Dutch vs. First-Price Auctions with Expectations-Based Loss-Averse Bidders*

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Abstract

We study Dutch and first-price auctions with expectations-based loss-averse bidders and show that the strategic equivalence between these formats no longer holds. Intuitively, as the Dutch auction unfolds, a bidder becomes more optimistic about her chances of winning; this stronger "attachment" effect pushes her to bid more aggressively than in the first-price auction. Thus, Dutch auctions raise more revenue than first-price ones. Indeed, the Dutch auction raises the most revenue among standard auction formats. Our results imply that dynamic mechanisms that make bidders more optimistic raise more revenue, thereby rationalizing the use of descending-price mechanisms by sellers in the field.

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1 Introduction

The static first-price auction (FPA) and its dynamic counterpart, the Dutch (or descending-price) auction, are among the most prominent auction formats. A central result in auction theory is that these two formats are strategically equivalent. The crucial insight, due originally to Vickrey (1961), is that the information bidders obtain during the Dutch auction does not affect their optimal strategies; therefore, bidders choose their bids solely based on their prior information. Indeed, the equivalence between these two formats holds in many different environments (e.g., with independent or correlated private values, pure common values, interdependent values, affiliated types, etc...), even under risk aversion. The strategic equivalence further implies that the two auction formats generate the same expected revenue. However, evidence from both laboratory and field experiments shows that revenue equivalence may fail. For instance, Lucking-Reiley (1999) conducts a field experiment by selling *Magic* game cards via Internet auctions and reports that the Dutch auction produces 30-percent higher revenues than the FPA. Katok and Kwasnica (2008) obtain similar results in a laboratory experiment when the price in the Dutch auction drops slowly. These studies suggest that the Dutch auction tends to generate more revenue than the FPA, especially if the price clock of the Dutch auction is relatively slow.[1]

In this paper, we provide a novel explanation for the strategic (and hence, revenue) non-equivalence between the FPA and the Dutch auction based on reference-dependent preferences and loss aversion. We analyze both auction formats in a symmetric environment where bidders have independent private values (IPV) and are expectations-based loss averse à la Kőszegi and Rabin (2006, 2007, 2009). We show that loss-averse bidders bid more aggressively in the Dutch auction than in the FPA. Intuitively, the larger the probability with which a loss-averse bidder expects to win the auction, the stronger her incentives to bid high in order to avoid experiencing disappointment from losing the auction. This is what Kőszegi and Rabin (2006) call the "attachment effect". We argue that, although the two auction formats select the same winner, they create different levels of attachment for the bidders. Consider, for instance, a bidder with a fairly low value. When submitting her bid in the FPA, she knows it is quite likely that one of her opponents has a higher value. Thus, she is rather pessimistic about her chances of winning the auction and not very attached to the prize; therefore, she does not have a strong incentive to bid high. In contrast, consider the same bidder participating in a Dutch auction and imagine the clock is only slightly above the price at which she had originally planned to buy. By now she has updated her beliefs about her strongest opponent's value and is very optimistic that it is below hers – after all, if (one of) her opponents had a much larger valuation than hers, they would have already stopped the clock. Thus, she is very much attached to the prize. In this case, the bidder has a strong incentive to raise her bid and stop the clock at an earlier price in order to reduce the chances

[1]However, earlier experiments by Coppinger et *al.* (1980) and Cox *et al.* (1982) report higher revenues for FPA than for Dutch; Cox *et al.* (1983) attribute this finding to probability miscalculations in the Dutch auction.

of experiencing a loss if another bidder stops the clock before her. In other words, the bidding strategy of a loss-averse bidder in the FPA is shaped by the attachment effect arising from her *initial* beliefs about how likely she is to win the auction. In the Dutch auction, in contrast, she becomes increasingly more optimistic about her chances of winning as the auction unfolds; this creates a stronger attachment effect inducing her to bid more aggressively than in the FPA.

As the theoretical equivalence between the Dutch auction and the FPA holds for many different environments, some authors have suggested that its empirical breakdown might be caused by non-standard risk preferences. Karni (1988) is the first to point out that these two formats are equivalent if and only if bidders are expected-utility maximizers. Nakajima (2011) considers bidders whose preferences exhibit the Allais paradox (Allais, 1953) and shows that the Dutch auction systematically yields more revenue than the FPA.[2] Auster and Kellner (2020) obtain the same result for the case of ambiguity-averse bidders. Another strand of literature, however, attributes the breakdown of the FPA-Dutch equivalence to bidders' time preferences. In fact, in those studies where the Dutch auction generates more revenue than the FPA, typically the clock of the Dutch auction moves rather slowly. Katok and Kwasnica (2008) and Carare and Rothkopf (2005) explain this observation by appealing to bidders' impatience. Our model, while featuring non-standard risk preferences, is also related to this second explanation as a slower clock allows more time for bidders' reference points to adjust.[3]

Section 2 describes the auction environment, bidders' preferences, and solution concept. We consider a standard symmetric environment with independent private values and bidders who have expectations-based reference-dependent preferences as in Kőszegi and Rabin (2009). Hence, in addition to classical material utility, a bidder experiences "gain-loss utility" from comparing her material outcomes to a reference point equal to her (rational) beliefs about these outcomes, as well as "news utility" from updating her reference point from old to new beliefs; both gain-loss and news utility attach a higher weight to losses than to equal-size gains. We focus on symmetric equilibria in increasing strategies; thus, the bidder with the highest value wins the auction.

In Section 3 we begin our analysis by characterizing the equilibrium strategy of loss-averse bidders in the FPA. We show that the attachment effect generates an upward pressure on the equilibrium bids. Indeed, because bidders hold their reference point fixed when submitting their bid, they are willing to pay more in order to reduce the chances of losing the auction.

Next, we turn to the Dutch auction. Here, the main intricacy in characterizing the equilibrium is a form of belief-based time inconsistency that arises even though bidders' preferences are time consistent.[4] That is, the price at which a bidder stops the clock in equilibrium need not be – and in general it is not – the price at which the bidder would have preferred to stop the clock from the outset. This happens because, as the auction unfolds and the reference point adjusts, the bidder

[2] Weber (1982) shows that the opposite holds if bidders' preferences exhibit the *counter* Allais paradox.

[3] For evidence on the adjustment of reference points, see Imas (2016), Heffetz (2018) and Thakral and Tô (2019).

[4] Kőszegi and Rabin (2009) and Pagel (2016, 2017) explore the implications of this belief-based time inconsistency for intertemporal consumption and saving decisions.

is tempted to "surprise" herself by stopping the clock earlier or later than originally planned. In equilibrium, however, the bidder's plan must be consistent with her expectations so that she stops the clock exactly at the price at which she had planned to do so. We show that there can be multiple consistent bidding plans and identify the symmetric plan that provides bidders with the highest utility from an ex-ante perspective.

We then compare the equilibrium strategies of the two formats and show that loss-averse bidders bid more aggressively in the Dutch auction than in the FPA. An immediate corollary is that the Dutch auction raises more revenue than the FPA. More generally, we argue that managing buyers' expectations is crucial for the performance of a selling mechanism and, by combining ours and previous results, we show that the expected revenue of the four standard auction formats ranks as follows: Dutch > FPA = second-price auction (SPA) > English.[5] Indeed, bidders' beliefs about their likelihood of winning at the time of bidding, and hence their attachment, coincide in the static FPA and SPA. By contrast, the attachment effect is weakest in the English auction where, as the auction unfolds, a bidder becomes more pessimistic about her likelihood of winning.

In Section 4 we analyze two extensions of our baseline model. First, we consider the case where, in the Dutch auction, a bidder's reference point does not immediately adjust to her current beliefs, but follows a more sluggish adjustment process. In particular, we posit that at any point during the auction, the reference point equals a convex combination between the bidder's initial beliefs (i.e., at the beginning of the auction) and her current ones. We find that also under this "stickier" formulation of the reference point, the Dutch auction raises more revenue than the FPA. In our second extension, we show that this revenue ranking continues to hold under the solution concept of choice-acclimating personal equilibrium (CPE), where a bidder's reference point immediately adjusts to her action, on and off the equilibrium path (see Kőszegi and Rabin, 2007).

Section 5 concludes the paper by discussing some implications of our results. In particular, we highlight that with expectations-based reference-dependent preferences, the dynamic evolution of beliefs endogenously impacts a bidder's valuation. Hence, the "Revelation Principle" (Myerson, 1979; 1981) does not necessarily hold since, when moving from a direct static mechanism to a dynamic one, a bidder's valuation may change. Therefore, a revenue-maximizing seller should opt for dynamic mechanisms which induce bidders to have optimistic beliefs, thereby increasing their willingness to pay. Finally, we discuss other contexts, beyond auctions, where the attachment effect is likely to play a role, like the dissolution of partnerships, bargaining and dynamic pricing.

[5] Balzer and Rosato (2021) establish the revenue equivalence between the FPA and the SPA, while von Wangenheim (2020) establishes the ranking between the English auction and the SPA. Using a different solution concept, Lange and Ratan (2010) show that the FPA raises more revenue than the SPA, while Eisenhuth (2019) shows that the all-pay auction yields the most revenue among all sealed-bid formats. Using different dynamic models of reference-dependent preferences than ours, Erhart and Ott (2017) compare Dutch and English auctions while Rosato (2019) analyzes sequential sealed-bid auctions. Fugger *et al.* (2020) characterize the optimal two-stage procurement auction. For related applications of reference-dependent preferences in industrial organization, contract theory, and matching see Heidhues and Kőszegi (2008, 2014), Herweg *et al.* (2010, 2018), Herweg and Mierendorff (2013), Karle and Peitz (2014, 2017), Daido and Murooka (2016), Rosato (2016, 2017), Karle and Schumacher (2017), Macera (2018), Dreyfuss *et al.* (2020), and Meisner and von Wangenheim (2020).

