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Strategic Information Revelation*

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Abstract

We analyze the problem in which agents have non-public information and are to play an asymmetric information game. The agents may reveal some or all of their information to other agents prior to playing this game. Revelation is via exogenously verifiable statements. The equilibria resulting from various revelation strategies are used to determine equilibrium revelation of information. Sufficient conditions are provided for complete revelation of all private information. A number of examples are provided illustrating when revelation will or will not occur in commonly analyzed games.

1. Introduction

Many economic problems are characterized by the presence of both strategic behavior on the part of individuals and by asymmetric information. Models encompassing both of these characteristics have been used sucessfully to examine questions in many fields, usually using the concept of Bayesian Nash equilibrium as the solution concept. Typically the agents' information is given exogenously and is fixed throughout the analysis. For many of the problems analyzed via this technique, it is appropriate to model the information structure as static, but not for all. We typically model situations which are inherently dynamic with static models for analytic convenience. Most economic encounters allow for some interchange between the agents before the outcome is settled irrevocably. In such circumstances there is at least the possibility that one or more of the agents possessing non-public information may be able to reveal part or all of his information to one or all of the other agents in the economy during the course of their contact. To the extent that such a communication is believed by the other agents, the priors of the other agents will change and we will have what is in essence a new asymmetric information game (new in that the priors of the agents have changed). An agent with such a possibility of changing the priors of the other agents in this way would then presumably behave in a strategic manner with respect to his private information: reveal some or all of it if the result is an increase in his payoff.

We should be interested in the question of strategic information revelation since there are many models which "explain" particular phenomena via models which have asymmetrically informed agents. Many of the conclusions of these models depend on the asymmetric information; if the asymmetry disappears so may the results. If agents behave strategically with respect to their private information, they may reveal all of their private information, eliminating the asymmetry.

We model the strategic revelation of private information by adding a first stage announcement game to a given game with asymmetric information. Agents' private information is modelled in the standard way of assigning a different type to each type of information that an agent might have. Agents are allowed to announce some or all of their private information by announcing a set of types, to be interpreted as "I am one of these types". In this work it is assumed that certain exogenously specified announcements are verifiable, that is, can be proved by the agent making it. For example, if the private information is whether a given agent owns a house, he may be able to verify that he owns one (by showing a deed, for instance) but may not be able to verify that he does **not** own a house. It is assumed that agents simultaneously make announcements and if an agent makes a verifiable announcement, other agents revise their beliefs according to Bayes rule. With these revised probability beliefs, the given asymmetric information game is played. We can associate to each set of beliefs, the equilibrium payoffs (assuming uniqueness) of the original game with these beliefs. In this way the first stage information game can be considered as an asymmetric information game by itself and the Bayes Nash equilibrium can be investigated.

In section 3 we provide sufficient conditions on the information structure and the given game such that the unique sequential Bayes Nash equilibrium of the information game is complete revelation, that is agents reveal **all** their private information. The conditions are quite strong, but there are some economic problems which are commonly modelled in a way which satisfies the conditions which are sufficient for revelation of private information. Also, while the conditions are not necessary for information revelation, for many problems failing to satisfy the conditions it can be shown that information revelation will not occur. We provide a number of examples which illustrate how information may not unravel in various cases that the conditions are not satisfied. This provides at least the beginning of an understanding of the circumstances under which information will **not** be revealed. The circumstances in which we can be sure that information will not be revealed is of importance in itself.

We will describe two specific areas of research to which our analysis applies. The first is the problem of information sharing among oligopolists. A number of recent papers (Clark (1983), Gal-Or (1985,1986), Novshek and Sonnenschein (1982), Palfrey (1985), and Vives (1985) among them) have dealt with the question of whether or not various firms, each possibly having information not held by any of the other firms, would have higher profits when the information is shared with other firms than when it is kept private. Typically, the problem is treated as a game with incomplete (asymmetric) information and the Bayesian equilibrium solution concept is used. The equilibrium for the game with the original information structure is compared to the equilibrium when each firm has the union of all information. The answer to the question of whether oligopolists find sharing information more profitable than not is of interest since the welfare comparison of the two equilibria is ambiguous.

The papers described above can be interpreted as investigating strategic information revelation; the firms calculate expected profits under both the sharing and the non sharing regimes. The difference between our approach and that taken in these papers is that the calculations in the previous literature were ex ante calculations of expected profits, the expectation being taken over all information that an agent might get. It may be (in fact it will usually be) the case that the ex ante calculation is at variance with the calculation made once the private information is known. Ex ante an agent may decide that it is better to share information than not but, upon getting his private information, finds that his expected profit given this particular private information (the expectation being over the others' private information) is lower than had his private information been revealed. In this case the information would not be revealed if the agent could prevent it. Most of the literature cited above assumes that the agents agree that the private information go directly to a central agency which distributes it to other agents. Alternatively, binding contracts among agents to share information might be feasible.

The opposite case could also occur: the agents agree not to share information based on their (ex ante) expected profits under each regime yet an agent finds that for the particular information he received, his profits would have been higher had he shared. Again, some sort of commitment not to reveal the information would have been necessary to enforce the agreement. A contract to prevent revelation might be very difficult to enforce. If the information were received by a central agency without the individual agent knowing it, there would be no problem. The possibility of such an agency may or may not be realistic.

The conditions we provide which are sufficient to guarantee complete revelation of private information, while quite strong, are satisfied in many of the oligopoly models in which information sharing has been investigated. Thus, in the absence of some way to commit **not** to share information, the information will be revealed. Ex ante it may be in the agents' interest to not share, but the only sequential Bayes equilibrium involves complete sharing.

The second area of research to which our results will be related is that of a seller with private information regarding the quality of a good facing a potential buyer who is uninformed about the quality. The question of how much information will be revealed to the buyer in a sequential equilibrium has been studied by Grossman (1981), Matthews and Postlewaite (1985), Milgrom (1981), and Milgrom and Roberts (1986). The model in our paper is similar to the models in these papers and extends previous results. More importantly, perhaps, our paper provides several examples pointing out how restrictive the conditions which guarantee the revelation of information are in this case. For example, when a seller may not be able to verify the information he has, or even cannot verify whether he has information, full revelation of information may not occur.

2. Model

Let $N = \{1,\ldots,n\}$ be the set of players $(n \geqq 2)$. The game is played in two consecutive stages numbered 1 and 2. In stage 1, players engage in a game of information exchange. In stage 2, a game is played taking as given the private information held by each player and the beliefs about other players' private information that result from information exchange in stage 1. We shall analyze a sequential equilibrium of the two stage game.

Each player i has private information summarized by his type t_i, an element of T_i. For simplicity, T_i is assumed to be a finite set $\{t^1,\ldots,t^\ell\}$ whose elements are ordered as $t^1 < t^2 <\cdots< t^\ell$, common to all players. For any variable (or any set or function), we denote its profile over all players by the corresponding bold letter and its profile over all players except that of player i by the corresponding letter with subscript $-i$. Thus, $\mathbf{t}$ denotes the information profile $(t_i)_{i\in N} \in \times_i T_i = \mathbf{T}$ and t_{-i} denotes $(t_j)_{j\neq i} \in \times_{j\neq i} T_j = T_{-i}$.

At the beginning of stage 1, each player i is informed of his true type

$t_i \in T_i$. All other players do not know other players' true types. They know, however, the probability distribution of other players' types, p_i over T_i for each i. The p_i's are assumed to be common knowledge.

After the informational exchange that takes place in the first stage, players have beliefs about other players' types. Let Q_i be the set of all probability distributions on T_i. A belief about player i's type is a probability distribution $q_i \in Q_i$. Belief q_i should be interpreted as other players believing that the probability of player i's true type being $t_i \in T_i$ is $q_i(t_i)$.

