97-F-10

Bounded Rationality in Economics: A Game Theorist's View

Hitoshi Matsushima
University of Tokyo

February 1997

Bounded Rationality in Economics: A Game Theorist's View*

Hitoshi Matsushima**
Faculty of Economics
University of Tokyo

February 1997

Abstract

In this paper, we will argue that it is essential to incorporate *bounded rationality* into game theory. Game theory has been applied to economics such as industrial organization on the basis of the *naïve* interpretation of game theory, which requires players to be *ideally rational* in an extremely unrealistic way. We will stress the importance of establishing the *perceptive* interpretation of game theory by taking boundedly rational players' *inductive* reasoning processes into account. We will explain my recent work, Matsushima (1997), which shows that the subjective games perceived by players in the long run are entirely different from the true objective game, and are *trivial games* in the sense that there exists a strictly dominant and subjectively Pareto-efficient strategy profile.

---

* The earlier version of this paper was based on the Invited Lecture presented at the Annual Meeting of the Japan association of Economics and Econometrics, Osaka, September 22-23, 1996. The author is grateful to Kazuya Kamiya, Michihiro Kandori, Satoko Nakano, Tadashi Shimizu, Hirokazu Takizawa, and Noriyuki Yanagawa for their helpful comments.

** Mailing Address: Hitoshi Matsushima, Faculty of Economics, University of Tokyo, Hongo, Bunkyo-ku, Tokyo 113, Japan.

1. Introduction

The neoclassical framework on the basis of the assumption that economic agents are *rational* has pervaded the whole of economic theory developed in the last century. However, despite its mathematical tractability of analyzing various problems, economists have been for a long time dissatisfied with the position of supporting this framework. Actual economic agents are not so rational as economists have assumed. Several laboratory experiments indicate that lots of the conclusions derived from ideal rationality are inconsistent with the actual behaviors.

Herbert A. Simon have pointed out in the 1950's that real individuals are not so rational to accomplish the payoff-maximization, and therefore, *ideal rationality* (or *substantive rationality* in his terminology) should be replaced by the more realistic view of *bounded rationality* (or *procedural rationality*).

Recently, game theorists have gradually had misgivings about the use of the ideal rationality assumption in a more serious way. In the 1980's, game theory has been extensively applied to various fields of economics such as industrial organization. Economists have interpreted game theory in a *naïve* way that a game is regarded as a full description of a state of the physical world as well as a state of mind, players are rational, and players' rationality and the structure of the game are both common knowledge among the players. Based on this naïve interpretation of game theory, economists have constructed the vast variety of complex extensive form games, and have required players to behave rationally in an exhaustive way. As a consequence, most of the predictions in these studies of applied game theory were criticized because they are far from reasonable.

In this paper, we will examine the possibility of incorporating bounded rationality into game theory, and emphasize that it is essential to formulate boundedly rational players' *inductive reasoning processes*. The *perceptive* interpretation of game theory will be introduced as an alternative to the naïve interpretation of game theory, according to which a game is regarded as a description of what players perceive in a complex and unstructured situation of conflict under uncertainty.

In section 2, we will briefly overview the development of applied game theory on the basis of the naïve interpretation studied in the 1980's, and will explain its drawbacks from the view-point of *the cognitive limits of rationality*, which is regarded as one aspect of bounded rationality. In Section 3, two topics in experimental game theory, i.e., ultimatum games and coordination games, will be examined from the view-point of *the motivational limits of rationality*, which is regarded as another aspect of bounded rationality.

In section 4, we will explain several approaches related to bounded rationality developed in the last decade such as automata games and evolutionary game theory, and clarify their drawbacks and the unsolved problems. We will emphasize that it is essential to establish the perceptive interpretation of game theory.

Finally, in section 5, we will present the contents of my recent work, Matsushima (1997), providing a formal theory in which how a boundedly rational player perceives an uncertain situation and constructs *a subjective game* is examined in an explicit way. It will be shown that the subjective games perceived in the long run are entirely different from the true objective game, and are *trivial games* in the sense that there is a strictly dominant, and also subjectively Pareto-efficient, strategy profile.

It is not the purpose of this paper to give a comprehensive survey.[1] We must admit that there are a number of important topics and papers which are not mentioned in this paper but are closely related to our concern, such as the incomplete contract approach explored by Grossman and Hart (1986).[2]

2. Applications of Game theory to Economics

In the 1980's, the applications of noncooperative game theory to economics have been intensively studied, especially in the area of industrial organization. Economists have contrived the vast variety of extensive form games for the descriptions of various situations of strategic conflict such as price competition, non-price competition, auction, bargaining, entry deterrence, tacit collusion, R & D competition, and the like. Nash equilibrium and its refinements such as perfect equilibrium have been put into the position of the basic behavioral hypothesis applicable to all of these situations in general. Moreover, by embracing the idea of Bayesian games pioneered by Harsanyi (1967, 1968), economists have succeeded in dealing with a wide range of situations of incomplete information. Many results are now well known among the economists in various fields, and regarded as the new standard for understanding the functioning of market

---

[1] There are many papers discussing the importance of bounded rationality in economics and game theory, for example, Binmore (1987, 1988), Selten (1990, 1991), North (1990), Aumann (1992), van Damme (1994), Rubinstein (1996), Conlisk (1996), and so on.

