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Katsuhito Iwai
University of Tokyo

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Katsuhito Iwai

Faculty of Economics

The University of Tokyo

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by Katsuhito Iwai

Faculty of Economics, The University of Tokyo

<ABSTRACT>

In the traditional economic theory, whether classical or neoclassical, the long-run state of the economy is an equilibrium state in which all profits in excess of normal rate vanish completely. If there is a theory of long-run profits, it is a theory about the determination of the normal rate of profit. This paper challenges this long-held tradition in economics. It introduces a series of new evolutionary models which are capable of studying the evolutionary process of an industry's technology as a dynamic interplay among many a firm's growth, imitation and innovation activities. It demonstrates that the economy will never dissipate positive profits even in the long-run, because what it will approach over a long passage of time is not a classical or neoclassical equilibrium of uniform technology but a statistical equilibrium of technological disequilibria which reproduces a relative dispersion of efficiencies in a statistically balanced form. As Joseph Schumpeter once remarked, "surplus values may be impossible in perfect equilibrium, but can be ever present because that equilibrium is never allowed to establish itself."

The paper also shows that our evolutionary models behave like a well-behaved neoclassical growth model if we ignore all the complexity of the evolutionary processes working at the microscopic level and look only at the microscopic performance. It thus provides a critique of the growth accounting technique which decomposes the overall growth process into a movement along an aggregate production function and an autonomous shift of that function.

0. Introduction.

The title of this paper may sound a contradiction in terms. In the traditional economic theory, by which I include both classical and neoclassical economics, the long-run state of an economy is an equilibrium state and the long-run profits (if they ever exist) are equilibrium phenomena. *Fig.1* illustrates this by drawing two supply curves that can be found in any textbook of economics. In the upper panel is an upward-sloping supply curve which aggregates diverse cost conditions of the existing firms in an industry. Its intersection with a downward-sloping demand curve determines an equilibrium price, which in turn determines the amount of profits (represented by the shaded triangle) accruing to the industry as a whole. As long as the supply curve is upward-sloping, an industry is able to generate positive profits.

<Insert *Fig.1* around here.>

In traditional theory, however, this is merely a description of the 'short-run' state of an industry. Whenever there are positive profits, existing firms are encouraged to expand their productive capacities and potential firms are induced to enter the industry, both making the supply curve flatter and flatter. This process will continue until the industry supply curve becomes totally horizontal, thereby wiping out any opportunity for positive profits. The lower panel of *Fig.1* describes this 'long-run' state of the industry.

This implies that if there are any profits in the long-run, it must be the 'normal' profits which have already been incorporated into cost calculations. In fact, it is how to explain the fundamental determinants of these normal profits which divides the traditional economic theory into classical and neoclassical approaches. Classical economics (as well as Marxian economics) has highlighted an inverse relationship between the normal profit rate and the real wage rate, and reduced the problem of determining the former to that of determining the latter and ultimately to that of distributional conflicts between classes. Neoclassical economics has identified the normal profit rate with the interest rate plus a risk premium and reduced the problem of its determination to that of characterizing equilibrium conditions for intertemporal resource allocation under uncertainty. But no matter how opposed their views might appear over the ultimate determinants of normal profits, they share the same 'equilibrium' perspective on long-run profits -- any profits in excess of the normal rate are 'disequilibrium' phenomena which are bound to disappear in the long-run.

It is Joseph Schumpeter who gave us a powerful alternative to this deep-rooted 'equilibrium' tradition in the theory of long-run profits. According to Schumpeter, it is through an "innovation" or "doing things differently" that positive profits emerge in the capitalist economy. "The introduction of new commodities..., the technological change in the production of commodities already in use, the opening-up of new markets or of new sources of supply, Taylorization of work, improved handling of material, the setting-up of new business organizations"[1] etc. allow the innovators to charge prices much higher than costs of production. Profits are thus the premium put upon innovation. Of course, the innovator's cost advantage does not last long. Once an innovation is successfully introduced into the economy, "it becomes much easier for other people to do the same thing."[2] A subsequent wave of imitations soon renders the original innovation obsolete and gradually wears out the innovator's profit rate. In the long-run, there is therefore an inevitable tendency towards classical or neoclassical equilibrium which does not allow any positive profits in excess of the normal rate. And yet Schumpeter argued that positive profits will never disappear from the economy because capitalism is "not only never but never can be stationary." It is an "evolutionary process" that "incessantly revolutionalizes the economic structure *from within*, incessantly destroying an old one, incessantly creating a new one."[3] Indeed, it is to destroy the tendency towards classical

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[1] Schumpeter [1939], p.84.
[2] Schumpeter [1939], p.100.
[3] Schumpeter [1950], p. 83.

or neoclassical equilibrium and to create a new industrial disequilibrium that is the function the capitalist economy has assigned to those who carry out innovations. "Surplus values [i.e., profits in excess of normal rate] may be impossible in perfect equilibrium, but can be ever present because that equilibrium is never allowed to establish itself. They may always tend to vanish and yet be always there because they are incessantly recreated."[4]

The main purpose of this paper is to formalize this grand vision of Joseph Schumpeter from the perspective of evolutionary economics.[5] Indeed, in my previous papers [1984a] and [1984b], I developed a mathematically tractable evolutionary model of industrial dynamics that is capable of analyzing the evolutionary process of an industry's technology as an aggregate outcome of dynamic interactions among innovations, imitations and growth at the micro level of firms. The present paper continues the task of these papers by presenting new evolutionary models of industrial dynamics which are again capable of analyzing (only with pencils and paper) the evolutionary process of an industry's technology as an aggregate outcome of dynamic interactions among innovation, imitation and growth at the micro level of firms. What differentiate these new models from the previous one is their assumption of the step-by-step nature of innovations. In addition, two of these models also deal with embodied type technological progress. But, as far as the general features are concerned, they all can be regarded as variations of the previous evolutionary model. In any case, mathematically tractable evolutionary models are very scarce because of the intrinsic difficulty in developing dynamic models without the help of optimization technique and equilibrium concept. It is hoped that just to present a series of mathematically tractable new models will in itself be a positive contribution to evolutionary economic theory.

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[4] Schumpeter [1950], p. 28.

[5] See, for instance, Nelson and Winter [1982] , Dossi, Freeman, Nelson, Silverberg and Soete [1988] and Anderson [1994] for the exposition of "evolutionary perspective" in economics.

However, this paper's main objective lies in its use of these evolutionary models for the purpose of formally demonstrating the Schumpeterian thesis that profits in excess of normal rate never disappear from the economy no matter how long it is run.[6] Indeed, the paper will show that what the economy approaches over a long passage of time is not a classical or neoclassical equilibrium of uniform technology but (at best) a statistical equilibrium of technological disequilibria which reproduces a relative dispersion of efficiencies among firms or capitals in a statistically balanced form. Although positive profits are impossible in perfect equilibrium, they can be ever present because that equilibrium is never allowed to establish itself.

This paper is organized as follows. It is divided into two Parts. Part I presents the basics of our evolutionary models. After setting up the static structure of the industry's technology in section I-1, the following two sections (I-2 and 3) examine how firms' imitative and innovative activities evolve the state of technology over time. It is argued that while swarm-like appearance of imitations pushes the state of technology towards uniformity (hence, the economic evolution has a Lamarckian feature), punctuated appearance of innovations disrupts the imitation's equilibriating tendency. Section I-4 then turns to the long-run of the industry's state of technology. It is shown that over a long passage of time these conflicting microscopic forces will balance each other in a statistical sense and give rise to the long-run average distributions of efficiencies across firms.

Now, the central core of any evolutionary theory worthy of the name is the Darwinian selection mechanism – the fittest survives and spreads its favored traits through higher reproduction rate. In the case of economic evolutionary models, this selection mechanism works itself out through differential growth rates between efficient and inefficient firms (or, more precisely, between

[6] In Iwai [1998] I have presented the same thesis, using an evolutionary model developed in Iwai [1984b].

efficient and inefficient capitals). In order to incorporate this Darwinian mechanism, Part II of the paper superimposes the process of capital growth upon the evolutionary models of Part I. Section II-1 sets up two alternative models of capital growth – one for disembodied technology and the other for embodied technology – and works out the economic selection mechanism. Then, the following two sections (II-2 and 3) analyze the dynamic interactions between capital growth on the one hand and technological imitations and innovations on the other and derive the long-run average efficiency distributions of capital stocks. This long-run average distribution in turn allows us to deduce in section II-4 the long-run average supply curves. Indeed, it is shown that they are all *upward-sloping* and thereby capable of generating *positive* profits even in the *long-run*. Hence, the title of this paper – "disequilibrium theory of long-run profits".

The present paper will adopt the 'satisficing' principle for the description of firms' behaviors – firms do not optimize a well-defined objective function but simply follow organizational routines in deciding their growth, imitation and innovation policies.[7] Indeed, the purpose of the penultimate section (section II-5) is to show that our evolutionary model is able to "simulate" all the macroscopic characteristics of neoclassical growth model without having recourse to the neoclassical assumption of full individual rationality. If we look only at the aggregative performance of our evolutionary economy, it is as if aggregate labor and aggregate capital together produce aggregate output in accordance with a well-defined aggregate production function with Harrod-neutral technological progress. Yet, if we zoomed into the microscopic level of the economy, what we would find is the complex and dynamic interactions among many a firm's growth, imitations and innovations. It is simply impossible to group these microscopic forces into a

[7] The term "satisficing" was first coined by Simon [1957] to designate the behavior of a decision maker who does not care to optimize but simply wants to obtain a satisfactory utility or return. The notion of "organizational routines" owes to Nelson and Winter [1982].

movement along an aggregate production function and an autonomous shift of that function itself. The so-called growth accounting method in neoclassical growth theory is thus seen to have no empirical content in our Schumpeterian world.

Section II-6 concludes the paper.

Part I: An evolutionary model of Imitation and innovation

I-1. The state of technology in the short-run

Consider an industry which consists of a large number of firms competing with each other. Some firms are active participants of the industry, busy turning out products; others are temporarily staying away from production but are ready to start it when the right time comes. In order to make our description of the industry as general as possible at this stage of analysis, I will not specify the market structure until section II-1; the industry may face a perfectly competitive market or a monopolistically competitive market or other form of market for its products.

The starting point of our evolutionary model is an observation that knowledge is not a public good freely available among firms and that technologies with a wide range of efficiency coexist even in the same industry. And one of the end points of our evolutionary model is to demonstrate that in most industries a wide range of efficiency will indeed coexist among firms even in the long-run. Let F stand for the total number of firms, both active and inactive, and assume it to be constant over time. Let us suppose that at time t there are N_t distinct technologies in an industry which can be labeled as $1, 2, \ldots, N_t\text{-}1, N_t$. Let us also suppose that their efficiency can be ordered linearly from the worst to the best, so that we can identify the index 1 as the least efficient and the index N_t as the most efficient technology in an industry at time t. In section I-3 I will indeed specify that the ith technology has labor productivity equal to $e^{\lambda n}$ with $\lambda > 0$, but we do not have to commit to this particular specification in the meantime. Note that the index of the best technology has a time subscript t, because firms' innovative activities are bringing in new technologies into the industry every now and then, as we will see soon.

Let me describe how these technologies are distributed over firms, or it comes to the same thing, how firms are distributed over these technologies. For this purpose, let $f_t(n)$ stand for the relative share of firms having access to the nth technology at time t. (Their total number is then equal to $Ff_t(n)$.) It of course satisfies an adding-up equation: $f_t(1)+\ldots+f_t(n)+\ldots+f_t(N_t) = 1$. I will call the set of these shares, $\{f_t(n)\}$, the 'efficiency distribution of firms' at time t. As is illustrated in *Fig. 2*, this gives us a snap-shot picture of the distribution of firms over a spectrum of technologies from the best to the worst. Unlike the paradigm of classical and neoclassical economics, however, the state of technology is never static in a capitalist economy. As time goes by, dynamic competition among firms over technological superiority constantly changes the efficiency distribution of firms from one configuration to another.

<Insert *Fig. 2* around here.>

I will now turn to the evolutionary process of the efficiency distribution of firms in our Schumpeterian industry.

I-2. Imitations and the evolution of the state of technology

There are basically two means by which a firm can advance its technology – by innovation and by imitation. A firm may succeed in putting a new and better technology into practice by its own R&D efforts. A firm may increase its efficiency by successfully copying another firm's technology. The evolution of the state of technology is then determined by the dynamic interaction of innovations and imitations. We take up the process of imitations first.

In the present paper I will hypothesize the process of imitations as follows[8]:

Hypothesis (IM-b): Firms seek to imitate only the best technology N_t, and the probability that one of the firms imitates the best technology during a small time interval dt is equal to $\mu Ff_t(N_t)dt$; where $Ff_t(N_t)$ is the number of the firms currently using the best technology and μ (> 0) is a small constant uniform across firms. ◊

One of the characteristic features of technology as a commodity is its non-excludability. It may be legally possible to assign property rights to the owners of technology. But, as Arrow has remarked in his classic paper [1962], "no amount of legal protection can make a thoroughly appropriable commodity of something so intangible as information," because "the very use of the information in any productive way is bound to reveal it, at least in part."[9] The above hypothesis captures such spill-over effects of new technology in the simplest possible way.

The imitation parameter μ in the above hypothesis represents the effectiveness of each firm's imitative activity. As was indicated in section I-1, the present paper follows the strict evolutionary perspective in supposing that firms do not optimize but only "satisfice" in the sense that they simply follow organizational routines in deciding their imitative, innovative and growth policies. Indeed, one of the purposes of this paper is to see how far we can go in our description of the economy's dynamic performances without the assumptions of individual optimality. We thus assume μ is an exogenously given parameter.

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[8] Both *Hypothesis (IM')* and *(IM)* in Iwai [1984a and b, 1998] suppose that firms imitate not only the best technology but also any of the technologies better than the ones currently used. *Hypothesis (IM')* then assumed that the probability of imitating the nth technology is equal to $\mu f_t(n)$ per unit of time and *Hypothesis (IM)* then assumed that probability equal to $\mu s_t(n)$, where $s_t(n)$ represents the capital share of the nth technology. Note that μF in our *Hypothesis (IM-b)* corresponds μ in *Hypotheses (IM')* and *(IM)* in those papers.

[9] p. 615.

Now, the hypothesis *(IM-b)* allows us to analyze the evolution of the efficiency distribution of firms in the following manner. First, consider the evolution of the share of the best technology firms $f_t(N_t)$. Its value increases whenever one of the firms using a lesser technology succeeds in imitating the best technology N_t. Since the relative share of those firms is $1-f_t(N_t)$ and the probability of such a success for each firm is $\mu F f_t(N_t)dt$ during a small interval dt, we can calculate the expected increase in $f_t(N_t)$ during dt as $(\mu F f_t(N_t)dt)(1-f_t(N_t))$. If the number of firms F is sufficiently large, we can apply the law of large numbers and deduce the following differential equation (as a good approximation) for the *actual* rate of change in $f_t(N_t)$:

$$(1)\quad \dot{f}(N_t) = \mu F f_t(N_t)(1-f_t(N_t))\ .$$

This is of course a well-known 'logistic differential equation' with a logistic parameter μF. It is not hard to solve it to obtain the following explicit formulae for its evolutionary process over time.[10] Setting T as an initial time, we have for $t \geq T$:

$$(2)\quad f_t(N_t) = \frac{1}{1+(1/f_T(N_t)-1)e^{-\mu F(t-T)}}\ ,$$

where e stands for the exponential. This is nothing but a 'logistic growth curve' which frequently appears in population biology and mathematical ecology.

Next, consider the evolution of the relative share of the firms employing one of the less efficient technologies or of $f_t(n)$ for $n=N_t-1,\ N_t-2,\ \ldots,\ 1$. This share never increases but decreases whenever one of those firms succeeds in imitating the best technology. Since the relative share of those firms is $f_t(n)$ and the probability of such a success for each firm is $\mu F f_t(N_t)dt$ during a time interval dt, we can calculate the expected decrease in $f_t(n)$ as $(\mu F f(N_t)dt)f_t(n)$.

[10] A logistic differential equation: $x' = ax(1-x)$ can be solved as follows. Rewrite it as: $x'/x-(1-x)'/(1-x)$ and integrate it with respect to t, we obtain: $\log(x)-\log(1-x) = \log(x_0)-\log(1-x_0)+at$, or $x/(1-x) = e^{at}x_0/(1-x_0)$. This can be rewritten as: $x = 1/(1+(1/x_0-1)e^{-at})$, which is nothing but a logistic equation given by *(2)*.

The law of large numbers then enables us to deduce the following differential equation (as a good approximation) for the *actual* rate of change in $f_t(n)$:

$$(3)\quad \dot{f}(n) = -\mu F f_t(N_t) f_t(n) \qquad (n = 1, 2, \ldots, N_t-1).$$

We can also solve this to obtain the following formula:

$$(4)\quad f_t(n) = \frac{f_T(n)(1 - f_t(N_t))}{1 - f_T(N_t)} \qquad (n = 1, 2, \ldots, N_t-1);$$

for $t \geq T$. This equation describes the way the share of the lesser technologies decays over time.

"If one or a few have advanced with success many of the difficulties disappear," so wrote Schumpeter, "others can then follow these pioneers, as they will clearly do under the stimulus of the success now attainable. Their success again makes it easier, through the increasingly complete removal of the obstacles ..., for more people follow suit, until finally the innovation becomes familiar and the acceptance of it a matter of free choice."[11] The set of growth equation *(2)* and decay equations *(4)* describes this swarm-like process of technological imitation in one of the simplest possible mathematical forms.

<Insert *Fig. 3* around here.>

Fig. 3 illustrates all these curves into a three-dimensional diagram. Its x-axis measures time, y-axis technology index, and z-axis the share of firms. The *S*-shaped curve in the front traces the growth pattern of the share of the best technology firms. Every other curve traces the decaying pattern of the share of each of the lesser technology firms. These curves give us a motion picture of the evolution of the state of technology under the pressure of imitative activities. When only a small fraction of firms use the best technology, imitation is difficult and the growth of its users is slow. But one imitation breeds another and a bandwagon soon sets in motion. The growth of the share of the best technology accelerates until a half of the firms come to adopt it. Then, the growth starts decelerating, while the share itself

[11] Schumpeter [1934], p. 228.

continues to grow until it absorbs the whole population in the industry. In the long-run, therefore, the best technology will dominate the entire industry. This technological diffusion process is nothing but an economic analogue of the "Lamarkian" evolutionary process.