The second stage game is defined as follows. For any player $i \in N$, S_i is his <u>set of actions</u>. $\pi_i: S \times \mathbf{T} \to R$ is his <u>payoff function</u> specifying his payoff $\pi_i(\mathbf{s},\mathbf{t})$ when action profile is $\mathbf{s} \in \mathbf{S}$ and the true information profile is $t \in T$. Player i's <u>second stage strategy</u> is a mapping $\sigma_i: T_i \to S_i$ that associates his choice of action $\sigma_i(t_i)$ to his own type $t_i \in T_i$. We denote by Σ_i the set of player i's second stage strategies.

For any player $i \in N$, his <u>expected payoff</u> of the second stage game is defined by a mapping $\Pi_i: \Sigma \times Q_{-i} \times T_i \to R$ such that, given a second stage strategy profile σ, beliefs about other players' types q_{-i}, and his own type t_i, his expected payoff is defined as:

$$\Pi_i(\sigma;q_{-i},t_i) = E[\pi_i(\sigma_i(t_i),\sigma_{-i}(t_{-i});\ t_i,t_{-i})|q_{-i}]$$

$$= \sum_{t'_{-i}\in T_{-i}} \prod_{j\neq i} q_j(t'_j)\pi_i(\sigma_i(t_i),\sigma_{-i}(t'_{-i}),t_i,t_{-i}),$$

where $E[\cdot|q_{-i}]$ is the expectation operator conditional upon belief q_{-i} and $\sigma_{-i}(t'_{-i}) = (\sigma_j(t'_j))_{j\neq i}$.

Given an information exchange in stage 1 that gives rise to a belief profile $\mathbf{q}$, a second stage game is played. We call this game a "subgame" although this is a game that stems from an information set (in the extensive form representation of the entire game associated with each given information exchange in stage 1) rather than the one that stems from a node.

Given a belief profile $\mathbf{q}$, a "subgame" equilibrium is a second stage strategy profile σ^* such that, for all $i \in N$, $t_i \in T_i$ and $s_i \in S_i$,

$$\Pi_i(\sigma^*;q_{-i},t_i) \geqq \Pi_i(s_i,\sigma^*_{-i};q_{-i},t_i).$$

Given a belief profile $\mathbf{q}$, the "subgame" equilibrium is called virtually unique if for any two subgame equilibria σ^* and σ^{**}, $\sigma^*_i(t_i) = \sigma^{**}_i(t_i)$ for any t_i for any i. We shall assume that for any possible belief profile, the "subgame" equilibrium is virtually unique. This is admittedly a very restrictive and critical assumption. However, without this assumption problems arise from the selection of one of several equilibria in the second stage which could arise from a single vector of beliefs. Moreover, our proof relies on a property which is guaranteed under virtual uniqueness but which may fail if we simply took a selection from a correspondence which was multiple valued.

Having assumed virtual uniqueness, the "subgame" equilibrium expected payoff to agent $i \in N$ of type $t_i \in T_i$ when belief profile is $\mathbf{q}$ is unambiguously defined. We shall write this payoff as $u_i(\mathbf{q},t_i)$.

In stage 1, players exchange information in the form of reporting a subset of T_i. Players may choose not to participate in information exchange. We interpret this choice as player i reports the whole set T_i. When i's true type is $t_i \in T_i$, a report $x_i \subset T_i$ is said to be truthful if $t_i \in x_i$. We assume that a report is not believed unless its truthfulness is verified. As we discussed in the previous section, some reports, though truthful, may not be verifiable. The report that a player is of the type "he does not know anything" is such an example in many situations (see example 2 in the next section.) In general, which sets are verifiable depends upon the particular problem we are analyzing. In this paper, we shall assume that the set of verifiable reports is exogeneously given and that verifiable reports can be made and verified without incurring any cost.

Formally, for any $i \in N$ let Δ_i be the collection of all non-empty subsets of T_i and $\Delta_i(t_i) = \{x_i \in \Delta_i | t_i \in x_i\}$, i.e., the set of truthful reports when his type is t_i. Let $\Delta_i^*(t_i) \subset \Delta_i(t_i)$ be the collection of truthful reports of i that are verifiable when his true type is t_i. For $i \in N$ his feasible <u>reporting strategy</u> is a mapping $\rho_i: T_i \to \Delta_i$ satisfying $\rho_i(t_i) \in \Delta_i^*(t_i)$ for all $t_i \in T_i$. We denote by $x_i = \rho_i(t_i) \in \Delta_i$ player i's report.

After the first stage, the report profile is verified and becomes common knowledge. Thus, for example, if a singleton set is verifiable and such a set is reported and verified, then it becomes common knowledge that the agent who reported the singleton must be of the reported type.

Players will make inferences about other players' types from the report profile. An <u>inference function</u> about the type for a player $i \in N$ is a mapping $b_i: \Delta_i \to Q_i$. Thus, given player i's report $x_i \in \Delta_i$ and inference function b_i, other players' beliefs about player i's type are represented by a posterior probability distribution $b_i(t_i | x_i)$.

Our assumption that only verifiable reports are sent gives rise naturally to the following definition of consistent beliefs. An inference function profile $\mathbf{b}$ is said to be <u>consistent</u> with a reporting strategy profile $\boldsymbol{\rho}$ if for all $i \in N$, $\mathbf{t} \in \mathbf{T}$, and for all $\mathbf{x} \in \Delta$,

(a) $b_i(t_i|x_i) = 0$ if $t_i \notin \Delta_i^*(t_i)$, and

(b) $$b_i(t_i|x_i) = \frac{p_i(t_i)}{\sum_{t_i' \in \rho_i^{-1}(x_i)} p_i(t_i')} \text{ if } t_i \in \rho_i^{-1}(x_i).$$

(a) is simply a statement that agents' beliefs reflect the assumption that a report is verifiable, and hence the posterior beliefs should put full probability on the verifiable set. (b) is a statement that, for reports that are actually sent for some type, the beliefs are consistent with Bayesian updating.

A sequential equilibrium is a pair $(\boldsymbol{\rho}^*, \mathbf{b}^*)$ satisfying:

(a) $\mathbf{b}^*$ is consistent with $\boldsymbol{\rho}^*$, and

(b) for all $i \in N$, $t_i \in T$ and $x_i \in \Delta_i^*(t_i)$,

$$\sum_{t_{-i}' \in T_{-i}} \times_{j \neq i} p_j(t_j') u_i(\mathbf{b}^*(\rho_i^*(t_i), \rho_{-i}^*(t_{-i}')), t_i) \geq$$

$$\sum_{t_{-i}' \in T_{-i}} \times_{j \neq i} p_j(t_j') u_i(\mathbf{b}^*(x_i, \rho_{-i}^*(t_{-i}')), t_i).$$

A sequential equilibrium $(\boldsymbol{\rho}^*, \mathbf{b}^*)$ is said to completely reveal private information if for all $i \in N$ and for all $t_i \in T_i$, $b_i^*(t_i|\rho_i^*(t_i)) = 1$.

3. Main Results

In the rest of the paper, we shall employ some or all of the following assumptions.

Assumption 1 (Dimensionality):

(a) For all $i \in N$, S_i is the closed interval $[0,\bar{s}]$ of the real line.

(b) For all $i \in N$, $s_{-i} \in S_{-i}$, and $t \in T$, $\pi_i(0,s_{-i},t) > \pi_i(\bar{s},s_{-i},t)$.

Assumption 2 (Interiority):

For any consistent belief profile, $\mathbf{q}$, the subgame equilibrium $\boldsymbol{\sigma}^*$ is interior, that is, for any $i \in N$, $t_i \in T_i$, $\sigma_i^*(t_i) \in (0,\bar{s})$.

Assumption 3:

For all $i \in N$ and for all $t_i \in T_i$, there exists $x \in \Delta_i^*(t_i)$ such that: $t_i = \min\{t_i' | t_i' \in x\}$.

