Elections accelerate inefficiencies in local public good provision with decentralized leadership

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Abstract

This paper introduces the election process to the traditional decentralized leadership model, where the central government does not have a pre-commitment ability, and interregional transfer is optimally designed ex post. In the traditional decentralized leadership model, it has been shown that local public good provision is distorted by ex post transfer. The purpose of this paper is to examine how the introduction of the election process affects inefficiencies in the decentralized leadership situation. Our results show that elections accelerate inefficiencies and the commitment environment affects the direction of this distortion, the degree of which depends on the degree of spillover.

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1 Introduction

In January 2015, the radical left-wing party, led by Alexis Tsipras, won the election in Greece with a promise of anti-austerity (increasing public spending or reducing tax rates). Why was anti-austerity supported by the people even when the European Union (EU) called for austerity in Greece as a condition of additional fiscal support?

The reason why the Greek people took this extreme position may be related to the rescue ex post by the EU. Even if Greece did not impose severe austerity, the EU may have been expected to support Greece ex post by increasing transfers. A very radical ex ante position might have been important to Greece in order to induce transfers from the EU. Given this expectation, the policy leader in Greece is likely to be strategically selected in an election.

The effect of the ex post interregional transfer by the central government has been analyzed theoretically in the context of the lack of commitment. When the central government cannot commit to the policy, it faces a chance to redesign its policy ex post after observing the result in the economy or society. This situation can be modeled for local governments as Stackelberg leaders and for the central government as a Stackelberg follower. Such a model structure is often called "decentralized leadership." Under decentralized leadership, it is shown that local public goods may be either under or overprovided, depending on local governments' ex ante policy. However, some studies show that resource allocation is socially efficient when local governments commit to the provision of pure public goods ex ante (e.g., Caplan et al. 2000; Koethenbuerger 2004, 2008; Akai and Sato 2008; Caplan and Silva 2011; Silva 2014, 2015; Akai and Watanabe 2020). In particular, in previous literature, the importance of the type of local governments' commitment in terms of social welfare has been focused on the decentralized leadership model (e.g., Akai and Watanabe 2020). [1]

In previous models, the election stage where the representative who promises the policy is elected is not considered. It is interesting to incorporate the election stage into the decentralized leadership model from the perspective of reality and analyze how the introduction of the election process affects inefficiencies previously presented in the decentralized leadership model.

In the absence of ex post interregional transfers, there exists some previous literature that considers

[1] For a comprehensive survey of research on central and local government policy making and their interactions, see Agrawal et al. (2021).

the election process in the model with interregional competitive interactions among local governments. The policy maker is strategically elected by voters, taking their effect on the election in other regions into account, the behavior of which is often called "strategic delegation." Several studies derive the effects on the local tax or expenditure policies, the degree of strategic delegation, and resource allocation in equilibrium. As for the public goods model with spillover, Besley and Coate (2003), Dur and Roelfsema (2005), Loeper (2017) and Kempf and Rota-Graziosi (2019) analyze strategic delegation and evaluate the efficiency in public goods provision in the non-cooperative or cooperative regime. While they focus on incentives for strategic delegation, they do not consider any ex post transfers. In contrast, we focus on the election process in the presence of ex post transfer and analyze how strategic delegation takes place, and the efficiency of public goods provision would be affected. [2] However, in these previous studies, no ex post interregional transfer is considered. [3]

This paper analyzes how the consideration of the election process improves or exacerbates the resource allocation in decentralized leadership where the central government cannot commit to its policy ex ante and can design interregional transfer ex post.

The main results are as follows. In the situation where the median voter is elected in the absence of ex post transfer, ex post transfer affects the political outcome, and the policy maker who prefers different policies from the median voter is elected. As a result, the resource allocation of local public good provision under the presence of the election process is further distorted, in addition to the original distortion by the ex post transfer in the model without the election process.

The direction of its distortion depends on whether the policy maker's ex ante policy is committed to local public good provision or the local tax rate. When the policy maker in each region commits to local public good provision ex ante, the policy maker with the stronger preference for local public

[2] As for a relationship between the election process and the efficiency in public goods provision, Ihori and Yang (2009) develop the model of tax competition under representative democracy. Theoretical models of tax competition often show that they cause inefficiently low taxation, called race to the bottom, in order to attract a tax base to their region. They show that a decisive voter (i.e., the median voter in majority voting) in the election delegates the authority of setting the tax rate to residents who prefer a higher tax rate compared to himself/herself, and since intraregional political competition and interregional tax competition work well, they suggest that the optimal provision of public goods could be realized. In other words, they suggest that strategic delegation may *improve* the inefficiency in public goods provision. In contrast, our decentralized leadership model suggests that strategic delegation may *accelerate* the inefficiency in public goods provision.

[3] One exception is Susa (2019), who analyzes electoral outcomes with equalization transfer in a tax competition framework. The equalization is based on the difference at the capital level, the total of which is constant. The incentive is not biased in equilibrium. By contrast, we analyze the transfer on the levels of private consumption or local public goods, the total of which is not constant in the model.

goods would be elected, the degree of which depends on the degree of the spillover effect of local public goods. Resource allocation is more distorted by the higher level of local public good provision, except for the case of maximal spillover. The increase of the degree of the spillover mitigates the inefficiency of the resource allocation, and this distortion can be fully canceled out in the case of maximal spillover.

By contrast, when the policy maker in each region commits to its local tax rate, the policy maker with the weaker preference for local public goods is elected, and resource allocation is more distorted with the lower level of local public good provision. In this case, this distortion cannot be canceled out even in the case of maximal spillover, while the efficiency of resource allocation is improved as the degree of the spillover increases.

This paper is structured as follows. The basic model is explained in Section 2. Then, we analyze two scenarios. In Section 3, we analyze the scenario where the policy maker commits to local public good provision. In Section 4, we analyze another scenario where the policy maker commits to the local tax rate. In Section 5, we compare the effect of ex post interregional transfer on the elected policy maker between scenarios. In Section 6, we compare the social welfare between scenarios. In Section 7, we make two extensions for the main model. Section 8 concludes the paper.

2 The model

Consider an economy with two identical regions except for residents' income, $i = A, B$. [4] There are two local governments and one central government. The population size of each region is normalized to one. Each region provides a local public good g_i, which is measured in per capita terms and may generate interregional spillovers. All individuals j in region i enjoy private consumption c_{ij}.

Individuals have different local public goods preference. We denote the local public goods preference of individual j in each region by θ_j. This preference in each region is symmetrically distributed over the interval $[\theta^{min}, \ \theta^{max}]$, so the median is equivalent to the mean. The higher an individual's θ_j is, the stronger his/her preference for local public goods is.

Each individual j in both regions is characterized by the local public goods preference parameter

[4] For simplicity, we analyze two regions. Our results would not change even if N regions were considered.

θ_j. The preference of a type θ_j citizen in each region is:

$$u(c_{ij}) + \theta_j v(g_i, g_{-,i}; \lambda)$$
$$= \log c_i + \theta_j[\log g_i + \lambda \log g_{-i}], \ i = A, B, \ \forall j, \ i \neq -i \quad (1)$$

We assume the additively separable utility function, which is a functional form adopted in Besley and Coate (2003), to clearly define the effect of ex post interregional transfer on the election of the policy maker. [5] λ represents the degree of spillover of local public goods. If $\lambda = 0$, spillover effects are absent: individuals in region i do not care about local public good provision in region $-i$. The higher the value of λ, the higher the degree of spillover. If $\lambda = 1$, individuals care about the local public good provided in the other region as well as the local public good produced in their own region. In order to an interior solution in the subgame-perfect equilibrium, we assume:

Assumption .

$$\frac{1}{1-\lambda} > \theta_M > \frac{1}{1+\lambda} \quad (2)$$

For the above assumption to be practical, λ must satisfy $0 < \lambda \leq 1$. [6]

Local government i levies a lump sum tax t_i on all residents who live in the region i, and the tax revenue is used for local public good provision g_i, which is measured in per capita terms. Suppose that y_i is the exogenous per capita income of individuals in region i, then private consumption of individuals in region i is given as follows.

$$c_{ij} = y_i - t_i = c_i, \ i = A, B, \ \forall j \quad (3)$$

Note that the decision t_i directly links to the level of private consumption, given the exogenous income.

[5]Under the additively separable utility function, the distortion does not occur in the model without ex post transfers. Therefore, it is possible to derive the effect of the introduction of ex post transfers clearly. By contrast, Kempf and Rota-Graziosi (2019) assume a utility function that satisfies $\partial^2 v(g_i, g_{-,i}; \lambda)/\partial g_i \partial g_{-i} \neq 0$. In their paper, strategic delegation takes place even without ex post transfers. See Kempf and Rota-Graziosi (2019) for detailed explanations.

[6]In the two scenarios we analyze in the following sections, $1/(1-\lambda) > \theta_M$ and $> \theta_M > 1/(1+\lambda)$ must be satisfied for private consumption and local public goods provision to be positive in the subgame-perfect equilibrium, respectively. See equations (21) and (37) for details.

Now the budget constraint of local government i can be written as:

$$g_i = t_i + s_i, \quad i = A, B \tag{4}$$

where s_i denotes the per capita transfer from the central government to the region. Turning to the budget constraint of the central government, the expression becomes:

$$s_A + s_B = 0 \tag{5}$$

s_i can be either positive or negative.[7] The central government can control the transfer to pursue its own objectives, but cannot commit to the transfer policy; therefore, the transfer is optimized ex post. Given the central and local budget constraints, we can describe the overall resource constraint as follows.

$$c_A + c_B + g_A + g_B = y_A + y_B \tag{6}$$

2.1 Benchmark

We first derive the socially optimal level of local public goods. The social optimum serves as a benchmark in order to evaluate the outcomes of political decision making under ex post interregional transfers. We define the social optimum as the outcome that maximizes the sum of utilities of all individuals in both regions. Because individuals in each region are symmetrically distributed over the interval $[\theta^{min},\ \theta^{max}]$, and with population size normalized to one, social welfare is equal to the sum of utilities of median voters in both regions. We denote the position of the median in both regions by θ_M. Hence,

$$\begin{aligned} \max_{\{c_i, g_i\}_{i=A,B}} \quad S = &\log c_A + \theta_M[\log g_A + \lambda \log g_B] \\ &+ \log c_B + \theta_M[\log g_B + \lambda \log g_A] \\ \text{s.t. } &c_A + c_B + g_A + g_B = y_A + y_B \end{aligned}$$

[7] In our model, negative transfer implies ex post taxation of the local government by the central government.

By solving this optimization problem, we obtain the socially optimal local public goods provision and private consumption as follows. [8]

$$g_i^{**} = g^{**} = \frac{(1+\lambda)\theta_M(y_A + y_B)}{2[(1+\lambda)\theta_M + 1]}, \ i = A, B \tag{7}$$

2.2 Timeline

The timeline of our model is defined as follows.

1. In each region, the policy maker is simultaneously elected from among the individuals through majority voting. The authority to determine the local policy within a region is delegated to the individual selected in this election.

2. Given the environment related to the committed policy variables, the local policy is determined by the policy maker selected through the election in Stage 1.

