International Firm Mobility, Comparative Advantage, and Long-run Growth*

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Abstract

This paper constructs a multi-country endogenous growth model, and investigates the interaction between comparative advantage and international firm mobility in determining global growth and wage inequality. We introduce international firm mobility into the endogenous model of Romer (1990) by allowing geographical separation between innovation and manufacturing. We find that in the long-run equilibrium with free trade, firm mobility raises global growth and the average skill premium without boosting international knowledge spillovers. An increase in global growth occurs as firm mobility induces countries with comparative advantage in research to invent more goods than they manufacture and those with comparative advantage in labor endowment to manufacture more goods than they invent. We also show that global growth falls if a country with comparative advantage in research subsidizes manufacturing and/or a country with large comparative advantage in labor endowment subsidizes research.

JEL Classification: F12, F21, F43, J31

Key Words: endogenous growth, innovation, manufacturing, foreign direct investment, offshoring

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* This paper is very preliminary. Comments are welcome.

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1. Introduction

International firm mobility has been a major factor behind the dramatic expansion of global value chains that occurred since 1990s. International firm mobility in this paper refers to geographical separation between the birthplace of a technology and the manufacturing site of products that use the technology, either through technology licensing or foreign direct investment. [1] The separation has increasingly been facilitated by the advancement of transportation and communication technology. It played a pivotal role in intensifying the expansion of global value chains and spreading industrialization and growth around the world.

This paper constructs an endogenous growth model of a world composed of countries with heterogeneous endowments and heterogeneous productivities, and investigates the interplay between comparative advantage and international firm mobility in determining global growth and wage inequality. We examine international firm mobility by modifying the endogenous growth model based on expanding input varieties by Romer (1990): we allow the number of intermediate goods varieties that a country manufactures to differ from the number of intermediate goods varieties that the country has invented. The model is highly stylized to capture in a minimal, but integrated framework the complicated relationship among trade, international firm mobility, growth, and skill premium (the ratio between the wages of skilled and unskilled workers). To highlight the tradeoff between innovation and manufacturing, we assume that final goods are produced by unskilled workers who assemble intermediate goods, and intermediate goods and their designs are produced by skilled workers alone.

We obtain the following results on the effect of globalization on balanced growth equilibrium. If the world shifts from the autarkic regime to free trade without international firm mobility, countries grow faster and their average skill premium rise if the scope of knowledge spillovers change from being national to global after trade. However, if knowledge spillovers are global before and after trade, trade has only level effects, and does not alter the growth rate and the skill premium of countries. In contrast, when the world shifts from free trade without international firm mobility to free trade with international firm mobility, global growth and the average skill premium always rise.

This occurs because international firm mobility intensifies the world division of labor based on comparative advantage. The mobility induces countries with comparative advantage in

[1] Imitation is another means of separation, but it will be ruled out in this paper.

research to invent more goods than they manufacture, and those with comparative advantage in labor endowment to manufacture more goods than they invent. As a consequence, a country becomes a net exporter (importer) of inventions if it has comparative advantage in research (labor endowment). The distribution of factor income across countries is also fundamentally altered with firm mobility. Under trade without firm mobility, labor endowment and comparative advantage in research jointly determine the cross-country distribution of skill premium and capital income. Under trade with firm mobility, (productivity-adjusted) skill premium is equalized across countries, and relative research productivity alone determines the cross-country distribution of capital income and wealth.

We also examine the effects of subsidy policies. We find that without firm mobility, global growth rises whenever a country subsidizes research more than manufacturing. However, with firm mobility, global growth falls if a country with comparative advantage in research subsidizes manufacturing (for reshoring) or a country with large comparative advantage in labor endowment subsidizes research (for technological independence).

A number of papers have examined complicated relationships among trade, growth, international firm mobility, and wage premium using various models. Naturally, this paper overlaps with them in many aspects. Three lines of research are directly related. One is the literature that studies the linkage between trade and growth. Two-country endogenous models by Grossman and Helpman (1990) and Rivera-Batiz and Romer (1991a, 1991b) are prominent examples. This paper has a direct bearing on Grossman and Helpman (1990), who examines the role of comparative advantage in research in determining long-run growth. However, as in most endogenous growth models, they restrict the number of produced varieties and the number of invented varieties to be equal in each country. We supplement their work by showing how their results change when they do not have to coincide.

Another line of research is the literature on trade and agglomeration. Among them, papers by Martin and Ottaviano (1999), Martin and Ottaviano (2001), Fujita and Thisse (2003), and Baldwin and Martin (2004) are most intimately connected because they also examine the role of international firm mobility in models of endogenous growth. As this paper does, these works emphasize the need to integrate two influential approaches that developed independently: the analysis of firm location developed by the new economic geography literature, and the analysis of firm creation emphasized by endogenous growth models. Works in new economic geography frequently abstract from differences among countries and focus on the possibility of catastrophic concentration of economic activities on a single location. In this paper, we repress agglomeration forces such as labor mobility and vertically linkages among intermediates producers, and instead focus on the effect of natural comparative advantages on the world distribution of economic activities.[2] Due to the difference, we obtain different results in a parallel setting. In contrast to Baldwin and Martin (2004), this paper finds that international firm mobility has a strong impact on global growth when the cross-country distribution of relative research productivity and that of labor endowment greatly differ from each other.

We also have a vast literature on trade and skill premium, especially papers that investigate the role of trade in intermediate inputs or tasks in determining skill premium, such as Feenstra and Hanson (1996) and Grossman and Rossi-Hansberg (2008). However, these studies do not deal with the causal connection between intermediate goods trade and skill premium that this paper focuses on: a higher global growth rate increases the demand for skilled workers, and hence raises skill premium. In this regard, Grossman and Helpman (2018) is the most closely related to this paper. They build an endogenous growth model that is similar to ours in basic structure, but incorporates a richer structure that allows sorting and matching between heterogenous workers and firms. However, they do not examine the role of international firm mobility.

The rest of the paper is organized as follows. Section 2 builds the basic model and determines the long-run equilibrium of an autarkic economy. Section 3 examines long-run equilibria under free trade with and without international firm mobility, and compare them. Section 4 adds brief concluding remarks.

2. Long-run equilibrium in autarky

2.1. The basic model

We assume that the head of the representative family in country i chooses the path of consumption for her family to maximize the following dynastic utility.

$$Max \int_t^{\infty} \ln C_i(s)\ \exp[-\rho(s-t)]\, ds$$

[2] Redding (2020) surveys the quantitative successes of economic geography models that explain the geographic distribution of economic activities based upon first-nature heterogeneity among regions. However, our paper, at the current stage, does not incorporate trade costs.

$$s.t \quad \dot{V}_i = r_i\, V_i + w_{Li}\, L_i + w_{Hi}\, H_i - C_i\,, \tag{1}$$

$$\lim_{s\to\infty} V_i(s) \exp[-\int_t^s r_i(\zeta)\, d\zeta] \geq 0. \tag{2}$$

Subscript i denote country i. C_i is the consumption level of the family at each moment, ρ the subjective discount rate, V_i financial wealth, and r_i the interest rate. $w_{Li}\, L_i + w_{Hi}\, H_i$ determines the total labor income of the family, where w_{Li} and w_{Hi} are the wages of unskilled and skilled workers, and L_i and H_i are the number of unskilled and skilled workers in the family, which we assume are fixed. (1) is the budget constraint, and (2) is the no-Ponzi game condition. The paths of wages and the interest rate are taken as given by the household. The necessary conditions for the maximization problem are:

$$\frac{\dot{C}_i}{C_i} = r_i - \rho, \tag{3}$$

$$\lim_{s\to\infty} V_i(s) \exp\left[-\int_t^s r_i(\zeta)\, d\zeta\right] = 0. \tag{4}$$

(3) is the standard Ramsey rule for optimal consumption, and (4) is the transversality condition.

The economy produces three kinds of goods: one final good, a set of intermediate goods, and designs for new intermediate goods. The market for the final good is competitive. It is produced by unskilled workers who assemble intermediate goods using the following Cobb-Douglas production function:

$$Y_i = \frac{1}{\alpha}\, (\eta_{Yi}\, L_i)^{1-\alpha} \int_0^{A_i} m_i(u)^{\alpha} du. \tag{5}$$

η_{Yi} is the level of (Harrod-neutral) productivity in the final good sector, and $m_i(u)$ is the input of intermediate good u. A_i is the number (measure) of intermediate goods varieties available for use in the country, which in autarky, is equal to the number of intermediate goods that the country has invented. In equilibrium, L_i must be equal to the number of unskilled workers that the household supplies. We take the final good as the numeraire, and set its price equal to 1. Therefore, all prices are real. Denoting the price of intermediate good u by $p_i(u)$, the demand for intermediate good u is given by

$$m_i(u) = p_i(u)^{\frac{-1}{1-\alpha}} \eta_{Yi} L_i \,. \tag{6}$$

Competitive firms make zero profit in in equilibrium. Therefore, the minimum unit cost of Y_i should be equal to 1. This requires that

$$\frac{w_{Li}}{\eta_{Yi}} = \frac{1-\alpha}{\alpha} \int_0^{A_i} p_i(u)^{-\frac{\alpha}{1-\alpha}} du. \tag{7}$$

Intermediate goods are produced by monopolistically competitive firms using only skilled workers. One skilled worker produces η_{Xi}/α units of an intermediate good. α is a scaler for notational simplicity, and η_{Xi} is the country-specific level of productivity in the intermediate goods sector. Indicating the output of intermediate good u by $x_i(u)$, the profit from manufacturing intermediate good u is given by:

$$\pi_{Xi}(u) = p_i(u)x_i(u) - w_{Hi}\frac{\alpha}{\eta_{Xi}}x_i(u). \tag{8}$$

In autarky, $x_i(u)$ must be equal to $m_i(u)$ in (6) in equilibrium. To maximize the profit, the firm sets the marginal revenue $p_i(1 - 1/\varepsilon)$ equal to the marginal cost $w_{Hi}\,\alpha/\eta_{Xi}$. ε is the price elasticity of the demand, and by (6), is given by $1/(1 - \alpha)$. Thus,

$$p_i = \frac{w_{Hi}}{\eta_{Xi}}, \tag{9}$$

$$x_i = (\frac{w_{Hi}}{\eta_{Xi}})^{\frac{-1}{1-\alpha}} \eta_{Yi} L_i, \tag{10}$$

$$\pi_{Xi} = (1-\alpha)\, p_i\, x_i = (1-\alpha)\, (\frac{w_{Hi}}{\eta_{Xi}})^{\frac{-\alpha}{1-\alpha}} \eta_{Yi} L_i. \tag{11}$$