Assumption 4:

(a) For all $i \in N$, $\mathbf{s} \in \mathbf{S}$, and $\mathbf{t} \in \mathbf{T}$,

π_i is concave and differentiable in s_i, and decreasing in s_{-i},

(b) For all $i \in N$, $\mathbf{s} \in \mathbf{S}$, and $\mathbf{t} \in \mathbf{T}$,

$\frac{\partial}{\partial s_i} \pi_i(\mathbf{s},\mathbf{t})$ is continuous and decreasing in s_{-i},

(c) (Monotonicity) For all $i \in N$, $\mathbf{s} \in \mathbf{S}$, and $\mathbf{t} \in \mathbf{T}$,

$\frac{\partial}{\partial s_i} \pi_i(\mathbf{s},\mathbf{t})$ is increasing in t_i and non-increasing in t_{-i}.

Assumption 4' (Linearity):

For all $i \in N$, $\pi_i(\mathbf{s},\mathbf{t}) = c\{a_i(t) - \sum_{j \neq i} ds_j - s_i\}s_i$, where $a_i(t)$ is increasing in t_i and non-increasing in t_{-i}, c and d are parameters satisfying $c > 0$ and $2 > d \geqq 0$.

Assumption 4 (and 4') is stated more restrictively than necessary for the sake of simplicity of the exposition. What is needed for 4(a) and 4(b) is the monotonicity of both π_i itself and the derivative of π_i with respect to s_{-i}. The most critical assumption is monotonicity, 4(c) (or monotone property of $a_i(t)$ in 4'). The weakest form of this assumption is that (i) the derivative of π_i is monotonic with respect to both t_i (with strict monotonicity) and t_{-i} (with weak monotonicity), and (ii) the monotonic order must be reversed between the two. The monotonic orders appearing in 4(a)-(b) and 4(c) (and the condition $c > 0$ in 4') can be altered by changing the ordering of t's itself.

An inference function profile **b** is called <u>skeptical</u> if for all $i \in N$, for all $t_i \in T_i$ and for all $x_i \in \Delta_i^*(t_i)$:

$$b_i(t_i|x_i) = 1 \quad \text{if } t_i = \min\{t_i' | t_i' \in x_i\}$$

$$= 0 \quad \text{otherwise.}$$

Theorem 1: *If $n = 2$ (i.e., $N = \{1,2\}$) and assumptions 1-4 are satisfied, all sequential equilibria completely reveal private information. Moreover, the equilibrium inference function profile is skeptical.*

Theorem 2:

For any N, if assumptions 1-3 and 4' are satisfied, then all sequential equilibria completely reveal private information and equilibrium inference function profile must be skeptical.

4. Examples

Example 1: (Sequential equilibrium in which private information is revealed.)

Consider a Cournot duopoly problem in which both firms have constant marginal cost. Firm 1's marginal cost is C and is known for certain. Firm

2's marginal cost can be one of two equally likely values--high, H, or low, L. Firm 2 knows its own marginal cost while firm 1 knows only the possible values and the probabilities that they might arise. The two firms face a linear inverse demand function $p(x) = a-bx$ where x is the combined quantities that the firms produce. It is assumed that this structure is common knowledge. We will look for a Bayes-Nash equilibrium for the incomplete information game they will play. An equilibrium is a quantity that firm 1 produces, s_1, and a pair of quantities s_H and s_L for firm 2 depending upon its actual costs. These choices should be optimal for each of the firms given its information at the time that it must make its decision. Hence, s_1 is a best response to the quantity that firm 2 puts on the market which is taken to be random with value s_H or s_L, each with probability 1/2. In figure 1, the best response functions, or reaction curves, are shown. When firm 2's marginal cost is high its best response to firm 1's choice s_1 is shown as R_2^H. It is linear due to the assumption of linear demand and constant marginal cost. Similarly, when firm 2's marginal cost is low its best response function is linear but is higher now due to its lower cost. It is shown as R_2^L.

In general, we would not be able to graph a best response functions for firm 1 in the same way. The normal way of calculating a firm's optimal level of output given a fixed level of output of the other firm would not make sense here. Since firm 1 does not know firm 2's cost and firm 2's optimal output will depend upon its cost, firm 1 will essentially be facing a lottery: firm 2 will produce one output when its cost is low and a different output when its cost is high. Thus in general firm 1's best response function will be a function mapping pairs of outputs into its optimal response. But the fact that the demand function is linear means that all that is necessary for firm 1 to calculate its optimal output given a choice by firm 2 is the expected output of the other firm. We have drawn firm 1's best response function R_1, where this should be interpreted in exactly this way: it represents firm 1's profit maximizing output given an expected output of firm 2. We have also drawn in the "average" of the two different best response functions for firm 2 given its two possible costs; this is denoted R_2^A. The equilibrium for this example will then be an output of s_1 for firm 1 and outputs s_2^H and s_2^L for firm 2 for costs H and L respectively. As can be seen from the diagram, s_2^H and s_2^L are each best responses to s_1 for firm 2 for each of the possible costs. For firm 1, s_1 is best response to the expectation of these two values.

If firm 2 can verify its marginal cost to firm 1, it will have an incentive to do so when its costs are low. In this case the equilibrium will not be determined by the intersection of R_1 and R_2^A but rather by the intersection of R_1 and R_2^L since the latter will then be known to be the relevant best response function for firm 2. Hence the equilibrium is the pair (s_1, s_2^L). The movement from that part of the original equilibrium which was relevant when firm 2 had low cost to this new point involves firm 1 producing less and firm 2 producing more. The combined output will be lower than before since both points will be on R_2^L which has slope less than 1. Thus the price will be higher, which in conjunction with the increase in firm 2's output guarantees that firm 2's profit will be higher; this means that firm 2 has an incentive to reveal (and verify of course) its marginal coat whenever it is low. The only reasonable inference that firm 1 can make when firm 2 does not reveal any information (in other words, announces that it is simply one of the two possible types) is to infer that its cost is high. With a first stage appended to the game in which firm 2 can announce and verify its cost to firm

1, the only sequential equilibrium will involve all the information that firm 2 has being revealed to firm 1.

__Example 2__: (Incomplete revelation of private information due to non-verifiability of some type.)

This example will be as in example 1 except that we will allow the possibility that firm 2 might not know its own cost. Firm 2 then has 3 possible types — t_2^1, that it has high cost and knows it, t_2^2, that it does not know its cost (but knows that it is high or low), and t_2^3, that it has low cost and knows it. We will assume that each of the three types is equally likely. We will assume as before that when firm 2 knows its cost it can verify it to firm 1, say by demonstrating its technology. This might not be possible; it is an assumption. We will assume, however, that if firm 2 is of type t_2^2 it cannot verify it. Again, this is an assumption although it does seem that it would be difficult to verify that one did **not** know something. The best response functions for firm 2 when it is of type t_1 or t_2 are exactly as in the previous case. When it is of type t_2, that is it does not know its own cost, firm 2 will maximize its expected profits. The assumptions of linear demand and constant marginal cost gives us a best response function in this case which is precisely the average of the two best response functions when it knows its costs. This will also be the average of the three best response functions over the three possible types that firm 2 can be. Firm 1's best response function is as it was in example 1. The equilibrium for this example involves firm 1 producing s_1 and firm 2 producing s_2^1, s_2^2, or s_2^3 depending upon whether its type is t_2^1, t_2^2, or t_2^3 respectively.

As before if firm 2 is of type t_2^3 it will have an incentive to reveal (and verify) this to the other firm and raise its profits. Thus when firm 1 hears no announcement, it knows that it must not be facing t_2^3. Then when firm 2 is of type 2, it has an incentive to reveal itself; the problem has reduced itself to that in example 1. There are two types which have different best response functions and the one with the higher best response function will increase its profits by revealing itself. But we have assumed that this type cannot verify itself. There is no way (by assumption) that it can prove that it is not a type 1, that its cost is high and it knows it. Thus there will be only partial revelation of private information: t_2^3 will announce his type and types 2 and 1 will announce nothing. Type 2 could of course make an announcement that he was type 2 but then so could type 1. Since it cannot be verified, it is equivalent to announcing nothing. The final equilibrium will then be of the form that type 3 of firm 2 reveals itself and a complete information Cournot game follows, while types 1 and 2 will not be separated and an incomplete information Cournot game follows in this case.

