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Environmental Externalities, Growth and Consumption Taxes

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Abstract

This paper investigates welfare implications of environmental externalities with a framework of voluntary provision of a public good by non-overlapping and overlapping generations. We show that economies in which consumption causes greater environmental degradation may not necessarily lead to poorer environment and lower welfare. Economic growth will raise welfare and improve environmental quality. Consumption taxes can internalize both intragenerational and intergenerational externalities.

Key words: Environmental externalities, consumption taxes, generational conflict

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1. Introduction

There has been much attention on the long-run effects of economic activities on the world environment. In the analysis of environment problems we could consider two types of a public good (or bad). First, consumption of agents accumulates pollution, which hurts environmental quality. Second, contributions of agents may improve environmental quality. Researchers have investigated mechanisms under which a decentralized economy might successfully internalize environmental externalities. The standard analysis is of intragenerational conflict, the free-rider problem within a generation. See Pigou (1920) and Varian (1995). Another important aspect of the analysis is an intergenerational conflict. See John et al. (1995), Farzin (1996), and Yoshida (1996a). Farzin (1996) showed how in the face of environmental stock externalities the static market-based policy instruments such as Pigouvian tax should be modified using a simple dynamic model. He showed that even if for an initial period there is going to be no pollution stock damage, the optimal policy still requires that abatement begins immediately and at increasing rate.

This paper investigates both conflicts in a simple theoretical framework of non-overlapping (and then overlapping) generations and explores the normative role of consumption taxes that could internalize both intragenerational and intergenerational externalities.

We show that economies in which consumption causes greater environmental degradation may not necessarily lead to poorer environment and lower welfare. Economic growth will raise welfare and improve environmental quality. An introduction of consumption taxes is always desirable. The optimal level of consumption taxes is increasing with the number of agents (the degree of intragenerational externalities) and the social generation preference factor (the degree of intergenerational externalities). We also explore some potential conflicts among generations of taxes on consumption. We finally incorporate both consumption and production externalities. By doing so, our model can exhibit various types of dynamic properties; a low-level equilibrium trap, catastrophe, convergence to a steady state, or steady state growth.

Section 2 presents the analytical model of non-overlapping generations. Section 3 investigates some comparative statics of changes in environmental factors. Section 4 considers the first-best solution. Section 5 introduces consumption tax policies and derives the optimal level of consumption taxes. Section 6 then extends the basic framework to an overlapping generations model with capital accumulation. Section 7 investigates dynamic properties of the model and considers how changes in environmental parameters would affect the dynamic system. Section 8 considers welfare implications of changes in environmental factors and then investigates normative aspects of the model by deriving the first-best solution and the optimal levels of consumption taxes. And section 9 concludes the paper.

2. Model

Assume that there are n (more than two) identical agents (or countries) of generation t at time t in the world. Agent i's utility is given by

$$U^i_t = U(c_{it}, G_t) \tag{1}$$

where U^i_t is welfare of agent i at time t, c_{it} is private consumption of agent i at time t, and G_t is the quality of environment for agent i at time t, which is common to all agents and may be regarded as a pure public good. (i = 1,2...,n). We assume that both c_i and G are normal goods. We adopt as a basic framework a non-overlapping generations model; an agent of generation t is born at time t, lives for one period and dies at time t+1. Each generation has n agents and there is no population growth.

The environmental quality will change over time. The dynamic process of G is given by

$$G_t = (1-b)G_{t-1} - \sum_{i=1}^{n} \beta c_{it} + \sum_{i=1}^{n} \gamma g_{it} \tag{2}$$

where g_{it} is voluntary payment of environmental maintenance and improvement provided by agent i at time t, b measures the autonomous evolution of environmental quality, β indicates the degree of environmental degradation by agent i's consumption, and γ is the degree of environmental improvement by agent i's environmental expenditures. g_i may be regarded as a voluntary provision of a public good of improving environmental quality of the Earth[1].

Agent i of generation t's budget constraint is given by

$$c_{it} + p g_{it} = Y \tag{3}$$

where Y is exogenously given identical income of agent i and p is the relative price of environmental expenditures in terms of private consumption. Low (high) p means high (low) productivity of providing the public good, which improves environmental quality.

Substituting (2) into (3), we have

$$\gamma c_{it} + pG_t = \gamma Y - p\sum_{i=1}^{n} \beta c_{it} + p\sum_{j \neq i} \gamma g_{jt} + (1-b)pG_{t-1}$$

Or

$$(\gamma + p\beta)c_{it} + pG_t = \gamma Y - p\sum_{j \neq i} \beta c_{jt} + p\sum_{j \neq i} \gamma g_{jt} + (1-b)pG_{t-1} \tag{3'}$$

We assume that each agent determines its public good provision g_i and consumption c_i, treating the other's spending c_j, g_j, the relative price of the public good p, environmental parameters, β, γ, b, G_{-1}, n, and income Y as given. As in the standard model of voluntary provision of a pure public good, we will exclude binding contracts or cooperative behavior between the agents (countries) and will explore the outcome of non-cooperative Nash behavior.

In this Cournot-Nash model, define the expenditure function:

Minimize $E^i_t = (\gamma + p\beta)c_{it} + pG_t$ subject to $U^i_t = \text{U}$.

Then, the following equation will determine agent i's welfare, U^i_t as a function of real income, $\gamma Y - p\sum_{j\neq i}\beta c_{jt} + p\sum_{j\neq i}\gamma g_{jt} + (1-b)pG_{t-1}$, which contains actual income, degradation of environment by consumption of other agents, the externalities from the other agent's provision of the environmental expenditure, and the previous quality of environment inherited from the previous generation.

$$E(U^i_t, \gamma + p\beta, p) = \gamma Y - p\sum_{j\neq i}\beta c_{jt} + p\sum_{j\neq i}\gamma g_{jt} + (1-b)pG_{t-1} \tag{4}$$

By a variant of Shephard's Lemma we know

$$G^i_t = G(U^i_t, \gamma + p\beta, p) \tag{5}$$

where G^i ($\equiv \partial E^i / \partial q^3$) is the compensated demand function for the public good of agent i, i.e., environmental quality of the Earth. Here, let us define respectively the effective prices of c and G as follows[2];

$$q^1 \equiv \gamma + p\beta \tag{6-1}$$

$$q^3 \equiv p \tag{6-3}$$

An increase in β will reduce real income. This income effect is captured by an increase in the effective price of c. An increase in p/γ will also reduce real income.

From (4) we have

$$\frac{1}{p}\sum_{i=1}^{n} E^{i}_{t} = \frac{\gamma n}{p}Y + (n-1)G_t + (1-b)G_{t-1} \tag{7}$$

(7) comes from (2) and (4). Namely, multiply (4) for E_i by $\frac{1}{p}$. Add them from i=1 to n and use (2). Then we will get (7).

Since all agents within a generation are identical, from (5)(6) and (7) the multi-agent Nash equilibrium model may be summarized by the following dynamic equation.

$$\begin{aligned} E(U_t, \gamma + p\beta, p) &= \gamma Y + \frac{n-1}{n} pG(U_t, \gamma + p\beta, p) \\ &+ \frac{1}{n}(1-b)pG(U_{t-1}, \gamma + p\beta, p) \end{aligned} \tag{8}$$

This dynamic equation determines U_t as a function of price p, environmental parameters, β, b, γ, U_{t-1}, n, and income Y. We assume the existence of a Nash equilibrium with $g_i > 0$.[3]

3. Steady state equilibrium

3. 1 Comparative statics

In the steady state substituting $U_t = U_{t-1} = U$ into (8), then the system reduces to

$$E(U, \gamma + p\beta, p) = \gamma Y + p\frac{n-b}{n}G(U, \gamma + p\beta, p) \tag{9}$$

which determines long-run welfare U as a function of $p, \beta,$ b, n, and Y.

