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# Industrial Revolutions and Global Imbalances

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## Abstract

Based on the historical data since 1845, we identify a stylized fact, that is, alternating waves in global imbalances generated by sequential industrial revolutions. We develop a new theory to explain this stylized fact. Our theory proposes a development-stage view for the optimal global imbalances. It explains the Lucas Paradox on capital flows as well as rises and falls of the external wealth of nations over time.

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# I. Introduction

The debate on global imbalance have become intense recently, fueled by large, consistent current account surpluses by countries like China, Japan, and Germany. Based on past 10 year movements, global imbalance seems indeed large and worrisome. Figure 1 shows the past 10 years (2005-2015) of current account movements in billion US dollars for G5 countries plus China. In particular, in the run-up to 2008 global financial crisis, China rapidly increased its current account surplus. Since then, Japan's surplus went down dramatically. On the other side was the US, almost alone providing a giant current account deficit to the world, though somewhat subsided since 2008.

While current account is a flow data, the IMF recently started to publish the international investment positions of countries. Figure 2 shows past 10 year movements (2006-2015) in billion US dollars for G5 countries plus China. Essentially, the net international investment position is the accumulated surpluses or deficits over time with valuation changes. Perhaps, the valuation changes matter a lot for the US, whose negative position has been widening rapidly even after 2008, unlike shrinking current account deficit suggests. Japan is clearly the largest creditor over the past 10 years but on a seemingly declining path, while China and Germany have been increased the position since 2011.

Persistent surplus countries like China, Japan, and Germany may invite suspicions on their policies and institutions. Some US politicians and commentators blame policies of those surplus countries as key culprits behind persistent large deficit of the US. Also, academic explorations often conclude the surplus countries as holding too large official reserves from the viewpoint of self insurance over business cycles (e.g., Jeanne and Rancière, 2011), or at least blaming weak domestic financial market development in countries like China for people's incentives to invest abroad (e.g., Ju and Wei, 2010).

However, looking back in history since 1970, a somewhat different picture emerges. Figure 3 shows net foreign assets of G5 countries plus China as a share of the world gross assets.[1] China emerges as a sizable creditor only since around 2000. Japan has been the world largest asset holders since

[1] The data for country's overall external positions have been created recently and maintained by Lane and Miliesi-Feretti (2007). Their data is available only from 1970. We define the world gross assets (the denominator) to make Figure 3 as the world assets plus liabilities, divided by 2.

mid-1980s though its relative significance lowered from 2000 along with China's emergence. Before 1990, however, Germany and the U.K were more important creditors of the world. But, by far the largest asset holder at least before 1980 was the US. Although its significance eroded since 1970, the US is the main source of global imbalance as the dominant creditor for many years. Indeed, the picture is omitted but the US played such a dominant role among the world creditors from the end of the World War II to around 1970.

The US as the giant creditor of the world after 1945 until 1980 could be perhaps viewed as just a unique non-economic event due to the World War II. And, if so, the global imbalance between 1845 and 1980 could be thought of as exogenous to a structural economic model. However, this view does not seem warranted if we look back further into history before 1940. Figure 4 shows the official gold holdings as the world share between 1845 to 1940. Note that before 1970, with the exception in 1930s, these G5 countries adopted the gold standard most of the time. Since the data for foreign assets is scarce before 1970, we use the official gold holdings data as a proxy for the net foreign assets. Actually, the gold is likely a good proxy for international financial assets between 1930 and 1970, because during these years the international capital markets were hardly active, worse than 1920, due to restrictive policy measures (Rajan and Zingales, 2003).

Some data before the World War II on net foreign assets is available by Goldsmith (1985). Table 1 shows that the correlation between the net foreign assets and the official gold holdings between 1850 and 1940 for G5 countries.[2] Though only several years (UK) to only two years (Germany) coverage, we compute the country specific correlations and then report the average correlation of G5 countries. It is about 0.35. Two year coverage of Germany are 1930 and 1940, during the interwar period when Germany's international asset position should be regarded as exogenously determined due to World War I reparations. So, if we take out them, the overall correlation is 0.69, implying the official gold holdings is a good proxy for the net foreign asset position under the gold standard.

Figure 4 shows that the global imbalances, seemingly caused by newly industrialized countries, have

[2]We can trace figures of externally issued government bonds (i.e., liabilities) of major countries back to 19the century but unfortunately such data is scarce on wealth side, i.e., which countries owns those bonds. On the other hand, partly because of the gold standard, the official gold holdings data and price data is well available back to 19th century.

always existed in the world since the United Kingdom experienced the Industrial Revolution. The UK accumulated tremendous external wealth by the mid-19th century. After the UK, France, and Germany, though somewhat to a lesser extent, took over the UK's position in the world after 1860. By the beginning of the 20th century, the US then took over the status of the world largest asset holders. Then, the US emerged in the inter-war period as the dominant external wealth holder.

A canonical pattern in these figures can be simply put as alternating large external wealth holdings generated by sequential industrial revolutions. Each global imbalance episode is quite persistent, nothing to do with business cycles, which however researchers often try to relate to the optimal foreign reserve levels (e.g., Jeanne and Rancière, 2011). Also, unlike a possible explanation for the current Chinese economy by Ju and Wei (2010), under-development of the domestic financial market relative to other countries cannot be a factor to explain the UK or the US as the biggest creditor of the world for decades in the history.

Here, a new theory is needed. From the next section, we present a theoretical model that is consistent with the identified stylized fact.

## II. MODEL SETUP

There is a continuum of agents live in finitely many countries (total $S$ countries). Countries with the same population are distributed uniformly in $[1, S]$, as if indexed by $s$ with the cumulative distribution denoted by $\Omega$. The world total population mass is normalized to be $1$, i.e., $\Omega(S) = 1$ and $\Omega(1) = 0$. Production of a representative firm in country $s$ in year $t$ is given as

$$Y_{s,t} = K_{s,t}^{\nu} \left(\gamma_{s,t} L_{s,t}\right)^{1-\nu}, \tag{1}$$

where $K_{s,t}$ is capital, $L_{s,t}$ is labor, and $\gamma_{s,t}$ is productivity of a representative firm of country $s$ in year $t$.

