Knowledge Barter

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July 2000
Revised: January 2001

*We gratefully acknowledge the financial support of the Social Sciences and Humanities Research Council of Canada, the UBC Centre for Real Estate and Urban Economics, and the Real Estate Foundation of British Columbia. We are also grateful for suggestions made by Iain Cockburn, Marty Puterman, Richard Romano, Tom Ross, and the participants of the UBC Summer Conference in Urban Land Economics.

Abstract

It is common to model the transfer of knowledge as an exogenous spillover. However, there are many instances in science, industry and education in which knowledge transfers are deliberate and reciprocal. This paper models these endogenous knowledge transfers. We show that knowledge transfers arise as a kind of barter under perfect information, with efficiency a likely outcome. Efficient knowledge barter can also take place under imperfect information if the interaction is repeated and if discount rates are not too high. However, the sustainability of efficient knowledge barter depends crucially on the number of agents. When the number of agents is large, it is easier for an agent who withholds knowledge to go unpunished. This means that the sustainable level of knowledge barter is smaller in a large group.

I. Introduction

There are many instances of knowledge being transferred between agents without the intermediation of a price system. Scientific collaboration sometimes involves small knowledge transfers (here are my comments on your paper) and sometimes involves large transfers (do you think we should work together on this?). The role of knowledge exchange in science is discussed in Merton (1973). Knowledge is also transferred in industry. Marshall (1890) discusses the spread of knowledge within an occupation, while Jacobs (1969) discusses the spread of knowledge across activities. Both argue that spatial concentration is essential to knowledge transfer, although they disagree about whether it is better to have specialized clusters or large and diverse cities. There are also knowledge transfers among students, referred to as peer-group effects. These are discussed in Summers and Wolfe (1977), and Epple and Romano (1998).

It is common to model the transfer of knowledge as an exogenous spillover. The word "spillover" seems to be used because the interactions are not price-mediated. The spillovers are sometimes referred to as being "uncompensated," which means that there is no payment for any knowledge transferred. In the literature on science, the typical approach is to follow Arrow (1962) and suppose that knowledge is a public good. All agents in the economy receive the knowledge transfer in this case. The key element of individual choice in this sort of analysis is the decision of how much knowledge to acquire.[1] If the knowledge cannot be contained by its producer, there will naturally be underprovision. In the literature on spillovers in industry, the typical approach is again to suppose that the amount of knowledge that spills over is exogenous. This is the approach taken in Glaeser's (1999) Marshallian model of workers learning from each other and by

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[1] Cohen and Levinthal (1989,1990) are an interesting exception to this approach. They focus on the ability of firms to increase their ability to learn from other firms by carrying out their own research and development.

Berliant, Wang, and Reed's (1998) more Jacobian model of matching and learning by workers. In work on education, de Bartolome (1990), Epple and Romano (1998), and Nechyba (2000) all treat the characteristics of agents as exogenous. The peer-group effect then depends on with whom a student interacts rather than on decisions made by the students doing the interacting.

While some knowledge may spill over exogenously, it is likely that the degree of spillover is at least in part a deliberate economic choice. If one thinks about the ability to foster successful academic collaboration in a department, it is difficult to avoid the conclusion that size is not all that matters. Any academic can identify groups of gifted individuals who appear to learn nothing from each other. More concretely, following the spatial analysis of Marshall and Jacobs, there are instances of similar industrial clusters that perform quite differently. One example of this is the difference between Boston's Route 128 and the Silicon Valley (Saxenian (1994)). In the case of schools, children make choices that influence the impact they have on their peers. Taking a different perspective, parents' decisions can be seen as endogenous choices that influence peer-interaction (Helsley and Strange (2000)). This paper considers endogenous knowledge transfers, and shows that endogeneity fundamentally changes the effects of the transfers on resource allocation.

The paper's main conclusions are as follows. First, knowledge transfers can arise as a kind of barter under perfect information. Unlike models where spillovers are completely exogenous or where they are characterized as Samuelsonian public goods, efficiency is a likely outcome. Second, knowledge transfers can also arise under imperfect information if the interaction is repeated and if discount rates are not too high. However, there are many equilibria in this game, so it is possible that no knowledge be transferred in equilibrium. This result can help explain some puzzling cross-sectional variance in knowledge spillovers like the divergence between Boston and the Silicon Valley. Third, membership in a larger group does not always enhance knowledge transfer. This is because when the number of agents is large, it is easier for an agent who withholds knowledge to go unpunished. Thus, the sustainable level of knowledge barter is smaller in a large group. This result suggests that the formation of exclusive groups may not be motivated only by sorting but also by the need to create groups small enough so that cooperation is possible.

The rest of the paper is organized as follows. Section II considers the possibility of knowledge barter under certainty. Section III examines repeated knowledge barter. Section IV considers group formation for the purpose of knowledge transmission. Section V concludes by considering the application of the analysis to consumption-oriented knowledge transfers.

II. A model of knowledge barter

A. Model

We have deliberately used the term "knowledge barter" instead of the more common "knowledge spillovers." Some discussion of usage is therefore warranted. We conceive of knowledge transfers as a general phenomenon where one agent learns something from another agent. Some knowledge transfers are accidental, which are of course knowledge spillovers. Others involve deliberate decisions to give knowledge to another agent. When the exchange is both deliberate and reciprocal, it is knowledge barter.

This section presents a simple model of knowledge barter. In contrast to the conventional assumption that knowledge transfers arise from exogenous spillovers, in this model individuals choose whether or not to exchange knowledge with others. When barter is desirable, the agents also decide how much knowledge to exchange. Thus, knowledge transfers are endogenous. We examine the conditions under which knowledge barter is feasible, the characteristics of efficient exchanges, and the levels of knowledge exchange that may occur in equilibrium.