Differentiating (9) with respect to p, β, γ, b, n, and Y, we have

$$\frac{dU}{dp} = \frac{1}{\Delta}[-\frac{bG}{n} - \beta c + p\frac{n-b}{n}(\beta G_1 + G_3)] \tag{10}$$

$$\frac{dU}{d\beta} = \frac{1}{\Delta}p[p\frac{n-b}{n}G_1 - c] \tag{11}$$

$$\frac{dU}{d\gamma} = \frac{1}{\Delta}[Y + p\frac{n-b}{n}G_1 - c] \tag{12}$$

$$\frac{dU}{db} = \frac{1}{\Delta}p[-\frac{1}{n}G] \tag{13}$$

$$\frac{dU}{dn} = \frac{1}{\Delta}pb\frac{1}{n^2}G \tag{14}$$

$$\frac{dU}{dY} = \frac{\gamma}{\Delta} \tag{15}$$

where $\Delta \equiv E_U - p\frac{n-b}{n}G_U > 0$, $G_1 \equiv \frac{\partial G}{\partial q^1} > 0$, $G_3 \equiv \frac{\partial G}{\partial q^3} < 0$.

It is easy to see that an increase in income (Y) raises welfare; (15) > 0. An increase in the number of agents (n) also raises welfare; (14) > 0. In both cases c and G increase. Economic growth will raise welfare and improve environmental quality. There is no trade-off between growth and environmental improvement in this model. This result is consistent with the divergence between the developed and the developing countries in both consumption and environmental quality if the environmental externalities are restricted within the developed and the developing countries respectively. An increase in the number of countries raises total consumption and total environmental expenditures. The former effect is harmful, while the latter effect is beneficial. (14) means that the overall effect is beneficial.

An increase in the degree of autonomous evolution of environmental quality (b) will reduce welfare; (13) < 0. Since $\beta G_1 + G_3 = -\frac{\gamma}{p} G_1 < 0,$ a decrease in the productivity of environmental expenditures (an increase in p) will reduce welfare; (10) < 0. In both cases G will also decline. An increase in the degree of environmental improvement by environmental expenditures (γ) is the same as a decrease in p and hence raises welfare. An increase in γ (a decrease in p) may be regarded as a combination of increases in Y and β. An increase in Y is beneficial, and this dominates the overall effect. These results are also intuitively plausible.

However, the sign of (11) is ambiguous; the effect of environmental degradation (β) on welfare is ambiguous. An increase in β reduces real income and hence welfare, which is the income effect. An increase in β raises q^1, the effective price of c, stimulating substitution from c to g, which is the price effect. If the price effect dominates the income effect, (11) becomes positive. In this case G will increase but the effect on c is ambiguous. (11) > 0 if and only if $\frac{n-b}{n}\frac{E}{q^1}\varepsilon_G(1-\alpha)-\alpha > 0,$ where ε_G $(\equiv G_1 q^1 / G)$ is the elasticity of G with respect to q^1 and α $(\equiv q^1 c / E)$ is the marginal propensity to spend on c. Suppose $\varepsilon_G = 2, \alpha = 0.5$, n>2, and $E/q^1>1$. Then, (12) becomes positive. Economies in which consumption causes greater environmental degradation (higher β) may not necessarily lead to poorer environment and lower welfare.[4]

The decentralized steady-state economy summarized by (9) has two kinds of market failures; the intragenerational and intergenerational externalities. Thus, consumption is too much, while environmental quality (or the voluntary contribution on improving environment) is too little.

3.2 Heterogeneous countries and country-specific changes

Let us next consider the case where countries (or agents) are not identical. For simplicity without loss of generality, we focus on the case of n=2. Suppose some exogenous parameters specific to country 1 change. How would these changes affect both country 1 and country 2?

As shown in Ihori (1996), in the voluntary provision of public good model a decrease in p (an increase in the productivity of providing the public good) is not beneficial to its own country. We may show that this result is applied to changes in the degree of environmental quality improvement (γ). Intuition is as follows. An increase in γ will raise g_1, which will benefit country 2. Then country 2 will react to decrease her supply of the public good, g_2, which is not beneficial to country 1. If this negative spillover price effect from country 2 is greater than the direct positive income effect of an increase in γ, country 1 will lose.

Our analysis suggests that a country may not have a strong incentive to raise the country-specific productivity of improving environmental quality as an increase in γ may reduce (not raise) welfare of the country.

If the preferences are the same between countries, then $G^i(U^i, \gamma_i + p_i\beta_i, p_i)$ is the same. Suppose also $p_1 = p_2$. Then we know that $U^1 > U^2$ (and hence $c_1 > c_2$) if and only if $\gamma_1 < \gamma_2$ or $\beta_1 < \beta_2$ Every country must have the same demand for G in the equilibrium. Since G_q is positive, a country with low γ (β) needs high welfare (i.e. high effective income) to demand the same level of G. A country with high γ does not necessarily enjoy high welfare. A country with low γ can enjoy high welfare, which is a seemingly paradoxical result.

If the effective productivity of improving environmental quality (γ / p) is the same between countries, then a country in which consumption causes smaller environmental degradation enjoys higher consumption and better welfare. This is intuitively plausible. This may explain the divergent experiences of Eastern Europe and the OECD countries. Eastern Europe has experienced low levels of consumption and welfare. In our model, even if income is equal and the productivity of improving environmental quality is the same, the East could have suffered because it had access only to inferior disposal (consumption) technologies. A country has a strong incentive to reduce country-specific environmental degradation of consumption, but may not have a strong incentive to raise the country-specific productivity of improving environmental quality.

4. The first best solution

Consumption taxes would be effective to internalize both externalities. In order to evaluate the normative role of consumption taxes, it is useful to investigate the first best solution of this economy where agents are identical. We analyze the optimal path which would be chosen by a central planner who maximizes an intertemporal social welfare function expressed as the sum of generational utilities discounted by the social generation preference factor, ρ, which is between 0 and 1.

Substituting (3) into (2) and considering $\sum_{i=1}^{n} c_{it} = nc_t$, we have

$$G_t = (1-b)G_{t-1} - n\beta c_t + \frac{\gamma n}{p}(Y - c_t) \qquad (2)'$$

Therefore the maximization problem faced by the planner is

$$Max \sum_{t=0}^{\infty} \rho^t U(c_t, G_t) \qquad \text{subject to (2)'}$$

In other words, the first best problem is to maximize the Lagrange function

$$W = \sum_{t=0}^{\infty} \rho^t \{U(c_t, G_t) + \lambda_t [G_t - (1-b)G_{t-1} + n(\frac{\gamma}{p} + \beta)c - \frac{\gamma n}{p} Y]\} \qquad (16)$$

where λ is the current shadow price of G and the Lagrange multiplier constraint at time t is $\rho^t \lambda_t$.

The optimality conditions with respect to c_t and G_t are given by

$$U_{ct} = -n(\frac{\gamma}{p} + \beta)\lambda_t \qquad (17\text{-}1)$$

$$U_{Gt} = -\lambda_t + (1-b)\rho\lambda_{t-1} \qquad (17\text{-}2)$$

along with the transversality condition

$$\lim_{t\to\infty} \rho^t \lambda_t G_t = 0 \qquad (17\text{-}3)$$

where $U_{ct} \equiv \frac{\partial U_t}{\partial c_t}$ and $U_{Gt} \equiv \frac{\partial U_t}{\partial G_t}$.

In the steady state from (17-1) and (17-2) we have as the optimal marginal rate of substitution between G and c

$$\frac{U_G}{U_c} = \frac{p[1-(1-b)\rho]}{(\gamma + p\beta)n} \qquad (18)$$

Note that in the decentralized market economy the marginal rate of substitution between G and c is given by

$$\frac{q^3}{q^1} = \frac{U_G}{U_c} = \frac{p}{\gamma + p\beta} \tag{19}$$

which is greater than (18). Since $\rho > 0$ and n>1, U_G / U_c given by (18) is less than U_G / U_c given by (19).

In the long run from (2)' we have as the feasibility condition

$$\gamma Y = \frac{pb}{n} G + (\gamma + p\beta)c$$

which is shown as line AB in Figure 1. Point S is the first best point associated with condition (19), while point E is the equilibrium point associated with condition (19). As shown in Figure 1, c is too much and G (or g) is too little in the laissez faire economy.