2 The Model

In this section, we describe the environment, bidders' preferences, and solution concept.

2.1 Environment

An indivisible item is auctioned off to $N \geq 2$ bidders. Each bidder $i \in \{1, \dots, N\}$ has a private value $\theta_i \in \Theta := \left[\underline{\theta}, \overline{\theta}\right] \subseteq \mathbb{R}_+$. Values (or types) are independently and identically distributed across bidders according to a CDF $F : \Theta \rightarrow [0, 1]$ admitting a continuous PDF f. Let F_1 and f_1 respectively denote the CDF and PDF of the highest order statistic among $N - 1$; similarly, let $F_1(\cdot|x)$ and $f_1(\cdot|x)$ respectively denote its CDF and PDF conditional on being lower than x.

In the FPA, bidders simultaneously submit sealed bids; the highest bidder wins the auction and pays her bid. Regarding the Dutch auction, we assume that the clock starts at some sufficiently high price and then drops in steps of size $\varepsilon > 0$. The first bidder who stops the clock wins the auction and pays the price displayed on the clock.

2.2 Preferences and Solution Concept

Throughout the paper, we restrict attention to symmetric pure-strategy equilibria with strictly increasing, differentiable bidding functions, $\beta : \Theta \rightarrow \mathbb{R}_+$. Consider a bidder with type θ bidding (in either the FPA or the Dutch auction) as if her type were $\tilde{\theta} \neq \theta$. If she wins the auction, she obtains an item she values θ and pays the price $\beta(\tilde{\theta})$; denote this outcome by $(\theta, \beta(\tilde{\theta}))$. If she loses the auction, she gets nothing and pays nothing; denote this outcome by $(0, 0)$. Hence, the set of material outcomes is $\tilde{\mathcal{O}} = \{(\theta, \beta(\tilde{\theta})), (0, 0)\}$ and the bidder's possible material payoffs are $\theta - \beta(\tilde{\theta})$ and 0, respectively. Following Kőszegi and Rabin (2006, 2007, 2009) we assume that, in addition to classical material utility, the bidder also derives psychological gain-loss utility from comparing her material outcomes to a reference outcome given by her recent expectations (probabilistic beliefs).[6] If the bidder *plans* to bid $\beta(\theta)$, her reference outcomes are $\mathcal{O} = \{(\theta, \beta(\theta)), (0, 0)\}$ and her reference point at any point in time is a distribution over the set of reference outcomes $\mathcal{O}$.

We first elaborate on the reference point of a type-θ bidder *at the beginning* of the auction as induced by the bidding strategy β. Since, in a symmetric equilibrium, the bidder wins with probability $F_1(\theta)$ if she plans to bid $\beta(\theta)$, her reference point is given by

$$r = \begin{cases} (\theta, \beta(\theta)) & \text{with probability} \quad F_1(\theta) \\ (0, 0) & \text{with probability} \quad 1 - F_1(\theta) \end{cases} .$$

[6] For experimental evidence on Kőszegi and Rabin's model see Abeler *et al.* (2011), Ericson and Fuster (2011), Gill and Prowse (2012), Banerji and Gupta (2014), Heffetz and List (2014), Karle *et al.* (2015), Sprenger (2015), Zimmermann (2015), Gneezy *et al.* (2017), Smith (2019), Cerulli-Harms *et al.* (2019), and Rosato and Tymula (2019). For evidence from the field, see Pope and Schweitzer (2011), Card and Dahl (2011) and Crawford and Meng (2011). While most of the evidence indicates that expectations play an important role in shaping reference points, a few studies have also documented some violations of the model's directional predictions.

Moreover, the bidder updates her reference point based on the arrival of new information about her material outcomes. In the static FPA, updating only takes place once the auction is over and the bidder learns whether or not she won so that her beliefs become degenerate.

In the Dutch auction, instead, at each price drop the bidder observes whether an opponent stopped the clock and instantaneously updates her beliefs about the opponents' types (and hence her likelihood of winning) accordingly.[7] Thus, if at price $\beta(\theta') > \beta(\theta)$ the auction is still running, a type-θ bidder updates her likelihood of winning — given her plan to stop the clock at price $\beta(\theta)$ — to $F_1(\theta|\theta')$. Similarly, a bidder updates her reference point if she decides to deviate to another strategy. For instance, if at price $\beta(\theta') > \beta(\theta)$ a type-θ bidder decides to deviate from the plan to stop the clock at price $\beta(\theta)$ to the plan of stopping the clock at $\beta(\tilde{\theta})$, then she instantaneously updates her likelihood of winning (and thus her reference point) to $F_1(\tilde{\theta}|\theta')$.

Such updating of the reference point by itself induces psychological gains and/or losses. In particular, following Kőszegi and Rabin (2009), we assume that the bidder makes an "ordered comparison" percentile-by-percentile between her previous beliefs and her new ones.[8] Formally, for any $p \in (0,1)$ let $c_r(p)$ and $c_{\tilde{r}}(p)$ denote the consumption levels at percentile p under two reference point's distributions r and $\tilde{r}$, respectively. The gain-loss utility arising from updating the reference point from $\tilde{r}$ to r in dimension $k \in \{g, m\}$ is defined as follows:

$$N(r, \tilde{r}) = \sum_{k \in \{g,m\}} \int_0^1 \mu^k(c_r(p) - c_{\tilde{r}}(p))dp.$$

Following most of the literature, we assume that the gain-loss function μ^k is piecewise linear:

$$\mu^k(x) = \begin{cases} \eta^k x & \text{if} \quad x \geq 0 \\ \eta^k \lambda^k x & \text{if} \quad x < 0 \end{cases}$$

with $\eta^k > 0$ and $\lambda^k > 1$ for $k \in \{g, m\}$.[9]

Consider the gain-loss utility of a type-θ bidder when the clock of the Dutch auction drops from price $\beta(\theta')$ to price $\beta(\theta'')$. If no opponent buys in this time interval, then the probability with which the bidder expects to win increases by $F_1(\theta|\theta'') - F_1(\theta|\theta')$. Hence, the bidder experiences a gain in the item dimension and a loss in the money dimension equal to:

$$N = \eta^g \left[F_1(\theta|\theta'') - F_1(\theta|\theta')\right] \theta - \eta^m \lambda^m \left[F_1(\theta|\theta'') - F_1(\theta|\theta')\right] \beta(\theta).$$

[7] Instantaneous updating implies that the bidder's reference point coincides with her *most* recent, i.e. current, beliefs. In Section 4 we consider a more sluggish adjustment process and show that the main insights are unchanged.

[8] Kőszegi and Rabin (2009) call this "news utility" or "prospective gain-loss utility" and allow this gain-loss utility to be discounted depending on how far in the future consumption outcomes will materialize. Since in our model the outcome will materialize soon, when the auction ends, we abstract away from this possibility and assume that bidders place the same weight on prospective and contemporaneous gain-loss utility. For a different definition of prospective gain-loss utility see Pagel (2019).

[9] Although, most of the literature assumes that $\eta^g = \eta^m$ and $\lambda^g = \lambda^m$, we do not impose such a "universal" gain-loss function. Indeed, in Section 3 we will often provide the intuition behind our results by assuming $\eta^m = 0$.

Let $U(\tilde{\theta}|\theta, \theta')$ denote a type-θ bidder's total expected utility when the current clock price is $\beta(\theta')$ and the bidder — who had planned to stop the clock at price $\beta(\theta) < \beta(\theta')$ — is considering to deviate by stopping the clock at price $\beta(\tilde{\theta}) < \beta(\theta')$. Apart from material utility, $U(\tilde{\theta}|\theta, \theta')$ consists of psychological gain-loss utility from both (i) the update of the winning probability due to the deviation to $\beta(\tilde{\theta})$, and (ii) the bidder's expected gain-loss at all future price drops.

We say that a bidder's strategy to stop the clock at price $\beta(\tilde{\theta})$ is *credible*, if she buys when the clock reaches time $\beta(\tilde{\theta})$, given her updated reference points at time $\beta(\tilde{\theta})$. That is, at time $\beta(\tilde{\theta})$ the bidder prefers buying to any other *credible* deviation.

Definition 1. *A strategy $\beta(\theta)$ is a personal equilibrium (PE) for a bidder with type θ if, taking as given the distribution of bids induced by $\beta(\theta)$, for all $\theta' \geq \theta$ it holds that*

$$U(\theta|\theta, \theta') \geq U(\tilde{\theta}|\theta, \theta'),$$

for any credible deviation $\tilde{\theta} < \theta'$.[10]

The restriction to credible strategies (and deviations) is important. Indeed, notice that at price $\beta(\theta')$ a bidder might be tempted to deviate from her equilibrium strategy of stopping the clock at price $\beta(\theta)$ to an alternative strategy — such as, for instance, stopping the clock at some other price $\beta\left(\widehat{\theta}\right)$ — even though she would not carry through with this plan when it is time to execute it. The reason is that the bidder might enjoy additional psychological gain-loss utility from the change in the reference point caused by non-credible deviations; once the reference point has adjusted to the new plan, however, the bidder might want to deviate again (and again...). The restriction to credible strategies implies that a bidder will only entertain a plan that she is willing to follow through given the reference point implied by the plan.

