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Abstract: Many theoretical and empirical studies have shown that in a wide range of industries, prices are rigid because frequent price changes are costly. In such industries, firms set price first and subsequently fine-tune their service quality. In other settings, service quality accompanying the product is determined before price. In this paper, we examine the effect of price rigidity on price and service quality in a duopolistic market. We show under weak conditions that when firms choose price first, both price and service quality is higher than when they choose service quality first.

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1. Introduction

In many industrial markets, prices remain rigid for long periods of time, sometimes for periods exceeding one year (Carlton 1986). In such instances, once a price is set, the firms are likely to compete on such non-price factors as service quality (e.g., prompt delivery, provision of information, flexible payment terms). In financial service industries such as banking, there is also evidence of rigidity of deposit interest rates (Hannan and Berger 1991). Banks, consequently, have attempted to differentiate themselves by providing more services (e.g., more convenient hours, bank-by-phone, friendly tellers, shorter response time on loan applications).

Interestingly in some industries, we are seeing a movement towards greater rigidity in price. U.S. auto dealers, for example, have traditionally provided prospective buyers such services as information about cars and test drives, after which they negotiated on price. Thus service quality was determined before price. Recently, however, some automobile dealerships have begun to sell cars at posted prices - no price bargaining is entertained. In this mode of selling, price is determined before level of service is set.[1]

Price rigidity and accompanying service-quality competition are not confined to isolated industries, but found throughout the U.S. economy. In fact, price stickiness is the core of macroeconomics. A recent study by Blinder (1991) suggests that less than 15 percent of the U.S. GNP is repriced more frequently than quarterly, and fully 55 percent is repriced no more often than once per year. Moreover, Blinder reports that there are substantial lags (mean lag of about 3.5 months) in price adjustment to changes in demand and cost.

[1] Post-purchase car servicing is viewed as independent of the initial purchase.

Among the various explanations for price rigidity, the following are often considered to be the most important (Blinder 1991).[2] First, firms whose buyers are less sensitive to price than to delivery lags and auxiliary services may find concentrating on the latter a more effective way to compete than cutting price (Carlton 1989). Such firms are likely to adjust their services more often than their prices.

Second, customers may demand "fair" pricing, and cost-based pricing is often considered to be fair (Okun 1981). A third explanation for price rigidity is also related to fairness. Some firms have an implicit understanding with customers that proscribes price increases when market conditions are tight. To renege on such understandings (often called an "invisible handshake") can be very costly to the extent that it antagonizes and leads to a substantial loss of customers (Okun 1981). Hence, firms tend to hold price constant until customers agree that the firms' costs have changed, and instead compete on the basis of services such as prompt delivery.

The fourth explanation is that because contract prices are written in nominal terms, to change prices involves negotiation costs. Similarly, to change prices on menus and in catalogs also involves fixed costs, especially in the mail order business. When the fixed costs of price adjustment are non-negligible, firms adjust their prices only occasionally.[3]

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[2] Corporate executives surveyed by Blinder (1991) consider the first four of the following explanations to be most important in their decisions about price adjustments. Blinder (1991) also considers other theories of price stickiness, which do not fare as well as these.

[3] Although for simplicity we assume away uncertainty in this paper, sticky prices are more likely when there is uncertainty. To organize an auction market that clears by price is costly (Carlton 1992). Thus, a firm must set its price in many markets, usually before knowing the demand condition perfectly (pre-determined price). See Nishimura (1992: ch.2) for an explanation of why a firm does not wait until all uncertainty is cleared to determine price.

Other explanations for price rigidity may be found in Rotemberg and Saloner (1987) for the monopoly case and Carlton (1991) for the competitive case. See also Nishimura (1986, 1992) for an explanation of how competition among firms affects degree of price rigidity.

In this paper, we do not attempt to explain why prices are rigid. Rather, given a market structure in which price is more lasting than service quality, we attempt to understand the effect of such price rigidity on price and the level of service quality as compared to traditional models in which quality is determined before price (Gabszewicz and Thisse 1979; Shaked and Sutton 1982; Bonanno 1986; Gal-Or 1983; Economides 1989, 1992; and Neven and Thisse 1990). Moreover, for industries in transition from flexible to rigid prices, it is important to know how price rigidity might change the nature of price and service quality competition. In the case of automobile distribution in the United States, for example, will the dominance of price-posting dealers result in a higher or lower quality of service and price? [4] On a broader level, we explore whether institutional restraints (e.g., regulation) that make price changes costly will increase or reduce the level of service quality and price.

In this paper, we contrast two different duopoly market structures: one in which service quality is set before price, the "quality-first model," and another in which price is set before service quality, the "price-first model." When one considers the quality of a manufactured product it is more natural to think that quality is determined before price, as price changes are less costly to make than product changes. Many firms change price in response to demand after a product is introduced into the market. But service quality can often be determined after price. Therefore, a model where price is determined first is more likely to apply to industries in which the provision of service is important.

We show under fairly general conditions that price and service quality are higher in the

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[4] For example, SATURN (a new company founded by General Motors) cars are sold at the posted price; dealers entertain no dickering over price (*MIT Management*, Spring 1992, pp 2-19).

price-first model. The intuition for this result is as follows. In a quality-first model, if Firm A increases its service quality in stage one, the best response of the competitor, Firm B, in stage two is to decrease its price. To the extent that this decreases demand and profits for Firm A, it diminishes its incentive to improve service quality in stage one. But in a price-first model, since price is already fixed, there is no need for a strategic reduction in service quality in the second stage. Thus, service quality is lower in the quality-first model. Moreover, in a price-first model, if Firm A decreases its price, the best response of Firm B in stage two is to improve its service quality. Firm B's service improvements decrease Firm A's demand and profits, and hence the latter's incentive to reduce price. Consequently, price is higher in the price-first model. To close the loop on the argument, if both firms' prices in the first stage are higher because of this dis-incentive to cut price, they will have to provide higher service quality in the second stage in order to compensate for the high prices in a symmetric equilibrium. Thus, both price and service quality are higher in the price-first model.