5 Taxes on consumption

5.1. Model

This section examines welfare implications of tax policies to depress polluting consumption and to stimulate environmental improving expenditures. Let us investigate the effect of taxes on consumption. When a tax τ_t is imposed on consumption c_i of agent i, the budget constraint (3) is rewritten as

$$c_{it} + \tau_t c_{it} + pg_{it} = Y - T_{it} \tag{20}$$

where T_i is a lump sum tax. Tax revenue from consumption taxes will be returned to the private sector by a lump sum transfer. Note that redistribution of income between agents of the same generation is neutral[5]. The government budget constraint is

$$\sum_{i=1}^{n} T_i = -\sum_{i=1}^{n} \tau c_i \tag{21}$$

Considering (2), (20) will reduce to

$$(\gamma + \gamma\tau_t + p\beta)c_{it} + pG_t = \gamma Y - \gamma T_{it} - \beta p\sum_{j\neq i} c_{jt} + p\sum_{j\neq i}\gamma g_{jt} + (1+b)pG_{t-1} \qquad (20)'$$

Thus, in place of (4), we have

$$E(U^i{}_t, \gamma + \gamma\tau_t + p\beta, p) = \gamma Y - \gamma T_{it} - \beta p\sum_{j\neq i} c_{jt} + p\sum_{j\neq i}\gamma g_{jt} + (1-b)pG_{t-1} \qquad (4)'$$

Considering (20), the dynamic Nash equilibrium model with consumption taxes will be summarized by the following equation.

$$E(U_t, \gamma + \gamma\tau_t + p\beta, p) = \gamma Y + \frac{n-1}{n}pG(U_t, \gamma + \gamma\tau_t + p\beta, p) + \frac{1}{n}(1-b)pG(U_{t-1}, \gamma + \gamma\tau_{t-1} + p\beta, p) + \gamma\tau c(U_t, \gamma + \gamma\tau_t + p\beta, p) \qquad (22)$$

5.2 Welfare effect of consumption taxes

The long-lived government's objective is to choose taxes to maximize an intertemporal social welfare function W expressed as the sum of generational utilities discounted by the factor of social time preference, ρ.

The associated Lagrange function is given as

$$W = \sum_{t=0}^{\infty}\rho^t\{U_t - \mu_t[E(U_t, \gamma + \gamma\tau_t + p\beta, p) - \gamma Y - \frac{n-1}{n}pG(U_t, \gamma + \gamma\tau_t + p\beta, p) - \frac{1}{n}(1-b)pG(U_{t-1}, \gamma + \gamma\tau_{t-1} + p\beta, p) - \gamma\tau c(U_t, \gamma + \gamma\tau_t + p\beta, p)]\} \qquad (23)$$

Differentiating the Lagrangian (23) with respect to τ_t, we have

$$\frac{\partial W}{\partial \tau_t} = \gamma\rho^t\{-\mu_t[-\frac{n-1}{n}pG_1 - \gamma\tau_t c_1] + \rho\mu_{t+1}\frac{1-b}{n}pG_1\} = 0 \qquad (24)$$

From (24) in the steady state we have

$$\tau c_1 + \frac{[(n-1)+\rho(1-b)]p}{n} G_1 = 0 \tag{25}$$

Since $c_1 \equiv \frac{\partial c}{\partial q^1} = -\frac{q^3}{q^1} G_1 < 0,\ G_1 > 0$, (25) gives the optimal level of consumption tax rate as

$$\tau^* = \frac{(\gamma + p\beta)[n-1+(1-b)\rho]}{\gamma[1-(1-b)\rho]} \tag{26}$$

An introduction of consumption taxation is always desirable. τ^* is increasing with the degree of environmental degradation (β), the factor of social time preference (ρ), and the number of countries (n), while decreasing with the autonomous evolution of environmental quality (b) and the effective productivity of environmental expenditures (γ / p).

Even if n=1 and $\beta = 0$, $\tau^* > 0$. When n= 1 and $\beta=0$, there are no intragenerational externalities and consumption does not produce intergenerational externalities. Still the myopic optimization produces intergenerational externalities, resulting in too much c and too little G. Thus, it is still desirable to raise the relative price of c in terms of G, stimulating G and depressing c.

Suppose that the short-lived government is myopic and only concerns the present generation. Substituting $\rho \Rightarrow 0$ into (26), the myopic optimal consumption tax rate for generation t, τ_m, is given by

$$\tau_m = (n-1)(\gamma + p\beta)/\gamma \tag{27}$$

τ_m is increasing with the number of agents, namely, the degree of intragenerational externalities. τ_m is always less than τ^*. When the government concerns welfare of future generations as well as the present generation, the optimal consumption tax rate must be raised to internalize the intergenerational externalities as well. Such an increase in τ will hurt the present generation. In this sense, there is a conflict between the present generation and the future generation with respect to the optimal level of consumption taxes.

The dynamically optimal level of consumption taxes (τ^*) internalizes both the intragenerational and intergenerational externalities. Thus, its level is higher, the higher the intragenerational externalities (n) and the higher the intergenerational externalities ($(1-b)\rho$).

Finally, several remarks are useful. First, let us compare the consumption tax policy with lump sum taxes. A lump sum redistribution policy between generations is effective in this model since the private sector is completely myopic. When the government imposes taxes on the present generation and subsidizes the future generation, the present generation loses, while the future generation gains. But, such lump sum taxes and subsidies cannot attain the first best solution. Even if the government imposes lump sum taxes and provides directly the public good, private provision of the public good would be reduced by the same amount. This is the well-known neutrality result. The lump sum tax policy is not effective to stimulate G. Such a change could be realized only by affecting the relative price of consumption.[6]

Second, we have assumed that the agent chooses c and g taking account the impact his choices will have on the environmental externality. The effective price of consumption, q^1, captures this externality. If the consumer is atomistic, he might not take into account his creation of pollution on the global pollution problem. In this case the consumer price would be 1. The analytical results would be almost the same as in the text. The optimal level of consumption tax rate is then given as

$$\tau^{**} = \tau^{*} + \frac{p\beta}{\gamma}$$

which is greater than τ^{*} given by (26). This is because the agent does not take into account the impact his consumption will have on the externality .

Third, we can show that an introduction of country-specific taxes on consumption may not benefit the home country. An increase in a consumption tax in country 1 only may hurt its own country, while benefiting other countries. This Prisoner's Dilemma situation is due to the spillover price effect. Since q_1 increases, country 1 will reduce c_1 and raise g_1, benefiting country 2. When $q_1 > q_2$, then $U^1 < U^2$. It should be noted that this result is not due to the tax burden effect.

Finally, we may consider the welfare effect of imposing subsidies on the provision of an environmental improving expenditure, g_i. In this case the budget constraint (3) is rewritten as

$$c_i + (1-\tau_i)g_i = Y_i - T_i \tag{28}$$

where τ_i is the subsidy rate. The subsidy policy is equivalent to the consumption tax policy. Thus, an increase in a country-specific subsidy rate on country 1's provision of environmental expenditures will benefit country 2 but it may hurt country 1. G and c_2 increase, while c_1 decreases. Country 2 enjoys higher consumption and better environment, while country 1 loses due to lower consumption. Each country has an incentive to subsidize environmental expenditures of the other country.

6. Overlapping generations economy

6.1 Model

In this section we extend the basic framework to an overlapping generations economy and investigate how the basic results would be affected. An agent of generation t born at time t, considers itself young in period t, old in period t+1, and dies at time t+2. When young an agent of generation t supplies one unit of labor inelastically and receives wages w_t out of which the agent consumes c^1_{it}, provides an environmental expenditure g_{it}, and saves s_{it}. An agent receives $(1+r_{t+1})s_{it}$ when old, which the agent then spends entirely on consumption c^2_{it+1}. r_t is the rate of interest in period t. There are no private bequests. As before there is no population growth.

Thus, a member of generation t faces the following budget constraints

$$c^1_{it} = w_{it} - pg_{it} - s_{it} \tag{29}$$

$$c^2_{it+1} = (1 + r_{t+1})s_{it} \tag{30}$$

His lifetime utility function is rewritten as

$$U^i_t = U^i(c^1_{it}, c^2_{it+1}, G_{t+1}) \tag*{(1)'}$$

It is assumed for simplicity that he is only concerned with environmental quality at the beginning of the old age, G_{t+1}.