There are N risk-neutral agents in the economy. In any period, each agent is randomly matched with one other agent. Assume that agent i is matched with agent j. Upon matching, they may choose to exchange knowledge. Each agent evaluates the other's knowledge offering before the exchange is consummated. Thus, we are assuming that each agent has perfect information about the knowledge offered by a partner. To assume that knowledge can be evaluated *a priori* is unappealing, but it serves as a useful benchmark. Section III examines the more realistic case where the quality or value of knowledge in exchange is not known before exchange occurs.

The utility of agent i in any period is $u_i(k_j,x)$, where k_j is the knowledge acquired from j in that period, x is the numeraire, and $u_i(-)$ is increasing in both arguments and strictly quasi-concave. We also suppose that $\lim_{k_j \to 0} \partial u_i(k_j,x)/\partial k_j = \infty$, so that small levels of knowledge are extremely valuable.[2] Knowledge exchange is costly. Let c_i be i's cost of transmitting a unit of knowledge to any another agent. Then, if y_i is income, the numeraire consumption of agent i is $y - ck_i$, and the utility of agent i can be written $u_i(k_j,y_i - c_ik_i)$.

We believe that this specification captures two important features of knowledge transfers: agents make choices about the knowledge that they transfer and knowledge transfers are costly.[3] There are other features that are not captured. One is that agents do not acquire knowledge for their own private use. It is straightforward to include this, and doing so changes none of our results. Another feature is that knowledge does not accumulate in this model. This is less straightforward to include, but the complications (discussed later) do not change anything fundamental. Our model also does not make a

[2] At one point we will also assume that $\partial^2u_i/\partial x\partial k_j > 0$. See the discussion around Proposition 4.
[3] Regarding the costs of knowledge transfers, Stephan (1996, p.1208) writes that "...techniques can often be transferred only at considerable cost, in part because their tacit nature makes it difficult to communicate in a written form (or codify)." The word tacit refers to knowledge that "cannot be...codified or made explicit in the form of blueprints or instructions, but instead must be learned through practice."

distinction between the acquisition and transfer of knowledge. One could imagine a different approach where agents acquired knowledge at the beginning of the game and later exchanged it. More learned agents could exchange knowledge at lower cost. Including this would add important complications to the analysis (also discussed later), but the paper's primary results would continue to hold.

B. Knowledge barter

When there is no knowledge exchange, $k_i = k_j = 0$, and the utilities of i and j equal $u_i(0,y_i)$ and $u_j(0,y_j)$, respectively. When there is knowledge exchange, the maximum amount of knowledge that i is willing to provide in order to receive an amount k_j from agent j is implicitly defined by $u_i(k_j,y_i - c_ik_i) = u_i(0,y_i)$. This defines the knowledge offer function for agent i, $k_i(k_j)$, where

$$\frac{dk_i}{dk_j} = \frac{\frac{\partial u_i}{\partial k_j}}{c\frac{\partial u_i}{\partial x}} . \tag{1}$$

The knowledge offer function corresponds to the indifference curve in (k_j,k_i) space that gives agent i utility level $u_i(0,y_i)$. The slope of $k_i(k_j)$ equals the marginal rate of substitution between transmitted knowledge k_i and received knowledge k_j. $k_i(k_j)$ is strictly concave since $u_i(-)$ is strictly quasi-concave. The knowledge offer functions for agents i and j are shown in Figure 1.

Of course, mutually beneficial knowledge exchange is not always possible. Agent i is willing to make a non-negative contribution $k_i \leq k_i(k_j)$ in return for receiving k_j. Symmetrically, the partner is willing to contribute $k_j \leq k_j(k_i)$. Thus, the set of mutually beneficial potential exchanges is $\{(k_j,k_i) : k_j \leq k_j(k_i), k_i \leq k_i(k_j), k_j \geq 0, k_i \geq 0\}$, the region between the knowledge offer functions in Figure 1. A sufficient condition for the feasible set to include points other than the origin is $\lim_{k_j \to 0} \partial k_i/\partial k_j \;\; \lim_{k_i \to 0} \partial k_j/\partial k_i > 1$, which is satisfied by the limiting condition on the marginal utility of knowledge. Thus there are always potential gains from knowledge exchange.[4]

This analysis may be summarized as follows:

Proposition 1: Knowledge transfers within a group can arise through individually rational decisions as a kind of barter.

The intuition here is that when the feasible region includes points other than the origin, then both agents can benefit from learning from each other, and so they may effect an exchange.[5] From this point on, we assume that agents have identical utility functions, incomes and knowledge transmission costs.

As noted in the introduction, there are numerous instances of knowledge barter. As the reader of this paper is no doubt aware, the free flow of knowledge is one of the highest ideals in science. Laband and Tollison (2000) note that knowledge can be transmitted either within a formal collaborative arrangement or informally. The formal arrangement gives both parties a property right in the research through co-authorship of papers. Informal cooperation is less well-rewarded, resulting instead in a footnoted acknowledgement. In their study of work in economics and biology, Laband and Tollison show that informal cooperation is productive by demonstrating that papers with more acknowledgements are cited more frequently. Acknowledgements of scholars at other institutions and acknowledgements of highly-cited scholars have the largest effects.

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[4] In the case where knowledge accumulates, an agent who already has a lot of knowledge derives less marginal utility from the potential contribution of another agent. However, it would also be reasonable to suppose that an agent who was already well-informed would have lower cost of transferring knowledge. The first effect shrinks the set of feasible knowledge exchanges, while the second enlarges it.

[5] The expenditures of agents on knowledge acquisition allow the barter described in Proposition 1 to occur. The possibility of knowledge barter, therefore, would encourage knowledge acquisition.