3. Having observed local policies within both regions committed in Stage 2, the central government designs the transfer s_i. Finally, given all policies of governments, each individual consumes and gains utility.

Regarding ex ante commitment of the policy maker in each region, we consider following two scenarios. In Scenario I, the policy maker commits to the level of local public provision g_i ex ante with the local tax rate t_i remaining as residual ex post. By contrast, the local tax rate is pre-committed in Scenario II, and g_i is determined ex post, after the central transfer is designed. In either scenario, the central government is the Stackelberg follower. Because the concept of a subgame-perfect equilibrium is applied, we solve each scenario backwardly.

[8] The socially optimal level of private consumption becomes:

$$c_i^{**} = c^{**} = \frac{y_A + y_B}{2[(1+\lambda)\theta_M + 1]}, \ i = A, B$$

3 Scenario I: g_i is committed ex ante

3.1 Stage 3: Ex post policy making of the central government

Given that g_i is already committed ex ante, t_i is adjusted ex post so as to balance the local budget $t_i = g_i - s_i$, with s_i transferred from the central government. [9] Then the central government chooses s_i to maximize the sum of utilities of median voters in both regions. [10] The optimization problem solved by the central government in the third stage is given as follows.

$$\max_{s_A, s_B} U_{AM} + U_{BM}$$
$$= \log(y_A - g_A + s_A) + \theta_M[\log g_A + \lambda \log g_B]$$
$$+ \log(y_B - g_B + s_B) + \theta_M[\log g_B + \lambda \log g_A]$$
$$\text{s.t. } s_A + s_B = 0$$

The first-order condition derived as

$$\frac{1}{c_A} - \frac{1}{c_B} = 0 \tag{8}$$

This implies that the ex post interregional transfer s_i is chosen to equalize the marginal utilities of private consumption across regions, namely $c_A = c_B = c$. Therefore, we obtain:

$$c = \frac{y_A + y_B - g_A - g_B}{2} \tag{9}$$

Substituting equation (9) into $c_i = y_i - g_i + s_i$ yields:

$$s_i = \frac{y_{-i} - g_{-i} - (y_i - g_i)}{2}, \ i = A, B, \ i \neq -i \tag{10}$$

[9] Then private consumption is adjusted by the transfer through the change of t_i.

[10] Here, we simply consider the case of a benevolent central government in order to focus on the distortion of the regional election under a benevolent central government.

Differentiating equation (10) by g_i yields: [11]

$$\frac{\partial s_i}{\partial g_i} = \frac{1}{2} > 0,\ i = A, B \tag{11}$$

$$\frac{\partial s_{-i}}{\partial g_i} = -\frac{1}{2} < 0,\ i = A, B,\ i \neq -i \tag{12}$$

This implies that increase of g_i can induce more transfers from the central government and the half of its cost is covered by its transfer, which affects the incentive of the policy maker in each region in the second stage.

3.2 Stage 2: The policy maker's ex ante policy

Let the utility level of the policy maker in region i be denoted by U_{iP}, and the policy maker's local public goods preference denoted by θ_{iP}. Taking local public good provision in the other region g_{-i} as given, the policy maker determines g_i in his/her region to maximize his/her utility. Therefore, the optimization problem solved by the policy maker in each region is given as follows. [12]

$$\max_{g_i} U_{iP} = \log c + \theta_{iP}[\log g_i + \lambda \log g_{-i}]$$
$$\text{s.t. } c = \frac{y_A + y_B - g_A - g_B}{2}$$

The first-order condition is: [13]

$$\frac{1}{c} \times \left(-\frac{1}{2}\right) + \theta_{iP}\frac{1}{g_i} = 0,\ i = A, B \tag{13}$$

[11] The denominator is 2 in equations (11) and (12) because we assume two regions.

[12] In this stage, the policy maker is able to commit to the policy he/she decides, and individuals know it in the first stage election.

[13] The first-order condition can be generally rewritten as follows.

$$MRS_{gc} \equiv \frac{\partial U_{iP}/\partial g_i}{\partial U_{iP}/\partial c} = \frac{1}{2}\ (< 1)$$

Since the marginal rate of substitution between local public good and private consumption is smaller than that in the absence of the transfer (i.e., 1), it implies that local public goods would be overprovided.

From condition (13) and equation (9), we obtain the reaction function of region i as follows.

$$g_i(g_{-i}) = -\frac{\theta_{iP}}{1+\theta_{iP}} g_{-i} + \frac{\theta_{iP}(y_A + y_B)}{1+\theta_{iP}}, \ i = A, B, \ i \neq -i \tag{14}$$

Solving equation (14) for $i = A, B$, we obtain:

$$g_i(\theta_{AP}, \theta_{BP}) = \frac{\theta_{iP}(y_A + y_B)}{1+\theta_{AP}+\theta_{BP}}, \ i = A, B \tag{15}$$

Combining equation (9) with (15) yields:

$$c(\theta_{AP}, \theta_{BP}) = \frac{y_A + y_B}{2(1+\theta_{AP}+\theta_{BP})} \tag{16}$$

3.3 Stage 1: Election of the policy maker

In the first stage, the policy maker is elected by majority voting in each region. Individuals in each region vote for a candidate based on their local public goods preference. Because of the median voter theorem, the individual located at the median of the distribution of the preference is the decisive voter in his/her region. Taking the result of the second stage into account and the choice of the other region as given, the median voter in each region decides to whom the authority to decide the level of local public good provision is delegated. Hence, the optimization problem considered by the median voter in region i is given as follows.

$$\begin{aligned} \max_{\theta_{iP}} U_{iM} &= \log c(\theta_{AP}, \theta_{BP}) + \theta_M[\log g_i(\theta_{AP}, \theta_{BP}) + \lambda \log g_{-i}(\theta_{AP}, \theta_{BP})] \\ \text{s.t. } c(\theta_{AP}, \theta_{BP}) &= \frac{y_A + y_B}{2(1+\theta_{AP}+\theta_{BP})} \\ g_i(\theta_{AP}, \theta_{BP}) &= \frac{\theta_{iP}(y_A + y_B)}{1+\theta_{AP}+\theta_{BP}}, \ i = A, B, \ i \neq -i \end{aligned}$$

The first-order condition can be derived as follows. [14]

$$-\frac{1}{1+\theta_{AP}+\theta_{BP}} + \theta_M \left[\frac{1+\theta_{-iP}}{\theta_{iP}(1+\theta_{AP}+\theta_{BP})} - \frac{\lambda}{1+\theta_{AP}+\theta_{BP}} \right] = 0, \ i = A, B, i \neq -i \tag{17}$$

[14] For confirmation of the second-order condition, see the appendix.

Therefore, we obtain the reaction function of region i as follows.

$$\theta_{iP}(\theta_{-iP}) = \frac{\theta_M}{1+\lambda\theta_M}\theta_{-iP} + \frac{\theta_M}{1+\lambda\theta_M}, \ \ i = A, B, i \neq -i \tag{18}$$

Solving equation (18) for $i = A, B$, we obtain:

$$\theta_{AP}^{\mathrm{I}*} = \theta_{BP}^{\mathrm{I}*} = \theta_P^{\mathrm{I}*} = \frac{\theta_M}{1-(1-\lambda)\theta_M} \tag{19}$$

Substituting equation (19) into (15) and (16), the equilibrium levels of local public goods and private consumption in each region are derived as follows.

$$g_A^{\mathrm{I}*} = g_B^{\mathrm{I}*} = g^{\mathrm{I}*} = \frac{\theta_M(y_A+y_B)}{(1+\lambda)\theta_M+1} \tag{20}$$

$$c^{\mathrm{I}*} = \frac{[1-(1-\lambda)\theta_M](y_A+y_B)}{2[(1+\lambda)\theta_M+1]} \tag{21}$$

We can also derive local public goods provision in the absence of election of the policy maker. In this case, local public good provision is determined directly by the median voter. We refer to this case as the “no election case.” [15] This level of the provision is derived by substituting $\theta_{AP} = \theta_{BP} = \theta_M$ into equation (15). Let $g^{\mathrm{I}*}|_{\theta_P^*=\theta_M}$ represent local public goods provision in the no election case, then we obtain the following.

$$g^{\mathrm{I}*}|_{\theta_P^*=\theta_M} = \frac{\theta_M(y_A+y_B)}{1+2\theta_M} \tag{22}$$

The results of Scenario I are summarized as follows.

Proposition 1.

(a) Assume $\lambda = 1$. Then, local public goods provision in the subgame-perfect equilibrium coincides with the social optimum, i.e., $g^{I*} = g^{**}$. The median voter is elected as the policy maker, i.e., $\theta_P^{I*} = \theta_M$.

[15] In the case without election, the median voter decides policies directly based on his/her utility maximization as in a direct democracy, rather than the benevolent local government decides policies based on the maximization of sum of residents' utility in the region. In other words, the median voter elects his/herself as the policy maker. This is also known as *self-representation*. Thus, the same process is applicable even for asymmetric distributions where the mean and median are different.

(b) Assume $\lambda \in (0,1)$. Then:

(b-1) The median voter delegates the authority to the individual with stronger local public goods preference to decide the policy, i.e., $\theta_P^{I*} > \theta_M$.

(b-2) Local public goods are overprovided relative to the social optimum, i.e., $g^{I*} > g^{**}$.

(b-3) Compared with the no election case, local public goods provision in the subgame-perfect equilibrium is greater, i.e., $g^{I*} > g^{I*}|_{\theta_P^{I*}=\theta_M} > g^{**}$.

The intuition of Proposition 1 is given as follows. First, as shown in (a), the degree of spillover has the crucial role of achieving the socially optimal allocation. This result can be interpreted as the combination of the results from both the decentralized leadership model and the strategic delegation model. In the decentralized leadership model, it has been shown that the incentive effect of ex post interregional transfer is fully canceled out by the incentive effect of the maximal spillover. In articles on strategic delegation, it has been shown that the strategically elected policy maker coincides with the median voter under the separable utility function. The special situation with both the separable utility function and maximal spillover makes resource allocation optimal.

By contrast, for (b) with $\lambda < 1$, the result dramatically changes. In addition to the fact that resource allocation is biased under $\lambda < 1$, because of the incentive effect of the interregional transfer, the strategically elected policy maker does not coincide with the median voter even under the separable utility function. In particular, the individual with stronger preference for local public goods is elected as the policy maker, because the higher level of local public good provision is desirable in order to induce the higher level of transfer ex post (see equation (11)). As a result, the level of local public goods provision is accelerated by the presence of the policy maker election stage. [16]

Figure 1 illustrates changes in electoral outcomes depending on the degree of spillover in Scenario I. For the slope and intercept of the best response function in the first stage represented by equation

[16] The results of Proposition 1 seem to be similar to Besley and Coate (2003) and Dur and Roelfsema (2005). in both papers, the reason for obtaining these results is due to the integration of regional budgets by *centralized (cooperative)* decision making, which is different from our results arising from cost sharing by ex post transfers under *decentralized* decision making.