The dynamic process of G is rewritten as

$$G_{t+1} = (1-b)G_t - \sum^n_{i=1} \beta(c^1_{it} + c^2_{it}) + \sum^n_{i=1} g_{it} \tag*{(2)'}$$

where the second period consumption of the previous generation reduces environmental quality of the present generation. From now on it is assumed for simplicity that $\gamma = 1$.

In place of (3)', the private budget constraint is given by

$$(1+p\beta)c_{it}^1+\frac{1}{1+r_{t+1}}c_{it+1}^2+pG_{t+1}=w_t-p\sum_{i\neq j}\beta c_{it}^1-p\sum_{i=1}^{n}\beta c_{it}^2$$
$$+p\sum_{i\neq j}g_{it}+(1-b)pG_t \tag{31}$$

The expenditure function is now defined by minimizing

$$E^i{}_t\equiv(1+p\beta)c_{it}^1+\frac{1}{1+r_{t+1}}c_{it+1}^2+pG_{t+1}$$

Let us define respectively the effective consumer prices of c_t^1, c_{t+1}^2 as follows;

$$q^1\equiv 1+p\beta \tag{6-1}$$

$$q_t^2\equiv\frac{1}{1+r_{t+1}} \tag{6-2}$$

Let us then formulate the aggregate production function. The firms are perfectly competitive profit maximizers who produce output using the production function

$$Y_t=A(G_t)F(K_t,n)=G_t^{\alpha}K_t^{1-\lambda}n^{\lambda} \qquad (\alpha>0,0<\lambda<1) \tag{32}$$

F( ) exhibits constant returns to scale. For simplicity we assume the Cobb-Douglas function. The function $A(G)=G^{\alpha}$ is a technological externality that captures enhancements to productivity from environmental quality. α measures the degree of production externality from environmental quality.[7] Because G_t is predetermined at time t, A(G) is a constant from the perspective of current producers. As for the standard first-order conditions from the firm's maximization problem in period t, we have

$$r_t=G_t^{\alpha}(1-\lambda)(\frac{n}{K_t})^{\lambda} \tag{33}$$

$$w_t=G_t^{\alpha}\lambda(\frac{K_t}{n})^{1-\lambda} \tag{34}$$

In an equilibrium agents can save by holding capital. We have

$$ns_t = K_{t+1} \tag{35}$$

The dynamic model may be summarized by (33)(34) and the following two equations.

$$\begin{aligned} E(U_t, q_t) = w_t + p\frac{n-1}{n}E_3(U_t, q_t) + \frac{1}{n}p(1-b)E_3(U_{t-1}, q_{t-1}) \\ - p\beta E_2(U_{t-1}, q_{t-1}) \end{aligned} \tag{36}$$

$$nq_t^2 E_2(U_t, q_t) = K_{t+1} \tag{37}$$

where $q_t = (q_t^1, q_t^2, q_t^3) = (1 + p\beta, \frac{1}{1+r_{t+1}}, p)$ is the consumer price vector for generation t. E_i denotes the partial derivatives of the expenditure function with respect to price q^i (i=1,2,3). (36) comes from (31) and (37) comes from (35).

7. Dynamics

In order to have concrete results with respect to dynamic properties, let us assume that the utility function (1)' is logarithmic.

$$U_t = \log c_t^1 + \log c_{t+1}^2 + \log G_{t+1} \tag{1)''$$

Then in this case we have

$$E_t = 3(q_t^1 q_t^2 q_t^3 U_t)^{\frac{1}{3}} = 3q_t^3 G_{t+1} \tag{38-1}$$

$$E_{1t} = \frac{q_t^3}{q_t^1} G_{t+1} \tag{38-2}$$

$$E_{2t} = \frac{q_t^3}{q_t^2} G_{t+1} \tag{38-3}$$

From (37) and (38-3) we have

$$npG_{t+1} = K_{t+1} \tag{39}$$

which means that G and K always move to the same direction. Substituting (39) into (33) and (34), we have

$$r_t = (1-\lambda)p^{-\lambda}G_t^{\alpha-\lambda} \tag{33'}$$

$$w_t = \lambda p^{1-\lambda}G_t^{\alpha-\lambda+1} \tag{34'}$$

Hence, considering (33)'(34)'(38-1) and (38-3), (36) may be rewritten as

$$G_{t+1} = \{p^{-\lambda}[\lambda-\beta(1-\lambda)]G_t^{\alpha-\lambda+1} + (\frac{1-b}{n}-\beta)G_t\}\frac{n}{2n+1}$$

$$\equiv \phi(G_t) \tag{40-1}$$

$$G_1 = \{\frac{1}{p}[w_0 - \frac{\beta(1+r_0)K_0}{n}] + \frac{1-b}{n}G_0\}\frac{n}{2n+1}$$

$$\equiv \varphi(G_0) \tag{40-2}$$

which are the fundamental dynamic equations of the model.

Figure 2 shows dynamics of (40-2). Substituting $G_1 = G_0 = \tilde{G}$ into (40-2), we have

$$\tilde{G} = \frac{nw_0 - \beta(1+r_0)K_0}{p(2n+b)}$$

If $G_0 > \tilde{G}$, then $\tilde{G} < G_1 < G_0$ and vice versa. If $nw_0 \le \beta(1+r_0)K_0$ ($\tilde{G} \le 0$), we always have $G_1 < G_0$. (40-2) also implies that G_1 is decreasing with b, p, β but may well be increasing with n.

Let us now investigate dynamics of (40-1). First of all, suppose $\frac{\lambda}{1-\lambda} > \beta,\ \frac{1-b}{n} \ge \beta$. Then we have the following three cases.

<u>Case A: $\alpha < \lambda$</u>

In this case $\phi' > 0,\ \phi'' < 0,\ \phi'(0) = \infty,\ \phi'(\infty) \ge 0$. Thus, as shown in Figure 3, there exists a unique stable steady-state point F. The economy departing from any initial point at time 1 converges to the long-run equilibrium point F. Substituting $G_{t+1}=G_t=G^*$ into (40-1), the long-run equilibrium G, G^*, is given by

$$G^* = \left[\frac{\frac{2n+b}{n} + \beta}{p^{-\lambda}[\lambda - \beta(1-\lambda)]} \right]^{\frac{1}{\alpha-\lambda}} \tag{41}$$

When the degree of production externality, α, is small, economies starting from any low levels of G_1 and K_1 can grow and move to the stationary state. All economies, which have different initial levels of G_1 and K_1, converge to the same stationary state.

Thus, if $G_0 < \tilde{G} < G^*$, then G will grow monotonously to point F. However, if $\tilde{G} < G_0 < G^*$, then $G_1 < G_0$ and $G_t > G_0$ for sufficiently large t. Economic growth is associated first with declines, then improvements, in environmental quality. The smaller K_0 and w_0, it is more likely to have this possibility.

Case B: $\alpha > \lambda$

In this case $\phi' > 0,\ \phi'' > 0,\ \phi'(0) = 0,\ \phi'(\infty) \geq 0$. Thus, as shown in Figure 4, the long-run equilibrium point F is unstable. If G_1 is greater than G^*, G grows infinitely. If G_1 is less than G^*, G converges to the zero stationary state, that is, point O. Economies with sufficient capital and environmental quality can take advantage of the increasing returns and experience sustained growth, while economies with worse initial conditions will move towards a low-level (zero) stable equilibrium. This model can exhibit a low-level equilibrium trap.[8]

Dynamic properties of this case depends on the initial levels of G_0 and K_0. If $\tilde{G} < G_0 < G^*$, G converges to point O monotonously. $\tilde{G} > G_0 > G^*$, G grows monotonously. On the other hand, if $G_0 < \tilde{G} < G^*$, $G_1 > G_0$ but then G declines. If

$G_0 > \tilde{G} > G^*$, $G_1 < G_0$ but then G grows. As in case A, economic growth is associated first with declines, then improvements, in environmental quality.