The equations above hold for all varieties of intermediate goods, and the argument u will be dropped henceforth. The Cobb-Douglass technology used in the final goods sector implies that the total cost of intermediate inputs $A_i\, p_i\, x_i = \alpha\, Y_i$, and the total wage bill $w_{Li}\, L_i = (1-\alpha)\, Y_i$. Therefore,

$$A_i\, p_i\, x_i = \frac{\alpha}{1-\alpha} w_{Li}\, L_i. \tag{12}$$

Finally, designs for new varieties of intermediate goods are created by research firms using the following technology:

$$\dot{A}_i(\zeta) = A_i \, D_i^{\lambda-1} \, D_i(\zeta) \text{ with } \lambda < 1. \tag{13}$$

$\dot{A}_i(\zeta)$ is the quantity of new designs invented by an individual research firm ζ at each moment, and $D_i(\zeta)$ is its input. A_i is the number of intermediate goods varieties invented so far by country i, and represents the stock of knowledge that research firms in country i have access to. We assume that $D_i(\zeta)$ is proportional to the employment of skilled labor, and thus (13) is identical to the knowledge spillover model of Romer (1990) with one difference. The efficiency of an individual research firm depends on D_i, which is the sum of $D_i(\zeta)$'s. With $\lambda < 1$, as aggregate research effort increases, the productivity of an individual research firm decreases. We have point-in-time diminishing returns to research at the aggregate level. Jones (1995) justifies the diminishing returns with the presence of negative externalities occurring from duplication in the R&D process. We introduce the diminishing returns into the model to make all countries active both in research and manufacturing in equilibrium.

An inventor of a new design acquires an exclusive and permanent right to use it, and can sell the right at the price of p_{Ai}. We assume that one skilled worker produces η_{Ai} units of $D_i(\zeta)$, η_{Ai} measuring the research productivity of the economy. Thus, the profit of each research firm can be written as $\pi_{Ai} = p_{Ai} \, A_i \, D_i^{\lambda-1} \, D_i(\zeta) - (w_{Hi}/\eta_{Ai}) \, D_i(\zeta)$. In a free-entry equilibrium with a positive and finite level of research, it must be equal to zero. Thus,

$$p_{Ai} = \frac{1}{A_i} \, D_i^{1-\lambda} \, \frac{w_{Hi}}{\eta_{Ai}}. \tag{14}$$

The left-hand side of (14) is the asset price of a design, and the right-hand side is the marginal cost of producing it. In addition, summating (13) over all research firms,

$$\dot{A}_i = A_i \, D_i^{\lambda}. \tag{15}$$

$\dot{A}_i$ is the sum of $\dot{A}_i(\zeta)$'s, and is equal to the number of new designs for intermediate goods produced by country i at each moment. It is also equal to the number of new intermediate goods that the economy starts to manufacture.

The ownership of a design for an intermediate good generates the income of π_{Xi} in (11) at each moment. Equilibrium in asset markets requires that the return on owning a design is equal to the interest rate.

$$\frac{\dot{p}_{Ai}+\pi_{Xi}}{p_{Ai}} = r_i. \tag{16}$$

(16), in conjunction with (4), implies that p_{Ai} is equal to the present value of π_{Xi}.

Now we impose market clearing conditions. We have already incorporated clearing conditions for intermediate goods and unskilled labor L_i. The market for skilled labor clears when

$$H_i = \frac{\alpha}{\eta_{Xi}} A_i\, x_i + \frac{1}{\eta_{Ai}} D_i. \tag{17}$$

The first term on the right-hand side is the number of skilled workers employed for manufacturing intermediate goods. The second term is the number of skilled workers employed for creating new designs for intermediate goods. In addition, the market for the final good should clear. Note that no physical capital is produced in the economy and all final goods produced should be consumed in equilibrium. Equations (1), (3), and (4) imply that consumption at each moment is equal to the subjective discount rate multiplied by the sum of the present value of labor income and the value of financial assets. The value of financial assets is equal to the asset value of designs $p_{Ai}\, A_i$. Therefore, the final good market clears when

$$Y_i = C_i = \rho[\int_t^\infty (w_{Li}\, L_i\ + w_{Hi}\, H_i) \exp\left[-\int_t^s r_i(\zeta)\, d\zeta\right]\ ds\ +\ p_{Ai}\, A_i]\ . \tag{18}$$

2.2. Long-run equilibrium in autarky

Before we compare equilibria under free trade in the next section, we first solve for the balanced growth path of the economy when it is in autarky. On balanced growth path, the interest rate r_i and the growth rates of all variables must be constant. Let us use g_v to denote the constant growth rate of variable v. By (3), $g_{C_i} = r_i - \rho$, and by (18) $g_{C_i} = g_{Y_i}$. Because

$w_{Li}\, L_i = (1-\alpha)\, Y_i$ and L_i is constant, $g_{w_{Li}} = g_{Yi}$. By (7) and (9), $g_{w_{Li}} = g_{A_i} - \alpha/(1-\alpha)\, g_{w_{Hi}}$. By (15), g_{Ai} is equal to D_i^{λ} and D_i is constant. Hence, $g_{p_{Ai}} = g_{w_{Hi}} - g_{A_i}$ by (14), and $g_{\pi_{Xi}} = -\,\alpha/(1-\alpha)\, g_{w_{Hi}}$ by (11). $g_{p_{Ai}}$ and $g_{\pi_{Xi}}$ must be identical to satisfy (16) at a constant interest rate. This implies that $g_{w_{Hi}} = (1-\alpha) g_{A_i}$ and $g_{p_{Ai}} = g_{\pi_{Xi}} = -\alpha\, g_{A_i}$. By (10), $g_{x_i} = -\,1/(1-\alpha)\, g_{w_{Hi}} = -g_{A_i}$. Because $A_i\, p_i\, x_i = (1-\alpha)\, Y_i$, $g_{Yi} = g_{Ai} + g_{pi} + g_{xi} = g_{w_{Hi}} = (1-\alpha) g_{A_i}$. To sum up, on BGP,

$$r_i = \rho + g_{C_i}, \tag{19}$$

$$g_{C_i} = g_{Y_i} = g_{w_{Hi}} = g_{w_{Li}} = (1-\alpha) g_{A_i}, \tag{20}$$

$$g_{x_i} = -g_{A_i},\ g_{p_{Ai}} = g_{\pi_{Xi}} = -\alpha\, g_{A_i}. \tag{21}$$

In addition, (16), (19), and (21) imply that

$$p_{Ai} = \frac{1}{\rho + g_{A_i}}\, \pi_{Xi}\ . \tag{22}$$

It is convenient to discuss equilibrium conditions in terms of variables in efficiency units. Indicating variables in efficieny units by tilde hats,

$$\widetilde{w}_{Li} \equiv \frac{w_{Li}}{\eta_{Yi}},\ \widetilde{w}_{Hi} \equiv \frac{w_{Hi}}{\eta_{Xi}},\ \widetilde{\omega}_i \equiv \frac{\widetilde{w}_{Hi}}{\widetilde{w}_{Hi}} = \frac{w_{Hi}}{w_{Li}}\, \frac{\eta_{Yi}}{\eta_{Xi}}, \tag{23}$$

$$\widetilde{L}_i \equiv \eta_{Yi}\, L_i,\ \widetilde{H}_i \equiv \eta_{Xi}\, H_i. \tag{24}$$

We will call $\widetilde{w}_{Li}$ and $\widetilde{w}_{Hi}$ effective unskilled wage and effective skilled wage. We will call $\widetilde{\omega}_i$ effective skill premium, which is proportional to the unadjusted skill premium w_H/w_L as long as η_Y/η_X is constant. $\widetilde{L}$ and $\widetilde{H}$ will be called effective unskilled labor and effective skilled labor. Additionally, we will use γ_i to denote the productivity of country *i* in research relative to that in manufacturing intermediate goods.

$$\gamma_i \equiv \frac{\eta_{Ai}}{\eta_{Xi}}. \tag{25}$$

By (14) and (15), $p_{Ai} = A_i^{-1}\, g_{A_i}^{\frac{1-\lambda}{\lambda}}\, \widetilde{w}_{Hi}\, \gamma_i^{-1}$. The asset price of a design is equal to the marginal cost of producing it, which is decreasing in the stock of knowledge, increasing in the rate of invention (due to the diminishing returns to research), increasing in the effective wage of skilled workers. and decreasing in the relative productivity in research. (11) and (12) imply that $\pi_{Xi} = \alpha\, A_i^{-1}\, \widetilde{w}_{Li}\, \tilde{L}_i$. The total profits of intermediates manufacturers $A_i\, \pi_{Xi}$ is proportional to their total sales $A_i\, p_i\, x_i$, which in turn is proportional to the total wage bill in the final goods sector. Plugging the two equations into (22) and rearranging terms:

$$\widetilde{\omega}_i = \gamma_i\, \tilde{L}_i\, \alpha\, \left(\rho + g_{A_i}\right)^{-1} g_{A_i}^{-\frac{1-\lambda}{\lambda}}. \tag{26}$$

We will call (26) the asset market equilibrium condition as it mainly derives from (22).

It is convenient for our discussion that the following relationship holds

$$D_i = \frac{g_{A_i}}{\rho + g_{A_i}} (1-\alpha)\, \gamma_i\, A_i\, x_i.^{3} \tag{27}$$

The clearing condition for skilled labor in (17) becomes $H_i = \left(\alpha + g_{A_i}\left(\rho + g_{A_i}\right)^{-1}(1-\alpha)\right) 1/\eta_{Xi}\ A_i\, x_i$. Multiplying both sides of the equation by w_{Hi} and rearranging terms using (12),

$$\widetilde{\omega}_i{}^{a}\, \frac{\tilde{H}_i}{\tilde{L}_i} = \alpha \left(\frac{\alpha}{1-\alpha} + \frac{g_{A_i}^{a}}{\rho + g_{A_i}^{a}} \right). \tag{28}$$

We can call this equation the labor market equilibrium condition. Superscript a indicates equilibrium values in autarky.