In section 2, we present a general model, and give conditions under which the price-first model gives rise to a higher level of service quality and price than the quality-first model. In section 3, we illustrate our main result using a linear demand function. In section 4, we summarize the implications of our model.

2. Models of Quality-first and Price-first Competition

We assume a duopoly market in which firms sell one product and compete on price and service quality. The firms' profit functions are defined as follows.

$$\Pi_1 = V(p_1, p_2, q_1, q_2), \text{ and} \tag{1}$$

$$\Pi_2 = W(p_1, p_2, q_1, q_2), \tag{2}$$

where p_i is the price set and q_i is the level of service quality provided, by firm i.[5] Here, $(p_1, p_2, q_1, q_2) \in D$, where D is a convex subset of R_+^4.

We assume that the following conditions are met.

A1. Π_1 is strongly concave in p_1 and q_1, which implies $V_{11} < 0$, $V_{33} < 0$, and $V_{11}V_{33} > V_{13}^2$, where V_{ij} denotes the partial differential of V with respect to the *i*th and then the *j*th variable.

A2. Π_2 is strongly concave in p_2 and q_2, which implies $W_{22} < 0$, $W_{44} < 0$, and $W_{22}W_{44} > W_{24}^2$.

A3. The products are substitutes, which implies that $V_2 > 0$, $V_4 < 0$, $W_1 > 0$, and $W_3 < 0$.

A4. In all games considered in this section, there is a unique interior solution for firms' strategy and a unique equilibrium of the game.

To compare price and level of service quality in both the quality-first and price-first models, we use an indirect approach that introduces a model in which quality and price are determined simultaneously. We show that price and service quality in the price-first model are higher than or equal to those of the simultaneous-determination model, which are, in turn greater than or equal to those of the quality-first model.

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[5] Because we interpret q_i as service quality, it might seem that we implicitly assume the observability of service quality before a consumer makes a purchase. But we can reinterpret q_i as service-related outlays of the firm, in which case the observability of service quality by all customers is not assumed. Rather, we assume simply that the service-related outlays influence demand and, subsequently, profit. Interpretation of q_i as either service quality or service-related outlays is reasonable. A retailer that advertises "delivery in 24 hours" is posting its level of service quality. A service company that advertises the intense training program its sales representatives must go through is posting the level of its service-related outlays.

Simultaneous-determination model. An equilibrium of the simultaneous-determination model, $(p_1^S, p_2^S, q_1^S, q_2^S)$, will satisfy the following conditions under assumptions A1 through A4.

$$V_1(p_1^S, p_2^S, q_1^S, q_2^S) = 0 \tag{3}$$

$$W_2(p_1^S, p_2^S, q_1^S, q_2^S) = 0 \tag{4}$$

$$V_3(p_1^S, p_2^S, q_1^S, q_2^S) = 0 \tag{5}$$

$$W_4(p_1^S, p_2^S, q_1^S, q_2^S) = 0. \tag{6}$$

Quality-first model. To compute a subgame perfect equilibrium, the game is solved by backward induction. In the second stage, given quality, firms will set their equilibrium prices, $(p_1^Q, p_2^Q) = (x(q_1, q_2), y(q_1, q_2))$, which are determined by solving:

$$V_1(p_1^Q, p_2^Q, q_1, q_2) = 0 \tag{7}$$

$$W_2(p_1^Q, p_2^Q, q_1, q_2) = 0. \tag{8}$$

Substituting $(p_1^Q, p_2^Q) = (x(q_1, q_2), y(q_1, q_2))$ into (1) and (2), we solve for the first-stage equilibrium service quality level, (q_1^Q, q_2^Q), which is given by:

$$q_1^Q = \underset{q_1}{argmax}\ V(x(q_1, q_2^Q), y(q_1, q_2^Q), q_1, q_2^Q) \tag{9}$$

$$q_2^Q = \underset{q_2}{argmax}\ W(x(q_1^Q, q_2), y(q_1^Q, q_2), q_1^Q, q_2)\ . \tag{10}$$

Price-first model. In the second stage of the game, given price, firms will set their equilibrium service quality levels, $(q_1^P, q_2^P) = (f(p_1, p_2), g(p_1, p_2))$, which are determined by solving:

$$V_3(p_1, p_2, q_1^P, q_2^P) = 0 \tag{11}$$

$$W_4(p_1, p_2, q_1^P, q_2^P) = 0. \tag{12}$$

Substituting $(q_1^P, q_2^P) = (f(p_1, p_2), g(p_1, p_2))$ into (1) and (2), we solve for the first stage equilibrium price, (p_1^P, p_2^P), which is given by:

$$p_1^P = \underset{p_1}{argmax}\ V(p_1, p_2^P, f(p_1, p_2^P), g(p_1, p_2^P)) \tag{13}$$

$$p_2^P = \underset{p_2}{argmax}\ W(p_1^P, p_2, f(p_1^P, p_2), g(p_1^P, p_2)). \tag{14}$$

Comparison of equilibrium price and service quality. We now state the conditions under which price and service quality are greater in the price-first model than in the quality-first model.

A5. Symmetry:

$$W(p_1,p_2,q_1,q_2) = V(p_2,p_1,q_2,q_1).$$

A6. Strong concavity, dominating strategic effects:

$$|V_{12}| < |V_{11}|,\ |V_{34}| < |V_{33}| \text{ and}$$

$$(V_{11} + V_{12})(V_{33} + V_{34}) - (V_{13} + V_{14})(V_{31} + V_{32}) > 0.$$

Profit function V is sufficiently concave with respect to its own strategic variables (i.e., $|V_{11}|$, $|V_{33}|$, and $V_{11}V_{33} - V_{13}^2$ are sufficiently large) in such a way as to dominate strategic dependence (i.e., V_{12}, V_{14}, V_{32}, V_{34}).

In addition, two conditions -- we call them "Complementarity between Symmetric Price and Symmetric Service Quality" and "Cross-Strategic Substitutability" -- are central to our results. These are defined as follows.