In the FPA, where bidders submit sealed bids at the beginning of the auction, updating of the reference point takes place only when the auction is over. Denote with $U(\tilde{\theta}|\theta)$ the expected utility of a type-θ bidder who has planned to submit a bid equal to $\beta(\theta)$ but deviates by bidding $\beta(\tilde{\theta})$. Then, in the FPA the requirement for a bidding strategy $\beta(\theta)$ to be a personal equilibrium reduces to the condition that at the beginning of the auction $U(\theta|\theta) \geq U(\tilde{\theta}|\theta)$ for all $\tilde{\theta}$. This equilibrium notion coincides with the concept of an unacclimating personal equilibrium (UPE) in Kőszegi and Rabin (2007), the special case of a PE in static environments.

We can now define our solution concept for the auction games:

Definition 2. *A bidding function β constitutes a symmetric personal equilibrium if for each type θ, given the knowledge that opponents bid according to β, the strategy $\beta(\theta)$ is a personal equilibrium.*

[10] As we will focus on symmetric increasing equilibria, the restriction in our definition to deviations to a bid $\widetilde{b}$ such that $\widetilde{b} = \beta(\tilde{\theta})$ rather than any arbitrary bid is without loss of generality. Indeed, any bid larger than $\beta(\overline{\theta})$ is dominated by $\beta(\overline{\theta})$ as it would increases the price the bidder would have to pay without any additional gain on the probability of winning. Similarly, any bid lower than $\beta(\underline{\theta})$ would result in a winning probability of zero, thereby yielding the same payoff as a bid equal to $\beta(\underline{\theta})$.

Finally, as there can be multiple symmetric personal equilibria, we assume bidders collectively select the one yielding the highest utility from an ex-ante perspective — the "preferred personal equilibrium" (PPE).[11]

3 Analysis

In this section, we derive the equilibrium bidding strategies in the FPA and Dutch auction and highlight how the attachment effect shapes the incentives of loss-averse bidders. In particular, the magnitude of the attachment effect depends on how optimistic a bidder is at the time of submitting her bid; this, in turn, will imply that the Dutch auction raises more revenue than the FPA.

3.1 Equilibrium Bidding in the FPA

Consider a type-θ bidder who has planned to bid $\beta_I(\theta)$ but deviates by mimicking a bidder with type $\tilde{\theta} \geq \theta$.[12] In this case, her expected payoff is:

$$\begin{aligned} U(\tilde{\theta}|\theta) \;&=\; F_1(\tilde{\theta})\left[\theta - \beta_I(\tilde{\theta})\right] - \eta^g\lambda^g\left[1 - F_1(\tilde{\theta})\right]F_1(\theta)\theta + \eta^g F_1(\tilde{\theta})\left[1 - F_1(\theta)\right]\theta \\ &+\eta^m\left[1 - F_1(\tilde{\theta})\right]F_1(\theta)\beta_I(\theta) - \eta^m\lambda^m F_1(\tilde{\theta})\left[1 - F_1(\theta)\right]\beta_I(\tilde{\theta}) - \eta^m\lambda^m F_1(\tilde{\theta})F_1(\theta)\left[\beta_I(\tilde{\theta}) - \beta_I(\theta)\right]. \quad (1) \end{aligned}$$

The first term in (1) represents the standard expected material payoff. The other terms capture expected gain-loss utility and are derived as follows. The second term captures the loss in the item dimension for a bidder who expected to win the auction with probability $F_1(\theta)$ but ends up losing it — an event happening with probability $1-F_1(\tilde{\theta})$ — and thus experiences a loss equal to $\eta^g\lambda^g F_1(\theta)\theta$. Similarly, the third term captures the gain in the item dimension for a bidder who expected to lose with probability $1 - F_1(\theta)$ but ends up winning — an event happening with probability $F_1(\tilde{\theta})$ — and thus experiences a gain equal to $\eta^g\left[1 - F_1(\theta)\right]\theta$. The fourth and fifth terms capture the corresponding expected gains and losses in the money dimension. The final term captures the loss in the money dimension when winning at a price higher than expected. Differentiating (1) with respect to $\tilde{\theta}$ and evaluating the resulting first-order condition at $\tilde{\theta} = \theta$ yields a differential equation whose solution provides us with the equilibrium bidding strategy:[13]

Proposition 1. *The symmetric PPE bidding strategy in the FPA is given by*

$$\beta_I(\theta) = \int_{\underline{\theta}}^{\theta} \frac{1 + \eta^g\lambda^g F_1(x) + \eta^g\left[1 - F_1(x)\right]}{F_1(\theta)\left(1 + \eta^m\lambda^m\right)} e^{\frac{\eta^m(\lambda^m-1)[F_1(\theta)-F_1(x)]}{1+\eta^m\lambda^m}} x f_1(x)dx. \quad (2)$$

[11] Notice that Kőszegi and Rabin (2006, 2007, 2009) propose PPE to address the issue of the multiplicity of personal equilibria in the context of *individual* decision problems. In our multi-player game, however, selecting the PPE is akin to assuming that all bidders are able to coordinate on the (symmetric) personal equilibrium that is best for the group. Notwithstanding this additional restriction, we think the PPE selection represents a reasonable benchmark in our model as it provides the auctioneer with a worst-case scenario.

[12] As shown by Balzer and Rosato (2021), upward deviations are the most relevant ones.

[13] The FOC is also sufficient since the bidder's payoff satisfies single crossing; see Balzer and Rosato (2021).

Balzer and Rosato (2021) derived the symmetric PPE bidding function for an environment with interdependent values and independent signals. As the IPV model is a special case of theirs, applying their result to our environment yields expression (2). It is easy to verify that $\beta_I(\theta)$ is increasing in the coefficient of loss aversion in the item dimension (λ^g) and decreasing in the coefficient of loss aversion in the money dimension (λ^m). Intuitively, if the bidder wins the auction she experiences a loss in money; this induces her to reduce her bid when loss aversion in the money dimension becomes stronger. Similarly, the bidder experiences a loss in the item dimension when she loses the auction; this, in turn, induces her to increase her bid when loss aversion in the item dimension becomes stronger.

Next, we compare the loss-averse bidding strategy with the risk-neutral benchmark. For $\eta^m = 0$, expression (2) reduces to

$$\beta_I(\theta) = \mathbb{E}_{F_1}[v(x)|x \leq \theta],$$

where

$$v(x) = \{1 + \eta^g \lambda^g F_1(x) + \eta^g [1 - F_1(x)]\} x.$$

The term $v(x)$ represents the belief-dependent "opportunity value" of winning the auction for a bidder with type x. Indeed, in addition to classical material utility, when winning the bidder also experiences a gain equal to $\eta^g [1 - F_1(x)] x$ while concurrently avoiding a loss equal to $\eta^g \lambda^g F_1(x)x$. Hence, as in the risk-neutral benchmark, bidders in equilibrium bid the expectation of their strongest opponent's valuation conditional on winning; with expectations-based loss aversion, however, this valuation equals the opportunity value of winning which also depends on the bidder's beliefs. Importantly, the belief-dependent part of this opportunity value increases in a bidder's type. This is the attachment effect: bidders with higher types expect to win with a higher probability and thus feel more attached to the prize. As a result, compared to the risk-neutral benchmark, high types overbid more strongly than low types.[14]

3.2 Equilibrium Bidding in the Dutch Auction

Differently from the FPA, the Dutch auction is a dynamic format where a bidder's beliefs (and hence her reference point) evolve throughout the auction. Moreover, when she submits her bid in the FPA, the bidder is unsure about whether she will win; in the Dutch auction, instead, when she submits her bid by stopping the clock, the bidder is sure to win.