[2] Hart (1995) is the excellent textbook on the incomplete contract approach.

institution.[3]

We, however, must admit that this research program of applied game theory has the following serious drawbacks which are tightly related to *bounded rationality*. Applied game theorists at that time, implicitly or explicitly, have taken a position of confirming to *the naïve interpretation* in a sense that a game is regarded as a full description of a state of the physical world as well as a state of mind. Moreover, in order to get an accurate prediction, they have incorporated the detail of the description as much as possible, with very little help of relevant empirical facts. Moreover, in order to overcome the potential multiplicity of Nash equilibria, economists have positively adopted the ideas of its refinements from the view-point of rationality. As a consequence, various situations were classified in an extremely flexible way, the predictions derived from the refinements depend substantially on the very detail of the description of the models, and it is generally impossible to judge which models are more appropriate. The flexibility in formulating situations as extensive form games and the tight dependence of the predictions on the very details of the models imply that it is quite difficult to get *general insights* with significant economic implications.[4]

Economists' persisting in this naïve interpretation as the origin of the failure of applied game theory is an inevitable consequence of the adaptation of the neoclassical framework based on ideal rationality widespread around economics. It is assumed that players are rational, the model is common knowledge among players, and the fact that they are rational is itself common knowledge. Needless to say, these assumptions are very restrictive in most economic environments, especially in situations of incomplete information represented by a Bayesian game which is inevitably very complicated. An actual player is not so rational as the game theorists assume, she cannot identify what the true model is, and she also have the serious bounded capabilities of computing the payoff-maximization.

Herbert A. Simon has introduced the concept of bounded rationality in the 1950's. He emphasized that the rationality postulate of payoff-maximization should be replaced by a more realistic behavioral hypothesis such as *satisficing with aspiration levels*, because the decisions in many economic environments are complicated enough to transcend the actual agents' cognitive capabilities (Simon (1955, 1957, 1982)). It should be proposed

---

[3] For the readers unfamiliar with this research program, see Tirole (1988), Schmalensee and Willig (1989), and Scherer and Ross (1990).

[4] For the criticisms on the theory of industrial organization, see Fisher (1989), Pelzman (1989), and van Damme (1994).

that the naïve interpretation of game theory be abandoned, because it contradicts *the cognitive limits of rationality*, which is regarded as one aspect of bounded rationality.

We have also another aspect of bounded rationality called *the motivational limits of rationality*, the importance of which was stressed by Selten (1978, 1990). Selten argued that actual economic agents have multiple motivations which are entirely different from the pursuit of maximizing their own economic gains, and several experimental facts in laboratories contradict the predictions induced by sub-game perfect equilibrium or the related solution concepts with the stronger rationality assumptions. In the next section, we will briefly overview some of the experimental results in laboratories relevant to our concerns.

3. Experimental Works in Laboratories

We have the growing literature of experimental works in the analysis of strategic interaction.[5] Our concern is to find a clue from various experimental results to understand what motivates actual players. We, however, must admit that it is difficult to control players' state of mind in laboratories in contrast to the flexibility of controlling the physical structure of the model. It might be unavoidable to fall back on forced interpretations about the functioning of motivations. We nevertheless have in fact lots of interesting experimental results which economists can never overlook.

In this section, the experimental studies on *ultimatum games* and *coordination games* will be discussed. Both are popular games frequently applied to economics such as industrial organization.

3.1. Ultimatum Games

Consider the following two-person two-stage game. At the first stage player 1 makes an offer about how to share a given cake. At the second stage player 2 either accepts or rejects player 1's offer. If player 2 accepts, they can eat their respective shares according to player 1's offer. If she rejects, both cannot eat at all.

Sub-game perfect equilibrium predicts that player 1's offer gives player 2 zero and player 2 accepts any offer. The experimental results, however, are different from this

[5] For the survey of this literature, see Crawford (1995), and Kagel and Roth (1995).

prediction. In laboratories, player 1 will make an offer to give player 2 near half, and player 2 accepts only if her share is close to or more than half.

Based on the related works such as Roth et al. (1991) and the arguments in Crawford (1995), the above experimental fact can be interpreted as follows. Player 2 is, at least partly, motivated by *fairness*, instead of pure maximization of her own economic gain. On the other hand, player 1 is rational enough: She takes the fact that player 1 is motivated by fairness into account in a sophisticated way, and maximizes her own economic gain with the constraint of the risk of rejection.

An actual player is not necessarily motivated by the pure maximization of her own economic gain, even though she understands the ideas of game theory very well (Selten (1978, 1990)). What motivates players depends in a coherent way on the social setting and *the context* such as how the game is presented, as well as the physical structure of the game.

3.2. Coordination Games and the Focal Point Principle

Next, we discuss the experimental works explored by Thomas Schelling (1960), which yields insight into the use of contextual principle of coordinating, called *the focal point principle*.