Complementarity between Symmetric Price and Symmetric Service Quality. Letting $p_1 = p_2 = p$ and $q_1 = q_2 = q$, there exists Complementarity between Symmetric Price and Symmetric Service Quality if:

$$\frac{dx(q,\, q)}{dq} \geq 0, \quad \frac{dy(q,\, q)}{dq} \geq 0, \quad \frac{df(p,\, p)}{dp} \geq 0, \quad \textit{and} \quad \frac{dg(p,\, p)}{dp} \geq 0.$$

This condition states that the best response functions of both firms are such that a higher symmetric price implies a higher or at least the same symmetric service quality, and vice versa. In a quality-first model, this condition ensures that, if both firms increase their service quality by the same amount in the first stage, both firms' optimal price in the second stage will increase or at least stay unchanged. Similarly, in a price-first model, if both firms increase price by the same amount in the first stage, both firms' optimal service quality in the second stage will increase or remain unchanged.

Cross-Strategic Substitutability.[6] Cross-Strategic Substitutability exists if:

$$\frac{\partial x(q_1,\, q_2)}{\partial q_2} \leq 0, \quad \frac{\partial y(q_1,\, q_2)}{\partial q_1} \leq 0, \quad \frac{\partial f(p_1,\, p_2)}{\partial p_2} \leq 0, \quad \textit{and} \quad \frac{\partial g(p_1,\, p_2)}{\partial p_1} \leq 0.$$

This definition states that the best response functions of both firms are such that the best response

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[6] Bulow, Geanakoplos, and Klemperer (1985) were the first to propose this notion of defining the product characteristics of an oligopolistic market by strategic substitutability and complementarity.

to a service quality increase of the competitor is to lower price or at least keep it unchanged.[7] Analogously, the best response to a competitor's price decrease is to improve service quality or at least keep it unchanged. In a quality-first model, Cross-Strategic Substitutability ensures that, given its competitor's level of service quality, a firm that improves its service quality induces the competitor to decrease its price or at least keep it constant in order to meet competition. In a price-first model, this condition ensures that, given the competitor's price, a firm that reduces its price induces the competitor to improve, or at least hold constant, its level of service quality.

We now state our main proposition.

Proposition 1. Suppose assumptions A1-A6 are satisfied; Complementarity between Symmetric Price and Symmetric Service Quality exists; and Cross-Strategic Substitutability exists at the symmetric price and symmetric quality. Then in a symmetric equilibrium, both price and service quality in the price-first model are greater than or equal to price and service quality in the quality-first model.

Sketch of Proof (A formal proof is given in Appendix A):

For purposes of this sketch of the proof, we assume strict Complementarity between Symmetric Price and Symmetric Service Quality and strict Cross-Strategic Substitutability. We first show that price and service quality are higher in the simultaneous-determination model than in the quality-first model. We then show that price and service quality are higher in the price-

[7] Of course, we do not preclude the possibility of a service quality improvement as well a price decrease as a best response.

first model than in the simultaneous-determination model.

Let us first compare the quality-first model with the simultaneous-determination model. In the quality-first model, if Firm A improves its service quality in stage one, the best response of its competitor, Firm B, is to decrease price in stage two, for a given level of Firm B's service quality (by Cross-Strategic Substitutability). Since Firm B's price reduction decreases Firm A's demand and profits, the latter's incentive to improve service quality in the first stage is reduced. By contrast, in the simultaneous-determination model, there is no need for a strategic reduction in service quality. Thus, service quality is lower in the quality-first model than in the simultaneous-determination model.

Moreover, since Firm A's service quality is lower in the first stage of the game because of the dis-incentive to improve, Firm A will have to compensate for its lower level of service quality with a lower price in stage two. Since the model is symmetric, both firms' service quality is lower by the same amount in the quality-first model. By Complementarity between Symmetric Price and Symmetric Service Quality, both firms will charge a lower price in the second stage to compensate for the lower service quality in stage one. Thus, both equilibrium price and service quality are lower in the quality-first model than in the simultaneous-determination model.

Next, we compare the price-first model with the simultaneous-determination model. In the price-first model, since price is already fixed, there is no need for a strategic reduction in level of service quality in the second stage. Moreover, in a price-first model, the best stage two response of Firm B to Firm A's price reduction is to improve its level of service quality (by Cross-Strategic Substitutability). Since an increase in Firm B's level of service quality will decrease Firm A's demand and profits, the latter's incentive to decrease its price will be reduced. As there is no such strategic dis-incentive in the simultaneous-determination model, price is higher in the price-first model than in the simultaneous-determination model.

Finally, since Firm A's price is higher in the first stage because of the dis-incentive to cut price, it will have to compensate for the higher price with a higher level of service quality in stage two. Since the model is symmetric, both firms' price is higher by the same amount in the price-first model. By Complementarity between Symmetric Price and Symmetric Service Quality, if both firms' price is increased by the same amount, both also increase level of service quality by the same amount. Thus, equilibrium price and level of service quality are higher in the price-first model than in the simultaneous-determination model. This ends the sketch of the proof.

The economic significance of Proposition 1 is that price and level of service quality will be higher in industries in which prices are rigid.

In the proof of Proposition 1, we made use of Complementarity between Symmetric Price and Symmetric Service Quality, and Cross-Strategic Substitutability. Although they are intuitive conditions, they are not easily verifiable in certain cases. However, we provide sufficient conditions for the two properties, which are based on the curvature of profit functions and thus more easily verifiable. In the remainder of this section we will present two lemmas giving the sufficient conditions.

The following assumptions are utilized in the subsequent lemmas. The consequence of each assumption is explicated immediately after the assumption. As will be shown in the proof of Lemma 1 and Lemma 2, assumptions A7 and A8 are directly related to Complementarity between Symmetric Price and Symmetric Service Quality, while A9 and A10 are related to Cross-Strategic Substitutability.

A7. $V_{31} + V_{32} \geq 0$.