This, of course, suggests a question: how is it possible to sustain valuable but poorly rewarded cooperation? Their admittedly speculative answer is that (p. 651), "As economists, we doubt that such activity is purely altruistic...It seems much more likely that collegial commentary by one party forms part of some sort of implicit transaction, or stream of transactions, between provider and recipient." This is exactly the sort of knowledge barter that we have modeled in this section.

There are also instances of knowledge barter in industry and education. In the case of knowledge transfers in industry, the discussions of learning in both Marshall and Jacobs conspicuously omit any treatment of price. Marshall's Sheffield cutlery workers find knowledge in the air, presumably the air through which they watch other workers go about their business or the air that their fellow workers breathe when they talk about their work. Jacobs describes "new work" being created through synergies between occupations (i.e., obsidian gathering and animal husbandry or aerospace engineering and the design of sliding doors). Again, the learning involves no exchange of money. In the case of education, knowledge is exchanged among members of a peer-group. There are certainly instances of knowledge exchange involving payment, but these are typically proscribed and are certainly not the norm.

There is an important question regarding knowledge barter that we have not yet addressed: why would knowledge exchange take place through transfers rather than through a price system? There are a number of possibilities. The most obvious answer is that knowledge barter can arise because it works. Jevons (1875) notes that money exchange has the advantage over barter that it does not require the "double coincidence of wants." However, knowledge is an unusual good. It is infinitely differentiated, and agents' endowments of knowledge are idiosyncratic. This means that it is likely that any pair of agents could successfully identify a mutually beneficial knowledge exchange. Thus, for knowledge, the possibility of there being a double coincidence of wants is not remote, and so barter may be a more natural way than a price system to effect an exchange. Barter may also be preferable for other reasons. For one thing, it may reduce transactions costs. Trading knowledge for knowledge may be less costly than two transactions where knowledge is traded for money. In addition, this kind of barter is not taxable, while price mediated transactions would be. Finally, there may be legal or ethical reasons why money is not involved. This is clearly true for peer-group effects in education.

C. Efficient exchanges

This section characterizes efficient knowledge exchanges. These are feasible exchanges that maximize the utility of one agent while providing the other with a given, fixed utility level. Efficient exchanges solve the following maximization program:

$$\max_{k_i,k_j} u(k_j, y - ck_i), \tag{2}$$

subject to

$$u(k_i, y - ck_j) \geq \bar{u}, \tag{3}$$

$$u(k_i, y - ck_j) \geq u(0,y), \tag{4}$$

$$u(k_j, y - ck_i) \geq u(0,y), \tag{5}$$

$$k_i \geq 0, \tag{6}$$

$$k_j \geq 0. \tag{7}$$

(3) is the requirement that the other agent j achieve at least utility level $\bar{u}$. (4) through (7) characterize the feasible region. Before solving this problem formally, note that equation (3) is redundant, given (5). Also, note that any exchange of the form $(0,k_i)$ where $k_i > 0$,

or $(k_j,0)$ where $k_j > 0$, violates (4) or (5). Thus, the only corner solution that we need to consider is (0,0).

Letting λ, μ, η, and ξ be the multipliers associated equations with (4) through (7), respectively, letting u_i represent $u(k_j,y - ck_i)$, and letting u_j represent $u(k_i,y - ck_j)$, the first order necessary conditions for this problem are:

$$-c\frac{\partial u_i}{\partial x} + \lambda\frac{\partial u_j}{\partial k_i} - \mu c\frac{\partial u_i}{\partial x} + \eta = 0\,, \tag{8}$$

$$\frac{\partial u_i}{\partial k_j} - \lambda c\frac{\partial u_j}{\partial x} - \mu\frac{\partial u_i}{\partial k_j} + \xi = 0\,, \tag{9}$$

$$\lambda \geq 0, \mu \geq 0, \eta \geq 0, \xi \geq 0\,, \tag{10}$$

$$u(k_i,y - ck_j) - u(0,y) \geq 0, u(k_j,y - ck_i) - u(0,y) \geq 0, k_i \geq 0, k_j \geq 0\,, \tag{11}$$

$$\lambda\left[u(k_i,y - ck_j) - u(0,y)\right] = 0, \mu\left[u(k_j,y - ck_i) - u(0,y)\right] = 0, \eta k_i = 0, \xi k_j = 0\,. \tag{12}$$

Suppose $k_i = k_j = 0$. Then $\partial u_i/\partial x = \partial u_j/\partial x$, $\partial u_i/\partial k_j = \partial u_j/\partial k_i$, and (8) and (9) can be written

$$-c\frac{\partial u_i}{\partial x} + \lambda\frac{\partial u_i}{\partial k_j} - \mu c\frac{\partial u_i}{\partial x} + \eta = 0\,, \tag{13}$$

$$\frac{\partial u_i}{\partial k_j} - \lambda c\frac{\partial u_i}{\partial x} - \mu\frac{\partial u_i}{\partial k_j} + \xi = 0\,. \tag{14}$$

Adding (13) and (14) and simplifying gives

$$\eta + \xi = (1 + \lambda + \mu)(c\frac{\partial u_i}{\partial x} - \frac{\partial u_i}{\partial k_j}) < 0, \tag{15}$$

where we have used the assumption that $\lim_{k_j \to 0} \partial k_i / \partial k_j > 1$. (15) in turn implies $\eta < 0$, or $\xi < 0$, or both, violating (10). Thus, $k_i > 0$ and $k_j > 0$, which implies $\eta = \xi = 0$ by (12). Then, using (1), (8) and (9) imply:

$$\frac{dk_i}{dk_j}\frac{dk_j}{dk_i} = 1. \tag{16}$$

Thus, an efficient exchange has the property that the product of the slopes of the knowledge offer functions (the product of the marginal rates of substitution) equals 1. (16) characterizes the contract curve, the locus of feasible efficient knowledge exchanges. The contract curve is labeled CC in Figure 1. The symmetric efficient allocation (k*,k*) occurs where the contract curve intersects the 45° line. From (16), $dk_i/dk_j = 1$ completely characterizes the unique symmetric efficient exchange.