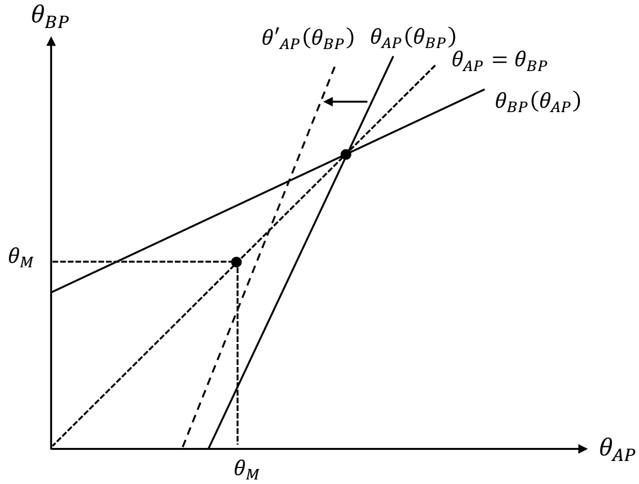

Figure 1: Best responses at the electoral stage in Scenario I.

(18), we obtain:

$$\frac{\partial}{\partial \lambda}\left(\frac{\theta_M}{1+\lambda\theta_M}\right) = -\frac{\theta_M^2}{(1+\lambda\theta_M)^2} < 0$$

Therefore, the degree of spillover shifts the best response function of region A downward. This is represented by the shift from $\theta_{AP}(\theta_{BP})$ to $\theta'_{AP}(\theta_{BP})$ in Figure 1. Of course, the best response function of region B also shifts downward, so that the intersection of best response functions of both regions shifts to the southwest on the 45-degree line. As a result, the local public goods preference of the policy maker elected in each region approaches the median voter's preference. This means that local public goods provision in equilibrium decreases. That is, we have:

$$\frac{\partial g^{\mathrm{I}*}}{\partial \lambda} = -\frac{\theta_M^2(y_A + y_B)}{[(1+\lambda)\theta_M + 1]^2} < 0$$

The reason for the relationship between the degree of spillover and the electoral outcome is that the higher spillover effect decreases the incentive to induce the transfer to its own region (in other words, the incentive for strategic delegation) due to the increased benefit from local public good provision in the other region. Under maximal spillover (i.e., $\lambda = 1$), the incentive for strategic delegation disappears, so the elected policy maker is consistent with the median voter. As it approaches the median voter, local public goods provision in equilibrium also decreases, while the socially optimum level increases with the increase of the degree of the spillover. Therefore, local public goods provision coincides with the social optimum.

4 Scenario II: t_i is committed ex ante

4.1 Stage 3: Ex post policy making of the central government

In this section, we consider another scenario in which the policy maker in each region commits to the local tax rate ex ante. Because t_i is already committed ex ante, in contrast to Scenario I, g_i is adjusted ex post such as to balance the local budget $g_i = t_i + s_i$. Therefore, the optimization problem solved by the central government in the third stage is given as follows.

$$\begin{aligned} \max_{s_A, s_B} \; & U_{AM} + U_{BM} \\ & = \log c_A + \theta_M [\log(t_A + s_A) + \lambda \log(t_B + s_B)] \\ & \quad + \log c_B + \theta_M [\log(t_B + s_B) + \lambda \log(t_A + s_A)] \\ \text{s.t. } & s_A + s_B = 0 \end{aligned}$$

The first-order condition derived as

$$(1 + \lambda)\theta_M \frac{1}{g_A} - (1 + \lambda)\theta_M \frac{1}{g_B} = 0 \tag{23}$$

This implies that the central transfer s_i is chosen to equalize the marginal utilities of local public goods across regions, namely $g_A = g_B = g$. Therefore, we obtain:

$$g = \frac{t_A + t_B}{2} \tag{24}$$

Substituting equation (24) into $g_i = t_i + s_i$ yields:

$$s_i = \frac{t_{-i} - t_i}{2}, \;\; i = A, B, \;\; i \neq -i \tag{25}$$

Differentiating equation (25) by t_i yields:

$$\frac{\partial s_i}{\partial t_i} = -\frac{1}{2} < 0, \ i = A, B \tag{26}$$

$$\frac{\partial s_{-i}}{\partial t_i} = \frac{1}{2} > 0, \ i = A, B, \ i \neq -i \tag{27}$$

This implies that decrease of t_i can induce more transfers from the central government and the half of its cost is covered by its transfer, which affects the incentive of the policy maker in each region in the second stage.

4.2 Stage 2: The policy maker's ex ante policy making

In the second stage, the policy maker determines t_i in his/her region to maximize his/her utility, taking the local tax rate in the other region t_{-i} as given. Therefore, the optimization problem solved by the policy maker in each region is given as follows.

$$\max_{t_i} U_{iP} = \log c_i + \theta_{iP}[\log g + \lambda \log g]$$

$$\text{s.t. } c_i = y_i - t_i, \ i = A, B$$

$$g = \frac{t_A + t_B}{2}$$

The first-order condition is: [17]

$$-\frac{1}{y_i - t_i} + \theta_{iP}\left[\frac{1}{g} \times \frac{1}{2} + \lambda \frac{1}{g} \times \frac{1}{2}\right] = 0, \ i = A, B \tag{28}$$

From condition (28) and equation (24), we obtain the reaction function of region i as follows.

$$t_i(t_{-i}) = -\frac{1}{(1+\lambda)\theta_{iP} + 1} t_{-i} + \frac{(1+\lambda)\theta_{iP} y_i}{(1+\lambda)\theta_{iP} + 1}, \ i = A, B, i \neq -i \tag{29}$$

[17] The first-order condition can be generally rewritten as follows.

$$MRS_{gc} \equiv \frac{\partial U_{iP}/\partial g}{\partial U_{iP}/\partial c_i} = 2(> 1)$$

Since the marginal rate of substitution between local public good and private consumption is larger than that in the absence of the transfer (i.e., 1), it implies that local public goods would be underprovided.

Solving equation (29) for $i = A, B$, we obtain:

$$t_i(\theta_{AP}, \theta_{BP}) = \frac{\theta_{iP} y_i[(1+\lambda)\theta_{-iP} + 1] - \theta_{-iP} y_{-i}}{(1+\lambda)\theta_{AP}\theta_{BP} + \theta_{AP} + \theta_{BP}}, \ i = A, B, i \neq -i \tag{30}$$

Substituting equation (30) into $c_i = y_i - t_i$ and equation (24) yields:

$$c_i(\theta_{AP}, \theta_{BP}) = \frac{\theta_{-iP}(y_A + y_B)}{(1+\lambda)\theta_{AP}\theta_{BP} + \theta_{AP} + \theta_{BP}}, \ i = A, B, i \neq -i \tag{31}$$

$$g(\theta_{AP}, \theta_{BP}) = \frac{1}{2} \times \frac{(1+\lambda)\theta_{AP}\theta_{BP}(y_A + y_B)}{(1+\lambda)\theta_{AP}\theta_{BP} + \theta_{AP} + \theta_{BP}} \tag{32}$$

4.3 Stage 1: Election of the policy maker

As presented in Section 3.3, the optimization problem considered by the median voter in region i is given as follows.

$$\begin{aligned}
\max_{\theta_{iP}} \ & U_{iM} = \log c_i(\theta_{AP}, \theta_{BP}) + \theta_M[\log g(\theta_{AP}, \theta_{BP}) + \lambda \log g(\theta_{AP}, \theta_{BP})] \\
\text{s.t. } & c_i(\theta_{AP}, \theta_{BP}) = \frac{\theta_{-iP}(y_A + y_B)}{(1+\lambda)\theta_{AP}\theta_{BP} + \theta_{AP} + \theta_{BP}}, \ i = A, B, \ i \neq -i \\
& g(\theta_{AP}, \theta_{BP}) = \frac{1}{2} \times \frac{(1+\lambda)\theta_{AP}\theta_{BP}(y_A + y_B)}{(1+\lambda)\theta_{AP}\theta_{BP} + \theta_{AP} + \theta_{BP}}
\end{aligned}$$

The first-order condition can be derived as follows. [18]

$$\begin{aligned}
& -\frac{[(1+\lambda)\theta_{-iP} + 1]}{(1+\lambda)\theta_{AP}\theta_{BP} + \theta_{AP} + \theta_{BP}} \\
& + \theta_M \left[\frac{(1+\lambda)\theta_{-iP}}{\theta_{iP}[(1+\lambda)\theta_{AP}\theta_{BP} + \theta_{AP} + \theta_{BP}]} \right] = 0, \ i = A, B, i \neq -i
\end{aligned} \tag{33}$$

Therefore, we obtain the reaction function of region i as follows.

$$\theta_{iP}(\theta_{-iP}) = \frac{(1+\lambda)\theta_M}{(1+\lambda)\theta_{-iP} + 1}\theta_{-iP}, \ i = A, B, i \neq -i \tag{34}$$

[18] For confirmation of the second-order condition, see the appendix as in Scenario I.

Solving equation (34) for $i = A, B$, we obtain:

$$\theta_{AP}^{\text{II}*} = \theta_{BP}^{\text{II}*} = \theta_{P}^{\text{II}*} = \frac{(1+\lambda)\theta_M - 1}{1+\lambda} \tag{35}$$

Substituting equation (35) into (31) and (32), the equilibrium levels of private consumption and local public good in each region are derived as follows.

$$c_A^{\text{II}*} = c_B^{\text{II}*} = c^{\text{II}*} = \frac{y_A + y_B}{(1+\lambda)\theta_M + 1} \tag{36}$$

$$g^{\text{II}*} = \frac{[(1+\lambda)\theta_M - 1](y_A + y_B)}{2[(1+\lambda)\theta_M + 1]} \tag{37}$$

As in Scenario I, we also derive local public goods provision in the no election case. This is derived by substituting $\theta_{AP} = \theta_{BP} = \theta_M$ into equation (32). Then, we obtain:

$$g^{\text{II}*}|_{\theta_P^* = \theta_M} = \frac{(1+\lambda)\theta_M(y_A + y_B)}{2[(1+\lambda)\theta_M + 2]} \tag{38}$$

The results of Scenario II are summarized as follows.

Proposition 2.

(a) Local public good provision in the subgame-perfect equilibrium never coincides with the social optimum, irrespective of the degree of spillover effect λ.

(b) For any λ:

(b-1) The median voter delegates the authority to the individual with weaker local public goods preference to decide the policy, i.e., $\theta_P^{II*} < \theta_M$.

(b-2) Local public goods are underprovided relative to the social optimum, i.e., $g^{II*} < g^{**}$.

(b-3) Compared with the no election case, local public goods in the subgame-perfect equilibrium are less provided, i.e., $g^{II*} < g^{II*}|_{\theta_P^* = \theta_M} < g^{**}$.

The intuition of Proposition 2 is given as follows. In this scenario, interregional transfer tends to depress local public good provision, and the election accelerates this downward incentive because the policy maker with weaker preference for local public goods who prefers a lower level of the local tax rate will induce a higher level of transfer ex post.