First, consider the game of "name a positive integer". Most people are motivated by their own personal *predilections*, and name their respective favored numbers like 3, 7, 13, 100 and 1. Next, consider the game of "pick the same integer the others will pick", where this question is equally asked to all players and everyone knows everyone else is trying to do so. In a coordination game like this, there are multiple Nash equilibria. Everyone's picking the same number is a Nash equilibrium irrespective of what this number is. Everyone's picking randomly can also be sustained by a mixed strategy Nash equilibrium. The results in laboratories, however, indicates the strong tendency to uniquely implement a particular equilibrium in which most people are likely to pick number 1.[6]

This experimental result is interpreted as follows. It must be noted that there is no unique "favored number", and because of the large variety of favored numbers like 3,7 and so on there is no obvious way to pick the most conspicuous one. Then players ignore the population frequency of predilections, and instead seek another contextual rule of

[6] For the formal study of this phenomenon, see Mehta et al. (1994).

selection, which would lead to unambiguous results. As a result they are struck with the fact that the most clearly unique number is a smallest number, i.e., number 1.

In a coordination game it is impossible to determine a unique solution only by its abstract structure. Schelling combined its abstract structure with some contextual labels, and proposed the focal point principle that players' public knowledge about these labels makes an action profile salient as a focal point in a contextual but strategically sophisticated way.[7]

4. Bounded Rationality: Recent Progress

In this section, we will present the recent development of several approaches for the investigation of bounded rationality such as *constrained maximization*, *evolution*, and *inductive reasoning*. Frankly speaking, we are at the very early stage of the establishment of the theory of bounded rationality. Most of the existing works have serious weak points. I, however, would like to say that these approaches will give substantial clues to the vast progress of this literature in the future.

4.1. Constrained Maximization

One way to deal with bounded rationality is that a player is regarded as the payoff-maximizer with the explicit constraints on the limitation of information processing ability. Fershtman and kalai (1993), for example, investigated multi-market oligopolists as the profit-maximizer with *limited attention*. Dow (1991) and Rubinstein (1993) are also related to this "constrained maximization" approach, where players have the limited ability of *memorizing* observed prices and have to decide which prices to classify as low and which as high.

The research program for bounded rationality which has been studied most intensively in the last decade is the analysis of *finite automata* playing repeated games, in

[7] A related idea is addressed in the argument about the analogy between professional investment and beauty competition by Keynes (1936, chapter 12) . We can also find the similar ideas in the historical study of institutional economics by Greif (1994), which presented historical evidences that players' common cultural trait makes a solution salient.

which players have limited abilities to *implement* complicated strategies.[8] Neyman (1985) assumed that players are limited to using strategies programmed on finite automata with exogeneously fixed number of states. He showed that cooperation might be a Nash equilibrium in the *finitely* repeated prisoner-dilemma. Rubinstein (1986) and Abreu and Rubinstein (1988) assumed that the size of the automaton as the number of state is costly and players care about the long-run payoffs and the complexity costs lexicographically by eliminating states which are not used in equilibrium. Abreu and Rubinstein showed that, in contrast with the folk theorem, any Nash equilibrium outcome in the infinitely repeated game played by finite automata is on the *diagonal* of the set of feasible outcomes.

The serious drawback is that these works do not take into account the cost of *computing* a constrained maximal strategy. Gilboa (1988) explained that the problem of computing an optimal automaton is hard to solve, i.e., NP complete, if the number of the opponents is unknown. More interestingly, Papadimitriou (1992) showed that computing an optimal automaton with the restriction of the number of states is much more difficult than computing with no such restrictions. Generally, incorporating additional complexity costs makes the maximization problem much more difficult to solve.[9]

Based on these observations, either constrained maximization must be replaced by other behavioral hypothesis such as satisficing with the aspiration levels addressed by Simon (1957), or a complicated problem must be replaced by a drastically simplified rule of selection such as the focal point principle addressed by Schelling (1960).

4.2. Nash Equilibrium, Evolution, and Learning

Nash equilibrium is needless to say the most basic notion in noncooperative game theory. Nash equilibrium is interpreted as the necessary requirement for the self-enforcing rational behavior, assuming that each game has a unique solution. However, it is well known that there are multiple Nash equilibria in general. In the 1980's, the study of the refinements of Nash equilibrium concept such as perfect equilibrium, sequential equilibrium, and stable equilibrium, has been intensively studied in order to eliminate Nash equilibria which are unreasonable from the view-point of rationality.[10] Most of the

---

[8] For the survey of this literature, see Osborne and Rubinstein (1994, chapter 9).

[9] For the *infinite regress* issue when maintaining maximization as the ultimate logical basis, see Lipman (1991) and Conlisk (1996).

[10] For the survey of this literature, see van Damme (1991).

refinements are based on the assumption of *persistent rationality* which requires that players behave rationally enough even *off* the equilibrium path. However, the inappropriateness of this assumption has been frequently pointed out (see Dekel and Gul (1996)) .