The knowledge barter that we have been analyzing is an instance of bargaining with certainty. It is generally agreed that optimum and equilibrium will coincide in this setting. Of course, there are many ways to define the equilibrium, and so it is possible to sustain various optima as equilibria of particular specifications of the bargain. For instance, the bargain may involve only one take-it-or-leave-it offer. The players would then be randomly assigned to the roles of offerer and responder. In this case, the equilibrium would maximize the utility of the offerer subject to the constraint that the responder be as well off accepting as by rejecting. This defines two equilibria, at each edge of the contract curve. If instead the bargain involved finitely repeated offers (Ståhl (1972)), then the equilibrium would be found by backward induction from a final stage corresponding exactly to the take-it-or-leave-it game described above. This would still give efficient allocations, but they would now be in the interior of the feasible region.

With infinite repetition (Rubinstein (1982)), the allocation would also be efficient and in the interior of the feasible region. Finally, one might choose to resolve the bargaining problem axiomatically (Nash (1950)) rather than by using game theory. In this case, the outcome would depend on the exogenous relative "bargaining power" that the agents are assumed to possess.

There is another important question that we have not yet addressed: how can knowledge be bartered when it is impossible verify value before the exchange? The next section will consider a resolution to this problem through repeated exchange. The key result is that repeated knowledge exchange can implement efficient transfers in some but not all situations.

III. Repeated knowledge barter

A. Model

In the previous section, we assumed that the quality or value of knowledge offered by a partner could be ascertained before any exchanged occurred. This is a critical assumption. If knowledge could not be evaluated *a priori*, then the dominant strategy of each agent would be to set $k_i = 0$. Thus, it might seem that knowledge barter is impossible in the absence of perfect information. However, most knowledge barter occurs in the context of repeated interactions. This section examines whether knowledge barter can be sustained in this setting.

We now suppose that interactions occur over an infinite number of time periods. In each period every agent is matched with another agent, with each agent choosing a level of knowledge to give the partner.[6] An agent may or may not know how the partner

[6] We suppose that this matching takes place with replacement, in order to avoid the problem of backward induction making a finitely repeated game into a game that is essentially played only once.

has behaved in the past. Specifically, we suppose that with probability ρ an agent knows the history of the partner's play. Otherwise, the agent knows nothing about the partner's history. In general, an agent's strategy will specify a level of knowledge exchange as a function of both the agent's own and the partner's play history.

It should not be surprising that there are many equilibria of this game. For instance, in contrast to the previous section's certainty analysis, with uncertainty there is an equilibrium where agents choose not to transfer any knowledge at all. If no other agent will transfer knowledge under any circumstances, then it does not pay to transfer knowledge oneself. There are many other equilibria as well.[7] Fortunately, our purposes do not require that we characterize all the game's equilibria. We are instead concerned with whether it is possible to sustain the efficient level of knowledge exchange. Failing that, we are interested in determining the maximum sustainable amount of knowledge exchange.

To achieve these ends, we suppose that agents play "grim strategies," in the following sense. An agent will provide the pairwise efficient amount of knowledge, k*, as long as the partner is known to have to have provided k* in the past. If the partner is known to have been less than completely cooperative in the past, then the agent will transmit no knowledge. In this situation, if an agent chooses to be uncooperative (by providing less than k*), setting k = 0 is dominant among uncooperative strategies. Thus, although the game allows for continuous choices of k, if the rest of the agents are playing the grim strategy, any given agent chooses between being cooperative and providing k* and being uncooperative and providing k = 0. We will refer to the uncooperative strategy as "withholding" knowledge, and we will use "W" to denote it. We will refer to the cooperative strategy as "providing" knowledge, and we will use "P" to denote it. In this

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[7] The essence of the so-called "Folk Theorem" (Fudenberg and Maskin (1986)) is that there are conditions under which any sequence of feasible payoffs can be sustained as the unique subgame perfect equilibrium of an infinitely repeated game.

setup, we will be able to make use of some existing results on repeated prisoner's dilemma games. Of course, unlike most repeated prisoner's dilemma games, here agents interact with each other sequentially and randomly, and agents do not necessarily know the history of the game.[8]

When all agents are assumed to play the grim strategies, any two agents who meet play the following normal form stage game:

	W	P
W	A,A	B,C
P	C,B	D,D

where the payoff to the row player is listed first, and A = u(0,y), B = u(k*,y), C = u(0,y - ck*) and D = u(k*,y - ck*). The payoffs are ranked as follows: C < A (since k* > 0), A < D (by the results of Section II), and D < B (since k* > 0), so C < A < D < B. The game would be exactly equivalent to a prisoner's dilemma if (B + C) < 2D, but this condition cannot be guaranteed. Below, we will consider conditions under which it is an equilibrium for the two agents to play cooperatively.

B. Sustaining efficient knowledge barter

If all other agents are providing knowledge, the expected utility of providing knowledge now and forever is given by

$$V_P = D + \sum_{t=1}^{\infty} \delta^t \big(\rho D + (1-\rho)D\big) = \frac{D}{1-\delta}, \tag{17}$$

[8] There are other approaches that we could have taken. In Carmichael and McLeod's (1997) analysis of reciprocal gift giving, there is no external knowledge of past defections. They show that the exchange of valueless gifts can support efficient relations within a matched pair because it prevents agents from defecting in the hope of finding another match partner to exploit.

where $\delta < 1$ is the discount factor. A player who cooperates receives the mutual cooperation payoff D in the current and in all future periods. The expected utility of withholding knowledge is given by

$$V_W = B + \sum_{t=1}^{\infty} \delta^t \left(\rho A + (1-\rho)B\right) = B + \frac{\delta}{1-\delta}\left(\rho A + (1-\rho)B\right). \tag{18}$$

A player who defects receives B in the current period, and then A or B in future periods, depending on whether the partners encountered know of the past uncooperative behavior. The agent will cooperate when $V_P \geq V_W$. Using (17) and (18) cooperation will occur provided

$$\rho\delta \geq \frac{B-D}{B-A}. \tag{19}$$

This analysis may be summarized as follows:

Proposition 2: When information about the knowledge offered by a partner is imperfect, efficient levels of knowledge exchange can sustained through repeated interactions, although a complete absence of knowledge exchange is also an equilibrium.