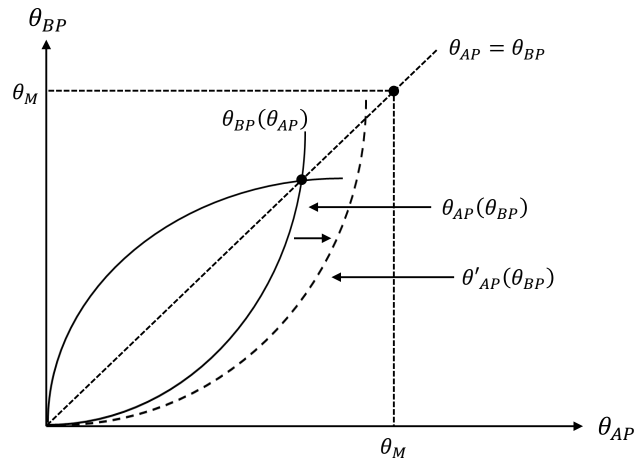

Figure 2: Best responses at the electoral stage in Scenario II.

Figure 2 illustrates changes in electoral outcomes depending on the degree of spillover in Scenario II. For the best response function in the first stage, represented by equation (34), we obtain:

$$\frac{\partial \theta_{iP}(\theta_{-iP})}{\partial \lambda} = \frac{\theta_M \theta_{-iP}}{[(1+\lambda)\theta_{-iP}+1]^2} > 0, \ \ i = A, B, i \neq -i$$

Therefore, with the increase of the degree of spillover, the best response function of region A shifts upward. This is depicted by the shift from $\theta_{AP}(\theta_{BP})$ to $\theta'_{AP}(\theta_{BP})$ in Figure 2. Of course, the best response function of region B also shifts upward, so that the intersection of best response functions of both regions shifts to the northeast on the 45-degree line. In equilibrium, the local public goods preference of the policy maker elected in each region approaches the median voter's preference. This shows that local public goods provision also increases in equilibrium. That is, we have:

$$\frac{\partial g^{\mathrm{II}*}}{\partial \lambda} = \frac{\theta_M (y_A + y_B)}{[(1+\lambda)\theta_M + 1]^2} > 0$$

Therefore, increasing the degree of spillover decreases the incentive for strategic delegation, namely the increase of $\theta_P^{\mathrm{II}*}$, and increases local public goods provision with the reduction of private consumption in equilibrium. This change might be desirable in the sense that local public goods gap between equilibrium and the social optimum becomes small as the degree of spillover increases (we can derive this by differentiating the right-hand side of equation (A.1) in the appendix with respect to λ).

Compared with Scenario I, even in the case of maximal spillover, local public goods provision in equilibrium does not coincide with the social optimum as shown in (a) in Scenario II. In Scenario I, the enhanced marginal incentive for providing local public good by ex post interregional transfer is perfectly compatible with the socially desirable incentive level under maximal spillover. By contrast, in Scenario II, the reduced marginal incentive for providing local public good by ex post interregional transfer cannot be compatible with the socially desirable incentive level, which requires enhancing the provision of local public goods. Therefore, each region in Scenario II has an incentive to keep private consumption at some level while the incentive for strategic delegation toward the higher level of private consumption is not perfectly eliminated, even if the degree of spillover is maximal. As a result, the elected policy maker still has a smaller incentive for raising the local tax rate (or providing the local public good) compared with the median voter, and local public goods provision in equilibrium is still less than the socially optimal level, even if the degree of spillover is maximal.

5 Opposite effects on electoral outcomes between scenarios

Table 1 summarizes the effect on the elected policy maker, θ_P^*, compared with no election case in the model with/without interregional transfer, θ_M. Under the setting adopted in this paper, the election is not distorted in the case without transfer, $\theta_P^* = \theta_M$. [19] In the existence of the transfer, the distorted policy maker is elected, except for the case with commitment to local public good provision with maximal spillover. This exception is caused by the fact that the distortion from the interregional transfer is fully canceled out by maximal spillover. Generally, effects from the two types of commitments considered in this paper are in the opposite direction to the electoral outcome. In the case with commitment to local public good provision, the policy maker with stronger preference for local public goods is elected. By contrast, in the case of commitment to the tax rate, namely the level of private consumption, the policy maker with weaker preference for local public goods is elected.

[19] The case without transfer corresponds to the decentralized outcome in Besley and Coate (2003) and Dur and Roelfsema (2005).

These perfectly opposite directions of effect are caused by the opposite direction of incentives due to different commitments.

6 Welfare comparison

In the above, two scenarios are analyzed separately. In this section, we compare the social welfare between Scenario I and Scenario II, which are calculated as follows. [20]

$$S^{\mathrm{I}*} - S^{\mathrm{II}*} = \log\left[\frac{1-(1-\lambda)\theta_M}{2}\right] + \theta_M(1+\lambda)\log\left[\frac{2\theta_M}{(1+\lambda)\theta_M - 1}\right] \tag{39}$$

$[1-(1-\lambda)\theta_M]/2$ in equation (39) represents the ratio of $c^{\mathrm{II}*}$ to $c^{\mathrm{I}*}$. We have $c^{\mathrm{I}*}/c^{\mathrm{II}*} = [1-(1-\lambda)\theta_M]/2 < 1$ because $c^{\mathrm{I}*}(c^{\mathrm{II}*})$ is under (over)provided. Also, $2\theta_M/[(1+\lambda)\theta_M - 1]$ represents the ratio of $g^{\mathrm{II}*}$ and $g^{\mathrm{I}*}$. This means that $g^{\mathrm{I}*}/g^{\mathrm{II}*} = 2\theta_M/[(1+\lambda)\theta_M - 1] > 1$ because $g^{\mathrm{I}*}(g^{\mathrm{II}*})$ is over (under)provided. As λ becomes smaller, the effect of the negative first term is stronger and the effect of the positive second term is weaker, so it would appear that $S^{\mathrm{I}*} < S^{\mathrm{II}*}$. On the other hand, θ_M becomes larger, the effect of the negative first term is weaker and the effect of the positive second term is stronger , so it would appear that $S^{\mathrm{I}*} > S^{\mathrm{II}*}$.

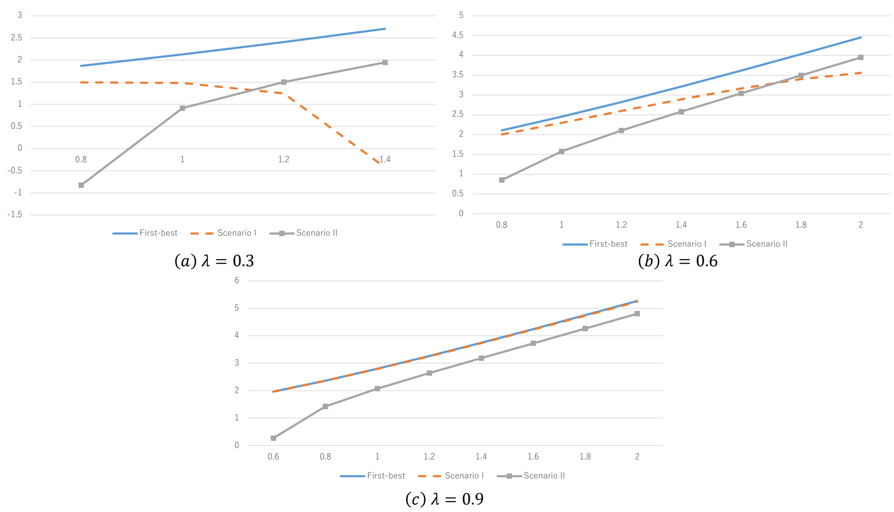

Figure 3: Welfare comparison in three cases: $\lambda = 0.3,\ 0.6$ and 0.9.

Figure 3 (a)-(c) represent the social welfare of the social optimum, Scenario I and Scenario II

[20]For the derivation, see Appendix D in the appendix.

regarding the position of the median voter θ_M at each level of λ. [21] As λ becomes higher, the incentives for strategic delegation become weaker in both scenarios, and local public goods provision approaches the social optimum. Especially, in Scenario I, the local public goods provision is consistent with the social optimum when $\lambda = 1$. In addition, as θ_M increases, local public goods provision increase in both scenarios. In Scenario I, where local public goods are overprovided (except for $\lambda = 1$), the distortion of local public goods provision are expanded, while the distortion is improved in Scenario II where the local public goods are underprovided. Therefore, when λ is low and θ_M is high, the social welfare of scenario II exceeds that of scenario I, as shown in Figure 3 (a) and (b). On the other hand, when λ approaches 1, the social welfare of Scenario I exceeds that of Scenario II for any θ_M, as shown in Figure 3 (c).

7 Extension

7.1 Asymmetric distribution

So far, we assume a symmetric distribution of local public goods preference. In this section, let us first assume an asymmetric distribution. In a symmetric distribution, the median and mean of the distribution are identical, as in a normal distribution. We assume that the distribution among regions are identical and the mean is represented by $\bar{\theta}$ (the median is represented by θ_M as before). In an asymmetric distribution, the distribution is skewed and median and mean do not coincide, i.e., $\theta_M \neq \bar{\theta}$. See the appendix for detailed derivation.

The difference between the symmetric and asymmetric distributions exists in the social welfare function (the sum of utilities of all individuals). In the case of a symmetric distribution, the median and mean of the distribution coincide, so the sum of the utilities of all residents could be expressed as the number of population times the utilities of the *median* resident of both regions. However, in the case of an asymmetric distribution, the median and mean of the distribution do not coincide, so the sum of the utilities of all residents is the number of population times the utilities of the *mean* residents of both regions. Therefore, in the case of asymmetric distribution, the benchmark (socially optimal) local public good provision and the central government's decisions are evaluated by the *mean*, while

[21] Note that in Figure 3 (a)-(c), ranges of θ_M are set such as to satisfy assumption $1/(1-\lambda) > \theta_M > 1/(1+\lambda)$.

the decisions of each region are made by the *median* through the election of the policy maker. Due to this discrepancy between the median and the mean, the social optimum is not achieved even in the case of $\lambda = 1$ in Scenario I. Whether local public good is over or underprovision in this situation depends on whether the median or the mean is greater when $\lambda = 1$. We can confirm the following results (see Appendix D in the appendix for details). In Scenario I, the results in the symmetric distribution hold perfectly when $\lambda \neq 1$ and $\theta_M > \bar{\theta}$. Even when $\lambda \neq 1$ and $\bar{\theta} > \theta_M$, if the gap between $\bar{\theta}$ and θ_M is small, the results still holds (see equation (A.4) in the appendix). However, if the gap between $\bar{\theta}$ and θ_M is sufficiently large, the results may be modified.

The implications are as follows. In Scenario I, local public goods are overprovided in the symmetric distribution. Now, when the environment is modified to an asymmetric distribution, namely $\theta_M > \bar{\theta}$, the gap between the provision of local public goods in equilibrium and that of the social optimum becomes larger than in the symmetric distribution (because the social optimum is evaluated at the *mean*). Therefore, the result that the local public goods are overprovided holds. However, if $\bar{\theta} > \theta_M$, the social optimum level is valued higher, so if the gap between $\bar{\theta}$ and θ_M is sufficiently large, strategic delegation to the individual with stronger local public goods preference increase local public goods provision in equilibrium and would improve the inefficiency in local public goods provision.