We should note that it is the failure of assumption 3 (or 3') which causes the information to not be revealed here. Monotonicity, as well as our other assumptions of theorem 1, hold for this example.

For the rest of examples, we shall assume that all the singletons are verifiable.

Example 3: (No revelation of private information due to corner equilibrium in the second stage.)

Let there be two firms playing a Cournot game in the second stage with inverse demand function $p(x) = 1-x$. Firm 1 has costless production while firm 2 has constant marginal cost of either 3 or 0, each believed equally likely. Firm 1 knows firm 2's marginal cost, but firm 2 does not. Firm 1's marginal cost is common knowledge. It is not as far-fetched that firm 1 knows firm 2's cost and firm 2 does not as it may seem. One can imagine an existing firm in an industry that, through experience, has determined the costs of various technologies for producing a good. Firm 2 may be a potential entrant with access to a technology about which firm 1 knows more than firm 2.

In the case that no information is revealed, it is as though firm 2 has a production cost of 3/2 per unit. Since the highest possible price per unit is 1, firm 2 chooses $s_2 = 0$ in equilibrium. Hence the equilibrium with no information revealed has firm 1 choosing the monopoly quantity and making monopoly profits. If firm 1 reveals its private information when firm 2's cost is low, the game will be a symmetric Cournot game in which firm 1's profit is lower than the monopoly profit level. Revealing the information when the cost is high gives rise to a situation as in no revelation: firm 2 chooses $s_2 = 0$ and firm 1 makes monopoly profit. In either case firm 1's profit is no larger under revelation than without revelation. Thus no revelation is an equilibrium.

Example 4: (No revelation of private information due to a multi-dimensional strategy space at the second stage.)

Suppose there are two firms, 1 and 2 and two goods, A and B. The demand functions for the two goods are uncertain. Firm 1 believes that each of the following cases is equally likely.

$$\{p_A=1-x_A,\ \ p_B=0\},\ \ \{p_A=0,\ p_B=1-x_B\},\ \ \{p_A=1-x_A,\ p_B=1-x_B\}.$$

Firm 2 knows which case is correct. We therefore associate firm 2's type with the demand configuration. Each firm has the same technology, characterized by the cost function $c(x_A, x_B) = 2/3\ (x_A + x_B)$. The Bayes-Cournot equilibrium has firm 1 producing 0, since from its point of view, the maximum expected price for either good which could arise is 2/3 while its marginal cost is 2/3. Thus firm 2 receives monopoly profits in the Bayes-Cournot equilibrium when its type remains unknown to firm 1. Whichever demand function arises (i.e. for whichever type firm 2 is), revealing this type leads to a symmetric Cournot equilibrium with the actual demand functions common knowledge. This yields lower profits for firm 2 than concealing its type, hence not revealing its type is the unique equilibrium.

It should be noted that this example does not satisfy monotonicity.

<u>**Example 5:**</u> (No revelation of private information due to a lack of monotonicity.)

Suppose there are two firms, 1 and 2, and a single good. Assume that the demand for the good is represented by $p(x) = a-bx$. Firm 1's marginal cost is $c_1(s_1)=0$ while firm 2's is either $c_2(s_2)=1-s_2$ or $c_2(s_2)=1+s_2$. It is assumed that neither firm knows 2's cost function but that each gets one of two signals regarding the marginal cost function. The relation of the signals to firm 2's marginal cost is as in the table below.

Firm 1 \ Firm 2	t^1	t^2
t^1	$1-s_2$	$1+s_2$
t^2	$1+s_2$	$1-s_2$

Each pair of signals is equally likely; thus, a single firm's signal gives no information about firm 2's marginal cost. With no further information, firm 2 simply maximizes expected profit and the Bayes-Cournot equilibrium is a deterministic outcome independent of signals. If firm 2 were to tell firm 1 its signal, nothing would change; only firm 2 will use additional information to alter its decision. If firm 2 learns firm 1's signal, however, it will know its own marginal cost and will make its output level dependent on this. Firm 2's choice is represented in figure 2. The marginal revenue function associated with the residual demand function is shown along with firm 2's possible marginal cost functions, $c_2(s_2) = 1+s_2$ and $c_2(s_2) = 1-s_2$; also shown is $c_2(s_2)=1$, which is the expected marginal cost which firm 2 uses, when its marginal cost is unknown, to choose an output level. Its choice in this case will be s_2^U. If firm 2 learns its marginal cost its choice will be either s_2^H or s_2^L depending upon whether the "high" or "low" marginal cost has occurred. It can be shown that s_2^U is less than the expected value of s_2^H and s_2^L (at least for some parameters). Thus for any choice of output by firm 1, firm 2's expected optimal response when it learns its true marginal cost is higher than its optimal response to this output level without the information. We have illustrated the reaction functions and the equilibria for each situation in figure 3. Firm 1's reaction curve, independent of whether firm 2 knows its marginal cost is R_1. Firm 2's reaction curve is R_2^U when it does not know its costs. R_2^H and R_2^L are firm 2's reaction curves when it learns its costs are "high" or "low" respectively; R_2^E is its expected quantity produced in response to firm 1's output. When firm 2 does not know its cost, s_1^U and s_2^U are the equilibrium quantities in the Bayes-Cournot equilibrium. If firm 2 knows its cost, firm 1's output drops to s_1 and firm 2's equilibrium output is s_2^H or s_2^L , depending upon its cost. With revelation of information, expected total quantity goes up, hence expected price goes down. Also firm 1's quantity goes down, and thus firm 1's expected profits go down with revelation. This is sufficient to verify that no information revelation is an equilibrium for the first stage.

Example 6: (Common value example in which monotonicity fails but private information is nonetheless revealed in equilibrium.)

Consider a Cournot duopoly problem with the two firms having costless production. They face a linear demand function $p(x) = a-bx$, where a is uncertain. Each of the two firms will get one of two signals which is correlated to this parameter. We will identify the firms' types with the signals they can receive. If both firms are of type 1, $a = 3$, if one is of type 1 and the other is of type 2, $a = 4$, and if both firms are of type 2, $a = 5$. The firms' types are independent and for each firm, its types are equally likely. Assumption 4 (and 4'), monotonicity, fails for this example. For firm 1, $\partial\pi_1/\partial s_1(s,t)$ is increasing in t_1. Intuitively, when firm 2's best response function is higher, it would like to reveal this to firm 2. But to reveal this is to reveal that firm 1 received the favorable signal regarding demand which would cause firm 2's best response function to shift out as well. Thus there are direct effects of revealing one's type (revealing that you are playing a best response function that is further out) and indirect effects (the shifting of your opponents best response function because of the information about his payoff function contained in the revelation). Monotonicity essentially requires that these two effects go in the same direction. While the two effects go in opposite directions in this example, and hence our proposition does not apply, it turns out that the direct effects dominate and the sequential equilibrium reveals private information.

Example 7: (Example in which information about a firm's cost does not get revealed due to the impossibility of partially revealing the firm's private information.)

Again consider a Cournot duopoly problem with a linear demand function for a single good. Firm 1 can be of four types, t^1, t^2, t^3, or t^4; firm 2 has a single type. Both firms' marginal costs depend on firm 1's type as shown in the table below.

firm 1's type	t^1	t^2	t^3	t^4
firm 1's mc	H	H	L	L
firm 2's mc with technology 1	H	L	H	L
firm 2's mc with technology 2	L	H	L	H

The interpretation of the example is as follows. There are two technologies available for producing the good in question. It is common knowledge that exactly one of these two technologies is "good," that is, that it leads to a low marginal cost. The other technology will lead to a high marginal cost. Firm 1 has chosen and committed to one of the technologies, but firm 2 does not know what this choice is. For example, when firm 1's type is t^1, technology 2 is the low cost technology, but firm 1 has chosen technology 1. Firm 2's cost depends upon which technology it chooses (not knowing, of course, which the the better technology). Thus, the game involves firm 2 choosing a technology and an output level and firm 1 choosing only an output level.