In this case, cooperation is supported by the sanction associated with the refusal of future knowledge barter. If the probability that a partner knows about past defections is too small, then this sanction is ineffective. The sanction is also ineffective if the discount factor is so small that the loss of future knowledge exchange is unimportant. Other things being equal, cooperation is more likely when the difference between the defection and mutual cooperation payoffs (B - D) is small. Intuitively, (B - D) is the utility difference associated with the cost of providing knowledge to others. Cooperation is also more likely when the difference between the defection and mutual non-cooperation payoffs (B - A) is large. Intuitively, (B - A) is the utility difference associated with the benefit of receiving knowledge from others. These comparative static results are sensible, and they are also familiar from the study of collusion in oligopoly (see Tirole (1988)).

C. Cooperation, trust, and social capital

The essence of Proposition 2 is that repeated interactions can engender trust, which can in turn result in efficient knowledge barter. There are many other situations both inside and outside of economics where repetition can lead to the possibility of efficient cooperation. This section will discuss some of them.

The classic example is probably Axelrod's (1984) analysis of repeated prisoner's dilemma experiments. These experiments were computer mediated tournaments between various strategies, with the very simple strategy of tit-for-tat usually outperforming all other strategies. Its success has been attributed to its ability to elicit cooperation from the cooperative, its forgiveness of defections at least some of the time, and its punishment of uncooperative rivals. The Axelrod analysis, of course, is based on the repeated play of the game by the same players, which is different than the interaction considered here.

That cooperation is a signal social virtue is well appreciated. Putnam (1993) uses the term "social capital" to refer to the ability of Northern Italy to support a positive civic environment and therefore growth. Spagnolo (1999) presents a model where cooperation in a firm is more likely if the workers also must interact in a separate social setting. The possibility of uncooperative behavior being sanctioned both within the firm and in the social setting makes cooperation more likely. He defines the ability to punish noncooperators as "social capital," an interesting formalization of Putnam's analysis. Lorenz (1999) observes that the share of British shipbuilders declined sharply after World War II. He argues that this was a consequence of a lack of administrative innovation, which was itself a direct consequence of inflexible work rules. This, in turn, was a consequence of industrial relations marked by a lack of trust, a close cousin to social capital.

Finally, the value of cooperation is a central concern in modern sociobiology. Trivers (1971), for example, argues that cooperation can be evolutionarily successful among animals, and that his insight applies to human interactions as well. In his words, "the natural selection of reciprocally altruistic behavior can readily explain the function of human altruistic behavior and the details of the psychological system underlying such behavior." See Ridley (1997) for a broad and accessible treatment of reciprocal cooperation.

IV. Groups and knowledge barter

A. Overview

Given the importance of knowledge exchange, it is not surprising that agents sort themselves into groups in order to foster learning. Professional or scientific associations, organizations (firms, universities) and their departments, and even cities or communities affect the ways that people interact for knowledge exchange. Indeed, in some cases, organizations form for the express purpose of facilitating knowledge exchange. For instance, Sakakibara (1997) and Branstetter and Sakakibara (1998) discuss the role of Japanese research consortia in internalizing knowledge spillovers. The role of knowledge exchange seems obvious in the case of scientific associations. It is also a common theme in models of the microfoundations of spatial agglomeration: one reason that activities cluster in space is to facilitate the exchange of information (Jacobs (1969)).

Clearly, one reason that these groups form is sorting. Universities are partitioned into academic departments in part in order to group the researchers who can learn the most from each other. Many cities specialize in the production of particular products in order that within-industry learning take place. Private schools may skim off the highest-ability students in order to form a productive and therefore valuable peer group.

Our model suggests an additional explanation. In a model where knowledge exchange is exogenous in the sense that one receives (or expects to receive) a certain fixed amount of knowledge from every group member, group size enhances knowledge exchange.[9] Interestingly, in our model, this is not necessary the case. The reason is that the cooperation that supports repeated knowledge exchange may break down in very large groups. This will be established below.

B. Group size and cooperation

To examine how group size affects knowledge exchange, we assume that the probability that past defections are known is a continuous and strictly decreasing function of N: $\rho(N)$, $\rho' < 0$, $\lim_{N\to\infty}\rho(N) = 0$. The idea we are trying to capture is that, other things being equal, defections are less conspicuous, and are therefore less likely to be punished, in large groups. There are several ways to motivate this supposition. A simple approach would be to suppose that a fixed number of agents $M < N$ learns about a defection and remembers it for one period. Then the probability that a random match brings a defector in contact with an informed agent is $\rho(N) = M/N$, which is strictly decreasing in N. A more complex approach would be to suppose that defections are only punished if an agent is subsequently matched with a partner against whom they defected in the past. Then the probability that a random match brings a defector in contact with an informed agent equals the probability of being matched with someone you have been matched with

[9]An increase in group size would also increase knowledge exchange if agents were idiosyncratically heterogeneous in their "demands" for knowledge. A larger group would enhance knowledge exchange by allowing the agent to be matched to a partner closer to his or her ideal.