This condition ensures that in the price-first model, if both firms increase their prices by the same amount in the first stage, their optimal level of service quality in the second stage will increase or at least remain unchanged.

A8. $V_{13} + V_{14} \geq 0$.

This condition ensures that in the quality-first model, if both firms improve their level of service quality by the same amount in the first stage, their optimal price in the second stage will increase or at least remain unchanged.

A9. $V_{12}V_{13} \leq V_{14}V_{11}$.

This condition ensures that in the quality-first model, given its competitor's level of service quality, a firm that improves its level of service quality induces its competitor to decrease or at least hold constant its price in order to meet competition.

A10. $V_{31}V_{34} \leq V_{32}V_{33}$.

This condition ensures that in the price-first model, given its competitor's price, a firm that decreases its price induces its competitor to improve or at least hold constant its level of service quality in order to meet competition.

The following lemmas give sufficient conditions for Complementarity between Symmetric Price and Symmetric Service Quality and Cross-Strategic Substitutability.

Lemma 1. If A1-A6, A7, and A8 hold, then there exists Complementarity between Symmetric

Price and Symmetric Service Quality.

Proof: See Appendix B.

Lemma 2. If A1-A6, A9, and A10 hold, then there exists Cross-Strategic Substitutability at symmetric price and symmetric service quality.

Proof: See Appendix B.

3. An Example of Price and Service Quality Competition in a Linear City

In this section, we illustrate our results in the context of a linear demand function that is commonly used in spatial models of price, quality and variety competition (Gabszewicz and Thisse 1979, Shaked and Sutton 1982, Wernerfelt 1986, Economides 1989, 1992, and Neven and Thisse 1990).

In fact, by using the same example as the one used by Economides (1989), we are able to show that his result that quality is lower in a quality-first model than in a simultaneous determination model while price is the same in both models is a special case of our result. By slightly perturbing his example, we show that price is also lower in a quality-first model than in a simultaneous-determination model. Another point of difference between our paper and the literature on spatial models is that we introduce notions of Complementarity between Symmetric Price and Symmetric Service Quality, and Cross-Strategic Substitutability, which give a stronger intuition behind our results. Furthermore, our economic intuition based on these two concepts gives results that are not dependent on the vagaries of a specific spatial model chosen.

We assume two firms located at opposite ends of a linear city stretching from 0 to 1. (Economides (1989) shows that this is equilibrium in the locational choice game.) Consumers are assumed to be distributed uniformly along this linear city (they are otherwise homogeneous) and to have linear unit transportation cost, t. Consumers have the following utility function for product i:

$$U_i = M + kq_i - p_i - d_i t, \tag{15}$$

where M is the value of the product at zero quality; k is the consumer's valuation of unit quality; and d_i is the distance from the consumer to firm i. The consumer always chooses a product with a higher U_i. To compute demand, we find a consumer who is indifferent between buying from firm 1 and firm 2. Letting firm 1 and firm 2 be located at 0 and 1, respectively, the boundary consumer is the one located at θ, where

$$M + kq_1 - p_1 - \theta t = M + kq_2 - p_2 - (1-\theta)t. \tag{16}$$

Demand of firm 1 is thus θ, and for firm 2, 1-θ. The profit function for firm i is:

$$\Pi_i = p_i \frac{p_j - p_i + k(q_i - q_j) + t}{2t} - cq_i^2, \qquad i \neq j \tag{17}$$

where cq_i^2 is the fixed cost of producing quality q_i.[8]

Quality-first model. Solving the second stage game first, we have:

$$p_i = t + k(q_i - q_j)/3. \qquad (18)$$

Substituting (18) into (17) gives the following first-stage profit function:

$$\Pi_i = \frac{1}{2t}\left[t + \frac{1}{3}k(q_i - q_j)\right]^2 - cq_i^2. \qquad (19)$$

When $k^2 < 18ct$, Π_i is concave in q_i; when $k^2 > 18ct$, Π_i is convex in q_i. We consider these two cases.

[Case 1] When $k^2 < 18ct$, assumption A4 is satisfied and the first-stage profit function is concave in the first-stage decision variable. In economic terms, this condition will be met when the product is sufficiently differentiated horizontally, since t being large indicates that the differences in the location of firms *does* matter. Since (19) is concave, we solve the first order conditions to get:

$$q_i^Q = k/(6c). \qquad (20)$$

Substituting (20) into (18) gives:

[8] Since we showed, in section 2, the general condition under which Proposition 1 will hold, the results of this particular example depend neither on the linearity of the demand function nor on the modeling of quality cost as being fixed instead of variable.

$$p_i^Q = t. \tag{21}$$

[Case 2] When $k^2 > 18ct$, assumption A4 is not satisfied and the first-stage profit function is convex in the first-stage decision variable. In economic terms, since t represents consumers' transportation cost, this condition will be met when consumers perceive differences in location of firms to be of little importance. Since (19) is convex and quadratic, there can be a solution to the problem only if we impose an arbitrary upper limit of $\bar{q}$ on the level of service quality that is allowed.[9] (Claim 1. See Appendix C for a proof.)

Price-first model/Simultaneous-determination model.

In this particular example, equilibrium of the price-first model and the simultaneous-determination model are the same.

(a) To solve a price-first model, the first-order conditions of the second-stage problem is considered first. We have:

$$q_i = p_i k/(4ct). \tag{22}$$

Substituting (22) into (17) gives the following first-stage profit function.

[9] In both the concave and the convex case, we have an equilibrium where price and service quality are symmetric for both players (See Appendix C for convex case). Recall that we exogenously decided on maximum horizontal differentiation by setting the location of the two firms to be at opposite ends of the linear city. Thus we have maximum horizontal differentiation and minimum vertical differentiation. This result is consistent with Neven and Thisse (1990) who show that when both vertical and horizontal location decisions are endogeneous, there is always maximum differentiation along one dimension and minimum differentiation along the other.

$$\Pi_i = p_i \frac{\left(1 - \frac{k^2}{4ct}\right)(p_j - p_i) + t}{2t} - c\left(\frac{k}{4ct}p_i\right)^2 . \tag{23}$$

The above profit function is concave if $k^2 < 8ct$; it is otherwise convex. We consider these two cases.

