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Moral Decision and Information Aversion

Hitoshi Matsushima
University of Tokyo

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Abstract

This paper investigates an individual who has the rule-based and consequence-based moral tastes and has time-inconsistent intertemporal preferences caused by immediate gratification. The individual decides whether to undertake the activity that maximizes her own self-interest, but is uncertain whether it harms others. The individual at the earlier stage decides whether to access the information channel as the commitment devise for avoidance of preference reversals between the earlier-stage and later-stage selves, but is uncertain about whether this access is beneficial. The decision whether to access the information channel greatly influences the probability that social harm occurs.

The relation between moral taste and pattern of information acquisition is clarified in the following way. The rule-based individual follows the morally bad pattern of information acquisition, while the consequence-based individual follows the morally good pattern. The individual with moral constraint follows the morally bad pattern if the moral constraint improves the earlier-stage self's morality, while she follows the morally good pattern if the moral constraint only serves to avoid preference reversals. It is shown that even if the access is beneficial, the individual is likely to misperceive it as being against her own interest and be averse to costless information.

Keywords: Immediate Gratification, Information Aversion, Learning, Moral Decision, Moral Rule.

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+ Faculty of Economics, University of Tokyo, Hongo, Bunkyo-ku, Tokyo 113, Japan. Fax: (03)3818-7082. e-mail: hitoshi@e.u-tokyo.ac.jp

1. Introduction

A human being has tastes for not harming others. A human being has not only the purely self-interested motive but also other motives such as morality, and sometimes refrains from pursuing her (or his) own self-interest.[1] This paper investigates the decision problem in which an individual who has moral tastes decides whether to undertake the activity that maximizes her own self-interest but may harm others.

The individual has two different kinds of moral tastes, i.e., the *rule-based moral taste* and the *consequence-based moral taste*. The rule-based moral taste is the taste for not violating the moral rule that the individual should not undertake the activity that causes social harm. The consequence-based moral taste is the taste for not experiencing the consequence of social harm. The individual obtains the *immediate* moral disutility by violating the moral rule at the same time that she decides to undertake the activity, while she obtains the *delayed* moral disutility by experiencing the consequence of social harm later on.

The individual is uncertain whether the activity actually harms others. When the individual expects that the activity is much likely to cause social harm, she refrains from undertaking it. Before deciding whether to undertake the activity, the individual has the opportunity at the earlier stage to access the information channel. The individual is, however, uncertain what kind of information channel she can access.

The access to the information channel greatly influences the probability that the individual decides to undertake the activity and social harm actually occurs. The access may inform the individual that the activity is much likely to cause social harm, and therefore, persuade her not to undertake the activity. The access may also inform the individual that the activity is not likely to cause social harm, and therefore, encourage her to undertake this harmful activity.

As is well known in non-expected utility theory, the individual does not necessarily have incentive to access the information channel even if it is costless.[2] How often and for what purpose the individual is willing to access the information channel crucially depends on which of two moral tastes, i.e., the rule-based and the consequence-based

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[1] It is the common assumption in philosophy and psychology that a human being has multiple motives other than the pure self-interest. Some economists have incorporated altruism and fairness into the framework of utility-maximization. For the literature on social preference in economics, see the survey on economics and psychology by Rabin (1998).

[2] See the survey on non-expected utility theory by Machina (1989).

moral tastes, motivates the individual in decisive ways. This paper shows that the rule-based individual accesses the information channel only to search for the possibility that the individual is morally admitted to undertake the activity, while the consequence-based individual accesses the information channel only to search for the possibility that the individual is not morally admitted to undertake the activity. Hence, we can say that *the rule-based individual follows the morally bad pattern of information acquisition, while the consequence-based individual follows the morally good pattern of information acquisition.*

Most parts of this paper assume that the individual has preference for *immediate gratification*. The individual has *hyperbolic discounting* intertemporal preferences which always overweigh the current payoff relative to the future payoffs, and therefore, suffers from the well-known *time-inconsistency* problem explored by Strotz (1955), Phelps and Pollak (1968), and Laibson (1997).[3] There are serious preference reversals between the earlier-stage and later-stage selves on whether to undertake the activity, because the later-stage self gives the less weight to the delayed payoff than the immediate payoff but the earlier-stage self gives the same weight to both.[4] The decision at the earlier stage whether to access the information channel serves as the commitment devise to avoid preference reversals.

The rule-based moral taste sometimes serves as the *moral constraint*. That is, there sometimes exists a threshold such that the moral rule strictly prohibits the individual from undertaking the activity irrespective of material benefit if she expects that the probability that the activity causes social harm is more than or equal to this threshold. It is the common theme in psychology that many of observed moral dispositions are rather ingrained as a set of such internal constraints. As is shown in the pioneering work by Rabin (1995), however, the individual with moral constraints who has *no* preference for immediate gratification accesses the information channel only to relax these constraints, i.e., follows the morally bad pattern of information acquisition.

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[3] See also Ainslie (1991), Loewenstein and Elster (1992), Carrillo and Mariotti (1997), O'Donoghue and Rabin (1999), and Benabou and Tirole (1999a, 1999b). There are many psychological researches which suggest that exponential discounting intertemporal preferences which economists typically assume is wrong. See the survey by Rabin (1998).

[4] In expected and non-expected utility theory, a human being is assumed to be a coherent entity. This paper is more close to the philosophical idea of *multiple-self* that the individual person is seen as a set of relatively autonomous selves and the process of decision making is regarded as the process of self-identification. See Elster (1985).

This paper clarifies the role of moral constraint when the individual does have preference for immediate gratification. We show that *the individual follows the morally bad pattern of information acquisition if the moral constraint is restrictive enough to improve the earlier-stage self's morality, whereas she follows the morally good pattern of information acquisition if the moral constraint only serves to avoid preference reversals caused by immediate gratification.*

Many propositions in this paper clarify whether and when there exist subjective beliefs on information channel which make the individual averse to costless information. On the similar ground to the well-known two-arm bandit problem in probability theory, we show that, in a repeated situation, whenever such subjective beliefs exist then the individual is very likely to stick to one of these beliefs in the long run and be averse to costless information forever. We show that *even if the access to the information channel is actually beneficial, the individual misperceives it as being against her own interest and the earlier-stage self loses incentive to acquire information in the long run.*

The individual is averse to costless information only if the intertemporal preferences are either time-inconsistent or is not reduced to the utilities of the consequences, i.e., do not satisfy consequentialism. Immediate gratification and the moral constraint cause time inconsistency and anti-consequentialism, respectively. We present also another reason why the intertemporal preferences are time-inconsistent, that is, the *moral rule on information acquisition.* The moral rule on information acquisition implies that the earlier-stage self should not access the information channel if this access induces the later-stage self to increase the probability that social harm occurs. We show that *the individual who has the moral taste for not violating the moral rule on information acquisition has strong tendency to prefer less information to more.*

The organization of this paper is as follows. Section 2 defines the decision problem. Section 3 defines consequentialism and time consistency. Section 4 introduces immediate gratification and the rule-based and consequence-based moral tastes. Section 5 investigates the pattern of action choice, and shows that the rule-based moral taste has some advantage over the consequence-based moral taste. Section 6 is the main section, which investigates the pattern of information acquisition and gives most of the main results. Section 7 introduces the moral rule on information acquisition. Finally, Section 8 shows that in the long run of a repeated situation the individual learns to stop acquiring information forever.

2. Decision Problem

We consider a decision problem which consists of stages 0, 1 and 2. Let Ω denote the finite set of *states*. The set of probability functions on Ω with full support is denoted by P. We denote by $p^* \in P$ the *prior* according to which a state $\omega \in \Omega$ is randomly determined. The individual knows p^* but can not observe the state until stage 2.

We denote by Λ the set of simple lotteries λ over P with countable support such that

$$\sum_{p \in P(\lambda)} \lambda(p)p(\omega) = p^*(\omega) \text{ for all } \omega \in \Omega,$$

where $P(\lambda) \equiv \{p \in P: \lambda(p) > 0\}$ is the support of λ. An element of P is sometimes called a *signal*. The *information channel* is denoted by $\lambda^* \in \Gamma$. At stage 0, the individual decides whether to access the information channel λ^*, i.e., choose $c = 1$, or not, i.e., choose $c = 0$. By choosing $c = 1$, the individual observes a signal $p \in P(\lambda^*)$ with probability $\lambda^*(p)$. By choosing $c = 0$, the individual always observes the signal $p = p^*$, i.e., can not observe any informative signal. When observing a signal p, the individual believes that the probability of state ω is $p(\omega)$ for each $\omega \in \Omega$.