before. Letting τ represent the number of time periods since the initial defection, for $\tau <$ N - 1, the probability of a repeat match is $\rho(N) = 1 - (N/(N-1))^{\tau}$, which is also strictly decreasing in N.[10]

The expected utility associated with providing knowledge in perpetuity is now

$$V_P = D + \sum_{t=1}^{\infty} \delta^t \big(\rho(N)D + (1-\rho(N))D\big) = \frac{D}{1-\delta}, \tag{20}$$

while the expected utility from withholding knowledge is

$$V_W = B + \sum_{t=1}^{\infty} \delta^t \big(\rho(N)A + (1-\rho(N))B\big) = B + \frac{\delta}{1-\delta}\big(\rho(N)A + (1-\rho(N))B\big). \tag{21}$$

Following (19), cooperation is sustainable under these conditions so long as $\rho(N)\delta \geq (B - D)/(B - A)$. Thus, now there is a critical group size N* beyond which cooperation will not occur. N* is implicitly defined by

$$\rho(N^*)\delta = \frac{B-D}{B-A}. \tag{22}$$

For example, when $\rho(N) = M/N$, the critical group size is $N^* = \delta M(B - A)/(B - D)$. N* increases with the discount factor δ, and decreases with the efficient level of knowledge exchange k*:

[10] For $\tau \geq N - 1$, all other agents have been met along some branches of the matching process, and the formula given in the text must be amended to eliminate the possibility of new matches along these branches. Calculations suggest that the probability that past defections are known is a strictly decreasing function of N in this case as well.

$$\frac{\partial N^*}{\partial \delta} = -\frac{\rho(N^*)}{\rho'(N^*)\delta} > 0,$$

$$\frac{\partial N^*}{\partial k^*} = \frac{1}{\rho'(N^*)\delta}\left(\frac{\left(u(k^*,y-ck^*)-u(0,y)\right)\frac{\partial u(k^*,y)}{\partial k^*}}{\left(u(k^*,y)-u(0,y)\right)^2}\right) < 0. \tag{23}$$

The latter result occurs because an increase in k* increases the difference between defection and mutual cooperation payoffs relative to the difference between the defection and mutual non-cooperation payoffs. In effect, the temptation to defect is stronger when the efficient level of knowledge exchange is larger. In groups larger than N*, the probability that defection is known is so small that that the sanction of foregoing future knowledge barter is ineffective. This analysis may be summarized as follows:

Proposition 3: When the probability that defection is known decreases with group size, cooperation is not sustainable in very large groups.

With exogenous knowledge spillovers, a large group always allows better knowledge exchange. Thus, in models with exogenous spillovers, group size must itself be either exogenous (Glaeser (1999)) or limited by the congestion of some fixed factor not directly related to knowledge (Helsley and Strange (2000)). With endogenous spillovers, group size may be limited as well by the ultimate loss of knowledge spillovers.

C. Incomplete knowledge barter

Even if it is not possible to sustain efficient knowledge barter, it may be possible to sustain knowledge barter at some lower level. This section considers the desirability and feasibility of knowledge exchange when individual agents provide less than k*. We refer to this as incomplete knowledge exchange.

It is easy to show that any positive level of knowledge exchange is desirable in this model. In the absence of knowledge exchange, each agent receives the autarky payoff $V_0 = u(0,y)/(1 - \delta)$. However, if a lower level of knowledge exchange $\underline{k} < k^*$ can be sustained, each agent would receive the payoff $\underline{V}_P = u(\underline{k},y - c\underline{k})/(1 - \delta)$. Quasi-concavity implies that $du(k,y - ck)/dk > 0$ for all $k < k^*$, which in turn implies $u(\underline{k},y - c\underline{k}) > u(0,y)$, and so $\underline{V}_P > V_0$. Thus, any positive level of knowledge exchange dominates autarky.

To see that incomplete knowledge exchange may be possible, suppose that agents play the following strategies: provide $\underline{k} < k^*$ unless a partner is known to have withheld knowledge, by providing less than $\underline{k}$. Otherwise provide zero. In this case, an agent again effectively plays a bimatrix game, choosing between providing and withholding knowledge. The payoff matrix for this game is as above, except that the payoffs in the various cells now depend on $\underline{k}$ rather than on k^*. Denote these payoffs as $\underline{A}$, $\underline{B}$, $\underline{C}$, and $\underline{D}$. Proceeding as before, the level of knowledge exchange $\underline{k}$ is sustainable as long as $\rho(N)\delta \geq (\underline{B} - \underline{D})/(\underline{B} - \underline{A})$.

The maximum sustainable level of knowledge exchange $\underline{k}$ is implicitly defined by

$$\rho(N)\delta = \frac{\underline{B} - \underline{D}}{\underline{B} - \underline{A}}, \tag{24}$$

or, substituting for the payoffs,

$$\rho(N)\delta = \frac{u(\underline{k},y) - u(\underline{k},y - c\underline{k})}{u(\underline{k},y) - u(0,y)}. \tag{25}$$

The comparative statics of $\underline{k}$ are:

$$\frac{\partial \underline{k}}{\partial \delta} = \frac{\rho(N)}{\Phi'(\underline{k})} > 0,$$
$$\frac{\partial \underline{k}}{\partial N} = \frac{\rho'(N)\delta}{\Phi'(\underline{k})} < 0, \tag{26}$$

where

$$\Phi(\underline{k}) = \frac{u(\underline{k},y) - u(\underline{k},y - c\underline{k})}{u(\underline{k},y) - u(0,y)}, \tag{27}$$

and $\Phi'(\underline{k}) > 0$ provided $\partial^2 u/\partial x \partial k > 0$.[11] Thus, $\underline{k}$ increases with the discount factor δ, and decreases with the size of the group N. This analysis may be summarized as follows:

Proposition 4: A smaller group can sustain a larger degree of knowledge exchange. Similarly, when the future is more important (larger δ), a larger degree of knowledge exchange can be sustained.