[Case 1] When $k^2 < 8ct$, we solve the first-order conditions to get:

$$p_i^P = t. \tag{24}$$

Substituting (24) into (23) gives:

$$q_i^P = k/(4c). \tag{25}$$

[Case 2] When $k^2 > 8ct$, (23) is convex and quadratic. Analogous to the quality-first model, there is an equilibrium only if an arbitrary upperbound is placed on price. (Claim 2. See Appendix C for a proof.)

(b) It can easily be checked that when $k^2 < 8ct$, the equilibrium of the simultaneous-determination model is the same as the equilibrium of the price-first model. Thus

$$P_i^S = t, \text{ and } q_i^S = k/(4c).$$

Comparison. We can easily verify that when the first-stage profit function is concave for both the quality-first and price-first models (i.e., $8ct > k^2$), assumptions A1 through A10 are satisfied. When the concavity of the first-stage profit functions are not assumed, the comparison of the price and service quality is meaningless because equilibrium price and service quality is determined by the upperbounds that are placed on price and service quality, which are quite arbitrary.

Comparing the symmetric solutions of the quality-first and price-first/simultaneous-determination models when the first period profit functions are concave, we have:

$$q_i^P = q_i^S = k/(4c) > k/(6c) = q_i^Q. \tag{26}$$

We also observe that:

$$p_i^P = p_i^S = p_i^Q = t. \tag{27}$$

Result (27) is an artifact of the linear demand function used here, which has the characteristic that A8 and A10 are satisfied with equality. If a demand function has the property that either A8 or A10 is satisfied with inequality, the dis-incentives for price-cutting will be larger and price will be higher in the price-first/simultaneous-determination models. (As will be shown subsequently, such an example can be obtained by slightly perturbing our demand function.)

Equations (26) and (27) support Economides' (1989) conclusion that quality is lower in the quality-first model while price is the same in both the quality-first and the simultaneous-determination models. Since price is the same and quality is lower in the quality-first model, consumers' welfare is lower when firms determine quality before price. However, we show that when we perturb our linear demand function in (17) such a way that we replace the expression $k(q_i - q_j)$ in the numerator of demand with $k_1q_i - k_2q_j$, where $k_1 = k_2 + \epsilon$ ($\epsilon > 0$), this conclusion no longer holds. The profit function of the perturbed game can be expressed as:

$$\Pi_i = p_i \frac{p_j - p_i + k_1 q_i - k_2 q_j + t}{2t} - c q_i^2. \qquad i \neq j \tag{28}$$

Under this profit function, we can show that both price and service quality in the price-first/simultaneous-determination models are strictly greater than the price and service quality in the quality-first model. Thus

$$q_i^P = q_i^S > q_i^Q \text{ and } p_i^P = p_i^S > p_i^Q.$$

(Claim 3. See Appendix D for a proof.)

In this case, the relative effect of a quality-first model on consumer welfare is indeterminate since the consumer is paying lower price for lower quality. Therefore, Economides' result is a special case of our model where $\epsilon = 0$.

4. Conclusion

In this paper, we examined a market structure in which product price is set before the level of service quality is established. We see instances of such behavior in industrial markets, banking, and recently in selected automobile dealers. At a general level, the analysis of the price-first model applies to industries in which the market does not clear by price due to costs associated with price changes (Carlton 1991).

We have shown that when there exist Complementarity between Symmetric Price and Symmetric Service Quality, and Cross-Strategic Substitutability, price and level of service quality are higher when firms set price first than when firms set service quality first.

Finally, it seems that the issue raised in this paper is related to the literature on resale price maintenance, where it is shown that a fixed price will encourage retailers to provide a higher level of service (Telser 1960). But Telser's result is based on an externality argument, whereby a consumer gathers information from a full service store and then buys from a discount store. Our model is not based on the externality argument. Moreover, the externality argument does not apply to such retailing institutions as banking and automobile dealerships.[10]

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[10] In banking, because one cannot distinguish service from product, it is not possible for the customer to receive services at one bank while transacting with another bank. Automobile dealers place great importance on drawing the prospective buyer to the showroom, inasmuch as once a customer is in the showroom, the salesperson has a fairly high probability of making the sale using a variety of selling techniques.

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Appendix A

Definition: Let p^Q and q^Q be the symmetric equilibrium of the quality-first game. Similarly, p^P and q^P is the symmetric equilibrium of the price-first game, and p^S and q^S is the symmetric equilibrium of the simultaneous-determination game. We first state three lemmas that are used subsequently in the proof of Proposition 1.

Lemma D1. If $W(p_1,p_2,q_1,q_2) = V(p_2,p_1,q_2,q_1)$ (i.e., A5), then $V_2(p_2,p_1,q_2,q_1) = W_1(p_1,p_2,q_1,q_2)$, $V_3(p_2,p_1,q_2,q_1) = W_4(p_1,p_2,q_1,q_2)$; $x(q_1, q_2) = y(q_2, q_1)$, $f(p_1, p_2) = g(p_2, p_1)$; $x_1(q_1, q_2) = y_2(q_2, q_1)$, $x_2(q_1, q_2) = y_1(q_2, q_1)$, $f_1(p_1, p_2) = g_2(p_2, p_1)$, and $f_2(p_1, p_2) = g_1(p_2, p_1)$.

Proof: Implicit in our symmetric framework. Proof omitted.