If firm 1 reveals no information about its type, firm 2 will choose either technology and the two firms will play the ensuing incomplete information Cournot game. If firm 1 could reveal only its marginal cost, it is straightforward to show that when its cost was low, the profits from the game arising when it revealed this to firm 2 are higher than when it does not. Thus, the only sequential equilibrium would completely reveal firm 1's private information in this case. This is tantamount to firm 1 revealing the set containing types 3 and 4 when either of them is the true type. But if we assume that the verifiable sets are only the singleton sets, this is not possible. Firm 1 here is faced with the choice of either revealing nothing or its precise type. Revealing its type pricisely when its marginal cost is low will lead to a choice by firm 2 of the technology which gives firm 2 a marginal cost exactly the same as firm 1's. This then gives rise to a symmetric, complete information Cournot game. The profits from the symmetric game may be lower than those in the game in which firm 2 chooses a technology and output level without knowing firm 1's type. Thus the only sequential equilibrium here involves no revelation of information.

This seems a very real problem in information sharing. The only way to verify that part of the information that you would like to share involves revealing information that you would like not to share. Many firms would like to share with their competitors (and could verify) that their costs are low. Many times this can only be verified by demonstrating the technology and thereby losing whatever advantage over competitors it had to begin with.

<u>Example 8</u>: (Application of our model to the quality revelation problem.)

Our framework is sufficiently broad to encompass economic problems other than oligopoly models. One such problem is that of a monopolist selling a good of exogenous quality which is known to him but not to the consumer. We can identify the seller's types with his information, that is, with the quality of the good he is selling. The buyer's types are associated with different valuations for the good in question. More specifically, the seller is to choose a price and the buyer is to choose a quantity to purchase given that price. The buyer has a demand function that shifts out with higher quality. We think of the buyer as having started with a given demand function (actually there is a set of demand functions-one for each quality assessment he has). The buyer has already acquired some of the commodity, and the quantity is not known to the seller. We thus associate the quantity already acquired with the buyer's types. Larger quantities of already acquired good shift the demand function the seller is facing in. Under at least some plausible parameters, assumption 4 will be satisfied (as well as the other assumptions) and the only sequential equilibrium of the game will involve complete revelation of each agent's private information.

In the case of a consumer with a known demand function, this problem is essentially that treated by Grossman (1981) and Milgrom (1981).

5. Proofs

We start by providing a few additional definitions and some additional notation.

Definition:

For any $i \in N$ and for any $t_i^k \in T_i$, the corresponding degenerate belief q_i^k is defined as $q_i^k(t_i) = 1$ if $t_i = t_i^k$ and $q_i^k(t_i) = 0$ otherwise.

Definition:

A game is said to satisfy the stochastic dominance property if: For any belief profile $\mathbf{q}$ and for any i, suppose an alternative belief about player i's type q_i' stochastically dominates q_i, i.e., for all $t_i \in T_i$, $\Sigma_{t_i' \leq t_i} q_i(t_i') \geq \Sigma_{t_i' \leq t_i} q_i'(t_i')$ and strict inequality for at least one $t_i \in T_i$. Then:

For all $t_i \in T_i$, $u_i(\mathbf{q};t_i) < u_i((q_i',q_{-i});t_i)$

must always hold.

Definition:

A game is said to satisfy the weak stochastic dominance property if: For any belief profile $\mathbf{q}$, and for any $i \in N$ and for any $q_i \in Q_i$, suppose that the minimum element of the support of q_i' is at least as large as the maximum element of the support of q_i, and the two supports are not the same singleton set, i.e.,

$$\text{(i)} \quad \min\{t_i \in T_i \mid q_i'(t_i) > 0\} \geqq \max\{t_i \in T_i \mid q_i(t_i) > 0\}$$

and

(ii) there exists $t_i, t_i' \in T_i$ such that

$$q_i(t_i) > 0,\ q_i'(t_i') > 0 \ \text{and} \ t_i < t_i'.$$

Then for all $t_i \in T$,

$$u_i(\mathbf{q};t_i) < ((q_i',q_{-i});t_i)$$

must always hold.

Definition:

Suppose $N = \{1,2\}$ and consider a subgame with belief profile $\mathbf{q}$. For $i,j \in N$ $(i \neq j)$, player i's optimal response function is defined as a mapping $\phi_i \colon \Sigma_j \times Q_j \times T_i \to S_i$ such that:

$$\phi_i(\sigma_j;\, q_j, t_i) = \arg\max_{s_i \in S_i}{}' \sum_{t_j \in T_j} q_j(t_j)\pi_i(s_i, \sigma_j(t_j); t_i, t_j),$$

and $\phi_i(\sigma_j;q_j) = \left(\phi_i(\sigma_j;q_j,t_i)\right)_{t_i \in T_i}$. It follows that the subgame equilibrium strategy σ_i^* for a belief profile **q** is a fixed point of the composite mapping $\phi_i(\phi_j(\cdot;q_j),q_i)$.

Lemma 1:

Under assumption 3, if the game satisfies the stochastic dominance property, then the only sequential equilibrium is that of complete revelation with the skeptical inference function profile.

Proof:

Let $(\mathbf{p}^*,\mathbf{b}^*)$ be any sequential equilibrium. Suppose for some $i \in N$, $b_i^*(x_i)$ does not put full probability on the lowest element of x_i. Let $t_i^M(x_i)$ be the highest type in the support of $b_i^*(x_i)$. Then, player i who finds his type to be $t_i^M(x_i)$ (or higher) will never report x_i, as he will be better off by reporting the set whose lowest element is his own type, for such a report will generate a belief that stochastically dominates the original belief. This implies that the beliefs associated with equilibrium reports must be degenerate and the only sequential equilibrium is that of complete revelation.

If, again, $b_i^*(x_i)$ does not put full probability on the lowest type in x_i, the lowest type will find himself better off by sending this report than the equilibrium report he is supposed to send. For the belief associated with x_i stochastically dominates the belief associated with his equilibrium report that, by the previous paragraph, has full probability on his true type. Hence the equilibrium inference function profile must be skeptical.

On the other hand, if an inference function profile, **b**, is skeptical and reporting strategy porfile, **p**, completely reveals private information, the pair **(p,b)** obviously constitutes a sequential equilibrium. Q.E.D.

Corollary:

Under assumption 3, if the game satisfies the weak stochastic dominance property, then the only sequential equilibrium is that of complete revelation with the skeptical belief.

Proof:

Straightforward from the proof of lemma 1. Q.E.D.

Lemma 2:

Suppose $N = \{1,2\}$ and assumptions 1,2, and 4 are satisfied. Then the game satisfies the weak stochastic dominance property.

Proof:

We first prove that for any belief profile $\mathbf{q}^0 = (q_i^0, q_j^0)$, for any $i \in N$, and for any $t_i^k \in T_i$ with the corresponding degenerate belief q_i^k;

$$u_i(\mathbf{q}^0; t_i^k) < u_i((q_i^k, q_j^0); t_i^k) \quad \text{if} \quad q_i^k \text{ stochastically dominates } q_i^0, \text{ and}$$

$$u_i(\mathbf{q}^0; t_i^k) > u_i((q_i^k, q_j^0); t_i^k) \quad \text{if} \quad q_i^0 \text{ stochastically dominates } q_i^k.$$

Let $\boldsymbol{\sigma}^0$ and $\boldsymbol{\sigma}^k$ be the subgame equilibrium when the belief profiles are $\mathbf{q}^0$ and (q_i^k, q_j^0) $(i \neq j)$, respectively.