I-3. Innovations and the evolution of the state of technology

Does this mean that the industry's long-run state is no more than the paradigm of classical and neoclassical economics where every market participant has complete access to the best technology available in the industry?

The answer is, however, a "No". And the key to this answer lies in the innovation -- the carrying out of what Schumpeter called a "new combination". Indeed, the function of firms' innovative activities is precisely to destroy this evolutionary tendency towards a static equilibrium.

Suppose that at some point in time one of the firms succeeds in introducing a new and better technology into the industry. From that time on this new technology takes over the best technology index N_t and the former best technology is demoted to the lesser index $N_t\text{-}1$. Let $T(N_t)$ stand for this epoch and call it the 'innovation time' for N_t. Since the total number of firms is F, this means that at $t = T(N_t)$ a new share $f_t(N_t)$ emerges out of nothing and takes the value of $1/F$.

No sooner does this innovation take place than do all the lesser technology firms start to seek the opportunities to imitate it. Under *Hypothesis (IM-b)*, this sets in motion a new logistic growth curve *(2)* of $f_t(N_t)$ from an initial share $1/F$ from $t = T(N_t)$ on. Hence, we have:

$$(5) \quad f_t(N_t) = \frac{1}{1+(F-1)e^{-\mu F(t-T)}} \qquad \text{for } t \geq T(N_t)\ .$$

As for the lesser technologies (including the former best technology which now has the index of $N_t\text{-}1$), each of their shares follows a decay curve which has the same mathematical form as *(4)*. We thus have:

$$(6)\quad f_t(n) = \frac{f_{T(N_t)}(n)(1 - f_t(N_t))}{1 - 1/F} \qquad (n = 1, 2, \ldots, N_t-1).$$

Note here that if the innovator used technology *m* just before the innovation, the share $f_t(m)$ loses $1/F$ at $t = T(N_t)$. But all the other shares traverse the innovation time $T(N_t)$ without any discontinuity.

Fig. 4 **squeezes all these processes (and more) into a three-dimensional diagram.**

<Insert *Fig. 4* around here.>

Innovation is not a single-shot phenomenon, however. No sooner than an innovation takes place, a new round of competition for a better technology begins. And no sooner than a new winner of this game is named, another round of technological competition is set out. The whole picture of *Fig.4* exhibits how the industry's state of technology evolves over time as a dynamic interplay between two opposing technological forces -- swarm-like appearance of imitations and creative destruction of innovations. While the former works as an equilibriating force which tends the state of technology towards uniformity, the latter works as a disequilibriating force which destroys this leveling tendency.

A new question then arises: is it possible to derive any law-like properties about the industry's state of technology out of this seemingly erratic movement?

In order to give an answer to this question, we need to characterize the nature of technological innovations in more detail. The first hypothesis concerning the nature of innovations is about its effect on labor productivity.

Hypothesis (PG)**: Each innovation causes the productivity of the industry's best technology N_t to grow by a fixed rate of λ (> 0). If we denote by *a(n)* the labor productivity of the *n*th innovation, then it is given by $e^{\lambda n}$. ◊**

The next assumption is concerned with the stochastic nature of the way innovations take place over time. Indeed, in this paper we present two alternative hypotheses.

Hypothesis (IN-a): All the firms have an equal chance for an innovation. The probability that each firm succeeds in an innovation during a small time interval dt is νdt; whereν (> 0) is a very small constant uniform across firms. ◊

Hypothesis (IN-b): Only the firms using the best technology are able to carry out a next innovation. The probability that each of the best technology firms succeeds in an innovation during a small time interval dt is ξdt; where ξ (> 0) is a very small constant uniform across firms. ◊

The above two hypotheses constitute two polar cases about the pool of potential innovators from which a next innovator is drawn. *Hypothesis (IN-a)* insists that there is no prerequisite knowledge for a firm to become an innovator, whereas *Hypothesis (IN-b)* insists that one has to practice the most advanced technology in order to make a further progress on it. The reality seems to lie somewhere in between.

The innovation parameter, ν or ξ, represents the effectiveness of each firm's innovative activity in the industry. In the present paper which has adopted en evolutionary perspective, their values are taken as exogenously given. Note that since the total number of firms is F, what *Hypothesis (IN-a)* implies is that the probability that an innovation occurs during a small time interval dt is νFdt. Note also that the total number of the best technology firms is given by $Ff_t(N_t)$, what *Hypothesis (IN-b)* implies is that the probability that an innovation occurs during a small time interval dt is $\xi Ff_t(N_t)dt$.

We have already defined $T(N_t)$ as the time at which a technology N_t is introduced for the first time into an industry. A difference between two adjacent innovation times, $T(N_t+1)-T(N_t)$, thus defines a 'waiting period' for the next innovation. Let $W(t)$ denote its probability distribution, or $W(t) \equiv Pr\{T(N_t+1)-T(N_t) \leq t\}$. Then, *Appendix A* shows that it can be expressed as:

$$(7a) \quad W(t) = 1- e^{-\nu Ft} \qquad \text{for } t \geq 0,$$

under *Hypothesis (IN-a)* and as:

$$(7b) \quad W(t) = 1-\left(\frac{(F-1)+e^{\mu F t}}{F}\right)^{-\xi/\mu} \text{ for } t \geq 0,$$

under *Hypothesis (IN-b)*. Let ω denote the 'expected waiting time' of the next innovation, or $\omega \equiv E(T(N_t+1)-T(N_t)) = \int_0^{\infty} t dW(t)$. Then, *Appendix A* also shows that it can be calculated as:

$$(8a) \quad \omega = \frac{1}{\nu F},$$

under *Hypothesis (IN-a)*, and the value of:

$$(8b) \quad \omega = \sum_{i=0}^{\infty} \frac{(1-1/F)^i}{(\xi+\mu i)F},$$

under *Hypothesis (IN-a)*. Note that an increase in ν decreases ω of *(8a)* and an increase in ξ and μ decreases ω of *(8b)*.

I-4. The efficiency distribution of firms in the long-run.

The state of technology given by $\{f_t(n)\}$ is a historical outcome of the dynamic interaction between imitations and innovations. A swarm-like appearance of imitations is an equilibriating force which pushes the industry's state of technology towards uniformity, whereas the intermittent arrival of an innovation is a disequilibrium force which destroys such tendency towards technological uniformity. Every time an innovation has taken place, a new round of imitative activities starts from scratch and resumes their pressure towards technological uniformity. As time goes by, however, innovations turn up over and over again and reset the process of imitations over and over again. In fact, under both *Hypothesis (IN-a)* and *Hypothesis (IN-b)*, the sequence of the waiting periods for the next innovation are mutually independent random variables having the same probability distribution $W(t)$, and the whole movement of the shares of technologies within each waiting period becomes a statistical replica of each other. This means that the entire evolutionary process of the state of technology now constitutes what is called a 'renewal process' in the probability theory.[12] (As

[12] A process E is called a 'renewal process' if after each occurrence of E the trials

a matter of fact, under *Hypothesis (IN-a)* it constitutes the simplest of all renewal processes -- a 'Poisson process'.) We can thus expect that over a long passage of time a certain statistical regularity will emerge out of its seemingly irregular patterns.

The first regularity we want to examine is about the productivity growth rate of the best technology. For this purpose let us note that N_t, the index of the best technology at time t, can also be identified with the number of innovations occurring from time 0 to time t. Then, the so-called 'renewal theorem' in the probability theory says that as t becomes very large, the random occurrence of innovations will be gradually averaged out and the expected rate of innovations $E(N_t)/t$ will approach the inverse of the expected waiting period $1/\omega$.[13] Since by *Hypothesis (PG)* each innovation raises the productivity by a rate λ, the productivity of the industry's best technology is expected to grow at the rate of λ/ω in the long-run. Or, by *(8a)* we have

$$(9a) \quad E\left(\frac{\log(a(N_t))}{t}\right) \rightarrow vF\lambda$$

when every firm can innovate (i.e. under *Hypothesis (IN-a)*), and by *(8b)* we have:

$$(9b) \quad E\left(\frac{\log(a(N_t))}{t}\right) \rightarrow \frac{F\lambda}{\sum_i((1-1/F)^i/(\xi+\mu i))},$$

when only the best technology firm can innovate (i.e. under *Hypothesis (IN-b)*). It is not hard to show that an increase in v and λ increases the long-run growth rate of the best technology under *Hypothesis (IN-a)* and that an increase in ξ, μ and λ increases the long-run growth rate of the best technology under *Hypothesis (IN-b)*. The industry's productivity growth rate is thus endogenously determined in the present evolutionary model.[14]

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start from scratch in the sense that the trials following an occurrence of E form a replica of the whole experiment, or if the waiting times $W(t)$ between successive events are mutually independent random variables having the same distribution. See, for instance, Feller [1966] Chap. 11 and Cox and Miller [1965], Chap. 9 for the general discussion on the renewal process.

[13] See Feller [1966].

[14] In contrast to the models in Iwai [1984a, 1984b, 1998] which have assumed that the

Indeed, not merely the process of innovations but also the entire evolution of the efficiency distribution of firms is expected to exhibit a statistical regularity in the long run. Of course, we cannot hope to detect any regularity just by tracking the motion of the efficiency distribution as it is, for it is incessantly shifted in the direction of higher productivity. If there is any statistical regularity at all, it must come out from the recurrent pattern of their *relative* structure over a long passage of time.

Accordingly, let us focus on the sequence of technology indices arranged in reverse order, $N_t, N_t\text{-}1, \ldots, N_t\text{-}i, \ldots$. As t moves forward, a technology occupying each of these indices becomes better and better. But the best technology N_t is always the best technology, the second-best technology $N_t\text{-}1$ the second-best, ..., the $i+1^{st}$-best technology $N_t\text{-}i$ the $i+1^{st}$-best, and so on, independently of their actual occupants. The set of the technology shares, $\{f_t(N_t), f_t(N_t\text{-}1), \ldots, f_t(N_t\text{-}i), \ldots\}$, thus represents the *relative* form of the efficiency distribution of firms. Let us now determine their long-run average configuration.

Here, I am omitting all the mathematical details and only reporting the results obtained in *Appendix B* first in the form of geometry and then in the form of algebraic equations.

<Insert *Fig.5a* and *Fig.5b* around here.>

Fig.5a above illustrates the relative form of long-run average efficiency distribution when every firm can innovate, and *Fig. 5b* the relative form of the long-run average state of technology when only the best technology firm's can innovate. The former has the form of geometric distribution, and the latter usually has a peak at the second-best technology and assumes the form of geometric distribution from that point on.

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productivity growth rate of the best possible technology is determined exogenously by the inventive activities of academic institutions, private firms, government agencies and amateur inventors outside of the industry.

The mathematical formulae for these curves are given as follows. Under *Hypothesis (IN-a),* we have:

$$(10a)\quad E\{f_t(N_t)\} \rightarrow \Phi,$$

$$E\{f_t(N_t\text{-}i)\} \rightarrow \Phi(1\text{-}\Phi)^i \qquad (i = 1, 2, \ldots, N_t\text{-}1),$$

where

$$(11)\quad \Phi \equiv \int_0^{\infty} \frac{\nu F e^{-\nu F z}}{1+(F\text{-}1)e^{-\mu F z}} dz = \int_{1/F}^{1} \left(\frac{x}{1/F}\right)^{1\text{-}\nu/\mu} \left(\frac{1\text{-}x}{1\text{-}1/F}\right)^{\nu/\mu} dx\ .$$

(Note that $0 < \Phi < \nu(F\text{-}1)^{1\text{-}\nu/\mu}/2 < \nu F/2$. This is of course much smaller than unity.) It is not hard to show that $\partial\Phi/\partial\nu > 0$ and $\partial\Phi/\partial\mu < 0$. Under *Hypothesis (IN-b),* we have:

$$(10b)\quad E\{f_t(N_t)\} \rightarrow \frac{1}{\xi F \omega} = \frac{1}{\sum_i (1 - 1/F)^i/(1+i(\mu/\xi))},$$

$$E\{f_t(N_t\text{-}i)\} \rightarrow \left(1-\frac{1}{\xi F\omega}\right)\left(\frac{\mu}{\xi+\mu}\right)\left(\frac{\xi}{\xi+\mu}\right)^{i-1} \qquad (i = 1, 2, \ldots N_t\text{-}1).$$

It is again not hard to show that $\partial E\{f_t(N_t)\}/\partial\xi < 0$ and $\partial E\{f_t(N_t)\}/\partial\mu > 0$.

We have thus seen that what our Schumpeterian industry approaches over a long passage of time is not a classical or neoclassical equilibrium of uniform technology but a statistical equilibrium of technological disequilibria which reproduces a relative dispersion of efficiencies among firms or capitals in a statistically balanced form.

Part II: Process of capital accumulation and the long-run industry supply curve

II-1. The mechanism of economic selection.

One of the main pillars of any evolutionary theory worthy of the name is the Darwinian selection mechanism – the fittest survives and spreads their traits through higher reproduction rates. In the case of economic evolutionary process, this Darwinian mechanism works through differential growth rates between efficient and inefficient capitals. If technology is not embodied in capital stock those firms which carry better technology generate higher profits and accumulate their capitals more rapidly than others, and if technology is embodied in capital stock those capital stocks which embody better technology generate higher profits and are accumulated more rapidly than others. Let us now introduce the process of capital accumulation into our evolutionary models.

Let $k_t(n)$ denote the total capital stock carrying technology n at time t and by K_t the total capital stock accumulated in the entire industry at time t. We of course have: $K_t = k_t(N_t)+k_t(N_t-1)+\ldots+k_t(1)$. We can then define the 'capital share' of the n^{th} technology by $k_t(n)/K_t$ and denote it by $s_t(n)$. We call the set of capital shares, $\{s_t(n)\}$, the 'efficiency distribution of capitals.' As is illustrated in *Fig. 6*, it gives us a snapshot picture of the way the industry's total capital stock is distributed over a spectrum of technologies ranging from the most to the least efficient.

<Insert *Fig. 6* around here>

As is the case of efficiency distribution of firms $\{f_t(n)\}$, the efficiency distribution of capital stocks $\{s_t(n)\}$ is never static in a capitalistic economy. Differential growth rates between efficient and inefficient capitals as well as technological competition among firms constantly change its configuration over time. In order to analyze this process, we now have to specify the structure of the markets as well as the structure of each technology much more concretely than in Part I.

First, let us assume that each technology is of Leontief-type fixed proportion technology with labor service as the sole variable input and capital stock as the sole fixed input. Let us also assume that only the labor productivity varies across technologies, so that the n^{th} technology can be written as:

(12) $y = Min[a(n)l, bk]$,

where y, l and k denote output, labor and capital, respectively, and $a(n)$ and b denote labor productivity and capital productivity, respectively. Because of *Hypothesis (PG)* we have $a(n) = e^{\lambda n}$, but b is assumed to be constant over time and uniform across technologies.

Next, let us suppose also that every firm in the industry produces the same product and hires homogeneous workers. They thus face the same price for the products they produce and the same money wage rate for workers they hire. Let P_t and W_t denote the product price and money wage rate at time t and let $r_t(n)$ denote the real rate of profit (in terms of product price) accruing from the use of the n^{th} technology at time t. If the price of capital equipment is equal to the price of product, the latter can be calculated as $r_t(n) = (P_t y - W_t l)/P_t k = b(1-(W_t/P_t)a(n))$. For analytical convenience, we approximate this as $b(log(a(n))-log(W_t/P_t))$. This is not a bad approximation, as long as the labor productivity $a(n)$ and the real wage rate W_t/P_t are not so wide apart. Since $a(n) = e^{\lambda n}$, this can be further rewritten as $b(\lambda n-log(W_t/P_t))$.

"Without development there is no profit, without profit no development," so said our Schumpeter. "For the capitalist system ... without profit there would be no accumulation of wealth."[15] Our next step is to relate the firms' capital growth policy to the rate of profit it is earning. Here, I would like to introduce two alternative hypotheses --- one for the case of disembodied

[15] Schumpeter [1961], p. 154.

technology and the other for the case of embodied technology. In the case of disembodied technology all the capital stocks accumulated in a firm have the same productivity, whereas in the case of embodied technology different capital stocks may carry different technologies even in the same firm. We have:

Hypothesis (CG-d): Technology is not embodied in capital goods, and the growth rate of a firm possessing n^{th} technology is linearly dependent on its rate of profit, or it is given by: $\gamma r_t(n) - \gamma_0$, where $\gamma (> 0)$ and $\gamma_0 (> 0)$ are given positive constants uniform across firms. ◊

Hypothesis (CG-e): technology is embodied in capital goods, and the growth rate of capital stocks embodying n^{th} technology is linearly dependent on their rate of profit, or it is given by: $\gamma r_t(n) - \gamma_0$, where $\gamma (> 0)$ and $\gamma_0 (> 0)$ are given positive constants uniform across both technologies and firms. ◊

Each of the above hypotheses tries to capture the Darwinian mechanism of economic selection in the simplest possible manner – the capital stock earning higher profit rates are expected to grow faster than the others and enlarge the shares of the technology they use. The parameter γ represents the sensitivity of the firm's growth rate to the rate of profits, and the parameter γ_0 represents the rate of capital depreciation of the break-even firm or break-even capital stock. As was remarked in section I-2, their values are taken as exogenously given in the present paper.

Now, all the hypotheses in my evolutionary models are finally laid out. First, *Hypothesis (IM-b)* concerning the spill-over effects of the best technology through imitations. Second, *Hypothesis (PG)* concerning the step-by-step nature of innovations. Third, *Hypotheses (IN-a)* and *(IN-b)* concerning the nature of potential innovators and their success probability – the one supposing that every firm has an equal chance for innovation and the other supposing that only the best technology firms can innovate. And finally, *Hypotheses (CG-d)* and *(CG-e)* concerning firms' capital accumulation process – the one for the case of disembodied technology and the other for the case of embodied technology. We are now in a position of analyzing how do these microscopic forces combine with each other and move the entirety of the efficiency distribution of capitals $\{s_t(n)\}$ over time. It is necessary to proceed step by step.