In contrast, in Scenario II, the results in the symmetric distribution hold perfectly when $\bar{\theta} > \theta_M$. Even when $\theta_M > \bar{\theta}$, if the gap between $\bar{\theta}$ and θ_M is small, the results still holds (see equation (A.7) in the appendix). The implications are as follows. In Scenario II, local public goods are underprovided in the symmetric distribution. When $\bar{\theta} > \theta_M$, the social optimum level is valued higher than in the symmetric distribution. Therefore, the result that the local public goods are underprovided holds. However, if $\theta_M > \bar{\theta}$, the social optimum level is valued lower, so if the gap between θ_M and $\bar{\theta}$ is sufficiently large, strategic delegation to the individual with weaker local public goods preference increase local public goods provision in equilibrium and would improve the inefficiency in local public goods provision. As a result, it seems to retain the result that elections accelerate inefficiencies in certain plausible situations.

7.2 Non-identical regions

We then assume non-identical regions. Here we assume that the median values of the distributions are different, $\theta_{AM} \neq \theta_{BM}$ under symmetric distribution, $\theta_{iM} = \bar{\theta}_i,\ i = A, B$. In the case of non-identical regions, it means that the preference for local public goods is different among regions (A higher θ_{iM} means that the preference for local public goods is stronger in the region). This means that, as a result, local public good provision and the elected policy maker may differ across regions. In particular, the results of Scenario I may differ significantly between the identical and non-identical regions. We can confirm the following results (see Appendix F in the appendix for details). In Scenario I, when $\lambda = 1$ and the preference of the median resident, θ_{iM}, is larger/smaller than the preference of the median resident in the other region, the policy maker with the stronger/weaker preference is elected (see equations (A.10) and (A.11) in the appendix) and over/underprovision of local public goods occur (see equations (A.13) and (A.14) in the appendix). We assume that $\theta_{AM} > \theta_{BM}$, then the main results in the symmetric regions hold for region A. For region B, they hold when λ is small and the gap between θ_{AM} and θ_{BM} is small (see equations (A.16) and (A.18) in the appendix). On the other hand, in Scenario II, the results in the symmetric regions always hold (see equation (A.28) in the appendix).

The reason why the results are not modified in Scenario II is that, unlike local public goods, private consumption has no spillover effect. In Scenario I, the policy maker determines local public good provision with spillover effect. Since the benefits arise from local public good provision in the other region, the results would be affected by the difference of θ_M and the degree of λ among regions when the environment is modified from symmetric to asymmetric regions. In contrast, noting that $c_i = y_i - t_i$, in Scenario II, the policy maker determines the local tax rate, or in other words, the level of private consumption without spillover effect. Since no benefits arise from private consumption in the other region, the modification from symmetric to asymmetric regions does not significantly affect outcomes. As a result, it seems to retain the result that elections accelerate inefficiencies in certain plausible situations.

8 Conclusion

We show that, in the situation where the median voter is elected without transfer, the representative who differs from the median voter is elected, and resource allocation is distorted more after the election process. The direction of this difference in the preference of the elected policy maker depends on whether ex ante policy can be committed to local public good provision or to the local tax rate by the policy maker.

When the policy maker in each region commits to local public good provision ex ante, then the policy maker with stronger preference for local public goods is elected, and resource allocation is more distorted with the higher level of local public good provision, except for the case of maximal spillover. In addition, the spillover mitigates the inefficiency of resource allocation, and this distortion can be fully canceled out in the case of maximal spillover. By contrast, when the policy maker in each region commits to the local tax rate ex ante, then the policy maker with weaker preference for local public goods is elected, and resource allocation is more distorted with a lower level of local public good provision. In this case, this distortion cannot be canceled out even in the case of maximal spillover.

To relate the Greek example in the introduction to the results of this paper, both the strategic delegation to the individual with stronger public goods preference in Scenario I and to individuals with weaker public goods preference (in other words, stronger preference for lower tax rates) in Scenario II can be seen as anti-austerity trends, despite the difference in whether the resulting local public goods are over or underprovision. Thus, our model would suggest that the existence of an ex post fiscal transfer in the form of a bailout from the EU led the Greek population to support anti-austerity.

Our results may lead to the following policy implications. In the situation where there exists ex post transfer due to the lack of central government's commitment ability, the election accelerates the inefficiency of local public good provision. The direction of this distortion depends on the commitment environment. Moreover, the degree of this distortion depends on the degree of spillover. Therefore, it might be desirable for the central government to regulate the policy of the policy maker elected, depending on the commitment environment and the degree of spillover.

Finally, this paper has some limitations. We assume a logarithmic form of utility function and no strategic interaction (strategic complementarity or substitutability) between local public goods for simplicity of analysis. Loeper (2017) shows that in the framework of cooperative games, the convexity of the demand function is related to the voters' incentive, and outcomes. In the framework with ex post fiscal transfers, considering the form of utility function and the strategic interaction between local public goods, and would bring more generality to our model and give new insights to our research in the future. Also, Pal and Sharma (2019) analyze and compare simultaneous and sequential determination of tax rates, expanding on Ihori and Yang (2009), which consider only simultaneous taxation. They show that in the framework of tax competition, the result shown by Ihori and Yang (2009) may change, depending on the timing of the tax rate determination (in other words, Stackelberg leader or follower). We only consider simultaneous elections and local policy making. By following Pal and Sharma (2019) and considering their sequential determinations, we may examine the election process and the efficiency in public goods provision from new perspectives in the context of ex post fiscal transfers.

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Appendix

Appendix A: Proof of Proposition 1

First, we examine the direction of strategic delegation. Comparing the policy maker in equilibrium with the median voter, we have:

$$\begin{aligned}\theta_P^{\mathrm{I}*} - \theta_M &= \frac{\theta_M}{1-(1-\lambda)\theta_M} - \theta_M \\ &= \frac{(1-\lambda)\theta_M}{1-(1-\lambda)\theta_M}\end{aligned}$$

Since $1 > (1-\lambda)\theta_M$ and $\lambda \in (0,1]$, we obtain the following conditions.

$$\begin{aligned}\theta_P^{\mathrm{I}*} &> \theta_M \quad \text{if} \quad 0<\lambda<1 \\ \theta_P^{\mathrm{I}*} &= \theta_M \quad \text{if} \quad \lambda = 1\end{aligned}$$

Next, we examine local public goods provision in equilibrium. Remembering that the socially optimal level of local public goods provision is $g^{**} = (1+\lambda)\theta_M(y_A+y_B)/\{2[1+(1+\lambda)\theta_M]\}$, we have:

$$\begin{aligned}g^{\mathrm{I}*} - g^{**} &= \frac{\theta_M(y_A+y_B)}{(1+\lambda)\theta_M+1} - \frac{(1+\lambda)\theta_M(y_A+y_B)}{2[(1+\lambda)\theta_M+1]} \\ &= \frac{(1-\lambda)\theta_M(y_A+y_B)}{2[(1+\lambda)\theta_M+1]}\end{aligned}$$

Comparing the case with and without election of the policy maker, we have:

$$\begin{aligned}g^{\mathrm{I}*} - g^{\mathrm{I}*}|_{\theta_P^*=\theta_M} &= \frac{\theta_M(y_A+y_B)}{(1+\lambda)\theta_M+1} - \frac{\theta_M(y_A+y_B)}{1+2\theta_M} \\ &= \frac{(1-\lambda)\theta_M^2(y_A+y_B)}{[(1+\lambda)\theta_M+1](1+2\theta_M)}\end{aligned}$$

Finally, comparing local public goods provision of the no election case with that of the social optimum, we have:

$$
\begin{aligned}
g^{\mathrm{I}*}|_{\theta_P^*=\theta_M} - g^{**} &= \frac{\theta_M(y_A+y_B)}{1+2\theta_M} - \frac{(1+\lambda)\theta_M(y_A+y_B)}{2[(1+\lambda)\theta_M+1]} \\
&= \frac{(1-\lambda)\theta_M(y_A+y_B)}{2(1+2\theta_M)[(1+\lambda)\theta_M+1]}
\end{aligned}
$$

Since $\lambda \in (0,1]$, we obtain the following conditions.

$$
\begin{aligned}
g^{\mathrm{I}*} > g^{\mathrm{I}*}|_{\theta_P^*=\theta_M} > g^{**} \quad &\text{if} \quad 0<\lambda<1 \\
g^{\mathrm{I}*} = g^{\mathrm{I}*}|_{\theta_P^*=\theta_M} = g^{**} \quad &\text{if} \quad \lambda=1 \quad \square
\end{aligned}
$$

Appendix B: Proof of Proposition 2

First, as in Scenario I, we examine the direction of strategic delegation. Comparing the policy maker in equilibrium with the median voter, we have:

$$
\begin{aligned}
\theta_P^{\mathrm{II}*} - \theta_M &= \frac{(1+\lambda)\theta_M - 1}{1+\lambda} - \theta_M \\
&= -\frac{1}{1+\lambda}
\end{aligned}
$$

Because $\lambda \in (0,1]$, we have:

$$
\theta_P^{\mathrm{II}*} < \theta_M
$$

Next, we compare local public good provision in equilibrium with the social optimum. Then, we have:

$$
\begin{aligned}
g^{\mathrm{II}*} - g^{**} &= \frac{[(1+\lambda)\theta_M - 1](y_A+y_B)}{2[(1+\lambda)\theta_M+1]} - \frac{(1+\lambda)\theta_M(y_A+y_B)}{2[(1+\lambda)\theta_M+1]} \\
&= -\frac{y_A+y_B}{2[(1+\lambda)\theta_M+1]} \qquad \text{(A.1)}
\end{aligned}
$$

Comparing the case with and without election of the policy maker, we have:

$$
\begin{aligned}
g^{\mathrm{II}*} - g^{\mathrm{II}*}|_{\theta_P^*=\theta_M} &= \frac{[(1+\lambda)\theta_M - 1](y_A + y_B)}{2[(1+\lambda)\theta_M + 1]} - \frac{(1+\lambda)\theta_M(y_A + y_B)}{2[(1+\lambda)\theta_M + 2]} \\
&= -\frac{y_A + y_B}{[(1+\lambda)\theta_M + 1][(1+\lambda)\theta_M + 2]}
\end{aligned}
$$

Finally, comparing local public goods provision of the no election case with that of the social optimum, we have:

$$
\begin{aligned}
g^{\mathrm{II}*}|_{\theta_P^*=\theta_M} - g^{**} &= \frac{(1+\lambda)\theta_M(y_A + y_B)}{2[(1+\lambda)\theta_M + 2]} - \frac{(1+\lambda)\theta_M(y_A + y_B)}{2[(1+\lambda)\theta_M + 1]} \\
&= -\frac{(1+\lambda)\theta_M(y_A + y_B)}{2[(1+\lambda)\theta_M + 2][(1+\lambda)\theta_M + 1]}
\end{aligned}
$$

Since $\lambda \in (0, 1]$, we obtain the following conditions.

$$
g^{\mathrm{II}*} < g^{\mathrm{II}*}|_{\theta_P^*=\theta_M} < g^{**} \quad \square
$$

Appendix C: Confirmation of the second-order conditions

The first-order condition in stage 1 of Scenario I is as follows.

$$
\Gamma = -\frac{1}{1+\theta_{AP}+\theta_{BP}} + \theta_M \left[\frac{1+\theta_{-iP}}{\theta_{iP}(1+\theta_{AP}+\theta_{BP})} - \frac{\lambda}{1+\theta_{AP}+\theta_{BP}}\right], \ i = A, B, i \neq -i
$$