Combining (26) and (28), we can determine the growth rate of the economy on BGP.[4]

[3] To see this, rewrite (14) as $p_{Ai} = D_i/A_i\ g_{A_i}^{-1}\ w_{Hi}/\eta_{Ai}$ and note that $\pi_{Xi} = (1-\alpha)\ w_{Hi}/\eta_{Xi}\ x_i$. Plugging these two equations into (22), we obtain (26).

[4] We can check that the clearing condition for the final good in (18) is automatically satisfied if (26) and (28) hold. The Walras law applies.

$$\left(\frac{\alpha}{1-\alpha} + \frac{g_{A_i}^{\alpha}}{\rho+g_{A_i}^{\alpha}}\right) = \gamma_i\, \widetilde{H}_i \left(\rho + g_{A_i}^{\alpha}\right)^{-1} g_{A_i}^{\alpha}{}^{-\frac{1-\lambda}{\lambda}}. \tag{29}$$

The left-hand side of (29) is increasing in g_{A_i}. The right-hand side is decreasing in g_{A_i}, and spans from infinity to zero as g_{A_i} increases from zero. Therefore, balanced growth path with a positive value of g_{A_i} is uniquely determined. Once the growth rate is determined, we can determine the skill premium using (28). Using (28) and (29), we can easily see that if the discount rate ρ increases, the growth rate and the skill premium fall. If α increases, the growth rate falls, but the skill premium increases. We can also see that if $\tilde{L}_i$ increases, the growth rate is not affected, but the skill premium increases.

We can also easily check the effects of other parameters using the same method. If γ_i increases, both the growth rate and the skill premium increases. When γ_i increases, it lowers the marginal cost of inventing an intermediate good relative to the marginal cost of producing it, and stimulates knowledge creation. This, in turn, put an upward pressure on effective skilled wage by increasing the demand for skilled labor. If $\widetilde{H}_i$ increases, the growth rate increases, but the skill premium falls. A larger stock of skilled labor depresses skilled wage and it stimulates research.[5] These results are as expected from a Romer-type endogenous growth model. What our model additionally shows is how the skill premium moves at the same time.

So far, we have assumed that the stock of knowledge that research firms in country i can use is limited by A_i, the number of intermediate goods varieties that country i has invented on its own. However, country i researchers may have access to knowledge created in foreign countries even without trade with foreign countries. Empirical evidence tends to support the view that trade or foreign direct investment stimulates cross-border knowledge spillovers (Keller; 2004, 2021) However, there is much controversy on the existence and size of effects that trade or foreign direct investment has on cross-country knowledge spillovers. (e.g., Branstetter, 2001) Investigating the role of trade or foreign direct investment in spreading knowledge across countries is not the purpose of this paper, though it is quite an important topic. Our objective is to highlight the role of trade and foreign direct investment (or technology

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[5] Our model inherits the much-debated property of the standard endogenous growth models: scale economies prevail and a country with a larger stock of human capital grows faster.

licensing) per se in the determination of world growth and income distribution. To isolate the effect of trade or international firm mobility from their effects on knowledge spillovers, we can use the assumption that knowledge spillovers are global before and after trade, and with or without international mobility. Then, all changes in equilibrium can be attributed to the effect of trade or international firm mobility per se. With global knowledge spillovers, (15) is replaced by:

$$\dot{A}_i = A\, D_i^{\lambda}. \tag{30}$$

The stock of knowledge that each country has access to is given by A, which is the sum of A_i over i's. If there is no international patent law that prohibits firms from using designs created by foreign firms, the set of intermediate goods invented by countries can easily overlap. In the autarkic world regime, this situation is likely to prevail, but again to isolate the pure effect of trade, let us assume that an international patent law prevails, and A_i's are disjoint. We focus on balanced growth equilibrium where all countries grow at the same rate: $g_{A_i} = \dot{A}_i / A_i = g_A$ for all i where g_A is the world grow rate of knowledge. Using the same method as before, we can show that the following equations hold on balanced growth path.

$$\sigma_i^a = \frac{(\gamma_i\, \widetilde{H}_i)^{\lambda}}{\sum_{i=1}^{J} (\gamma_i\, \widetilde{H}_i)^{\lambda}}. \tag{31}$$

$$\widetilde{\omega}_i^a\, \frac{\widetilde{H}_i}{\widetilde{L}_i} = \alpha \left(\frac{\alpha}{1-\alpha} + \frac{g_A^a}{\rho + g_A^a} \right). \tag{32}$$

$$\frac{\alpha}{1-\alpha} + \frac{g_A^a}{\rho + g_A^a} = \left(\sum_{i=1}^{J} \left(\gamma_i\, \widetilde{H}_i \right)^{\lambda} \right)^{\frac{1}{\lambda}} (\rho + g_A^a)^{-1}\, g_A^{a\,-\frac{1-\lambda}{\lambda}}. \tag{33}$$

σ_i will be used to dempte the share of country i in the creation of intermediate goods designs: A_i / A. Because all countries grow at the same rate, the share of each country in world invention is constant. (32) and (33) correspond to (28) and (29). Note that (32) is identical to (28) except in that g_{Ai} is replaced by g_A because g_{Ai} is identical across countries with global knowledge spillovers. In (33), $\gamma_i\, \widetilde{H}_i$ in (29) is replaced by their sum in CES form $\left(\sum_{i=1}^{J} \left(\gamma_i\, \widetilde{H}_i \right)^{\lambda} \right)^{\frac{1}{\lambda}}$. The proofs for (31), (32), and (33) are provided in the Appendix.

3. Long-run equilibrium under free trade

In this section, we extend the previous model to a multi-country model. There are J countries in the world, and they costless trade with each other in all goods: the final good and all intermediate goods. We assume that all countries are identical except in five parameters: skilled labor endowment $\widetilde{H}_i$, unskilled labor endowment $\tilde{L}_i$, final goods productivity η_{Yi}, intermediate goods productivity η_{Xi}, and research productivity η_{Ai}. The other parameters for preferences and technologies are identical across countries. We also assume that knowledge spillovers are global with free trade regardless of firm mobility. We will compare the results under two different regimes: one with trade but without international firm mobility and the other with trade and international firm mobility. With international firm mobility, an inventor of a design for an intermediate good can transfer the ownership of the technology or license its use to an intermediate good manufacturer located in another country. Thus, the number of intermediate goods varieties manufactured in a country may exceed or fall short of the number of varieties that has been invented in the country. We first investigate the case where there is no firm mobility and the number of varieties that a country produces is restricted to be equal to the number of varieties that it has invented. This is the case that most papers on endogenous growth in open economies investigate. As in the previous section, we focus only on balanced growth path, ignoring transitional dynamics.

3.1. Long-run equilibrium without international firm mobility

With free trade, most equilibrium conditions that we derived for autarky are still valid, but several of them should be modified to reflect international transactions. First, an intermediate good producer can now sell its product to all firms in the world. Instead of (6), a producer of intermediate good u located in country i faces the following world-wide demand:

$$x_i(u) = p_i(u)^{\frac{-1}{1-\alpha}}\tilde{L}\,. \tag{34}$$

$\tilde{L}$ is equal to $\sum_{j=1}^{J}\tilde{L}_j$. Because the price elasticity is identical and equal to $1/(1-\alpha)$ in all countries, a firm in country *i* sets an identical price for all destinations and

$$p_i(u) = \widetilde{w}_{Hi}. \tag{35}$$

Hence,

$$\pi_{Xi} = (1-\alpha)\, p_i\, x_i = (1-\alpha)\, \widetilde{w}_{Hi}{}^{\frac{-\alpha}{1-\alpha}}\, \tilde{L}. \tag{36}$$

With free trade, the prices of the final good and all intermediate goods are equalized across countries. This implies that $\widetilde{w}_{Li}$ is also equalized across countries. From (7),

$$\widetilde{w}_L = \frac{1-\alpha}{\alpha} \int_0^A p(u)^{-\frac{\alpha}{1-\alpha}}\, du = \frac{1-\alpha}{\alpha} \sum_{j=1}^{J} A_j\, \widetilde{w}_{Hi}{}^{\frac{-\alpha}{1-\alpha}} = \frac{1-\alpha}{\alpha}\, A \sum_{j=1}^{J} \sigma_j\, \widetilde{w}_{Hi}{}^{\frac{-\alpha}{1-\alpha}}. \tag{37}$$

I dropped subscript i in $\widetilde{w}_{Li}$ because it is identical in all countries.

With global knowledge spillovers, (14) is modified to

$$p_{Ai} = \frac{1}{A}\, D_i^{1-\lambda}\, \widetilde{w}_{Hi}\, \gamma_i^{-1}\,. \tag{38}$$

As in the balanced growth equilibrium for the world of autarkic economies with global knowledge spillovers, g_{Ai} should be identical to g_A in every country, and σ_i and the relative size of each country in consumption and outputs are all maintained constant. By (30), $g_A = D_i^{\lambda}/\sigma_i$, and D_i^{λ} should be proportional to σ_i on balanced growth path. In addition, the relationships between g_A and other variables set in (19) through (21) should continue to hold in each country. (23) also holds, but with asset prices and profits different for countries.

