Equity Premium in a World Without Uncertainty

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Introduction:

Our paper, “The Equity Premium: A Puzzle” demonstrated that the premium for bearing non-diversifiable aggregate risk is small. This came as a surprise to the profession, since the accepted tenet of the neoclassical paradigm was (and perhaps still is) that the observed differences in the rates of return of financial assets, in particular, the large difference between the average returns on corporate equity and T-bills, was a premium for bearing non-diversifiable aggregate risk.

Over the last 20 years or so, attempts to resolve the ‘Equity Premium Puzzle’ have become a major research impetus in finance and economics. Researchers have proposed modifications and creative iterations to explain away the puzzle but with limited success. A majority of these efforts have focused on ways to increase the premium for bearing non-diversifiable risk[1].

In this paper, in contrast to the approaches mentioned above, we take as given the findings in Mehra and Prescott (1985) – that the premium for bearing non-diversifiable risk is small - and examine the extent to which the differences in asset returns can be accounted for by *factors other than aggregate risk.* Our thinking is that a combination of these two approaches will

[1] For an elaboration of these efforts see Mehra (2003) or Mehra and Prescott (2003).

provide a cogent explanation of the observed differences in asset returns– a task that modern Finance theory has, heretofore been unsuccessful in addressing.

Our point of departure is twofold. First, we reexamine the appropriate benchmark risk free rate used in defining the equity premium. Next we incorporate agent heterogeneity and examine its impact on the equity premium.

The Choice of a Riskless Asset:

In the two decades since 'The Equity Premium: a Puzzle' was published, the way we think about economic phenomena has undergone a fundamental change. In our earlier paper we used T-bills as a proxy for a riskless asset that is used by agents to smooth consumption. We now examine the appropriateness of this choice and whether we can come up with a more appropriate measure.

In our model an implicit assumption was that agents use both equity and the riskless asset to *smooth intertemporal consumption*. Only then is it meaningful to compare the returns on the assets. Is this assumption reasonable for T-bills? Do people actually hold T-bills to finance their retirement? Only if this were empirically true would it be reasonable to equate their marginal rate of substitution of consumption to the rate of return on T-bills.

A natural question then is who holds T-bills and do the holders use them to intertemporally smooth consumption in the face of relatively predictable long-term variations in income? A large fraction of the stock of short-term U.S. T-bills are held by foreign central banks and are used to smooth exchange rates. These central banks *are not* using these assets to equate the marginal utility of consumption today to the expected discounted marginal utility of consumption tomorrow.

U.S. households also hold large amounts of short-term U.S. T-bills and close substitutes. However, these short-term assets with low expected return are held, in significant part, for reasons *other than saving for retirement.* These include:

Liquidity:

A part of these assets are held for liquidity purposes and as precautionary balances. The later serve as a substitute for insurance against idiosyncratic risk, which is both large and very costly to insure against because of the associated moral hazard. When this is the case, in the household's maximization problem there is Lagrange multiplier that is not zero and as a result the expected returns are less than that predicted by a theory that abstracts from the holding of liquid assets as a substitute for insurance against idiosyncratic risk.

Transaction balances:

There are large transaction costs associated with moving into and out of high yielding assets. A large component is record keeping. With assets whose price varies, there are capital gains or loses with *every sale* of these assets. Every gain or loss must be reported on the annual tax form. This is why money market accounts hold debt securities until they mature. Hence there are no capital gains or loses associated when an individual buys or sells shares in his money market account. All that the money market mutual fund need report to its shareholders is an annual statement of the interest they received on the 1099 form. This is a major reason that sizable quantities of currency and non-interest bearing demand deposits are held. We note that Ml is about 15 percent of GNP.

Brokerage charges were (and perhaps continue to be for some) another significant cost associated with getting into and out of higher yielding securities such as stocks and bonds. It is true that discount brokers now exist, but gaining the prerequisite knowledge is costly in terms of time. If the holding period is short, even for a risk neutral investor it makes no sense for the investor to buy and later sell stocks unless the expected return on stocks is huge.

It is interesting to look at the assets held by American households. The four big asset holding categories of households are real estate holdings, business capital, both tangible and intangible, and debt assets. Although there are large amounts of debt assets held, most of these are in the form of pension fund and life insurance reserves. Some are in the form of demand deposits for which free services are provided. There is very little government debt and much of this is in the form of savings bonds that people gift to their grandchildren.