Lemma D2. If $q_1 = q_2 = q$, then $x_1 + x_2 = y_1 + y_2 = - (V_{13} + V_{14})/(V_{11} + V_{12})$

Proof: From Lemma D1, we have

$$V_1(p_2,p_1,q_2,q_1) = W_2(p_1,p_2,q_1,q_2) \tag{L1}$$

Rewriting (7) and substituting (L1) into (8), we get

$$V_1(x(q_1, q_2), y(q_1, q_2), q_1, q_2) = 0 \tag{L2}$$

$$V_1(y(q_1, q_2), x(q_1, q_2), q_2, q_1) = 0. \tag{L3}$$

Differentiating (L2) and (L3) by q_1, we get

$$V_{11}x_1 + V_{12}y_1 + V_{13} = 0 \tag{L4}$$

$$V_{11}y_1 + V_{12}x_1 + V_{14} = 0. \tag{L5}$$

Letting $q_1 = q_2 = q$, from Lemma D1 we have $x(q, q) = y(q, q)$, so that $V_{11}(x(q, q), y(q, q),$

$q, q) = V_{11}(y(q, q), x(q, q), q, q)$, and $V_{12}(x(q, q), y(q, q), q, q) = V_{12}(y(q, q), x(q, q), q, q)$. Solving (L4) and (L5),

$$y_1(q, q) = \frac{V_{12}V_{13} - V_{14}V_{11}}{V_{11}^2 - V_{12}^2} \tag{L6}$$

$$x_1(q, q) = \frac{V_{12}V_{14} - V_{11}V_{13}}{V_{11}^2 - V_{12}^2}. \tag{L7}$$

By Lemma D1 $x_1 = y_2$ and $x_2 = y_1$. Thus, we obtain

$$x_1 + x_2 = -\frac{V_{13} + V_{14}}{V_{11} + V_{12}} = y_1 + y_2 . \tag{L8}$$

Lemma D3. If $p_1 = p_2 = p$, then $f_1 + f_2 = g_1 + g_2 = -(V_{31} + V_{32})/(V_{33} + V_{34})$

Proof: From Lemma D1, we have

$$V_3(p_2,p_1,q_2,q_1) = W_4(p_1,p_2,q_1,q_2) \tag{L9}$$

Rewriting (11) and substituting (L9) into (12), we get

$$V_3(p_1, p_2, f(p_1, p_2), g(p_1, p_2)) = 0 \tag{L10}$$

$$V_3(p_2, p_1, g(p_1, p_2), f(p_1, p_2)) = 0. \tag{L11}$$

Differentiating (L10) and (L11) by p_1, we get

$$V_{31} + V_{33}f_1 + V_{34}g_1 = 0 \tag{L12}$$

$$V_{32} + V_{33}g_1 + V_{34}f_1 = 0. \tag{L13}$$

Using the symmetry argument as in the proof of Lemma D2 and solving (L12) and (L13), we get

$$g_1(p,\, p) \;=\; \frac{V_{31}V_{34} \;-\; V_{32}V_{33}}{V_{33}^2 \;-\; V_{34}^2} \tag{L14}$$

$$f_1(p,\, p) \;=\; \frac{V_{34}V_{32} \;-\; V_{31}V_{33}}{V_{33}^2 \;-\; V_{34}^2}\;. \tag{L15}$$

By Lemma D1, $f_1 = g_2$ and $f_2 = g_1$. Thus, we obtain

$$f_1 \;+ f_2 \;=\; -\;\frac{V_{31} \;+\; V_{32}}{V_{33} \;+\; V_{34}} \;=\; g_1 \;+\; g_2 \tag{L16}$$

Proof of Proposition 1:

(i) First we show that $q^S \geq q^Q$, and $p^S \geq p^Q$.

By the first order conditions from (9) and (10), and the fact that $V_1 = 0$ and $W_2 = 0$ (envelope relation), we have

$$V_1x_1 + V_2y_1 + V_3 = V_2y_1 + V_3 = 0 \tag{B1}$$

$$W_1x_2 + W_2y_2 + W_4 = W_1x_2 + W_4 = 0. \tag{B2}$$

From Lemma D1, if $q_1 = q_2 = q^Q$, then $V_2 = W_1$; $V_3 = W_4$; and $x_2 = y_1$. Therefore, if $q_1 = q_2 = q^Q$, (B1) and (B2) are the same. Rewriting (B1), we get

$$V_2(x(q^Q,q^Q),\, y(q^Q,q^Q),\, q^Q,\, q^Q)\; y_1(q^Q,\, q^Q) \;+\; V_3(x(q^Q,q^Q),\, y(q^Q,q^Q),\, q^Q,\, q^Q) = 0. \tag{B3}$$

By Cross-Strategic Substitutability we have $\partial y(q_1,\, q_2)/\partial q_1 = y_1(q_1,\, q_2) \leq 0$. Since by A3 we have $V_2 > 0$, $V_2y_1 \leq 0$. Therefore from (B3)

$$V_3(x(q^Q,q^Q),\, y(q^Q,q^Q),\, q^Q,\, q^Q) \geq 0. \tag{B4}$$

Note that the equilibrium quality q^S of the simultaneous-determination model satisfies

$$V_3(x(q^S, q^S), y(q^S, q^S), q^S, q^S) = 0. \tag{B5}$$

(This relation is obtained by solving (3) and (4) for p_1 and p_2, by substituting the result into (5) and (6), and then by taking account of symmetry.)

From Lemma D2, we have

$$x_1 + x_2 = y_1 + y_2 = -(V_{13} + V_{14})/(V_{11} + V_{12}). \tag{B6}$$

Differentiationg V_3 with respect to q we have

$$\frac{d}{dq} V_3(x(q, q), y(q, q), q, q) = V_{31}(x_1 + x_2) + V_{32}(y_1 + y_2) + V_{33} + V_{34}. \tag{B7}$$

Substituting (B6) into (B7), and by A6 we have,

$$\frac{d}{dq} V_3 = \frac{(V_{11} + V_{12})(V_{33} + V_{34}) - (V_{13} + V_{14})(V_{31} + V_{32})}{(V_{11} + V_{12})} < 0. \tag{B8}$$

From (B4), (B5), and (B8), we obtain

$$q^S \geq q^Q.$$

By Complementarity between Symmetric Price and Symmetric Service Quality we have $dx(q, q)/dq = x_1 + x_2 \geq 0$. Since $q^S \geq q^Q$, $p^S = x(q^S, q^S)$, $p^Q = x(q^Q, q^Q)$, and $dx(q, q)/dq = x_1 + x_2 \geq 0$, we obtain

$$p^S \geq p^Q.$$

(ii) Next, we show that $q^P \geq q^S$, and $p^P \geq p^S$.