In a symmetric equilibrium, a type-θ bidder stops the clock at price $\beta_D(\theta)$. In particular, the bidder prefers executing this plan over switching to another credible plan at any point in time. Suppose the current clock price is $\beta_D(\theta') > \beta_D(\theta)$ and a type-θ bidder considers deviating to

[14] Straight overbidding compared to risk neutrality is driven by the assumption that $\eta^m = 0$, which reduces the weight over money relative to the item dimension in a bidder's utility. Yet, the intuition that a stronger attachment effect leads to more aggressive bids for high types equally applies when bidders are loss averse in both dimensions.

another plan, $\beta_D(\tilde{\theta}) > \beta_D(\theta)$.[15] In this case, her expected payoff is:

$$
\begin{aligned}
U(\tilde{\theta}|\theta,\theta') \;&=\; F_1(\tilde{\theta}|\theta')[\theta-\beta_D(\tilde{\theta})] + \eta^g\theta\left[F_1(\tilde{\theta}|\theta') - F_1(\theta|\theta')\right] \\
&\quad -\eta^m\lambda^m[F_1(\tilde{\theta}|\theta')\beta_D(\tilde{\theta}) - F_1(\theta|\theta')\beta_D(\theta)] + \mathbb{E}\left[N(\tilde{\theta}|\theta,\theta')\right],
\end{aligned} \tag{3}
$$

where $\mathbb{E}\left[N(\tilde{\theta}|\theta,\theta')\right]$ is the sum of total expected news utility a type-θ bidder expects to experience from all updates between price $\beta_D(\theta')$ and price $\beta_D(\tilde{\theta})$ given the new plan to buy at price $\beta_D(\tilde{\theta})$ (its functional form is derived in Lemma 1 below). The first term on the right-hand side of (3) is the standard expected material payoff. The other terms capture expected gain-loss utility. By deviating from her plan to buy at price $\beta_D(\theta)$ to the new plan of buying at price $\beta_D(\tilde{\theta})$, the bidder's probability of winning increases from $F_1(\theta|\theta')$ to $F_1(\tilde{\theta}|\theta')$. Hence, by deviating she experiences a gain in the item dimension equal to $\eta^g\theta[F_1(\tilde{\theta}|\theta') - F_1(\theta|\theta')]$. At the same time, however, the bidder also increases her expected payment from $F_1(\theta|\theta')\beta_D(\theta)$ to $F_1(\tilde{\theta}|\theta')\beta_D(\tilde{\theta})$, thereby experiencing a loss in the money dimension equal to $\eta^m\lambda^m[F_1(\tilde{\theta}|\theta')\beta_D(\tilde{\theta}) - F_1(\theta|\theta')\beta_D(\theta)]$. The last term on the right-hand side of (3) captures news utility; that is, the expected gain-loss utility stemming from changes in beliefs and the resulting updating of the reference point as the auction unfolds. The next result allows us to re-write this expression in terms of the model's primitives.

Lemma 1. *Let the current clock price be $\beta_D(\theta')$ and consider a bidder of type θ planning to stop the clock at price $\beta_D(\tilde{\theta}) < \beta_D(\theta')$. For $\varepsilon \to 0$, the following equality holds:*

$$
\mathbb{E}\left[N(\tilde{\theta}|\theta,\theta')\right] = -\left[\eta^g(\lambda^g-1)\theta + \eta^m(\lambda^m-1)\beta_D(\tilde{\theta})\right]\int_{\tilde{\theta}}^{\theta'} f_1(x|\theta')F_1(\tilde{\theta}|x)dx. \tag{4}
$$

In order to keep the analysis tractable and simplify the notation as much as possible, for the remainder of the analysis we will focus on the limit case where $\varepsilon \to 0$, which can be interpreted as an arbitrarily fine price grid. The term on the right-hand side of (4) is a natural generalization of static expected gain-loss utility to a dynamic setting. We discuss it by focusing on the risk in the item dimension, but a similar intuition applies for the money dimension. From the perspective of a bidder who is active at price $\beta_D(\theta')$, at any future price $\beta_D(x) \in \left[\beta_D(\tilde{\theta}), \beta_D(\theta')\right]$ only one of the two following events can realize. The auction may continue and the bidder learns that her strongest opponent's type is below x. This event, given the current price is $\beta_D(\theta')$, happens with probability $F_1(x|\theta')$; in this case, the bidder updates her beliefs and her probability of winning increases by $-\frac{\partial}{\partial x}F_1(\tilde{\theta}|x) = f_1(x|x)F_1(\tilde{\theta}|x)$, generating a gain equal to $\eta^g f_1(x|\theta')F_1(\tilde{\theta}|x)$. Alternatively, the auction may end and the bidder learns that her strongest opponent's type is exactly x. This event, given the current price is $\beta_D(\theta')$, happens with (marginal) probability $f_1(x|\theta')$; in this case, she learns that she lost and her beliefs about winning drop from $F_1(\tilde{\theta}|x)$ to zero, generating a loss equal to $\eta^g\lambda^g f_1(x|\theta')F_1(\tilde{\theta}|x)$.

[15] As for the FPA, upward deviations are the most relevant ones.

An equilibrium bid is a credible plan about when to stop the clock such that, at any point during the auction, the bidder prefers executing it over switching to another credible plan. Verifying that an equilibrium bid is indeed a credible plan is technically tedious and so we relegate it to Appendix A. Yet, equilibrium behavior is rather intuitive: at any price $\beta_D(\theta') > \beta_D(\theta)$ a type-θ bidder prefers to stay in the auction instead of buying immediately; hence, $U(\theta|\theta, \theta') \geq U(\theta'|\theta, \theta')$. In Appendix A we show that letting $\theta' \to \theta$ yields a lower bound on the derivative of the bidding function. Letting this lower bound bind and solving the resulting differential equation provides us with the equilibrium bidding strategy:

Proposition 2. *The symmetric PPE bidding strategy in the Dutch auction is given by*

$$\beta_D(\theta) = \int_{\underline{\theta}}^{\theta} \frac{1 + \eta^g \lambda^g}{F_1(\theta)\,(1 + \eta^m \lambda^m)} \left[\frac{F_1(\theta)}{F_1(x)}\right]^{\frac{\eta^m(\lambda^m - 1)}{1 + \eta^m \lambda^m}} x f_1(x) dx. \tag{5}$$

Again, it is easy to see that $\beta_D(\theta)$ is increasing in the coefficient of loss aversion in the item dimension (λ^g) and decreasing in the coefficient of loss aversion in the money dimension (λ^m). Moreover, for $\eta^m = 0$ expression (5) simplifies to

$$\beta_D(\theta) = (1 + \eta^g \lambda^g)\mathbb{E}_{F_1}[x|x \leq \theta].$$

Hence, compared to the risk-neutral benchmark every type overbids by a factor $1 + \eta^g \lambda^g$. As in the FPA, bidders bid the expectation of their strongest opponent's opportunity value of winning, where this value is now given by

$$v(x) = (1 + \eta^g \lambda^g)x.$$

Indeed, bidding behavior in the Dutch auction is driven by a bidder's incentives shortly before she buys and, at this time, the bidder is effectively certain to win. This magnifies the attachment effect (and therefore the opportunity value of winning) compared to the FPA, where a bidder's attachment is pinned down by her ex-ante beliefs.

The fact that bidding strategy in the Dutch auction is different than in the FPA immediately suggests that bidders' ex-ante utility will also differ between the two formats. Remarkably, however, loss-averse bidders would be worse off in the Dutch auction *even if they were to bid the same in both formats.* The next proposition formally states this result.

Proposition 3. *If bidders use the same strategy in the FPA and the Dutch auction, their ex-ante utility is lower in the Dutch auction than in the FPA.*

The intuition for the above result is as follows. If bids are the same in both formats, a bidder's expected payment and probability of winning are also the same; this, in turn, implies that from an ex-ante perspective a bidder's reference points coincide in the two formats. Yet, while in the FPA uncertainty is resolved all at once, the Dutch auction entails a more gradual resolution of uncertainty. Such a gradual resolution of uncertainty exposes a bidder to the risk of first becoming more optimistic and then suddenly learn that an opponent just bought the item. A loss-averse bidder dislikes these fluctuations in beliefs. Indeed, if possible, the bidder would prefer to commit to a bid at the beginning and avoid seeing the auction unfolds.[16]

The dislike for fluctuations in beliefs is also the source of a belief-based form of time inconsistency that arises in the bidders' plans even though their preferences are time consistent. In particular, bidders with rather low types, who are exposed to fluctuations in beliefs for a relatively long time, would like to mitigate the expected losses from such fluctuations during the auction by committing to a lower bid. Indeed, as the next proposition shows, such deviation would make low-type bidders better off.

Proposition 4. *In the Dutch auction there exists a cut-off type $\theta^* \in (\underline{\theta}, \overline{\theta})$ such that all types $\theta < \theta^*$ would ex-ante increase their utility by deviating to a lower bidding strategy.*

Yet, such a mitigation strategy is not dynamically consistent and, therefore, cannot be part of an equilibrium. The reason is that, when a bidder decides to stop the clock, her losses from the gradual resolution of uncertainty are "sunk" and hence do not affect her incentives. Hence, even though a bidder may ex-ante prefer a lower bidding strategy, she anticipates that she would never actually execute such a plan.

The results in both Proposition 3 and Proposition 4 reveal a demand for commitment in the Dutch auction on the part of loss-averse bidders. Such a commitment could be achieved via proxy bidding whereby bidders could effectively transform the Dutch auction into a first-price one. Yet, while auction sites like eBay and others usually provide proxy bidding services in English (or ascending-price) auctions — effectively turning them into second-price ones — such services are much less common for Dutch auctions. Indeed, in the next section we will see that it is in the seller's interest for bidders to engage in proxy bidding in English auctions, but not Dutch ones; hence, our model provides a potential reason why proxy bidding is much more prevalent in English than in Dutch auctions.[17]

3.3 Revenue Comparison

We now show that, by creating a stronger attachment effect, the Dutch auction raises more revenue than the FPA. Suppose $\eta^m = 0$ and consider a type-θ bidder who contemplates mimicking type $\theta' > \theta$. In equilibrium, $U(\theta|\theta) \geq U(\theta'|\theta)$; hence, using expression (1), the following must hold:

[16] Pagel (2018) makes a similar observation for a loss-averse investor who prefers to ignore and not rebalance his portfolio because he dislikes bad news more than he likes good news.