Thus much of intertemporal savings done in the form of debt assets is in assets such as annuities and mortgage debt held in retirement accounts and as pension fund reserves. *Other assets, not T-bills are held to finance consumption when old.*

Once we learned more about who held short-term liquid debt and how much of this debt is held, we concluded that the assumption that the return on this debt is equal to the marginal rate of substitution for an important group of agents was not a reasonable one.

What then would be a better proxy for a risk free debt instrument? We believe that an inflation indexed default free bond portfolio that has a similar duration to a well-diversified equity portfolio would be a reasonable choice. For most of the 20th century equity has had an implied duration of about 25 years so a portfolio of TIPS of a similar duration would be a reasonable proxy.

We realize that such an asset has only relatively recently (1997) been introduced in the US capital markets. This makes it difficult to get accurate estimates of the mean return on this asset class. Nonetheless as argued above we feel that the use of T-bills as a proxy is inappropriate and a better (though imperfect) proxy would be to use the return on indexed mortgages guaranteed by Ginnie Mae or issued by Fannie Mae.

Yet another option would be to use the effective return on annuities. For a 65 year old individual, the annuity premium (mortality premium) is about 4-5% above the rate on long term government debt.

Using these indexed default free securities or annuities as a benchmark will reduce the magnitude of the equity premium from about 6% to something in the range of 1% - 3.5%, depending on the benchmark used.

Adjustments to the Historical Average Returns on Debt

Quite independent of the above arguments, we note that the estimated average return on debt assets in U.S (including T-bills) over the period 1926-2000 is biased downwards. **Table 1** shows that the return on debt securities during the 1941-54 period was considerably lower than their long term average value. This is directly attributable to interest rate regulations during this period and serves as a reminder that governments can pursue regulatory policies that result in negative interest rates over an extended period of time. Clearly these rates have little to do with the with the agents' marginal rate of substitution that would be inferred were there no regulations. Such regulatory periods should be excluded in estimating the long term average rates on debt securities. The third column in Table 2 shows how the conventionally used numbers (in column two) change when the 1941-54 period is excluded. The estimated average rates increase by about 1%.

Table 1

U.S. Inflation Adjusted Average Return on Debt

Period	1926-2000	1926-40 + 1955-2000	1941-1954
U.S. Treasury bills	0.8%	1.8%	-3.6%
Intermediate-Term Government Bonds	2.4%	3.6%	-2.7%
Long-Term Government Bonds	2.7%	3.8%	-1.9%
Long-Term Corporate Bonds	3.0%	4.1%	-1.9%

Source: Ibbotson *2001*

In the case of T-bills a further adjustment needs to be made to the returns in the 1930's. During that period, in some states, T-bills were exempt from personal property taxes while cash was not. This created an excess demand for the T-bills and they sold at a premium. Again these rates on return have little to do with the marginal rate of substitution of consumption over time. The effect of these adjustments is to further reduce the magnitude of the equity premium.

To summarize, equity securities are held to save for retirement and smooth intertemporal consumption; using a comparable riskfree asset as a bench mark -- one that is used for saving significantly reduces the equity premium.

The Impact of Agent Heterogeneity and Intermediation Costs

Next, we incorporate agent heterogeneity and intermediation costs and examine their impact on the equity premium. In the neoclassical growth model, there is no borrowing and lending, as all economic agents are assumed to be homogenous. The shadow price of consumption at date t+1 in terms of consumption at date t is the price such that borrowing and lending is zero.

In reality, however, there is a large amount of borrowing and lending between households, in particular amongst the over 50 year olds. Most of this is intermediated through financial intermediaries such as banks and the government. This leads to a divergence between borrowing and lending rates that can lead to an equity premium even in a world of certainty.

We capture this intra generational borrowing and lending by considering a heterogeneous agent economy without *aggregate uncertainty.* The only uncertainty that agents face is idiosyncratic uncertainty about the length of their lifetime remaining *after* retirement. Agents enter the retirement phase of their life cycle with wealth saved during their working years. They have identical preferences for consumption, however, they differ with respect to their preference for bequests. In equilibrium, this leads the agents with a "low intensity" for bequests to buy an annuity (lend) and hold no equity. While those with a strong preference for bequests hold equity and borrow. This borrowing is done either directly, by issuing mortgages to finance owner occupied housing or indirectly, by owning partially debt financed rental properties through direct or limited partnerships or REITS.