Plugging (36) and (38) into the asset market equilibrium condition in (22), which now holds for a common growth rate g_A, and imposing the condition that $\sum_{i=1}^{J} \sigma_i = 1$ yields:

$$\sigma_i = \frac{\widetilde{\omega}_i{}^{\frac{-1}{1-\alpha}\frac{\lambda}{1-\lambda}}\, \gamma_i^{\frac{\lambda}{1-\lambda}}}{\sum_{i=1}^{J} \widetilde{\omega}_i{}^{\frac{-1}{1-\alpha}\frac{\lambda}{1-\lambda}}\, \gamma_i^{\frac{\lambda}{1-\lambda}}}. \tag{39}$$

The share of each country in the production and invention of intermediate goods is increasing in γ_i and decreasing in the skilled wage. When γ_i rises or $\widetilde{w}_{Hi}$ falls, the marginal cost of producing a design in the left-hand side of (38) decreases, and thus raises the country's share in world knowledge creation. Using (37) and (39), we can also show that

$$\frac{\sum_{i=1}^{J} \tilde{\omega}_i^{\frac{-1}{1-\alpha}\left(\alpha+\frac{\lambda}{1-\lambda}\right)} \gamma_i^{\frac{\lambda}{1-\lambda}}}{\left(\sum_{i=1}^{J} \tilde{\omega}_i^{\frac{-1}{1-\alpha}\frac{\lambda}{1-\lambda}} \gamma_i^{\frac{\lambda}{1-\lambda}}\right)^{\frac{1}{\lambda}}} = \tilde{L}\,\alpha\,(\rho+g_A)^{-1}\,g_A^{-\frac{1-\lambda}{\lambda}}, \tag{40}$$

The proofs for (39) and (40) can be found in the Appendix.

(40) is the asset market equilibrium condition for the world economy. To find the labor market equilibrium condition, we first note that (12) does not hold for an individual economy, and holds only for the world economy.

$$\sum_{i=1}^{J} A_i\, p_i\, x_i = \frac{\alpha}{1-\alpha} \sum_{i=1}^{J} w_{Li}\, L_i. \tag{41}$$

To see this, note that in the final good production of country i, the total cost of intermediate inputs is still proportional to the wage bill: $\sum_{j=1}^{J} A_j\, p_j\, m_{ji} = \frac{\alpha}{1-\alpha} w_{Li}\, L_i$, where m_{ji} is the amount of an intermediate good that is produced in country *j* and used in country *i* in the production of the final good. However, the world market for each intermediate good should be cleared, meaning $\sum_{i=1}^{J} m_{ji} = x_j$. (41) follows.

(27) still holds in each country with global knowledge spillovers.

$$D_i = (1-\alpha)\frac{g_A}{\rho+g_A}\gamma_i\, A_i\, x_i.^{6} \tag{42}$$

Therefore, the clearing of skilled labor requires

$$H_i = \frac{\alpha}{\eta_{Xi}} A_i\, x_i + \frac{1}{\eta_{Ai}} D_i = \left(\alpha + (1-\alpha)\frac{g_A}{\rho+g_A}\right)\frac{1}{\eta_{Xi}}\, A_i\, x_i. \tag{43}$$

Multiplying both sides by w_{Hi}, and making a summation over i's,

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[6] $g_A = A/A_i\, D_i^{\lambda} = A\, D_i^{\lambda-1}\, D_i/A_i$. Thus, $D_i^{1-\lambda} = g_A^{-1}\, A\; D_i/A_i$. Plugging the equation into (38) and (22), $p_{Ai} = g_A^{-1}\, D_i/A_i\, w_{Hi}/\eta_{Ai} = (\rho+g_A)^{-1}\,\pi_{Xi} = (\rho+g_A)^{-1}(1-\alpha)\; w_{Hi}/\eta_{Xi}\; x_i$. (42) follows.

$$\sum_{i=1}^{J} w_{Hi} H_i \ = \left(\alpha \ + (1-\alpha)\frac{g_A}{\rho+g_A} \ \right)\sum_{i=1}^{J} A_i \, p_i \, x_i \ . \tag{44}$$

Using (41),

$$\frac{\sum_{i=1}^{J} w_{Hi} H_i}{\sum_{i=1}^{J} w_{Li} \, L_i} = \frac{\sum_{i=1}^{J} \widetilde{\omega}_i \, \widetilde{H}_i}{\widetilde{L}} = \ \alpha \ \left(\frac{\alpha}{1-\alpha} \ + \frac{g_A}{\rho+g_A} \ \right). \tag{45}$$

For the first equality, we used the fact that $\widetilde{w}_{Li}$ is identical in all countries.

(39) and (45) are the key equations for determining equilibrium under free trade without international firm mobility. We still have to find a vector $\{\widetilde{\omega}_i; \ i = 1, \dots, J\}$ satisfying the two equations. We can show that $\widetilde{\omega}_i$'s should be distributed across countries by the following formula:

$$\widetilde{\omega}_i = \widetilde{\omega}_M^n \frac{\widetilde{H}}{\sum_{i=1}^{J} \widetilde{H}_i^{\alpha} \left(\frac{\eta_{Ai}}{\eta_{Xi}} \widetilde{H}_i\right)^{\lambda(1-\alpha)}} \ \widetilde{H}_i^{-(1-\alpha)(1-\lambda)} \gamma_i^{\lambda(1-\alpha)}. \tag{46}$$

where $\widetilde{\omega}_M^n$ is a positive constant and $\widetilde{H}$ is equal to $\sum_{j=1}^{J} \widetilde{H}_j$. Note that $\sum_{i=1}^{J} \widetilde{\omega}_i \, \widetilde{H}_i / \sum_{i=1}^{J} \widetilde{H}_i \ = \widetilde{\omega}_M^n$. Therefore, $\widetilde{\omega}_M^n$ is a weighted average of $\widetilde{\omega}_i$'s, and can be called the average skill premium under free trade without firm mobility. Superscript n indicates the case of no firm mobility.

Plugging (46) into (39),

$$\sigma_i^n = \frac{(\gamma_i \, \widetilde{H}_i)^{\lambda}}{\sum_{i=1}^{J} (\gamma_i \, \widetilde{H}_i)^{\lambda}}. \tag{47}$$

Note that $\gamma_i \, \widetilde{H}_i = \eta_{Ai} \, H_i$, which is equal to the maximum amount of effective labor that country *i* can allocate in the research sector and can be called research potential of the country. The share of each country in the world knowledge creation is determined by its relative research potential.

Plugging (47) into (40) and (45), we can obtain that

$$\tilde{\omega}_M^n \frac{\tilde{H}}{\tilde{L}} = \alpha \left(\frac{\alpha}{1-\alpha} + \frac{g_A^n}{\rho + g_A^n} \right). \tag{48}$$

$$\frac{\alpha}{1-\alpha} + \frac{g_A^n}{\rho + g_A^n} = \left(\sum_{i=1}^{J} \left(\gamma_i \, \tilde{H}_i \right)^{\lambda} \right)^{\frac{1}{\lambda}} (\rho + g_A^n)^{-1} \, g_A^{n^{-\frac{1-\lambda}{\lambda}}}. \tag{49}$$

(48) and (49) together determines the equilibrium value of $\tilde{\omega}_M^n$ and g_A^n. We made them isomorphic to equations (28) and (29) or equations (32) and (33) for easy comparisons later. Once we know the value of $\tilde{\omega}_M^n$, we can determine all $\tilde{\omega}_i$'s using (46). The derivation of (46) through (49) is provided in the Appendix.

3.2. Long-run equilibrium with international firm mobility

Deriving equilibrium conditions for the case of free trade with international firm mobility is much simpler because it is a special case of free trade without international firm mobility. Most equations that we derived for the case of firm immobility are still valid. We have only to make two modifications. One comes from the fact that $\tilde{w}_{Hi}$ should be equalized across countries with firm mobility. If $\tilde{w}_{Hi}$ is lower in one country than in other countries, by (36), the profit of an intermediate good manufacturer located in the country is higher than the profit of firms located in other countries. Thus, the owner of a design can increase its asset value by moving production to the country with a lower $\tilde{w}_{Hi}$. Thus, in an equilibrium where all countries produce a positive amount of intermediate goods, $\tilde{w}_{Hi}$ should be identical everywhere. Because $\tilde{w}_{Li}$ is already equalized among countries by the free trade of goods, $\tilde{\omega}_i$ should also be identical in all countries. We denote this common skill premium by $\tilde{\omega}^m$. Superscript m indicates the case of firm mobility.

When $\tilde{\omega}_i$'s are identical everywhere, (39) is reduced to

$$\sigma_i^m = \frac{\gamma_i^{\frac{\lambda}{1-\lambda}}}{\sum_{i=1}^{J} \gamma_i^{\frac{\lambda}{1-\lambda}}}. \tag{50}$$

If all $\tilde{\omega}_i$'s are equal, (40) becomes

$$\tilde{\omega}^m = \left(\sum_{i=1}^{J} \gamma_i^{\frac{\lambda}{1-\lambda}} \right)^{\frac{1-\lambda}{\lambda}} \tilde{L} \, \alpha \, (\rho + g_A^m)^{-1} \, g_A^{m^{-\frac{1-\lambda}{\lambda}}}. \tag{51}$$

The other modification necessary is the labor market equilibrium condition. (43) should be altered to reflect the fact that the number of goods produced can differ from the number of goods invented.

$$H_i = \frac{\alpha}{\eta_{Xi}} N_i\, x_i + \frac{1}{\eta_{Ai}} D_i = \left(\alpha\, N_i + (1-\alpha)\frac{g_A}{\rho+g_A}\, A_i\right)\frac{1}{\eta_{Xi}}\, x_i. \tag{52}$$

N_i is the number of intermediate goods that country i produces, which may be different from A_i, the number of intermediate goods invented by the country. Multiplying both sides of the equation by w_{Hi} and summating over i 's, we get $\sum_{i=1}^{J} w_{Hi} H_i = (\alpha + (1-\alpha)\ g_A/(\rho+g_A)\,)\, A\, p\, x$. We used the fact that $\sum_{i=1}^{J} N_i = \sum_{i=1}^{J} A_i = A$, and p_i and x_i is identical for any i because the effective skilled wage is equalized. Using (41), $A\, p\, x = \sum_{i=1}^{J} A_i\, p_i\, x_i = \alpha/(1-\alpha)\ \sum_{i=1}^{J} w_{Li}\, L_i$. Therefore, the labor market equilibrium condition becomes

$$\frac{\sum_{i=1}^{J} w_{Hi} H_i}{\sum_{i=1}^{J} w_{Li}\, L_i} = \widetilde{\omega}^m \frac{\widetilde{H}}{\widetilde{L}} = \alpha\left(\frac{\alpha}{1-\alpha} + \frac{g_A^m}{\rho+g_A^m}\right). \tag{53}$$

(51) and (53) imply that

$$\left(\frac{\alpha}{1-\alpha} + \frac{g_A^m}{\rho+g_A^m}\right) = \left(\sum_{i=1}^{J} \gamma_i^{\frac{\lambda}{1-\lambda}}\right)^{\frac{1-\lambda}{\lambda}} \widetilde{H}\, (\rho+g_A^m)^{-1}\, g_A^{m^{-\frac{1-\lambda}{\lambda}}}. \tag{54}$$

(54) determines the long-run growth rate of the world economy. Then, from (53), we can read the corresponding value of the skill premium.[7]

3.3. Comparisons

[7] We have not checked for the clearing of the final goods market in the two free-trade regimes, but we can show that the world final goods market always clears once the equilibrium conditions that we presented are satisfied.