The second-order condition is as follows.

$$
\begin{aligned}
\frac{\partial \Gamma}{\partial \theta_{iP}} &= \frac{1}{\gamma^2} + \theta_M \left[-\frac{(1+2\theta_{iP}+\theta_{-iP})(1+\theta_{-iP})}{\theta_{iP}^2\gamma^2} + \frac{\lambda}{\gamma^2}\right] \\
&= \frac{1}{\theta_{iP}^2\gamma^2}\left[(1+\lambda\theta_M)\theta_{iP}^2 - \theta_M(1+2\theta_{iP}+\theta_{-iP})(1+\theta_{-iP})\right], \ i = A, B, i \neq -i \qquad \text{(A.2)}
\end{aligned}
$$

where $\gamma = 1 + \theta_{AP} + \theta_{BP}$. Evaluating the second-order condition in equilibrium, since $\theta_{AP}^{\mathrm{I}*} = \theta_{BP}^{\mathrm{I}*} = \theta_P^{\mathrm{I}*} = \theta_M/[1-(1-\lambda)\theta_M]$, we obtain:

$$
\left.\frac{\partial \Gamma}{\partial \theta_{iP}}\right|_{\theta_{iP}=\theta_P^{\mathrm{I}*}} = -\frac{1}{(\theta_P^*)^2\gamma^2}\left(\frac{\theta_M}{1-(1-\lambda)\theta_M}\right)^2 \left[\left(1+2\lambda+\frac{1}{\theta_M}\right) + \lambda\theta_M(1-\lambda)\right] < 0
$$

The first-order condition in stage 1 of Scenario II is as follows.

$$\Phi = -\frac{[(1+\lambda)\theta_{-iP}+1]}{(1+\lambda)\theta_{AP}\theta_{BP}+\theta_{AP}+\theta_{BP}} + \theta_M\left[\frac{(1+\lambda)\theta_{-iP}}{\theta_{iP}[(1+\lambda)\theta_{AP}\theta_{BP}+\theta_{AP}+\theta_{BP}]}\right], \; i=A,B, i\neq -i$$

The second-order condition is as follows.

$$\frac{\partial\Phi}{\partial\theta_{iP}} = \frac{1}{\theta_{iP}^2\phi^2}\left\{\theta_{iP}^2[(1+\lambda)\theta_{-iP}+1]^2 - \theta_M[(1+\lambda)^2\theta_{iP}\theta_{-iP}^2+(1+\lambda)\theta_{iP}\theta_{-iP}+(1+\lambda)(2+\lambda)\theta_{-iP}^2+(1+\lambda)\theta_{-iP}]\right\}, \; i=A,B, i\neq -i \tag{A.3}$$

where $\phi=(1+\lambda)\theta_{AP}\theta_{BP}+\theta_{AP}+\theta_{BP}$. Evaluating the second-order condition in equilibrium, since $\theta_{AP}^{\mathrm{II}*}=\theta_{BP}^{\mathrm{II}*}=\theta_P^{\mathrm{II}*}=[(1+\lambda)\theta_M-1]/(1+\lambda)$, we obtain:

$$\left.\frac{\partial\Phi}{\partial\theta_{iP}}\right|_{\theta_{iP}=\theta_P^{\mathrm{II}*}} = -\frac{\theta_M}{\theta_P^*\phi^2}[(1+\lambda)(2+\lambda)\theta_M-1]<0$$

We find that the second-order conditions in the first stage of both scenarios are *locally* satisfied in the neighbourhood of equilibrium. For each second-order condition to be *globally* satisfied, we need conditions on θ_M such that equations (A.2) and (A.3) are negative, respectively.

Appendix D: Welfare comparison

The social welfare in Scenario I is expressed as follows.

$$\begin{aligned} S^{\mathrm{I}*} &= \log c^{\mathrm{I}*} + \theta_M[\log g^{\mathrm{I}*}+\lambda\log g^{\mathrm{I}*}] \\ &= \log\left[\frac{[1-(1-\lambda)\theta_M](y_A+y_B)}{2[(1+\lambda)\theta_M+1]}\right] + \theta_M(1+\lambda)\log\left[\frac{\theta_M(y_A+y_B)}{(1+\lambda)\theta_M+1}\right] \end{aligned}$$

Also, the social welfare of Scenario II is expressed as follows.

$$
\begin{aligned}
S^{\mathrm{II}*} &= \log c^{\mathrm{II}*} + \theta_M[\log g^{\mathrm{II}*} + \lambda \log g^{\mathrm{II}*}] \\
&= \log\left[\frac{y_A + y_B}{(1+\lambda)\theta_M + 1}\right] + \theta_M(1+\lambda)\log\left[\frac{[(1+\lambda)\theta_M - 1](y_A + y_B)}{2[(1+\lambda)\theta_M + 1]}\right]
\end{aligned}
$$

Thus, we obtain as follows.

$$
\begin{aligned}
S^{\mathrm{I}*} - S^{\mathrm{II}*} =& \log\left[\frac{[1-(1-\lambda)\theta_M](y_A + y_B)}{2[(1+\lambda)\theta_M + 1]}\right] - \log\left[\frac{y_A + y_B}{(1+\lambda)\theta_M + 1}\right] \\
&+ \theta_M(1+\lambda)\log\left[\frac{\theta_M(y_A + y_B)}{(1+\lambda)\theta_M + 1}\right] - \theta_M(1+\lambda)\log\left[\frac{[(1+\lambda)\theta_M - 1](y_A + y_B)}{2[(1+\lambda)\theta_M + 1]}\right] \\
=& \log\left[\frac{1-(1-\lambda)\theta_M}{2}\right] + \theta_M(1+\lambda)\log\left[\frac{2\theta_M}{(1+\lambda)\theta_M - 1}\right]
\end{aligned}
$$

Appendix E: Asymmetric distribution

Denoting the mean of the distribution by $\bar{\theta}$, in the case of asymmetric distribution, the condition for obtaining the socially optimal local public good provision is given by as follows.

$$
\begin{aligned}
\max_{\{c_i, g_i\}_{i=A,B}} \quad S_{AD} &= \log c_A + \bar{\theta}[\log g_A + \lambda \log g_B] \\
&+ \log c_B + \bar{\theta}[\log g_B + \lambda \log g_A] \\
\text{s.t. } & c_A + c_B + g_A + g_B = y_A + y_B
\end{aligned}
$$

Therefore, the socially optimal local public goods provision is as follows.

$$
g_{AD}^{**} = \frac{(1+\lambda)\bar{\theta}(y_A + y_B)}{2[(1+\lambda)\bar{\theta} + 1]}
$$

Since policy decisions at the local level are invariant from the symmetric distribution case, comparing local public good provision in equilibrium with the with the social optimum, we obtain:

$$
\begin{aligned}
g^{\mathrm{I}*} - g_{AD}^{**} &= \frac{(y_A + y_B)}{2[(1+\lambda)\theta_M + 1][(1+\lambda)\bar{\theta} + 1]}\{2\theta_M[(1+\lambda)\bar{\theta} + 1] - (1+\lambda)\bar{\theta}[1+\lambda)\theta_M + 1]\} \\
&= \frac{(y_A + y_B)}{2[(1+\lambda)\theta_M + 1][(1+\lambda)\bar{\theta} + 1]}[(1-\lambda)(1+\lambda)\theta_M\bar{\theta} + 2\theta_M - (1+\lambda)\bar{\theta}]
\end{aligned}
$$

Therefore, we obtain the following conditions.

$$g^{\mathrm{I}*} \geq g^{**}_{AD} \quad \text{if} \quad (1-\lambda)(1+\lambda)\theta_M\bar{\theta} + 2\theta_M \geq (1+\lambda)\bar{\theta}$$
$$g^{\mathrm{I}*} < g^{**}_{AD} \quad \text{if} \quad (1-\lambda)(1+\lambda)\theta_M\bar{\theta} + 2\theta_M < (1+\lambda)\bar{\theta}$$

In particular, the following conditions hold when $\lambda = 1$.

$$g^{\mathrm{I}*} \geq g^{**}_{AD} \quad \text{if} \quad \theta_M \geq \bar{\theta}$$
$$g^{\mathrm{I}*} < g^{**}_{AD} \quad \text{if} \quad \theta_M < \bar{\theta}$$

Also, comparing with the no election case as well as the symmetric distribution case, we obtain:

$$\begin{aligned} g^{\mathrm{I}*}|_{\theta^*_P=\theta_M} - g^{**}_{AD} &= \frac{(y_A+y_B)}{2(1+2\theta_M)[(1+\lambda)\bar{\theta}+1]}\{2[(1+\lambda)\bar{\theta}+1]\theta_M - (1+2\theta_M)(1+\lambda)\bar{\theta}\} \\ &= \frac{(y_A+y_B)}{2(1+2\theta_M)[(1+\lambda)\bar{\theta}+1]}\{2\theta_M - (1+\lambda)\bar{\theta}\} \end{aligned}$$

Therefore, we obtain the following conditions.

$$g^{\mathrm{I}*}|_{\theta^*_P=\theta_M} \geq g^{**}_{AD} \quad \text{if} \quad 2\theta_M \geq (1+\lambda)\bar{\theta}$$
$$g^{\mathrm{I}*}|_{\theta^*_P=\theta_M} < g^{**}_{AD} \quad \text{if} \quad 2\theta_M < (1+\lambda)\bar{\theta}$$

Based on the comparison so far, we obtain the following conditions.

$$g^{\mathrm{I}*} > g^{\mathrm{I}*}|_{\theta^*_P=\theta_M} > g^{**}_{AD} \quad \text{if} \quad (1-\lambda)(1+\lambda)\theta_M\bar{\theta} + 2\theta_M > 2\theta_M > (1+\lambda)\bar{\theta} \tag{A.4}$$
$$g^{\mathrm{I}*} > g^{**}_{AD} > g^{\mathrm{I}*}|_{\theta^*_P=\theta_M} \quad \text{if} \quad (1-\lambda)(1+\lambda)\theta_M\bar{\theta} + 2\theta_M > (1+\lambda)\bar{\theta} > 2\theta_M \tag{A.5}$$
$$g^{**}_{AD} > g^{\mathrm{I}*} > g^{\mathrm{I}*}|_{\theta^*_P=\theta_M} \quad \text{if} \quad (1+\lambda)\bar{\theta} > (1-\lambda)(1+\lambda)\theta_M\bar{\theta} + 2\theta_M > 2\theta_M \tag{A.6}$$

Figure A.1 illustrates the above conditions. The horizontal axis in this figure is $\bar{\theta}$, which represents the value of each condition. When $\lambda = 1$ and $\theta_M \geq (<)\bar{\theta}$, then $g^{\mathrm{I}*} = g^{\mathrm{I}*}|_{\theta^*_P=\theta_M} \geq (<) g^{**}_{AD}$ holds. In $\lambda \neq 1$, when (1) $\theta_M > \bar{\theta}$ or (2) $\bar{\theta} > \theta_M$ and the gap between them is sufficiently small, namely (A.4), then the results in the symmetric distribution are retained. [22]

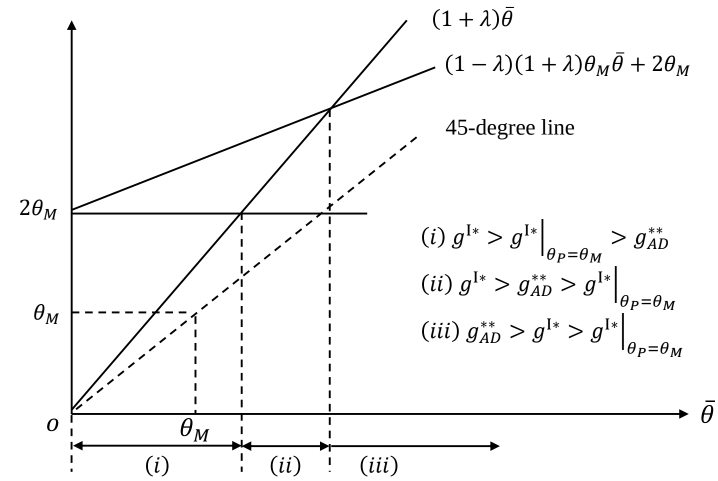

Figure A.1: Scenario I in asymmetric distribution.