Each country starts with very low level of productivity, as in pre-industrial age. However, at some point of time, each country start to be industrialized. Here, $t = 0, 1, 2, \cdots$ indicate the calendar time and $s$ indicate the entry year of a country into industrial, modern growth. In other words, the country indexed by $s$ is the country that starts industrialization from year $s$. Country $s = 1$ is the "frontier" country, which is industrialized at time 1. All countries $s = 1, 2, 3, \cdots$ have a chance of industrialization with probability $\rho$ in every year until they begin industrialization. In other words, mass $\rho$ of people, as citizens of a country, starts industrialization every year.

For a country that is not yet industrialized, productivity stays at the lowerbound, pre-industrial level, i.e., for $t < s$,

$$\gamma_{s,t} = \underline{\gamma}. \tag{2}$$

The productivity of the frontier country is assumed to follow a simple Solow-type exogenous growth, with growth rate $\alpha$,

$$\gamma_{1,t} = (1+\alpha)\gamma_{1,t-1} = (1+\alpha)^t \underline{\gamma}. \tag{3}$$

Following Lucas (2004), the productivity of any follower country $s \geq 2$ in $t$ is assumed to catch up to that of the frontier country in each year gradually, i.e., for $t \geq s$[3]

$$\gamma_{s,t} = (1+\alpha)\gamma_{1,t}^{\theta}\gamma_{s,t-1}^{1-\theta}. \tag{4}$$

Firms are set up and shut down each year. We assume that capital is freely mobile across countries but that labor cannot move out of country borders. The capital and labor are owned by households, who work for local firms but lend capital to firms anywhere in the world. The real rental price of capital is given by $r_t$, the same for all countries but varies over years, and the real wage is given by $w_{s,t}$ for each country in each year.

A country s's representative firm's problem is, for each year,

$$\max_{K_{s,t}, L_{s,t}} K_{s,t}^{\nu} \left(\gamma_{s,t} L_{s,t}\right)^{1-\nu} - r_t K_{s,t} - w_{s,t} L_{s,t}. \tag{5}$$

[3]The formula below is a bit simpler than Lucas (2004).

A representative household in each country maximizes their discounted sum of utilities over time. For each country, the initial endowment of gold in ton is denoted as $M_{s,0}$ and capital in real term $k_{s,0}$. Also, one unit of time to work $l_{s,t}$ is endowed in each period for a representative household in each country. There is no disutility from working. Income of a representative household consists of labor income $w_{s,t}l_{s,t}$ and capital income $r_t k_{s,t}$.

At the beginning of each period, households start working for and lend capital to firms. In the middle of each period, firms produce outputs. At the end of each period, firms deliver their outputs as consumption $c_{s,t}$ and investments $i_{s,t}$, and next-period gold $M_{s,t+1}$ (as investment), to households in exchange for payments.

We assume that consumption goods $c_{s,t}$ are cash goods, required to be paid by gold $M_{s,t}$ (i.e., the gold-in-advance constraint). On the other hand, households are assumed to be able to use credits to pay for investment goods $i_{s,t}$ and next-period gold $M_{s,t+1}$ (i.e., credit goods). Firms use the credits from sales to pay wages and capital rents to households and settle the credits and the debits with the households by the end of each period.

Apparently, the financial system is incomplete. Consumption goods can be transacted only with gold, while investments can be settled by within-period credits, presumably because creditors can seize capital and gold easily. The capital stock can be lent or borrowed also with within-period rental contracts, though leases, loans, or equity contracts are indistinguishable in this model. What is not assumed available is the market for contingent claims, contingent on all countries' starting years of industrialization, which is the only source of risk in this model. Gold is then accumulated as a vehicle for self insurance against industrialization status of the world.

In summary, a country $s$'s representative household's problem after starting industrialization depends on year $s$, when they start industrialization, and gold and capital stocks as state variables. It can be written recursively for a representative household living in country $s$, for $t \geq s$ with $u(\cdot)$ denoting the period utility,

$$V_s(M_s, k_s) = \max_{M'_s, k'_s} u(c_s) + \beta V_s(M'_s, k'_s), \tag{6}$$

subject to the budget constraint

$$c_s + i_s + QM'_s = rk_s + w_s l_s + QM_s, \tag{7}$$

the gold-in-advance (GIA) constraint

$$c_s \leq m_s = QM_s \tag{8}$$

and the law of motion of capital

$$k'_s = (1 - \delta)k_s + i_s. \tag{9}$$

For the sake of simplicity, we assume the CRRA period utility function, i.e., $c^{1-\sigma}/1 - \sigma$ with assuming $\sigma > 1$.

A country $s$'s representative household's problem before industrialization cannot depend on time $s$ of industrialization, as it is not yet known. Instead, the value function depends on the current period $t = n$, because the country has lost chances to become industrialized up to $t = n$. We distinguish the pre-industrialization variables by subscript $p$. Then, for all pre-industrialized countries in $t = n < s$, we can write a typical value function of them as

$$W_n(M_p, k_p) = \max_{M'_p, k'_p} u(c_p) + \beta\{(1 - \rho)W_{n+1}(M'_p, k'_p) + \rho V_{s=n+1}(M'_{s=n+1}, k'_{s=n+1})\}, \tag{10}$$

with no difference in capital and gold stock in the beginning of the next period with or without industrialization,

$$M'_p = M_{s=n+1} \quad \text{and} \quad k'_p = k_{s=n+1}, \tag{11}$$

and subject to the same constraints (7) - (9) above.[4] Note that, if a country starts industrialization in the next period $t = n + 1$, then the country will be assigned $s = n + 1$ as its industrialization-order

[4] Without prime indicating the next period values, these equations are written with the additional time subscript $n + 1$ as

$$M_{p,n+1} = M_{s=n+1,n+1} \quad \text{and} \quad k_{p,n+1} = k_{s=n+1,n+1}.$$

index. The value function (10) depends on $n$, total periods that a country missed to become industrialized. And $n = t$, the current period, until a country starts industrialization, that is, this value is time dependent. However, all the remaining countries share the same value $W_n$, as if they were assigned, but not informed, the industrialization order index $s$. On the other hand, the value of industrialized countries (6) depends on $s$, specific period that a country starts industrialization, that is, this value differs for each country. However, it is not time dependent, i.e., the value $V_s$ does not depend on current period $t$.