In the first step, let us ignore the existence of both technological imitations and technological innovations for the time being so as to place the process of economic selection in full relief. To begin with, let us note that both *Hypothesis (CG-d)* and *Hypothesis (CG-e)* imply that the growth rate of capital stock with technology n can be expressed as:

$$(13) \quad \frac{\dot{k}_t(n)}{k_t(n)} = \gamma(\lambda n - \log(W_t/P_t)) - \gamma_0 \qquad (n = 1, 2, \ldots, N_t-1, N_t).$$

If we substitute this relation into the right-hand-side of an identity: $\dot{s}_t(n)/s_t(n) \equiv \dot{k}_t(n)/k_t(n) - \dot{K}_t/K_t$ and rewrite the resulting expression as $\gamma(\lambda n - \log(W_t/P_t)) - \gamma(\lambda \sum_n n s_t(n) - \log(W_t/P_t)) = \gamma\lambda(n - \sum_n n s_t(n)) = \gamma\lambda(n - N_t s_t(N_t) - \sum_{n \neq Nt} n s_t(n)) = \gamma\lambda(n - N_t + (1-s_t(N_t))N_t - (1-s_t(N_t))\sum_{n \neq Nt} n s_t(n)/(1-s_t(N_t)))$, we can deduce the following set of differential equations for the capital shares $\{s_t(n)\}$:

$$(14) \quad \dot{s}_t(N_t) = (\gamma\lambda\zeta_t)s_t(N_t)(1-s_t(N_t)),$$

$$(15) \quad \dot{s}_t(N_t - i) = -(\gamma\lambda)(\zeta_t s_t(N_t) + (i-\zeta_t))s_t(N_t-i) \qquad (i = 1, 2, \ldots, N_t-1);$$

where ζ_t represents the gap between the best technology index and the average index of all the rest and is defined by:

$$(16) \quad \zeta_t \equiv N_t - \sum_{n}^{N_t-1} \frac{n s_t(N_t)}{1-s(N_t)}.$$

The value of ζ_t in general depends on t. But we can also expect it to move only slowly over time.[16] Indeed, for the sake of simplicity, we will from now on proceed our exposition as if it were actually an exogenously given constant ζ. Then, *(14)* takes the same logistic form as *(1)*, and *(15)* the same mathematical form as *(2)* except for an additional term $-(\gamma\lambda)(i-\zeta)s_t(N_t-i)$. We can thus solve them to obtain:

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[16] It would be useful to approximate the value of ζ explicitly by using a fixed-point method recently proposed by Franke [1998] for the model of Iwai [1984b].

$$(17)\quad s_t(N_t) = \frac{1}{1+(1/s_{T(N_t)}-1)e^{-\gamma\lambda\zeta F(t-T(N_t))}},$$

$$(18)\quad s_t(N_t-i) = e^{-\gamma\lambda(i-\zeta)(t-T(N_t))}\frac{1-s_t(N_t)}{1-s_{T(N_t)}(N_t)}s_{T(N)}(N_t-i) \qquad (i = 1, 2, \ldots, N_t-1),$$

for $T(N_t) \leq t$.

<Insert *Fig. 7* around here.>

Fig. 7 illustrates the movement of the whole set of capital shares $\{s_t(N_t-i)\}$ under the sole pressure of economic selection in a three-dimensional diagram whose x-axis measures time, z-axis technology index, y-axis capital shares. It looks very much like *Fig. 3* of section I-2 which illustrated the diffusion process of the best technology among firms. In fact, we have again encountered a now familiar *S*-shaped logistic growth curve, this time tracing out the motion of the best technology's capital share $s_t(N_t)$. Yet, the logic behind *Fig. 7* is entirely different from that of *Fig. 3*. In contrast to the Lamarkian evolutionary process depicted in *Fig. 3*, what *Fig. 7* illustrates is a Darwinian evolutinary process which constantly shifts the distribution of capital shares from the lesser technology to the best technology through the relative difference in their capital growth rates. When the capital share of the best technology is very small, that share can grow almost exponentially by constantly absorbing the shares of the lesser technologies. But, as the best technology begins to occupy a larger and larger capital share, the shares of the lesser technologies it absorbs become smaller and smaller. It gradually loses its growth momentum, but keeps growing nevertheless until it finally swallows the whole industry. If there is neither technological imitation nor technological innovation, only the fittest will survive in the long-run state of the industry, and this of course reproduces the mechanism of natural selection in the world of economics.

II-2. Capital growth and technological diffusion.

In our Schumpeterian industry, there is a continuous wave of technological imitations as well as an intermittent arrival of technological innovations, incessantly interfering the way capital stocks are accumulated over time. We thus have to modify the economic selection process discussed in the preceding section in order to take account of such technological interference. In the present section I will re-introduce the process of technological diffusion, leaving the re-introduction of technological innovations to the next section.

As we will now see, the impact of technological imitation on capital accumulation process is different between when it is embodied in capital goods and when it is not. Let me examine the case of disembodied technology first.

Now, we know from *(1)* that under *Hypothesis (IM-b)* during $F df_t(N_t)/dt = \mu F^2 f_t(N_t)(1-f_t(N_t))$ firms come to use the best technology N_t by imitation each time unit. In the case of disembodied technology, these firms can transform all their capital stocks into the most efficient ones. Since their average capital share is $(1-s_t(N_t))/(F(1-f_t(N_t)))$, their successful imitations on average increase the best technology's capital share $s_t(N_t)$ by $((1-s_t(N_t))/(F(1-f_t(N_t))))\mu F^2 f_t(N_t)(1-f_t(N_t)) = \mu F f_t(N_t)(1-s_t(N_t))$. Adding this to the right-hand-side of *(14)*, we have:

(19d) $\dot{s}_t(N_t) = ((\gamma\lambda\zeta)s_t(N_t)+\mu F f_t(N_t))(1-s_t(N_t))$,

for $T(N_t) \leq t < T(N_t+1)$. By the same token, we know from *(2)* that $-F\dot{f}_t(N_t-i) = -\mu F^2 f_t(N_t) f_t(N_t-i)$ firms abandon technology N_t-i by imitating the best technology N_t during each time unit. Since the average capital share of these firms is equal to $s_t(N_t-i)/(Ff_t(N_t-i))$, they on average subtract $\mu F^2 f_t(N_t) f_t(N_t-i) s_t(N_t-i)/(Ff_t(N_t-i)) = \mu F f(N_t) s_t(N_t-i)$ from the right-hand-side of *(15)*. Hence, under both *Hypothesis (CG-d)* and *Hypothesis (IM-b)* we have:

(20d) $\dot{s}_t(N_t-i) = -((\gamma\lambda\zeta)s_t(N_t)+(\gamma\lambda)(i-\zeta)+\mu F f_t(N_t))s_t(N_t-i)$ $\quad (i = 1, 2,\ldots,N_t-1)$,

for $T(N_t) \leq t < T(N_t+1)$. As will be shown in *Appendix C*, it is possible to solve these two differential equations and derive (after some hard work) the following rather formidable expressions:

$$(21d)\ s_t(N_t) = 1-\frac{\left(\frac{f_t(N_t)}{1/F}\right)^{-\gamma\lambda\zeta/\mu F}\left(\frac{1-f_t(N_t)}{1-1/F}\right)^{1+\gamma\lambda\zeta/\mu F}}{\frac{1}{1-s_{T(N_t)}}-\left(\frac{\gamma\lambda\zeta}{\mu F}\right)\int_{1/F}^{f_t(N_t)}\left(\frac{x}{1/F}\right)^{-\gamma\lambda\zeta/\mu F}\left(\frac{1-x}{1-1/F}\right)^{1+\gamma\lambda\zeta/\mu F}dx},$$

where $f_t(N_t)$ is a logistic curve $1/(1+(F-1)e^{\mu F(t-T)})$ defined by *(5)*; and

$$(22d)\ s_t(N_t-i) = e^{-\gamma\lambda(i-\zeta)(t-T(N_t))}\frac{1-s_t(N_t)}{1-s_{T(N_t)}(N_t)}s_{T(N)}(N_t-i) \qquad (i = 1,\ 2,\ \ldots,\ N_t-1),$$

for $T(N_t) \le t < T(N_t+1)$.

Let us next examine the case of embodied technology. Again under *Hypothesis (IM-b)* during each time unit $Fdf_t(N_t)/dt$ firms come to imitate the best technology N_t. In the case of disembodied technology, however, these firms have to invest in new capital stocks in order to be able to use the newly adopted technology. Let us denote by σK_t the minimum capital stock that is necessary to start a new production process and assume that the coefficient σ is invariant over time. (This initial capital share is assumed to be financed by bank credit.) Then, the firms' successful imitations increase the best technology capital stock $k_t(N_t)$ by $\sigma F\dot{f}_t(N_t)$. If we note that they also increase the total quantity of capital stock K_t by the same magnitude, we can calculate the contribution of these embodied imitations to the rate of change in the best technology's capital share as: $\dot{s}_t(N_t) = \dot{k}_t(N_t)/K_t - s_t(N_t)\dot{K}_t/K_t = \sigma F\dot{f}_t(N_t)(1-s_t(N_t))$. If we add this to *(15)*, we have:

$$(19e)\quad \dot{s}_t(N_t) = ((\gamma\lambda\zeta)s_t(N_t)+\sigma F\dot{f}_t(N_t))(1-s_t(N_t)),$$

for $T(N_t) \le t < T(N_t+1)$. By the same token, we can calculate the rate of change in the lesser technology's capital share as:

$$(20e)\quad \dot{s}_t(N_t-i) = -((\gamma\lambda\zeta)s_t(N_t)+(\gamma\lambda)(i-\zeta)+\sigma F\dot{f}_t(N_t))s_t(N_t-i) \qquad (i = 1,\ 2,\ldots,N_t-1)$$

for $T(N_t) \le t < T(N_t+1)$. As will be seen in *Appendix C*, it is again possible to solve these two differential equations and deduce the following expressions for the motion of capacity shares:

$$(21e)\ s_t(N_t) = \frac{\nu F e^{-\gamma\lambda\zeta(t-T(N_t))-\sigma F(f_t(N_t)-1/F)}}{(1+\sigma)-(\gamma\lambda\zeta)\int_0^{t-T(N_t)} e^{-\gamma\lambda\zeta s-\sigma F(f_s(N_s)-1/F)}ds}.$$

$$(22e)\ s_t(N_t-i) = e^{-\gamma\lambda(i-\zeta)(t-T(N_t))}\frac{1-s_t(N_t)}{1+\sigma}s_{T(N)}(N_t-i) \quad (i = 1, 2, \ldots, N_t-1),$$

for $T(N_t) \leq t < T(N_t+1)$. (Here we have used the fact that $s_{T(N_t-i)}(N_t-i) = \sigma/(1+\sigma)$ and $s_{T(N_t)}(N_t-i) = s_{T(N_t)-0}(N_t-i)/(1+\sigma)$, because at each innovation time not only the capital stock of the best technology jumps from 0 to σK_t but also the total capital stock increases from K_t to $(1+\sigma)K_t$.)

<Insert *Fig. 8d* and *Fig. 8e* around here.>

Fig. 8d illustrates the motion of the capital shares in the case of disembodied technology (i.e., under *Hypothesis (CG-d)*), given by *(21d)* and *(22d)*, and *Fig. 8e* in the case of embodied technology (i.e., under *Hypothesis (CG-e)*), given by *(21e)* and *(22e)*. In particular, the left-most portions of these two diagrams show how the Darwinian mechanism of economic selection and the Lamarkian process of technological diffusion jointly contribute to the logistic-like growth process of the best technology's capital share -- the former by growing the most efficient capital stocks relative to the other and the latter by diffusing the best technology throughout the industry. While the Darwinian mechanism of economic selection represents a centralizing force, the Lamarkian process of technological diffusion represents a decentralizing force in the industry. But, no matter how opposed the underlying logic might be, their effects upon the efficiency distribution of capital stocks are the same – the best technology tends to dominate the industry's entire capital stocks in the long-run, other things being equal. But, of course, other things will not be equal in the long-run.

II-3. Growth, imitation and innovation in the long-run.

Finally, let me re-introduce the process of technological innovations into our picture. As is seen by simply tracing out the evolutionary curves of various capital shares from left to right in *Fig. 8d* and *Fig. 8e* of the preceding section, it is again the recurrence of innovations that destroys the joint force of economic selection and technological diffusion that moves the efficiency distribution of industry's capital shares or capitals towards uniformity. Innovation is a disequilibrium force of the industry structure.

As time goes by, however, innovations turn up over and over again and reset the processes of economic selection and technological diffusion over and over again. We have already shown in section I-4 how the efficiency distribution of firms will in the long-run exhibit a certain statistical regularity. It is the task of the present section to examine whether the efficient distribution of capitals will also exhibit some statistical regularity over a long passage of time. We are thus concerned with the long-run average configuration of $\{s_t(N_t),\ s_t(N_t-1),\ \ldots,\ s_t(N_t-i),\ \ldots\}$.

Since before us are four different versions of evolutionary models as 2×2 combinations of *Hypotheses (IN-a)* and *(IN-b)* on the one hand and *Hypotheses (CG-d)* and *(CG-e)* on the other, and since the required calculations are rather lengthy, we only report here the results obtained for each model in *Appendix D*.

(i): The case where technology is disembodied and every firm can innovate, i.e., where *Hypotheses (IN-a)* and *(CG-d)* hold.

As $t \to \infty$, we have:

$$(23ad)\quad E\{s_t(N_t)\} \to 1-\Gamma_\zeta\ ;$$

$$E\{s_t(N_t-i)\} \to (1-\Gamma_\zeta)\Gamma_1 \ldots \Gamma_i \qquad (i = 1, 2, \ldots, N_t-1),$$

where

$$(24ad)\quad \Gamma_i \equiv \int_0^\infty \frac{(\varphi(z)/(1/F))^{-\gamma\lambda\zeta/\mu F}((1-\varphi(z))/(1-1/F))^{1+\gamma\lambda\zeta/\mu F}\,\nu F e^{-(\gamma\lambda(i-\zeta)+\nu F)z}dz}{F/(F-1)-(\gamma\lambda\zeta/\mu F)\int_{1/F}^{\varphi(z)}(x/(1/F))^{-\gamma\lambda\zeta/\mu F}((1-x)/(1-1/F))^{1+\gamma\lambda\zeta/\mu F}dx},$$

and $\varphi(z) \equiv 1/(1+(F-1)\,e^{-\mu F z})$. Note that $1 > \Gamma_1 > \Gamma_2 > \ldots > \Gamma_{i-1} > \Gamma_i > 0$.

(ii): The case where technology is embodied and every firm can strike an innovation, i.e., where *Hypotheses (IN-a)* and *(CG-e)* hold.

As $t \to \infty$, we have:

(23ae) $E\{s_t(N_t)\} \rightarrow 1-\Lambda_\zeta$;

$E\{s_t(N_t-i)\} \rightarrow (1-\Lambda_\zeta)\Lambda_1 \ldots \Lambda_i$ $\quad (i = 1, 2, \ldots, N_t-1),$

where

$$(24ae)\ \Lambda_i \equiv \int_0^\infty \frac{\nu F e^{-\gamma\lambda(i-\zeta+\zeta z)-\sigma F(\varphi(z)-1/F)-\nu F z}dz}{1/(1-\sigma)-(\gamma\lambda\zeta)\int_0^z e^{-\gamma\lambda\zeta y-\sigma F(\varphi(y)-1/F)}dy}.$$

Note that $1 > \Lambda_1 > \Lambda_2 > \ldots > \Lambda_{i-1} > \Lambda_i > 0$.

(iii): The case where technology is embodied and only the best technology firms can strike an innovation, i.e., where *Hypotheses (IN-b)* and *(CG-e)* hold.

As $t \rightarrow \infty$, we have:

(23be) $E\{s_t(N_t)\} \rightarrow 1-\Omega_\zeta$;

$E\{s_t(N_t-1)\} \rightarrow (1-\Psi_\zeta)\Omega_1$;

$E\{s_t(N_t-i)\} \rightarrow (1-\Psi_\zeta)\Psi_1 \ldots \Psi_{i-1}\Omega_i$ $\quad (i = 2, \ldots, N_t-1);$

where

$$(24be)\ \Omega_i \equiv \int_0^\infty \frac{e^{-\gamma\lambda(i-\zeta+\zeta z)-\sigma F(\varphi(z)-1/F)}(1/\omega)((1-\varphi(z))/(1-1/F))^{\xi/\mu}}{(1+\sigma)-(\gamma\lambda\zeta)\int_0^z e^{-\gamma\lambda\zeta y-\sigma F(\varphi(y)-1/F)}dy}dz$$

$$(24be')\ \Psi_i \equiv \int_0^\infty \frac{e^{-\gamma\lambda(i-\zeta+\zeta z)-\sigma F(\varphi(z)-1/F)}\xi F\varphi(z)((1-\varphi(z))/(1-1/F))^{\xi/\mu}}{(1+\sigma)-(\gamma\lambda\zeta)\int_0^z e^{-\gamma\lambda\zeta y-\sigma F(\varphi(y)-1/F)}dy}dz$$

Note that $1 > \Omega_1 > \ldots > \Omega_i > 0$ and $1 > \Psi_1 > \ldots > \Psi_i > 0$.

(iv): Unfortunately, I have not been able to deduce any explicit formulae for the long-run average efficiency distribution of capitals for the case where technology is disembodied and only the best technology firms can strike an innovation, i.e., where *Hypotheses (IN-a)* and *(CG-e)* hold.

Note in passing that what is important about these mathematical formulae is not that they have these particular forms but that they can be obtained by pencils and paper without having recourse to computer simulation.

<Insert *Figs. 9ad, 9ae* and *9be* around here.>

Fig. 9ad, *Fig. 9ae* and *Fig. 9be* illustrate the long-run average efficiency distribution of capital stocks for the first three cases above -- *(i)* the case where technology is disembodied and every firm can strike an innovation; *(ii)* the case where technology is embodied and every firm can strike an innovation; and *(iii)* the case where technology is embodied and only the best technology firms can strike an innovation. The first and the second cases have monotonically declining distributions, with the declining rates initially slower than but later getting faster than that of geometric distribution. The third case has a distribution which peaks at the second-best technology and then declines at the rate initially slower but later faster than that of geometric distribution.