Since agents who hold equity are also the borrowers, in a world of certainty the return on equity will be equal to the household borrowing rate. The household lending rate is lower than the borrowing rate; the difference reflects the amount of intermediation costs, which we calibrate with the help of statistics from National Income and Product Accounts.

Since government debt is not intermediated it will be priced such that its return is equal to the lending rate. Hence in this economy, the difference between the return on equity and government securities – the equity premium-- will be positive.

The Economy

We analyze an overlapping generations economy in steady state. Each period a set of agents of measure one enter the economy. We term the working years as stage one of the life cycle. All agents work for the same number of years and retire. The wealth of these agents at retirement may differ for a number of reasons. Some may have received bequests during their working years; they may have been endowed with differing amounts of human capital and may have made different occupational choices – leading to differential wealth at retirement. In this economy the wealth that individuals have when they retire is a *sufficient statistic* for their decisions during stage one of the life cycle.

At retirement agents face idiosyncratic uncertainty about the length of their remaining lifetime. Their lifetimes are exponentially distributed. The probability of dying in any given period is a constant δ resulting in a mean life expectancy of $1/\delta$ years.

During their retirement years, agents order their preferences by

$$\sum_{t=0}^{\infty} \beta^t (1-\delta)^t \log c_t + \alpha \sum_{t=1}^{\infty} \beta^t (1-\delta)^{t-1} \delta \log b_t$$

Where t=0 is the retirement date, β is the subjective discount factor, c_t is the period consumption, b_t is the bequest, which is given at time of death and α is a parameter that governs the strength of bequest motive.

The Optimal No Annuity Strategy

The optimal consumption and bequest strategy for an agent with a given α and wealth w at retirement, in the absence of annuity markets, is obtained by choosing c_t and b_t that solves the following post-retirement utility maximization problem:

$$
\begin{aligned}
&max \ \{ \sum_{t=0}^{\infty} \beta^t (1-\delta)^t \log c_t + \alpha \sum_{t=1}^{\infty} \beta^t (1-\delta)^{t-1} \delta \log b_t \} \\
&subject \ to \\
&w_{t+1} = (w_t - c_t)(1 + r_e) \\
&b_t = w_t \quad t = 1, 2.... \\
&w_0 = w
\end{aligned}
$$

Where r_e is the return on equity.

This can be solved using recursive methods. The associated functional equation is

$$
\begin{aligned}
v(w) = max\{\log c + \delta\,\alpha\,\beta \log b + (1-\delta)\,\beta\, v(w')\} \\
s.t. \quad c + \frac{w'}{1 + r_e} \le w
\end{aligned}
$$

It is straightforward to show that the value function (and hence the maximum utility) of the form:

$$v(w) = a + \theta \log w$$

where

$$\theta = \frac{1+\alpha\beta\delta}{1-(1-\delta)\beta}$$

and

$$a = \frac{A\log A - (1+A)\log(1+A) + A\log(1+r_e)}{1-(1-\delta)\beta}$$

with $$A = \frac{\alpha\beta\delta + (1-\delta)\beta}{1-(1-\delta)\beta}$$

The optimal consumption and bequest strategies are:

$$c_t = w_t \,/\, (1+A)$$

$$b_t = w_t$$

The Optimal Annuity Strategy

In the presence of annuity markets (and no equity markets) the agent's optimal consumption and bequest strategies can be characterized as a solution to the following problem:

$$\begin{aligned} &max\ \{ \sum_{t=0}^{\infty} \beta^t (1-\delta)^t \log c_t + \alpha \sum_{t=1}^{\infty} \beta^t (1-\delta)^{t-1} \delta \log b_t \} \\ &subject \text{ to} \\ &\qquad \sum_{t=0}^{\infty} \frac{\beta^t (1-\delta)^t c_t}{(1+r)^t} + \sum_{t=1}^{\infty} \frac{\beta^t (1-\delta)^{t-1} \delta b_t}{(1+r)^t} = w \end{aligned}$$

r is the lending rate

Here the law of large numbers is used to determine the payments each period in formulating the budget constraint. This can be solved using standard non-recursive techniques. The resulting optimal consumption and bequest strategies in this instance are