Harsanyi and Selten (1982) constructed a coherent theory of rational reasoning process which solves the selection problem among multiple Nash equilibria, where the *risk-dominant* Nash equilibrium is regarded as the unique solution in a two-person 2×2 game. The drawback is, as Harsanyi and Selten themselves have pointed out, that a theory of equilibrium selection inevitably contradicts criteria of rationality which are not required in this theory but should be required generally, and therefore, there exist potentially multiple theories of equilibrium selection which are contradictory to each other.[11]

This rationalistic justification also relies on the assumption that all players know the unique rational solution. This assumption is restrictive in many cases, because it needs the strong *epistemic* conditions such that the structure of the game, players' rationality, and also the knowledge structure of all players, are common knowledge among the players (see Osborne and Rubinstein (1994, chapter 5), Aumann and Brandenberger (1995), and Dekel and Gul (1996)).

Moreover, Binmore (1988) argued that it is also difficult to justify the relevance of Nash equilibrium from the computational aspects of rational players. Games were modeled as pairs of computable algorithms called *Turing machines*, and it was investigated the possibility that the abilities to mimic the opponents' thinking processes make their behaviors predictable and lead to a Nash equilibrium. Binmore showed that it is at best possible to reach Nash equilibria in a quite limited class of situations.[12]

We will explain below another way to justify the realization of a Nash equilibrium from the aspects of *evolution* and *learning*. The idea that no rationality is required to reach a Nash equilibrium has been first developed by Maynard Smith and Price (1973) and Maynard Smith (1982) in evolutionary biology, and their followers have tried to apply it to economics. In a population, players are programmed to play certain strategies, more successful strategies reproduce faster, and eventually the most successful strategies survive. Hence, the stable strategies in the long run must be a Nash equilibrium as *a population equilibrium*. Mayard Smith and Price introduced the notion of *evolutionary stable strategy (ESS)* in symmetric two-person games which is a refinement of Nash

---

[11] See Carlsson and van Damme (1993).

[12] See also Anderlini (1990) and Canning (1992).

equilibrium with the robustness of small portion of *mutational deviations*. Roughly speaking, ESS is characterized by the asymptotically stable fixed point of the reproduction process as a *replicator dynamics*.

Evolutionary theory and related ideas developed in biology may be applicable to economic contexts in which players have the limited knowledge about the structure of the game and there is little room of conscious reasoning, such as the development of the QWERTY typewriter keyboad explained by David (1986). However, in economic contexts, it might be essential to incorporate the aspects of players' *inductive learning* with bounded rationality into the models. Players are supposed to accumulate the relevant empirical evidences, based on which they decide what to do at the current time. A player is modeled as *an information processing rule* which translates memorized observations into a decision in every period. Hence, a combination of players' information processing rules determines how players' behaviors evolve in the long run.

Much of the related researches have investigated the question of whether the concepts of rationalizability, Nash equilibrium, or its refinements such as perfect equilibrium are justified by some inductive learning processes. Kandori et al. (1993), for example, showed that some evolutionary dynamics with stochastic mutations approaches the risk-dominant equilibrium addressed by Harsanyi and Selten (1988) in the context of single-valued theory of rationality. In place of the requirements of sophisticated rational reasoning, *psychological natures* such as the law of inertia and the motivation of trial and error will give the substantial influences on the convergence to the equilibrium points.

The literature of evolution and learning has grown up vastly in various ways. Because of the shortage of space and time, we can not give a survey of the whole aspects.[13] It, however, must be admitted that what consequence will be arrived through the evolutionary pressures depends crucially on how learning rules are specified, but we still have serious lack of empirical researches, and in fact a restricted class of recurrent situations with mathematically tractable learning rules have been exclusively studied. Empirical and theoretical studies as well as simulation studies should walk in step with each other in the future.

---

[13] For the surveys of evolutionary game theory and its related literature, see van Damme (1991), Fudenberg and Levine (1995), Kandori (1996), Weibull (1996), and Vega-Redondo (1996).

4.3. Decision under Uncertainty and Perceptive Interpretation

Most game theorists have studied from the standpoint of the naïve interpretation of game theory, in which a game is regarded as a full description of a state of the physical world as well as a state of mind. It is assumed that the well-specified game is common knowledge among players, and player's rationality is also common knowledge. Harsanyi has advocated the doctrine that every situation of incomplete information can be described by a state of nature in a Bayesian game with nature's move artificially added at the top of the game. He has emphasized that the common knowledge assumption of a Bayesian game and the common prior assumption is always satisfied if the players are *ideally* rational.[14]

This *Harsanyi-doctrine* is, however, strongly objected as follows: The common prior assumption is not necessarily justified only by ideal rationality, and more importantly, the requirement of ideal rationality itself is extremely restrictive.[15] The early Bayesian theorists such as Savage (1954) have, implicitly or explicitly, paid attentions to a limited class of single-person decision making under uncertainty in which the sets of possible states are naturally given. However, Harsanyi and his followers have extended the Bayesian framework into a wide class of environments including multi-person decision making in which states can not be simply formulated.

Most of real economic situations such as industrial organization are complex and not well-structured. Players have limited prior knowledge about the true model, and therefore, must spend most time to understand it *subjectively*. Players' subjective models depend crucially on how they *perceive* the situation, and therefore, are not necessarily identical to the true objective model, as well as the other players' perceptions.