The individual does *not* know λ^*, and therefore, estimates a *subjective belief* $\lambda \in \Lambda$ at the beginning of stage 0. At stage 1, the individual chooses an action a among the finite set of actions A. At stage 2, the individual observes the state.

A *strategy* is defined as a pair of functions (α, β), where $\alpha: \Lambda \to \{0,1\}$ is called a *pattern of information acquisition* and $\beta: P \to A$ is called a *pattern of action choice.* Given subjective belief λ, the individual chooses $c = \alpha(\lambda) \in \{0,1\}$ at stage 0, chooses $a = \beta(p^*) \in A$ at stage 1 when choosing $c = 0$ at stage 0, and chooses $a = \beta(p) \in A$ at stage 1 when choosing $c = 1$ and observing signal $p \in P$ at stage 0.

The utility evaluated at stage 0 when the individual forms a subjective belief λ and behaves according to a strategy (α, β) is given by

$$\sum_{p \in P(\lambda)} \lambda(p) u_0(\beta(p), p) \quad \textit{if } \alpha(\lambda) = 1,$$

and

$$u_0(\beta(p^*), p^*) \qquad \textit{if } \alpha(\lambda) = 0,$$

where $u_0: A \times P \to R$. The utility evaluated at stage 0 when the subjective belief λ assigns probability one to signal p^* is equal to $u_0(\beta(p^*), p^*)$, i.e., the utility evaluated at stage 0 when she does not access the information channel. This implies that *there is no fixed physical and cognitive cost of accessing the information channel.*

The utility evaluated at stage 1 when the individual observes signal $p \in P$ and chooses action $a \in A$ is given by

$$u_1(a, p),$$

where $u_1 : A \times P \to R$ and u_1 is not necessarily equivalent to u_0.

We sometimes call the incarnation at stage 0 the *stage-0 self* and the incarnation at stage 1 the *stage-1 self*. The stage-0 self maximizes the utility evaluated at stage 0, while the stage-1 self maximizes the utility evaluated at stage 1. Hence, the individual behaves according to the strategy (α^*, β^*) which is defined by

$$\alpha^*(\lambda) = 1 \text{ if and only if } \sum_{p \in P(\lambda)} \lambda(p) u_0(\beta^*(p), p) \geq u_0(\beta^*(p^*), p^*),$$

and for every $p \in P$,

$$u_1(\beta^*(p), p) \geq u_1(a, p) \text{ for all } a \in A.$$

3. Consequentialism and Time Consistency

Let $\Gamma(A)$ denote the set of simple lotteries over A. Let B denote the set of all functions from Ω to $\Gamma(A)$. We introduce the following two conditions on (u_0, u_1).

Consequentialism: There exists a function $U: B \rightarrow R$ such that for every $\lambda \in \Lambda$, every strategy (α, β), and every $b \in B$, if

$$\sum_{p \in P(\lambda): \beta(p)=a} \lambda(p) p(\omega) = b(\omega)(a) \text{ for all } \omega \in \Omega \text{ and all } a \in A,$$

then

$$\sum_{p \in P(\lambda)} \lambda(p) u_0(\beta(p), p) = U(b).$$

Time Consistency: For every $p \in P$,

$$u_0(\beta^*(p), p) \geq u_0(a, p) \text{ for all } a \in A.$$

Consequentialism implies that the utility evaluated at stage 0 is reduced to the utilities on the consequences. Time consistency implies that the pattern of action choice maximizes not only the utility evaluated at stage 1 but also the utility evaluated at stage 0.

We can check that *if the individual satisfies consequentialism and time consistency, then the stage-0 self never prefers less information to more, i.e.,*

$$\alpha^*(\lambda) = 1 \text{ for all } \lambda \in \Lambda.$$

Indeed, time consistency and consequentialism imply that for every $\lambda \in \Lambda$,

$$\sum_{p \in P(\lambda)} \lambda(p) u_0(\beta^*(p), p) \geq \sum_{p \in P(\lambda)} \lambda(p) u_0(\beta^*(p^*), p) = u_0(\beta(p^*), p^*),$$

and therefore, $\alpha^*(\lambda) = 1$.

The following proposition characterizes cases in which there exists a subjective belief $\hat{\lambda} \in \Lambda$ such that $\alpha^*(\hat{\lambda}) = 0$, i.e., with which the stage-0 self prefers less information to more.

Proposition 1: *There exists* $\hat{\lambda} \in \Lambda$ *such that* $\alpha^*(\hat{\lambda}) = 0$, *if, and only if, there exist* $(p, p') \in P^2$ *and* $e \in (0,1)$ *such that* $p^* = ep + (1-e)p'$ *and*

$$u_0(\beta^*(p^*), p^*) > e u_0(\beta^*(p), p) + (1-e) u_0(\beta^*(p'), p'). \qquad (1)$$

Proof: Let $\hat{\lambda} \in \Lambda$ be defined by

$$P(\hat{\lambda}) = \{p, p'\}, \ \hat{\lambda}(p) = e, \text{ and } \lambda(p') = 1 - e.$$

Inequality (1) implies $\alpha^*(\hat{\lambda}) = 0$.

Suppose that $\hat{\lambda} \in \Lambda$ satisfies $\alpha^*(\hat{\lambda}) = 0$, i.e.,

$$\sum_{p'' \in P(\hat{\lambda})} \hat{\lambda}(p'') u_0(\beta^*(p''), p'') < u_0(\beta^*(p^*), p^*).$$

This inequality says that there exist $(p, p') \in P(\lambda)^2$ and $e \in (0,1)$ which satisfy $p^* = ep + (1-e)p'$ and inequality (1).

Q.E.D.

We define

$$u_0^m(p) \equiv \max_{a \in A} u_0(a, p).$$

When the individual satisfies time consistency, inequality (1) is replaced by

$$u_0^m(p^*) > eu_0^m(p) + (1-e)u_0^m(p'). \tag{2}$$

Hence, there exists a subjective belief $\hat{\lambda} \in \Lambda$ such that $\alpha^*(\hat{\lambda}) = 0$ if the individual is time-consistent and $u_0^m(p)$ is *strictly concave* at $p = p^*$.

The following proposition characterizes cases in which there exists a subjective belief with which the individual who satisfies consequentialism prefers less information to more.

Proposition 2: *Suppose that the individual satisfies consequentialism and there exists* $\varepsilon > 0$ *such that*

$$\beta^*(p) = \beta^*(p^*) \text{ if } |p - p^*| \le \varepsilon.^5$$

Then, there exists $\hat{\lambda} \in \Lambda$ *such that* $\alpha^*(\hat{\lambda}) = 0$, *if, and only if, there exists* $\hat{p} \in P$ *such that*

$$u_0(\beta^*(p^*), \hat{p}) > u_0(\beta^*(\hat{p}), \hat{p}). \tag{3}$$

Proof: Choose $p' \in P$ and $e \in (0,1)$ such that $p^* = e\hat{p} + (1-e)p'$ and p' is near p^* enough to satisfy $\beta^*(p') = \beta^*(p^*)$. Consequentialism and inequality (3) imply

$$\begin{aligned} & eu_0(\beta^*(\hat{p}), \hat{p}) + (1-e)u_0(\beta^*(p'), p') \\ &= eu_0(\beta^*(\hat{p}), \hat{p}) + (1-e)u_0(\beta^*(p^*), p') \\ &< eu_0(\beta^*(p^*), \hat{p}) + (1-e)u_0(\beta^*(p^*), p') = u_0(\beta^*(p^*), p^*), \end{aligned}$$

which, together with Proposition 1, implies the "if" part of this proposition.

Suppose that there exists $\hat{\lambda} \in \Lambda$ such that $\alpha^*(\hat{\lambda}) = 0$. Consequentialism says

$$\sum_{p \in P(\hat{\lambda})} \hat{\lambda}(p) u_0(\beta^*(p), p) < u_0(\beta^*(p^*), p^*) = \sum_{p \in P(\hat{\lambda})} \hat{\lambda}(p) u_0(\beta^*(p^*), p),$$

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[5] Here, we define $|r - r'| \equiv \max_{k \in \{1,\ldots,K\}} |r_k - r_k'|$, where $r = (r_1, \ldots, r_K)$.

which implies that there exists $\hat{p} \in P(\hat{\lambda})$ with inequality (3), i.e., the "only if" part.

Q.E.D.

Proposition 2 says that the individual who satisfies consequentialism may prefer less information to more if there exists at least one signal which induces the stage-1 self to decrease the utility evaluated at stage 0.

4. Immediate Gratification and Moral Taste

We incorporate two psychological natures of human being into the decision problem. We assume that the individual has preference for *immediate gratification*, which says that the individual overweighs the current payoff relative to the future payoffs in a *time-inconsistent* way. We assume also that the individual sometimes refrains from earning the self-interested material benefit because she has *moral tastes* according to which she does not like to harm others in a society if possible.