Claim 1: For all i, his optimal response function is increasing in t_i, i.e., for all $t_i, t_i' \in T$, $\sigma_i^0(t_i) = \phi_i(\sigma_j^0; q_j^0, t_i) > \phi_i(\sigma_j^0; q_j^0, t_i') = \sigma_i^0(t_i')$ if and only if $t_i > t_i'$.

By the first order condition, if $s_i = \phi_i(\sigma_j;q_j,t_i)$, then;

$$0 = \sum_{t_j \in T_j} q_j^0(t_j) \frac{\partial}{\partial s_i} \pi_i\left(s_i,\sigma_j^0(t_j);t_i,t_j\right)$$

$$> \sum_{t_j \in T_j} q_j^0(t_j) \frac{\partial}{\partial s_i} \pi_i\left(s_i,\sigma_j^0(t_j);t_i',t_j\right),$$

where the equality holds from the concavity of π_i (assumption 4(a)) and interiority, while the inequality holds from the monotonicity (assumption 4(c)). However by the same reasoning, if $s_i' = \phi_i(\sigma_j;q_j t_i')$,

$$0 = \sum_{t_j \in T_j} q_j^0(t_j) \frac{\partial}{\partial s_i} \pi_i\ \left(s_i',\sigma_j^0(t_j);t_i',t_j\right).$$

It follows from the concavity of π_i that $s_i > s_i'$, which proves Claim 1.

<u>Claim 2:</u> $\phi_j\ (\sigma_i^0;q_i^k) < \phi_j(\sigma_i^0;q_i^0)$ if q_i^k stochastically dominates q_i^o and $\phi_j(\sigma_i^0;q_i^k) > \phi_j(\sigma_i^0;q_i^0)$ if q_i^0 stochastically dominates q_i^k.

By assumption 4(b)-(c) and the previous lemma, $\frac{\partial}{\partial s_j} \pi_j\left(S_j,\sigma_i^0(t_i),t_j,t_i\right)$ is decreasing in t_i. Because q_i^k stochastically dominates q_i^0, it follows that, if $\phi_j(\sigma_i^0;q_i^0) = s_j$:

$$0 = \sum_{t_i \in T_i} q_i^0(t_i) \frac{\partial}{\partial s_j} \pi_j\left(S_j,\sigma_i^0(t_i),t_j,t_i\right)$$

$$> \sum_{t_i \in T_i} q_i^1(t_i) \frac{\partial}{\partial s_j} \pi_j(S_j,\sigma_i^0(t_i),t_j,t_i).$$

The first half of the claim follows immediately. The second half is proved similarly.

<u>Claim 3</u>: $\sigma_i^k(t_i^k) > \sigma_i^0(t_i^k)$ if q_i^k stochastically dominates q_i^0, and $\sigma_i^0(t_i^k) > \sigma_i^k(t_i^k)$ if q_i^0 stochastically dominates q_i^k.

Note first that:

(a) Since q_i^k is degenerate, $\phi_j(\sigma_i;q_i^k)$ depends only upon $\sigma_i(t_i^k) = s_i^k$ and can be written as $\phi_j(s_i^k;q_i^k)$,

(b) by the definition of subgame equilibrium and the virtual uniqueness, $\sigma_i^k(t_i^k)$ is the unique fixed point of the composite mapping $\phi^k(s_i^k) = \phi_i(\phi_j(s_i^k;q_j^k);q_j^0,t_i^k)$.

Suppose that q_i^k stochastically dominates q_i^0. By the previous claim:

$$\sigma_j^0 = \phi_j(\sigma_i^0;q_i^0) > \phi_j(\sigma_i^0;q_i^k).$$

It follows that:

$$\sigma_i^0(t_i^k) = \phi_i(\sigma_j^0;q_j^0,t_i^k) < \phi_i(\phi_j(\sigma_i^0;q_i^k);q_j^0,t_i^k)$$

$$= \phi_i(\phi_j(\sigma_i^0(t_i^k);q_i^k);q_j^0,t_i^k) = \psi^k(\sigma_i^0(t_i^k)),$$

where all equalities are definitional and the inequality holds from the previous claim and the fact that $\phi_i(\sigma_j;q_j,t_i)$ is a decreasing function of σ_j in view of assumption 4(b). By the uniqueness of the fixed point, it trivially follows that the fixed point of ψ^k is larger than $\sigma_i^0(t_i^k)$. This proves the first half of the claim. The second half is similarly proved.

<u>**Claim 4**</u>: $\sigma_j^k < \sigma_j^0$ if q_i^k stochastically dominates q_i^0, and $\sigma_j^0 > \sigma_j^k$ if q_i^0 stochastically dominates q_i^k.

Suppose that q_i^k stochastically dominates q_i^0. Then;

$$\sigma_j^0 = \phi_j(\sigma_i^0;q_i^0) > \phi_j(\sigma_i^0;q_i^k) = \phi_j(\sigma_i^0(t_i^k);q_i^k) > \phi_j(\sigma_i^k(t_i^k);q_i^k) = \sigma_j^k.$$

where the first inequality holds from Claim 2, while the second inequality holds from Claim 3 and the fact that ϕ_j is a decreasing function of σ_i. The second part of the claim is similarly proved.

Suppose q_i^k stochastically dominates q_i^0 and consider the following chain of (in)equalities:

$$\begin{aligned} u_i((q_j^0,q_i^k),t_i^k)) &= \sum_{t_j \in T_j} q_j^0(t_j)\pi_i(\sigma_i^k(t_i^k),\sigma_j^k,t_i^k,t_j) \\ &> \sum_{t_j \in T_j} q_j^0(t_j)\pi_i(\sigma_i^k(t_i^k),\sigma_j^0,t_i^k,t_j) \\ &\geqq \sum_{t_j \in T_j} q_j^0(t_j)\pi_i(\sigma_i^0(t_i^k),\sigma_j^0,t_i^k,t_j) \\ &= u_i(q^0,t_i^k). \end{aligned}$$

The first inequality holds from the previous claim and assumption 4(a), the second inequality holds from the fact that $\sigma_i^0(t_i^M)$ is the optimal response against σ_j^0 when his type is t_i^M. This proves the first half of the assertion. The second half is similarly proved. The lemma then follows immediately. Q.E.D.

Proof of Theorem 1:

By lemma 2, under assumptions 1-2 and 4, the game trivially satisfies weak stochastic dominance property. But then by corollary to lemma 1, the only equilibrium is that of complete revelation with skeptical inference.

Q.E.D.

Lemma 3:

For any N with $n \geq 2$, suppose assumptions 1-3 and 4' are satisfied. Then the game satisfies the stochastic dominance property.

Proof:

Let a belief profile $\mathbf{q}$ and a second stage strategy profile $\boldsymbol{\sigma}$ be given. Then for any $i \in N$, i's second stage expected payoff when his true type is $t_i \in T_i$ is written as:

$$\Pi_i(\sigma:q,t_i) = E[c\{a_i(t_i,t'_{-i}) - d\sum_{j\neq i}\sigma_j(t_j) - \sigma_i(t_i)\}\sigma_i(t_i)|q_{-i}]$$

$$= c\{\alpha_i(t_i,q_{-i}) - d\sum_{j\neq i}\sum_{t_j\in T_j} q_j(t_j)\sigma_j(t_j) - \sigma_i(t_i)\}\sigma_i(t_i), \qquad (1)$$

where $\alpha_i(t_i,q_{-i}) = E[a_i(t)|t_i,q_{-i}] = \sum_{t_{-i}\in T_{-i}} \prod_{j\neq i} q_j(t_j)a_i(t_{-i},t_i)$.