Selten (1978) has emphasized that formulating a model in a perceptive way is the most important step for an actual agent to reach a final decision. We must develop an explicit model of boundedly rational players' inductive reasoning processes, according to which the perceived elements relevant to the current problem, together with the empirical experiences accumulated in the previous periods, are translated into the construction of a subjective model. Selten presented an informal model of inductive reasoning process which takes into account the cognitive steps of perception, problem solving, investigation, implementation, and learning. Gilboa and Schmeidler (1995) presented a more formal theory as an alternative to the expected utility theory, called *the case-based*

---

[14] See also Aumann (1976) and Aumann and Brandenberger (1995).

[15] See the discussion in Dekel and Gul (1996).

decision theory. They provided an axiomatization of a decision procedure which chooses the best action evaluated by the sum of the utility levels resulted from using this action in the previous *cases*, each weighted by *the similarity* between that previous case and the current problem.[16] Rubinstein (1991) presented an idea of the *perceptive* interpretation of game theory in which a combination of a game and a strategy profile is viewed as a *common perception* among the relevant players.

Despite its unquestionable importance, the perceptive approach of modeling conflicting situations is at this time very immature, and we have little knowledge about the consequences of this interpretation of game theory. We will present the lists of questions which it is substantial to answer.

(1) Are the subjective games different from the objective model ?
(2) In what way are the subjective games connected with the objective model ?
(3) What are the characteristics of the class of subjective games which a player confronted with various objective models can perceive ?
(4) Can several games which have been intensively studied in the game theory literature such as Prisoner-dilemma games, coordination games, and Hawk-Dove games be perceived as subjective games ?
(5) Does a subjective game have a unique solution as apparently obvious among the relevant players ?
(6) Is there any welfare implication from the perceptive view-point ?

In order to answer these questions, we will present in the next section a formal model explored by Matsushima (1997).

5. Towards a Theory of Subjective Games

Matsushima (1997) considers the situation in which n players play a noncooperative game $G = (N, A_1, \dots, A_n, u_1, \dots, u_n)$ in some period, say, period T. $N = \{1, \dots, n\}$ is the

[16] The drawback of Gilboa and Schmeidler is that the axioms are required from the mathematical tractability, and are difficult to interpret from the view-point of psychological human nature. This point parallels the well-known criticism to the expected utility theory that the required axioms are not necessarily satisfied by laboratory experiments. See Allais (1953), Ellsberg (1961), Kagel and Roth (1995), and Hey (1995).

finite set of players, A_i is the finite set of actions for player i, $A = \times_{i \in N} A_i$, and $u_i : A \to R$ is the payoff function for player i. For convenience of the argument, we will assume that G has a unique *maxmin* action profile $a^* \in A$ such that for every $i \in N$,

$$\min_{a_{-i} \in A_{-i}} u_i(a_i^*, a_{-i}) > \min_{a_{-i} \in A_{-i}} u_i(a_i, a_{-i}) \text{ for all } a_i \neq a_i^* .$$

Players know a priori the sets of actions $A_1, \ldots, A_n$, but have no prior knowledge about the payoff functions $u_1, \ldots, u_n$, and also have no prior knowledge about which actions the others will choose. Each player i instead has her own subjective payoff function $\hat{u}_i : A \to R$. The game $\hat{G} = (N, A_1, \ldots, A_n, \hat{u}_1, \ldots, \hat{u}_n)$ constructed on the basis of these players' perceptions will be called *the subjective game* in distinction from *the objective game* G.

Player i also has a subjective probability function $\rho_i : A_{-i} \to R_+$ on the set of the other players' action profiles $A_{-i} = \times_{j \neq i} A_j$. If player i is the subjective expected payoff-maximizer, she will choose an action a_i which maximize the weighted sum $\sum_{a_{-i} \in A_{-i}} \rho_i(a_{-i}) \hat{u}_i(a_i, a_{-i})$.

To answer the questions addressed in Section 4.3, we will introduce the following stochastic dynamics according to which players' objective models evolve: Each player i repeatedly plays the same game G confronting different opponents randomly matched in all periods, and chooses actions $a_i(t), t = 1,2,\ldots$ among A_i infinitely many times. In a period t, player i observes the realized payoffs $v_i(t)$ and observes the randomly determined signals $\phi_i(t) \in \Phi_i$, but can not observe the action profile $b_i(t) \in A_{-i}$ chosen by the opponents in that period. Here, $v_i(t) = u_i(a_i(t), b_i(t))$, and given an arbitrary history $h_i^{t-1} \in H_i^{t-1}$, $b_i(t)$ and $\phi_i(t)$ are randomly determined by

$$b_i(t) = a_{-i} \qquad \text{with probability } p_i(a_{-i} | h_i^{t-1}),$$

and

$$\phi_i(t) = \phi_i \qquad \text{with probability } q_i(\phi_i | h_i^{t-1}, a_i(t), b_i(t)),$$

where $\sum_{a_{-i} \in A_{-i}} p_i(a_{-i} | h_i^{t-1}) = 1$ and $\sum_{\phi_i \in \Phi_i} q_i(\phi_i | h_i^{t-1}, a_i(t), b_i(t)) = 1$.