II-4. The long-run average supply curve.

The efficiency distribution of capital stocks we have so far been concerned with represents only the technological "possibility" of the industry. However, not every firm is actually engaged in production, and some of the existing capital stocks are left idle or simply discarded. Since money wage rate divided by labor productivity $W_t/a(N_t\text{-}i) = W_t e^{-\lambda(N\text{-}i)}$ represents the unit cost of production of each technology, the firm operates the capital stocks of technology $N_t\text{-}i$ at its full capacity only if they generate a surplus, or only if $P_t > W_t e^{-\lambda(N\text{-}i)}$, and the firm operates a portion of those capital stocks, depending on demand, if they are just break-even, or if $P_t = W_t e^{-\lambda(N\text{-}i)}$. (We have here ignored the cost associated with shutting-down of a production line as well as the cost associated with setting-up of a new production line.) Then, by summing up all the profitable productive capacities and adding a portion of the break-even productive capacities, we can determine the industry's supply curve (or the supply correspondence, to be precise) for each price P_t. If we let Y_t denote the industry's total product supply at time t, it can be given by:

$$(25)\quad Y_t = bk_t(N_t)+\ldots+bk_t(N_t\text{-}i) \qquad \text{if } e^{-\lambda(N\text{-}i)}W_t < P_t < e^{-\lambda(N\text{-}i\text{-}1)}W_t\,,$$

$$\in [bk_t(N_t)+\ldots+bk_t(N_t\text{-}i),\ bk_t(N_t)+\ldots+bk_t(N_t\text{-}i\text{-}1)] \qquad \text{if } P_t = e^{-\lambda(N\text{-}i\text{-}1)}W_t\,,$$

with an understanding that $Y_t = 0$ if $P_t < e^{-\lambda Nt}\,W_t$.

In a Schumpeterian industry, however, the above supply curve is constantly shifting rightwards by capital accumulation and constantly shifting downwards by technological progress. In order to neutralizes the effect of capital accumulation, let us divide the industry's output Y_t by its maximum productive capacity bK_t.[17] And in order to neutralize (at least a part of) the effects of technological progress, let us divide both the product price P_t and the wage cost $e^{-\lambda(N-i)}W_t$ by the efficiency money wage of the best technology $W_t/e^{\lambda Nt}$. If we let $y_t \equiv Y_t/bK_t$ denote the output-capacity ratio and $p_t \equiv P_t e^{\lambda Nt}/W_t$ the product price-efficiency wage ratio, we get the following "relative" form of industry supply curve:

$$(26)\ y_t = s_t(N_t)+\ldots+s_t(N_t-i) \qquad \text{if } e^{\lambda i} < p_t < e^{\lambda(i+1)},$$
$$\in [s_t(N_t)+\ldots+s_t(N_t-i),\ s_t(N_t)+\ldots+s_t(N_t-i-1)] \quad \text{if } p_t = e^{\lambda(i+1)}.$$

with an understanding that $y_t = 0$ if $p_t \leq e^{\lambda 0} = 1$.

<Insert *Fig. 10* around here.>

Fig. 10 depicts the relative form of industry supply curve in a Marshallian diagram whose vertical axis measures the logarithm of price-efficiency wage ratio $log p_t$ and horizontal axis output-capacity ratio y_t. Indeed, it is nothing but the left-hand cumulative distribution of capital shares – a partial sum of capital shares depicted in *Fig. 6* added from the best to the worst technology. As long as a multitude of technologies with different efficiencies coexist within the industry, it remains an upward-sloping curve. And as long as it is upward-sloping, the industry is able to generate positive profits, as is indicated by the shaded area in *Fig. 10*.

With the help of our analyses in sections II-2, 3 and 4 of the effects of accumulation, imitations and innovations on the efficiency distribution of capitals, a mere perusal of *Fig. 10* allows us to discuss how each of these three dynamic forces influences the industry's profit rate in the short-run.

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[17] It is easy to show from *(16)* that: $(dK_t/dt)/K_t = \gamma\{\lambda\sum_n n s_t(n)-log(W_t/P_t)\}-\gamma_o$. In other words, the accumulation rate of the industry's total capitals K_t is linearly dependent on the proportional gap between the average labor productivity and real wage rate. If, on the one hand, $(dK_t/dt)/K_t$ is pre-determined (probably by the growth rate of the demand for this industry's products), this equation can be used to determine W_t/P_t. If, on the other hand, W_t/P_t is pre-determined (probably by the labor market conditions in the economy as a whole), this equation can be used to determine $(dK_y/dt)/K_t$. In either case, the forces governing the motion of K_t are in general of different nature from those governing the motion of $\{s_t(n)\}$.

For instance, the arrival of an innovation lowers the leftmost potion of the supply curve and boosts the industry's profit rate, and the subsequent growth of the best technology capital stocks and the subsequent diffusion of the best technology itself both expand the lowered portion of the supply curve horizontally to the right, thereby further increasing the profit rate at first. However, as the best technology capital stocks continue to grow either by economic selection or by technological diffusion, the effect of the rightward shift of the whole supply curve starts to take its effect on price. The industry price declines rapidly and soon arrests the upward movement of the profit rate. The profit rate then follows a downward path, until another innovation again reverses this trend.

Fig. 10 also allows us to say something interesting about the impact of an increase in demand on the rate of profit. Of course, everybody knows that an increase in output raises the industry's profit rate. But, what I would like to suggest here is not merely that an increase in output raises the profit but that an increase in output raises the profit roughly by the order of its *square*. This is most easily seen by approximating the industry supply curve in *Fig. 10* by a linear curve. Then, the shaded area representing profits can be identified as a triangle, and the elementary school arithmetic tells us that the magnitude of the industry's profit given by that triangle becomes proportional to the square of the level of output. This simple fact implies that if the firms' fixed investment is linearly dependent on the rate of profit, as we have supposed in *Hypothesis (CG-d)* or *(CG-e)*, the so-called investment function becomes an increasing and quadratic (more generally convex) function of output level. If this is so, it is very likely to violate the short-run stability condition for investment-saving equilibrium of the economy as a whole, suggesting its instability at least in the short-run. This would certainly connect our models to the recent works on non-linear economic dynamics. In the present paper, however, I only mention this possibility in passing and must resume our analysis of the long-run performance of the Schumpeterian industry.

Indeed, the fact that the industry's state of technology will retain features of disequilibrium even in the long-run does have an important implication for the nature of the industry's supply curve in the long-run. For, as is seen from *(26)*, the relative form of industry supply curve is a partial sum of the capital shares $s_t(n)$ added from the best technology to the break-even technology. Hence, if each of the capital shares tends to exhibit a statistical regularity in the long-run, the relative form of the industry supply curve should also exhibit a statistical regularity in the long-run.

Now, it is possible to express the expectation of the relative supply curve of the industry as:

$$(27)\quad \lim_{t\to\infty} E(y_t) = \lim_{t\to\infty} (E(s_t(N_t))+\ldots+E(s_t(N_t-i))) \quad \text{if } \lambda i < \log p_t < \lambda(i+1),$$
$$\in \lim_{t\to\infty} [E(s_t(N_t))+\ldots+E(s_t(N_t-i)), E(s_t(N_t))+\ldots+E(s_t(N_t-i-1))] \quad \text{if } \log p_t = \lambda(i+1).$$

Then, in view of *(23ad)*, we can indeed calculate the long-run average industry supply curve under *Hypotheses (IN-a)* and *(CG-d)* as:

$$(28ad)\quad \lim_{t\to\infty} E(y_t)$$
$$= 0 \quad \text{if } \log p_t < \lambda 0 = 0,$$
$$\in [0, 1-\Gamma_\zeta] \quad \text{if } \log p_t = 0,$$
$$= (1-\Gamma_\zeta)(i\Gamma_1+(i-1)\Gamma_2+\ldots+\Gamma_i) \quad \text{if } \lambda(i-1) < \log p_t < \lambda i,$$
$$\in [(1-\Gamma_\zeta)(i\Gamma_1+\ldots+\Gamma_i), (1-\Gamma_\zeta)((i+1)\Gamma_1+\ldots+\Gamma_{i+1})] \quad \text{if } \log p_t = \lambda i.$$

In view of *(23ae)*, we can also calculate the long-run average industry supply curve under *Hypotheses (IN-a)* and *(CG-e)* as:

$$(28ae)\quad \lim_{t\to\infty} E(y_t)$$
$$= 0 \quad \text{if } \log p_t < \lambda 0 = 0,$$
$$\in [0, 1-\Lambda_\zeta] \quad \text{if } \log p_t = 0,$$
$$= (1-\Lambda_\zeta)(i\Lambda_1+(i-1)\Lambda_2+\ldots+\Lambda_i) \quad \text{if } \lambda(i-1) < \log p_t < \lambda i,$$
$$\in [(1-\Lambda_\zeta)(i\Lambda_1+\ldots+\Lambda_i), (1-\Lambda_\zeta)((i+1)\Lambda_1+\ldots+\Lambda_{i+1})] \quad \text{if } \log p_t = \lambda i.$$

Finally, in view of *(23be)* the long-run average industry supply curve becomes under *Hypotheses (IN-b)* and *(CG-e)* as:

(28be) $\lim_{t\to\infty} E(y_t)$

$$= 0 \qquad \text{if } \log p_t < \lambda 0 = 0 ,$$

$$\in [0, 1-\Omega_\zeta] \qquad \text{if } \log p_t = 0 ,$$

$$= 1-\Omega_\zeta \qquad \text{if } 0 < \log p_t < \lambda ,$$

$$\in [1-\Omega_\zeta , (1-\psi_\zeta)\Omega_1] \qquad \text{if } \log p_t = \lambda ,$$

$$= 1-\Omega_\zeta+(1-\psi_\zeta)(\Omega_1+\psi_1(\Omega_2+\psi_2(\Omega_3+...+\psi_{i-1}\Omega_i)...)) \qquad \text{if } \lambda i < \log p_t < \lambda(i+1) ,$$

$$\in [1-\Omega_\zeta+(1-\psi_\zeta)(\Omega_1+\psi_1(\Omega_2+..+\psi_{i-1}\Omega_i)..), 1-\Omega_\zeta+(1-\psi_\zeta)(\Omega_1+\psi_1(\Omega_2+..+\psi_i\Omega_{i+1})..)]$$

$$\text{if } \log p_t = \lambda(i+1) .$$

<Insert *Figs. 11 ad, ae* and *be* around here.>

Each of *Figs. 11ab, 11ae* and *11be* exhibits the relative form of the industry's long-run average supply curve for each of the above three versions of our evolutionary model. As in the case of short-run industry supply curve, the horizontal axis measures output-capacity ratio y_t and the vertical axis measures the logarithm of price-efficiency wage ratio $\log p_t$. What is most striking about the three long-run average supply curves in *Figs. 11ad, ae* and *be* is that they are all *upward-sloping*!

Let us now recall the lower panel of *Fig. 1* of the introductory section. It has reproduced a typical shape of the long-run supply curve which can be found in any textbook of economics. This horizontal curve is supposed to describe the long-run state of the industry in which the least cost technology is available to every firm in the industry and all the opportunities for positive profits are completely wiped out by competition among firms. However, the long-run average supply curves we have drawn in *Fig. 11* have nothing to do with such traditional picture. There will always be a multitude of diverse technologies with different productivity conditions and the industry supply curve will never lose its upward-sloping tendency. There will, therefore, always be some firms which are capable of earning positive profits, no matter how competitive the industry is and no matter how long it is run, as long as there will be enough demand for the industry's product.

I will thus conclude that positive profits are not only the short-run phenomenon but also the long-run phenomenon of the industry. It is true that the positivity of profits is a symptom of disequilibrium. But, if what the industry will approach is at best a statistical equilibrium of technological disequilibria, it will never stop generating positive profits from within even in the never-never-land of the long-run.

II-5. *Pseudo* aggregate production functions.

Since the publication of Robert Solow's "Technical Change and the Aggregate Production Function" in 1957, it has become a standard technique in neoclassical growth theory to decompose the growth rate of an economy's per capita GNP into the effect of capital/labor substitution along an aggregate production function due to capital accumulation and the effect of continuous shift in the aggregate production function itself due to technological progress. Solow found that more than 80 % of per capital GNP growth rate in the United States from 1909 to 1949 could be attributed to the technological progress and less than 20 % to the capital deepening, and opened the eyes of economists to the importance of technological progress in understanding the economic growth process. At the same time it gave rise to a well-known controversy – the so-called Cambridge-Cambridge controversy – on the concepts of aggregate production function and aggregate capital stocks, on which Solow's technique of growth accounting relied heavily. This controversy, however, has died out, perhaps because of its degeneration into such esoteric problems as re-switching and all that. The purpose of this section is to provide a new critique of the neoclassical growth theory. This time, however, the critique is much more "constructive" than its predecessor. Because what I am now showing is that our evolutionary models are capable of "simulating" all the characteristics of neoclassical growth models.

Let me begin our "simulation" of neoclassical aggregate production functions by constructing the industry's labor demand function. As is seen in *Fig. 10*, when demand is small and the price (in terms of efficiency wage) just covers the wage cost of the best technology or when $logp_t = 0$, only the capital stocks carrying the best technology N_t are operated and output Y_t is determined by the level of demand. Because of the fixed proportion technology *(12)*, we can represent the level of total employment associated with this output as $L_t = e^{-\lambda Nt} Y_t$. When the demand reaches the total capacity of the best technology $bk_t(N_t) = s_t(N_t)bK_t$, then a further increase in demand is absorbed solely by an increase in price, while output is kept at the capacity level. But when the price reaches the wage cost of the second-best technology or when $logp_t = \lambda$, the second-best capital stocks start to join the production and all the increase in demand is absorbed by a corresponding increase in output. Then, the relation between output Y_t and employment L_t can be given by $L_t = e^{-\lambda Nt} s_t(N_t)bK_t + e^{-\lambda(Nt-1)}(Y_t - s_t(N_t)bK_t)$ until Y_t reaches the total productive capacity of the first- and second-best technology $(s_t(N_t)+s_t(N_t-1))bK_t$. In general, when $(s_t(N_t)+\ldots+s_t(N_t-i))bK_t \leq Y_t < (s_t(N_t)+\ldots+s_t(N_t-i-1))bK_t$, the relation between Y_t and L_t can be given by $L_t = (s_t(N_t)+\ldots+e^{\lambda i}s_t(N_t-i)+e^{\lambda(i+1)}(Y_t/bK_t-(s_t(N_t)+..+s_t(N_t-i))))bK_t e^{-\lambda Nt}$. If we divide this relation by $bK_t e^{-\lambda Nt}$, we can express the industry-wide efficiency labor-capacity ratio $x_t \equiv e^{\lambda Nt} L_t/bK_t$ as a function of the industry-wide output-capacity ratio $y_t \equiv Y_t/bK_t$ as:

$$(29)\quad x_t = s_t(N_t)+\ldots+e^{\lambda i}s_t(N_t-i)+e^{\lambda(i+1)}(y_t-(s_t(N_t)+..+s_t(N_t-i))),$$

$$\text{when } s_t(N_t)+..+s_t(N_t-i) \leq y_t < s_t(N_t)+..+s_t(N_t-i-1).$$

<Insert *Fig. 12* around here.>

Fig.12 depicts the inverse of the above relation in a Cartesian diagram which measures efficiency labor-capacity ratio x_t along horizontal axis and output-capacity ratio y_t along a vertical axis. It is not hard to see that this relation satisfies all the properties a neoclassical production function is

supposed to satisfy.[18] Y_t is linearly homogeneous in L_t and K_t, because $y_t \equiv Y_t/bK_t$ is a function only of $x_t \equiv e^{\lambda Nt} L_t/bK_t$. Though not smooth, this relation also allows a substitution between K_t and L_t and satisfies the marginal productivity principle: $\partial^- y_t/\partial x_t \leq 1/p_t = e^{-\lambda Nt} W_t/p_t$ (= efficiency real wage rate) $\leq \partial^+ y_t/\partial x_t$. (Here, $\partial^- y/\partial x$ and $\partial^+ y/\partial x$ represent left- and right-partial differential, respectively.) Yet, the important point is that this is *not* a production function in the proper sense of the word! It is a mere theoretical construct summarizing the production structure of the industry as a whole, and has little to do with the actual technological conditions of the individual firms working in the industry. As a matter of fact, the technology each firm uses is a Leontief-type fixed proportion technology which does not allow any capital/labor substitution. It is in this sense that we call the relation *(32)* a 'short-run *pseudo* aggregate production function.'

The shape of this function is determined by the efficiency distribution of capital shares $\{s_t(n)\}$. Hence, as this distribution changes, the shape of the pseudo production function also changes. And in our Schumpeterian industry, the efficiency distribution of capitals is incessantly changing over time as the result of dynamic interplay among technological innovations, technological imitations and capital accumulation. The most conspicuous feature of the short-run pseudo aggregate production function is, therefore, its instability.

In the long-run, however, we know we can detect a certain statistical regularity in the relative form of capital share distribution out of its seemingly unpredictable movement. We can thus expect to detect a certain statistical regularity in the relative form of pseudo aggregate production function out of its seemingly unpredictable movement as well.

To see this, let us first note that by *(29)* we have $x_t = e^{\lambda 0} s_t(N_t) + \ldots + e^{\lambda i} s_t(N_t - i)$ when $y_t = s_t(N_t) + .. + s_t(N_t - i)$. Taking expectation, we then have:

(30) $E(x_t) = E\{s_t(N_t) + \ldots + e^{\lambda i} s_t(N_t - i)\}$ when $E(y_t) = E\{s_t(N_t) + .. + s_t(N_t - i)\}$.

[18] See Sato [1975] for the general discussion on the aggregation of micro production functions.