By interiority, the subgame equilibrium strategy $\boldsymbol{\sigma}^*$ given $\mathbf{q}$ must satisfy the following set of first-order conditions for all $i \in N$ and $t_i \in T_i$;

$$\alpha_i(t_i,q_{-i}) - d\sum_{j\neq i}\sum_{t_j\in T_j} q_j(t_j)\sigma^*_j(t_j) - 2\sigma^*_i(t_i) = 0, \quad \text{or} \qquad (2)$$

$$\alpha_i(t_i,q_{-i}) - d\sum_{j\neq i}\sum_{t_j\in T_j} q_j(t_j)\sigma^*_j(t_j) - \sigma^*_i(t_i) = \sigma^*_i(t_i). \qquad (3)$$

In view of (1) and (3), the subgame equilibrium payoff is written as:

$$u_i(\mathbf{q},t_i) = \Pi_i(\sigma^*;\mathbf{q},t_i) = c(\sigma_i^*(t_i))^2. \tag{4}$$

Thus, the subgame equilibrium payoff will increase if and only if his subgame equilibrium strategy $\sigma_i^*(t_i)$ increases as a result of change in beliefs.

Moreover, the equilibrium strategy configuration $\boldsymbol{\sigma}^*$ under $\mathbf{q}$ can be expressed as a solution to the matrix equation, $Q\cdot\sigma^* = \boldsymbol{\alpha}(\mathbf{q})$, where Q, σ^* and $\boldsymbol{\alpha}(\mathbf{q})$ are defined as follows.

Let $\nu_i = \#T_i$, i.e., the cardinality of the set T_i, and let $\nu = \sum_{i \in N} \nu_i$. A $\nu \times \nu$ matrix Q, which consists of n^2 submatrices Q_{ij} each of which is a $\nu_i \times \nu_j$ matrix, is defined as:

(i) $Q_{ii} = 2I_{\nu_i}$, and

(ii) $Q_{ij} = d\mathbf{e}_{\nu_i}\mathbf{q}_j'$ if $i \neq j$,

where I_n is a n-dimensional identity matrix, $\mathbf{e}_n$ is an n-dimensional unit column vector, and $\mathbf{q}_i$ is a ν_i-dimensional column vector $\{q_i(t_i)\}_{t_i \in T_i}$. A $\nu \times 1$ vector $\alpha(\mathbf{q})$, which consists of n subvectors $\boldsymbol{\alpha}_i(\mathbf{q})$ each of which is a $\nu_i \times 1$ vector, is defined as:

$$\boldsymbol{\alpha}_i(\mathbf{q}) = \{\alpha_i(t_i,q_{-i})\}_{t_i \in T_i} = \{E[a_i(\mathbf{t})|t_i,q_{-i}]\}_{t_i \in T_i}.$$

Finally, with an abuse of notation, a $\nu_i \times 1$ vector, $\boldsymbol{\sigma}^*$, consisting of n subvectors $\boldsymbol{\sigma}_i^*$ each of which is a $\nu_i \times i$ vector is defined as:

$$\sigma_i^* = \{\sigma_i^*(t_i)\}_{t_i \in T_i}.$$

Note that the inverse matrix of $\mathbf{Q}$ exists and we denote it as $\mathbf{R}$, for otherwise the virtual uniqueness is violated. Hence, the equilibrium $\boldsymbol{\sigma}^*$ is denoted as $\boldsymbol{\sigma}^* = \mathbf{R}\cdot\boldsymbol{\alpha}(\mathbf{q})$.

Moreover, in view of Theorems 8.3.3. and 9.3.3. of Graybill [1969], $\mathbf{R}$ can be written as follows:

(i) $\mathbf{R}$ is also partiotioned into n^2 matrices of the same order as $\mathbf{Q}$,

(ii) $\mathbf{R}_{ii} = \frac{1}{2} I_{\nu_i} + (\gamma - \delta)\, \mathbf{e}_{\nu_i} \mathbf{q}_i'$, and

(iii) $\mathbf{R}_{ij} = -\delta\, \mathbf{e}_{\nu_i} \mathbf{q}_j'$ if $i \neq j$,

where $\gamma = d/2(2-d) > 0$ and $\gamma > \delta = \dfrac{d(\frac{1}{2}+\gamma)^2}{1+dn(\frac{1}{2}+\gamma)} = \dfrac{\gamma}{4+(n-2)d} > 0.$

It then follows that for any $t_i \in T_i$:

$$\sigma_i^*(t_i) = \frac{1}{2}\alpha_i(t_i,q_{-i}) + (\gamma-\delta)\, \mathbf{q}_i'\boldsymbol{\alpha}_i(q) - \sum_{j\neq i} \delta\, \mathbf{q}_j'\boldsymbol{\alpha}_j(\mathbf{q}).$$

Now consider the belief profile changes from $\mathbf{q}$ to (q_i',q_{-i}) where q_i' stochastically dominates q_i. By assumption 4', $\alpha_i(t_i,q_{-i})$ is an increasing function of t_i and the second term of the right hand side of the equation increases. By the same assumption, $a_j(\mathbf{t})$ is non-decreasing in t_i for all $j \neq i$ and $\alpha_j(t_j,q_{-j})$ decreases or remains the same as a result of this belief change. Hence, $\sigma_i^*(t_i)$ increases whenever the proposed belief change occurs. In view of our earlier ovservation that the subgame equilibrium payoff increases if and only if the subgame equilibrium strategy increases, this proves the lemma.

Q.E.D.

Proof of Theorem 2:

Straightforward from Lemma 1 and Lemma 3. Q.E.D.

6. Conclusion

We should care about information sharing for two reasons. There are public policies developed with the notion that normally operating competitive markets may provide too little information; consumer information laws are promulgated with this problem in mind. There is also a recognition that certain kinds of information sharing may be socially bad; the case of firms within an industry sharing certain information may fall into this category. Before policies are drawn up to correct problems of too little or too much information sharing in markets, we need models in which the agents in the market are presumed to make choices optimally as to the amount of information they share with the other economic agents. As we have pointed out, whether information sharing is good or bad for the participants is independent of whether such information can be part of a sequential equilibrium.

We have provided theorems giving sufficient conditions for information to be fully revealed in a sequential equilibrium. The conditions of the theorems, in particular monotonicity, are quite strong. The examples show that weakening these conditions may easily upset the conclusion that the information will be revealed in equilibrium. As example 2 shows, when the incomplete information including whether other information is known or not known by other agents is typically not verifiable; such problems may well lead to less than full revelation of private information. Example 7 shows that if the information structure becomes complex, agents may prefer to reveal nothing to revealing all they know, if those are the alternatives.

The theorems and examples give a rough feeling for problems which may prevent full revelation of information in equilibrium. Some of the examples, such as example 7, suggest that intermediaries may arise to collect and verify information and pass along to other agents an accurate but less revealing summary of an agent's private information. In this example, firm 1 would like to reveal its cost when it is low but will not do so if by doing so it reveals the optimal technology for the other firm. An intermediary (a manufacturers' association or a governmental agency?) might arise to solve this problem. The analysis of the role that such intermediaries might play in information sharing seems to us to be a very interesting problem.

As seen in some of the examples, sequential equilibria may lead to no information sharing even when it is ex-ante efficient to do so. If information sharing is more easily accomplished within a firm than between firms, alternative firm structures may emerge as a result of the potential benefits of information sharing. Future research should investigate the relationship between the alternative institutional structures of firms and industries and the possibilities of information sharing.