Now we proceed to compare equilibria with and without firm mobility. Before doing that, we first compare autarky and free trade equilibria.

[Proposition 1] Suppose that knowledge spillovers in research are national in autarky, but become global after trade. On balanced growth path, every country grows faster under trade without firm mobility than in autarky. The skill premium is, on average, higher under trade without firm mobility than in autarky.

Proof) By comparing (29) and (49), we can see that $g_A^n \geq g_A^a$ if and only if $\left(\sum_{i=1}^{J}\left(\gamma_i \widetilde{H}_i\right)^{\lambda}\right)^{\frac{1}{\lambda}} \geq \gamma_i \widetilde{H}_i$. The inequality always holds. By (28) and (48), $\widetilde{\omega}_M^n \widetilde{L}_i/\widetilde{L} \geq \widetilde{\omega}_i^a \widetilde{H}_i/\widetilde{H}$. Thus, $\widetilde{\omega}_M^n \equiv \sum_{i=1}^{J} \widetilde{\omega}_i^n \widetilde{H}_i/\widetilde{H} \geq \sum_{i=1}^{J} \widetilde{\omega}_i^a \widetilde{H}_i/\widetilde{H}$. The weighted average of countries' skill premiums under free trade without firm mobility is greater that the weighted average of skill premiums in autarky where weights are given by countries' shares of effective skilled labor.

[Proposition 2] Suppose that knowledge spillovers in research are global both in autarky and under trade. On balanced growth path, the world growth rate and the average skill premium of countries do not change when the world shifts from autarky to trade without firm mobility.

Proof) Compare (33) and (49). They are identical and hence g_A^a and g_A^n must be equal. Using (32) and (48), we can see that $\widetilde{\omega}_M^n \widetilde{L}_i/\widetilde{L} = \widetilde{\omega}_i^a \widetilde{H}_i/\widetilde{H}$. Then, $\widetilde{\omega}_M^n = \sum_{i=1}^{J} \widetilde{\omega}_i^n \widetilde{H}_i/\widetilde{H} = \sum_{i=1}^{J} \widetilde{\omega}_i^a \widetilde{H}_i/\sum_{i=1}^{J} \widetilde{H}_i$. Thus, $\widetilde{\omega}_M^n$ is identical to the weighted average of $\widetilde{\omega}_i^a$, where weights are given by countries' shares of effective skilled labor.

Therefore, trade has no effect on global growth. We can also see by comparing (31) and (47), the share of each country in the world knowledge creation does not change with trade. Of course, this does not mean that trade has no effect. It has level effects. By increasing the number of intermediate good varieties available for the production of the final good, it raises the total factor productivity of the final good sector and hence welfare.

Before we go to the next proposition, we prove the following lemma.

[Lemma 1]

$$\left(\sum_{i=1}^{J}\left(\gamma_i\,\widetilde{H}_i\right)^{\lambda}\right)^{\frac{1}{\lambda}} \leq \left(\sum_{i=1}^{J}\gamma_i^{\frac{\lambda}{1-\lambda}}\right)^{\frac{1-\lambda}{\lambda}}\widetilde{H}.$$

Proof)

$$\frac{\left(\sum_{i=1}^{J}(\gamma_i\,\widetilde{H}_i)^{\lambda}\right)^{\frac{1}{\lambda}}}{\left(\sum_{i=1}^{J}\gamma_i^{\frac{\lambda}{1-\lambda}}\right)^{\frac{1}{\lambda}}} = \left(\sum_{i=1}^{J}\frac{\gamma_i^{\frac{\lambda}{1-\lambda}}}{\sum_{i=1}^{J}\gamma_i^{\frac{\lambda}{1-\lambda}}}\left(\gamma_i^{\frac{-\lambda}{1-\lambda}}\,\widetilde{H}_i\right)^{\lambda}\right)^{\frac{1}{\lambda}} \leq \left(\sum_{i=1}^{J}\frac{\gamma_i^{\frac{\lambda}{1-\lambda}}}{\sum_{i=1}^{J}\gamma_i^{\frac{\lambda}{1-\lambda}}}\,\gamma_i^{\frac{-\lambda}{1-\lambda}}\,\widetilde{H}_i\right)^{\lambda\frac{1}{\lambda}} = \frac{\widetilde{H}}{\sum_{i=1}^{J}\gamma_i^{\frac{\lambda}{1-\lambda}}}.$$

The inequality comes from Jensen's inequality $E\left[X^{\lambda}\right] \leq (E[X])^{\lambda}$ for $\lambda < 1$, where E is an expected value operator.

[Proposition 3] On balanced growth path under trade, a country grows faster with firm mobility than without. The skill premium is, on average, higher with firm mobility than without.

Proof) The first part of the proposition follows from (49), (54), and Lemma 1. Because $g_A^m \geq g_A^n$, $\widetilde{\omega}^m \geq \widetilde{\omega}_M^n = \sum_{i=1}^{J}\widetilde{\omega}_i^n\,\widetilde{H}_i/\sum_{i=1}^{J}\widetilde{H}_i$ by (48) and (53).

In this case, effective skill premiums that were different among countries in the absence of firm mobility converge to a common value with firm mobility. According to Proposition 3, the common skill premium with firm mobility must be higher than the average premium without firm mobility. However, the premium may fall with firm mobilty in a country whose skill premium was relatively high in the absence of firm mobility. According to (46), these countries are those that have a lower endowment of skilled labor or a higher relative research productivity (γ_i) compared to others.

The following proposition also follows.

[Proposition 4] Under trade with firm mobility, a country becomes a net exporter of inventions if and only if the country has comparative advantage in research. Formally,

$$(A_i - N_i)\,(1-\alpha)\,p\,x = \frac{1-\alpha}{\alpha}\,\widetilde{w}_H\,\widetilde{H}\left(\frac{\gamma_i^{\frac{\lambda}{1-\lambda}}}{\sum_{i=1}^{J}\gamma_i^{\frac{\lambda}{1-\lambda}}} - \frac{\widetilde{H}_i}{\widetilde{H}}\right). \tag{55}$$

The proofs are in the Appendix. We define that a country has comparative advantage in research if $\gamma_i^{\frac{\lambda}{1-\lambda}} \Big/ \sum_{i=1}^{J} \gamma_i^{\frac{\lambda}{1-\lambda}} > \widetilde{H}_i / \widetilde{H}$, and it has comparative advantage in labor endowment if the reverse holds. Note that $(1-\alpha)\, p\, x$ is the amount of operating profits that each design of intermediate goods generates. This is also equal to dividends (in the case of FDI), license fees (in the case of technology licensing) or financial return on the asset value of a design (in the case of patent sales) depending on the method of technology transfer. Thus, $(A_i - N_i)\,(1-\alpha)\, p\, x$ is equal to total income from abroad that country *i* receives by offshoring the production of intermediate goods that it has invented.

Proposition 4 helps us understand why world growth increases with firm mobility. The possibility of geographical separation between innovation and manufacturing allows a more efficient world division of labor between innovation and manufacturing. It induces countries with comparative advantage in research to invent more goods than it manufactures, and those with comparative advantage in labor endowment to manufacture more goods than it invents.

The following proposition demonstrates that international firm mobility has a big impact on global growth when the cross-country distribution of relative research productivity γ_i and that of effective skilled labor $\widetilde{H}_i$ greatly differ from each other.

[Proposition 5] On balanced growth path under trade without international firm mobility,

1) if $d\,\widetilde{H}_i = -d\,\widetilde{H}_j > 0$ and $\gamma_i^{\frac{\lambda}{1-\lambda}} \Big/ \gamma_j^{\frac{\lambda}{1-\lambda}} > \widetilde{H}_i / \widetilde{H}_j$, the world growth rate rises.
2) the world growth rate is maximized for a given value of $\sum_{i=1}^{J} \widetilde{H}_i$ when $\widetilde{H}_i = k\,\gamma_i^{\frac{\lambda}{1-\lambda}}$ for every i for a positive constant k. In this case, the world economy under trade without firm mobility is identical to that under trade with firm mobility.

1) can be proved by differentiating the term $\sum_{i=1}^{J} \left(\gamma_i\, \widetilde{H}_i\right)^{\lambda}$ in (49). 1) tells that if we move effective skilled labor from a country with comparative advantage in endowment to a country with comparative advantage in research, global growth increases. In fact, we can execute this reallocation and raise global growth until the distribution of $\gamma_i^{\frac{\lambda}{1-\lambda}}$ is perfectly aligned with that of $\widetilde{H}_i$. That is when the world growth rate is maximized as 2) argues. The statement can also be proven by examining the proof for Lemma 1. The Jensen's inequality holds with equality

only when $\gamma_i^{\frac{-\lambda}{1-\lambda}} \widetilde{H}_i$ is constant. Note also from (54) that in the case of firm mobility, the reallocation of effective skilled labor across countries has no effect on growth.

We can also make some observations about the effect of firm mobility on the cross-country distribution of wages and capital income, and on the pattern of trade. As we have seen, free trade equalizes the effective unskilled wage $\widetilde{w}_{Li}$ across countries with or without international firm mobility. In the absence of firm mobility, the effective skilled wage $\widetilde{w}_{Hi}$ and the effective skill premium $\widetilde{\omega}_i$ are proportional to $\widetilde{H}_i^{-(1-\alpha)(1-\lambda)} \gamma_i^{\lambda(1-\alpha)}$ according to (46). They depend negatively on effective skilled labor endowment, and positively on relative research productivity. With firm mobility, the influence of skilled labor endowment and research productivity on the cross-country distribution of the effective skilled wage vanishes. It is completely equalized across countries.

