma_1 \in \Gamma_1^r$, we have the following lemma.

Lemma 2. For each γ, if $\gamma_1^{Y_1} = c(\gamma_1)$, then $(\pi^*(\gamma), s^1(\gamma), \beta^*(\gamma))$ is rational to $(\gamma_1^{Y_1}, \gamma_2)$. That is, each agent i has no incentive to deviate from $(\pi_i^*(\gamma), s_i^1(\gamma))$ if he has belief system $\beta_1^*(\gamma)$, given agent $-i$ follows $(\pi_{-i}^*(\gamma), s_{-i}^1(\gamma))$.

Recall that $c(\gamma_1) = \gamma_1$ if $\gamma_1 \in \Gamma_1^r$ and that $c(\gamma_1^*) = \bar{\gamma}_1$. Therefore, it follows from this lemma that $(\pi^*(\gamma), s^1(\gamma), \beta^*(\gamma))$ is rational to γ if $\gamma_1 \in \Gamma_1^r$, and that $(\pi^*(\gamma_1^*, \gamma_2), s^1(\gamma_1^*, \gamma_2), \beta^*(\gamma_1^*, \gamma_2))$ is rational to $(\bar{\gamma}_1, \gamma_2)$, and thus, the response rule of the agents is rational.

The proof of this lemma is as follows, but we investigate only the incentive of agent 1 without loss of generality.

Suppose contrarily that there exists another response of agent 1, say $(\pi_1'(\gamma), \sigma_1'(\gamma))$, strictly prefered to $(\pi_1^*(\gamma), s_1^1(\gamma))$ by the agent. A necessarily condition of this supposition is that there exists θ_1, p such that agent 1 strictly prefers $\sigma_{J1}'(\theta_1, \gamma, p)$ to $s_{J1}^1(\theta_1, \gamma, p)$. Otherwise, his expected utility never decreases if he changes his strategy from $(\pi_1'(\gamma), \sigma_1'(\gamma))$ to $(\pi_1'(\gamma), s_1^1(\gamma))$. However, since $\pi_1^*(\theta_1, \gamma)$ is his best response for each θ_1, γ given that agent 2 follows $\pi_2^*(\gamma)$ and that both agents follow $s^1(\gamma)$ afterward, agent 1's expected utility never decreases if he change his strategy from $(\pi_1'(\gamma), s_1^1(\gamma))$ to $(\pi_1^*(\gamma), s_1^1(\gamma))$. It contradicts that he strictly prefers $(\pi_1'(\gamma), \sigma_1'(\gamma))$ to $(\pi_1^*(\gamma), s_1^1(\gamma))$.

Given θ_1, p, when he follows $s_{J1}^1(\theta_1, \gamma, p)$, by lemma 1, his expected utility is as follows.

$$\begin{aligned}
& \sum_{\theta_2} \beta_1^*(\theta_2|\theta_1,\gamma,p)U_1(\theta,\gamma_{1I}^{Y_1}(s_{1I}^1(\theta,\gamma,p)),\gamma_{2I}(s_{2I}^1(\theta,\gamma,p)) \\
= & \sum_{\theta_2} \beta_1^*(\theta_2|\theta_1,\gamma,p)U_1(\theta,\gamma_{1I}(\sigma_{1I}^*(\theta,\gamma,p)),\gamma_{2I}(\sigma_{2I}^*(\theta,\gamma,p)).
\end{aligned}$$

When he follows $\sigma'_{J1}(\theta_1,\gamma,p)$, his expected utility becomes as follows, in each case where p satisfies (1)$1 \notin p_1$, (2)$I_1(p) = \{1\}$, or (3)$I_1(p) = I$.