From (13) and (14), the fact that $V_3 = 0$ by the envelope relation, and Lemma D1, we have

$$V_1 + V_3 f_1 + V_4 g_1 = V_1 + V_4 g_1 = 0. \tag{B9}$$

By Cross-Strategic Substitutability, $\partial g(p_1, p_2)/\partial p_1 = g_1(p_1, p_2) \leq 0$. Since by A3 we have $V_4 < 0$, we have $V_4 g_1 \geq 0$. Therefore if $p_1 = p_2 = p^P$, we obtain from (B9)

$$V_1(p^P, p^P, f(p^P, p^P), g(p^P, p^P)) \leq 0. \qquad \text{(B10)}$$

Note that the equilibrium price p^S in the simultaneous determination game satisfies

$$V_1(p^S, p^S, f(p^S, p^S), g(p^S, p^S)) = 0. \qquad \text{(B11)}$$

By Lemma D3, we get

$$f_1 + f_2 = g_1 + g_2 = - (V_{31} + V_{32})/(V_{33} + V_{34}). \qquad \text{(B12)}$$

Since we have

$$\frac{d}{dp} V_1(p, p, f(p, p), g(p, p)) = V_{11} + V_{12} + V_{13}(f_1 + f_2) + V_{14}(g_1 + g_2), \qquad \text{(B13)}$$

Substituting (B12) into (B13), A6 implies

$$\frac{d}{dp} V_1 = \frac{(V_{11} + V_{12})(V_{33} + V_{34}) - (V_{13} + V_{14})(V_{31} + V_{32})}{(V_{33} + V_{34})} < 0. \qquad \text{(B14)}$$

By (B10), (B11) and (B14), we get

$$p^P \geq p^S.$$

By Complementarity between Symmetric Price and Symmetric Service Quality we have $df(p, p)/dp = f_1 + f_2 \geq 0$. Since $p^P \geq p^S$, $q^P = f(p^P, p^P)$, $q^S = f(p^S, p^S)$, and $df(p, p)/dp = f_1 + f_2 \geq 0$, we obtain

$$q^P \geq q^S.$$

Q.E.D.

Appendix B

Proof of Lemma 1: From Lemma D2, and Lemma D3,

$$x_1 + x_2 = -\frac{V_{13} + V_{14}}{V_{11} + V_{12}} = y_1 + y_2$$

$$f_1 + f_2 = -\frac{V_{31} + V_{32}}{V_{33} + V_{34}} = g_1 + g_2 \;.$$

Since $V_{11} + V_{12} < 0$, $V_{33} + V_{34} < 0$ by (A6);

$V_{31} + V_{32} \geq 0$ (A7); and $V_{13} + V_{14} \geq 0$ (A8), we have

$$\frac{dx(q, q)}{dq} = x_1 + x_2 \geq 0$$

$$\frac{dy(q, q)}{dq} = y_1 + y_2 \geq 0$$

$$\frac{df(p, p)}{dp} = f_1 + f_2 \geq 0$$

$$\frac{dg(p, p)}{dp} = g_1 + g_2 \geq 0.$$

Proof of Lemma 2:

In a symmetric case $q_1 = q_2 = q$ and $p_1 = p_2 = p$. Therefore, from (L6) in Lemma D2 and (L14) in Lemma D3, we have

$$y_1(q, q) = \frac{V_{12}V_{13} - V_{14}V_{11}}{V_{11}^2 - V_{12}^2}$$

$$g_1(p, p) = \frac{V_{31}V_{34} - V_{32}V_{33}}{V_{33}^2 - V_{34}^2} .$$

From Lemma D1, $y_1(q, q) = x_2(q, q)$ and $g_1(p, p) = f_2(p, p)$.

Since $V_{11}^2 - V_{12}^2 > 0$ and $V_{33}^2 - V_{34}^2 > 0$ by A6, when $V_{12}V_{13} \leq V_{14}V_{11}$ (A9), and $V_{31}V_{34} \leq V_{32}V_{33}$ (A10)

$$\frac{\partial x(q, q)}{\partial q_2} = x_2(q, q) = y_1(q, q) = \frac{\partial y(q, q)}{\partial q_1} = \frac{V_{12}V_{13} - V_{14}V_{11}}{V_{11}^2 - V_{12}^2} \leq 0$$

$$\frac{\partial f(p, p)}{\partial p_2} = f_2(p, p) = g_1(p, p) = \frac{\partial g(p, p)}{\partial p_1} = \frac{V_{31}V_{34} - V_{32}V_{33}}{V_{33}^2 - V_{34}^2} \leq 0.$$

Appendix C

Proof of Claim 1: When $k^2 > 18ct$, Π_i is convex $(i=1,2)$. Letting $\bar{q}$ $(< 3t/k)$ be the upper limit of q, we have

$$\frac{\partial \Pi_i}{\partial q_i}(q_i, \bar{q}) = \frac{1}{t}\left[t + \frac{k}{3}(q_i - \bar{q})\right]\frac{k}{3} - 2cq_i \qquad i = 1, 2.$$

$$\geq \frac{k}{3t}\left(\frac{k}{3}q_i\right) - 2cq_i$$

$$= q_i\left(\frac{k^2 - 18ct}{9t}\right) > 0. \quad (for\ q_i > 0)$$

Therefore, if $\bar{q} < 3t/k$, $(q_1^Q, q_2^Q) = (\bar{q}, \bar{q})$ is an equilibrium.