The world does not produce any physical capital. Thus, wealth and capital income are totally governed by the value of intangible capital A_i. More precisely, under free trade without firm mobility,

$$\frac{(1-\alpha)\, A_i\, p_i\, x_i}{\sum_{i=1}^{J}(1-\alpha)\, A_i\, p_i\, x_i} = \frac{p_{Ai}\, A_i}{\sum_{i=1}^{J} p_{Ai}\, A_i} = \frac{\sigma_i\, \widetilde{\omega}_i^{-\frac{\alpha}{1-\alpha}}}{\sum_{i=1}^{J} \sigma_i\, \widetilde{\omega}_i^{-\frac{\alpha}{1-\alpha}}} = \frac{\widetilde{H}_i^{\alpha+\lambda(1-\alpha)}\, \gamma_i^{\lambda(1-\alpha)}}{\sum_{i=1}^{J} \widetilde{H}_i^{\alpha+\lambda(1-\alpha)}\, \gamma_i^{\lambda(1-\alpha)}}. \tag{56}$$

(56) follows from (30), (37), (38), (41), (46), and (47). The share of country i in world capital income (operating profits) or in world wealth is jointly determined by effective skilled labor $\widetilde{H}_i$ and relative research productivity γ_i. With international firm mobility, the influence of labor endowment disappears, and the cross-country distribution of capital income and wealth is solely determined by relative research productivity. From (50),

$$\frac{(1-\alpha)\, A_i\, p_i\, x_i}{\sum_{i=1}^{J}(1-\alpha)\, A_i\, p_i\, x_i} = \frac{p_{Ai}\, A_i}{\sum_{i=1}^{J} p_{Ai}\, A_i} = \sigma_i = \frac{\gamma_i^{\frac{\lambda}{1-\lambda}}}{\sum_{i=1}^{J} \gamma_i^{\frac{\lambda}{1-\lambda}}}. \tag{57}$$

The contrast between the two regimes in terms of wealth distribution leads to difference in trade pattern. Before we make some comparisons, we impose an additional assumption. So far, we have not specified whether there is financial capital mobility or not. In fact, our results on balanced growth equilibrium do not depend on financial capital mobility. However, consumption and hence trade pattern on balanced growth path depend on the presence of international transactions in bonds and stocks. To simplify our discussion, let us assume that the right to use intermediate goods designs is transferred solely through a royalty contract while cross-border holdings of stocks or bonds are not feasible. In other words, every manufacturer of an intermediate good pays royalty fees equal to its operating profit π_{Xi} to the inventor of the design, whether it is located in the domestic country or in a foreign country, for the use of the design. In addition, we assume that all international transactions in royalty services are recorded in trade services account. Formally, we impose the following condition.

[Assumption 1] Balanced trade

$$Y_i - C_i + N_i\, p_i\, x_i - \sum_{j=1}^{J} A_j\, p_j\, m_{ji} \;+\; (1-\alpha)\,(A_i - N_i)\, p_i\, x_i = 0.$$

$Y_i - C_i$ is equal to the net exports of the final good, $N_i\, p_i\, x_i - \sum_{j=1}^{J} A_j\, p_j\, m_{ji}$ is the net exports of intermediate goods, and $(1-\alpha)\,(A_i - N_i)\, p_i\, x_i$ is the net export of royalty services (or service account balance). In the case where there is no international firm mobility, the net export of royalty services must be zero.

Using Assumption 1 and Proposition 4, we can make the following observations on trade pattern without and with international firm mobility. Under free trade without international firm mobility,

$$A_i\, p_i\, x_i - \sum_{j=1}^{J} A_j\, p_j\, m_{ji} \;=\; C_i - Y_i = \frac{\alpha}{1-\alpha}\, \widetilde{w}_L\, \widetilde{L} \left(\frac{\widetilde{H}_i^{\alpha+\lambda(1-\alpha)}\, \gamma_i^{\lambda(1-\alpha)}}{\sum_{i=1}^{J} \widetilde{H}_i^{\alpha+\lambda(1-\alpha)}\, \gamma_i^{\lambda(1-\alpha)}} - \frac{\widetilde{L}_i}{\widetilde{L}} \right). \tag{58}$$

From (55), $\widetilde{H}_i^{\alpha+\lambda(1-\alpha)}\, \gamma_i^{\lambda(1-\alpha)} / \sum_{i=1}^{J} \widetilde{H}_i^{\alpha+\lambda(1-\alpha)}\, \gamma_i^{\lambda(1-\alpha)}$ is the share of country i in world wealth. An increase of this share raises the demand for final goods through wealth effect on consumption, while an increase of effective unskilled labor stimulates their supply. A country becomes a net importer of final goods and a net exporter of intermediate goods when the wealth share is greater than the share of effective unskilled labor.

In the case of firm mobility, country i's net exports of intermediate goods are given by

$$N_i\, p_i\, x_i - \sum_{j=1}^{J} A_j\, p_j\, m_{ji} \;= \frac{\alpha}{1-\alpha}\widetilde{w}_L \tilde{L}\left(\frac{\gamma_i^{\frac{\lambda}{1-\lambda}}}{\sum_{i=1}^{J}\gamma_i^{\frac{\lambda}{1-\lambda}}} - \frac{\tilde{L}_i}{\tilde{L}}\right) - \frac{1-\alpha^2}{\alpha}\widetilde{w}_H \widetilde{H}\left(\frac{\gamma_i^{\frac{\lambda}{1-\lambda}}}{\sum_{i=1}^{J}\gamma_i^{\frac{\lambda}{1-\lambda}}} - \frac{\widetilde{H}_i}{\widetilde{H}}\right). \tag{59}$$

The first term on the right-hand side inherits (57), now with the wealth share solely determined by comparative advantage in research. The negative influence of the second term reflects Proposition 4. When a country has comparative advantage in research, the domestic production of intermediate goods decreases relative to the number of designs that it creates, depressing the exports of intermediate goods. (59) also implies that a country that has comparative advantage in research can become a net importer of both final and intermediate goods. The proofs for (58) and (59) can be found in the Appendix.

Finally, we examine the effects of subsidy policies. Recently, there has been a lot of discussions on supply chain independence from potential adversary counties. The concern about the security of supply chains is leading the U.S. to adopt policies for bringing back the manufacturing of critical products. The production subsidies on semiconductors or electric vehicles and their components are notable examples. At the same time, China, partly in response to the export controls of key technologies by the United States, is pursuing policies for technological independence: heavy subsidies on research in advanced technologies. We can capture the effects of these policies by introducing manufacturing and research subsidies into our model. Let s_{Xi} be the rate of subsidy on the cost of manufacturing intermediate goods. Then, the after-subsidy wage faced by intermediates manufactures becomes $(1 - s_{Xi})\, w_{Hi}$. Likewise, if we denote the rate of subsidy on the cost of research by s_{Ai}, the after-subsidy wage faced by research firms becomes $(1 - s_{Ai})\, w_{Hi}$. By repeating derivations that we conducted in 3.1 and 3.2, we can derive the following equations that determine the growth rate under free trade without and with international firm mobility.

$$1 = \left(\sum_{i=1}^{J}\left(\frac{\gamma_i \frac{1-s_{Xi}}{1-s_{Ai}}\widetilde{H}_i}{\frac{\alpha}{1-\alpha} + \frac{g_A^n}{\rho + g_A^n}\frac{1-s_{Xi}}{1-s_{Ai}}}\right)^{\lambda}\right)^{\frac{1}{\lambda}} (\rho + g_A^n)^{-1}(g_A^n)^{-\frac{1-\lambda}{\lambda}}. \tag{60}$$

$$1 = \left(\sum_{i=1}^{J}\gamma_i^{\frac{\lambda}{1-\lambda}}\; \left(\frac{1-s_{Xi}}{1-s_{Ai}}\right)^{\frac{\lambda}{1-\lambda}}\right)^{\frac{1-\lambda}{\lambda}} \sum_{i=1}^{J}\frac{\widetilde{H}_i}{\frac{\alpha}{1-\alpha} + \frac{g_A^m}{\rho + g_A^m}\frac{1-s_{Xi}}{1-s_{Ai}}}\; (\rho + g_A^m)^{-1}(g_A^m)^{-\frac{1-\lambda}{\lambda}}\,. \tag{61}$$

Again, superscript n refers to the case of no firm mobility, and superscript m the case of firm mobility. A solution for a positive growth rate always exist because the left-hand side is decreasing in the growth rate and spans from infinite to zero. From the two equations, we can derive our final proposition.

[Proposition 5] On balanced growth path under free trade,

1) without international firm mobility, global growth increases whenever $\frac{1-s_{Xi}}{1-s_{Ai}}$ increases.
2) with international firm mobility, an increase in $\frac{1-s_{Xi}}{1-s_{Ai}}$ starting from the position of no subsidies raises global growth if and only if

$$\frac{\gamma_i^{\frac{\lambda}{1-\lambda}}}{\sum_{i=1}^{J}\gamma_i^{\frac{\lambda}{1-\lambda}}} > \frac{(1-\alpha)\frac{g_A}{\rho+g_A}}{\alpha+(1-\alpha)\frac{g_A}{\rho+g_A}}\frac{\widetilde{H}_i}{\widetilde{H}}.$$

1) immediately follows from (60). 2) can be obtained by differentiating the term $\left(\sum_{i=1}^{J}\gamma_i^{\frac{\lambda}{1-\lambda}}\left(\frac{1-s_{Xi}}{1-s_{Ai}}\right)^{\frac{\lambda}{1-\lambda}}\right)^{\frac{1-\lambda}{\lambda}}\sum_{i=1}^{J}\widetilde{H}_i/\left(\frac{\alpha}{1-\alpha}+\frac{g_A^m}{\rho+g_A^m}\frac{1-s_{Xi}}{1-s_{Ai}}\right)$ in (61) with $s_{Xi}=s_{Ai}=0$. Note that $(1-s_{Xi})/(1-s_{Ai})$ increases when the subsidy rate on research increases relatively more than the subsidy rate on manufacturing. Note that the term $(1-\alpha)\frac{g_A}{\rho+g_A}/(\alpha+(1-\alpha)\frac{g_A}{\rho+g_A})$ is less than 1. Thus, global growth increases even when a country with comparative advantage in labor endowment adopts subsidies favoring research if $\gamma_i^{\frac{\lambda}{1-\lambda}}/\sum_{i=1}^{J}\gamma_i^{\frac{\lambda}{1-\lambda}}$ is greater than the term multiplied by $\widetilde{H}_i/\widetilde{H}_i$. This reflects the presence of externalities in research coming from knowledge spillovers. There is a tendency that research-biased subsidies stimulate growth in our model as 1) shows. Proposition 5 shows that if a country who has comparative advantage in research adopts a subsidy policy that favors manufacturing and/or a country who has large comparative advantage in labor endowment adopts a subsidy policy that favors, the world growth rate falls. The intuition is clear. The set of subsidy policies partly reverses the more efficient world division of labor between innovation and manufacturing enabled by international firm mobility. Thus, the proposition can be considered as a warning against the current development of the world trade system.