Player i chooses according to *a decision procedure* denoted by $d_i : \bigcup_{t=0}^{\infty} H_i^t \to \Delta(A_i)$, where H_i^t is the set of possible histories $h_i^t = (a_i(\tau), v_i(\tau), \phi_i(\tau), b_i(\tau))_{\tau=1}^t$ up to period t, $\Delta(A_i)$ is the set of mixed actions for player i, and $d_i(h_i^t)$ is independent of the history of her opponents' choices $(b_i(1),\ldots,b_i(t-1))$ because player i can not observe it. Hence, given an arbitrary history $h_i^{t-1}\in H_i^{t-1}$, $a_i(t)$ is randomly determined by

$$a_i(t)=a_i \qquad \text{with probability } d_i(h_i^{t-1})(a_i).$$

We assume that there exists a positive, but probably near-zero, real number $\varepsilon>0$ such that the following properties are always satisfied:

$$p_i(a_{-i}|h_i^t)\geq\varepsilon,$$
$$q_i(\phi_i|h_i^{t-1},a_i(t),b_i(t))\geq\varepsilon,$$

and

$$[d_i(h_i^{t-1})(a_i)\geq\varepsilon]\Leftrightarrow[d_i(h_i^{t-1})(a_i)>0].$$

Let $v_i^a:\bigcup_{t=0}^{\infty}H_i^t\to R$ be *player* i*'s evaluation rule for action profile* $a=(a_i,a_{-i})$. Player i expects in period t that she can get payoff $v_i^a(h_i^{t-1})$ when she chooses a_i and her opponents choose a_{-i}, provided history h_i^{t-1} was realized. We assume that $v_i^a(h_i^t)\geq\min_{\tau\in\{1,\ldots,t\}}v_i(\tau)$. Next, let $\delta_i:\bigcup_{t=0}^{\infty}H_i^t\to\Delta(A_{-i})$ be a probability functional according to which player i expects the opponents to choose a_{-i} with probability $\delta_i(h_i^{t-1})(a_{-i})$ in period t, provided history h_i^{t-1} was realized.

Moreover, we assume that the decision procedure d_i is *consistent with* $((v_i^a)_{a\in A},\delta_i)$ in the sense that

(i) $$d_i(h_i^{t-1})(a_i)>0 \text{ if } \sum_{a_{-i}\in A_{-i}}\delta_i(h_i^{t-1})(a_{-i})v_i^a(h_i^{t-1})\geq\sum_{a_{-i}\in A_{-i}}\delta_i(h_i^{t-1})(a_{-i})v_i^{(a_i',a_{-i})}(h_i^{t-1})$$
$$\text{for all } a_i'\in A_i,$$

(ii) $$d_i(h_i^{t-1})(a_i)>0 \text{ if } a_i=a_i(t-1),$$

and

(iii) $$d_i(h_i^{t-1})(a_i)=0 \text{ if } \sum_{a_{-i}\in A_{-i}}\delta_i(h_i^{t-1})(a_{-i})v_i^a(h_i^{t-1})<\sum_{a_{-i}\in A_{-i}}\delta_i(h_i^{t-1})(a_{-i})v_i^{(a_i',a_{-i})}(h_i^{t-1})$$
$$\text{for some } a_i'\in A_i \text{ and } a_i\neq a_i(t-1).$$

Property (i) implies that player i chooses the action which maximizes the subjective expected payoff with positive probability. Property (ii) implies "the law of inertia" that player i chooses the same action as the action chosen in the last period with positive probability. Property (iii) implies that player i never chooses the actions which are not the same as the action chosen in the last period and do not maximize the subjective expected payoff.

Given an arbitrary profile of histories $(h_i^{T-1})_{i\in N}$ up to period $T-1$, the subjective game $\hat{G}=(N,A_1,\ldots,A_n,\hat{u}_1,\ldots,\hat{u}_n)$ will be specified by

$$\hat{u}_i(a)=v_i^a(h_i^{T-1}) \text{ for all } a\in A \text{ and all } i\in N.$$

Hence, what is the subjective game relies crucially on which profile of histories is realized. Moreover, for every player $i\in N$, the subjective probability function ρ_i will be specified

$$\rho_i(a_{-i})=\delta_i(h_i^{T-1})(a_{-i}) \text{ for all } a_{-i}\in A_{-i}.$$

Hence, how player i expects the opponents to behave in period T also relies crucially on which profile of histories is realized.

Assume $|\Phi_i|=|A_{-i}|$, and let $\pi_i: A_{-i}\to\Phi_i$ be a bijection. By observing a signal $\phi_i(t)=\pi_i(a_{-i})\in\Phi_i$, player i always believes that the opponents have chosen $a_{-i}\in A_{-i}$, which is, with strictly positive but probably very small probability, not the same as the true action profiles $b_i(t)\in A_{-i}$ chosen by the opponents.

We will introduce the following two conditions on $((v_i^a)_{a\in A},\delta_i)_{i\in N}$.