(41) means that G* is increasing with β, p, b and decreasing with n if $\alpha > \lambda$. Thus, the higher β, p, and b or the smaller n, it is more likely to have a low-level trap. A high level of production externality, α, does not necessarily benefit the economy if initial levels of G and K are small. On the other hand, a high level of population normally stimulate accumulation of environmental quality.

Case C: $\alpha = \lambda$

In this case $\phi' = \{p^{-\lambda}[\lambda - (1-\lambda)\beta] + \frac{1-b}{n} - \beta\}\frac{n}{2n+1}$ >0 and $\phi'' = 0$. Thus, as shown in Figure 5-1, the only stable equilibrium is point O if $\phi' < 1$. Economies departing from any initial point converges to the zero steady state. We have a catastrophic case. On the other hand, as shown in Figure 5-2, if $\phi' > 1$, G grows infinitely at the constant rate of ϕ'-1. The greater b, p, β or the smaller n, it is more likely to have the catastrophic case. Long-run dynamic properties of this case is independent of the initial state. When n is large or b, p, β are small, any economies can enjoy positive growth forever. When p=1, we always have $\phi' < 1$; the catastrophic case. If $G_0 < \tilde{G}$ and $\phi' > 1$, economic growth is associated first with declines, then improvements, in environmental quality.

Case D: $\lambda = \beta(1-\lambda)$

Let us next consider the case of $\lambda = \beta(1-\lambda)$. We still assume $\frac{1-b}{n} > \beta$.

Then we have $\phi' > 0$, $\phi'' = 0$, $\phi' = (\frac{1-b}{n} - \beta)\frac{n}{2n+1} < 1$. In this case economies departing from any initial point converges to the zero steady state. We have a catastrophic case. This dynamic property is independent of values of $\alpha - \lambda$ or n, b, p. Even if the degree of production externality, α, is high, no economy can have positive long-run growth when β is relatively high.

If $\lambda < (1-\lambda)\beta$, G_t becomes to be negative, which is not consistent with non-negative constraints. The economy cannot work in the long run in this case even if the degree of production externality is sufficiently large.

Our model exhibits various types of dynamic properties (convergence to the stationary state, a low-level equilibrium trap, catastrophe, steady state growth) for positive possible values of α, the degree of production externality. Both production externalities (α) and consumption disexternalities (β) have important roles to determine dynamic properties. The lower β, p, b and the higher n, it is more likely to have accumulation of environmental quality and capital. In this sense, population growth will normally improve environmental quality.[9]

Two remarks are useful. First, if we do not incorporate the production externality from environmental quality (α =0), the dynamic property is qualitatively the same as in the case of $\alpha < \lambda$. In other words, an introduction of the production externality can produce some interesting results mostly in the case where the degree of production externality is very large.

Second, it may be interesting to consider the case where environmental quality does not affect utility; $U_t = U(c_t^1, c_{t+1}^2)$. In this case, an agent does not provide environmental improving expenditures at the Nash equilibrium; g_i=0. Then, G will decline but K can grow and it is possible that utility increases for a while. However, sooner or later, a decrease in G reduces production and hence utility. We could have a situation where environmental quality decreases but capital accumulation occurs for a while[10].

8. Welfare

8.1 Welfare implications of changes in environmental parameters

From (33)' and (38-1), utility of generation t, U_t, is given by a function of G_{t+1} as well as environmental properties.

$$U_t = G_{t+1}^3[1+(1-\lambda)G_{t+1}^{\alpha-\lambda}p^{-\lambda}]\frac{p^2}{1+p\beta} \tag{42}$$

It is easy from (42) that U_t is increasing with G_{t+1}. Growth of physical capital and environmental quality always raises welfare. In other words, positive growth of G makes all future generations better off, while negative growth of G makes them worse off. U_t is also increasing with p and decreasing with β for given level of G_{t+1}.

Let us then investigate welfare implications of changes in environmental parameters. First of all suppose that the economy is at point F_0 in Figure 6 in case A ($\alpha < \lambda$). An increase in n or decreases in b, p, and β will raise G^*, and hence the long-run equilibrium point moves from F_0 to F_1. G (and hence K) will grow during transition, which enhances welfare as expressed in (42). This is called the growth effect. (42) also means that the direct effect of an increase in β at given G reduces U, while the direct effect of an increase in p raises U. (40-2) also means that G_1 is decreasing with p, b, β and may well be increasing with n. Hence, when n increases or b decreases at time 0, U_0 increases due to an increase in G_1 (the growth effect) and U_t (>0) will increase. When β increases at time 0, U_0 decreases due to a decrease in G_1 (the growth effect) and the direct effect of an increase in β, and hence U_t will also decrease. However, when p increases at time 0, U_0 may increase (if the direct effect dominates the growth effect). But U_t will decrease due to the negative growth effect for t>1. There are no conflicts between present and future generations when b, n or β changes. But there may exist a conflict between present and future generations when p changes. The present generation does not have a strong incentive to raise the productivity of environmental improving expenditures although it will benefit future generations.

Next, let us consider case B ($\alpha > \lambda$). As shown in Figure 7, when n increases or b, p, β decrease, curve ϕ shifts upwards and F_0 moves to F_1. When the economy is initially at point F, G starts to grow. Thus we have qualitatively the same results as in case A. Namely, when n increases or b decreases at time 0, U_0 increases due to an increase in G_1 and U_t (>0) will also increase. When β decreases at time 0, U_0 increases and U_t will also increase. However, when p increases at time 0, U_0 may increase (if the direct effect dominates the growth effect) but U_t will decrease due to the negative growth effect. A change in p is a once-for-all change, while G grows forever. Hence, the effect of changes in G dominates the total welfare effect in the long run. On the other hand, when n decreases or b, p, and β increase, future generations as well as the present generation are worse off. The present generation may be worse off when p decreases, while it will benefit future generations.

Finally, let us consider case C ($\alpha = \lambda$). In this case, an increase in n or decreases in p, b, β raise G. Hence, the welfare impacts of changes in these parameters are qualitatively the same as before. Population growth normally enhances environmental quality and welfare, despite the adverse effect of high consumption.

8.2 The first best solution

In order to investigate the normative aspect of the model, it is useful to derive the first best solution[11]. The feasibility condition is given as

$$pG_{t+1} = (1-b)pG_t + Y_t + K_t - K_{t+1} - n(1+p\beta)(c_t^1 + c_t^2) \tag{43}$$

Hence, the first best problem is to maximize the Lagrange function

$$W = \sum_{t=0}^{\infty} \rho^t \{U(c_t^1, c_{t+1}^2, G_{t+1}) + \mu_t [pG_{t+1} - p(1-b)G_t - Y_t - K_t + K_{t+1} + n(1+p\beta)(c_t^1 + c_t^2)]\} \tag{44}$$

where $\rho^t \mu_t$ is a Lagrange multiplier at time t.

The first order conditions are as follows.

$$U_{1t} + \mu_t n(1+p\beta) = 0 \tag{45-1}$$

$$U_{2t+1} + \mu_{t+1}(1+p\beta)n\rho = 0 \tag{45-2}$$

$$U_{3t+1} + p\mu_t - p(1-b)\rho\mu_{t+1} - \alpha G_{t+1}^{\alpha-1} K_{t+1}^{1-\lambda} n^{\lambda} \mu_{t+1}\rho = 0 \tag{45-3}$$

$$-\mu_{t+1}(1+r_{t+1})\rho + \mu_t = 0 \tag{45-4}$$

where $U_{1t} = \partial U_t / \partial c_t^1$, $U_{2t+1} = \partial U_t / \partial c_{t+1}^2$, and $U_{3t+1} = \partial U_t / \partial G_{t+1}$.