Figure 2 illustrates the dependence of payoffs on group size. For groups smaller than N*, $V_P > V_W$, so efficient knowledge barter is sustainable, and k = k*. In this region, $\partial V_P/\partial N = 0$, by (20), and

$$\frac{\partial V_W}{\partial N} = \frac{\delta}{1-\delta}\rho'(N)\left(u(0,y) - u(k^*,y)\right) > 0, \tag{28}$$

by (21). For groups larger than N*, efficient knowledge barter is not sustainable, so $k = \underline{k} < k^*$. In this region, the payoffs associated with providing and withholding knowledge are equal by construction, $\underline{V}_P = \underline{V}_W = u(\underline{k},y - c\underline{k})/(1 - \delta) \equiv \underline{V}$, and

[11] The proof is given in the appendix.

$$\frac{\partial \underline{V}}{\partial N} = \frac{\frac{d}{dk}u(k^*, y - ck^*)}{1-\delta}\frac{\partial \underline{k}}{\partial N} < 0\,, \tag{29}$$

by (26). The utility possibility frontier is given by the maximum payoff associated with each value of N. This is V_P for $N \leq N^*$, and $\underline{V}$ for $N > N^*$. As shown in the figure, the payoff to an agent eventually declines because of the difficulty of sustaining cooperation, and the resulting decrease in the level of knowledge barter.

Thus, there is a natural limit to group size in this context. However, this analysis is quite different than in standard models of group formation. It is typically the case that group size is limited by the congestion of a fixed factor. For instance, the size of a club or community is limited by the congestion of fixed facilities (Buchanan (1965), Tiebout (1956)). Similarly, it has been argued that firms eventually face increasing costs in part because of the congestion of managerial control (Lucas (1978)). Although it is certainly true that congestion of fixed factors limits group size, our model illustrates another constraint: group size is limited by the difficulty of sustaining knowledge barter in large groups.

This result has a distant relationship to some results on the stability of cartels. Stigler (1950) points out that firms outside a cartel enjoy the high price that the cartel realizes but need not restrict their own output. Thus, outsiders are likely to earn higher profits than firms inside the cartel, since the latter enjoy a high price only by restricting output . This obviously makes it difficult to assemble a stable cartel. d'Aspremont et al (1983) show that this obstacle is not insurmountable, since firms might prefer to be exploited by fringe free-riders than to exercise no restraint at all on competition. The parallel to our analysis is the endogeneity of cooperation. Our result is quite different, however, in that there is no incentive for an agent to leave a group and do better as a member of some kind of fringe.

The result that there is less knowledge exchange in large groups suggests that sorting by type is not the only reason for the formation of these small, exclusive groups. Since large groups have less cooperation, the formation of a small group has the additional advantage of ensuring cooperation. The fact that private schools are smaller than public schools -- only 18.6% with student bodies greater than 300, compared to 69.2% of public schools -- is consistent with this (National Center for Education Statistics (1995)). That optimal group size may be determined by the ability to sustain cooperation is a conclusion that is also reached by Ridley (1997). He goes so far as to argue that the optimal group size is 150 whatever the sharing activity might be. He supports this claim by noting that that is roughly the number of people anyone is likely to know well. More concretely, he writes that it is also the number of cards in a typical rolodex.

V. Consumption-oriented knowledge transfers

This paper has considered knowledge transfers in science, industry, and education. If one conceives of knowledge transfers broadly, there are also a number of consumption-oriented activities that can usefully be thought of as knowledge transfer. Many of these are new-economy activities like trading digitally recorded music or video. These are knowledge transfer activities in that it is literally information (bits) that are being exchanged rather than the atoms of which physical products are made.

It is easy to see that these knowledge transfers are often instances of deliberate knowledge barter. Creating files for trading and allowing other users to access them involves a cost. Although money is sometimes involved in these exchanges, the transfers are instead frequently barters. There are good reasons for this. First, there may be legal reasons why money is not involved. For instance, the Audio Home Recording Act has been interpreted to allow fair use of music, which includes giving tapes to friends, but it explicitly does not allow their sale. Second, there are sometimes ethical barriers to selling "knowledge" of this kind. An example of this is the stringently observed edict not to trade the official recordings of musicians who allow taping of their live performances.

In addition to new economy exchanges of bits, there are many old-economy activities that involve knowledge transfers. Clubs that allow like-minded people to interact with each other are at least in part vehicles for knowledge exchange. Similarly, in hobbies like gardening and cooking, there are significant knowledge transfers.

What does this paper's analysis have to say about knowledge transfers for the purpose of consumption? Propositions 1 and 2 illustrate the possibility of efficient knowledge barter, and this result has direct application: bilateral knowledge barter can be efficient. However, in many cases, the knowledge exchange is not really bilateral, and there is a third party to the transaction who must incur costs in order that the knowledge be transferred. Specifically, music must be composed and performed and movies and television programs must be produced in order that they be traded. This is absent in the model. In this case, it is likely that Arrow's public good problem would remain, with knowledge production inefficient. This is not certain, however, since the existence of barter could increase the demand for the output of the third party. This is consistent with recent increases in sales of CDs at the same time that digital music trading has exploded, for example.

Proposition 3 argues that knowledge barter is not possible for large groups. Thus, in order to realize consumption-oriented knowledge transfers, it is necessary for consumers to sort themselves into small communities. This result can be seen as contrary to the new-economy ethic of disintermediation. One need not be online very long, however, to realize that universality is sometimes linked to incivility. Thus, although the knowledge transfers are accomplished through bilateral interactions, some intermediation may be necessary in order that cooperation be maintained.

