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A No-Speculation Theorem for Economies with Intertemporal Asset Markets*#

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ABSTRACT

This paper considers the possibility of speculation in economies with intertemporal asset markets. The main theorem, called the no-speculation theorem, asserts that the speculative premiums defined by Harrison and Kreps are ruled out in economies consisting of a single risk-averse (or risk-neutral) individual if his day-by-day consumption/portfolio decisions are based on rational expectations on future prices. To put it differently, rational expectations and homogeneity of the individual traders (to the extent that the so-called "demand-aggregation" is possible) exclude price bubbles. It thus endorses and clarifies the conventional observation. A theorem in dynamic programming relating path-conserving, path-unimprovable, and path-optimal strategies enables us to present the no-speculation theorem in a very general format and makes the proof straightforward.

1. INTRODUCTION.

Equilibrium valuation models for financial assets must deal with two kinds of complexities. One is the complexity associated with aggregating individual consumption/portfolio decisions to a set of total demand functions. If individuals are very different in their preferences and endowments, it becomes extremely difficult to solve the market-clearing condition analytically with respect to prices. The Sharpe-Lintner-Mossin capital asset pricing model and its variations[1/] resolve this problem by assuming "sufficient" homogeneity among investors. If each inventor evaluates any portfolio of assets only in the mean and the variance of the portfolio's rate of return, then, as is well-known, the market portfolio must be mean-variance efficient in equilibrium. That is, one can construct a "surrogate" individual representing the investor universe such that the market portfolio is the equilibrium choice of this hypothetical agent. More generally, if individual demands can be aggregated in such a way that the total demand function for each asset coincides with the individual demand function of some hypothetical surrogate investor, the issue of solving the equilibrium prices turns out simply as the one of finding the supporting hyperplane of his preference surface at the endowment point. As one can easily speculate, sufficient homogeneity in investors' time and risk preferences as well as in their probabilistic beliefs must be assumed for such an aggregation to be possible. Wilson [1967], [1968] and Rubinstein [1974] investigated this problem and provided several versions of the so-called "aggregation theorem".

The second complexity is caused by intertemporal considerations. Unless the market is "complete" in the Arrow-Debreu sense, investors have incentives to retrade in the subsequent markets. This is in fact what we observe in reality. Multiperiod equilibrium models with incomplete markets are significantly harder than single period models, not only in that one must solve dynamic optimization problems, but also in that one must "endogeneize" investors' expectations on future prices of assets. Radner [1972] gives a general formulation of such an equilibrium model.

The latter burden disappears if one could assume that all investors are "fundamentalists". If investors hold financial assets only for the purpose of receiving interests and dividends and never "speculate" to earn future capital gains, the problem degenerates essentially to equilibrium valuation in a one-shot market. At each moment of time portfolio decisions of investors are merely based on the forecast of future interest and dividend payments: they have no concern about future price movements of their portfolios. Thus the equilibrium model need not "endogeneize" price expectations. One can simply investigate the interaction of investors' preferences over uncertain but exogenously given cash streams to determine equilibrium prices.

We show in this paper that this simplifying assumption is perfectly compatible with an economy consisting of more active, fully rational investors if investors are homogeneous to the extent that the aggregation theorem applies.

We consider a multiperiod exchange economy consisting of a single representative consumer/investor. The <u>fundamental equilibrium</u> of this economy is defined to be a time-sequence of competitive equilibria under the assumption that the investor acts as a fundamentalist. The <u>speculative equilibrium</u> (or the <u>Radner equilibrium</u>) of this economy is defined to be a time-sequence of equilibria under the assumption that the investor is fully active, basing his day-by-day portfolio decisions upon rational expectations on the uncertain future price movements. We relate these two equilibria to the concepts of a path-unimprovable and a path-conserving strategy, which were proposed in Kobayashi [1986]. We apply the main propositions of that paper to show that the equilibrium prices of the two equilibria are identical under a mild set of assumptions on the investor's utility function.