Country $s$'s representative firm is assumed to attract capital from everywhere in the world, even before the industrialization, while labor comes locally by assumption. Then, the resource constraint for capital used in country $s$'s representative firm is expressed as

$$K_s = \int_1^S \psi_{j,s} k_j d\Omega(j), \tag{12}$$

where $\psi_{j,s}$ is the ownership of capital for country $s$'s representative firm by country $j$'s representative household. And, for the world wide, capital demand by all representative firms $s \in [1, S]$ should be equal to capital supply by all representative households $j \in [1, S]$,

$$\int_1^S K_j d\Omega(s) = \int_1^S k_j d\Omega(j). \tag{13}$$

As for the labor, the resource constraint matters in each country,

$$L_s = l_s. \tag{14}$$

Regarding the goods market, supply by firms $s \in [1, S]$ should be met by demand by households $j \in [1, S]$,

$$\int_1^S Y_j d\Omega(s) = \int_1^S (c_j + i_j)\, d\Omega(j). \tag{15}$$

The world-wide gold quantity in ton is fixed from the beginning to the end.

$$\int_1^S M_s d\Omega(s) = \overline{M}. \tag{16}$$

## III. EQUILIBRIUM

*Competitive equilibrium* is, given $t = 0, 1 \cdots, \infty$ sequence of world gold price $Q_t$, world real interest rate $r_t$, and real wage $w_{s,t}$, each representative firm solves its problem (5), each representative household solves her problem (6) in industrialized countries and problem (10) in pre-industrial countries, subject to constraints (7) to (9), and markets clear, i.e., resource constraints (12) to (16) are met. We assume initial gold price $Q_0 = 1$ arbitrarily.

Almost straightforward prediction is that capital flows from the poor to the rich.

**Lemma 1.** *Capital employed in each country's representative firm is proportional to the productivity level.*

*Proof.* Our assumption of free capital mobility means that there is one world-wide interest rate, $r_t$. The first order condition of the firm's problem (5) implies equalization of the marginal product of capital across countries,

$$r_t = MPK_{s,t} = \nu \left( \frac{K_{s,t}}{\gamma_{s,t} L_{s,t}} \right)^{\nu - 1} = \nu \left( \frac{K_{1,t}}{\gamma_{1,t} L_{1,t}} \right)^{\nu - 1}, \tag{17}$$

implying that

$$K_{s,t} = \frac{\gamma_{s,t}}{\gamma_{1,t}} K_{1,t}. \tag{18}$$

*Q.E.D.*

We denote the gross return from the capital investment less depreciation as

$$R_t \equiv \nu K_{s,t}^{\nu-1} \left( \gamma_{s,t} L_{s,t} \right)^{1-\nu} + (1 - \delta) = MPK_{s,t} + (1 - \delta) = r + (1 - \delta). \tag{19}$$

We also write

$$R_{mt} \equiv Q_t / Q_{t-1} \tag{20}$$

as the gross real return from investing in one unit of gold in the last period, realized in the current period.

The household problem (6) for industrializing countries can be solved by the first order conditions and the envelope theorem. We slightly modify the budget constraint (7) to eliminate investment $i$ using the law of motion of capital and the gross return relation (19), i.e.,

$$c_s + k_s' + QM_s' = Rk_s + w_s l_s + QM_s. \tag{21}$$

Denoting the Lagrange multiplier for this budget constraint by $\mu_{V,s}$ and that for the GIA constraint by $\lambda_{V,s}$, the first order condition with respect to next period capital $k'$ is expressed as

$$\mu_{V,s} = \beta V_{s,K}', \tag{22}$$

and as for the next period gold $M'$,

$$Q\mu_{V,s} = \beta V_{s,M}'. \tag{23}$$

The envelope theorem gives the following relation for the current capital,

$$V_K = \mu_{V,s} R, \tag{24}$$

and for the current gold,

$$V_M = Q(\mu_{V,s} + \lambda_{V,s}). \tag{25}$$

Note that, if the GIA constraint binds, $\lambda_{V,s} > 0$ but otherwise it is zero. For the capital relation, the first order condition and the envelope condition are combined into

$$\frac{\mu_{V,s}}{\mu_{V,s}'} = \beta R', \tag{26}$$

and as for the gold holdings,

$$\frac{\mu_{V,s}}{\mu_{V,s}' + \lambda_{V,s}'} = \beta R_m'. \tag{27}$$

This implies that, when deciding investment on next period gold, the next period GIA constraint matters.

The first order condition with respect to consumption is, if constrained by the GIA,

$$u'(c_s) = \mu_{V,s} + \lambda_{V,s}. \tag{28}$$

Hence, the Euler equation regarding capital becomes,

$$\frac{u'(c_s) - \lambda_{V,s}}{u'(c'_s) - \lambda'_{V,s}} = \beta R'. \tag{29}$$

The Euler equation regarding gold can be also written as,

$$\frac{u'(c_s) - \lambda_{V,s}}{u'(c'_s)} = \beta R'_m < \beta R'. \tag{30}$$

Next, we solve for the household problem of pre-modern economy (10). In period $t = n$, we assign the Lagrange multiplier $\mu_{W,n}$ for the budget constraint and $\lambda_{W,n}$ for the GIA constraint. But, as usual, subscript $n$ is omitted as much as possible below. Their envelope conditions and the first order condition for consumption are essentially identical to those of the industrialized countries. However, the first order conditions with respect to the next-period capital $k'$ becomes slightly different,

$$\mu_W = \beta\{(1-\rho)W'_K + \rho V'_{s=n+1,K}\}, \tag{31}$$

and that for the next-period gold $M'$, if constrained, becomes

$$Q\mu_W = \beta\{(1-\rho)W'_M + \rho V'_{s=n+1,M}\}. \tag{32}$$

Together with the envelop conditions,

$$\frac{\mu_W}{(1-\rho)\mu'_W + \rho\mu'_{V,s=n+1}} = \beta R', \tag{33}$$

and for the gold holdings

$$\frac{\mu_W}{(1-\rho)\left(\mu'_W+\lambda'_W\right)+\rho\left(\mu'_{V,s=n+1}+\lambda'_{V,s=n+1}\right)}=\beta R'_m. \tag{34}$$