Thus, under *Hypotheses (IN-a)* and *(CG-d)*, if we let $t \to \infty$, we obtain by *(23ad)*:

$$(31ad)\ E(x_t) \to (1-\Gamma_\zeta)(1+e^{\lambda 1}\Gamma_1+\ldots+e^{\lambda i}\Gamma_i)$$

$$\text{when } E(y_t) \to 1-\Gamma_\zeta \qquad \text{for } i = 0,$$

$$\to (1-\Gamma_\zeta)(i\Gamma_1+(i-1)\Gamma_2+\ldots+\Gamma_i) \qquad \text{for } i = 1, 2, \ldots N_t-1,$$

with an understanding that $E(x_t) = 0$ when $E(y_t) = 0$. Next, under *Hypotheses (IN-a)* and *(CG-e)*, if we let $t \to \infty$ we obtain by *(23ae)*:

$$(31ae)\ E(x_t) \to (1-\Lambda_\zeta)(1+e^{\lambda 1}\Lambda_1+\ldots+e^{\lambda i}\Lambda_i),$$

$$\text{when } E(y_t) \to 1-\Lambda_\zeta \qquad \text{for } i = 0,$$

$$\to (1-\Lambda_\zeta)(i\Lambda_1+(i-1)\Lambda_2+\ldots+\Lambda_i) \qquad \text{for } i = 1, 2, \ldots N_t-1,$$

with an understanding that $E(x_t) = 0$ when $E(y_t) = 0$. Finally, under *Hypotheses (IN-b)* and *(CG-e)*, if we let $t \to \infty$ we obtain by *(23be)*:

$$(31ae)\ E(x_t) \to 1-\Omega_\zeta+(1-\Psi_\zeta)(e^{\lambda 1}\Omega_1+\ldots+e^{\lambda i}\Psi_1\ldots\Psi_{i-1}\Omega_i),$$

$$\text{when } E(y_t) \to 1-\Omega_\zeta \qquad \text{for } i = 0,$$

$$\to 1-\Omega_\zeta+(1-\psi_\zeta)(\Omega_1+\psi_1(\Omega_2+\ldots+\psi_{i-1}\Omega_i)\ldots)) \qquad \text{for } i = 1, 2, \ldots N_t-1,$$

with an understanding that $E(x_t) = 0$ when $E(y_t) = 0$. If we span a convex hull of the points $(E(x_t), E(y_t))$ defined by each of the above long-run relations respectively, we are able to generate the "long-run average *pseudo* aggregate production functions" for the three versions of our evolutionary model.

<Insert *Fig. 13ad*, *13ae* and *13be* around here.>

Fig. 13ad, *13ae* and *13be* illustrate these curves in a Cartesian diagram which measures the expected efficiency labor-capacity ratio $E(x_t)$ along abscissa and the expected output-capacity ratio $E(y_t)$ along ordinate. Now, these long-run average pseudo aggregate production functions exhibit all the properties that neoclassical production functions should have! Indeed, they show that the long-run average output-capacity ratio $E(Y_t/bK_t)$ is an increasing and concave function of the long-run average efficiency labor-capacity $E(e^{\lambda N_t}L_t/K_t)$. Thus, it is as if the total work force L_t and total capital stock K_t were jointly producing the total output Y_t, subject to an aggregate neoclassical production function: $Y = bKy(e^{\lambda N_t}L/K)$ under Harrod-neutral (or pure labor augmenting) technological progress: $e^{\lambda Nt}$. It is as if we had entered the Solovian world of neoclassical economic growth in which the growth process of the economy could be decomposed into the capital-labor substitution along an aggregate neoclassical production function due to capital accumulation and the constant outward shift of the aggregate neoclassical production function itself due to the manna-like technological progress. This is, however, a mere macroscopic illusion! If we zoomed in the microscopic level of the industry, the picture we would get is entirely different. What we would find out is the complex and dynamic interplay of many a firm's innovation, imitation and accumulation activities. It is just impossible to disentangle these microscopic forces and decompose the overall growth process into a movement along a well-defined aggregate productio function and an outward shift of the function itself. As a matter of fact, the basic parameters, λ, μ and ν or ξ, which determine the rate of pseudo Harrod-neutral technical progress $e^{\lambda Nt}$, are also the parameters that determine the very shape of the pseudo aggregate production function. (We already know from *(9a)* and *(9b)* that the long-run average rate of technological change is equal to $\lambda\nu F$ under *Hypothesis (IN-a)* and $\lambda F/\sum_i((1-1/F)^i/(\xi+\mu i))$ under *Hypothesis (IN-b)*.) We are after all living in a Schumpeterian world where the incessant reproduction of technological disequilibrium prevents the pseudo aggregate production function from collapsing into the fixed proportion technology of individual firms. It is, in other words, its non-neoclassical features that give rise to the illusion that the industry is behaving like a neoclassical growth model. Neoclassical growth accounting thus has no empirical content in our Schumpeterian world.

II-6. Concluding Remarks.

In the traditional economic theory, whether classical or neoclassical, the long-run state of the economy is an equilibrium state and the long-run profits are equilibrium phenomena. If there is a theory of long-run profits, it must be a theory about the determination of the normal rate of profit.

This paper has challenged this long-held tradition in economics. It has introduced a series of new simple evolutionary models which are capable of analyzing (without having recourse to computer simulation) the evolutionary process of the state of technology as a dynamic interplay among many a firm's growth, imitation and innovation activities. And it has used these models to demonstrate that what the economy will approach over a long passage of time is not a classical or neoclassical equilibrium of uniform technology but a statistical equilibrium of technological disequilibria which maintains a relative dispersion of efficiencies in a statistically balanced form. Positive profits will never disappear from the economy no matter how long it is run. 'Disequilibrium' theory of 'long-run profits' is by no means a contradiction in terms.

Not only is a disequilibrium theory of long-run profits possible, but it is also 'operational.' Indeed, our evolutionary model would allow us to calculate (only with pencils an paper) the economy's long-run profit rate as an explicit function of the model's basic parameters which represent the forces of economic selection, technological diffusion and recurrent innovations. "Without development there is no profit, without profit no development," to quote Joseph Schumpeter once more.[19] The model we have presented in this paper can thus serve as a foundation, or at least as a building block, of the theory of 'long-run development through short-run fluctuations' or 'growth through cycles.' To work out such a theory in more detail is of course an agenda for the future research.

The present paper has adopted the so-called 'satisficing' principle in its description of the firms' behaviors – firms do not optimize a well-defined objective function but simply follow fixed organizational routines in deciding their growth, imitation and innovation policies. Indeed, one of the purposes

[19] Schumpeter [1961], p. 154.

of this paper was to see how far we could go in our representation of the economy's dynamic performance without relying on the neoclassical assumption of full individual rationality. And it has even succeeded in 'simulating' all the macroscopic characteristics of neoclassical growth model. And yet, there is no denying that our strict evolutionary assumption of fixed organizational routines is as unrealistic as the neoclassical assumption of fully rational decision-making is. Where have all these organizational routines come from? What are their determinants? How will they change over time? Another important agenda for the future research is to study the very evolutionary process of these routines by injecting at least a modicum of rationality into our firms' head-quarters. This will not turn our evolutionary model into a neoclassical model. But it will, I hope, furnish us with a common ground with the recently emerged and rapidly growing literature on endogenous growth in neoclassical economics.[20]

[20] On endogenous growth literature, see Aghion and Howitt [1992, 1997], Grossman and Helpman [1993], Romer [1990] and Segerstrom [1991]. They are all based on the assumption of individual rationality which extends over an infinite horizon.

<*Appendix A*>

The purpose of this *Appendix* is to indicate how *(7a)* and *(7b)* in the main text can be obtained.

Let $W(t)$ denote the probability distribution of a "waiting period" for the next innovation $T(N_t+1)-T(N_t)$, or $W(t) \equiv Pr\{T(N_t+1)-T(N_t) \leq t\}$. Then, the probability that an innovation takes place during a small interval $[t, t+dt]$ *for the first time since time 0* can be written as $W(t+dt)-W(t) = dW(t)$. This is also the probability that no innovation has taken place during $[0, t)$ *and* an innovation takes place during $[t, t+dt]$. Since the probability of the former event equals $(1-W(t))$ and the probability of the latter event equals $\nu F dt$ under *Hypothesis (IN-a)* and $\xi F f_t(N_t)dt$ under *Hypothesis (IN-b)*, we get an equation of the form of:

$$(A1a) \quad dW(t) = (1-W(t))\nu F dt$$

under *Hypothesis (IN-a)*, and an equation of the form of:

$$(A1b) \quad dW(t) = (1-W(t))\xi F f_t(N_t)dt$$

under *Hypothesis (IN-b)*. *(A1-a)* can be solved as:

$$(A2a) \quad W(t) = 1- e^{-\nu F t} \qquad \text{for } t \geq 0.$$

This is *(7a)* of the main text. On the other hand, if we rewrite *(A1-b)* as $d(1-W(t))/(1-W(t)) = -\xi F f_t(N_t)dt$, integrate it and rearrange terms, we obtain:

$$(A2b) \quad W(t) = 1- e^{-\xi F \int_0^t fs(Ns)ds} = 1-(1-1/F)^{-\xi/\mu}(1-f_t(N_t))^{\xi/\mu} = 1-\left(\frac{(F-1)+e^{\mu F t}}{F}\right)^{-\xi/\mu},$$

for $t \geq 0$. This is *(7b)* of the main text.

Let ω be the expected waiting time. Then, we can calculate it under *Hypothesis (IN-a)* as:

$$(A3a) \quad \omega = \int_0^\infty t dW(t) = \int_0^\infty t\nu F \exp(\nu F t)dt = 1/\nu F ,$$

and under *Hypothesis (IN-b)* as:

$$(A3b) \quad \omega = \int_0^\infty t dW(t) = \int_0^\infty (1-W(t))dt = \int_0^\infty \left(\frac{F-1+e^{\mu F s}}{F}\right)^{-\xi/\mu} ds = \sum_{i=0}^{\infty} \frac{(1-1/F)^i}{(\xi+\mu i)F} .$$

Note that an increase in ν decreases ω in *(A3a)* and an increase in ξ and μ decreases ω in *(A3b)*.

<Appendix B>

The purpose of this *Appendix* is to deduce the long-run average efficiency shares of firms, given by *(10a)* and *(10b)* in the main text.

Let us begin by examining the share of the best technology $f_t(N_t)$. This share emerges from 0 at $T(N_t)$ and moves along a logistic growth curve *(5)*: $1/(1+(F-1)\,e^{-\mu F(t-T(N_t))})$ from that time on. Its value is thus determined by how far back its innovation time was. Accordingly, let $B_t(z)$ denote the cumulative probability of the length of time measured backwards from t to $T(N_t)$, or $B_t(z) \equiv Pr\{s-T(N_t) \leq z\}$. We may call this 'the backward waiting period distribution'. In terms of this distribution, we can express the expected share of the best technology as:

$$(A4)\quad E\{f_t(N_t)\} = \int_0^{t-T(N_t)} \frac{1}{1+(F-1)e^{-\mu F z}}\,dB_t(z).$$

Now, since the sequence of waiting times, $T(2)-T(1), \ldots, T(N_{t+1})-T(N_t)$, constitutes a 'renewal process,' the renewal theory tells us that the distribution of the backward waiting time $B_t(z)$ will in the long-run approach a steady-state distribution, or as $t \to \infty$:

$$(A5)\quad B_t(z) \to \int_0^z \frac{1-W(s)}{\omega}\,ds.$$

independently of t.[21] Hence, under *Hypothesis (IN-a),* we obtain by *(A2a)* and *(A3a)*:

$$(A6a)\quad E\{f_t(N_t)\} \to \int_0^\infty \frac{\nu F e^{-\nu F z}}{1+(F-1)e^{-\mu F z}}\,dz = \int_{1/F}^{1} \left(\frac{x}{1/F}\right)^{1-\nu/\mu}\left(\frac{1-x}{1-1/F}\right)^{\nu/\mu} dx$$

This is nothing but the first line of *(10a)*. By the same token, under *Hypothesis (IN-b),* we obtain by *(A2b)*:

$$(A6b)\quad E\{f_t(N_t)\} \to \int_0^\infty \frac{(1-W(z))/\omega}{1+(F-1)e^{-\mu F z}}\,dz = \int_0^\infty \frac{1}{\xi F\omega}\left(\frac{dW(z)}{dz}\right)dz = \frac{1}{\xi F\omega}$$

$$= \frac{1}{\sum_i (1-1/F)^i/(1+(\mu/\xi)i)}.$$

---

[21] See Feller [1966], p. 355.

This is nothing but the first line of *(10b)*.

Next, let us turn to the examination of the shares of the less than best technologies $f_t(N_t\text{-}i)$ for $i = 1, 2, \ldots, N_t\text{-}1$. Rewrite *(6)* recursively as:

(A7) $f_t(N_t\text{-}i) =$

$$\frac{1\text{-}f_t(N_t)}{1\text{-}f_{T(N_t)}(N_t)}\,\frac{f_{T(N_t)}(N_t\text{-}i)}{f_{T(N_t\text{-}0)}(N_t\text{-}i)}\,\frac{1\text{-}f_{T(N_t\text{-}0)}(N_t\text{-}1)}{1\text{-}f_{T(N_t\text{-}1)}(N_t\text{-}1)}\,\frac{f_{T(N_t\text{-}1)}(N_t\text{-}i)}{f_{T(N_t\text{-}1\text{-}0)}(N_t\text{-}i)}\cdots\cdots$$

$$\frac{1\text{-}f_{T(N_t\text{-}i\text{-}0)}(N_t\text{-}i+1)}{1\text{-}f_{T(N_t\text{-}i+1)}(N_t\text{-}i+1)}\,\frac{f_{T(N_t\text{-}i+1)}(N_t\text{-}i)}{f_{T(N_t\text{-}i+1\text{-}0)}(N_t\text{-}i)}\,f_{T(N_t\text{-}i+1\text{-}0)}(N_t\text{-}i)$$

The share of a lesser technology $f_s(N_t\text{-}i)$ $(i = 1, 2, 3, \ldots)$ thus goes through i different phases. First, it emerges at its own innovation time $T(N_t\text{-}i)$ and moves along a logistic growth curve: $1/(1+(F\text{-}1)e^{-\mu F(s\text{-}T(N_t-i))})$ until the next innovation time $T(N_t\text{-}i\text{+}1)$. If the innovator of $N_t\text{-}i\text{+}1$ does not belong to its members, it traverses $T(N_t\text{-}i\text{+}1)$ smoothly and starts to follow a decay curve: $f_{T(N_t\text{-}i+1)}(N_t\text{-}i)(1\text{-}f_s(N_s))/(1\text{-}1/F)$ until $T(N_t\text{-}i\text{+}2)$. If, however, the innovator turns out to be one of its former members, it loses a share of $1/F$ at $T(N_t\text{-}i\text{+}1)$ and follows the same decay curve from a reduced initial value: $f_{T(N_t\text{-}i+1)}(N_t\text{-}i) = f_{T(N_t\text{-}i+1\text{-}0)}(N_t\text{-}i)\text{-}1/F$. This process is repeated for $T(N_t\text{-}i\text{+}2) \leq s < T(N_t\text{-}i\text{+}3)$, ..., for $T(N_t\text{-}1) \leq s < T(N_t)$, and finally in a truncated form for $T(N_t) \leq s \leq t$.

Now, under *Hypothesis (IN-a)* the probability that the innovator belongs to one of their members is equal to its very share, so that its expected reduction at each innovation time becomes equal to $(1/F)f_{T(N_t\text{-}m)}$, or $E\{f_{T(N_t\text{-}m)}(N_t\text{-}i)/f_{T(N_t\text{-}m\text{-}0)}(N_t\text{-}i)\} = (1\text{-}1/F)$ at least in the long-run, $m = i\text{-}1, i\text{-}2, \ldots, 0$. Noting that $T(N_t\text{-}i\text{+}1)\text{-}T(N_t\text{-}i) \leq z$ obeys the probability distribution $W(z)$ and $t\text{-}T(N_t) \leq z$ the probability distribution $B_t(z)$, we can express the expected value of the share of the $i\text{+}1^{st}$ best technology as follows.

$$(A8a)\quad E\{f_t(N_t\text{-}i)\} = \int_0^{t-T(N_t-i)} \varphi(z)dW(z)(1-1/F) \int_0^{t-T(N_t-i+1)} \frac{1-\varphi(z)}{1-1/F}dW(z)(1-1/F) \cdots \int_0^{t-T(N_t-1)} \frac{1-\varphi(z)}{1-1/F}dW(z)(1-1/F) \int_0^{t-T(N_t)} \frac{1-\varphi(z)}{1-1/F}dB_t(z)$$

where $\varphi(z)$ is a logistic function $1/(1+(F-1)e^{-\mu F z})$.

As $t \to \infty$, $t\text{-}T(N_t\text{-}j) \to \infty$ and $dB_t(z) \to (1\text{-}W(z))dz/\omega$. Noting that since we have $(1\text{-}W(z))/\omega = dW(z) = \nu F exp(-\nu F z)$ in the case of Poisson process, we can easily rewrite the above expression as:

$$(A9a)\quad E\{f_t(N_t\text{-}i)\} \to \Phi(1-\Phi)^i \quad \text{with } i = 1, 2, \ldots, N_t\text{-}1,$$

where $\Phi \equiv \int_0^{\infty} \varphi(z)\nu F e^{-\nu F z}dx = \int_{1/F}^{1} (\frac{x}{1/F})^{1-\nu/\mu}(\frac{1-x}{1-1/F})^{\nu/\mu}dx$.

This is the second line of $(10a)$ in the main text.

Next, under *Hypothesis (IN-b)* the innovator always comes from among the users of the former best technology, so that the discontinuous phase transition occurs only once and at the second innovation time $T(N_t\text{-}i+1)$. We thus have:

$$(A8b)\quad E\{f_t(N_t\text{-}i)\} = \left(\int_0^{t-T(N_t-i)} \varphi(z)dW(z)-1/F\right) \int_0^{t-T(N_t-i+1)} \frac{1-\varphi(z)}{1-1/F}dW(z) \cdots \int_0^{t-T(N_t-1)} \frac{1-\varphi(z)}{1-1/F}dW(z) \int_0^{t-T(N_t)} \frac{1-\varphi(z)}{1-1/F}dB_t(z).$$

As $t \to \infty$, $t\text{-}T(Nt\text{-}j) \to \infty$ and $dB_t(z) \to (1\text{-}W(z))dz/\omega$. Now, in view of $(A1b)$ and $(A3b)$ we have:

$$\int_0^{\infty} (1-\varphi(z))dW(z) = \int_0^{\infty} (1-\varphi(z))\xi F\varphi(z)(1-W(z))\frac{dz}{d\varphi(z)}d\varphi(z)$$

$$= (\xi/\mu)\int_{1/F}^{1} (1-W(z))d\varphi(z) = (\xi/\mu)\int_{1/F}^{1} (1-1/F)^{-\xi/\mu}(1-\varphi(z))^{\xi/\mu}d\varphi(z)$$

$$= (\xi/\mu)(1-1/F)^{-\xi/\mu}(1+\xi/\mu)^{-1}[0-(1-1/F)^{(1+\xi/\mu)}] = (1-1/F)(\xi/(\xi+\mu)).$$

Substituting this expression as well as $(A6b)$ into $(A8b)$ and rearranging terms, we obtain:

$$(A9b)\quad E\{f_t(N_t\text{-}i)\} \rightarrow \frac{1}{F\lambda/(\sum_i (1-1/F)^i/(\xi+\mu i))}$$

with $i = 1, 2, \ldots, N_t\text{-}1$. This is the second line of *(10b)* in the main text.