The individual has two different kinds of moral tastes, i.e., the *rule-based moral taste* and the *consequence-based moral rule.* The rule-based moral taste is the taste for avoiding the *consequence* of social harm caused by her activities, whereas the consequence-based moral taste is the taste for not violating the *moral rule* which says that the individual should not undertake an activity if she believes that it causes social harm with high probability.

In the next two sections, we investigate the decision problem specified as follows. Let

$$A=\{a_0,a_1\} \text{ and } \Omega=\{\omega_0,\omega_1\},$$

and we will write $P=[0,1]$ and regard $p\in P$ as the probability of state ω_1. The individual decides whether to undertake *the stage-1 activity*, i.e., to choose action a_1, or not, i.e., to choose action a_0. If the individual undertakes the activity at stage 1 and observes state ω_1 at stage 2, then social harm certainly occurs at stage 2. Otherwise, there is no social harm at stage 2.

When the individual undertakes the activity at stage 1, she obtains at stage 1 not only the *immediate material benefit* v_1 but also the *immediate moral disutility* $-w_1(p)$. The immediate moral disutility is induced by the rule-based moral taste. We assume that $w_1(p)$ is nondecreasing, $w_1(0)=0$, and $\lim_{p'\downarrow p} w_1(p')=w_1(p)$ for all $p\in[0,1)$.

When the individual undertakes the activity at stage 1, she obtains at stage 2 not only the *delayed material benefit* v_2 but also the *delayed moral disutility* $-w_2(p)$. The delayed moral disutility is induced by the consequence-based moral taste. We assume that $w_2(p)$ is nondecreasing and $w_2(0)=0$.

Since the delayed moral disutility is derived from experiencing the harmful consequence, it is natural to assume that it is reduced to the disutilities on the consequences. Hence, for every $\lambda\in\Lambda$,

$$\sum_{p\in P(\lambda)}\lambda(p)w_2(p)=w_2(p^*),$$

and therefore, $w_2(p)$ is expressed by the expected value

$$w_2(p) \equiv pz_2,$$

where $z_2 > 0$.

When the individual does not undertake the activity at stage 1, she obtains no material benefits and no moral disutilities.

The Individual has preference for immediate gratification which is described by the parameter of *hyperbolic discounting* $\delta \in (0,1]$, i.e., the salience of the current payoff relative to the future payoffs. The stage-0 self gives the same weight to the immediate payoff and the delayed payoff, whereas the stage-1 self gives weight to the immediate payoff $\frac{1}{\delta}$ times as much as the delayed payoff. Hence, the intertemporal preferences are time-inconsistent. This is in contrast to the exponential discounting which economists typically assume.

Based on these observations, we specify u_0 and u_1 by

$$u_0(a_0, p) = 0,$$
$$u_0(a_1, p) = \delta\{v_1 - w_1(p) + v_2 - pz_2\},$$
$$u_1(a_0, p) = 0,$$

and

$$u_1(a_1, p) = v_1 - w_1(p) + \delta\{v_2 - pz_2\}.$$

When $\delta < 1$, the individual has preference for immediate gratification.

5. Pattern of Action Choice

In this section, we investigate the pattern of action choice β^* according to which the individual chooses an action at stage 1.

We denote by $p^+ = p^+(\delta, v_1, v_2, w_1, w_2) \in P$ *the minimal signal with which the individual does not undertake the activity at stage 1, or, with which the stage-1 self prefers not to undertake the activity.* That is,

$$u_1(a_1, p) > u_1(a_0, p) \text{ for all } p < p^+,$$
$$u_1(a_1, p) \leq u_1(a_0, p) \text{ for all } p \geq p^+,$$

and therefore, β^* is given by

$$\beta^*(p) = a_1 \text{ for all } p < p^+,$$

and

$$\beta^*(p) = a_0 \text{ for all } p \geq p^+.$$

We denote by $p^{++} = p^{++}(\delta, v_1, v_2, w_1, w_2) \in P$ *the minimal signal with which the stae-0 self prefers not to undertake the activity.* That is,

$$u_0(a_1, p) > u_0(a_0, p) \text{ for all } p < p^+,$$

and

$$u_0(a_1, p) \leq u_0(a_0, p) \text{ for all } p \geq p^+.$$

We can easily check that when $\delta < 1$, i.e., the individual has preference for immediate gratification,

$$p^+ > p^{++} \text{ and } p^+ \text{ is nonincreasing with respect to } \delta \text{ if}$$
$$v_1 > w_1(\frac{v_2}{z_2}),$$

and

$$p^+ < p^{++} \text{ and } p^+ \text{ is nondecreasing with respect to } \delta \text{ if}$$
$$v_1 < w_1(\frac{v_2}{z_2}).$$

Inequality $v_1 > w_1(\frac{v_2}{z_2})$ means that $v_1 - w_1(p^+) > 0 > v_2 - p^+ z_2$, i.e., the immediate payoff is larger than the delayed payoff when the individual observes the signal p^+, while inequality $v_1 < w_1(\frac{v_2}{z_2})$ means that $v_1 - w_1(p^+) < 0 < v_2 - p^+ z_2$, i.e., the immediate payoff is less than the delayed payoff when the individual observes the signal p^+.

The individual is said to be *consequence-based* if $v_1 > w_1(\frac{v_2}{z_2})$, whereas she is said to be *rule-based* if $v_1 < w_1(\frac{v_2}{z_2})$. When the individual is consequence-based, the stage 0-self regards the stage-1 self as being too active, whereas when the individual is rule-based, the stage 0-self regards the stage-1 self as being too moral.

We can check below that *the individual who gives the more weight to the immediate moral disutility follows the morally better pattern of action choice than the individual who gives the less weight.* Fix $\delta < 1$ arbitrarily. Suppose that (v_1, v_2, w_1, w_2) and $(\widetilde{v}_1, \widetilde{v}_2, \widetilde{w}_1, \widetilde{w}_2)$ satisfy that w_1 and $\widetilde{w}_1$ are continuous,

$$v_1 + v_2 = \widetilde{v}_1 + \widetilde{v}_2, \quad v_1 \le \widetilde{v}_1,$$

$$w_1(p) + w_2(p) = \widetilde{w}_1(p) + \widetilde{w}_2(p), \text{ and } w_1(p) > \widetilde{w}_1(p).$$

Then, the individual whose preference is described by $(\delta, v_1, v_2, w_1, w_2)$ has the same utility function evaluated at stage 0 as the individual whose preference is described by $(\delta, \widetilde{v}_1, \widetilde{v}_2, \widetilde{w}_1, \widetilde{w}_2)$, and therefore, the stage-0 selves make the same moral judgment on action choice, i.e.,

$$p^{++}(\delta, v_1, v_2, w_1, w_2) = p^{++}(\delta, \widetilde{v}_1, \widetilde{v}_2, \widetilde{w}_1, \widetilde{w}_2).$$

However, the former individual has the more moralistic utility function evaluated at stage 1 and makes the morally better action choice than the latter, i.e.,

$$p^{+}(\delta, v_1, v_2, w_1, w_2) < p^{+}(\delta, \widetilde{v}_1, \widetilde{v}_2, \widetilde{w}_1, \widetilde{w}_2).$$

6. Pattern of Information Acquisition

In this section, we investigate the pattern of information acquisition α^* according to which the individual decides whether to access the information channel at stage 0.

The pattern of information acquisition α^* is said to be *morally good with respect to* $\lambda \in \Lambda$ if

$$\alpha^*(\lambda) = \begin{cases} 0 & \text{whenever } \beta^*(p^*) = a_0 \\ 1 & \text{whenever } \beta^*(p^*) = a_1 \end{cases},$$

while it is said to be *morally bad with respect to* $\lambda \in \Lambda$ if

$$\alpha^*(\lambda) = \begin{cases} 0 & \text{whenever } \beta^*(p^*) = a_1 \\ 1 & \text{whenever } \beta^*(p^*) = a_0 \end{cases}.$$

Given the pattern of action choice, the morally good pattern of information acquisition minimizes the probability that social harm occurs, whereas the morally bad pattern of information acquisition maximizes it.