Using the first order condition for consumption, similar to (28), we substitute $\mu$ and $\lambda$ by the marginal utilities. Note that, for a unconstrained country, $\mu'_W$ denotes the next-period marginal utility of consumption when this country stays in the pre-modern economy, while $\mu'_{V,s=n+1}$ denotes the next-period marginal utility of consumption when this country starts industrialization in the next period, $n+1$. Therefore, weighted average of them with $\rho$ probability weights can be expressed as the expected marginal utility, $E[u(c')]$. However, in the next period, consumption of countries at the pre-modern stage in the current period will be identical because it is constrained by the same value of gold holdings $Q'M'$. Thus, this is no need to use expectation operation for the next-period utility. Then, the Euler equation can be expressed as

$$\frac{u'(c_p)-\lambda_W}{u'(c'_p)-E[\lambda'_n]}=\beta R', \tag{35}$$

where $E[\lambda']=(1-\rho)\lambda'_W+\rho\lambda'_{V,s=n+1}$, and for the gold holdings,

$$\frac{u'(c_p)-\lambda_W}{u'(c'_p)}=\beta R'_m<\beta R'. \tag{36}$$

## IV. Period 0: Pre-Modern World

**Assumption 1.** *At $t=0$, when no country is industrialized yet, every country is assumed to be identical and in the steady state expecting probability $\rho$ of starting industrialization next period.*

Assumption 1 simplifies the analysis of the pre-modern world focusing on the steady state. Because every country knows they would be industrialized with small chances from the next period on, this pre-modern steady state can be considered as a dawn of industrialization. As assumed, the initial gold price is $Q_0=1$, and moreover, we assume here that period zero is steady state, as if the world would stay in period 0 for a long run. These assumptions automatically pin down the return on gold in period 0 as $R_m = 1 < R$ because the gold price is stable at $1$. Hence, no country would like to hold gold more than needed to purchase consumption goods. In other words, consumption is just constrained by the gold holdings.[5]

## V. Period 1: Industrialization by the First Country

**Lemma 2.** *In period 1, the shadow price of the current GIA constraint of the first country should be greater than the other countries, $\lambda_{V,1} > \lambda_{W,1}$.*

The proof is obvious and omitted. The first country wants to consume more than other countries in the first period, if it were not for the GIA constraint, to smooth consumption over time (i.e., the permanent income hypothesis). This immediately implies that the shadow price of the GIA constraint for the first country is higher than that for the other countries. However, the next period shadow price $\lambda'$ can be chosen optimally.

**Proposition 1.** *In period 1, with probability $\rho$ of industrialization, the first industrializing country attracts capital and buys gold from the rest of the world. Consumption of the first country is higher than the other countries that remains at the pre-modern stage.*

*Proof.* That the first country attracts capital is straightforward from Lemma 1. The question is why the first country buys gold from the rest of the world in period 1.

Note that the consumption in period 1 is the same for all the countries, including the first country,

$$c_1 = c_p. \tag{37}$$

[5] We could think of another pre-modern steady state, in which every country would not expect industrialization happens in future. If this were the case, however, there would be a jump in consumption and other endogenous variables, for even yet-industrialized countries, when suddenly industrialization probability becomes positive. And, we do not consider this case.

This is because all the countries are constrained by the same gold holdings, which were already determined in the previous period (i.e., period 0). However, the Euler equations regarding gold holdings (30) and (36) means the equality of stochastic discount factors of all the countries,

$$\frac{u'(c_1) - \lambda_{V,1}}{u'(c_1')} = \frac{u'(c_p) - \lambda_W}{u'(c_p')} = \beta R_m'. \tag{38}$$

The same current consumption level with higher shadow price for the first country (Lemma 2) implies lower next period marginal utility for the first country, $u'(c_1') < u'(c_p')$, that is, higher consumption for the first country

$$c_1' > c_p'. \tag{39}$$

Combining (37) and (39), the consumption growth of the first country is also higher the other countries in period 1.

Because of the GIA constraint, the investment in next-period gold should match with the next-period consumption,

$$M_1' > M_p'. \tag{40}$$

In other words, the first country buys gold from others by the end of the first period to achieve higher consumption in the next period. *Q.E.D.*

**Corollary 1.** *The gold price goes up in period 1.*

*Proof.* The supply of the gold is fixed at $\overline{M}$ and the demand for the gold is the world-wide consumption. Thus the market clear condition for the gold (16) implies the equality between the world consumption and the value of gold in each period,

$$\overline{C} \equiv \int_1^S c_j d\Omega(j) = QM \tag{41}$$

This immediately implies that the world consumption growth equals to the return on gold,

$$\overline{g}_c \equiv \frac{\overline{C}'}{\overline{C}} = \frac{Q'}{Q} = R_m'. \tag{42}$$

Defining the gross growth rate of consumption as $g_c \equiv c'/c$, we can rewrite the Euler equations (30) and (36) as

$$g_{c,1}^{\sigma} - \frac{\lambda_{V,1}}{u'(c_1')} = g_{c,p}^{\sigma} - \frac{\lambda_W}{u'(c_p')} = \beta R_m' \tag{43}$$

By substituting $R_m$ in (43) by $\overline{g}_c$ and summing up over the world, we obtain the relationship between the world consumption growth rate as the average of each country's consumption growth rate, which possibly varies due to the GIA constraint,

$$\overline{g}_c = \int_1^S g_{c,j} d\Omega(j). = \int_1^S \left(\beta R_m' + \frac{\lambda_i}{u'(c_j')}\right)^{\frac{1}{\sigma}} d\Omega(j). \tag{44}$$

Overall, we have four unknowns, i.e., next period consumption $c_1'$ and $c_p'$ and the current period Lagrange multipliers for the GIA constraints, $\lambda_{V,1}$ and $\lambda_W$. But, we can identify them as we have also four equations, i.e., two Euler equations in (43), definition of world consumption growth rate (42), and the world consumption growth rate represented by an average of each country's consumption growth rate (44).

Importantly, the return on gold becomes higher $R_m > 1$ in period 1 because of the positive consumption growth of the first country in (42). Given our initial price assumption $Q_0 = 1$, this means $Q_1 = R_{m,1} > 1$. That is, the gold price goes up in period 1. *Q.E.D.*

**Corollary 2.** *The expected shadow price of the next period GIA constraint is smaller for the first country than those for the other countries, i.e.,* $\lambda_{V,1}' < E[\lambda_1']$.