$$c_t = \overline{c}\beta^t(1+r)^t w$$

with $$\overline{c} = \frac{1-\beta(1-\delta)}{1+\alpha\beta\delta}$$

and $$b_t = \frac{\alpha}{1+r}c_t \qquad \text{t=1,2 ...}$$

The maximum utility that can be achieved in this case is

$$V(w) = \frac{(1+\alpha\delta)(1-\delta)\beta}{(1-(1-\delta)\beta)^2}\log(\beta(1+r)) - \lambda w\log(\lambda) + \alpha\delta\log(\alpha/(1+r))\frac{\beta(1-\delta)}{1-(1-\delta)\beta}$$

with $\lambda = \dfrac{1+\alpha\beta\delta}{(1-\beta(1-\delta))w}$

We define $\Delta U = v(w) - V(w)$ as the difference between the maximum utility in the no annuity and annuity cases discussed above.

It can be shown that as long as $r_e \le r + \delta$ for each agent **independent of the wealth at retirement** w there is an α^* such that for $0 \le \alpha \le \alpha^*$ the agent chooses to annutize rather than to hold equity i.e. $\Delta U \le 0$.

An agent with $\alpha > \alpha^*$ prefers not to hold annuities. α^* is a function of r, r_e, δ and β but is independent of w. The key result is that an agent with any given w will either annutize or hold equity *but not both.* This decision will depend on the agent's α relative to α^*.

The finding that agents with a low "intensity" for bequest will annutize is consistent with the Yaari (1965) result.

Intermediation Technology

The intermediation technology is constant returns to scale with intermediation costs ϕ per unit of value intermediated. Thus

$$r_e = r + \phi$$

Calibration

The key parameters to calibrate are r and ϕ. We calibrate the later by constructing balance sheets for the principle classes of agents in the economy. The entries in the balance sheets are consistent with Flow-of-Funds statistics and match the U.S. household net worth. While most of the debt assets are pensions, they also include M2 and government debt.

Balance Sheets

Table 2

Pre-Retiree's Balance Sheet

Assets (GNP)		Liabilities (GNP)	
Equity Debt	0.5 0.8	Mortgage and other Borrowing Net Worth	0.5 0.8
Total	**1.3**	**Total**	**1.3**

Table 3

Balance Sheet of Retirees Who Annutize

Assets (GNP)		Liabilities (GNP)	
Equity (houses) Debt assets	0.3 1.0	Debt Liabilities Net Worth	0.2 1.1
Total	**1.3**	**Total**	**1.3**

Table 4

Balance Sheet of Retirees Who Don't Annutize

Assets (GNP)		**Liabilities (GNP)**	
Equity Debt assets	2.0 0.2	Debt Liabilities Net Worth	1.0 1.2
Total	**2.2**	**Total**	**2.2**

Table 5

Government

Assets (GNP)		**Liabilities (GNP)**	
Equity Debt assets	0.3 0	Debt Liabilities Net Worth	0.3 0
Total	**0.3**	**Total**	**0.3**

We assume that government debt is not intermediated so the amount of capital intermediated is the non-government debt which is seen to be 2 GDP from the balance sheets above. A two percent spread between borrowing and lending rates implies annual intermediation costs of 4 percent of GDP. This is consistent with the U.S experience.

We calibrate the return on government debt r to be 4%. This implies a capital annual-output ratio of about 3.5, which is also consistent with observation.

We use $r = 4\%$, $\phi = 2\%$ and hence $r_e = 6\%$ in our subsequent analysis.