Next, we compare each local public goods provision in Scenario II. Comparing local public goods provision in equilibrium with the social optimum, we obtain:

$$\begin{aligned}
g^{\mathrm{II}*} - g^{**}_{AD} &= \frac{(y_A + y_B)}{2[1+\lambda)\theta_M + 1][(1+\lambda)\bar{\theta}+1]}\{[(1+\lambda)\theta_M - 1](1+\lambda)\bar{\theta}+1] - (1+\lambda)\bar{\theta}[(1+\lambda)\theta_M + 1]\\
&= \frac{(y_A + y_B)}{2[(1+\lambda)\theta_M + 1][(1+\lambda)\bar{\theta}+1]}[(1+\lambda)\theta_M - 2(1+\lambda)\bar{\theta} - 1]
\end{aligned}$$

Therefore, we obtain the following conditions.

$$\begin{aligned}
g^{\mathrm{II}*} \geq g^{**}_{AD} \quad \text{if} \quad (1+\lambda)\theta_M \geq 2(1+\lambda)\bar{\theta}+1\\
g^{\mathrm{II}*} < g^{**}_{AD} \quad \text{if} \quad (1+\lambda)\theta_M < 2(1+\lambda)\bar{\theta}+1
\end{aligned}$$

Comparing with the no election case, we obtain:

$$\begin{aligned}
g^{\mathrm{II}*}|_{\theta^*_P=\theta_M} - g^{**}_{AD} &= \frac{(y_A + y_B)}{2[(1+\lambda)\theta_M + 2][(1+\lambda)\bar{\theta}+1]}\{(1+\lambda)\theta_M[(1+\lambda)\bar{\theta}+1] - (1+\lambda)\bar{\theta}[(1+\lambda)\theta_M + 2]\}\\
&= \frac{(y_A + y_B)}{2[(1+\lambda)\theta_M + 2][(1+\lambda)\bar{\theta}+1]}\{(1+\lambda)\theta_M - 2(1+\lambda)\bar{\theta}\}
\end{aligned}$$

[22] From the assumption $1/(1-\lambda) > \theta_M$ we impose, we have $(1+\lambda) > (1-\lambda)(1+\lambda)\theta_M$.

Therefore, we obtain the following conditions.

$$g^{\mathrm{II}*}|_{\theta_P^*=\theta_M} \geq g_{AD}^{**} \quad \text{if} \quad (1+\lambda)\theta_M \geq 2(1+\lambda)\bar{\theta}$$
$$g^{\mathrm{II}*}|_{\theta_P^*=\theta_M} < g_{AD}^{**} \quad \text{if} \quad (1+\lambda)\theta_M < 2(1+\lambda)\bar{\theta}$$

Based on the comparison so far, we obtain the following conditions.

$$g_{AD}^{**} > g^{\mathrm{II}*}|_{\theta_P^*=\theta_M} > g^{\mathrm{II}*} \quad \text{if} \quad 2(1+\lambda)\bar{\theta}+1 > 2(1+\lambda)\bar{\theta} > (1+\lambda)\theta_M \tag{A.7}$$
$$g^{\mathrm{II}*}|_{\theta_P^*=\theta_M} > g_{AD}^{**} > g^{\mathrm{II}*} \quad \text{if} \quad 2(1+\lambda)\bar{\theta}+1 > (1+\lambda)\theta_M > 2(1+\lambda)\bar{\theta} \tag{A.8}$$
$$g^{\mathrm{II}*}|_{\theta_P^*=\theta_M} > g^{\mathrm{II}*} > g_{AD}^{**} \quad \text{if} \quad (1+\lambda)\theta_M > 2(1+\lambda)\bar{\theta}+1 > 2(1+\lambda)\bar{\theta} \tag{A.9}$$

Figure A.2 illustrates the above conditions. The horizontal axis in this figure is $\bar{\theta}$, which represents the value of each condition. When (1) $\bar{\theta} > \theta_M$ or (2) $\theta_M > \bar{\theta}$ and the gap between them is sufficiently small, namely (A.7), then the results in the symmetric distribution are retained.

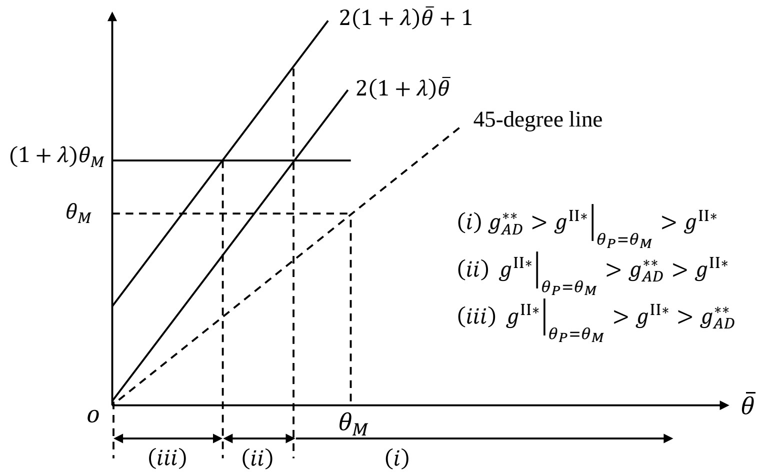

Figure A.2: Scenario II in asymmetric distribution.

Appendix F: Non-identical regions

Let the medians of the distributions for region A and B denote θ_{AM} and θ_{BM}, respectively. Since we assume a symmetric distribution in both regions, we have $\theta_{AM} = \bar{\theta}_A$ and $\theta_{BM} = \bar{\theta}_B$. The optimization problem of deriving the socially optimal local public goods provision is defined as follows.

$$\max_{\{c_i, g_i\}_{i=A,B}} \quad S = \log(c_A) + \theta_{AM}[\log(g_A) + \lambda \log(g_B)]$$
$$+ \log(c_B) + \theta_{BM}[\log(g_B) + \lambda \log(g_A)]$$
$$\text{s.t. } c_A + c_B + g_A + g_B = y_A + y_B$$

Therefore, the socially optimal local public goods provision is as follows.

$$g_i^{**} = \frac{(\theta_{iM} + \lambda\theta_{-iM})(y_A + y_B)}{(1+\lambda)(\theta_{AM} + \theta_{BM}) + 2}, \; i = A, B, i \neq -i$$

In the case of non-identical regions, local public goods provision and the elected policy makers in equilibrium in each scenario would be different among regions because preferences for local public goods differ among regions. The procedure for derivation is the same as in the case of identical distribution. In Scenario I, policy makers in equilibrium are as follows.

$$\theta_{iP}^{\mathrm{I}*} = \frac{\theta_{iM}(1 + (1+\lambda)\theta_{-iM})}{1 + \lambda(\theta_{AM} + \theta_{BM}) - (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM}}, \; i = A, B, i \neq -i$$

The direction of strategic delegation is as follows.

$$\begin{aligned}\theta_{iP}^{\mathrm{I}*} - \theta_{iM} &= \frac{\theta_{iM}(1 + (1+\lambda)\theta_{-iM})}{1 + \lambda(\theta_{AM} + \theta_{BM}) - (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM}} - \theta_{iM} \\ &= \frac{\theta_{iM}[\theta_{-iM} - \lambda\theta_{iM} + (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM}]}{1 + \lambda(\theta_{AM} + \theta_{BM}) - (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM}}, \; i = A, B, i \neq -i\end{aligned}$$

Therefore, we obtain the following conditions.

$$\theta_{iP}^{\mathrm{I}*} \geq \theta_{iM} \quad \text{if} \quad \theta_{-iM} - \lambda\theta_{iM} + (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM} \geq 0$$
$$\theta_{iP}^{\mathrm{I}*} < \theta_{iM} \quad \text{if} \quad \theta_{-iM} - \lambda\theta_{iM} + (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM} < 0$$

In particular, the following conditions hold when $\lambda = 1$.

$$\theta_{iP}^{\mathrm{I}*} \geq \theta_{iM} \quad \text{if} \quad \theta_{-iM} \geq \theta_{iM} \tag{A.10}$$

$$\theta_{iP}^{\mathrm{I}*} < \theta_{iM} \quad \text{if} \quad \theta_{-iM} < \theta_{iM} \tag{A.11}$$

The equilibrium levels of local public goods and private consumption in each region are derived as follows.

$$g_i^{\mathrm{I}*} = \frac{\theta_{iM}(y_A + y_B)}{1 + (1+\lambda)\theta_{iM}}, \;\; i = A, B$$
$$c^{\mathrm{I}*} = \frac{1 + \lambda(\theta_{AM} + \theta_{BM}) - (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM}}{2[1 + (1+\lambda)\theta_{AM}][1 + (1+\lambda)\theta_{BM}]}$$

We assume the following to ensure the interior solution.

$$1 + \lambda(\theta_{AM} + \theta_{BM}) > (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM} \tag{A.12}$$

In addition, local public goods provision in the no election case is as follows.

$$g_i^{\mathrm{I}*}|_{\theta_P^* = \theta_M} = \frac{\theta_{iM}(y_A + y_B)}{1 + \theta_{AM} + \theta_{BM}}$$

Comparing the provision of local public goods in equilibrium with the social optimum, we obtain:

$$\begin{aligned} g_i^{\mathrm{I}*} - g_i^{**} &= \frac{\theta_{iM}(y_A + y_B)}{1 + (1+\lambda)\theta_{iM}} - \frac{(\theta_{iM} + \lambda\theta_{-iM})(y_A + y_B)}{(1+\lambda)(\theta_{AM} + \theta_{BM}) + 2} \\ &= \frac{[\theta_{iM} - \lambda\theta_{-iM} + (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM}](y_A + y_B)}{[1 + (1+\lambda)\theta_{iM}][(1+\lambda)(\theta_{AM} + \theta_{BM}) + 2]}, \;\; i = A, B, i \neq -i \end{aligned}$$

Therefore, we obtain the following conditions.

$$g_i^{\mathrm{I}*} \geq g_i^{**} \quad \text{if} \quad \theta_{iM} - \lambda\theta_{-iM} + (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM} \geq 0$$

$$g_i^{\mathrm{I}*} < g_i^{**} \quad \text{if} \quad \theta_{iM} - \lambda\theta_{-iM} + (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM} < 0$$