(1)If $1 \notin p_1$, then $\sigma'_{11}(\theta_1,\gamma,p) = \bar{m} = s_{11}^1(\theta_1,\gamma,p)$. Therefore, his expected utility is,

$$\begin{aligned}
& \sum_{\theta_2} \beta_1^*(\theta_2|\theta_1,\gamma,p)U_1(\theta,\gamma_{1I}^{Y_1}(s_{1I}^1(\theta,\gamma,p)),\gamma_{2I}(\sigma'_{21}(\theta_1,\gamma,p),\sigma_{22}^*(\theta_2,\gamma,p)) \\
= & \sum_{\theta_2} \beta_1^*(\theta_2|\theta_1,\gamma,p)U_1(\theta,\gamma_{1I}(\sigma_{1I}^*(\theta,\gamma,p)),\gamma_{2I}(\sigma'_{21}(\theta_1,\gamma,p),\sigma_{22}^*(\theta_2,\gamma,p)) \\
\leq & \sum_{\theta_2} \beta_1^*(\theta_2|\theta_1,\gamma,p)U_1(\theta,\gamma_{1I}(\sigma_{1I}^*(\theta,\gamma,p)),\gamma_{2I}(\sigma_{2I}^*(\theta,\gamma,p)).
\end{aligned}$$

(2)If $I_1(p) = \{1\}$, then $s_{12}^1(\theta_2,\gamma,p) = \bar{m} = \sigma_{12}^*(\theta_2,\gamma,p)$ for each θ_2. Since he never likes to be punished, $\sigma'_{11}(\theta_1,\gamma,p)$ must be an element in Y_{11}. Since $\gamma_1^{Y_1} = c(\gamma_1)$, we have $Y_{11} = \gamma_{11}(M_1)$. That is, there exists $m'_{11} \in M_1$ such that $\gamma_{11}^{Y_1}(\sigma'_{11}(\theta_1,\gamma,p),\bar{m}) = \gamma_{11}(m'_{11},\bar{m})$, which equals to $\gamma_{11}(m'_{11},\sigma_{12}^*(\theta_2,\gamma,p))$. Therefore, agent 1's expected utility is,

$$\begin{aligned}
& \sum_{\theta_2} \beta_1^*(\theta_2|\theta_1,\gamma,p)U_1(\theta,\gamma_{1I}(m'_{11},\sigma_{12}^*(\theta_2,\gamma,p)),\gamma_{2I}(\sigma'_{21}(\theta_1,\gamma,p),\sigma_{22}^*(\theta_2,\gamma,p)) \\
& \leq \sum_{\theta_2} \beta_1^*(\theta_2|\theta_1,\gamma,p)U_1(\theta,\gamma_{1I}(\sigma_{1I}^*(\theta,\gamma,p)),\gamma_{2I}(\sigma_{2I}^*(\theta,\gamma,p)).
\end{aligned}$$

(3)If $I_1(p) = I$, then $s_{12}^1(\theta_2,\gamma,p) = (\theta_2,a_{1I})$ for each θ_2, where $a_{1I}(\theta) = \gamma_{1I}(\sigma_{1I}^*(\theta,\gamma,p))$. Since agent 1 never wants to be punished, $\sigma'_{11}(\theta_1,\gamma,p)$ must equal to (θ'_1,a_{1I}) for some $\theta'_1 \in \Theta_1$. Then $\gamma_{1I}^{Y_1}(\sigma'_{11}(\theta_1,\gamma,p),s_{12}^1(\theta_2,\gamma,p)) = a_{1I}(\theta'_1,\theta_2)$, which equals to $\gamma_{1I}(\sigma_{11}^*(\theta'_1,\gamma,p),\sigma_{12}^*(\theta_2,\gamma,p))$. Therefore, agent

1's expected utility is,

$$\sum_{\theta_2} \beta_1^*(\theta_2|\theta_1,\gamma,p)U_1(\theta,\gamma_{1I}(\sigma_{11}^*(\theta_1',\gamma,p),\sigma_{12}^*(\theta_2,\gamma,p)),\gamma_{2I}(\sigma_{21}'(\theta_1,\gamma,p),\sigma_{22}^*(\theta_2,\gamma,p))$$
$$\leq \sum_{\theta_2} \beta_1^*(\theta_2|\theta_1,\gamma,p)U_1(\theta,\gamma_{1I}(\sigma_{1I}^*(\theta,\gamma,p)),\gamma_{2I}(\sigma_{2I}^*(\theta,\gamma,p)).$$

Thus, agent 1 never strictly prefers $\sigma_{J1}'(\theta_1,\gamma,p)$ to $s_{J1}^1(\theta_1,\gamma,p)$ for any θ_1,p. It is a contradicton.(The end of the proof of lemma 2.)