First of all, we investigate the decision problem in which $\beta^*(p^*) = a_0$, i.e., *the prior induces the individual not to undertake the stage-1 activity.*

Proposition 3: *Suppose* $\beta^*(p^*) = a_0$. *Then, for every* $\lambda \in \Lambda$,

$$\alpha^*(\lambda) = 0 \text{ if } P(\lambda) \cap [0, p^{++}] = \phi \text{ and } P(\lambda) \cap (p^{++}, p^+) \neq \phi,$$

whereas

$$\alpha^*(\lambda) = 1 \text{ if } P(\lambda) \cap (p^{++}, p^+) = \phi.$$

Proof: We must note that

$$u_0(\beta^*(p^*), p^*) = 0,$$

and for every $\lambda \in \Lambda$,

$$u_0(\beta^*(p), p) \begin{cases} \geq 0 & \text{if } p \leq p^{++} \\ < 0 & \text{if } p^{++} < p < p^+ \\ = 0 & \text{if } p \geq p^+ \end{cases}.$$

Hence, one gets that for every $\lambda \in \Lambda$,

$$\sum_{p \in P(\lambda)} \lambda(p) u_0(\beta(p), p) < 0 = u_0(\beta(p^*), p^*) \quad \text{if} \quad P(\lambda) \cap [0, p^{++}] = \phi$$

$$\text{and } P(\lambda) \cap (p^{++}, p^+) \neq \phi,$$

$$\sum_{p \in P(\lambda)} \lambda(p) u_0(\beta(p), p) \geq 0 = u_0(\beta(p^*), p^*) \text{ if } P(\lambda) \cap (p^{++}, p^+) = \phi,$$

and therefore, Proposition 3 holds.

Q.E.D.

Proposition 3 says that, given $\beta^*(p^*) = a_0$:

(i) If $p^+ > p^{++}$, i.e., $v_1 > w_1(\frac{v_2}{z_2})$, then there exists $\lambda \in \Lambda$ such that $\alpha^*(\lambda) = 0$.

(ii) If $p^+ \leq p^{++}$, i.e., $v_1 \leq w_1(\frac{v_2}{z_2})$, then $\alpha^*(\lambda) = 1$ for all $\lambda \in \Lambda$.

Hence, we can say that *when the prior induces the individual not to undertake the activity, there exists a subjective belief $\lambda \in \Lambda$ with respect to which the consequence-based individual follows the morally good pattern of information acquisition, whereas the individual who is not consequence-based always follows the morally bad pattern of information acquisition.*

In the next three subsections, we investigates the decision problem in which $\beta^*(p^*) = a_1$, i.e., the prior induces the individual to undertake the activity.

6.1. Consequentialism

In this subsection, we assume that the immediate moral disutility can be reduced to the utilities of the consequences, i.e., for every $\lambda \in \Lambda$,

$$\sum_{p \in P(\lambda)} \lambda(p) w_1(p) = w_1(p^*).$$

Hence, the individual satisfies consequentialism and $w_1(p)$ is expressed by the expected value

$$w_1(p) \equiv p z_1, \tag{4}$$

where $z_1 > 0$. We can easily check that

$$p^+ = \frac{v_1 + \delta v_2}{z_1 + \delta z_2} \text{ and } p^{++} = \frac{v_1 + v_2}{z_1 + z_2},$$

and, given $\delta < 1$,

$$p^+ > p^{++} \text{ if } v_1 z_2 > v_2 z_1,$$

and

$$p^+ < p^{++} \text{ if } v_1 z_2 < v_2 z_1.$$

The cases of $p^+ > p^{++}$ and $p^+ < p^{++}$ are illustrated in Figures 1 and 2 respectively.

[Figure 1]

[Figure 2]

Proposition 4: *Suppose that equality (4) holds and $\beta^*(p^*) = a_1$. Then, for every $\lambda \in \Lambda$,*

$$\alpha^*(\lambda) = 0 \ \textit{if} \ P(\lambda) \cap (p^{++}, 1] = \phi \ \textit{and} \ P(\lambda) \cap [p^+, p^{++}) \neq \phi,$$

whereas

$$\alpha^*(\lambda) = 1 \ \textit{if} \ P(\lambda) \cap [p^+, p^{++}) = \phi.$$

Proof: We must note that

$$u_0(\beta^*(p^*), p^*) = \delta\{v_1 - p^* z_1 + v_2 - p^* z_2\},$$

and for every $\lambda \in \Lambda$,

$$u_0(\beta^*(p), p) \begin{cases} = \delta\{v_1 - pz_1 + v_2 - pz_2\} & \textit{if } p \leq p^{++} \\ = 0 < \delta\{v_1 - pz_1 + v_2 - pz_2\} & \textit{if } p^{++} < p < p^+ \\ = 0 \geq \delta\{v_1 - pz_1 + v_2 - pz_2\} & \textit{if } p \geq p^+ \end{cases}.$$

Hence, one gets that for every $\lambda \in \Lambda$,

$$\sum_{p \in P(\lambda)} \lambda(p) u_0(\beta(p), p) < \sum_{p \in P(\lambda)} \lambda(p) \delta\{v_1 - p^* z_1 + v_2 - p^* z_2\} = u_0(\beta^*(p^*), p^*)$$

$$\text{if } P(\lambda) \cap [p^{++}, 1] = \phi \text{ and } P(\lambda) \cap (p^{++}, p^+) \neq \phi,$$

and

$$\sum_{p \in P(\lambda)} \lambda(p) u_0(\beta(p), p) \geq \sum_{p \in P(\lambda)} \lambda(p) \delta\{v_1 - p^* z_1 + v_2 - p^* z_2\} = u_0(\beta^*(p^*), p^*)$$

$$\text{if } P(\lambda) \cap (p^{++}, p^+) = \phi,$$

because $\sum_{p \in P(\lambda)} \lambda(p) p = p^*$. Hence, Proposition 4 holds.

Q.E.D.

Proposition 4 says that, given $\beta^*(p^*) = a_1$:

(iii) If $p^+ > p^{++}$, i.e., $v_1 > w_1(\frac{v_2}{z_2})$, then $\alpha^*(\lambda) = 1$ for all $\lambda \in \Lambda$.

(iv) If $p^+ < p^{++}$, i.e., $v_1 < w_1(\frac{v_2}{z_2})$, then there exists $\lambda \in \Lambda$ such that $\alpha^*(\lambda) = 0$.

Hence, we can say that *when the individual satisfies consequentialism and the prior induces the individual to undertake the activity at stage 1, the consequence-based individual always follows the morally good pattern of information acquisition, whereas there exists a subjective belief $\lambda \in \Lambda$ with respect to which the rule-based individual follows the morally bad pattern of information acquisition.*

The contents of Propositions 3 and 4 are summarized as follows: When the individual satisfies consequentialism, the consequence-based individual is likely to follow the morally good pattern of information acquisition irrespective of prior, whereas the rule-based individual is likely to follow the morally bad pattern of information acquisition irrespective of prior.

6.2. Moral Constraint

This and the next subsections will be devoted to investigating the decision problem in which the immediate moral disutility can *not* be reduced to the utilities on the consequences.

In this subsection, we assume that the rule-based moral taste serves as the *moral constraint*. The individual maximizes the hyperbolic-discounted sum of the material benefits and the delayed moral disutility subject to the constraint which strictly prohibits her from undertaking the activity if the probability that it causes social harm is more than or equal to a threshold $\hat{p} \in P$.

The moral constraint is described by the immediate moral utility function w_1 specified by

$$w_1(p) = \begin{cases} -\underline{w}_1 & \text{if } p \geq \hat{p} \\ 0 & \text{otherwise} \end{cases}, \tag{5}$$

where $\underline{w}_1$ is large enough to satisfy

$$v_1 - \underline{w}_1 + \delta(v_2 - pz_2) < 0.$$

We can easily check that

$$p^{+} = \begin{cases} \dfrac{v_1 + \delta v_2}{\delta z_2} & \text{if } \hat{p} > \dfrac{v_1 + \delta v_2}{\delta z_2} \\ \hat{p} & \text{otherwise} \end{cases},$$

$$p^{++} = \begin{cases} \dfrac{v_1 + v_2}{z_2} < \dfrac{v_1 + \delta v_2}{\delta z_2} & \text{if } \hat{p} > \dfrac{v_1 + v_2}{z_2} \\ \hat{p} & \text{otherwise} \end{cases},$$

and given $\delta < 1$,

$$p^{+} > p^{++} \text{ if } \hat{p} > \frac{v_1 + v_2}{z_2},$$

and

$$p^{+} = p^{++} \text{ if } \hat{p} \leq \frac{v_1 + v_2}{z_2}.$$

Inequality $\hat{p} > \dfrac{v_1 + v_2}{z_2}$ means that the individual is consequence-based and the moral constraint only serves to avoid preference reversals caused by immediate gratification. Inequality $\hat{p} \leq \dfrac{v_1 + v_2}{z_2}$ means that the individual is not consequence-based and the moral constraint not only serves to avoid preference reversals but also strengthens the stage-0 self's morality. The cases of $p^{+} > p^{++}$ and $p^{+} = p^{++}$ are illustrated in Figures 3 and 4.