This result has relevance to the issue of the "speculative bubbles" discussed by Harrison and Kreps [1978]. Assuming risk-neutral investors and a Markov chain structure in the probabilistic movements of dividends, they showed that stock prices associated with the speculative equilibrium are higher than those associated with the fundamental equilibrium unless all investors agree on the assessment of the transition probabilities of the dividends. They call the difference between the two prices the "speculative premium". In another paper Tirole [1982] showed that differential information alone does not generate this speculative premium: namely, disagreement on investors' prior assessments of "states" are indispensable for positive speculative premium to arise. From this perspective, the no-speculation theorem of this paper extends their findings to economies with risk-averse investors. It asserts that speculative bubbles in the sense of Harrison and Kreps never arise if the investors' preferences and probability assessments are homogeneous in such a way that an aggregation theorem applies to their preference structure.

2. DESCRIPTION OF THE ECONOMY.

Consider a one-good, pure exchange economy consisting of a single individual. The good in this economy is non-durable. It is produced in a number of different firms; an asset is a claim to all or part of the output of one of these firms. Markets for the good and for the assets open at a discrete sequence of times. Productivity in each firm fluctuates stochastically through time, so that equilibrium asset prices will fluctuate as well. The equilibrium analysis concerns the relationship between the exogenously determined productivity changes and market determined movements in asset prices.

We assume that economic activity takes place at discrete <u>dates</u> $t = 0, 1, \cdots$. Let $\{\xi_t : t \geq 0\}$ be a stochastic process of "the state" which determines the movements of the economy. Denoting $\xi^t := (\xi_0, \cdots, \xi_t)$, the transition probabilities of states are given by $Q_t(\xi_{t+1} | \xi^t)$ for $t = 0, 1, \cdots$. To simplify the exposition we assume that ξ_t, for each $t \geq 0$, lies in a discrete <u>state space</u> X_t.

Let n be the number of firms in this economy. For $i = 1, \cdots, n$ and $t = 0, 1, \cdots$, let $y_{it}(\xi^t)$ be the output of firm i at date t if ξ^t prevails until date t. Let $\underline{\underline{y}}_t(\xi^t) := (y_{1t}(\xi^t), \cdots, y_{nt}(\xi^t))$ be the output vector at date t. Thus, production is entirely exogenous.

Ownership in these firms is determined at each date in competitive "asset" market. Each firm has outstanding one unit of perfectly divisible equity share. A share entitles its owner (who holds the share from date t-1 until t) to all of the firm's output at date t. The individual is initially endowed with one unit of each firm's share. We assume that he has no "earned income", that is, he derives income only from the assets. This simplifies the exposition, but nothing changes if we assume otherwise.

Shares as well as the consumption good are traded after "dividends" are paid to the shareholder. The consumption good is taken to be the <u>numeraire</u>: the asset prices are measured in units of the good. A <u>price system</u> is a nonnegative vector stochastic process $\{\underline{P}_t(\xi^t): t \geq 0\}$, in which $\underline{P}_t(\xi^t) := (P_{1t}(\xi^t), \cdots, P_{nt}(\xi^t))$. We write $\underline{P}$ for $\{\underline{P}_t : t \geq 0\}$.

At each date $t \geq 0$ the consumer chooses his consumption c_t and his portfolio of assets $\underline{s}_t := (s_{1t}, \cdots, s_{nt})$, where s_{it} represents the number of shares of asset i held from date t until t+1. For each t, c_t must be in a set Γ_t, which is an interval in the real line $\underline{R}$. The portfolio vector $\underline{s}_t$ must be in a set Σ_t, which is a convex set in $\underline{R}^n$ containing the vector $\underline{1} := (1, \cdots, 1)$ in its interior.[2/] Naturally, we assume that for each t and ξ^t the total output of the economy $\sum_{i=1}^n y_{it}(\xi^t)$ lies in Γ_t.

The consumer has a von-Neumann-Morgenstern utility function $U: \Gamma \to \underline{R}$, where $\Gamma := X_{t=0}^{\infty} \Gamma_t$. The objective for his consumption and investment decisions is to maximize the expected value of this utility function. We assume that $U(c_0, c_1, \cdots)$ is concave and once differentiable in c_t for each $t \geq 0$.