4. Concluding remarks

This paper contributes to the literature by filling the gap left open in researches on the relationship between trade, firm mobility, growth and skill premium. The paper does it by constructing a multi-country growth model that can examine all these variables in a simple integrated framework. We have derived a number of simple results that have a potential to accommodate more extensions. However, the model is highly stylized and is difficult to link directly to other theoretical works or to data from the real world.

One of the features that should be included in our future study is trade costs. Only by including them can our results be directly compared with important works in new economic geography. In addition, the structure of our model should be enriched to describe better the real world. This may include allowing skilled labor in manufacturing final goods, or allowing physical capital in manufacturing intermediate goods. Introducing sorting and matching between heterogenous workers and firms is another direction for extension. This paper is a work in progress, and we hope to address some of these issues in a future version of the paper.

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Mathematical Appendix

A.1 Proofs for (31), (32), and (33)

With global spillovers, (14) is replaced by (14)', and (15) is replaced by (15)'.

$$p_{Ai} = \frac{1}{A}\ D_i^{1-\lambda}\ \frac{w_{Hi}}{\eta_{Ai}}. \tag{14'}$$

$$\dot{A}_i = A\ D_i^{\lambda} = \frac{A_i}{\sigma_i}\ D_i^{\lambda}. \tag{15'}$$

Putting them together, $p_{A_i} = A^{-1}A_i^{-1}\ g_{A_i}^{\frac{1-\lambda}{\lambda}}\widetilde{w}_{Hi}\ \gamma_i^{-1}$. Using (11), (12), and (16), we can derive that $\sigma_i = \left((\rho + g_A)^{-1}g_A^{-\frac{1-\lambda}{\lambda}}\ \alpha\right)^{\lambda}\ \left(\widetilde{\omega}_i^{-1}\tilde{L}_i\ \frac{\eta_{Ai}}{\eta_{Xi}}\right)^{\lambda}$.

Summing over countries,

$$1 = \left((\rho + g_A)^{-1}g_A^{-\frac{1-\lambda}{\lambda}}\ \alpha\right)^{\lambda}\ \sum_{i=1}^{J}\left(\widetilde{\omega}_i^{-1}\tilde{L}_i\ \gamma_i\right)^{\lambda}. \tag{A1}$$

Thus,

$$\sigma_i = \frac{\left(\widetilde{\omega}_i^{-1}\tilde{L}_i\ \gamma_i\right)^{\lambda}}{\sum_{i=1}^{J}\left(\widetilde{\omega}_i^{-1}\tilde{L}_i\ \gamma_i\right)^{\lambda}}. \tag{A2}$$

We can show that (27) still holds with g_{A_i} is replaced by the common growth rate g_A. Therefore, $H_i = \frac{\alpha}{\eta_{Xi}}A_i\ x_i + \frac{1}{\eta_{Ai}}D_i = \left(\alpha + (1-\alpha)\frac{g_A}{\rho+g_A}\right)\frac{1}{\eta_{X_i}}\ A_i$. Using (12), we obtain (32) in the text.

$$\widetilde{\omega}_i\frac{\widetilde{H}_i}{\tilde{L}_i} = \alpha\left(\frac{\alpha}{1-\alpha} + \frac{g_A}{\rho+g_A}\right). \tag{32}$$

Or $\widetilde{\omega}_i^{-1}\ \tilde{L}_i = \left(\frac{\alpha}{1-\alpha} + \frac{g_A}{\rho+g_A}\right)^{-1}\ \alpha^{-1}\ \widetilde{H}_i$. Plugging this equation into (A1) and (A2), we obtain (31) and (33).

A.2 Proofs for (39) and (40)

Using (22), (30), (36), and (38),

$$p_{Ai} = \frac{1}{A}\ (\sigma_i\ g_A)^{\frac{1-\lambda}{\lambda}}\widetilde{w}_{Hi}\ \frac{\eta_{Xi}}{\eta_{Ai}} = (\rho + g_A)^{-1}(1-\alpha)\ \widetilde{w}_{Hi}^{\frac{-\alpha}{1-\alpha}}\ \tilde{L}.$$

The equation implies that

$$\sigma_i\ g_A = (\rho + g_A)^{\frac{-\lambda}{1-\lambda}}\ (1-\alpha)^{\frac{\lambda}{1-\lambda}}\ A^{\frac{\lambda}{1-\lambda}}\ \tilde{L}^{\frac{\lambda}{1-\lambda}}\ \widetilde{w}_{Hi}^{\frac{-1}{1-\alpha}\frac{\lambda}{1-\lambda}}\gamma_i^{\frac{\lambda}{1-\lambda}}. \tag{A3}$$

$$g_A = (\rho + g_A)^{\frac{-\lambda}{1-\lambda}} (1-\alpha)^{\frac{\lambda}{1-\lambda}} \ A^{\frac{\lambda}{1-\lambda}} \tilde{L}^{\frac{\lambda}{1-\lambda}} \left(\sum_{i=1}^{J} \widetilde{w}_{Hi}{}^{\frac{-1}{1-\alpha}\frac{\lambda}{1-\lambda}} \gamma_i^{\frac{\lambda}{1-\lambda}}\right). \tag{A4}$$

Dividing (A3) by (A4) and using the fact that $\widetilde{w}_{Li}$ is identical in all countries, (39) follows.

From (37),

$$\widetilde{w}_L{}^{\frac{1}{1-\alpha}} = \frac{1-\alpha}{\alpha} \ A \ \sum_{j=1}^{J} \sigma_j \ \widetilde{\omega}_i{}^{\frac{-\alpha}{1-\alpha}}. \tag{A5}$$

(A4) and (A5) imply that

$$g_A^{\frac{1-\lambda}{\lambda}} = (\rho + g_A)^{-1} (1-\alpha) \ A \ \tilde{L} \ \widetilde{w}_L{}^{\frac{-1}{1-\alpha}} \left(\sum_{i=1}^{J} \widetilde{\omega}_i{}^{\frac{-1}{1-\alpha}\frac{\lambda}{1-\lambda}} \gamma_i^{\frac{\lambda}{1-\lambda}}\right)^{\frac{1-\lambda}{\lambda}}. \tag{A6}$$

Plugging (A5) into (A6),

$$\sum_{i=1}^{J} \sigma_i \ \widetilde{\omega}_i{}^{\frac{-\alpha}{1-\alpha}} = (\rho + g_A)^{-1} \ g_A^{-\frac{1-\lambda}{\lambda}} \alpha \ \tilde{L} \left(\sum_{i=1}^{J} \widetilde{\omega}_i{}^{\frac{-1}{1-\alpha}\frac{\lambda}{1-\lambda}} \gamma_i^{\frac{\lambda}{1-\lambda}}\right)^{\frac{1-\lambda}{\lambda}}.$$

By (39),

$$\sum_{i=1}^{J} \sigma_i \ \widetilde{\omega}_i{}^{\frac{-\alpha}{1-\alpha}} = \frac{\sum_{i=1}^{J} \widetilde{\omega}_i{}^{\frac{-1}{1-\alpha}(\alpha + \frac{\lambda}{1-\lambda})} \gamma_i^{\frac{\lambda}{1-\lambda}}}{\sum_{i=1}^{J} \widetilde{\omega}_i{}^{\frac{-1}{1-\alpha}\frac{\lambda}{1-\lambda}} \gamma_i^{\frac{\lambda}{1-\lambda}}} \ .$$

(40) follows.

A.3 Proofs for (46), (47), (48) and (49)

From (43),

$$\sum_{i=1}^{J} \widetilde{w}_{Hi} \ \widetilde{H}_i \ = \left(\alpha \ + (1-\alpha) \frac{g_A}{\rho + g_A} \ \right) \sum_{i=1}^{J} A_i \ \widetilde{w}_{Hi} \ x_i \ .$$

Thus, using (34), (35), and (39),

$$\frac{\widetilde{w}_{Hi} \ \widetilde{H}_i}{\sum_{i=1}^{J} \widetilde{w}_{Hi} \ \widetilde{H}_i} = \frac{\widetilde{\omega}_i \ \widetilde{H}_i}{\sum_{i=1}^{J} \widetilde{\omega}_i \ \widetilde{H}_i} = \frac{A_i \ \widetilde{w}_{Hi} \ x_i}{\sum_{i=1}^{J} A_i \ \widetilde{w}_{Hi} \ x_i} = \frac{\sigma_i \ \widetilde{w}_{Hi}{}^{\frac{-\alpha}{1-\alpha}}}{\sum_{i=1}^{J} \sigma_i \ \widetilde{w}_{Hi}{}^{\frac{-\alpha}{1-\alpha}}} = \frac{\widetilde{\omega}_i{}^{\frac{-1}{1-\alpha}(\alpha + \frac{\lambda}{1-\lambda})} \gamma_i^{\frac{\lambda}{1-\lambda}}}{\sum_{i=1}^{J} \widetilde{\omega}_i{}^{\frac{-1}{1-\alpha}(\alpha + \frac{\lambda}{1-\lambda})} \gamma_i^{\frac{\lambda}{1-\lambda}}}. \tag{A7}$$

Therefore, $\widetilde{\omega}_i \ \widetilde{H}_i = \delta_0 \ \widetilde{\omega}_i{}^{\frac{-1}{1-\alpha}(\alpha + \frac{\lambda}{1-\lambda})} \gamma_i^{\frac{\lambda}{1-\lambda}}$ for some constant δ_0. Or, for some constant δ,

$$\widetilde{\omega}_i = \delta \ \widetilde{H}_i^{-(1-\alpha)(1-\lambda)} \gamma_i^{(1-\alpha)\lambda}. \tag{A8}$$

Plugging (A8) into (39), we obtain (47).