[Figure 3]

[Figure 4]

Proposition 5: *Suppose that equalities (5) hold and* $\beta^*(p^*) = a_1$. *Then, for every* $\lambda \in \Lambda$,

$$\alpha^*(\lambda) = 0 \text{ if } P(\lambda) \cap (\frac{v_1 + v_2}{z_2}, 1] = \phi \text{ and } P(\lambda) \cap [\hat{p}, \frac{v_1 + v_2}{z_2}) \neq \phi,$$

whereas

$$\alpha^*(\lambda) = 1 \text{ if } P(\lambda) \cap [\hat{p}, \frac{v_1 + v_2}{z_2}) = \phi.$$

Proof: We must note that

$$u_0(\beta^*(p^*), p^*) = \delta\{v_1 + v_2 - p^* z_2\}.$$

Suppose $\hat{p} \leq \frac{v_1 + v_2}{z_2}$. Then, for every $\lambda \in \Lambda$,

$$u_0(\beta^*(p), p) \begin{cases} = \delta\{v_1 + v_2 - p z_2\} & \text{if } p < \hat{p} \\ = 0 < \delta\{v_1 - p z_1 + v_2 - p z_2\} & \text{if } \hat{p} \leq p < \frac{v_1 + v_2}{z_2}, \\ = 0 \geq \delta\{v_1 - p z_1 + v_2 - p z_2\} & \text{if } p \geq \frac{v_1 + v_2}{z_2} \end{cases}$$

and therefore,

$$\sum_{p \in P(\lambda)} \lambda(p) u_0(\beta(p), p) < \sum_{p \in P(\lambda)} \lambda(p) \delta\{v_1 - p^* z_1 + v_2 - p^* z_2\} = u_0(\beta^*(p^*), p^*)$$

$$\text{if } P(\lambda) \cap [\frac{v_1 + v_2}{z_2}, 1] = \phi \text{ and } P(\lambda) \cap [\hat{p}, \frac{v_1 + v_2}{z_2}) \neq \phi,$$

and

$$\sum_{p \in P(\lambda)} \lambda(p) u_0(\beta(p), p) \geq \sum_{p \in P(\lambda)} \lambda(p) \delta\{v_1 - p^* z_1 + v_2 - p^* z_2\} = u_0(\beta^*(p^*), p^*)$$

$$\text{if } P(\lambda) \cap [\hat{p}, \frac{v_1 + v_2}{z_2}) = \phi,$$

because $\sum_{p \in P(\lambda)} \lambda(p) p = p^*$.

Next, suppose $\hat{p} > \frac{v_1 + v_2}{z_2}$. Then, for every $\lambda \in \Lambda$,

$$u_0(\beta^*(p),p)\begin{cases} = \delta\{v_1 + v_2 - pz_2\} & if\ p < \min[\hat{p}, \frac{v_1+v_2}{z_2}] \\ = 0 \geq \delta\{v_1 - pz_1 + v_2 - pz_2\} & if\ p \geq \min[\hat{p}, \frac{v_1+v_2}{z_2}] \end{cases},$$

and therefore,

$$P(\lambda)\cap[\hat{p}, \frac{v_1+v_2}{z_2}) = \phi,$$

and

$$\begin{aligned} &\sum_{p\in P(\lambda)} \lambda(p) u_0(\beta(p),p) \\ &\geq \sum_{p\in P(\lambda)} \lambda(p)\delta\{v_1 - p^* z_1 + v_2 - p^* z_2\} = u_0(\beta^*(p^*),p^*), \end{aligned}$$

because $\sum_{p\in P(\lambda)} \lambda(p)p = p^*$. Hence, Proposition 5 holds.

Q.E.D.

Proposition 5 says that, given $\beta^*(p^*) = a_1$:

(v) If $p^+ > p^{++}$, i.e., $\hat{p} > \frac{v_1+v_2}{z_2}$, then $\alpha^*(\lambda) = 1$ for all $\lambda \in \Lambda$.

(vi) If $p^+ = p^{++}$, i.e., $\hat{p} \leq \frac{v_1+v_2}{z_2}$, then there exists $\lambda \in \Lambda$ such that $\alpha^*(\lambda) = 0$.

Hence, *when the prior induces the individual to undertake the activity and the rule-based moral taste serves as the moral constraint, the individual always follows the morally good pattern of information acquisition if the moral constraint only serves to avoid preference reversals caused by immediate gratification, whereas there exists a subjective belief* $\lambda \in \Lambda$ *with respect to which the individual follows the morally bad pattern of information acquisition if the moral constraint strengthens the stage-0 self's morality.*

The contents of Propositions 3 and 5 is summarized as follows: When the rule-based moral taste serves as the moral constraint, the individual is likely to follow the morally good pattern of information acquisition irrespective of prior if the moral constraint only serves to avoid preference reversals, whereas the individual is likely to follow the morally bad pattern of information acquisition irrespective of prior if the moral constraint strengthens the stage-0 self's morality.

6.3. Differentiable Case

Finally, we consider general cases in which the immediate moral disutility function

$w_1(p)$ is differentiable. The following proposition gives the necessary and sufficient condition under which the individual follows the morally good pattern of information acquisition when the prior induces the individual to undertake the activity.

Proposition 6: *Suppose that* $\beta^*(p^*) = a_1$ *and* $w_1(p)$ *is differentiable. Then,*

$$\alpha^*(\lambda) = 1 \textit{ for all } \lambda \in \Lambda,$$

if and only if

$$w_1(p^*) + (p - p^*)w_1'(p^*) \geq w_1(p) \textit{ for all } p \in [0, p^+). \tag{6}$$

and

$$v_1 - w_1(p^*) - (p^+ - p^*)w_1'(p^*) + v_2 - p^+ z_2 \geq 0. \tag{7}$$

Proof: Suppose that inequalities (6) and (7) hold. Then, for every $p \in P$, if $p < p^+$, then

$$\begin{aligned} &u_0(\beta^*(p^*), p^*) = \delta\{v_1 - w_1(p) + v_2 - pz_2\} \\ &\geq \delta\{v_1 - w_1(p^*) - (p - p^*)w_1'(p^*) + v_2 - pz_2\}, \end{aligned}$$

and if $p \geq p^+$, then

$$\begin{aligned} &u_0(\beta^*(p^*), p^*) = 0 \\ &\geq \delta\{v_1 - w_1(p^*) - (p^+ - p^*)w_1'(p^*) + v_2 - p^+ z_2\} \\ &\geq \delta\{v_1 - w_1(p^*) - (p - p^*)w_1'(p^*) + v_2 - pz_2\}. \end{aligned}$$

This implies that for every $\lambda \in \Lambda$,

$$\begin{aligned} &\sum_{p \in P(\lambda)} \lambda(p) u_0(\beta(p), p) \\ &\geq \sum_{p \in P(\lambda)} \lambda(p) \delta\{v_1 - w_1(p^*) - (p - p^*)w_1'(p^*) + v_2 - p^* z_2\} \\ &= \sum_{p \in P(\lambda)} \lambda(p) \delta\{v_1 - w_1(p^*) + v_2 - p^* z_2\} = u_0(\beta^*(p^*), p^*), \end{aligned}$$

and therefore, the "if" part of Proposition 6 holds.

Next, suppose that there exists $p = p' \in [0, p^+)$ which does not satisfy inequality (6), i.e.,

$$u_0(\beta^*(p'), p') < \delta\{v_1 - w_1(p^*) - (p' - p^*)w_1'(p^*) + v_2 - p' z_2\}.$$

Let $p'' \in [1, p^+)$ and $\lambda \in \Lambda$ satisfy that $P(\lambda) = \{p', p''\}$, and p'' is close to p^* such that $u_0(\beta^*(p''), p'')$ is approximated by $\delta\{v_1 - w_1(p^*) - (p'' - p^*)w'(p^*) + v_2 - p'' z_2\}$. Hence, one gets

$$\begin{aligned} &\sum_{p \in P(\lambda)} \lambda(p) u_0(\beta(p), p) \\ &< \sum_{p \in P(\lambda)} \lambda(p) \delta\{v_1 - w_1(p^*) - (p - p^*)w_1'(p^*) + v_2 - p^* z_2\} \\ &= \sum_{p \in P(\lambda)} \lambda(p) \delta\{v_1 - w_1(p^*) + v_2 - p^* z_2\} = u_0(\beta^*(p^*), p^*), \end{aligned}$$

which means $\alpha^*(\lambda) = 0$.