An additional assumption is imposed on the utility function U.[3/] For T = 0, 1, ⋯ define $\bar{U}^T$ and $\underline{U}^T$ on Γ by

$$\bar{U}^T(c) := \sup\{U(c') \mid c' \in \Gamma,\ c^T(c') = c^T\},$$

and

$$\underline{U}^T(c) := \inf\{U(c') \mid c' \in \Gamma,\ c^T(c') = c^T\},$$

where $c := (c_0, c_1, \cdots)$, $c^T := (c_0, c_1, \cdots, c_T)$, and $c^T(c')$ denotes the projection of c' from Γ to $X_{t=0}^T \Gamma_t$. The function $\bar{U}^T(c)$ ($\underline{U}^T(c)$) measures the overall utility given $c^T = (c_0, \cdots, c_T)$ with the most "optimistic" ("pessimistic") estimate of the subsequent consumption after date T.

It should be obvious that

(a) $\bar{U}^0(c) \geq \bar{U}^1(c) \geq \cdots \geq U(c)$, and

(b) $\underline{U}^0(c) \leqq \underline{U}^1(c) \leqq \cdots \leqq U(c)$.

We say that U is <u>upper convergent</u> if $\lim_{T\to\infty}\bar{U}^T(c) = U(c)$ for all $c \in \Gamma$; U is <u>lower convergent</u> if $\lim_{T\to\infty} \underline{U}^T(c) = U(c)$ for all $c \in \Gamma$; U is <u>convergent</u> if U is both upper- and lower convergent. In this paper we assume that U is convergent.

If the consumer's time horizon, say, T, is finite, the value of U(c) only depends on $c^T(c)$. Therefore U is trivially convergent in this case. Also, if U is time-additive in the form $U(c) = \sum_{t=0}^{\infty} U_t(c_t)$ and $U_t(c_t)$ is bounded both from above and from below for all $t \geq 0$, then U is convergent.[4/]

3. DEFINITION OF EQUILIBRIA.

We now define two concepts of equilibria for the economy. Both concepts require that, at each date and at each history of the state until that date, if the consumer currently holds the market portfolio of assets his optimal decision is not to reshaffle his portfolio and to consume all of the current dividends; that is, the prevailing prices are market clearing. The two concepts differ in the assumption concerning the consumer's future strategy. Our first concept of equilibrium assumes that the portfolio of assets chosen today will be maintained throughout the time horizon. In contrast, our second concept of equilibrium assumes that the consumer will act optimally in future dates. If the prices of some of the assets appreciate sufficiently, he may sell them to realize capital gains. The second concept requires market-clearing at each date even if the consumer incorporates these future possibilities in choosing his current consumption and portfolio of assets.

Given a price system $\mathbf{P}$ the consumer's multiperiod decision problem corresponding to the two equilibrium concepts can be given a dynamic programming formulation. Define the *initial history* by $h_0 := (\mathbf{1}, \xi_0)$. It consists of the vector of endowed shares and the initial state. Let H_0 be the space of h_0 and call it the *initial history space*. For each $t = 1, 2, \cdots$ define the *partial history* by $h_t := (c^{t-1}, \mathbf{s}_{t-1}, \xi^t)$ and let H_t denote the *partial history space*. Thus, the partial history at date t records the sequence of past consumption until date t-1, the vector of shares held from date t-1 until t, and the sequence of the state until date t.

If the initial history is $h_0 = (\mathbf{1}, \xi_0)$, the set of *feasible actions at a price system* $\mathbf{P}$ for date 0 is the set $A_0(h_0; \mathbf{P})$ defined by

$$A_0(h_0; \mathbf{P}) := \{(c_0, \mathbf{s}_0) \mid c_0 \in \Gamma_0, \mathbf{s}_0 \in \Sigma_0,$$

$$c_0 + \mathbf{P}_0(\xi_0) \cdot (\mathbf{s}_0 - \mathbf{1}) \leqq \mathbf{y}_0(\xi_0) \cdot \mathbf{1}\}.$$

Here the symbol "$\cdot$" indicates the inner-product on $\mathbf{R}^n \times \mathbf{R}^n$. For $t \geqq 1$, if the partial history is $h_t = (c^{t-1}, \mathbf{s}_{t-1}, \xi^t)$, the set of *feasible actions at* $\mathbf{P}$ for date t is the set $A_t(h_t; \mathbf{P})$ defined by

$$A_t(h_t; \mathbf{P}) := \{(c_t, \mathbf{s}_t) \mid c_t \in \Gamma_t, \mathbf{s}_t \in \Sigma_t,$$

$$c_t + \mathbf{P}_t(\xi^t) \cdot (\mathbf{s}_t - \mathbf{s}_{t-1}) \leqq \mathbf{y}_t(\xi^t) \cdot \mathbf{s}_{t-1}\}.$$