Condition 1: For every $i\in N$, every $t\geq 1$, every $h_i^t\in H_i^t$, and every $a\in A$,

$$[(a_i(t),\phi_i(t))\neq(a_i,\pi_i(a_{-i}))]\Rightarrow[v_i^a(h_i^t)=v_i^a(h_i^{t-1})].$$

Condition 2: There exists an integer s such that for every $i\in N$, every $t>s$, every $h_i^{t-1}\in H_i^{t-1}$, and every $a\in A$,

$$\delta_i(h_i^{t-1})(a_{-i})=1 \text{ if } \phi_i(\tau)=\pi_i(a_{-i}) \textit{ for all } \tau=t-s,\ldots t-1,$$

and for every $r\in R$,

$$v_i^a(h_i^{t-1})\leq r \text{ if } (a_i(\tau),\phi_i(\tau))=(a_i,\pi_i(a_{-i})) \textit{ and } v_i(\tau)\leq r \textit{ for all } \tau=t-s,\ldots t-1.$$

Condition 1 implies that player i will not change the evaluation for an action profile $a=(a_i,a_{-i})$, if either she did not choose a_i or she did not observe the relevant signal $\pi_i(a_{-i})$. Condition 2 implies that irrespective of what is the current period and which history was realized, player i expects with certainty that the opponents will choose a_{-i}

and the payoff realized when she chooses a_i will be less than or equal to r, if she has repeatedly chosen a_i, observed signal $\pi_i(a_{-i})$, and all of the realized payoffs have been less than or equal to r, for the last s periods.

With Conditions 1 and 2, we will give answers to the six questions addressed in section 4.3: For every sufficiently large T, the following properties hold almost surely.

(1) The subjective game $\hat{G}$ is totally different from the objective game G.

(2) The subjective game $\hat{G}$ is connected with the objective game G in a way that

$$\hat{u}_i(a_i^*, a_{-i}) \geq \min_{a'_{-i} \in A_{-i}} u_i(a_i^*, a'_{-i}) > \hat{u}_i(a_i, a_{-i}) \text{ for all } a_{-i} \in A_{-i} \text{ and all } a_i \neq a_i^*.$$

(3) Hence, the subjective game $\hat{G}$ is a *trivial game* in the sense that a^* is a unique *strictly dominant* strategy equilibrium and is *Pareto-efficient* in $\hat{G}$.

(4) Several games which have been studied in the game theory literature such as Prisoner-dilemma games, coordination games, and Hawk-Dove games can *not* be perceived as subjective games, because these are not trivial games.

(5) The subjective game $\hat{G}$ has a unique solution as apparently *obvious* among the players, which is the maximin action profile a^* in the objective game G. The profile of the decision procedures $(d_i)_{i \in N}$ eventually continue to assign a^* almost surely.

(6) Moreover, there is a very significant welfare implication such that the obvious solution a^* may be inefficient in the objective game G, but is (subjectively) Pareto-efficient in the subjective game $\hat{G}$.

References

Abreu, D. and A. Rubinstein (1988): "The Structure of Nash Equilibrium in Repeated Games with Finite Automata," *Econometrica* 56, 1259-1282.

Allais, M. (1952): "Fondemonts et d'une Theorie Positive des Choix Comportant un Risque et Critique des Postulats et Axioms de L'Ecole Americaine," *Econometrica* 15, 257-332.

Anderlini, L. (1990): "Some Notes on Church's Thesis and the Theory of Games," *Theory and Decision* 29, 19-52.

Aumann, R. (1976): "Agreeing to Disagree," *Annals of Statistics* 4, 1236-1239.

Aumann, R. (1992): "Rationality and Bounded Rationality," Nancy L. Schwartz Lecture, Kellogg School of Management, Northwestern University.

Aumann, R. and A. Brandenburger (1995): "Epistemic Conditions for Nash Equilibrium," *Econometrica* 63.

Binmore, K. (1987, 1988): "Modeling Rational Players I and II," *Economics and Philosophy* 3 and 4, 179-214 and 9-55.

Canning, D. (1992): "Rationality, Computability, and Nash Equilibrium," *Econometrica* 60, 877-888.

Carlsson, H. and E. van Damme (1993): "Global Games and Equilibrium Selection," *Econometrica* 61, 989-1018.

Conlisk, J. (1996): "Why Bounded Rationality ?," *Journal of Economic Literature* 36, 669-700.

Crawford, V. (1995): "Theory and Experiment in the Analysis of Strategic Interaction," forthcoming in *Advances in Economic Theory: the Seventh World Congress*, Cambridge University Press.

David, P. (1985): "Clio and the Economics of QWERTY," *American Economic Review* 75, 332-337.

Dekel, E. and F. Gul (1996): "Rationality and Knowledge in Game Theory," forthcoming in *Advances in Economic Theory: the Seventh World Congress*, Cambridge University Press.

Dow, J. (1991): "Search Decisions with Limited Memory," *Review of Economic Studies* 58, 1-14.

Ellsberg, D. (1961): "Risk, Ambiguity, and the Savage Axioms," *Quarterly Journal of Economics* 75, 643-669.

Fershtman, C. and E. Kalai (1993): "Complexity Consideration and Market Behavior," *RAND Journal of Economics* 24, 224-235.

Fisher, F. (1989): "Games Economists Play: A Noncooperative View," *RAND Journal of Economics* 20, 113-137.

Fudenberg, D. and D. Levine (1995): "Theory of Learning in Games," mimeo.

Gilboa, I. (1988): "The Complexity of Computing Best Response Automaton in Repeated Games," *Journal of Economic Theory* 45, 342-352.