Finally, suppose that inequality (7) does not hold, i.e.,

$$u_0(\beta^*(p^+), p^+) = 0 < \delta\{v_1 - w_1(p^*) - (p^+ - p^*)w_1'(p^*) + v_2 - p^+ z_2\}.$$

Let $\lambda \in \Lambda$ satisfy that $P(\lambda) = \{p^+, p''\}$, where $p'' \in [0, p^+)$ is the signal defined above. Hence, one gets

$$\begin{aligned}
&\sum_{p \in P(\lambda)} \lambda(p) u_0(\beta(p), p) \\
&< \sum_{p \in P(\lambda)} \lambda(p)\delta\{v_1 - w_1(p^*) - (p - p^*)w_1'(p^*) + v_2 - p^* z_2\} \\
&= \sum_{p \in P(\lambda)} \lambda(p)\delta\{v_1 - w_1(p^*) + v_2 - p^* z_2\} = u_0(\beta^*(p^*), p^*),
\end{aligned}$$

which means $\alpha^*(\lambda) = 0$.

Q.E.D.

A sufficient condition under which inequalities (6) hold is that $w_1(p)$ is weakly convex. Hence, one gets from Propositions 3 and 6 that the consequence-based individual follows the morally good pattern of information acquisition if $w_1(p)$ is weakly convex. A sufficient condition under which inequalities (6) do not hold is that $w_1(p)$ is strictly concave. Hence, one gets from Propositions 3 and 6 that the rule-based individual follows the morally bad pattern of information acquisition if $w_1(p)$ is strictly concave.

7. Moral Rule on Information Acquisition

This section takes account of another moral rule, i.e., the *moral rule on information acquisition*, which says that the stage-0 self should not access the information channel if this access induces the stage-1 self to increase the probability that social harm occurs. The moral rule on information acquisition is another primary factor for time-inconsistency. We show that the individual who has the moral taste for not violating the moral rule on information acquisition may prefer less information to more even if she satisfies consequentialism and has no preference for immediate gratification.

We investigate the decision problem specified as follows. Let

$$A = \{a_0, \ldots, a_m\} \text{ and } \Omega = \{\omega_0, \omega_1\}.$$

If the individual chooses action a at stage 1 and observes state ω_1 at stage 2, then social harm occurs at stage 2 with probability $s(a)$. If she observes state ω_0, social harm never occurs. We assume that $s(a_l)$ is increasing with respect to $l \in \{0, \ldots, m\}$. Hence, the action a_0 is regarded as the *morally best action*, whereas the action a_m is regarded as the *morally worst action*.

When the individual chooses action a at stage 1, she obtains the immediate material benefit $v_1(a)$ and the immediate moral disutility $-ps(a)z_1$ at stage 1, and obtains the delayed material benefit $v_2(a)$ and the delayed moral disutility $-ps(a)z_2$ at stage 2. We assume that z_1 and z_2 are positive, and that $v_1(a_l)$ and $v_2(a_l)$ are increasing with respect to $l \in \{0, \ldots, m\}$.

The point of departure from the previous sections is that the individual obtains the moral disutility at stage 0 which is induced by the moral taste for violating the moral rule on information acquisition. We define this moral disutility as being equivalent to the probability that social harm occurs . That is, the moral utility at stage 0 when the individual decides to access the information channel is

$$\sum_{p \in \lambda} \lambda(p) ps(\beta^*(p)),$$

whereas the moral utility at stage 0 when she decides not to access it is

$$p^* s(\beta^*(p^*)).$$

Based on these observations, we specify u_0 and u_1 by

$$u_0(a, p) = -ps(a) + v_1(a) - ps(a)z_1 + v_2(a) - ps(a)z_2,$$

and

$$u_1(a_l, p) = v_1(a) - ps(a)z_1 + v_2(a) - ps(a)z_2.$$

The individual satisfies consequentialism and has no preference for immediate gratification. However, the intertemporal preferences are time-inconsistent because the stage-1 self does not take account of the moral disutility at stage 0.

The following proposition says that there exists a subjective belief according to which the individual prefers not to access the information channel when the action which the prior induces the stage-1 self to choose is not the morally worst.

Proposition 7: *Suppose that* $\beta^*(p^*) \neq a_m$,

$$u_1(\beta^*(p^*), p^*) > u_1(a, p^*) \text{ for all } a \neq \beta^*(p^*), \tag{8}$$

and for every $a \in A$*, there exists at least one signal* $p \in P$ *such that* $\beta^*(p) = a$*. Then, there exists* $\lambda \in \Lambda$ *such that* $\alpha^*(\lambda) = 0$.

Proof: We can easily check from the definitions of u_0 and u_1 that there exist p_0^+, ..., p_{m+1}^+ and p_0^{++}, ..., p_{m+1}^{++} such that

$$p_{m+1}^+ \equiv 0 \leq p_m^+ \leq \cdots \leq p_1^+ \leq p_0^+ \equiv 1,$$

$$p_{m+1}^+ \equiv 0 \leq p_m^{++} \leq \cdots \leq p_1^{++} \leq p_0^+ \equiv 1,$$

$$p_l^{++} < p_l^+ \text{ for all } l \in \{1, \ldots, m\},$$

for every $p \in P$ and every $l \in \{0, \ldots, m\}$,

$$\beta^*(p) = a_l \text{ if } p_{l+1}^+ < p < p_l^+,$$

and

$$u_0(a_l, p) > u_0(a, p) \text{ for all } a \in A / \{a_l\} \text{ if } p_{l+1}^{++} < p < p_l^{++}.$$

Let $l^* \in \{0, \ldots, m-1\}$ denote the integer such that $\beta^*(p^*) = a_{l^*}$. Inequalities (8) say that $p_{l^*+1}^+ \leq p^* < p_{l^*}^+$. We can choose $p' \in P$ and $p'' \in P$ such that $p_{l^*+1}^{++} < p' < p_{l^*+1}^+$ and $p^* < p'' < p_{l^*}^+$. Let $\lambda \in \Lambda$ satisfy that $P(\lambda) = \{p', p''\}$. Clearly,

$$\sum_{p \in P(\lambda)} \lambda(p) u_0(\beta^*(p), p) = \lambda(p') u_0(a_{l^*+1}, p') + \lambda(p'') u_0(a_{l^*}, p'')$$

$$< \lambda(p') u_0(a_{l^*+1}, p') + \lambda(p'') u_0(a_{l^*+1}, p'') = u_0(\beta^*(p^*), p^*),$$

which implies $\alpha^*(\lambda) = 0$.

Q.E.D.

According to the subjective belief specified in the proof of Proposition 7, the stage-0 self believes that the access to the information channel induces the stage-1 self to choose the morally worse action than the action $a = \beta^*(p^*)$. Hence, the stage-0 self believes that the access increases the moral disutility at stage 0 of which the stage-1 self does not take account, and therefore, decides not to access the information channel. On the other hand, the individual always prefer to access the information when $\beta^*(p^*) = a_m$ because there exists no morally worse action than $\beta^*(p^*)$.

8. Learning

This section gives an affirmative answer to the question about how often the individual estimates a subjective belief $\hat{\lambda}$ such that $\alpha(\hat{\lambda})=0$. We consider the situation in which the individual plays the decision problem infinitely many times. We show that the stage-0 self learns to believe that the access induces the stage-1 self to decrease the utility evaluated at stage 0 and stop acquiring information in the long run.

In every period $t \geq 1$, the individual accesses the time-dependent information channel $\lambda_t^* \in \Lambda$ by choosing $c=1$ at stage 0. We assume that there exist k *subchannels* $\{\lambda_1,\dots,\lambda_k\} \subset \Lambda$ such that for every $t \geq 1$, the information channel λ_t^* is expressed by a convex combination of $\{\lambda_1,\dots,\lambda_k\}$, i.e.,

$$\lambda_t^* = q_{t,1}^*\lambda_1 + \cdots + q_{t,k}^*\lambda_k,$$

where $q_t^* \equiv (q_{t,1}^*,\dots,q_{t,k}^*) > 0$. By choosing $c=1$, the individual accesses a subchannel λ_l and observes a signal $p \in P(\lambda_l)$ with probability $q_{t,l}^*\lambda_l(p)$. We assume that the individual can observe not only the signal but also the accessed subchannel. By choosing $c=0$, the individual accesses no informative subchannel, i.e., accesses the *null channel* λ_0 such that

$$\lambda_0(p^*)=1.$$

Let $h^{(t)} = (c^{(\tau)},\lambda^{(\tau)})_{\tau=1}^t$ denote a history up to period t of the choices of whether to access the information channel and the accessed subchannels, where

$$\lambda^{(\tau)} \in \{\lambda_1,\dots,\lambda_k\} \text{ if } c^{(\tau)}=1,$$

and

$$\lambda^{(\tau)} = \lambda_0 \text{ if } c^{(\tau)}=0.$$

Let $H^{(t)}$ denote the set of histories up to period t, where $H^{(0)}=\{h^{(0)}\}$ and $h^{(0)}$ is the null history.