<Appendix C>

The purpose of this *Appendix* is to solve the differential equations *(19d), (20d), (19e)* and *(20e)*.

Let $y(t) = 1/(1\text{-} s_t(N_t))$ and $\varphi(t) = f_t(N_t)$, then *(19d)* becomes $y(t)' = (\gamma\lambda\zeta+\mu F\varphi(t))y(t)\text{-}\gamma\lambda\zeta$. This is a first-order ordinary differential equation of $y(t)$ and has the solution:

$$(A10d)\ y(t) = e^{\int_T^t(\gamma\lambda\zeta+\mu F\varphi(s))ds}\left(y(T) - \int_T^t(\gamma\lambda\zeta)e^{-\int_0^s(\gamma\lambda\zeta+\mu F\varphi(r))dr}ds\right).$$

By *(1)* we have: $e^{\int_T^t \mu F\varphi(s)ds} = e^{\int_{\varphi(T)}^{\varphi(t)}\mu F\varphi(ds/d\varphi)d\varphi} = e^{\int_T^t(1-\varphi)^{-1}d\varphi} = e^{\log(1\text{-}\varphi(t))+\log(1\text{-}1/F)} = (1\text{-}1/F)/(1\text{-}\varphi(t))$ and $e^{\gamma\lambda\zeta(t\text{-}T)} = ((F\text{-}1)\varphi(t)/(1\text{-}\varphi(t)))^{\gamma\lambda\zeta}$. If we substitute these two expressions into the above solution of $y(t) = 1/(1\text{-}s_t(N_t))$, we can obtain *(21d)* in the main text. On the other hand, *(19e)* can be transformed into another first-order ordinary differential equation of the form: $y(t)' = (\gamma\lambda\zeta+\sigma F\varphi(t)')y(t)\text{-}\gamma\lambda\zeta$, which again has a solution:

$$\begin{aligned}(A10e)\quad y(t) &= e^{\int_T^t(\gamma\lambda\zeta+\sigma F\varphi(s)')ds}\left(y(T) - \int_T^t(\gamma\lambda\zeta)e^{-\int_0^s(\gamma\lambda\zeta+\sigma F\varphi(r)')dr}ds\right) \\ &= e^{-\gamma\lambda\zeta(t-T)-\sigma F(\varphi(t)-1/F)}\left(y(T) - (\gamma\lambda\zeta)\int_T^t e^{-\gamma\lambda\zeta s-\sigma F(\varphi(s)-1/F)}ds\right).\end{aligned}$$

If we substitute this into $s_t(N_t) = 1\text{-}1/y(t)$ and note that $s_{T(Nt\text{-}i)}(N_t\text{-}i) = \sigma/(1+\sigma)$ in the case of an embodied innovation, we obtain *(21e)* in the main text.

As for *(20d)* and *(20e)*, since they themselves are both are linear first-order ordinary differential equations of $s_t(N_t\text{-}i)$, they can easily be solved to obtain *(22d)* and *(22e)*.

<Appendix D>

The purpose of this *Appendix* is to deduce *(23ad)*, *(23ae)* and *(23be)*. We have to deal with each of them separately.

(i): The case where *Hypotheses (IN-a)* and *(CG-d)* hold.

Let us begin by examining the capital share of the best technology $s_t(N_t)$. This share emerges at $T(N_t)$ and moves along a curve *(21d)*. Let $\Theta(z;s_{T(Nt)})$ define the expression *(21d)* as a function of $z = t-T(N_t)$ and $s_{T(Nt)}(N_t)$. Then, we have:

$$(A11d)\ E\{s_t(N_t)\} = \int_0^{t-T(N_t)} \Theta(z; s_{T(N_t)}(N_t))dB_t(z).$$

Now, under the assumption of disembodied technology (*Hypothesis (CG-d)*) an innovator can implement a new technology into all of its capital stocks at the time of its success. This means that unlike the initial value of the firm share $f_{T(Nt)}(N_t)$ which always equals $1/F$, the initial value of the capital share $s_{T(Nt)}(N_t)$ is a variable whose value depends on the historical path of the innovator's capital share. Nonetheless, since under *Hypothesis (IN-a)* every firm has an equal chance for innovation, we also know that its expected value must be equal to the average capital share which is tautologically equal to $1/F$. In this paper I will use this average capital share as an approximation of $s_{T(Nt)}(N_t)$. (In the case where only the best technology firms can innovate I have not come up with a good approximation of $s_{T(Nt)}(N_t)$, and this is the reason why I have not been able to deduce any explicit formulae for the long-run average capital shares for that case under the assumption of disembodied technology.) Then, if we note that under *Hypothesis (IN-a)* we have $dB_t(z) = dW(z) = \nu F e^{-\nu Fz}dz$, *(A11d)* is seen to be equal to $1-\Gamma_\zeta$, where Γ_i is defined by *(24ad)*, as in the first line of *(23ad)* in the main text.

Next, in order to examine the capital shares of the lesser technologies $s_t(N_t-i)$ for $i = 1, 2, \ldots, N_t-1$, we have to rewrite *(22d)* recursively as:

$$(A12d)\quad s_t(N_t\text{-}i) =$$

$$e^{-\gamma\lambda(i-\zeta)(t-T(N_t))}\frac{1-s_t(N_t)}{1-s_{T(N_t)}(N_t)}\frac{s_{T(N_t)}(N_t-i)}{s_{T(N_t-0)}(N_t-i)}$$

$$\times e^{-\gamma\lambda(i-1-\zeta)(T(N_t)-T(N_t-1))}\frac{1-s_{T(N_t-0)}(N_t-1)}{1-s_{T(N_t-1)}(N_t-1)}\frac{s_{T(N_t-1)}(N_t-i)}{s_{T(N_t-1-0)}(N_t-i)}$$

$$\times\ldots\ldots$$

$$\times e^{-\gamma\lambda(1-\zeta)(T(N_t-i+2)-T(N_t-i+1))}\frac{1-s_{T(N_t-i-0)}(N_t-i+1)}{1-s_{T(N_t-i+1)}(N_t-i+1)}\frac{s_{T(N_t-i+1)}(N_t-i)}{s_{T(N_t-i+1-0)}(N_t-i)}$$

$$\times s_{T(N_t-i+1-0)}(N_t-i).$$

Now, under *Hypothesis (IN-a)* the probability that the innovator belongs to one of the users of technology $N_t\text{-}i$ is its very share $f_t(N_t\text{-}i)$ and the expected capital share of each member is $s_t(N_t\text{-}i)/(Ff_t(N_t\text{-}i))$, so that the expected reduction of its capital share at each innovation time, $E\{s_{T(N_t-m)}(N_t\text{-}i)/s_{T(N_t-m-0)}(N_t\text{-}i)\}$, is equal to $(1\text{-}1/F)$ for $m = i\text{-}1, i\text{-}2, \ldots, 0$. Again approximating $s_{T(N_t-m)}(N_{t-m})$ by $1/F$, we can express the expected value of the first, the second, ..., the penultimate and the last line of the R-H-S of *(A12d)* respectively as:

$$\int_0^{t-T(N_t)}(1-\Theta(z;1/F))e^{-\gamma\lambda(i-\zeta)z}dB_t(z)\frac{1}{1-1/F}(1-1/F);$$

$$\int_0^{t-T(N_t-1)}(1-\Theta(z;1/F))e^{-\gamma\lambda(i-1-\zeta)z}dW(z)\frac{1}{1-1/F}(1-1/F);$$

$$\ldots\ldots\ldots\ldots$$

$$\int_0^{t-T(N_t-i-1)}(1-\Theta(z;1/F))e^{-\gamma\lambda(1-\zeta)z}dW(z)\frac{1}{1-1/F}(1-1/F);$$

$$\int_0^{t-T(N_t-i)}\Theta(z;1/F)dW(z).$$

If we arrange terms and let $t \to \infty$, then they respectively converge to Γ_i, Γ_{i-1}, ..., Γ_1 and $1\text{-}\Gamma_\zeta$, where Γ_i is defined by *(24ad)*. Hence, we obtain the second line of *(23ad)*.

(ii): The case where *Hypotheses (IN-a)* and *(CG-e)* hold.

In the case of embodied technological change, the capital share of the best technology $s_t(N_t)$ emerges with a mass of $\sigma/(1+\sigma)$ at $T(N_t)$ and moves along a curve *(21e)*. Let $\Pi(z)$ denote $s_t(N_t)$ given by *(18e)* as a function of $z = t-T(N_t)$. Then, we have:

$$(A11d)\ E\{s_t(N_t)\} = \int_0^{t-T(N_t)} \Pi(z)dB_t(z).$$

Since as $t \to \infty$ $dB_t(z) \to \nu Fe^{-\nu Fdz}$ under *Hypothesis (IN-a)*, this expression converges to the first line of *(23ae)*. Next, since *(22e)* is identical with *(22d)*, the capital shares of the lesser technologies $s_t(N_t-i)$ can also be expressed by *(A12d)*. And under *Hypothesis (CG-e)* an innovation creates a new capital share of the best technology equal to $\sigma/(1+\sigma)$ and is expected to reduce all the capital shares of the lesser technologies uniformly by the factor of $1/(1+\sigma)$. Hence, the expected value of the terms:
$(1/(1-s_{T(N_t-i+1)}(N_t-i+1))(s_{T(N_t-i+1)}(N_t-i)/s_{T(N_t-i+1-0)}(N_t-i))$ in *(A11d)* becomes all equal to 1, and the expected value of the first, the second, ..., the penultimate and the last line of the R-H-S of *(A11d)* respectively becomes equal to:

$$\int_0^{t-T(N_t)} (1-\Pi(z))e^{-\gamma\lambda(i-\zeta)z}dB_t(z);\quad \int_0^{t-T(N_t-1)} (1-\Pi(z))e^{-\gamma\lambda(i-1-\zeta)z}dW(z);\ldots;$$

$$\int_0^{t-T(N_t-i-1)} (1-\Pi(z))e^{-\gamma\lambda(1-\zeta)z}dW(z);\quad \int_0^{t-T(N_t-i)} \Pi(z)dW(z).$$

If we note that $dW(z) = \nu Fe^{-\nu Fdz}$ and $dB_t(z) \to \nu Fe^{-\nu Fdz}$ under *Hypothesis (IN-a)*, it is easy to see that these expressions converge to $\Lambda_i, \Lambda_{i-1}, \ldots, \Lambda_i$ and $1-\Lambda_\zeta$, where Λ_i is defined by *(24ae)*, as $t\to\infty$. We can then obtain the second line of *(23ae)* in the main text.

(iii): The case where *Hypotheses (IN-b)* and *(CG-e)* hold.

All the formulae for this case is identical with the previous case, except that $W(z) = 1-((1-\varphi(z))/(1-1/F))^{\xi/\mu}$ and $dB_t(z) \to (1-W(z))dz/\omega$. Hence, $E\{s_t(N_t)\} \to 1-\Omega_\zeta$ and this is the first line of *(23be)*. Also the expected value of the first, the second, ..., the penultimate and the last line of the R-H-S of *(A13d)* respectively converge to Ω_i $\Psi_{i-1},\ldots,\Psi_1$ and $1-\Psi_\zeta$, where Ω_i and Ψ_i are defined by *(24be)* and *(24be')*. We can then obtained the second line of *(23be)*.

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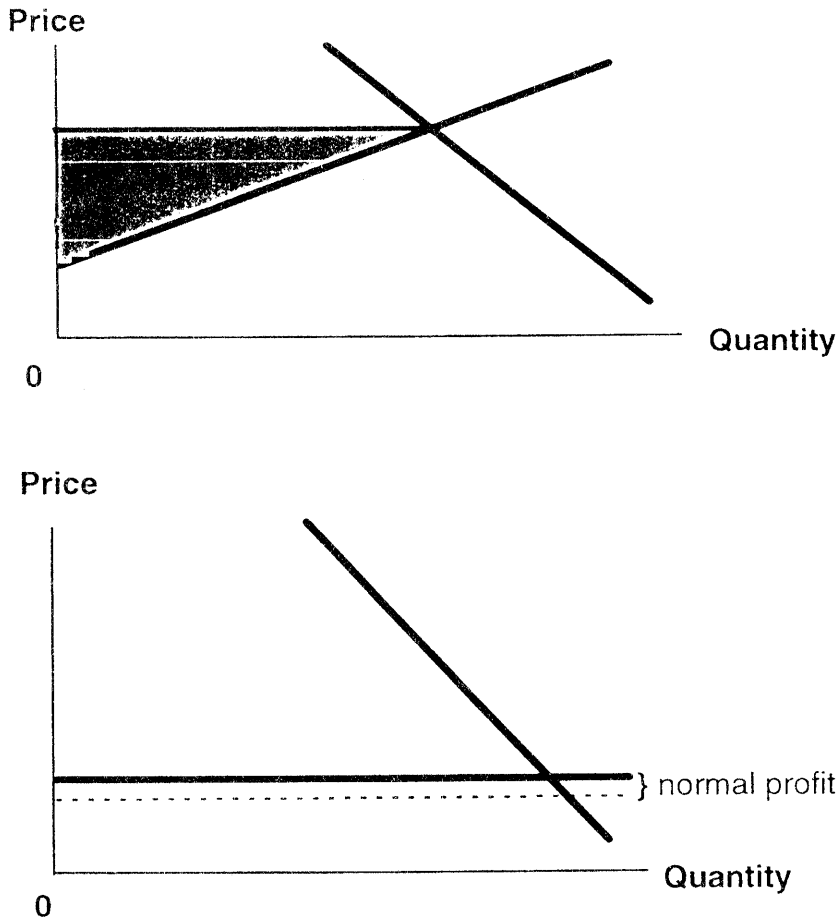

Fig. 1 : Industry supply curve in the short-run and in the long-run.

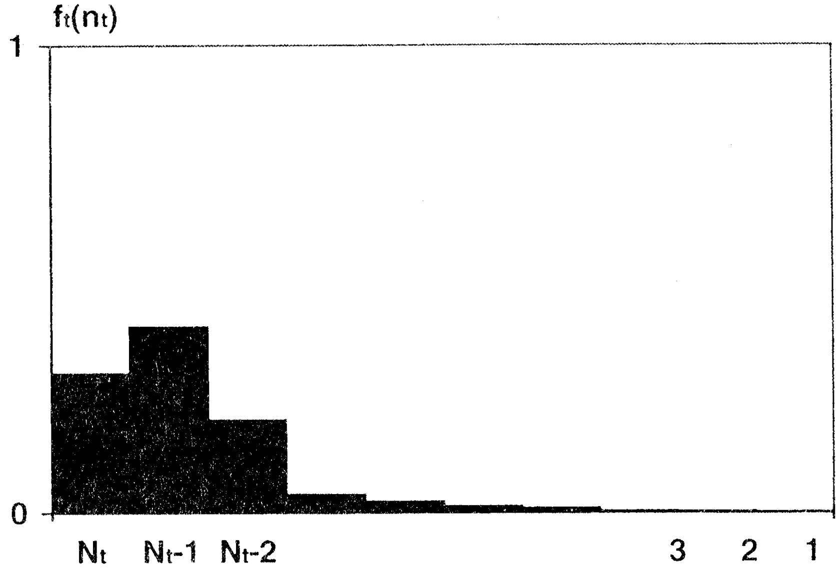

Fig. 2 : Efficiency distribution of firms.

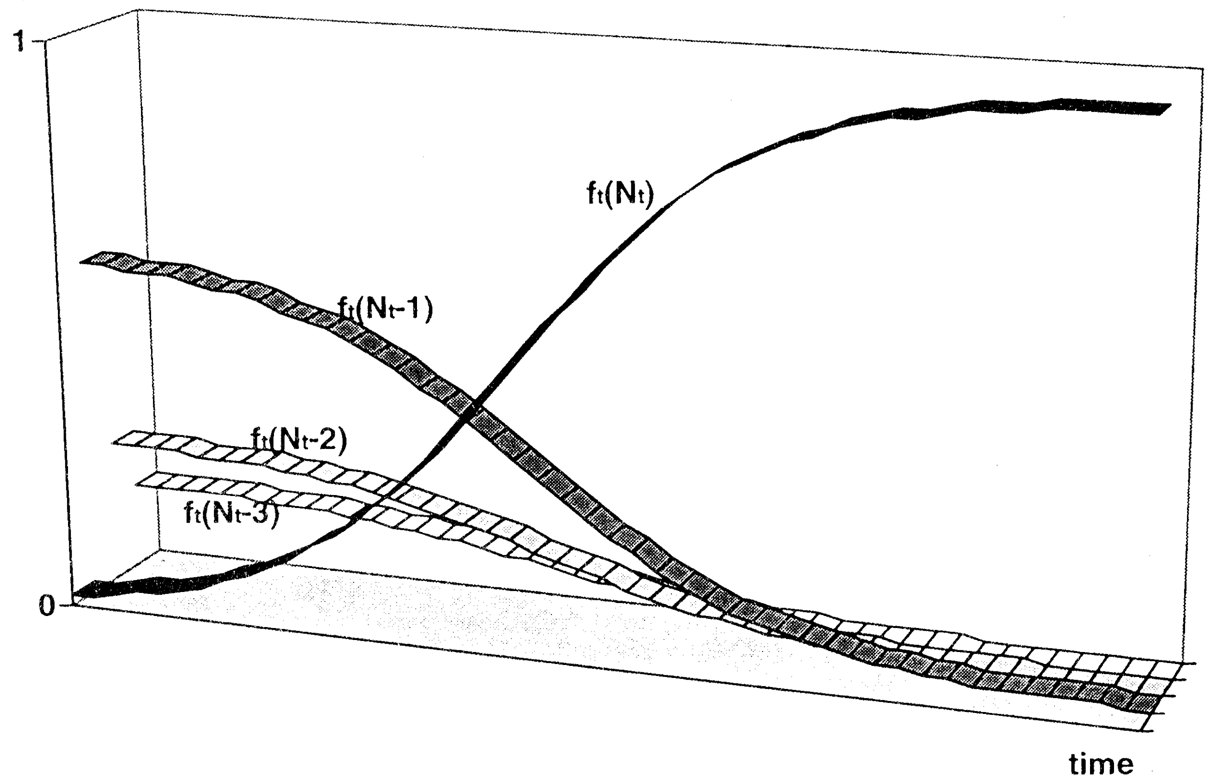

Fig. 3: Evolution of the efficiency distribution of firms under the sole pressure of technological diffusion.