*Proof.* The Euler equations regarding capital investments (29) and (35) means

$$\frac{u'(c_1) - \lambda_{V,1}}{u'(c_1') - \lambda_{V,1}'} = \frac{u'(c_p) - \lambda_W}{u'(c_p') - E[\lambda_1']} = \beta R'. \tag{45}$$

Comparing the above equation with (38), and using (39), it must be the case that

$$\frac{\lambda_{V,1}'}{E[\lambda_1']} = \frac{u'(c_1')}{u'(c_p')} < 1. \tag{46}$$

*Q.E.D.*

**Claim.** *Not only accumulating gold, the first country has a positive net foreign asset position under reasonable parameter values.*

The capital flows into the first country (Lemma 1). Then, the net foreign asset position is equal to the gold holdings minus capital inflow in our model. The capital inflow is roughly about the difference between the capital of a pre-modern country and the capital of the first country, which in period 1 is, using equation (18),

$$K_1 - K_p = \left(\frac{\gamma_1}{\overline{\gamma}} - 1\right) K_p. \tag{47}$$

As for the gold accumulation in value by country 1,

$$Q_1 M_1' - Q_1 M_1 = \frac{c_1'}{R_m'} - c_1 = \left(\frac{g_{c1}'}{R_m'} - 1\right) c_1 = \left(\frac{g_{c1}'}{\overline{g}_c'} - 1\right) c_1. \tag{48}$$

where the last equation is implied by equation (42). Here, the question is whether the capital inflow is less than the gold accumulation, i.e., in terms of per capita GDP,

$$\left(\frac{\gamma_1}{\overline{\gamma}} - 1\right) \frac{K_{p,1}}{Y_{1,1}} < \left(\frac{g_{c1}'}{\overline{g}_c'} - 1\right) \frac{c_{1,1}}{Y_{1,1}}. \tag{49}$$

Because the output growth of the first country is higher than the capital growth of a pre-modern country, $K_{p,1}/Y_{1,1} < K_{p,0}/Y_{p,0}$. In the modern US, presumably around the steady state, the capital to output ratio is known to be about 2, but in the pre-modern world, it can be much smaller, even smaller than one (see e.g., Klenow and Rodríguez-Clare, 1997). On the other hand, consumption to output ratio $c_{1,1}/Y_{1,1}$ can be also near one, especially for poor countries where many people live hand to mouth. In the modern US, it is known to be around 0.95. Then, inequality (49) is likely to hold if

$$\frac{g_{c1}'}{\overline{g}_c'} > \frac{\gamma_1}{\overline{\gamma}}. \tag{50}$$

This is probably satisfied. The left hand side is the consumption growth difference of the first country relative to the world average growth from period 1 to 2, while the right hand side is the TFP difference between the industrializing country and the pre-modern countries in period 1. Under the closed economy assumption they should be about the same, but under our open economy assumption, the consumption growth jumps in the period of starting industrialization.

Indeed, we can argue more clearly. In the left hand side of (50), the growth differential is equal to the next-period consumption difference given the same current consumption due to the GIA constraint. And, the TFP ratio in the right hand side is equal to the output ratio. Hence, inequality (50) can be rewritten as the relation in the next period (i.e., period 2),

$$\frac{c_{1,2}}{\bar{c}_2} > \frac{Y_{1,2}}{\overline{Y}_2}. \tag{51}$$

Noting that the average consumption and output are approximately the same as those of the pre-modern countries, this becomes

$$\frac{c_{1,2}}{Y_{1,2}} > \frac{c_{p,2}}{Y_{p,2}}. \tag{52}$$

This relation in the next period should be satisfied because the permanent income hypothesis implies a larger propensity to consume in a country with a higher income growth trend.[6]

## VI. PERIOD 2: INDUSTRIALIZATION BY THE SECOND COUNTRY

The comparative analysis between the second country and $s \geq 3$ countries in period 2 is almost identical to what described in Proposition 1 for the first country and the other countries. Hence, we focus the relation between the first country and the second country here.

**Proposition 2.** *The growth of consumption, and gold holdings, by the second country are higher than those of the first country from period 2 to 3.*

*Proof.* In period 2, Corollary 2 suggests that $\lambda_{V,1} \leq E[\lambda_1']$ is determined already in teh previous period (i.e., period1). However, Lemma 2 should also apply to period 2 for the second country and

[6]We do not assume wealth effects of consumption as we use the CRRA period-utility function. Note that, in the first period when a country starts industrialization, the first country limits consumption due to the severe GIA constraint, but from the next period on, the first country can choose the gold optimally to mitigate the severity of the GIA constraint (Corollary 2).

$s \geq 3$ countries and thus $\lambda_{V,2} > \lambda_W$. Because $E[\lambda_1']$ is the weighted average of $\lambda_{V,2}$ and $\lambda_{W,2}$,

$$\lambda_{V,1} < \lambda_{V,2}. \tag{53}$$

In period 2, the first country and the second country share the same Euler equation (30) as industrializing countries,

$$.\frac{u'(c_1) - \lambda_{V,1}}{u'(c_1')} = \frac{u'(c_2) - \lambda_{V,2}}{u'(c_2')}, \tag{54}$$

implying that

$$.\frac{u'(c_2')}{u'(c_1')} = \frac{u'(c_2) - \lambda_{V,2}}{u'(c_1) - \lambda_{V,1}} < \frac{u'(c_2)}{u'(c_1)}, \tag{55}$$

where the last inequality comes from (53). Divide the right hand side by the left hand side, and obtain

$$.1 > \frac{\frac{u'(c_2')}{u'(c_1')}}{\frac{u'(c_2)}{u'(c_1)}} = \frac{g_{c,1}^{\sigma}}{g_{c,2}^{\sigma}}, \tag{56}$$

that is,

$$g_{c,1} < g_{c,2}. \tag{57}$$

As their consumption both constrained by the value of gold holdings $QM_s$, it must be also the case that the growth rate of gold holdings by the second country must be higher than that of the first country, i.e.,

$$\frac{M_1'}{M_1} < \frac{M_2'}{M_2} \tag{58}$$

*Q.E.D.*

Note that, although the growth of consumption (and gold holdings) is higher in country 2 than in country 1, the level of consumption (and gold holding) is not necessarily higher in country 2 than country 1. Rather, the consumption level country 2 catches up to that of country 1 from below. Still, apparently, the world share of consumption (and gold holdings) of country 2 increases as the share of country 1 declines.