[17] Pricefalls is the only auction site running Dutch auctions with proxy bidding — called "buy if it hits" — that we are aware of. The company was founded in 2009 in response to users' frustrations with the eBay model and with a pronounced emphasis on buyer's satisfaction.

$$F_1(\theta')\beta_I(\theta') - F_1(\theta)\beta_I(\theta) \geq [F_1(\theta') - F_1(\theta)]\,\theta + \eta^g\theta[F_1(\theta') - F_1(\theta)]\,[1 + (\lambda^g - 1)\,F_1(\theta)]\,. \quad (6)$$

Similarly, in the Dutch auction in equilibrium it holds that $U(\theta|\theta,\theta') \geq U(\theta'|\theta,\theta')$ for any price $\beta_D(\theta') > \beta_D(\theta)$; multiplying both sides in (3) by $F_1(\theta')$ and re-arranging yields:

$$\begin{aligned} F_1(\theta')\beta_D(\theta') - F_1(\theta)\beta_D(\theta) \quad \geq \quad & [F_1(\theta') - F_1(\theta)]\,\theta \\ & + \eta^g\theta[F_1(\theta') - F_1(\theta)] + \eta^g(\lambda^g - 1)\theta F_1(\theta') \int_\theta^{\theta'} F_1(\theta|x) f_1(x|\theta')dx. \quad (7) \end{aligned}$$

In both (6) and (7), the term on the left-hand side and the first term on the right-hand side represent the familiar material costs and benefits associated with mimicking a bidder with a higher type — trading off a higher probability of winning against paying a higher price. The additional terms on the right-hand sides represent the additional incentives that a loss-averse bidder has to raise her bid; i.e., to realize gains and/or avoid losses. These incentives are stronger in the Dutch auction because

$$F_1(\theta') \int_\theta^{\theta'} F_1(\theta|x) f_1(x|\theta')dx > \int_\theta^{\theta'} F_1(\theta) f_1(x)dx = [F_1(\theta') - F_1(\theta)]F_1(\theta).$$

In the Dutch auction the incentives to raise one's bid are stronger since, when doing so, a bidder is (almost) sure to win; hence, by bidding more aggressively, she can reduce the expected losses caused by the fluctuations in beliefs that arise when updating the reference point.[18] Indeed, we have the following result:

Proposition 5. *$\beta_D(\theta) \geq \beta_I(\theta)$ and this inequality is strict for all $\theta > 0$.*

Thus, loss-averse bidders bid more in the Dutch auction than in the FPA. The next result then follows immediately from Proposition 5.

Corollary 1. *With loss-averse bidders the Dutch auction yields a higher revenue than FPA.*

The attachment effect in the FPA depends on a bidder's ex-ante likelihood of winning. In the Dutch auction, instead, the attachment effect grows over time. Indeed, as the price at which a bidder had planned to stop the clock approaches, her beliefs about her chances of winning — and hence her willingness to pay — increase. This, in turn, pushes a bidder to (plan to) stop the clock at a price which is higher than her bid in the FPA.

Figure 1 displays the loss-averse bidding strategies in the Dutch auction and the FPA, along with the risk-neutral one, for the case of equal gain-loss utility across dimensions. Notice that $\beta_I(\underline{\theta}) = \left(\frac{1+\eta}{1+\eta\lambda}\right)\underline{\theta}$ while $\beta_D(\underline{\theta}) = \left(\frac{1+\eta\lambda}{1+\eta}\right)\underline{\theta}$. These bids coincide with the maximum price at which a

[18] This effect similarly applies to gain-loss utility over money since, by stopping the clock earlier, a bidder also avoids fluctuations in her expected payment.

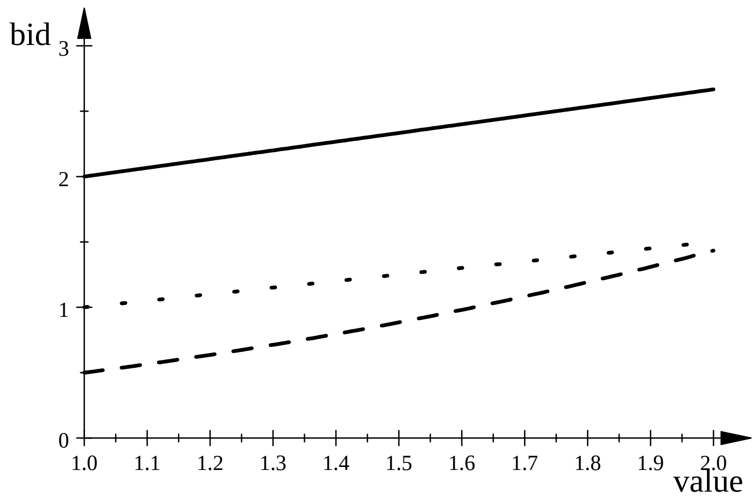

Figure 1: Bidding functions in the FPA (dashed) and Dutch auction (solid) for $N = 2$, $\eta^g = \eta^m = 1$ and $\lambda^g = \lambda^m = 3$ compared to risk neutrality (dotted) with θ distributed uniformly on $[1, 2]$.

loss-averse buyer with intrinsic valuation $\underline{\theta}$ is willing to buy when her expectations are to never or, respectively, always get the item (see Kőszegi and Rabin, 2006). Hence, at the time of submitting her bid, the attachment effect of a bidder with the lowest type is at its minimum in the FPA, but at its maximum in the Dutch auction. Moreover, Figure 1 also shows that, with uniformly distributed values, in the Dutch auction all types overbid whereas in the FPA all types underbid compared to the risk-neutral benchmark. The next proposition shows that the first observation holds independently of the distribution of values, while the second one is robust only for low types.

Proposition 6. *Let $\eta^k = \eta > 0$ and $\lambda^k = \lambda > 1$ for $k \in \{g, m\}$. In the Dutch auction all types overbid compared to the risk-neutral benchmark. In the FPA there exists a cutoff $\hat{p} \in ((0.5)^{\frac{1}{N-1}}, 1]$ such that a bidder overbids compared to the risk-neutral benchmark if and only if $F(\theta) > \hat{p}$.*

Hence, while low types underbid in the FPA independently of the distribution of values, some types at the top of the distribution might overbid. Yet, as the number of bidders increase, the share of types who underbid approaches one.

Combining Corollary 1 with results by Balzer and Rosato (2021) and von Wangenheim (2020), we obtain that the Dutch auction raises the most revenue among the four main auction formats:

Corollary 2. *With loss-averse bidders, in terms of expected revenue, the four main auction formats can be ranked as follows:*

$$\text{Dutch} > \text{FPA} = \text{SPA} > \text{English}.$$

Intuitively, the FPA and SPA are revenue equivalent as, since they are both static formats where a bidder's reference point depends on her ex-ante likelihood of winning, they induce the same level of attachment. The English auction raises the least revenue since a bidder becomes less optimistic about her chances of winning as the auction unfolds; this in turn lowers the bidder's reference point, inducing her to bid less aggressively than in the SPA. In other words, while in a Dutch auction a bidder's initial attachment grows as the auction evolves, in an English auction a bidder becomes less attached to the item as the auction continues. Hence, by creating the strongest attachment for bidders, the Dutch auction raises the most revenue among standard formats.

4 Extensions and Robustness

In this section, we investigate the robustness of our main result by analyzing two extensions of our baseline model. In the first one, we relax the assumption of instantaneous updating of the reference point by considering a more sluggish adjustment process. In the second one, we derive the bidding strategies under the alternative solution concept of choice-acclimating personal equilibrium (CPE), whereby a bidder's reference point immediately adjust to her bid — capturing the notion that bidders can commit to a bid before uncertainty fully resolves. For both extensions, we find that the Dutch auction continues to generate a stronger attachment effect — and hence a higher revenue — than the FPA.

4.1 Adjustment of the Reference Point

In our model, bidders update their reference points instantaneously; that is, a bidder's reference point coincides with her most recent beliefs. While convenient for the purpose of illustrating the attachment effect, this assumption might be too extreme as some time might be required for updated beliefs to "sink in" as reference points; see Heffetz (2018) and Smith (2019). However, none of our qualitative results concerning the comparison between the FPA and the Dutch auction hinge on the updating of the reference point to current beliefs being instantaneous. Indeed, as we will show in this section, as long as changes in beliefs cause an update, however small, of the reference point, the revenue ranking between the two auction formats still holds.