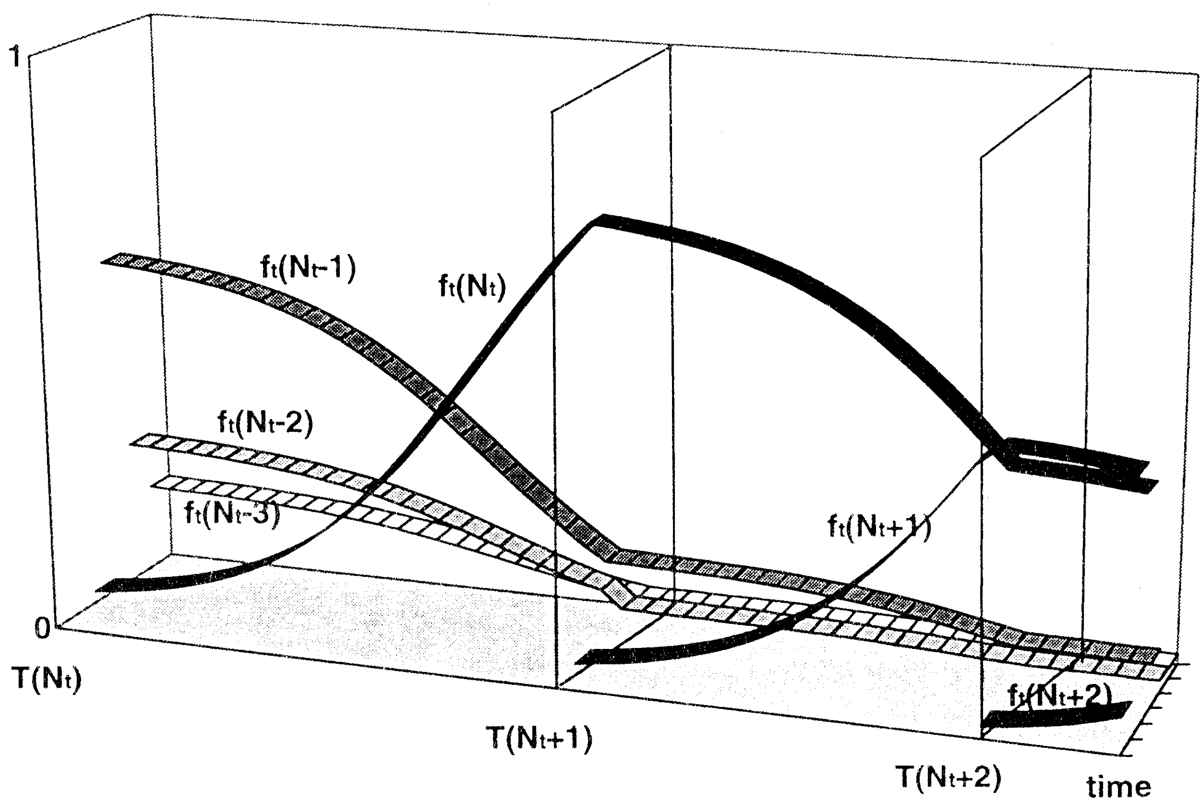

Fig. 4: Evolution of the efficiency distribution of firms under the joint pressure of technological diffusion and recurrent innovations.

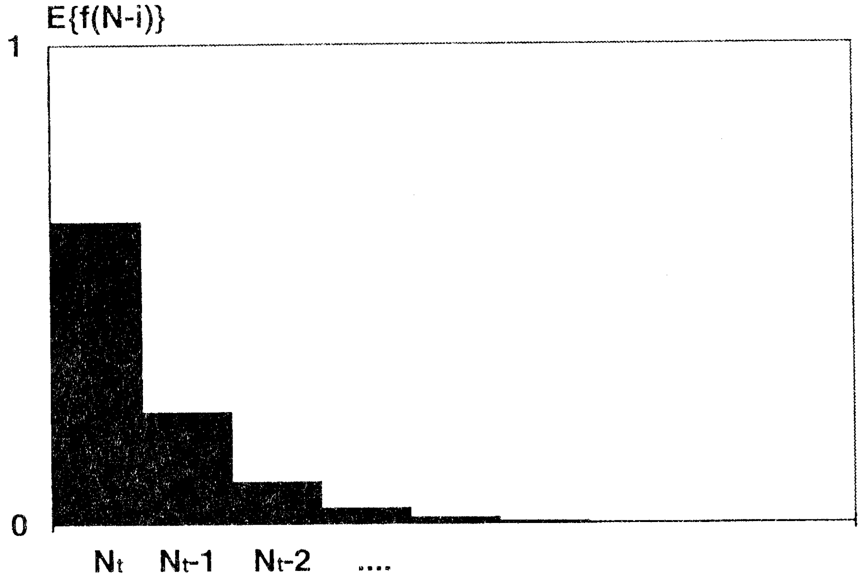

Fig. 5a: Long-run average efficiency distribution of firms when every firm can strike an innovation.

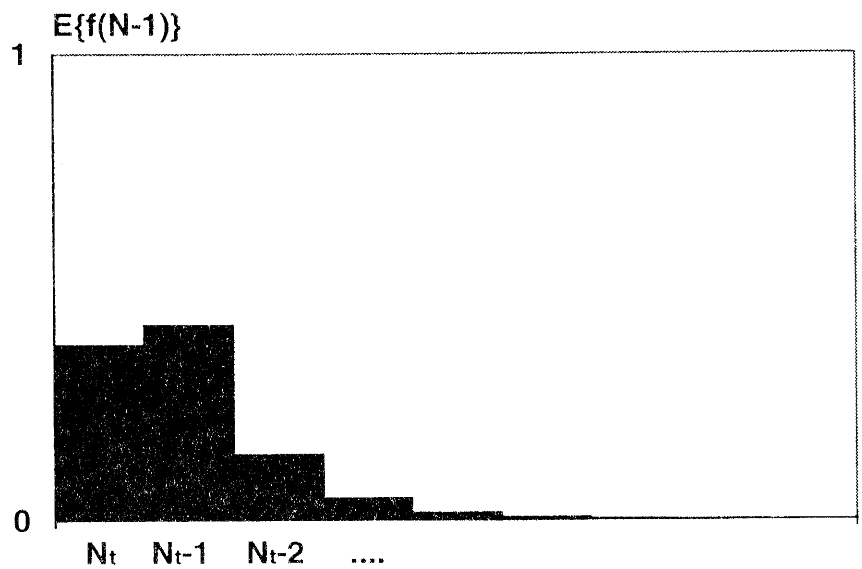

Fig. 5b: Long-run average efficiency distribution of firms when only one of the best technology firms can strike an innovation.

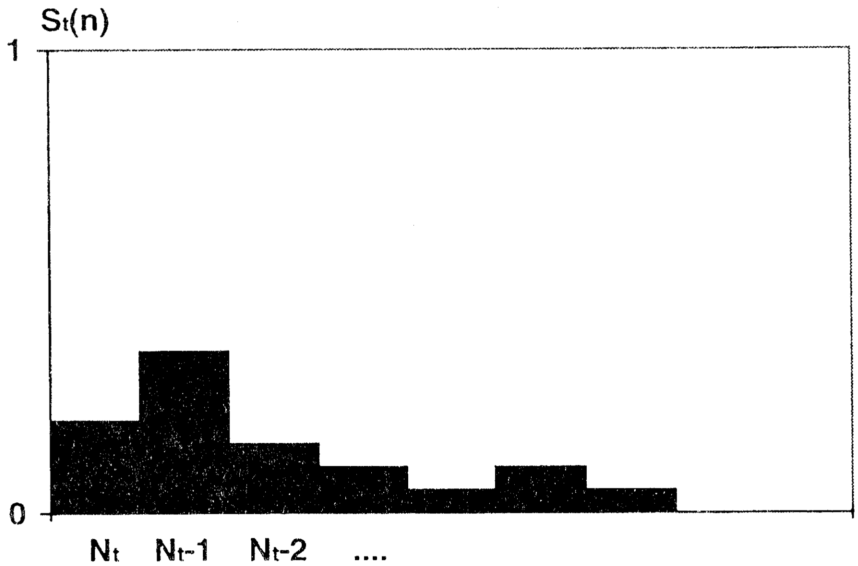

Fig. 6: Efficiency distribution of capitals.

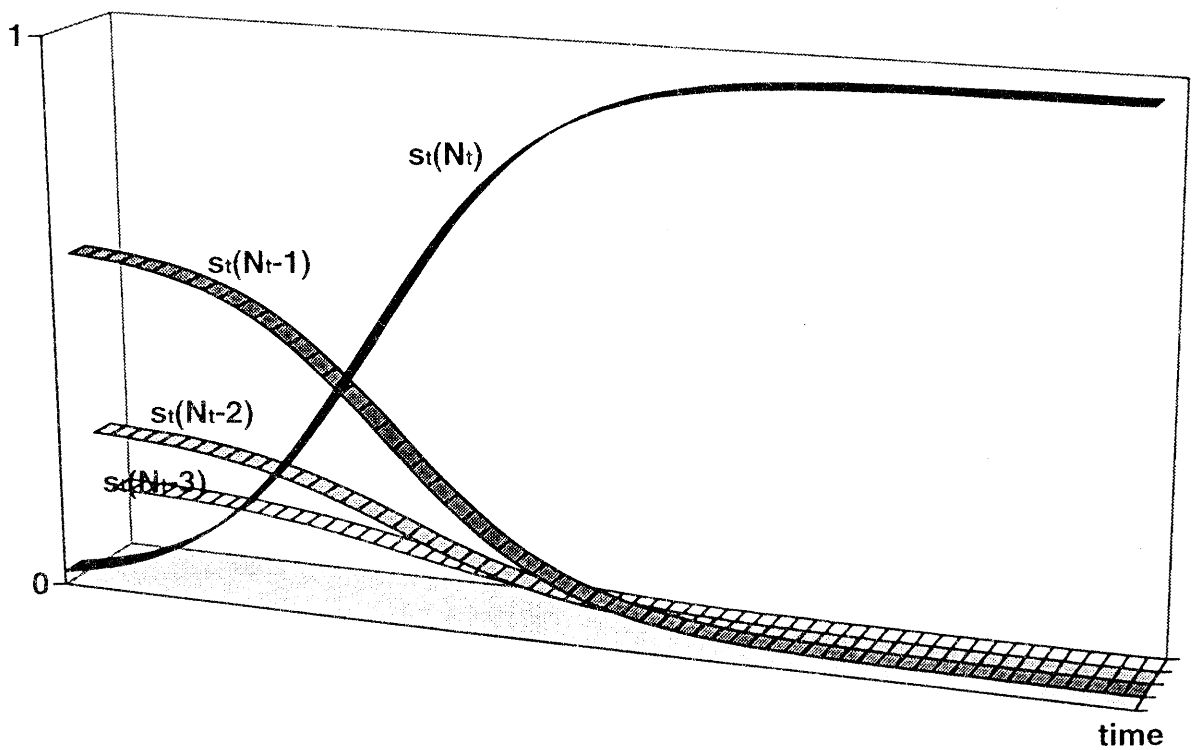

Fig. 7: Evolution of the efficiency distribution of capitals under the sole pressure of economic selection.

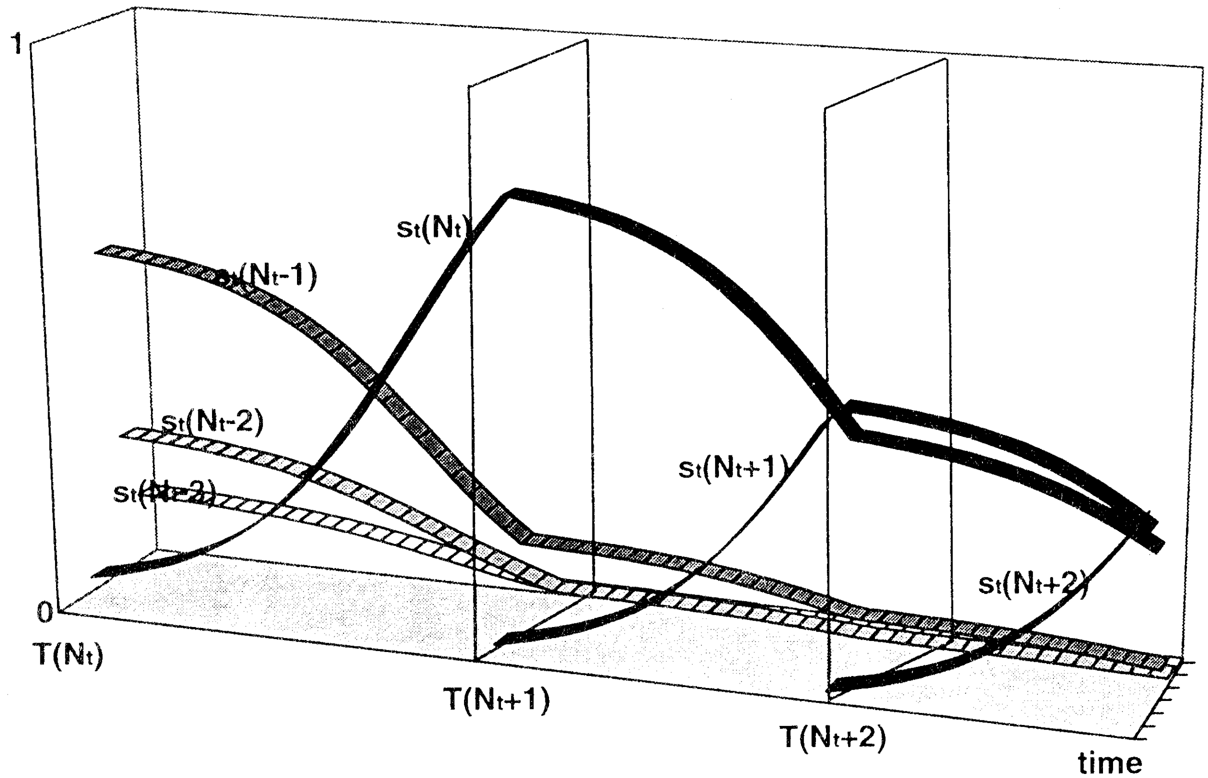

Fig. 8d: Evolution of the efficiency distribution of capitals under the joint pressure of economic selection, technological diffusion and recurrent innovations in the case where technology is not embodied in capital stocks.

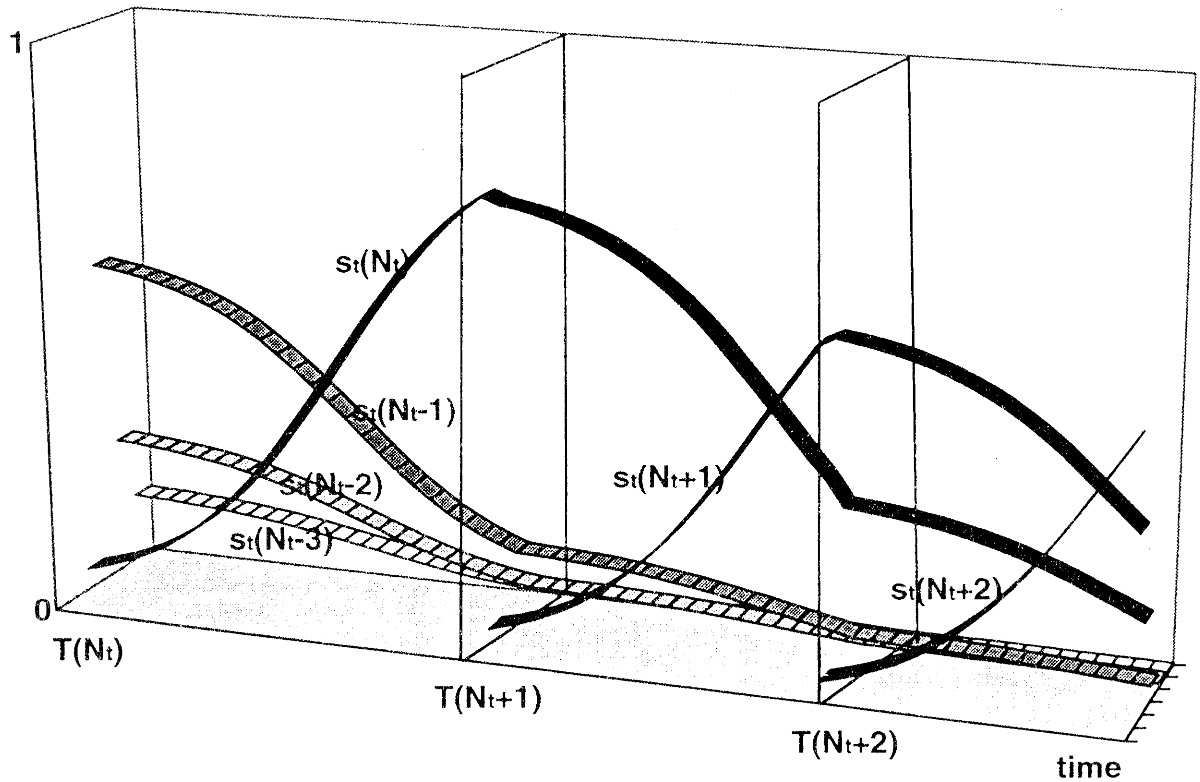

Fig. 8e: Evolution of the efficiency distribution of capitals under the joint pressure of economic selection, technological diffusion and recurrent innovations in the case where technology is embodied in capital stocks.