Period 3 and later can be characterized as the similar way as for period 2: a new taking-off country grows most rapidly and accumulates international assets most quickly.

## VII. CONCLUSION

We have identified a stylized fact, that is, alternating waves in historical global imbalance generated by sequential industrial revolutions. Newly industrialized countries often accumulate foreign assets as they grow rapidly. This pattern occurs several times since at least mid-19th century.

We propose a new theoretical model to explain this stylized fact by applying the sequential industrial revolution model of Lucas (2004) to an open economy setup, combined with the hard currency (gold) constraint to buy consumption goods. In the period when a country takes off, the country faces the severe gold constraint to limit consumption. As the country becomes sure about receiving a higher income, it saves and invests more rapidly than the others including already industrialized countries, and than the pre-industrialized days of itself.

Note that the catch-up process of the GDP and consumption in Lucas (2004) occurs solely due to the assumed sequential TFP growth under the closed economy assumption. In this paper, we consider an open economy, which seems more consistent with the real-world experience of industrial revolutions.

If we assumed the perfect markets, contingent claims can insure all the risks, including the timings of industrial revolutions, at time zero. Such perfect insurance would mean that the consumption over time would be the same for all ex ante identical countries. The capital still flows to the higher TFP countries (i.e., the Poor to the Rich, the Lucas Paradox (Lucas, 1990)) to equate the marginal product of capital among countries. Then, without additional international assets, the net foreign asset positions are negative for earlier industrializing countries, inconsistent with the stylized facts.

On the contrary, in this paper, we assume an incomplete market to insure against timings of industrial revolutions and the hard currency constraint for consumption goods purchase. These international finance frictions create strong needs for the hard currency by a newly industrializing country. Under reasonable parameter values, such needs for gold is higher than the capital inflows and brings a positive net foreign asset position.

Moreover, such gold accumulation speed becomes lower after the initial period. The older industrialized countries are proven to lose the world share of gold holdings, as a new country begins industrial revolution and accumulate gold rapidly. This theoretical prediction is consistent with the observation.

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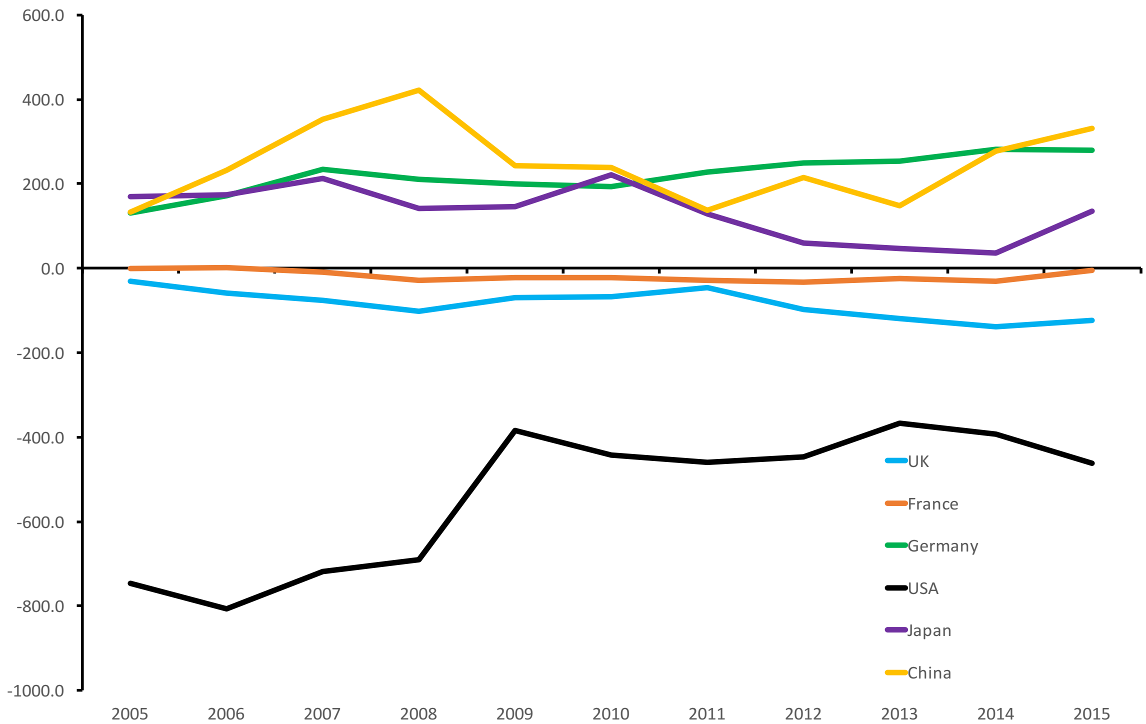
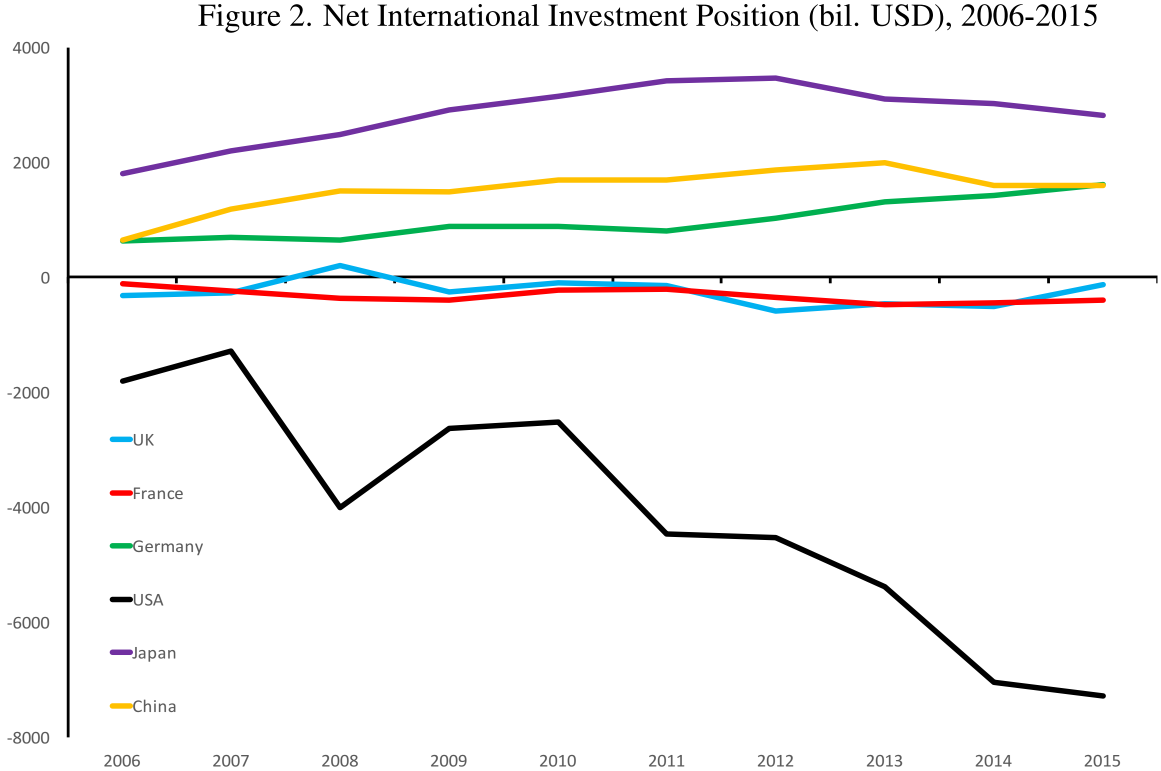
Figure 1. Current Account (bil. USD), 2005-2015