Consider an active bidder in the Dutch auction who plans to buy at price $\beta_D(\theta)$. A tractable way of modeling the updating of the reference point is to assume that while the auction unfolds, for any clock price $\beta_D(\theta')$, the bidder's reference point is a convex combination between her most recent beliefs $F_1(\theta|\theta')$ and her beliefs at the beginning of the auction, $F_1(\theta)$; then, when the auction terminates, the beliefs about the final allocation "sink in" and each bidder updates her reference point with respect to the final allocation. Let $\alpha \in [0, 1]$ be the weight on a bidder's most recent beliefs in her reference point as the Dutch auction unfolds. Evidently, $\alpha = 1$ corresponds to the situation analyzed in Section 3, where the Dutch auction raises more revenue than the FPA. By contrast, for $\alpha = 0$ there is no updating of the reference point during the Dutch auction so that a bidder's reference point stays equal to his ex-ante beliefs; it's easy to see then that in this case the two auction formats are revenue equivalent. For $\alpha \in (0, 1)$, we have the following result:

Proposition 7. *For all $\theta > \underline{\theta}$ the symmetric PPE bidding strategy $\beta_D(\theta)$ in the Dutch auction is increasing in α.*

Thus, equilibrium bids in the Dutch auction increase in how strongly the reference point adjusts; this, in turn, implies that for any $\alpha > 0$ a loss-averse bidder behaves more aggressively in the Dutch auction than in the FPA. In a controlled laboratory experiment, Katok and Kwasnica (2008) find that the Dutch auction generates more revenue than the FPA if the clock moves rather slowly. This finding is consistent with our model, as a slow clock provides more time for the reference point to adjust; i.e., a slower clock corresponds to a larger α in our model.[19]

4.2 Solution Concept

Up until now, we have employed the solution concept of (unacclimating) "personal equilibrium" (Kőszegi and Rabin, 2006; 2009) to analyze bidding behavior for both the FPA and the Dutch auction. According to this concept, a bidder's equilibrium action determines her reference point and, when deviating to an off-path action, she experiences psychological (dis-)utility from the changed distribution of final outcomes; in other words, in a personal equilibrium, a bidder keeps her reference point fixed when considering deviations. Moreover, under such a solution concept, a bidder also experiences psychological (dis-)utility when her beliefs change because of the arrival of new information (i.e. "news utility"). Hence, this solution concept best applies to auctions in which bidders form their plans sufficiently in advance for their expectations to become a reference point, but are not able to commit to a particular bid until shortly before uncertainty is resolved. The Dutch auction, given its dynamic nature, clearly meets these criteria. Moreover, we think that our solution concept can also be appropriate for sealed-bid auctions, especially if bidders have been looking ahead to the auction for quite some time but can still wait until the last minute to submit their actual bids. Furthermore, in some sealed-bid formats, like so-called "silent" or "secret-bid" auctions, the bidding phase lasts for a prespecified period of time during which bidders are required to be physically present and can revise their (sealed) bids multiple times.

Nevertheless, most of the prior literature on sealed-bid auctions with loss-averse bidders has employed the alternative solution concept, introduced by Kőszegi and Rabin (2007), of choice-acclimating personal equilibrium (CPE); see Lange and Ratan (2010) and Eisenhuth (2019). In a CPE, a bidder's reference point immediately adjusts to her action, both on and off the equilibrium path. That is, a bidding function $\beta(\theta)$ is a CPE in the FPA if and only if $U_\theta(\theta|\theta) \geq U_\theta(\tilde{\theta}|\tilde{\theta})$ where, with a slight abuse of notation, $U_\theta(\tilde{\theta}|\tilde{\theta})$ denotes a type-θ bidder's expected utility when bidding $\beta(\tilde{\theta})$ and having a reference point induced by this action. In other words, a bidder fully internalizes how a potential deviation affects her reference point and she experiences psychological (dis-)utility only from comparing the final realized outcomes to her most recent beliefs; thus, there is no "news

[19] However, they also find that with a fast clock the FPA raises more revenue than the Dutch auction whereas in our model, for the limiting case of $\alpha = 0$, the two auction formats are revenue equivalent.

utility" in CPE. In what follows, we show that also under CPE the Dutch auction continues to generate a stronger attachment effect and to raise a higher revenue than the FPA.

The original definition of CPE in Kőszegi and Rabin (2007) applies only to static decision problems. Hence, for the Dutch auction, we rely on the concept of sequential CPE (SCPE) — an extension of CPE to dynamic decision problems introduced by Rosato (2019). For any chosen strategy $\beta_D(\theta)$, at each price $\beta_D(\theta') > \beta_D(\theta)$ a type-θ bidder evaluates the expected utility of the final allocation with respect to the reference point generated by her current beliefs. Hence, the expected payoff of a type-θ bidder who considers stopping the clock at price $\beta_D(\tilde{\theta})$ when the current price is $\beta_D(\theta')$ is given by

$$\begin{aligned} U_\theta(\tilde{\theta}, \theta') \;\; = \;\; & F_1(\tilde{\theta}|\theta') \left[\theta - \beta_D(\tilde{\theta})\right] - \eta^g \left(\lambda^g - 1\right) F_1(\tilde{\theta}|\theta') \left[1 - F_1(\tilde{\theta}|\theta')\right] \theta \\ & - \eta^m \left(\lambda^m - 1\right) F_1(\tilde{\theta}|\theta') \left[1 - F_1(\tilde{\theta}|\theta')\right] \beta_D(\tilde{\theta}) \end{aligned} \tag{8}$$

where the first term on the right-hand side of (8) is classical expected material utility, whereas the other two terms represent expected gain-loss utility in the item and money dimensions, respectively. The strategy $\beta_D(\theta)$ is an SCPE for a type-θ bidder if, for any $\theta' > \theta$ and any credible deviation $\tilde{\theta} < \theta'$, the CPE condition $U_\theta(\theta, \theta') \geq U_\theta(\tilde{\theta}, \theta')$ holds. In other words, a bidder participating in the Dutch auction does not experience news utility from the arrival of new information, but at each time evaluates her action with respect to her current beliefs over the final allocation. Moreover, the only difference between an SCPE in the Dutch auction and a CPE in the FPA is that in the Dutch the plan must be CPE-optimal among all credible plans as beliefs evolve throughout the auction, whereas in the FPA it must be optimal only for ex-ante beliefs, $F_1(\theta)$. Then, we have the following result:

Proposition 8. *Let $\Lambda^k := \eta^k(\lambda^k - 1) \leq 1$. Then, under CPE, the Dutch auction raises more revenue than the FPA.*

Proposition 8 shows that also under CPE the Dutch auction raises more revenue than the FPA.[20] The intuition for this result is, again, the attachment effect. Indeed, under CPE a bidder's expected gain-loss utility is U-shaped in her likelihood of winning, so that bidding more aggressively increases the bidder's expected gain-loss utility only if this likelihood is at least 50%; yet, conditional on meeting this threshold, the larger is her likelihood of winning, the more the bidder is tempted to raise her bid. Thus, bidders still bid more aggressively in the Dutch auction since, at the time of stopping the clock, a bidder's likelihood of winning is 100%.[21]

[20] The condition $\Lambda^k \leq 1$, known as "non-dominance of gain-loss utility", ensures that a loss-averse agent does not select first-order stochastically-dominated options; see Masatlioglu and Raymond (2016). This condition is necessary for the existence of a separating equilibrium under CPE, but it is not needed under UPE.

[21] The only different prediction of CPE is that the FPA revenue dominates the SPA as the latter exposes bidders to additional risk in the money dimension, thereby pushing their bids down; see Lange and Ratan (2010).

5 Conclusion

We have shown that, in both the Dutch auction and the FPA, the incentives of expectations-based loss-averse bidders are driven by the attachment effect: the higher the probability with which a bidder expects to win the auction, the larger her disappointment if she loses and hence her willingness to pay. In the Dutch auction, bidders become more optimistic about their chances of winning as the auction unfolds; in the FPA, instead, the strategy of a loss-averse bidder depends on her ex-ante likelihood of winning. Hence, the Dutch auction induces a stronger attachment than the FPA. We expect this intuition to hold also in more general auction environments than the one considered in this paper. Indeed, as shown by Balzer and Rosato (2021), the attachment effect continues to exert an upward pressure on the bidding strategy of expectations-based loss-averse bidders even in an environment with common or interdependent values.

The key insight emerging from our analysis is that when bidders are expectations-based loss averse, managing their level of attachment is crucial for the performance of a selling mechanism. Indeed, a seller can increase his revenue by making bidders more optimistic at the time of submitting their bid. Using this general insight we were able to rank the four main standard auction formats: the Dutch auction raises more revenue than the FPA which is revenue equivalent to the SPA; and the latter two formats yield a higher revenue than the English auction. The evidence from both the lab and the field seems broadly consistent with this ranking. Indeed, Lucking-Reiley (1999) and Katok and Kwasnica (2008) find that the Dutch auction raises more revenue than the FPA. Moreover, several studies show that with private values the SPA tends to raise more revenue than the English auction; see Kagel *et al.* (1987) and Harstad (2000). Finally, Cheema *et al.* (2012) find that the Dutch auction yields a higher revenue than the English auction, and even more so when the clocks of the two auctions are relatively slow. Therefore, expectations-based loss aversion provides a novel rationale for the use of descending prices by sellers. Indeed, besides actual Dutch auctions, in practice several market negotiations feature descending prices as in, for instance, the real estate market where the asking prices of listed properties decline over time until an offer arrives; similarly, the pricing of tickets for sporting or entertainment events also typically follows a descending path. Descending prices can also be used to resolve financial disputes or for the dissolution of partnerships.[22]

More generally, with expectations-based reference-dependent preferences, two mechanisms that allocate the prize to the same bidder might still result in different payoffs for both the bidders and the seller, depending on how the allocation is implemented. In other words, the "Revelation Principle" might fail. Our results, then, imply that when agents are expectations-based loss-averse, focusing on static mechanisms is not without loss of generality.

[22] Qin and Zhang (2013) experimentally compare clock and sealed-bid auctions to dissolve partnerships. Consistent with our model, they find that subjects bid more aggressively in Dutch auctions than in first-price ones, but less aggressively in English auctions than in second-price ones.