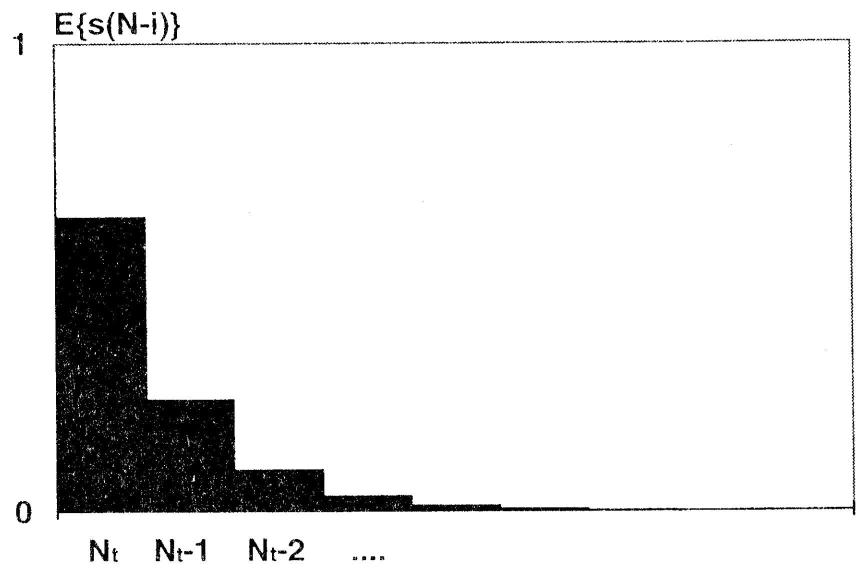

Fig. 9ad: Long-run average distribution of capitals in the case where technology is disembodied and every firm can strike an innovation.

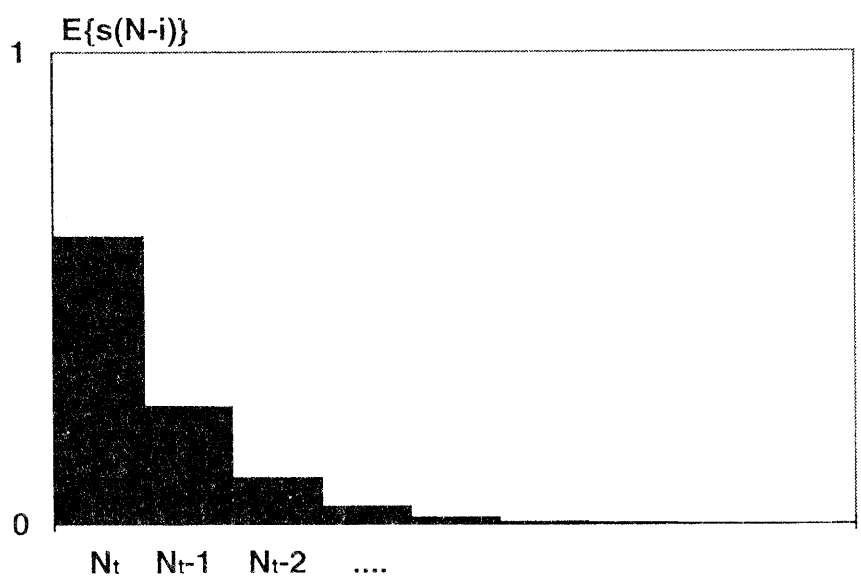

Fig. 9ae: Long-run average distribution of capitals in the case where technology is embodied and every firm can strike an innovation.

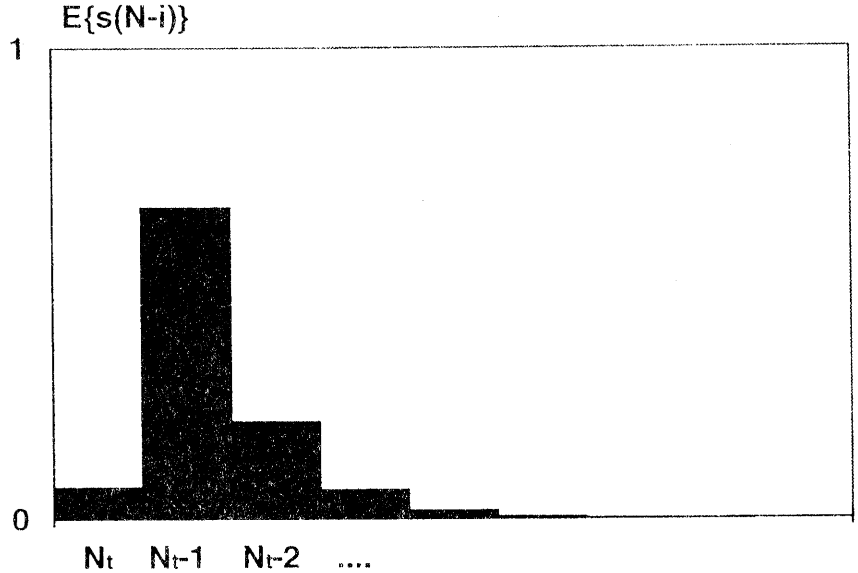

Fig. 9be: Long-run average distribution of capitals in the case where technology is embodied and only one of the best technology firms can strike an innovation.

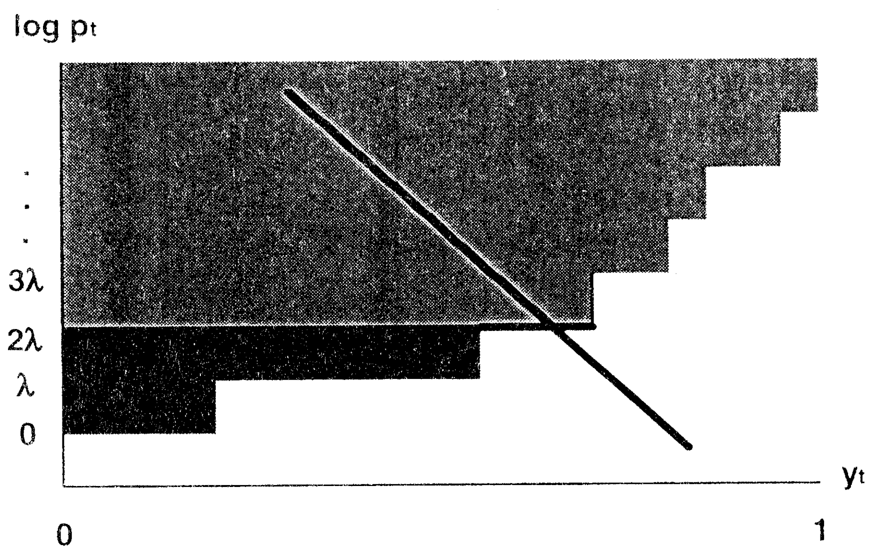

Fig.10: Short-run industry supply curve and determination of profit.

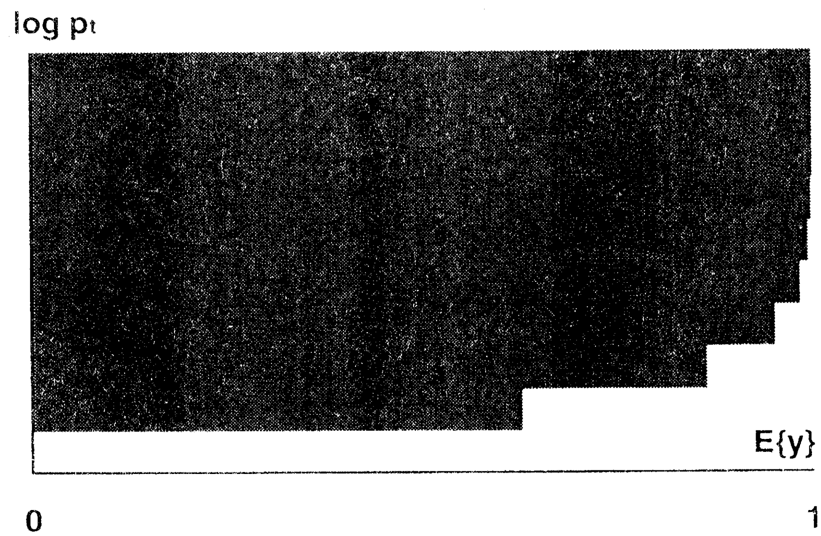

Fig. 11ad: Long-run average industry supply curve in the case where technology is disembodied and every firm can strike an innovation.

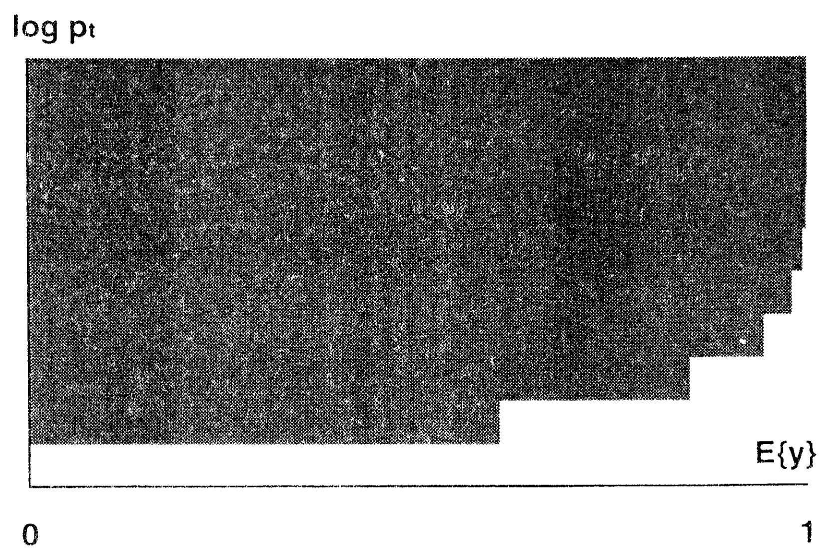

Fig. 11ae: Long-run average industry supply curve in the case where technology is embodied and every firm can strike an innovation.

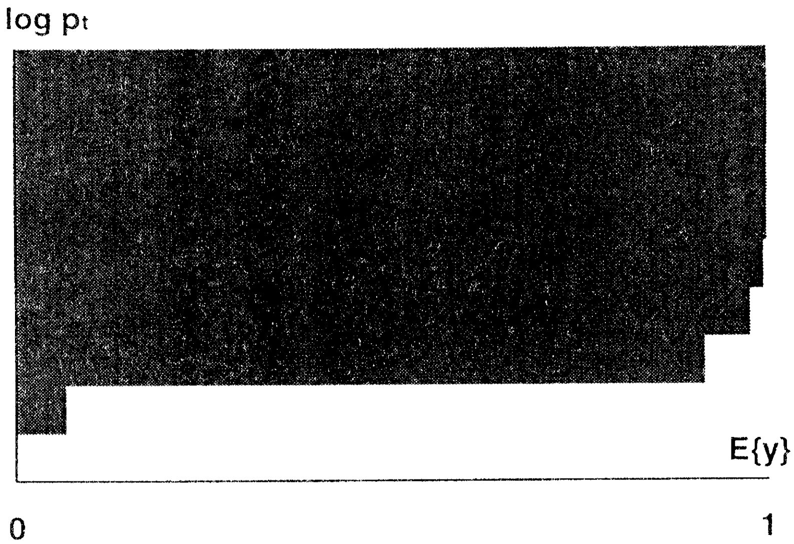

Fig. 11be: Long-run average industry supply curve in the case where technology is embodied and only one of the best technology firms can strike an innovation.

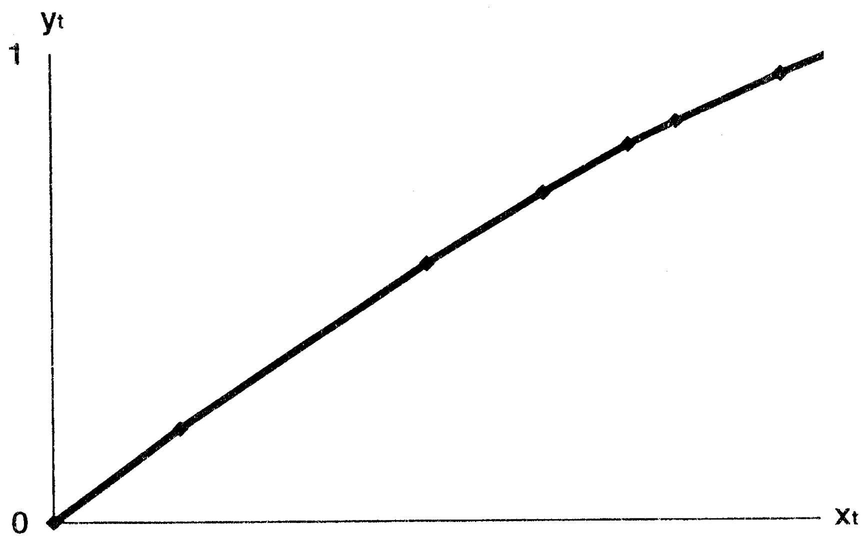

Fig. 12: Short-run 'pseudo' aggregate production function.

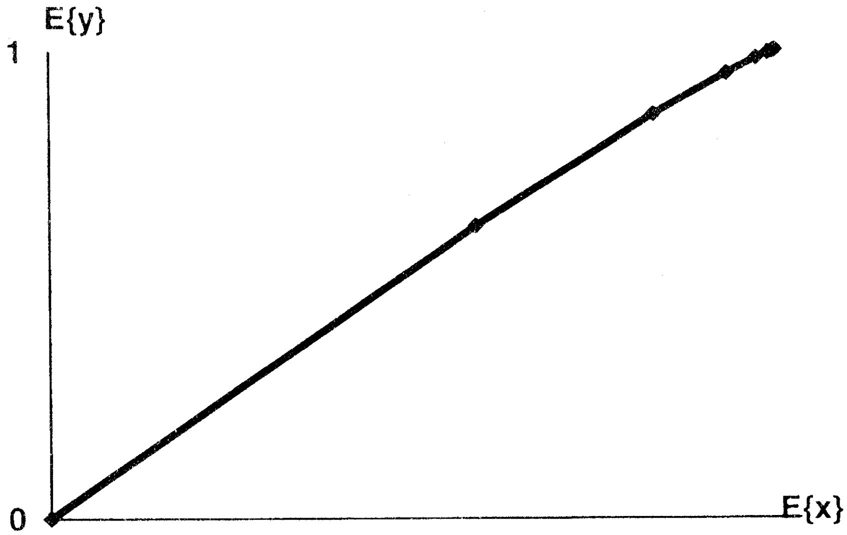

Fig. 13ad: Long-run average 'pseudo' aggregate production function in the case where technology is disembodied and every firm can strike an innovation.

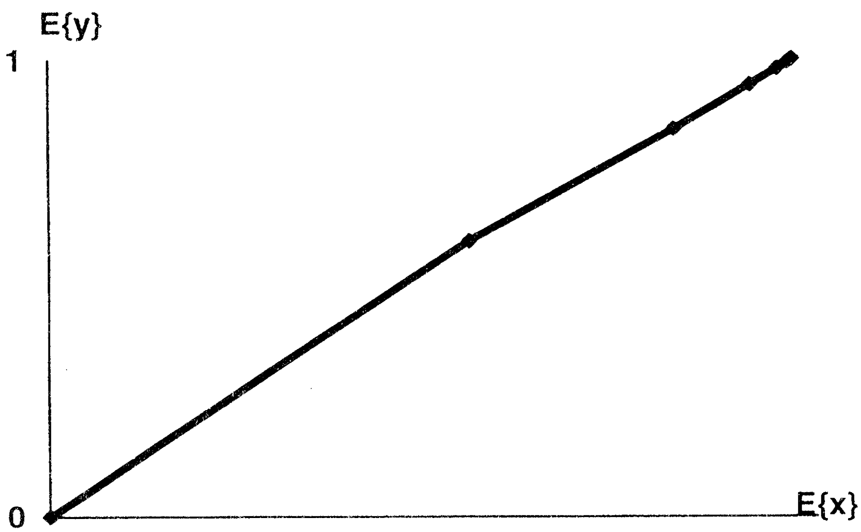

Fig. 13ae: Long-run average 'pseudo' aggregate production function in the case where technology is disembodied and every firm can strike an innovation.

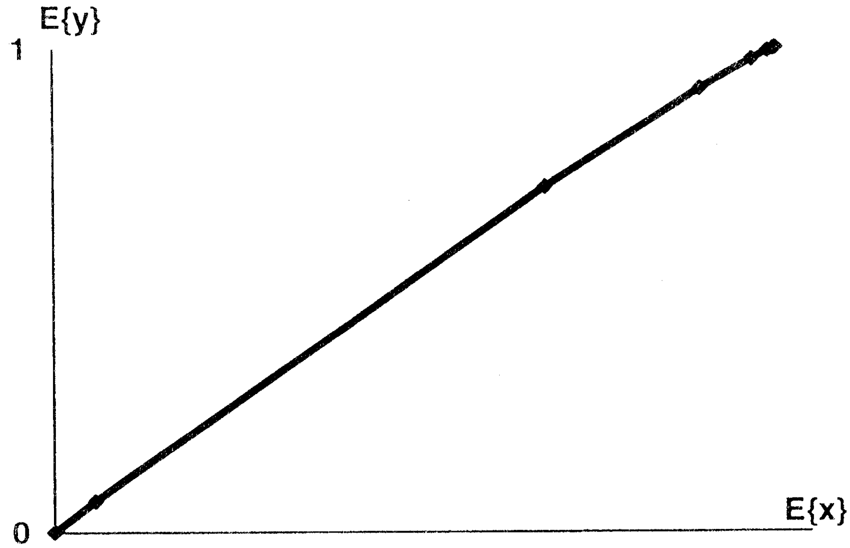

Fig. 13be: Long-run average 'pseudo' aggregate production function in the case where technology is embodied and only one of the best technology firms can strike an innovation.