A *strategy* for the consumer is a consumption plan and a rule for holding the n assets. This is represented by $\{c_t(h_t) \mid t \geqq 0, h_t \in H_t\}$ and $\{\mathbf{s}_t(h_t) \mid t \geqq 0, h_t \in H_t\}$. A strategy is *feasible at a price system* $\mathbf{P}$ if, for each $t = 0, 1, \cdots$, $(c_t(h_t), \mathbf{s}_t(h_t)) \in A_t(h_t; \mathbf{P})$. The set of strategies which are feasible at $\underline{\underline{P}}$ is denoted by $\Pi(\underline{\underline{P}})$, and a strategy in $\Pi(\underline{\underline{P}})$ is generically written as π.

The evolution of history is described as follows. If the partial history at date t is $h_t = (c^{t-1}, \underline{\underline{s}}_{t-1}, \xi^t)$ and an action $(c_t, \underline{\underline{s}}_t) \in A_t(h_t; \underline{\underline{P}})$ is taken, the partial history at date t+1 is $h_{t+1} = (c^t, \underline{\underline{s}}_t, \xi^{t+1})$ with probability $Q_t(\xi_{t+1}|\xi^t)$, in which $c^t = (c^{t-1}, c_t)$ and $\xi^{t+1} = (\xi^t, \xi_{t+1})$. For each strategy $\pi \in \Pi(\underline{\underline{P}})$ and partial history $h_t \in H_t$, *conditional probabilities* $P^\pi(\cdot|h_t)$ and *conditional expectations* $E^\pi(\cdot|h_t)$ *using* π (at date t and thereafter) *given* h_t are constructed on H_t from the transition probabilities in the usual fashion. *Unconditional probabilities* $P^\pi(\cdot)$ using π are constructed in the same manner.

For each t and h_t, the *expected utility using* π *given* h_t is defined by

$$v_t(\pi, h_t) := E^\pi[U(c_0, c_1, \cdots)|h_t],$$

and the *optimal expected utility given* h_t is defined by

$$f_t(h_t; \underline{\underline{P}}) := \sup_{\pi \in \Pi(\underline{\underline{P}})} v_t(\pi, h_t).$$

Note that $v_t(\pi, h_t)$ is the expected utility of the *entire* consumption sequence given the consumption sequence from date 0 until t-1. It is not the expected additional utility accruing after date t, which is more conventional in the dynamic programming literature but makes sense only if U is time-additive. Given π the value of $v_t(\pi, h_t)$ depends on h_t only through (c^{t-1}, ξ^t) but not through $\underline{\underline{s}}_{t-1}$. In contrast, the optimal expected utility $f_t(h_t; \underline{\underline{P}})$ depends on $\underline{\underline{s}}_{t-1}$, since $\underline{\underline{s}}_{t-1}$ restricts the feasible set of actions $A_t(h_t; \underline{\underline{P}})$ at date t.

Given a strategy one can distinguish between two classes of partial histories; those which will be reached with positive probabilities and those which will never be reached. The former class of partial histories plays an important role in the definition of the equilibria. For a strategy $\pi \in \Pi(\underline{\underline{P}})$ we denote the former class by $\mathcal{R}(\pi)$ and call it the set of all π-reacheable partial histories; namely,

$$\mathcal{R}(\pi) := \bigcup_{t=0}^{\infty} \{h_t \in H_t | P^{\pi}(h_t) > 0\}.$$

Let π^H denote the strategy which at every date and on every contingency instructs the consumer to consume all of the current dividends and not to reshaffle his asset portfolio; namely, if $h_t = (c^{t-1}, \underline{\underline{s}}_{t-1}, \xi^t)$ then

$$c_t^H(h_t) = \underline{\underline{y}}_t(\xi^t) \cdot \underline{\underline{s}}_{t-1}$$

and

$$\underline{\underline{s}}_t^H(h_t) = \underline{\underline{s}}_{t-1} \ .$$

Clearly, this strategy is feasible at any price system $\underline{\underline{P}}$. We call π^H the "buy and hold" strategy. The two types of equilibria are defined in relation to π^H.