A Proofs

Proof of Proposition 1: See Balzer and Rosato (2021). ■

Proof of Lemma 1: Consider a bidder planning to stay in the auction until $\beta_D(\tilde{\theta})$. At price $\beta_D(x) > \beta_D(\tilde{\theta})$, she expects to win with probability $F_1(\tilde{\theta}|x)$. Suppose the price drops to $\beta_D(x-\Delta) = \beta_D(x) - \varepsilon$. If an opponent stops the clock at price $\beta_D(x-\Delta)$ — an event happening with conditional probability $1 - F_1(x-\Delta|x)$ — the bidder loses the auction in which case her gain-loss utility is

$$-\eta^g \lambda^g F_1(\tilde{\theta}|x)\theta + \eta^m F_1(\tilde{\theta}|x)\beta_D(\tilde{\theta}).$$

With probability $F_1(x-\Delta|x)$ no opponent buys and the probability with which the bidder expects to win increases by $F_1(\tilde{\theta}|x-\Delta) - F_1(\tilde{\theta}|x)$. In this case, her gain-loss utility is

$$\eta^g \left[F_1(\tilde{\theta}|x) - F_1(\tilde{\theta}|x-\Delta)\right]\theta - \eta^m\lambda^m \left[F_1(\tilde{\theta}|x) - F_1(\tilde{\theta}|x-\Delta)\right]\beta_D(\tilde{\theta}).$$

Hence, her expected news utility when the price drops from $\beta_D(x)$ to $\beta_D(x-\Delta)$ is

$$\begin{aligned}
\mathbb{E}\left[N(x-\Delta|\theta,x)\right] &= \left[1 - F_1\left(x-\Delta|x\right)\right] F_1\left(\tilde{\theta}|x\right)\left[-\eta^g\lambda^g\theta + \eta^m\beta_D\left(\tilde{\theta}\right)\right] \\
&\quad + F_1\left(x-\Delta|x\right)\left[F_1\left(\tilde{\theta}|x-\Delta\right) - F_1\left(\tilde{\theta}|x\right)\right]\left[\eta^g\theta - \eta^m\lambda^m\beta_D\left(\tilde{\theta}\right)\right] \\
&= \left[1 - F_1\left(x-\Delta|x\right)\right] F_1\left(\tilde{\theta}|x\right)\left[-\eta^g\lambda^g\theta + \eta^m\beta_D\left(\tilde{\theta}\right)\right] \\
&\quad + \left[-F_1(x-\Delta|x)F_1(\tilde{\theta}|x) + F_1(\tilde{\theta}|x)\right]\left[\eta^g\theta - \eta^m\lambda^m\beta_D(\tilde{\theta})\right] \\
&= -\left[1 - F_1(x-\Delta|x)\right] F_1(\tilde{\theta}|x)[\eta^g(\lambda^g - 1)\theta + \eta^m(\lambda^m - 1)\beta_D(\tilde{\theta})].
\end{aligned}$$

When the current clock price is $\beta_D(\theta')$, for $x \in [\tilde{\theta}, \theta']$ price $\beta_D(x)$ is reached with probability $F_1(x|\theta')$. Hence, from the perspective of price $\beta_D(\theta')$, the expected news utility associated with a price drop from $\beta_D(x)$ to $\beta_D(x-\Delta)$ is given by:

$$F_1(x|\theta')\mathbb{E}\left[N(x-\Delta|\theta,x)\right] = -F_1(x|\theta')\left[\frac{F_1(x) - F_1(x-\Delta)}{F_1(x)}\right] F_1(\tilde{\theta}|x)[\eta^g(\lambda^g-1)\theta + \eta^m(\lambda^m-1)\beta_D(\tilde{\theta})]. \tag{9}$$

Total expected news utility is the sum of all these incremental expected gain-loss utility terms for all prices from $\beta_D(\theta')$ to $\beta_D(\tilde{\theta})$. Notice that, since β is continuously increasing, as $\varepsilon \to 0$ we have $\Delta \to 0$ and $\frac{F_1(x)-F_1(x-\Delta)}{\Delta F_1(\theta')} \to f_1(x|\theta')$. Hence, the expected news utility, (9), approaches $-\left[\eta^g(\lambda^g - 1)\theta + \eta^m(\lambda^m - 1)\beta_D(\tilde{\theta})\right] \int_{\tilde{\theta}}^{\theta'} f_1(x|\theta')F_1(\tilde{\theta}|x)dx$. ■

Proof of Proposition 2: We prove Proposition 2 in three steps. First, using only necessary conditions, we derive a lower bound on the equilibrium bid. Then, we focus on sufficient conditions and show that the lower bound is indeed attainable and thus constitutes a PE. Finally, we show that the PPE is the PE that involves the lowest bid.

Step 1. In a symmetric equilibrium, a type-θ bidder prefers executing her plan of buying at price $\beta_D(\theta)$ over buying at price $\beta_D(\tilde{\theta})$ at any clock price $\beta_D(\theta') > \beta_D(\theta)$ if and only if $\Delta U(\tilde{\theta}|\theta,\theta') := F_1(\theta')[U(\tilde{\theta}|\theta,\theta') - U(\theta|\theta,\theta')] \leq 0$ for all $\theta' \geq \theta$ and all credible deviations $\tilde{\theta} \leq \theta'$. For any upward deviation $\tilde{\theta} \geq \theta$ we have

$$\begin{aligned}\Delta U(\tilde{\theta}|\theta,\theta') = \;& (1+\eta^g)\,[F_1(\tilde{\theta}) - F_1(\theta)]\theta + \eta^g(\lambda^g - 1)\theta \left(\int_\theta^{\theta'} F_1(\theta|x) f_1(x)dx - \int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x) f_1(x)dx \right) \\ & - (1+\eta^m\lambda^m)\,[F_1(\tilde{\theta})\beta_D(\tilde{\theta}) - F_1(\theta)\beta_D(\theta)] \\ & + \eta^m(\lambda^m - 1) \left(\beta_D(\theta) \int_\theta^{\theta'} F_1(\theta|x) f_1(x)dx - \beta_D(\tilde{\theta}) \int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x) f_1(x)dx \right). \qquad (10)\end{aligned}$$

Differentiating $\Delta U(\tilde{\theta}|\theta,\theta')$ with respect to $\tilde{\theta}$ yields

$$\begin{aligned}\frac{\partial \Delta U(\tilde{\theta}|\theta,\theta')}{\partial \tilde{\theta}} &= (1+\eta^g\lambda^g)\theta f_1(\tilde{\theta}) - (1+\eta^m)\beta_D(\tilde{\theta}) f_1(\tilde{\theta}) - (1+\eta^m\lambda^m) F_1(\tilde{\theta}) \beta'_D(\tilde{\theta}) \\ & -\eta^g(\lambda^g - 1)\theta \int_{\tilde{\theta}}^{\theta'} f_1(\tilde{\theta}|x) f_1(x)dx - \eta^m(\lambda^m - 1)\beta_D(\tilde{\theta}) \int_{\tilde{\theta}}^{\theta'} f_1(\tilde{\theta}|x) f_1(x)dx \\ & -\eta^m(\lambda^m - 1)\beta'_D(\tilde{\theta}) \int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x) f_1(x)dx. \qquad (11)\end{aligned}$$

In equilibrium, the bidder does not want to deviate upwards locally, i.e. $\lim_{\theta' \searrow \theta} \frac{\partial \Delta U(\tilde{\theta}|\theta,\theta')}{\partial \tilde{\theta}} \leq 0$ for $\tilde{\theta} = \theta'$, which leads to the necessary condition

$$(1+\eta^m\lambda^m)\, F_1(\theta)\beta'_D(\theta) + (1+\eta^m)\,\beta_D(\theta) f_1(\theta) \geq (1+\eta^g\lambda^g)\, f_1(\theta)\theta. \qquad (12)$$

Imposing that (12) holds with equality and solving the resulting differential equation using the initial condition $\beta_D(\underline{\theta})F_1(\underline{\theta}) = 0$ yields a lower bound on any PE bid; call this lower bound $\underline{\beta_D}$. This is expression (5) in the main text.

Step 2. We now show that $\underline{\beta_D}$ satisfies the sufficient conditions for a PE. For upward deviations, note that $\frac{\partial^2 \Delta U(\tilde{\theta}|\theta,\theta')}{\partial \tilde{\theta} \partial \theta'} < 0$. Hence, a deviation to $\tilde{\theta} > \theta$ is profitable at price $\underline{\beta_D}(\theta') > \underline{\beta_D}(\tilde{\theta})$ if and only if it is profitable at price $\underline{\beta_D}(\theta') = \underline{\beta_D}(\tilde{\theta})$. But for any $\tilde{\theta} = \theta' > \theta$, we have from (11) that the (right-)derivative is

$$\begin{aligned}\left.\frac{\partial \Delta U(\tilde{\theta}|\theta,\theta')}{\partial \tilde{\theta}}\right|_{\tilde{\theta}=\theta'} &= (1+\eta^g\lambda^g)\theta f_1(\tilde{\theta}) - (1+\eta^m\lambda^m)\, F_1(\tilde{\theta})\beta'_D(\tilde{\theta}) - (1+\eta^m)\,\beta_D(\tilde{\theta}) f_1(\tilde{\theta}) \\ &= (1+\eta^g\lambda^g)(\theta - \tilde{\theta}) f_1(\tilde{\theta}) < 0\end{aligned}$$

where the second equality follows since (12) holds with equality for type $\tilde{\theta}$.