Also, the following conditions hold when $\lambda = 1$.

$$g_i^{\mathrm{I}*} \geq g^{**} \quad \text{if} \quad \theta_{iM} \geq \theta_{-iM} \tag{A.13}$$

$$g_i^{\mathrm{I}*} < g^{**} \quad \text{if} \quad \theta_{iM} < \theta_{-iM} \tag{A.14}$$

Next is a comparison with no election case. In the non-identical regions, local public goods provision in the no election case is represented as $g_i^{\mathrm{I}*}|_{\theta_P^*=\theta_M} = [\theta_{iM}(y_A + y_B)]/[1 + \theta_{AM} + \theta_{BM}],\ i = A, B$. Therefore, comparing local public goods provision in the no election case with the social optimum, we obtain:

$$\begin{aligned} g_i^{\mathrm{I}*}|_{\theta_P^*=\theta_{iM}} - g_i^{**} &= \frac{\theta_{iM}(y_A + y_B)}{1 + \theta_{AM} + \theta_{BM}} - \frac{(\theta_{iM} + \lambda\theta_{-iM})(y_A + y_B)}{(1+\lambda)(\theta_{AM} + \theta_{BM}) + 2} \\ &= \frac{[\theta_{iM} - \lambda\theta_{-iM} + \lambda(\theta_{AM} + \theta_{BM})(\theta_{iM} - \theta_{-iM})](y_A + y_B)}{[1 + \theta_{AM} + \theta_{BM}][(1+\lambda)(\theta_{AM} + \theta_{BM}) + 2]},\ i = A, B, i \neq -i \end{aligned}$$

Also, we compare the case with and without election of the policy maker. Then, we obtain:

$$\begin{aligned} g_i^{\mathrm{I}*} - g_i^{\mathrm{I}*}|_{\theta_P^*=\theta_M} &= \frac{\theta_{iM}(y_A + y_B)}{1 + (1+\lambda)\theta_{iM}} - \frac{\theta_{iM}(y_A + y_B)}{1 + \theta_{AM} + \theta_{BM}} \\ &= \frac{\theta_{iM}(\theta_{-iM} - \lambda\theta_{iM})(y_A + y_B)}{[1 + (1+\lambda)\theta_{iM}][1 + \theta_{AM} + \theta_{BM}]},\ i = A, B, i \neq -i \end{aligned}$$

Hereafter, we assume that $\theta_{AM} > \theta_{BM}$. Then, we consider the following two cases.

case (i): $\theta_{AM} > \theta_{BM} > \lambda\theta_{AM}$

The following inequalitys hold:

$$\theta_{AM} - \lambda\theta_{BM} + (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM} > 0$$

$$\theta_{BM} - \lambda\theta_{AM} + (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM} > 0$$

Therefore, we obtain:

$$\theta_{AP}^{\mathrm{I}*} > \theta_{AM} \tag{A.15}$$

$$g_A^{\mathrm{I}*} > g_A^{\mathrm{I}*}|_{\theta_P^*=\theta_M} > g_A^{**} \tag{A.16}$$

and

$$\theta_{BP}^{\mathrm{I}*} > \theta_{BM} \tag{A.17}$$

$$g_B^{\mathrm{I}*} > g_B^{\mathrm{I}*}|_{\theta_P^*=\theta_M} > g_B^{**} \quad \text{if} \quad \theta_{BM} - \lambda\theta_{AM} + \lambda(\theta_{AM} + \theta_{BM})(\theta_{BM} - \theta_{AM}) > 0 \tag{A.18}$$

$$g_B^{\mathrm{I}*} > g_B^{**} > g_B^{\mathrm{I}*}|_{\theta_P^*=\theta_M} \quad \text{if} \quad \theta_{BM} - \lambda\theta_{AM} + \lambda(\theta_{AM} + \theta_{BM})(\theta_{BM} - \theta_{AM}) < 0 \tag{A.19}$$

case (ii): $\theta_{AM} > \lambda\theta_{AM} > \theta_{BM}$

The following inequalitys hold:

$$\theta_{AM} - \lambda\theta_{BM} + (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM} > 0$$

$$\theta_{BM} - \lambda\theta_{AM} + (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM} \gtreqless 0$$

Therefore, we obtain:

$$g_A^{\mathrm{I}*}|_{\theta_P^*=\theta_M} > g_A^{\mathrm{I}*} > g_A^{**} \tag{A.20}$$

$$\theta_{AP}^{\mathrm{I}*} \geq \theta_{AM} \quad \text{if} \quad \theta_{BM} - \lambda\theta_{AM} + (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM} \geq 0 \tag{A.21}$$

$$\theta_{AP}^{\mathrm{I}*} < \theta_{AM} \quad \text{if} \quad \theta_{BM} - \lambda\theta_{AM} + (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM} < 0 \tag{A.22}$$

and

$$\theta_{BP}^{\mathrm{I}*} > \theta_{BM} \tag{A.23}$$

$$g_B^{\mathrm{I}*} \geq g_B^{**} > g_B^{\mathrm{I}*}|_{\theta_P^*=\theta_M} \quad \text{if} \quad \theta_{BM} - \lambda\theta_{AM} + (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM} \geq 0 \tag{A.24}$$

$$g_B^{**} > g_B^{\mathrm{I}*} > g_B^{\mathrm{I}*}|_{\theta_P^*=\theta_M} \quad \text{if} \quad \theta_{BM} - \lambda\theta_{AM} + (1+\lambda)(1-\lambda)\theta_{AM}\theta_{BM} < 0 \tag{A.25}$$

For Scenario II, we provide similar comparisons to Scenario I. In Scenario II, policy makers in equilibrium are as follows.

$$\theta_{iP}^{\mathrm{II}*} = \frac{(1+\lambda)^2\theta_{AM}\theta_{BM} - 1}{(1+\lambda)[(1+\lambda)\theta_{-iM} + 1]},\ i = A, B, i \neq -i$$

The direction of strategic delegation is as follows.

$$\begin{aligned}\theta_{iP}^{\mathrm{II}*} - \theta_{iM} &= \frac{(1+\lambda)^2\theta_{AM}\theta_{BM} - 1}{(1+\lambda)[(1+\lambda)\theta_{-iM} + 1]} - \theta_{iM} \\ &= -\frac{(1+\lambda)\theta_{iM} + 1}{(1+\lambda)[(1+\lambda)\theta_{-iM} + 1]} < 0,\ i = A, B, i \neq -i\end{aligned}$$

Therefore, we obtain:

$$\theta_{iP}^{\mathrm{II}*} < \theta_{iM},\ i = A, B \tag{A.26}$$

The equilibrium levels of private consumption and local public goods in each region are derived as follows.

$$\begin{aligned}c_i^{\mathrm{II}*} &= \frac{(y_A + y_B)}{(1+\lambda)\theta_{iM} + 1},\ i = A, B \\ g_i^{\mathrm{II}*} &= \frac{\theta_{iM} + \lambda\theta_{-iM}}{(1+\lambda)(\theta_{AM} + \theta_{BM})} \times \frac{[(1+\lambda)^2\theta_{AM}\theta_{BM} - 1](y_A + y_B)}{[(1+\lambda)\theta_{AM} + 1][(1+\lambda)\theta_{BM} + 1]},\ i = A, B, i \neq -i\end{aligned}$$

We assume the following to ensure the interior solution.

$$(1+\lambda)^2\theta_{AM}\theta_{BM} > 1 \tag{A.27}$$

In order for equations (A.12) and (A.27) to be practical, we assume:

$$\lambda > 0$$

Comparing the provision of local public goods in equilibrium with the social optimum, we obtain:

$$g_i^{\mathrm{II}*} - g_i^{**} = \frac{\theta_{iM} + \lambda\theta_{-iM}}{(1+\lambda)(\theta_{AM}+\theta_{BM})} \times \frac{[(1+\lambda)^2\theta_{AM}\theta_{BM} - 1](y_A + y_B)}{[(1+\lambda)\theta_{AM}+1][(1+\lambda)\theta_{BM}+1]} - \frac{(\theta_{iM} + \lambda\theta_{-iM})(y_A + y_B)}{(1+\lambda)(\theta_{AM}+\theta_{BM})+2}$$
$$= -\frac{(\theta_{iM} + \lambda\theta_{-iM})[(1+\lambda)^2(\theta_{AM}^2+\theta_{BM}^2) + 2(1+\lambda)(\theta_{AM}+\theta_{BM}) + 2](y_A + y_B)}{(1+\lambda)(\theta_{AM}+\theta_{BM})[(1+\lambda)\theta_{AM}+1][(1+\lambda)\theta_{BM}+1][(1+\lambda)(\theta_{AM}+\theta_{BM})+2]} < 0$$

Therefore, we obtain:

$$g_i^{\mathrm{II}*} < g_i^{**},\ i = A, B$$

In Scenario II, local public goods provision in the no election case is represented as follows.

$$g_i^{\mathrm{II}*}|_{\theta_P^*=\theta_M} = \frac{\theta_{iM} + \lambda\theta_{-iM}}{(1+\lambda)(\theta_{AM}+\theta_{BM})} \times \frac{(1+\lambda)\theta_{AM}\theta_{BM}(y_A + y_B)}{(1+\lambda)\theta_{AM}\theta_{BM} + \theta_{AM} + \theta_{BM}},\ i = A, B, i \neq -i$$

Comparing local public goods provision in the no election case with the social optimum, we obtain as follows.

$$g_i^{\mathrm{II}*}|_{\theta_P^*=\theta_M} - g_i^{**} = -\frac{(\theta_{iM} + \lambda\theta_{-iM})(\theta_{AM}^2+\theta_{BM}^2)(y_A + y_B)}{(\theta_{AM}+\theta_{BM})[(1+\lambda)\theta_{AM}\theta_{BM} + \theta_{AM} + \theta_{BM}][(1+\lambda)(\theta_{AM}+\theta_{BM})+2]} < 0$$

Therefore, we obtain:

$$g_i^{\mathrm{II}*}|_{\theta_P^*=\theta_M} < g_i^{**},\ i = A, B$$

Also, we compare the provision of local public goods in the case with and without election of the policy maker. Then, we obtain as follows.

$$g_i^{\mathrm{II}*} - g_i^{\mathrm{II}*}|_{\theta_P^*=\theta_M} = -\frac{\theta_{iM} + \lambda\theta_{-iM}}{(1+\lambda)(\theta_{AM}+\theta_{BM})} \times \frac{[2(1+\lambda)\theta_{AM}\theta_{BM} + \theta_{AM} + \theta_{BM}](y_A + y_B)}{[(1+\lambda)\theta_{AM}+1][(1+\lambda)\theta_{BM}+1][(1+\lambda)\theta_{AM}\theta_{BM} + \theta_{AM} + \theta_{BM}]} < 0$$

Therefore, we obtain:

$$g_i^{**} > g_i^{\mathrm{II}*}|_{\theta_P^*=\theta_M} > g_i^{\mathrm{II}*},\ i = A, B \tag{A.28}$$

Thus, in Scenario II, the results in the symmetric regions always hold.