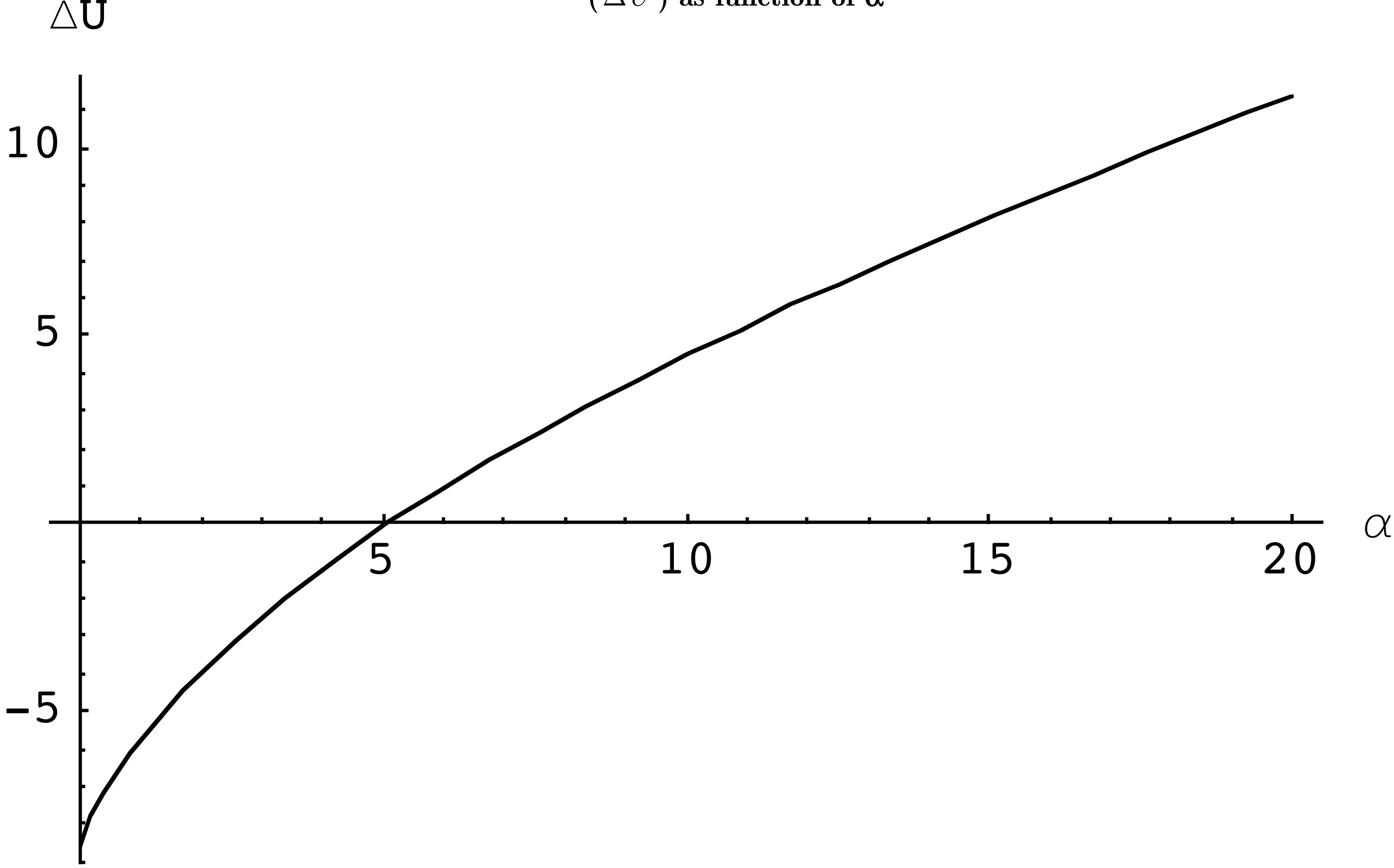
Results

For the parameters $\beta = 0.99, \delta = 0.05, r = 0.04$ and $r_e = 0.06$, figure 1 shows the utility difference between the optimal no annuity and annuity strategies (ΔU) plotted as function of α. As expected, we find that agents with a low α use annuities while those with a high α hold equity. In particular, with the above parameterization agents with $0 \leq \alpha^* \leq 5$ buy annuities. The implied equity premium is 2%.

Figure 1

Utility Difference Between the Optimal No Annuity and Annuity Strategies

(ΔU) as function of α



Concluding Comments

We have presented plausible arguments that can rationalize the observed equity premium. We argue that using an appropriate benchmark for the risk free rate and incorporating intermediation costs can account for a large part of the equity premium. That we have done so without resorting to risk supports the conclusion of our 1985 paper that the premium for bearing systematic risk is small.

This is a first step in what we conjecture will a big and fruitful research program. Extensions would include building in differential survival rates, addressing the issues of adverse selection and moral hazard when addressing the pricing of annuities.

In addition to matching intermediation spreads, the theory should match stocks of assets held. However, we will need better statistics on individual asset holdings to investigate this.

References

To be added