DEFINITION 1. A price system $\underline{\underline{P}}$ is said to be a fundamental equilibrium if for any $t \geq 0$ and $h_t \in \mathcal{R}(\pi^H)$,

$$(c_t^H(h_t), \underline{\underline{s}}_t^H(h_t)) \in \underset{a_t \in A_t(h_t;\, \underline{\underline{P}})}{\arg\max} \ E^{a_t}[v_{t+1}(\pi^H, h_{t+1}) | h_t].^{5/} \qquad (3.1)$$

DEFINITION 2. A price system $\underline{\underline{P}}$ is said to be a speculative equilibrium if for any $t \geq 0$ and $h_t \in \mathcal{R}(\pi^H)$,

$$(c_t^H(h_t),\ \underline{\underline{s}}_t^H(h_t)) \in \underset{a_t \in A_t(h_t;\ \underline{\underline{P}})}{\arg\max}\ E^{a_t}[f_{t+1}(h_{t+1};\ \underline{\underline{P}})|h_t]. \tag{3.2}$$

It is essential that for an equilibrium $\underline{\underline{P}}$ we require (3.1) or (3.2) only at all $h_t \in \mathcal{R}(\pi^H)$, i.e., at all π^H-reacheable partial histories. To take DEFINITION 1, for example, at each date t and given the history of the state ξ^t, we essentially look for a supporting hyperplane (or the price vector $\underline{\underline{P}}_t(\xi^t)$) at the endowment point ($c_t = \underline{\underline{y}}_t(\xi^t)\cdot\underline{\underline{1}}$, $\underline{\underline{s}}_t = \underline{\underline{1}}$). If we were to require (3.1) at every partial history $h_t \in H_t$, the equilibrium would not exist, since it is equivalent to looking for a price vector which supports the upper level set of the expected utility function not only at the endowment point but also at other points $(c_t,\ \underline{\underline{s}}_{t-1})$.[6/]

4. THE NO-SPECULATION THEOREM.

The two types of equilibria in the previous section can be given an alternative representation by using the concepts of path-unimprovable and path-conserving strategies in Kobayashi [1986].

Given a general dynamic programming problem with dates t=0, 1, 2, $\cdots$, a strategy space Π, partial history spaces $\{H_t\}$, conditional expectation operators $\{E^\pi(\cdot|h_t)\}$, the set of reacheable partial histories $\{\mathcal{R}(\pi)\}$, expected utility functions $\{v_t(\pi,\ h_t)\}$, and optimal expected utility functions $\{f_t(h_t)\}$, these concepts are defined as follows:

(a) A strategy $\pi \in \Pi$ is said to be <u>path-unimprobable</u> if it satisfies for all $t \geq 0$ and $h_t \in \mathcal{R}(\pi)$,

$$v_t(\pi,\ h_t) = \sup_{\pi' \in \Pi} E^{\pi'}[v_{t+1}(\pi,\ h_{t+1})|h_t]. \tag{4.1}$$

(b) A strategy $\pi \in \Pi$ is said to be <u>path-conserving</u> if it satisfies for all $t \geq 0$ and $h_t \in \mathcal{R}(\pi)$,

$$f_t(h_t) = E^{\pi}[f_{t+1}(h_{t+1}) | h_t]. \qquad (4.2)$$

We also say that a strategy $\pi \in \Pi$ is <u>path-optimal</u> if it satisfies for all $t \geq 0$ and $h_t \in \mathcal{R}(\pi)$,

$$v_t(\pi, h_t) = f_t(h_t). \qquad (4.3)$$

The supremand in (4.1) is the expected utility given h_t of using π' at date t and using π after date t+1. So (4.1) implies that strategy π cannot be improved upon by deviating only at date t from the strategy π. Condition (a) requires that this property holds at all dates and given any partial history which is reacheable by the strategy π.

The right-hand-side of (4.2) is the expected utility given h_t of using π at date t and tracing the optimal path afterwards. Condition (b) requires that this expected utility equals the optimal expected utility at all dates and at all partial histories which are reacheable by the strategy π.