We assume that the individual knows λ_1, ..., λ_k, but does not knows q_t^*. We denote by Q the set of possible probability distributions on $\{\lambda_1,\dots,\lambda_k\}$, an element of which is denoted by $q=(q_1,\dots,q_k)$. At the beginning of every period t, the individual estimates a subjective belief according to a *learning rule* $\eta\colon \bigcup_{t=0}^{\infty} H^{(t)} \to Q$. That is, the individual sets the subjective belief equal to

$$\eta_1(h^{(t-1)})\lambda_1 + \cdots + \eta_k(h^{(t-1)})\lambda_k,$$

where $\eta(h^{(t-1)}) = (\eta_1(h^{(t-1)}),\dots,\eta_k(h^{(t-1)})) \in Q$.

We introduce the following two conditions on learning rule η.

Condition 1: For every $\varepsilon > 0$, there exists a positive integer $T=T(\varepsilon)$ which satisfies the following property: For every $l \in \{1,\ldots,k\}$, every $t>0$, and every $h^{(t)} \in H^{(t)}$, there exists $\widetilde{h}^{(t+T)} \in H^{(t+T)}$ such that $\widetilde{h}^{(t)} = h^{(t)}$,

$$(c^{(\tau)}, \lambda^{(\tau)}) = (1, \lambda_l) \text{ for all } \tau \in \{t+1,\ldots,t+T\},$$

and

$$\eta_l(\widetilde{h}^{(t+T)}) \geq 1-\varepsilon .$$

Condition 2: For every $t>0$ and every $h^{(t)} \in H^{(t)}$,

$$\eta(h^{(t)}) = \eta(h^{(t-1)}) \text{ if } c^{(t)} = 0 .$$

Condition 1 says that by continuing to observe the same subchannel finitely many times, the individual believes that she can almost surely access this subchannel. Condition 2 says that whenever the individual decides not to access the information channel then she never changes the subjective belief.

The following proposition characterizes learning rules according to which the individual stops accessing the information channel in the long run.

Proposition 8: *Suppose that* η *satisfies Conditions 1 and 2 and there exist* $\hat{l} \in \{1,\ldots,k\}$ *and* $\eta > 0$ *such that* $\alpha(\lambda_{\hat{l}}) = 0$ *and* $q^*_{t,\hat{l}} \geq \eta$ *for all* $t \geq 1$. *Then, for every* $\varepsilon > 0$, *there exists a positive integer* $\hat{T} = \hat{T}(\varepsilon)$ *such that it holds at least with probability* $1-\varepsilon$ *that*

$$c^{(t)} = 0 \textit{ for all } t \geq \hat{T} .$$

Proof: Let $\varepsilon > 0$ be given as close to zero as possible. Condition 1 says that for every $t>0$, by continuing to choose $c=1$ from period t to period $t+T(\varepsilon)-1$, it is at least with time-independent positive probability $\eta^{T(\varepsilon)} > 0$ that the individual's subjective belief in period $t+T(\varepsilon)$ is well approximated by $\lambda_{\hat{l}}$. This implies that by continuing to choose $c=1$ infinitely many times, there almost surely exists a period τ in which the individual's subjective belief is well approximated by $\lambda_{\hat{l}}$, and therefore, the individual decides not to access the information channel. Condition 2 says that once the individual's subjective belief is approximated by $\lambda_{\hat{l}}$, she never accesses the information channel afterwards. Hence, we have proven this proposition.

Q.E.D.

Proposition 8 says that the individual stops accessing the information channel forever, even though the access to the information channel is beneficial for the stage-0 self. We can easily check that the following learning rule $\hat{\eta}$ satisfies Conditions 1 and 2: Fix $\theta \in (0,1)$ arbitrarily:

$$\hat{\eta}(h^{(t)}) = \hat{\eta}(h^{(t-1)}) \text{ if } c^{(t-1)} = 0,$$
$$\hat{\eta}_l(h^{(t)}) = (1-\theta)\hat{\eta}_l(h^{(t-1)}) + \theta \text{ if } c^{(t)} = 1 \text{ and } \lambda^{(t)} = \lambda_l,$$

and

$$\hat{\eta}_l(h^{(t)}) = (1-\theta)(1-\hat{\eta}_{l'}(h^{(t-1)}))\frac{\hat{\eta}_l(h^{(t-1)})}{\sum_{l''\neq l'}\hat{\eta}_{l''}(h^{(t-1)})} \text{ if } c^{(t)} = 1,\ \lambda^{(t)} = \lambda_{l'},$$
$$\text{and } l' \neq l.$$

Suppose that θ is close to zero, and $\lambda_t^* \equiv \lambda^*$ and $q_t^* \equiv q^*$ are time-independent. When individual continues to choose $c = 1$ infinitely many times, she almost certainly sets the subjective belief approximating λ^* in the long run. However, Proposition 8 says that the individual almost certainly fails to estimate λ^* correctly and stops choosing $c = 1$ in the long run.

The above argument indicates that the reason why individual fails to estimate correctly is not that the learning rule has some statistical bias but that the individual loses incentive to acquire all the available information. In this respect, this argument is closely related to the well-known two-arm bandit problem in probability theory.[6]

[6] See DeGroot (1970) and Rothschild (1974). There is the difference between the our argument and the two-arm bandit problem that models of Bayesian statistics do not satisfy the correspondent to Condition 1. Sarin and Vahid (1997) and Matsushima (1997) introduced conditions closely related to Condition 1 and showed similar results in different contexts.