Appendix

In this appendix, we show that if $\partial^2 u/\partial x \partial k > 0$, then $\Phi'(\underline{k}) > 0$, where $\Phi(k)$ is defined in (27). Applying the mean value theorem to the numerator of (27) gives

$$u(\underline{k},y) - u(\underline{k},y - c\underline{k}) = \frac{\partial u(\underline{k},\hat{x})}{\partial x} \; c\underline{k} \, ,$$

for some $\hat{x} \quad (y - c\underline{k}, y)$. Let $\hat{x} = \theta(y - c\underline{k}) + (1-\theta)y = y - \theta c\underline{k}, \ \theta \quad (0,1)$. Applying the mean value theorem to the denominator of (27) gives

$$u(\underline{k},y) - u(0,y) = \frac{\partial u(\hat{k},y)}{\partial k} \; \underline{k} \, ,$$

for some $\hat{k} \quad (0,\underline{k})$. Let $\hat{k} = \lambda(0) + (1-\lambda)\underline{k} = (1-\lambda)\underline{k}, \ \lambda \quad (0,1)$. Then

$$\Phi(\underline{k}) = \frac{c\dfrac{\partial u(\underline{k},y - \theta c\underline{k})}{\partial x}}{\dfrac{\partial u((1 - \lambda)\underline{k},y)}{\partial x}} \, ,$$

and so

$$\Phi'(\underline{k}) = \frac{u_k\left(cu_{xk} - \theta c^2 u_{kk}\right) - c(1 - \lambda)u_x u_{kk}}{u_k^2} \, .$$

Thus, $\Phi'(\underline{k}) > 0$ for $u_{xk} > 0 \quad \theta \quad (0,1) \quad \lambda \quad (0,1)$. QED

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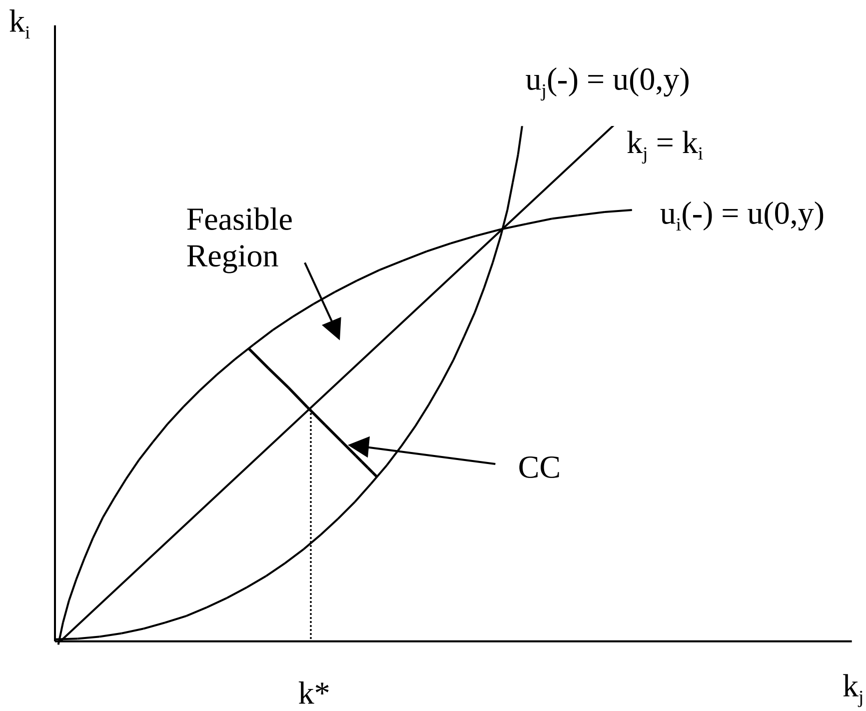

Figure 1

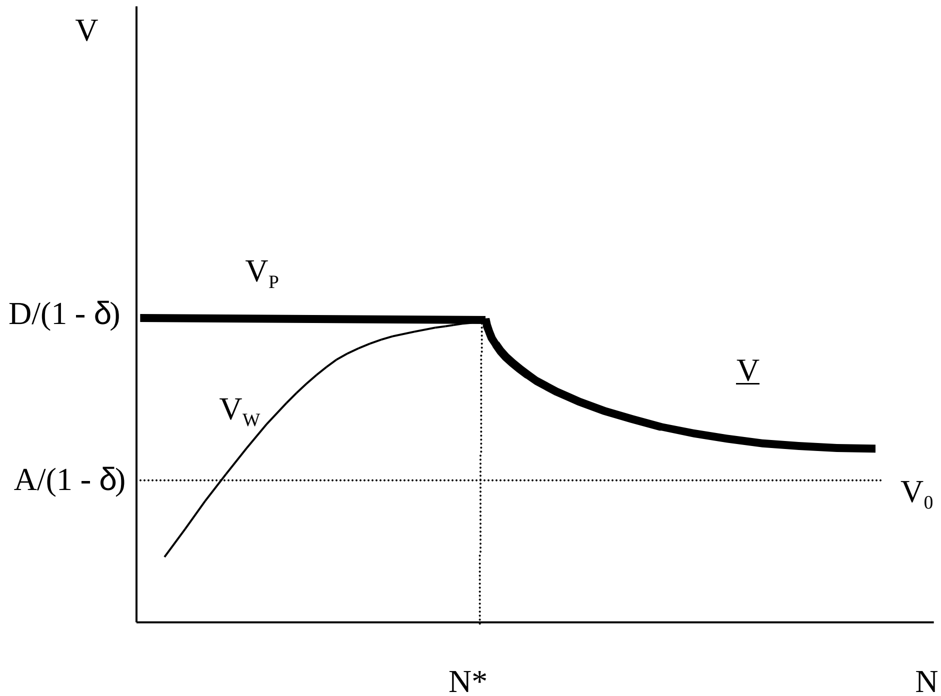

Figure 2