Eq. (4.3) implies that by using strategy π at and after date t one can attain the optimal expected utility given h_t. It should be clear that a strategy π is path-optimal if and only if it is optimal for the entire problem, i.e., $v_0(\pi, h_0) = f_0(h_0)$.

Now we relate the two equilibria of the previous section to these concepts of dynamic programming. Eq. (3.1) is rewritten as

$$v_t(\pi^H, h_t) = \max_{a_t \in A_t(h_t;\ \underline{\underline{P}})} E^{a_t}[v_{t+1}(\pi^H, h_{t+1}) | h_t]$$

$$= \max_{\pi \in \Pi(\underline{\underline{P}})} E^{\pi}[v_{t+1}(\pi^H, h_{t+1}) | h_t].$$

Hence, DEFINITION 1 can be rephrased as:

DEFINITION 1′: A price system $\underline{\underline{P}}$ is a fundamental equilibrium if the strategy π^H is a path-unimprovable strategy given $\underline{\underline{P}}$.

To rephrase DEFINITION 2 we use the well-known <u>optimality equation</u>, which states[7]

$$f_t(h_t;\ \underline{\underline{P}}) = \max_{\pi \in \Pi(\underline{\underline{P}})} E^{\pi}[f_{t+1}(h_{t+1};\ \underline{\underline{P}}) \mid h_t].$$

Using this relation (3.2) is rewritten as

$$\begin{aligned} E^{\pi^H}[f_{t+1}(h_{t+1};\ \underline{\underline{P}}) \mid h_t] &= \max_{a_t \in A_t(h_t;\ \underline{\underline{P}})} E^{a_t}[f_{t+1}(h_{t+1};\ \underline{\underline{P}}) \mid h_t] \\ &= \max_{\pi \in \Pi(\underline{\underline{P}})} E^{\pi}[f_{t+1}(h_{t+1};\ \underline{\underline{P}}) \mid h_t] \\ &= f_t(h_t;\ \underline{\underline{P}}). \end{aligned}$$

Thus DEFINITION 2 is rephrased as:

DEFINITION 2′: A price system $\underline{\underline{P}}$ is a speculative equilibrium if the strategy π^H is a path-conserving strategy given $\underline{\underline{P}}$.

Given these interpretations to the two types of equilibria and the main propositions in Kobayashi [1986], it is straightforward to prove the following theorem[8].

THE NO-SPECULATION THEOREM. *The following three conditions are equivalent*:

(1) *$\underline{\underline{P}}$ is a fundamental equilibrium;*

(2) *$\underline{\underline{P}}$ is a speculative equilibrium;*

(3) *π^H is a path-optimal strategy given $\underline{\underline{P}}$.*

PROOF. Proposition 2 of Kobayashi [1986] asserts that any path-optimal strategy is both path-unimprovable and path-conserving. So (3) implies (1) and (2). Since the utility function U is convergent, corollary to proposition 5 implies that any path-conserving strategy is path-optimal. Hence (2) implies (3). We assumed that U: $X_{t=0}^{\infty}\Gamma_t \rightarrow \underline{\underline{R}}$ is concave and differentiable in each argument c_t which is an element in a convex set Γ_t. We further assumed that the convex set Σ_t, for each t, contains the vector $\underline{\underline{1}}$ in its interior. Given this and the convergence property of U, corollary to proposition 6 implies that any path-unimprovable strategy is path-optimal. Hence (1) implies (3). ■

Fig. 1 below depicts the no-speculation theorem in more detail. Solid arrows indicate that no assumption is needed to "cross the bridge". The corollary to proposition 5 actually requires that U is upper convergent. This is indicated by the dotted arrow from (2) to (3). To cross from (1) to (3) one actually needs lower convergence, concavity and differentiability of U. One must also note that the set Σ_t of portfolio vectors must contain $\underline{\underline{1}}$ in its interior for this part of the theorem to hold.