From these conditions we have

$$\frac{U_{3t+1}}{U_{1t}} = \frac{p(r_{t+1} + b)}{(1+p\beta)n(1+r_{t+1})} - \frac{\alpha G_{t+1}^{\alpha+1} K_{t+1}^{1-\lambda} n^{\lambda}}{n(1+p\beta)(1+r_{t+1})} \tag{46-1}$$

$$\frac{U_{3t+1}}{U_{2t}} = \frac{p(r_{t+1}+b)}{(1+p\beta)n\rho(1+r_{t+1})} - \frac{\alpha G_{t+1}^{\alpha+1} K_{t+1}^{1-\lambda} n^{\lambda}}{(1+p\beta)n} \tag{46-2}$$

Hence,

$$\frac{U_{3t+1}}{U_{1t}} + \frac{U_{3t+1}}{U_{2t+1}} < \frac{p(r_{t+1}+b)}{(1+p\beta)nb(1+r_{t+1})} + \frac{p(r_{t+1}+b)}{(1+p\beta)n\rho(1+r_{t+1})} < \frac{p}{1+p\beta} + p(1+r_t) \tag{47}$$

Note that b<1, n>1, $\rho < 1$. Since in the competitive economy we always have

$$\frac{U_{3t+1}}{U_{1t}} = \frac{p}{1+p\beta},$$

$$\frac{U_{3t+1}}{U_{2t}} = p(1+r_t),$$

inequality (47) means that $(c_t^1 + c_t^2)/G_{t+1}$ in the competitive economy is greater than in the first best economy. In this sense, environmental quality is too little and private consumption is too much in the competitive economy. In the standard overlapping generations growth model it is well known that capital may be too much in the competitive equilibrium. Capital may be too much in this model as well when the economy is on the inefficient path ($(1+r)\rho < 1$). However, environmental quality is always too little in this model.

8.3 Consumption taxes

We now introduce consumption taxes and lump sum taxes. The private budget constraints (29) and (30) are rewritten as

$$w_t = (1+\tau_{1t})c_{it}^1 + s_{it} + pg_{it} + T_{it}^1 \tag{29'}$$

$$(1+r_{t+1})s_{it} = (1+\tau_{2t+1})c_{it+1}^2 + T_{it+1}^2 \tag{30'}$$

where τ_{it} is consumption tax rate on the i-th period consumption at time t (i=1,2), T^1_t is the lump sum tax levied on the young at time t, and T^2_t is the lump sum tax levied on the old at time t.

The present value of lifetime lump-sum tax payment on the agent of generation t (T_t) is given by

$$T_t = T_t^1 + \frac{1}{1+r_{t+1}} T_{t+1}^2 \tag{48}$$

The government budget constraint in period t is given as

$$\sum_{i=1}^{n} (\tau_{1t} c_{it}^1 + \tau_{2t} c_{it}^2 + T_{it}^1 + T_{it}^2) = 0 \tag{49}$$

For simplicity we do not incorporate public spending; tax revenues from consumption taxes will be returned to the private sector as a lump sum transfer.

The consumer price vector q_t is now rewritten as

$$q_t = (q_t^1, q_t^2, q^3) = (1 + \tau_{1t} + p\beta, \frac{1+\tau_{2t+1}}{1+r_{t+1}}, p)$$

The dynamic model will be summarized by the following three equations in addition to (32) (33) and (34).

$$E(U_t, q_t) = w_t + p\frac{n-1}{n} E_3(U_t, q_t) + \frac{1}{n}(1-b) p E_3(U_{t-1}, q_{t-1}) - p\beta E_2(U_{t-1}, q_{t-1}) + T_t \tag{50}$$

$$(1+p\beta)E_1(U_t, q_t) + (1+p\beta)E_2(U_{t-1}, q_{t-1}) + p\frac{1}{n}E_3(U_t, q_t) - p\frac{1-b}{n}E_3(U_{t-1}, q_{t-1}) = \frac{Y_t + K_{t+1} - K_t}{n} \tag{51}$$

$$q_{2t} E_2(U_t, q_t) + T_t - T_t^1 = \frac{K_{t+1}}{n} \tag{52}$$

(50) comes from the private budget constraint. (51) means the production feasibility condition (43); output is divided into consumption, saving, and provision of the environmental expenditure. (52) is the capital accumulation equation.

Considering (46-1)(46-2), the dynamically optimal consumption tax rates are given as

$$\frac{p}{1+\tau_{1t}^{*}+p\beta}=\frac{p(r_{t+1}+b)}{(1+p\beta)n(1+r_{t+1})}-\frac{\alpha G_{t+1}^{\alpha+1}K_{t+1}^{1-\lambda}n^{\lambda}}{n(1+p\beta)(1+r_{t+1})} \tag{53-1}$$

$$\frac{p(1+r_t)}{1+\tau_{2t}^{*}}=\frac{p(r_{t+1}+b)}{(1+p\beta)np(1+r_{t+1})}-\frac{\alpha G_{t+1}^{\alpha+1}K_{t+1}^{1-\lambda}n^{\lambda}}{(1+p\beta)n} \tag{53-2}$$

In this competitive economy the optimal lump-sum redistribution policy is needed to attain the modified golden rule (45-4), the dynamic efficiency condition. Namely, lump sum taxes and transfers are required to attain the dynamically efficient level of capital accumulation. On the other hand, consumption taxes are required to attain the first best level of environmental quality. (53-1) means τ_1 >0. The production externality results in a higher level of τ_1^*, compared with $\alpha=0$. Consumption taxes on the first-period consumption is needed to internalize both the intragenerational and intergenerational externalities. From (53-1)(53-2), we have

$$\tau_{2t}^{*}=\tau_{1t}^{*}+p\beta+\frac{(1+p\beta)n(1+r_{t+1})}{r_{t+1}+b}[(1+r_t)\rho-1] \tag{54}$$

Higher consumption taxes on the second-period consumption is required to internalize another intergenerational externality. An increase in c^2_t will reduce G_{t+1}, which is not considered by the private maximizing behavior.

When consumption taxes cannot discriminate consumption by age, then an additional interest income tax would be useful to tax the second period consumption more heavily than the first period consumption.

9. Conclusion

This paper has developed a general equilibrium model of nonoverlapping and then overlapping generations that provide a public good of improving environmental quality and also their consumption produces pollution. We have incorporated both intergenerational and intragenerational conflicts into the overlapping generations growth model with both consumption and production externalities.

It has been shown that an introduction of consumption taxes is always desirable. The optimal consumption tax rate is increasing with the degree of environmental degradation and the number of agents. It is decreasing with the autonomous evolution of environmental quality, the effective productivity of environmental expenditures, and the social rate of time preference. Consumption taxes may internalize both intragenerational and intergenerational externalities. If a myopic government is only concerned with the intragenerational free-rider problem, the optimal tax rate is too low. A further increase in consumption taxes will benefit the future generation but it will hurt the present generation. There is a potential conflict between the present and future generations with respect to the optimal level of consumption taxes.

A country may not have a strong incentive to impose country-specific taxes on polluting consumption or to subsidize its provision of environmental contributions as such policies may reduce (not raise) welfare of the country. We have explored some potential fiscal conflicts (the Prisoner's Dilemma situation) among countries by incorporating international environmental externalities.

When we allow for capital accumulation, the optimal lump-sum redistribution policy is also needed to attain the modified golden rule, the dynamic efficiency condition of capital accumulation. When consumption taxes cannot discriminate consumption by age, then an additional interest income tax would be useful to tax the second period consumption more heavily than the first period consumption.

Our model exhibits various types of dynamic properties (convergence to the stationary state, a low-level equilibrium trap, catastrophe, steady state growth) for positive values of α, the degree of production externality. Both production externalities and consumption disexternalities have important roles for dynamic properties. We have shown that the economy may be associated first with declines, then improvements, in environmental quality in the interior equilibrium. We have also clarified how the environmental parameters would affect dynamic properties. The lower the degree of environmental degradation by consumption (β), the autonomous evolution of environmental quality (b), the productivity of environmental improving expenditures (p), and the larger the number of agents (n), it is more likely to have accumulation of environmental quality and capital. There are no conflicts between present and future generations when b, n or β changes. Population growth will normally stimulate accumulation of environmental quality and enhance welfare. But there may exist a conflict between present and future generations when p changes. The present generation does not have a strong incentive to reduce p although it will benefit future generations.

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1 . One interpretation of G_t is the quality of soil or groundwater. Another interpretation is the inverse of the stock of greenhouse gases in the atmosphere. Or we could consider national parks, which have amenity value and which also require maintenance. If agents are countries, G may be regarded as an international public good.