Gilboa, I. and D. Schmeidler (1995): "Cased Based Decision Theory," *Quarterly Journal of Economics*, 605-639.

Greif, A. (1994): "Cultural Beliefs and the Organization of Society: A Historical and Theoretical Reflection on Collectivist and Individualist Societies," *Journal of Political Economy* 102, 912-950.

Grossman, S. and O. Hart (1986): "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," *Journal of Political Economy* 94, 691-719.

Harsanyi, J. (1967, 1968): "Games with Incomplete Information Played by Bayesian Players," *Management Science* 14, 159-182, 320-334, 486-502.

Harsanyi, J. and R. Selten (1988): *A General Theory of Equilibrium Selection in Games*, Cambridge: MIT Press.

Hart, O. (1995): *Firms, Contracts, and Financial Structure*, Oxford: Clarenton Press.

Hey, J. (1995): "Experiments and the Economics of Individual Decision Making under Risk and Uncertainty," forthcoming in *Advances in Economic Theory: the Seventh World Congress*, Cambridge University Press.

Kagel, J. and A. Roth (eds.)(1995): *Handbook of Experimental Economics*, Princeton University Press.

Kandori, M. (1996): "Evolutionary Game Theory in Economics," forthcoming in *Advances in Economic Theory: the Seventh World Congress*, Cambridge University Press.

Kandori, M., G. Mailath, and R. Rob (1993): "Learning, Mutation, and Long Run Equilibria in Games," *Econometrica* 61, 29-56.

Keynes, J. M. (1936): *The General Theory of Employment, Interest, and Money*, London: Macmillan.

Lipman, B. (1991): "How to Decide How to Decide How to ... : Modeling Limited Rationality," *Econometrica* 59, 1105-1126.

Matsushima, H. (1997): "Procedural Rationality and Learning Behavior," mimeo.

Maynard Smith, J. (1982): *Evolution and the Theory of Games*, Cambridge University Press.

Maynard Smith, J. and G. Price (1973): "The Logic of Animal Conflicts," *Nature* 246, 15-18.

Mehta, J. C. Starmer, and R. Sugden (1994): "The Nature of Salience: An Experimental Investigation of Pure Coordination Games," *American Economic Review* 84, 658-673.

Neyman, A. (1985): "Bounded Complexity Justifies Cooperation in the Finitely Repeated Prisoners' Dilemma," *Economics Letters* 19, 227-229.

North, D. (1990): *Institutions: Institutional Change and Economic Performance*, Cambridge University Press.

Osborne, M. and A. Rubinstein (1994): *A Course in Game Theory*, MIT Press.

Papadimitriou, C. (1992): "On Players with a Bounded Number of States," *Games and Economic Behavior* 4, 122-131.

Pelzman, S. (1991): "The Handbook of Industrial Organization: A Review Article," *Journal of Political Economy* 99, 201-217.

Roth, A., V. Prasnikar, M. Okuno-Fujiwara, and S. Zamir (1991): "Bargaining and Market Behavor in Jerusarem, Ljubljana, Pittsburgh, and Tokyo: An Experimental Study," *American Economic Review* 73, 362-372.

Rubinstein, A. (1986): "Finite Automata Play the Repeated Prisoners' Dilemma," *Journal of Economic Theory* 39, 83-96.

Rubinstein, A. (1991): "Comments on the Interpretation of Game Theory," *Econometrica* 59, 909-924.

Rubinstein, A. (1993): "On Price Recognition and Computational Complexity in a Monopolistic Model," *Journal of Political Economy* 101, 473-484.

Rubinstein, A. (1996): "Lectures on Modeling Bounded Rationality," *CORE Lecture Series*.

Savage, L. (1954): *The Foundation of Statistics*, New York: Wiley.

Schelling, T. (1960): *The Strategy of Conflict*, Cambridge: Harvard University Press.

Scherer, F. and D. Ross (1990): *Industrial Market Structure and Economic Performance: Third Edition*, Boston: Houghton Mifflin Company.

Schmalensee, R. and R. Willig (eds.) (1989): *Handbook of Industrial Organization*, Amsterdam: North-Holland.

Selten, R. (1978): "The Chain Store Paradox," *Theory and Decision* 9, 127-159.

Selten, R. (1990): "Bounded Rationality," *Journal of Institutional and Theoretical Economics* 146, 649-658.

Selten, R. (1991): "Evolution, Learning and Economic Behavior," *Games and Economic Behavior* 3, 3-24.

Simon, H. (1955): "A Behavioral Model of Rational Choice," *Quarterly Journal of Economics* 69, 99-118.

Simon, H. (1957): *Models of Man*, New York: Wiley.

Simon, H. (1982): *Models of Bounded Rationality,* MIT Press.

Tirole, J. (1988): *The Theory of Industrial Organization*, Cambridge: MIT Press.]

van Damme, E. (1987): *Stability and Perfection of Nash equilibria*, Springer Verlag: Berlin.

van Damme, E. (1994): "Game Theory: The Next Stage," mimeo.

Vega-Redondo, F. (1996): *Evolution, Games, and Economic Behavior*, Oxford University Press.

Weibull, J. (1995): *Evolutionary Game Theory*, Cambridge: MIT Press.