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Figure 1 : $p^{++} < p^{+}$

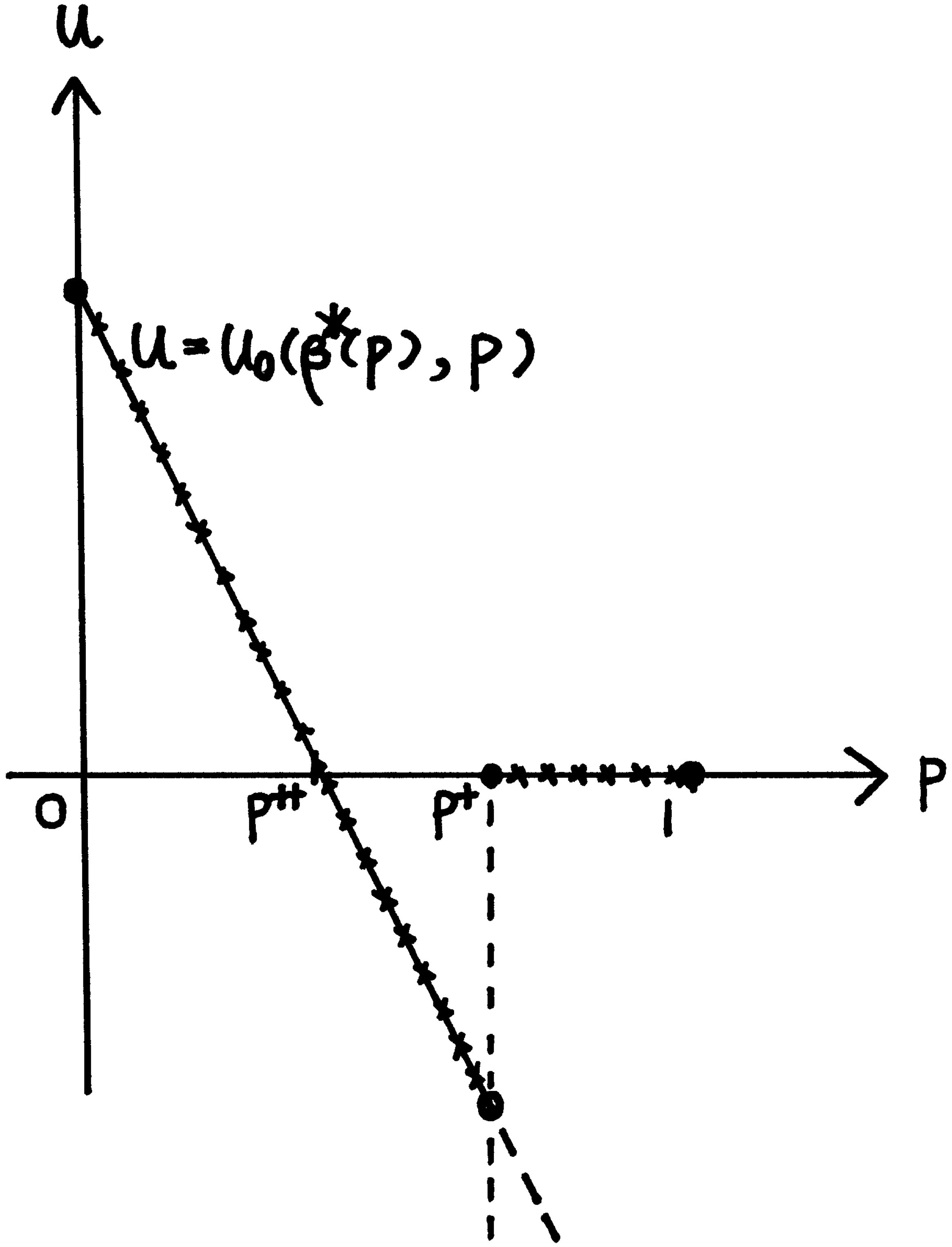

Figure 2 : $p^{+} < p^{++}$

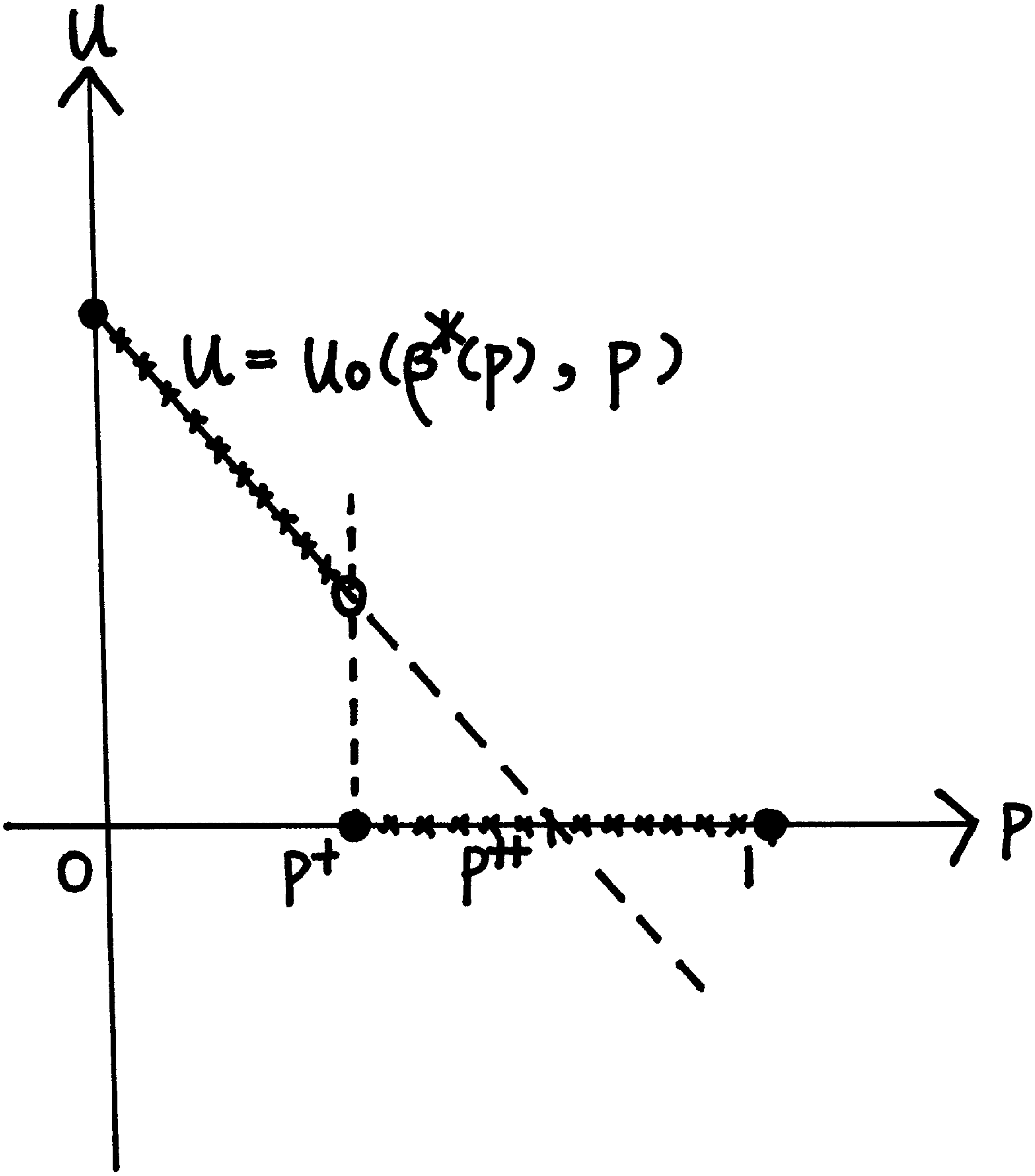

Figure 3 : $\frac{V_1+V_2}{Z_2} < \hat{p} < \frac{V_1+\delta V_2}{\delta Z_2}$

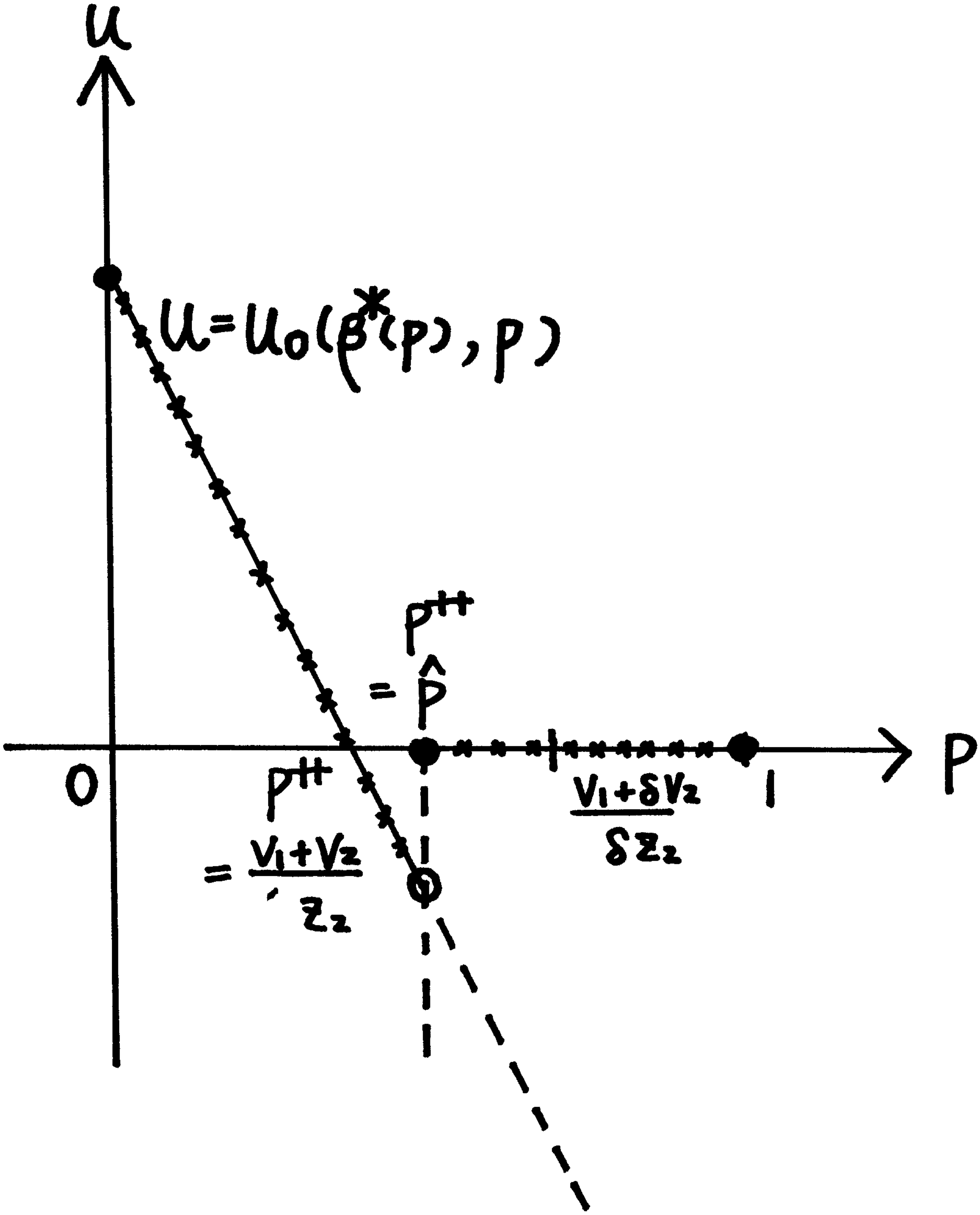

Figure 4 : $\hat{p} < \frac{V_1 + V_2}{Z_2}$