2 . (6-2) is defined in section 6.

3. In order to present the results in the simplest way and in their strongest form, we assume that non-negativity constraints on providing public goods are non-binding in

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equilibrium. As remarked by Bergstrom et al. (1986) and Boadway et al. (1989), this assumption is relatively weak in some of the situations we analyze. When all agents are identical, we always have the interior solution.

4 . This comparative static result is different from John et al. (1995). They showed that an increase in β has poorer environment and welfare because they abstracted from the free-rider problems among countries. Our analysis suggests that such a paradoxical case could occur even if n=1, i.e., we abstract from the intragenerational externalities.

5 . As for the neutrality theorem, see Shibata (1971) and Warr (1983). Ihori (1992, 1994, 1996) , Batina and Dion (1994), Buchholz and Konrad (1995), and Cornes and Sandler (1994) discussed several interesting cases where the neutrality result does not hold.

6. Subsidies on environmental expenditures could attain the first best solution as consumption taxes do.

7. The inclusion of current period's environmental quality in the current production technology is motivated by the recent literature on external increasing returns in endogenous growth models. See Romer (1986) among others.

8 . If we have in place of (4)

$$G_t = (1-b)G_{t-1} - \sum_{i=1}^{n} \beta c_{it} + \sum_{i=1}^{n} g_{it} + B, \quad B>0$$

the low level equilibrium has positive G and K.

9 . John and Pecchenino (1994) presented a model which exhibits various types of dynamic properties. But, they did not clarify how these dynamic properties are related with environmental parameters.

10 . John and Pecchenino (1994) also obtained a negative correlation between environmental quality and growth under the zero maintenance constraint (g_i=0). Our dynamic analysis has shown that a negative correlation between environmental quality and growth can be obtained at interior equilibrium. The economy may be associated with first declines, then improvements, in G if K is initially small.

11 . John and Pecchenino (1994) investigated the golden rule allocation in the steady state only.