Proof of Claim 2: When $k^2 > 8ct$, Π_i is convex ($i=1,2$). Letting $\bar{p}$ ($< 4ct^2/k^2$) be the upper limit of p, we have

$$\frac{\partial \Pi_i}{\partial p_i}(p_i, \bar{p}) = \frac{1}{2t}\left[\left(1 - \frac{k^2}{4ct}\right)\bar{p} + t - \left(2 - \frac{k^2}{4ct}\right)p_i\right] \qquad i = 1, 2$$

$$\geq \frac{1}{2t}\left[\frac{t\,(4ct-k^2)}{k^2} + t - \frac{8ct-k^2}{4ct}\,p_i\right]$$

$$= \frac{1}{2t}\left[\frac{k^2-8ct}{4ct}\,p_i + \frac{4ct^2}{k^2}\right] > 0.$$

Therefore, if $\bar{p} < 4ct^2/k^2$, $(\bar{p}, \bar{p})$ is an equilibrium.

Appendix D

Proof of Claim 3: If we perturb the demand function (17) such that $k(q_i - q_j)$ is replaced by $k_1q_i - k_2q_j$ where $k_1 = k_2 + \epsilon$ ($\epsilon > 0$), we have the following profit function. (Note that it is reasonable to have $\epsilon > 0$, since it is likely that demand will increase if the average quality is increased and prices remain the same.)

$$\Pi_i = p_i \frac{p_j - p_i + k_1q_i - k_2q_j + t}{2t} - cq_i^2, \qquad i \neq j \qquad \text{(D1)}$$

Quality-first model. Solving the second stage game first, we have:

$$p_i = t + \frac{2k_1q_i + k_1q_j - k_2q_i - 2k_2q_j}{3} \qquad \text{(D2)}$$

Substituting (D2) into (D1) gives the following first-stage profit function:

$$\Pi_i = \frac{1}{2t}\left[t + \frac{k_1}{3}(2q_i + q_j) - \frac{k_2}{3}(q_i + 2q_j)\right]^2 - cq_i^2. \tag{D3}$$

(D3) is concave when

$$18ct > 4k_1^2 + k_2^2 - 4k_1k_2. \tag{D4}$$

We assume hereafter that this condition holds. Since (D3) is concave, we solve the first order conditions to get:

$$q_i^Q = \frac{t\,(2k_1 - k_2)}{6ct + 3k_1k_2 - 2k_1^2 - k_2^2}\ . \tag{D5}$$

Substituting (D5) into (D2) gives

$$p_i^Q = \frac{6ct^2}{6ct + 3k_1k_2 - 2k_1^2 - k_2^2}\ . \tag{D6}$$

Price-first model/Simultaneous-determination model.

In this example, equilibrium of the price-first model and the simultaneous-determination model are the same.

(a) To solve a price-first model, the first-order conditions of the second-stage problem is considered first. We have:

$$q_i = p_ik_1/(4ct). \tag{D7}$$

Substituting (D7) into (D1) gives the following first-stage profit function.

$$\Pi_i = p_i \frac{p_j - p_i + \dfrac{k_1^2}{4ct} p_i - \dfrac{k_1 k_2}{4ct} p_j + t}{2t} - c \left(\frac{k_1}{4ct} p_i \right)^2 . \tag{D8}$$

The above profit function is concave if

$$8ct > k_1^2. \tag{D9}$$

Assuming inequality (D9) holds, we solve the first-order conditions to get:

$$p_i^P = \frac{4ct^2}{4ct + k_1 k_2 - k_1^2} \tag{D10}$$

Substituting (D10) into (D7) gives:

$$q_i^P = \frac{k_1 t}{4ct + k_1 k_2 - k_1^2} \tag{D11}$$

(b) It is easy to show that the equilibrium of the simultaneous-determination game is the same as that of the price-first game.

Comparison. It is easy to verify that A1 through A10 hold if the following conditions, (D12) and (D13), hold in addition to (D4) and(D9), which guarantee the concavity of the first-stage profit functions:

$$4ct + k_1k_2 - k_1^2 > 0, \tag{D12}$$

$$k_1 - 2k_2 < 0. \tag{D13}$$

The condition (D12) guarantees A6 (i.e., strong concavity, dominating strategic effects), and (D13) implies that A9 holds with strict inequality (i.e., *strict* cross-strategic substitutability). It is straightforward to show that A8 holds with strict inequality since $k_1 > k_2$.

By applying (D12) to (D10) and (D11), we can check that $p_i^P > 0$, and $q_i^P > 0$. We may also check that $p_i^Q > 0$ and $q_i^Q > 0$ as follows.

$$\begin{aligned}
&6ct + 3k_1k_2 - 2k_1^2 - k_2^2 \\
&= (18ct - 4k_1^2 - k_2^2 + 4k_1k_2 + 5k_1k_2 - 2k_1^2 - 2k_2^2)/3 \\
&> (5k_1k_2 - 2k_1^2 - 2k_2^2)/3 \qquad \text{(by D4)} \\
&= (2k_1 - k_2)(2k_2 - k_1)/3 \\
&> 0 \qquad \text{(by } k_1 > k_2 \text{ and D13)}
\end{aligned}$$

Therefore, we have $p_i^Q = 6ct^2/(6ct + 3k_1k_2 - 2k_1^2 - k_2^2) > 0$, and $q_i^Q = t(2k_1 - k_2)/(6ct + 3k_1k_2 - 2k_1^2 - k_2^2) > 0$. Finally, comparing the equilibrium prices and service quality levels (recall that $p_i^P = p_i^S$ and $q_i^P = q_i^S$), we have

$$p_i^P - p_i^Q = p_i^S - p_i^Q = \frac{2ct^2 (k_1 - k_2)(2k_2 - k_1)}{(4ct + k_1k_2 - k_1^2)(6ct + 3k_1k_2 - 2k_1^2 - k_2^2)} > 0,$$

$$q_i^P - q_i^Q = q_i^S - q_i^Q = \frac{2ct^2 (2k_2 - k_1)}{(4ct + k_1k_2 - k_1^2)(6ct + 3k_1k_2 - 2k_1^2 - k_2^2)} > 0,$$

by $k_1 > k_2$, $6ct + 3k_1k_2 - 2k_1^2 - k_2^2 > 0$, (D12), and (D13).