Figure 1

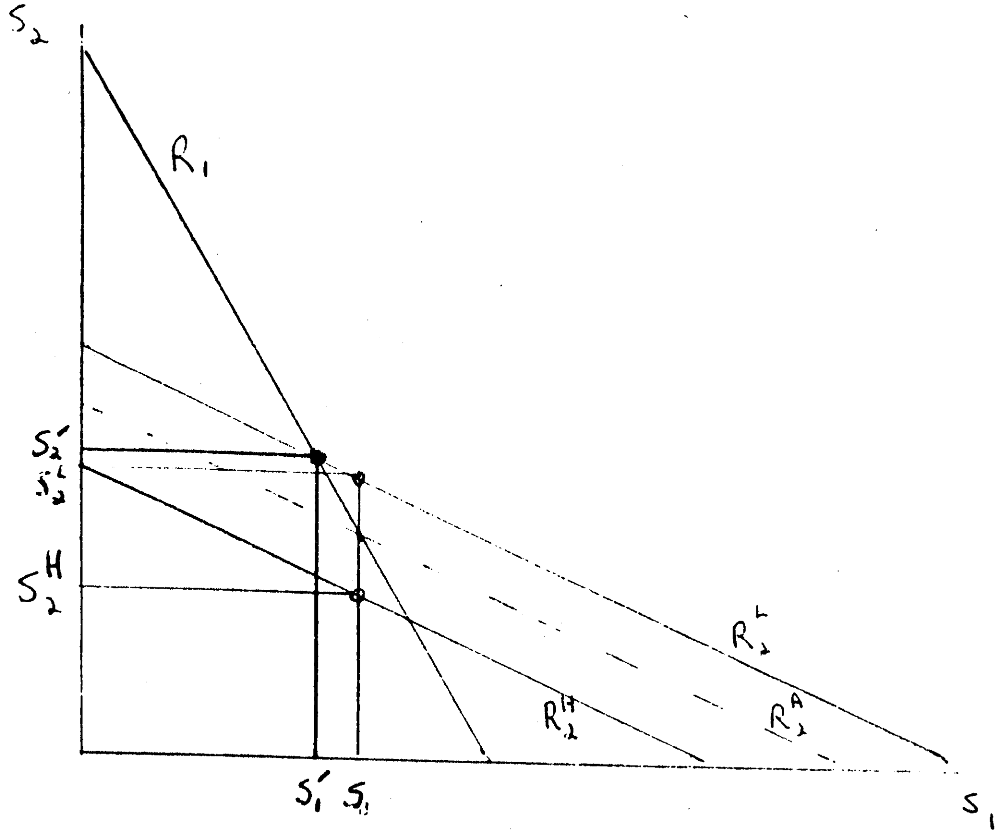


Figure 2

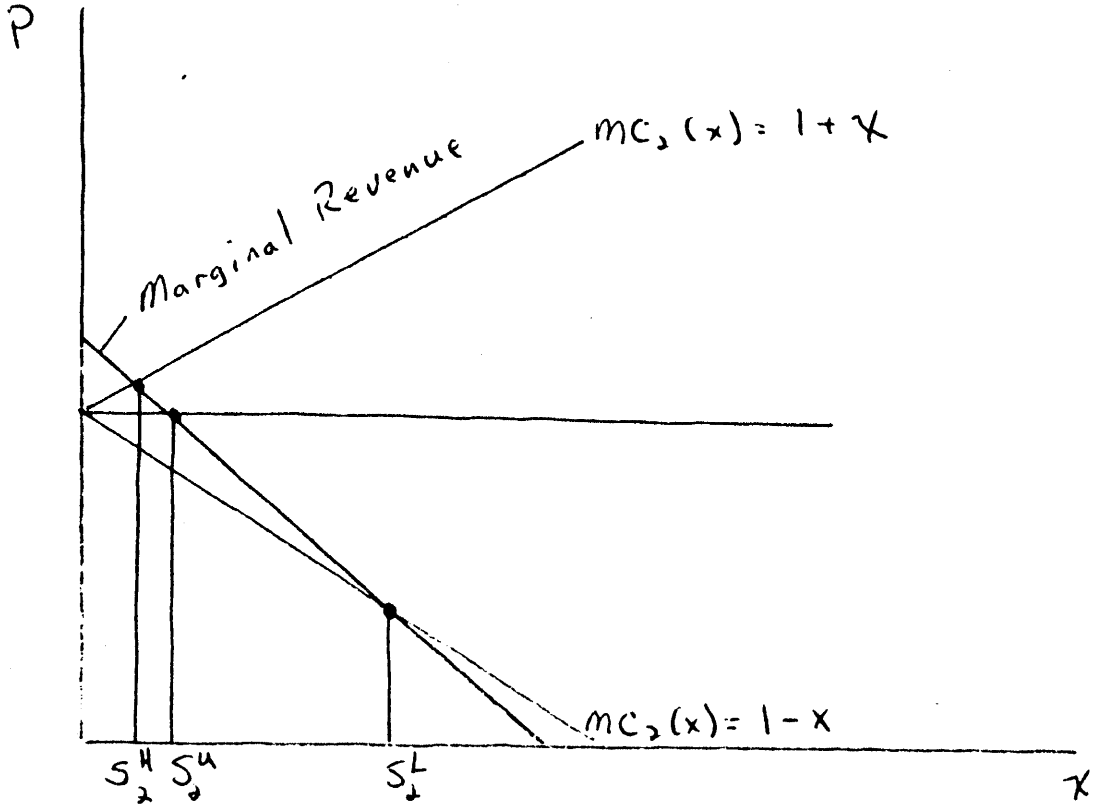
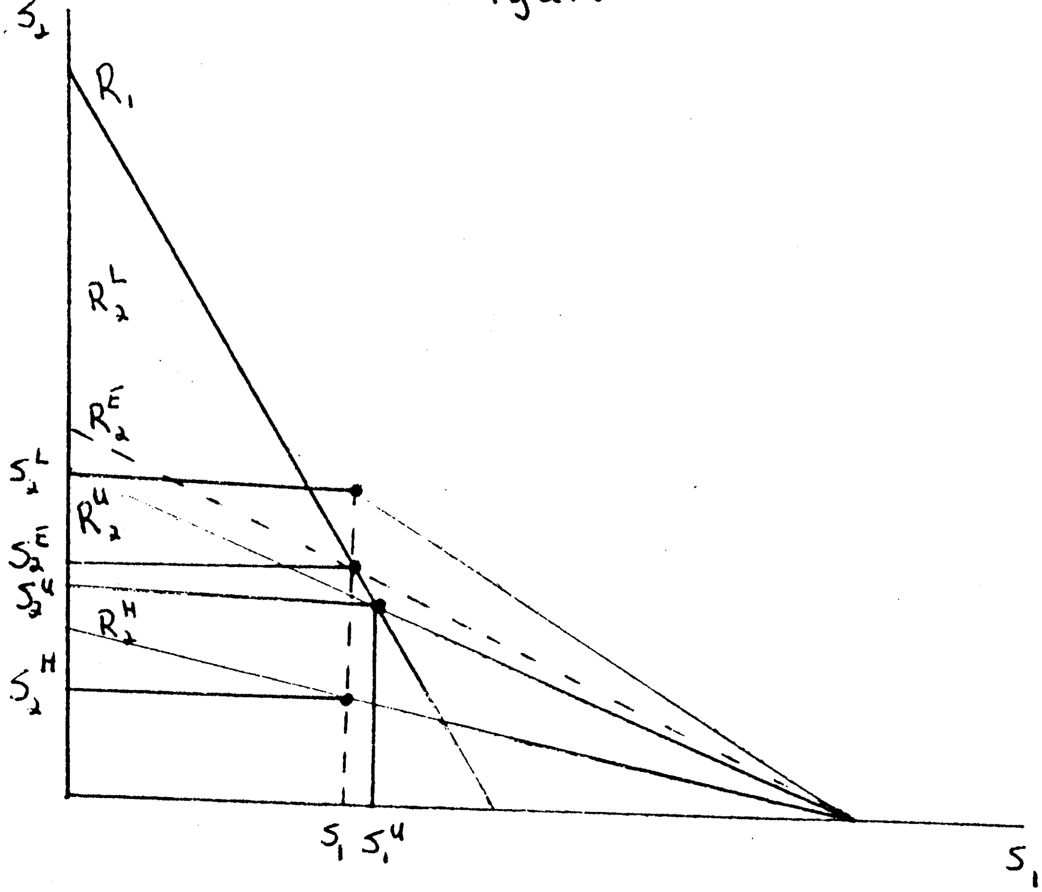


Figure 3



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