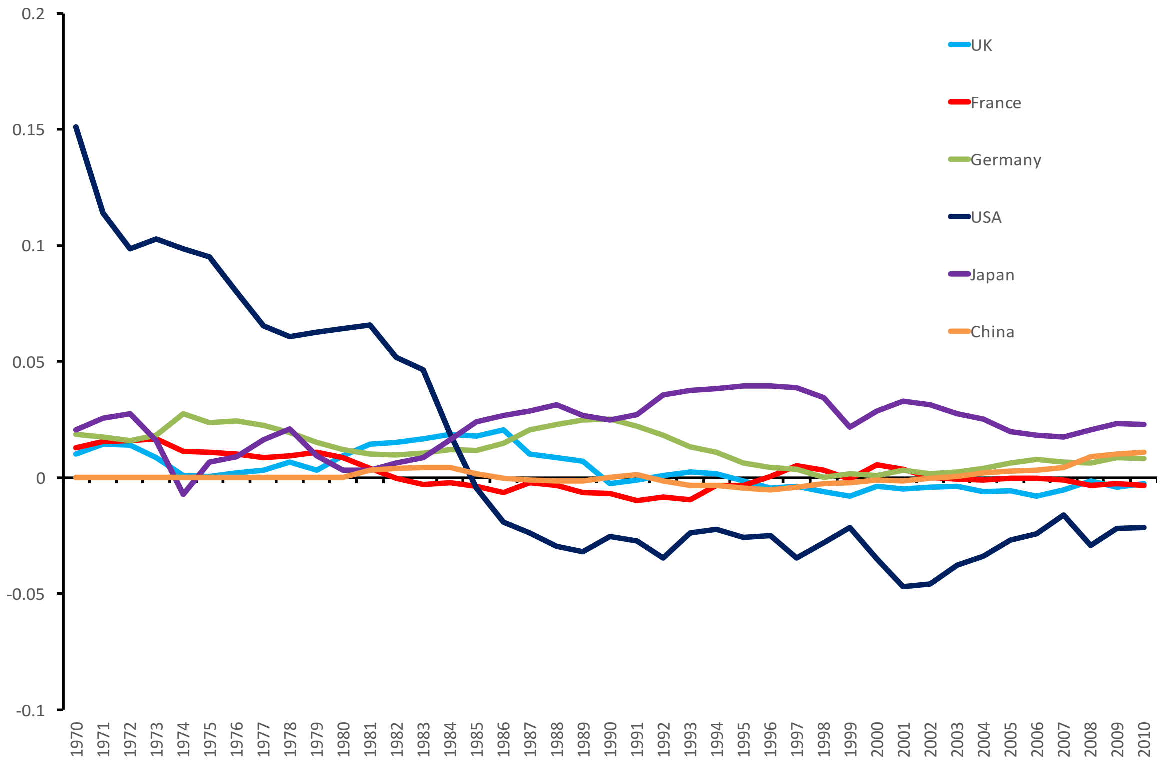

Source: IMF (billion USD)