Lastly, we check rationality of the strategy profile of the principals. Given principal 2 offers γ_2^* and the agents follow $(\tilde{\pi},\tilde{\sigma})$, principal 1's expected utility if she offers $\bar{\gamma}_1$ is as follows.

$$\begin{aligned}
& \sum_{\theta} f(\theta)V_1(\theta,\bar{\gamma}_{1I}(\tilde{\sigma}_{2I}(\theta,\bar{\gamma}_1,\gamma_2^*,\tilde{\pi}(\theta,\bar{\gamma}_1,\gamma_2^*))),\gamma_{2I}^*(\tilde{\sigma}_{2I}(\theta,\bar{\gamma}_1,\gamma_2^*,\tilde{\pi}(\theta,\bar{\gamma}_1,\gamma_2^*))))\\
= & \sum_{\theta} f(\theta)V_1(\theta,\bar{\gamma}_{1I}(s_{1I}^1(\theta,\gamma^*,\pi^*(\theta,\gamma^*))),\gamma_{2I}^*(s_{2I}^1(\theta,\gamma^*,\pi^*(\theta,\gamma^*))))\\
= & \sum_{\theta} f(\theta)V_1(\theta,\gamma_{JI}^*(\sigma_{JI}^*(\theta,\gamma^*,\pi^*(\theta,\gamma^*))))\\
\geq & \sum_{\theta} f(\theta)V_1(\theta,\gamma_{1I}(\sigma_{1I}^*(\theta,\gamma_1,\gamma_2^*,\pi^*(\theta,\gamma_1,\gamma_2^*))),\gamma_{2I}^*(\sigma_{2I}^*(\theta,\gamma_1,\gamma_2^*,\pi^*(\theta,\gamma_1,\gamma_2^*))))\\
= & \sum_{\theta} f(\theta)V_1(\theta,\gamma_{1I}(\tilde{\sigma}_{2I}(\theta,\gamma_1,\gamma_2^*,\tilde{\pi}(\theta,\gamma_1,\gamma_2^*))),\gamma_{2I}^*(\tilde{\sigma}_{2I}(\theta,\gamma_1,\gamma_2^*,\tilde{\pi}(\theta,\gamma_1,\gamma_2^*)))),
\end{aligned}$$

for any $\gamma_1 \in \Gamma_1$.

The first and the last equality follow from lemma 1. Therefore, she achieves her highest expected utility by offering $\bar{\gamma}_1$.

Given principal 1 offers $\bar{\gamma}_1$ and the agents follow $(\tilde{\pi},\tilde{\sigma})$, principal 2 achieves her highest expected utility by offering γ_2^* by the same logic. Her expected utility if she offers γ_2^* is as follows.

$$
\begin{aligned}
& \sum_\theta f(\theta)V_2(\theta, \bar{\gamma}_{1I}(\tilde{\sigma}_{2I}(\theta, \bar{\gamma}_1, \gamma_2^*, \tilde{\pi}(\theta, \bar{\gamma}_1, \gamma_2^*))), \gamma_{2I}^*(\tilde{\sigma}_{2I}(\theta, \bar{\gamma}_1, \gamma_2^*, \tilde{\pi}(\theta, \bar{\gamma}_1, \gamma_2^*))))\\
= & \sum_\theta f(\theta)V_2(\theta, \gamma_{JI}^*(\sigma_{JI}^*(\theta, \gamma^*, \pi^*(\theta, \gamma^*))))\\
\geq & \sum_\theta f(\theta)V_2(\theta, \gamma_{1I}^*(\sigma_{1I}^*(\theta, \gamma_1^*, \gamma_2, \pi^*(\theta, \gamma_1^*, \gamma_2))), \gamma_{2I}(\sigma_{2I}^*(\theta, \gamma_1^*, \gamma_2, \pi^*(\theta, \gamma_1^*, \gamma_2))))\\
= & \sum_\theta f(\theta)V_1(\theta, \bar{\gamma}_{1I}(\tilde{\sigma}_{2I}(\theta, \gamma_1^*, \gamma_2, \tilde{\pi}(\theta, \gamma_1^*, \gamma_2))), \gamma_{2I}^*(\tilde{\sigma}_{2I}(\theta, \gamma_1^*, \gamma_2, \tilde{\pi}(\theta, \gamma_1^*, \gamma_2)))),
\end{aligned}
$$

for any $\gamma_2 \in \Gamma_2$.(The end of (Step 3) of the proof of theorem 1.)