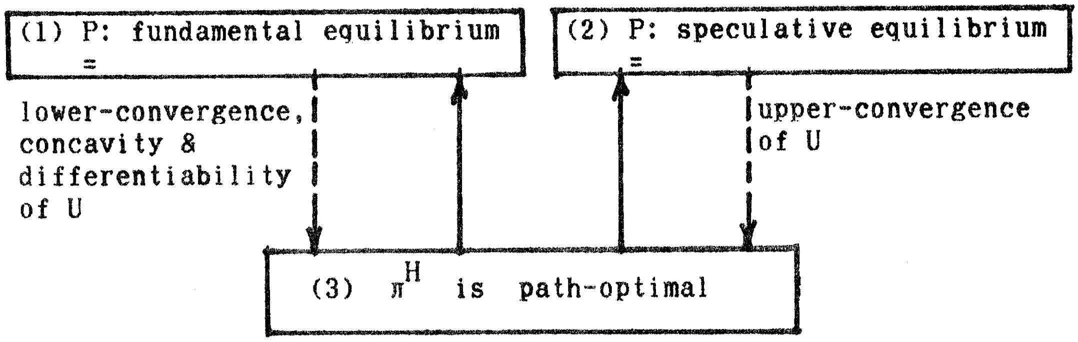

Fig. 1

The no-speculation theorem states that under the imposed assumptions the price system associated with the fundamental equilibrium coincides with the price system associated with the speculative equilibrium. This in turn implies that even if the consumer is a fully rational "speculator", the equilibrium price is the same as the equilibrium price in an economy where he acts as a naive fundamentalist. In other words, the speculative premium in the sense of Harrison and Kreps never arises in a single-individual economy or, more generally, in an economy in which the individuals are homogeneous to the extent that an appropriate form of the aggregation theorem applies. Fig. 1 clarifies the more exact relationship between these equilibria.

5. Topics For Further Study.

The essential assumption for our no-speculation theorem is that of a single-individual economy. Although Wilson and Rubinstein's works make it clear that this assumption does not necessarily require perfect symmetry among the individuals, the exact form of homogeneity which makes the aggregation possible to our model of a multi-period, incomplete-market economy is unknown. This surely serves as an interesting topic for future inquiry.

From a mathematical point of view the no-speculation theorem significantly lessens the burden for the equilibrium valuation of financial securities. It enables one to concentrate his effort on identifying the fundamental equilibria. This is much easier than to investigate the equilibrium condition by solving the stochastic optimization problems, which is required to investigate the fully rational, speculative equilibrium. An example was given in Kobayashi [1983]. One can also apply the technique to the finance literature such as Breeden [1979], Grossman and Shiller [1982] on the intertemporal capital asset pricing model, Cox, Ingersoll and Ross [1985b] on the term structure of interest rates, and more general valuation models in Lucas [1978] and Cox, Ingersoll and Ross [1985a]. They all essentially assume a single-individual economy.

FOOTNOTES

The No-Speculation Theorem of this paper was originally presented in Kobayashi [1983]. The current version, which is significantly more general and relies on a different line of proof, was made possible with a private conversation with David Kreps. The author wishes to acknowledge his suggestion. Any remaining errors, needless to say, are the author's. The preparation of this paper was partially supported by a research grant from the Kikawada Foundation.

1/ Sharpe [1964], Lintner [1965], Mossin [1966]. For the variations of the capital asset pricing model, see for example Elton and Gruber [1981], Chapter 12.

2/ Thus, to hold more than one unit of the share must be admissible.

3/ The following concept of a convergent utility is due to Kreps [1977].

4/ For more examples of upper- and lower convergent utilities, see Kreps, ibid.

5/ Given a strategy π, the expected utility $E^{\pi}[v_{t+1}(\pi^H, h_{t+1})|h_t]$ depends on π only through the action taken at date t given partial history h_t. This motivates the notation $E^{a_t}[v_{t+1}(\pi^H, h_{t+1})|h_t]$.

6/ This is the reason why we must rely on the notions of a path-unimprovable and a path-conserving strategy proposed in Kobayashi [1986]. See footnote 8/.

7/ For the proof, see Kreps, ibid.

8/ Kreps, ibid., deals with the concepts of unimprovable and conserving strategies, which were originally proposed by D. Blackwell and R. E. Strauch, respectively. These concepts require (4.1) or (4.2) to hold at every partial history $h_t \in H_t$; therefore his construction is not applicable to our problem.

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