Plugging (A8) into (40),

$$\frac{\sum_{i=1}^{J} \widetilde{\omega}_i{}^{\frac{-1}{1-\alpha}\left(\alpha + \frac{\lambda}{1-\lambda}\right)} \ \gamma_i^{\frac{\lambda}{1-\lambda}}}{\left(\sum_{i=1}^{J} \widetilde{\omega}_i{}^{\frac{-1}{1-\alpha}\frac{\lambda}{1-\lambda}} \ \gamma_i^{\frac{\lambda}{1-\lambda}}\right)^{\frac{1}{\lambda}}} \ = \ \delta \ \frac{\sum_{i=1}^{J} \widetilde{H}_i^{\alpha} \ (\gamma_i \ \widetilde{H}_i)^{\lambda(1-\alpha)}}{\left(\sum_{i=1}^{J} (\gamma_i \ \widetilde{H}_i)^{\lambda}\right)^{\frac{1}{\lambda}}} \ = \tilde{L} \ \alpha \ (\rho + g_A)^{-1} \ g_A^{-\frac{1-\lambda}{\lambda}}. \tag{A9}$$

By (A8) and (45),

$$\sum_{i=1}^{J} \widetilde{\omega}_i \, \widetilde{H}_i = \delta \, \sum_{i=1}^{J} \widetilde{H}_i^{\alpha+(1-\alpha)\lambda} \gamma_i^{(1-\alpha)\lambda} = \delta \, \sum_{i=1}^{J} \widetilde{H}_i^{\alpha} \left(\gamma_i \, \widetilde{H}_i\right)^{(1-\alpha)\lambda}$$

$$= \tilde{L} \, \alpha \left(\frac{\alpha}{1-\alpha} + \frac{g_A}{\rho+g_A}\right) \tag{A10}$$

Define a constant $\widetilde{\omega}_M^n$ such that $\delta = \widetilde{\omega}_M^n \, \frac{\widetilde{H}}{\sum_{i=1}^{J} \widetilde{H}_i^{\alpha} \, (\gamma_i \, \widetilde{H}_i)^{(1-\alpha)\lambda}}$. Then, (A8) becomes (46), (A10) becomes (48), and (A9) becomes

$$\widetilde{\omega}_M^n \, \frac{\widetilde{H}}{\left(\sum_{i=1}^{J} \, (\gamma_i \, \widetilde{H}_i)^{\lambda}\right)^{\frac{1}{\lambda}}} = \tilde{L} \, \alpha \, (\rho + g_A)^{-1} \, g_A^{-\frac{1-\lambda}{\lambda}}. \tag{A11}$$

(48) and (A11) implies (49).

A.4 Proof for Proposition 4

From (49),

$$\left(\alpha \, N_i \, + (1-\alpha) \frac{g_A}{\rho+g_A} \, A_i \right) p \, x = \frac{w_{Hi} \, H_i}{\sum_{i=1}^{J} w_{Hi} \, H_i} \, \sum_{i=1}^{J} w_{Hi} \, H_i \, = \frac{\widetilde{H}_i}{\widetilde{H}} \sum_{i=1}^{J} w_{Hi} \, H_i. \tag{A12}$$

Summing over i's and multiplying both sides by σ_i yields

$$\left(\alpha \, A_i \, + (1-\alpha) \frac{g_A}{\rho+g_A} \, A_i \right) \, p \, x = \sigma_i \sum_{i=1}^{J} w_{Hi} \, H_i. \tag{A13}$$

Substracting (A12) from (A13), (55) follows.

A.5 Proofs for (58) and (59)

With no firm mobility, by (18),

$$C_i = \rho(\, \int_t^{\infty} (w_{Li} L_i + w_{Hi} H_i) \, e^{-r\,(s-t)} \, ds + p_{A_i} \, A_i$$

$$= w_{Li} L_i + w_{Hi} H_i + \frac{\rho}{\rho+g_A} (1-\alpha) \, A_i \, p_i \, x_i.$$

By (41),

$$\left(\alpha \, + (1-\alpha) \frac{g_A}{\rho+g_A}\right) \, A_i \, p_i \, x_i = w_{Hi} \, H_i.$$

$$\left(\alpha \, + (1-\alpha) \frac{g_A}{\rho+g_A}\right) \sum_{i=1}^{J} A_i \, p_i \, x_i \, = \sum_{i=1}^{J} w_{Hi} H_i.$$

Thus,

$$A_i \, p_i \, x_i = \frac{w_{Hi} \, H_i}{\sum_{i=1}^{J} w_{Hi} H_i} \sum_{i=1}^{J} A_i \, p_i \, x_i = \frac{\alpha}{1-\alpha} \frac{w_{Hi} \, H_i}{\sum_{i=1}^{J} w_{Hi} H_i} \, \sum_{i=1}^{J} w_{Li} \, L_i. \tag{A14}$$

Using (A14),

$$C_i = w_{Li} L_i + \alpha \, A_i \, p_i \, x_i + \, (1-\alpha) \, A_i \, p_i \, x_i$$

$$= w_{Li}L_i + \frac{\alpha}{1-\alpha}\frac{w_{Hi}\,H_i}{\sum_{i=1}^{J} w_{Hi}H_i}\ \sum_{i=1}^{J} w_{Li}\ L_i.$$

On the other hand, $\sum_{j=1}^{J} A_j\ p_j\ m_{ji}\ =\ \frac{\alpha}{1-\alpha}\ w_{Li}\ L_i$, and

$$Y_i = w_{Li}L_i + \sum_{j=1}^{J} A_j\ p_j\ m_{ji}\ = \frac{1}{1-\alpha}\ w_{Li}L_i. \tag{A15}$$

Thus,

$$C_i - Y_i = \frac{\alpha}{1-\alpha}\left(\frac{w_{Hi}\,H_i}{\sum_{i=1}^{J} w_{Hi}H_i}\ \sum_{i=1}^{J} w_{Li}\ L_i - w_{Li}L_i\right) = \frac{\alpha}{1-\alpha}\ \sum_{i=1}^{J} w_{Li}\ L_i\left(\frac{\widetilde{\omega}_i\,\widetilde{H}_i}{\sum_{i=1}^{J}\widetilde{\omega}_i\,\widetilde{H}_i} - \frac{\tilde{L}_i}{\tilde{L}}\right).$$

Because $\widetilde{\omega}_i = \delta\ \widetilde{H}_i^{-(1-\alpha)(1-\lambda)}\gamma_i^{\lambda(1-\alpha)}$,

$$\frac{\widetilde{\omega}_i\,\widetilde{H}_i}{\sum_{i=1}^{J}\widetilde{\omega}_i\,\widetilde{H}_i} = \frac{\widetilde{\omega}_i\,\widetilde{H}_i}{\sum_{i=1}^{J}\widetilde{\omega}_i\,\widetilde{H}_i} = \frac{\widetilde{H}_i^{\alpha+\lambda(1-\alpha)}\ \gamma_i^{\lambda(1-\alpha)}}{\sum_{i=1}^{J}\widetilde{H}_i^{\alpha+\lambda(1-\alpha)}\ \gamma_i^{\lambda(1-\alpha)}}.$$

With balanced trade, $N_i\ p_i\ x_i - \sum_{j=1}^{J} A_j\ p_j\ m_{ji}\ = C_i - Y_i$ and (58) follows.

In the case of firm mobility, from (52),

$$w_{Hi}\ H_i = \left(\alpha\ N_i\ + (1-\alpha)\frac{g_A}{\rho+g_A}\ A_i\right)\ p_i\ x_i. \tag{A16}$$

By (18), (A12) and (55)

$$C_i = w_{Li}L_i + w_{Hi}H_i + \frac{\rho}{\rho+g_A}(1-\alpha)\ A_i\ p_i\ x_i$$

$$= w_{Li}L_i + A_i\ p\ x - \alpha\ (A_i - N_i)\ A_i\ \ p\ x.$$

$$= w_{Li}L_i + \frac{\alpha}{1-\alpha}\sigma_i\ \sum_{i=1}^{J} w_{Li}\ L_i - (1-\alpha)\left(\sigma_i - \frac{\widetilde{H}_i}{\widetilde{H}}\right)\ \sum_{i=1}^{J} w_{Hi}H_i.$$

Using (A15),

$$C_i - Y_i =\ \frac{\alpha}{1-\alpha}\sum_{i=1}^{J} w_{Li}\ L_i\ (\sigma_i - \frac{\tilde{L}_i}{\sum_{j=1}^{J}\ \tilde{L}_{i_i}}\ ) - (1-\alpha)\sum_{i=1}^{J} w_{Hi}H_i\left(\sigma_i - \frac{\widetilde{H}_i}{\widetilde{H}}\right).$$

With balanced trade,

$$N_i\ p_i\ x_i - \sum_{j=1}^{J} A_j\ p_j\ m_{ji}\ = C_i - Y_i - (A_i - N_i)\ (1-\alpha)\ p\ x$$

$$=\ \frac{\alpha}{1-\alpha}\sum_{i=1}^{J} w_{Li}\ L_i\ (\sigma_i - \frac{\tilde{L}_i}{\sum_{j=1}^{J}\ \tilde{L}_{i_i}}\ ) - \frac{1-\alpha^2}{\alpha}\sum_{i=1}^{J} w_{Hi}H_i\ \left(\sigma_i - \frac{\widetilde{H}_i}{\widetilde{H}}\right).$$