Figure 2. Net International Investment Position (bil. USD), 2006-2015



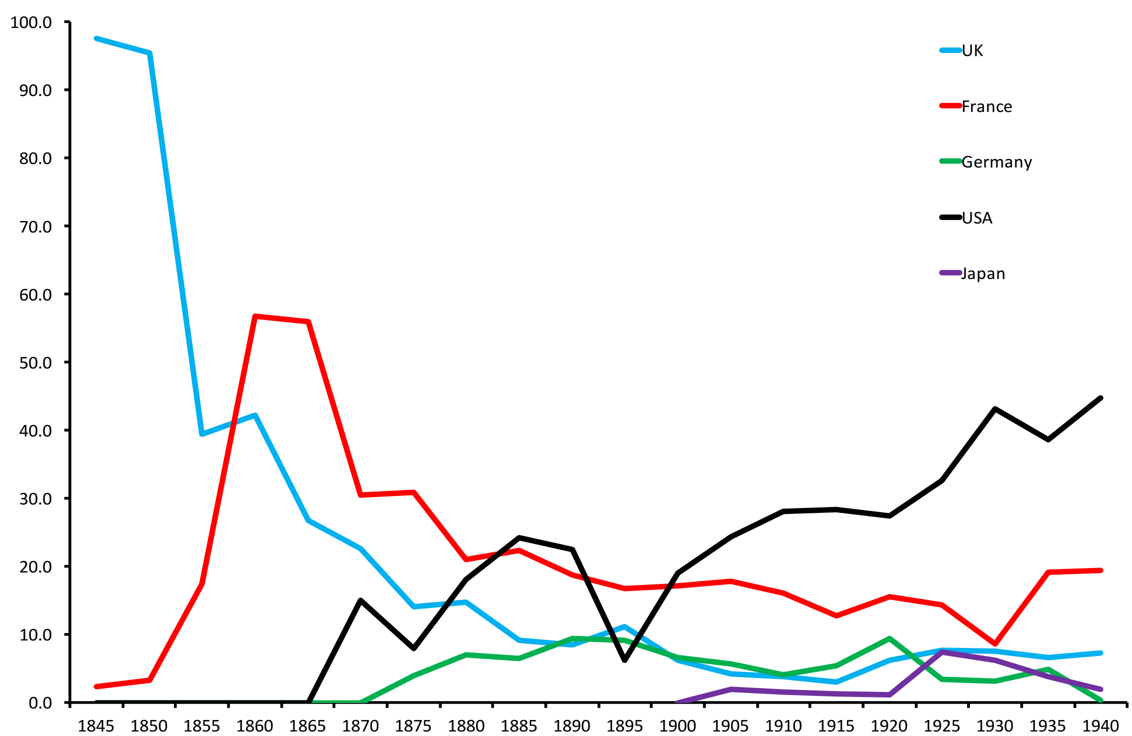

Source: IMF (billion USD)

Figure 3. Net Foreign Assets, World Share (%), 1970-2010



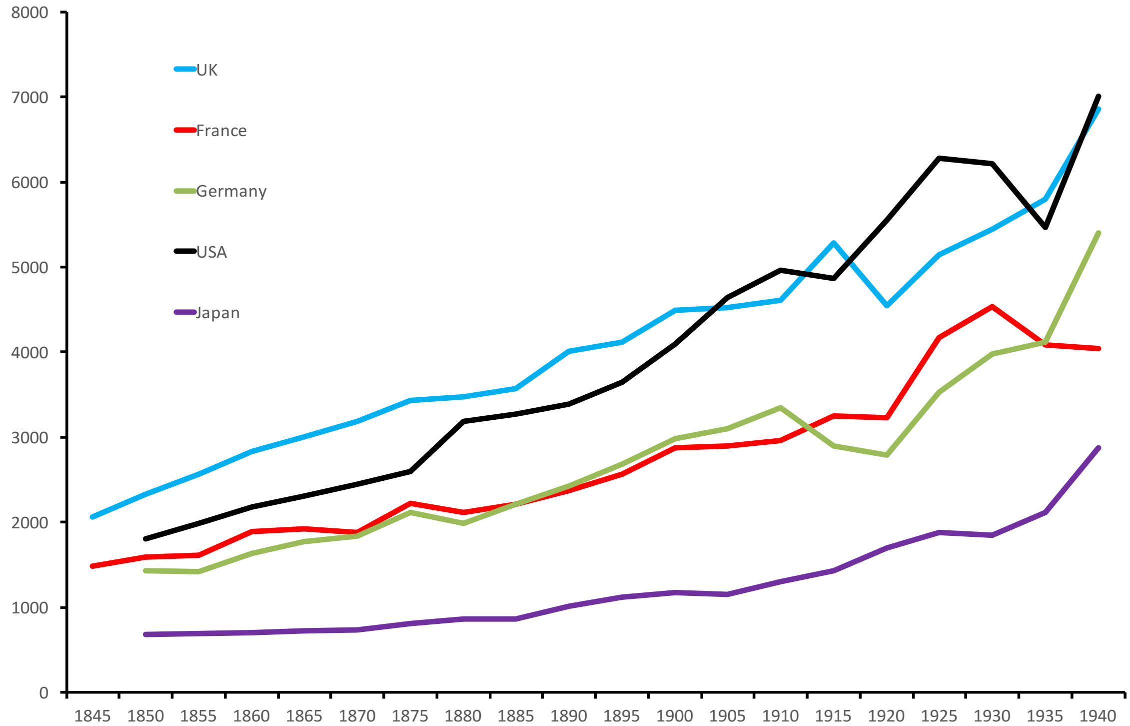

Source: Updated data of Lane and Milesi-Ferretti (2007) by authors (the denominator is world assets plus liabilities, divided by 2)

Figure 4. World Gold Holdings Share (%), 1845-1940




Source: World Gold Council

Figure 5. GDP per Capita (1990 Int'l USD), 1845-1940




Source: Maddison Project

Table 1. Net Foreign Assets and Gold Holdings, 1845-1940

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Year | 1850 | 1875 | 1895 | 1915 | 1930 | 1940 | *Correlation* |
| UK | Net Foreign Assets | 3.1 | 9.7 | 14.6 | 17.8 | 10.8 | 9.0 | |
| | Official Gold Holdings | 0.07 | 0.11 | 0.20 | 0.16 | 0.67 | 1.57 | *-0.12* |
| | Year | 1850 | 1880 | 1915 | | | | |
| France | Net Foreign Assets | 1.7 | 5.1 | 8.5 | | | | |
| | Official Gold Holdings | 0.00 | 0.16 | 0.66 | | | | *0.96* |
| | Year | 1930 | 1940 | | | | | |
| Germany | Net Foreign Assets | -1.3 | 0.9 | | | | | |
| | Official Gold Holdings | 0.28 | 0.06 | | | | | *-1.00* |
| | Year | 1900 | 1910 | 1930 | 1940 | | | |
| USA | Net Foreign Assets | -1.5 | -0.7 | 1.3 | 2.4 | | | |
| | Official Gold Holdings | 0.39 | 1.07 | 3.86 | 9.66 | | | *0.94* |
| | Year | 1915 | 1930 | 1940 | | | | |
| Japan | Net Foreign Assets | -2.5 | 1.0 | 0.3 | | | | |
| | Official Gold Holdings | 0.06 | 0.56 | 0.41 | | | | *0.99* |
| *Average Correlation* | | | | | | | | *0.35* |
| *Average Correlation without Germany (interwar years)* | | | | | | | | *0.69* |

Source: World Gold Council and Goldsmith (1985)