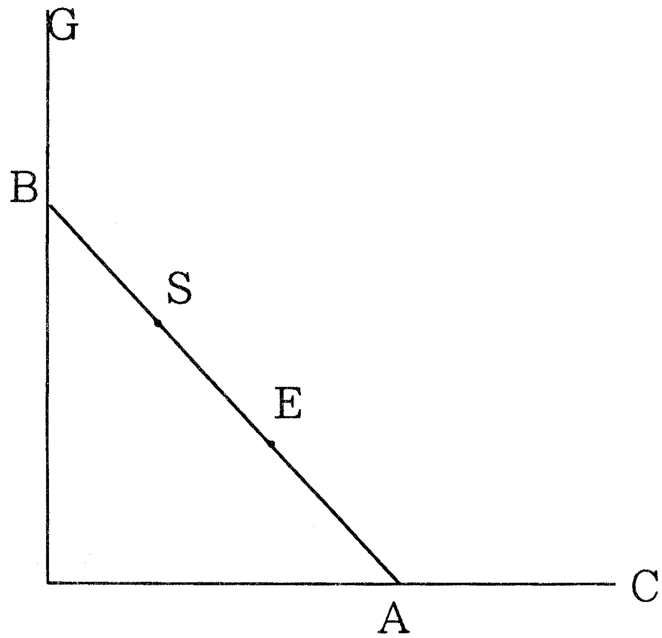

Figure 1

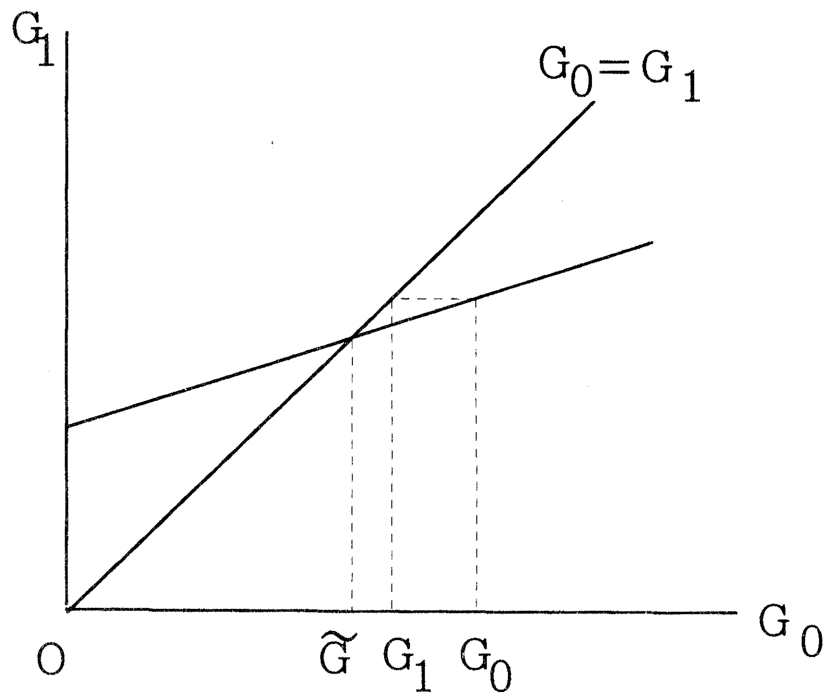

Figure 2

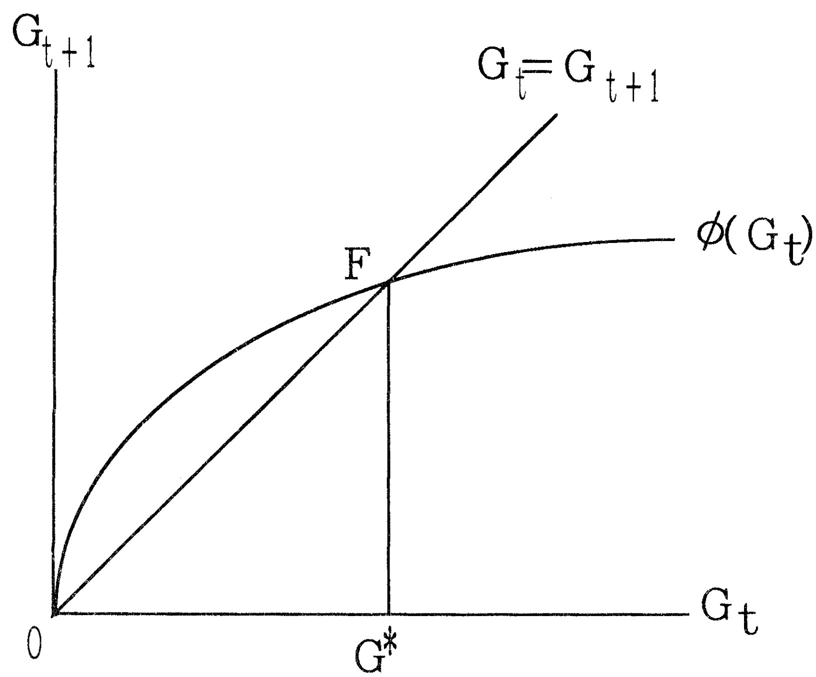

Figure 3

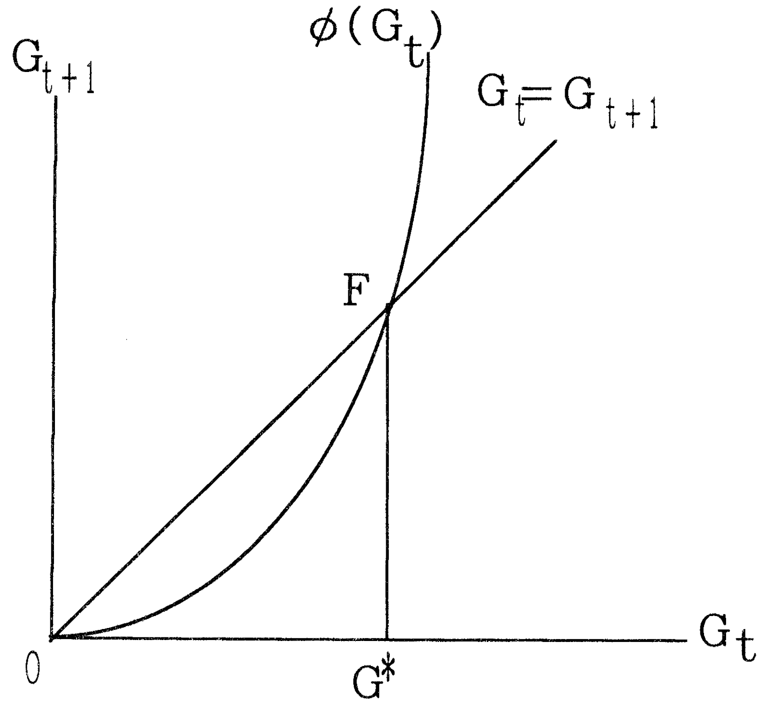

Figure 4

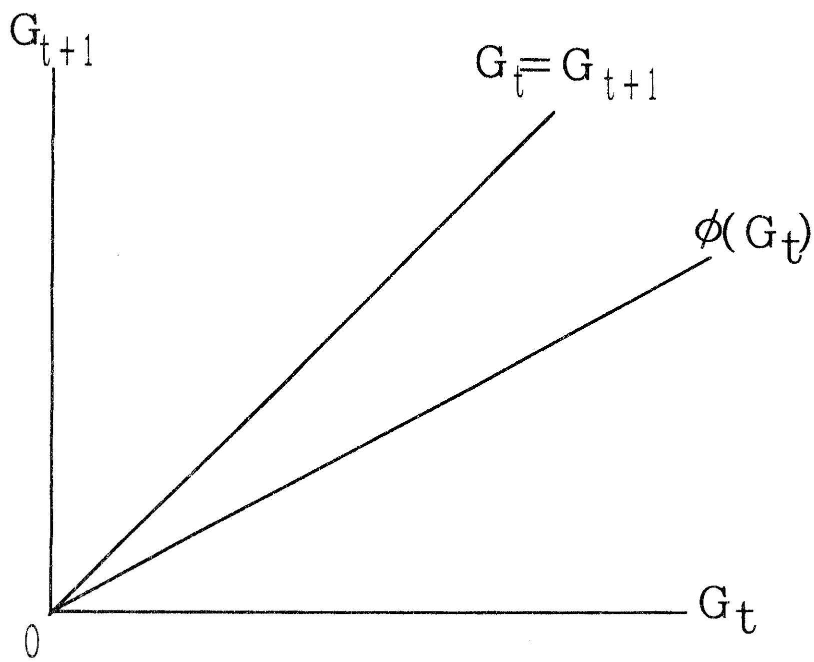

Figure 5-1

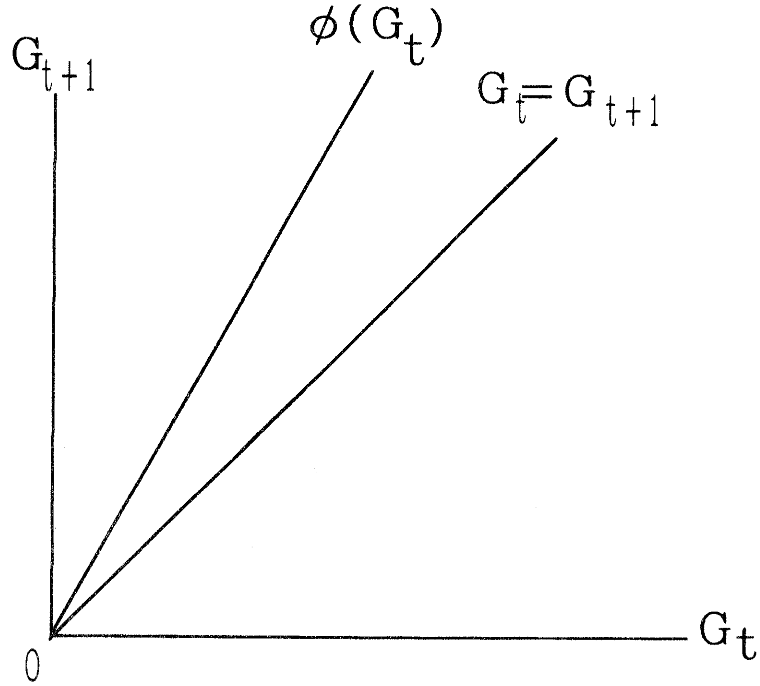

Figure 5-2

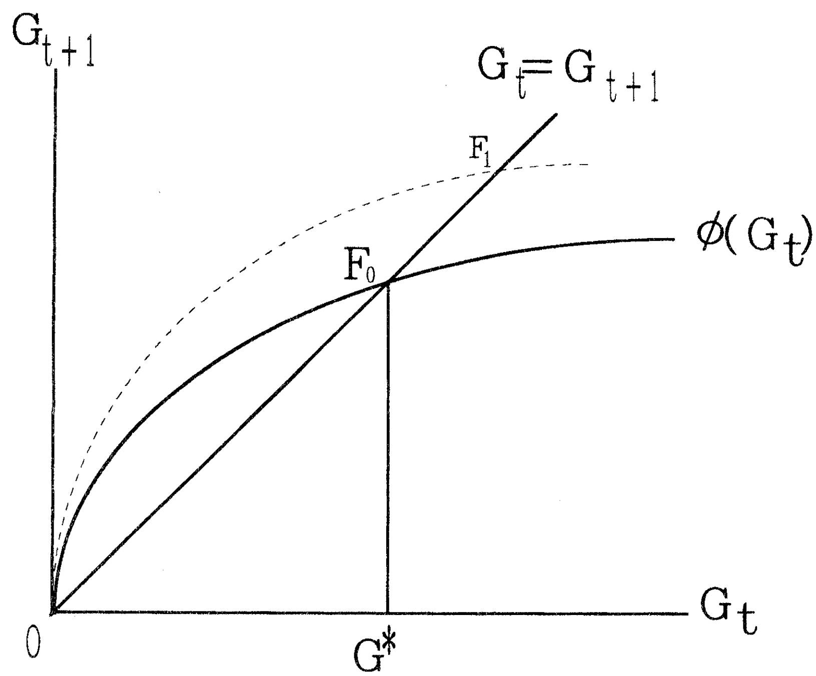

Figure 6

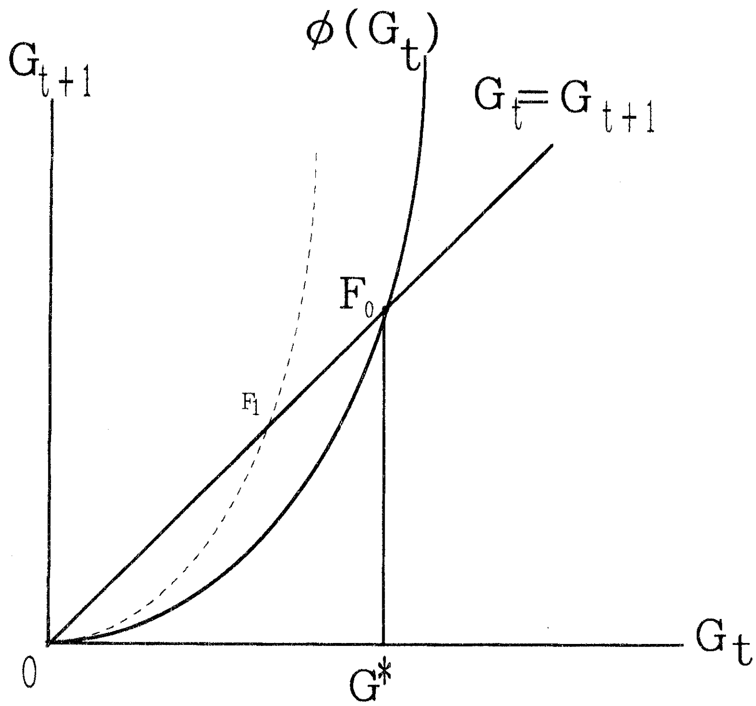

Figure 7