Next, we show that the bidding function $\underline{\beta_D}$ is immune to downward deviations. Fix $\tilde{\theta} < \theta < \theta'$ and suppose that when the clock price is $\underline{\beta_D}(\theta')$ a type-θ bidder deviates to the plan of buying at price $\underline{\beta_D}(\tilde{\theta}) < \underline{\beta_D}(\theta)$. Such a deviation is only a concern if it is a credible plan; that is, if the bidder actually carries it through. This, however, is not the case. Indeed, since (12) holds with equality for a type-$\tilde{\theta}$ bidder, such bidder would be indifferent towards a local upward deviation around price $\beta_D(\tilde{\theta})$. As the right-hand side of (11) is strictly increasing in θ, and $\theta > \tilde{\theta}$, a type-θ bidder strictly benefits from such a local upward deviation at $\underline{\beta_D}(\tilde{\theta})$.

Step 3. In Step 1 we showed that $\underline{\beta_D}$ is the lowest PE bid. Moreover, notice that all other strictly increasing PE bidding functions that arise in a symmetric equilibrium lead to the same allocation of the good. In equilibrium, no bidder deviates from her strategy and therefore, using (3), it is easy to see that a bidder's equilibrium payoff decreases in her bid. Thus, $\underline{\beta_D}$ is every bidder type's preferred symmetric PE bidding function and hence the PPE. ■

Proof of Proposition 3: In the FPA, the equilibrium utility of bidder with type θ is

$$F_1(\theta)\left[\theta - \beta_I(\theta)\right] - \left[\eta^g\left(\lambda^g - 1\right)\theta + \eta^m\left(\lambda^m - 1\right)\beta_I(\theta)\right] F_1(\theta)\left[1 - F_1(\theta)\right].$$

In the Dutch auction, the equilibrium utility of bidder with type θ is

$$F_1(\theta)[\theta - \beta_D(\theta)] - [\eta^g(\lambda^g - 1)\theta + \eta^m(\lambda^m - 1)\beta_D(\theta)] \int_\theta^{\overline{\theta}} f_1(x)F_1(\theta|x)dx.$$

Suppose that $\beta_I(\theta) = \beta_D(\theta)$. The result then follows since $\int_\theta^{\overline{\theta}} f_1(x)F_1(\theta|x)dx > F_1(\theta)\left[1 - F_1(\theta)\right]$ for $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. ■

Proof of Proposition 4: Recall that $\Delta U(\tilde{\theta}|\theta, \theta') := F_1(\theta')[U(\tilde{\theta}|\theta, \theta') - U(\theta|\theta, \theta')]$. Hence, for any downward deviation $\tilde{\theta} < \theta$ we have

$$\begin{aligned}
\Delta U(\tilde{\theta}|\theta, \theta') &= \left(1 + \eta^g\lambda^g\right)\left[F_1(\tilde{\theta}) - F_1(\theta)\right]\theta + \eta^g(\lambda^g - 1)\theta\left(\int_\theta^{\theta'} F_1(\theta|x)f_1(x)dx - \int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x)f_1(x)dx\right) \\
&- \left(1 + \eta^m\right)\left[F_1(\tilde{\theta})\beta_D(\tilde{\theta}) - F_1(\theta)\beta_D(\theta)\right] \\
&+ \eta^m(\lambda^m - 1)\left(\beta_D(\theta)\int_\theta^{\theta'} F_1(\theta|x)f_1(x)dx - \beta_D(\tilde{\theta})\int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x)f_1(x)dx\right). \qquad (13)
\end{aligned}$$

Notice that the above expression differs from (10) only for its first and third gain-loss utility terms. Differentiating (13) with respect to $\tilde{\theta}$ yields

$$\frac{\partial \Delta U(\tilde{\theta}|\theta,\theta')}{\partial \tilde{\theta}} = (1+\eta^g\lambda^g+\eta^g(\lambda^g-1))\theta f_1(\tilde{\theta}) - (1+\eta^m-\eta^m(\lambda^m-1))\beta_D(\tilde{\theta})f_1(\tilde{\theta})$$
$$-(1+\eta^m)F_1(\tilde{\theta})\beta'_D(\tilde{\theta}) - \eta^g(\lambda^g-1)\theta\int_{\tilde{\theta}}^{\theta'} f_1(\tilde{\theta}|x)f_1(x)dx$$
$$-\eta^m(\lambda^m-1)\beta_D(\tilde{\theta})\int_{\tilde{\theta}}^{\theta'} f_1(\tilde{\theta}|x)f_1(x)dx - \eta^m(\lambda^m-1)\beta'_D(\tilde{\theta})\int_{\tilde{\theta}}^{\theta'} F_1(\tilde{\theta}|x)f_1(x)dx. \quad (14)$$

As in the PPE condition (12) binds, it follows that for a local downward deviation at the beginning of the auction

$$\left.\frac{\partial \Delta U(\tilde{\theta}|\theta,\overline{\theta})}{\partial \tilde{\theta}}\right|_{\tilde{\theta}=\theta} = \eta^g(\lambda^g-1)\theta f_1(\theta) + \eta^m(\lambda^m-1)\beta_D(\theta)f_1(\theta)$$
$$+\eta^m(\lambda^m-1)F_1(\theta)\beta'_D(\theta) - \eta^g(\lambda^g-1)\theta\int_{\theta}^{\overline{\theta}} f_1(\theta|x)f_1(x)dx$$
$$-\eta^m(\lambda^m-1)\beta_D(\theta)\int_{\theta}^{\overline{\theta}} f_1(\theta|x)f_1(x)dx - \eta^m(\lambda^m-1)\beta'_D(\theta)\int_{\theta}^{\overline{\theta}} F_1(\theta|x)f_1(x)dx.$$

This expression is positive for $\theta = \overline{\theta}$. For $\theta = \underline{\theta} > 0$ we have

$$\begin{aligned}
&\lim_{\theta\to\underline{\theta}} \left.\frac{\partial \Delta U(\tilde{\theta}|\theta,\overline{\theta})}{\partial \tilde{\theta}}\right|_{\tilde{\theta}=\theta} \\
= &\lim_{\theta\to\underline{\theta}} f_1(\theta)\left\{\eta^g(\lambda^g-1)\left[1+\ln\left(F_1(\theta)\right)\right]\theta + \eta^m(\lambda^m-1)\left[1+\ln\left(F_1(\theta)\right)\right]\beta_D(\theta)\right\} \\
&+ \lim_{\theta\to\underline{\theta}} \eta^m(\lambda^m-1)\beta'_D(\theta)\left[F_1(\theta) - \int_{\theta}^{\overline{\theta}} F_1(\theta|x)f_1(x)dx\right] \\
= &-\infty,
\end{aligned}$$

whereas for $\theta = \underline{\theta} = 0$ we have

$$\frac{\partial \Delta U(\tilde{\theta}|\underline{\theta},\overline{\theta})}{\partial \tilde{\theta}}|_{\tilde{\theta}=\underline{\theta}} = -\eta^m(\lambda^m-1)\beta'_D(\underline{\theta})\int_{\theta}^{\overline{\theta}} F_1(\theta|x)f_1(x)dx < 0.$$

Therefore, by continuity, there exists a $\theta^* \in (\underline{\theta},\overline{\theta})$ such that for a local downward deviation, $\frac{\partial \Delta U(\tilde{\theta}|\theta,\overline{\theta})}{\partial \tilde{\theta}}|_{\tilde{\theta}=\theta} < 0$ for all $\theta < \theta^*$. Hence, the stated result follows. ■

Proof of Proposition 5: We first show that $\beta_I(\underline{\theta}) \le \beta_D(\underline{\theta})$ and with strict inequality if $\underline{\theta} > 0$. Applying integration by parts to $\beta_I(\theta)$ and $\beta_D(\theta)$, it is easy to see that $\beta_I(\underline{\theta}) = \frac{1+\eta^g}{1+\eta^m\lambda^m}\underline{\theta}$ and $\beta_D(\underline{\theta}) = \frac{1+\eta^g\lambda^g}{1+\eta^m}\underline{\theta}$. Thus, the claim follows.

Next, observe that the bid in the FPA, (2), is bounded from above by

$\int_{\underline{\theta}}^{\theta} \frac{1+\eta^g\lambda^g}{F_1(\theta)[1+\eta^m\lambda^m]} e^{\frac{\eta^m(\lambda^m-1)}{1+\eta^m\lambda^m}[F_1(\theta)-F_1(s)]} f_1(s)sds$. Thus, it is sufficient to show that

$$\left[\frac{F_1(\theta)}{F_1(s)}\right]^{\frac{\eta^m(\lambda^m-1)}{1+\lambda^m\eta^m}} \geq e^{\frac{\eta^m(\lambda^m-1)[F_1(\theta)-F_1(s)]}{1+\eta^m\lambda^m}}$$

which is equivalent to

$$\begin{aligned} & \ln(F_1(\theta)) - \ln(F_1(s)) \geq F_1(\theta) - F_1(s) \\ \Leftrightarrow \quad & \ln(F_1(\theta)) - F_1(\theta) \geq \ln(F_1(s)) - F_1(s). \end{aligned} \tag{15}$$

As $\theta \geq s$, (15) holds if $\ln(F_1(x)) - F_1(x)$ is increasing in x. This is the case since $\frac{f_1(x)}{F_1(x)} - f_1(x) = f_1(x)\frac{1-F_1(x)}{F_1(x)} \geq 0$. ■

Proof of Proposition 6: In the Dutch auction, for $x < \theta$ we have that $\left[\frac{F_1(\theta)}{F_1(x)}\right]^{\frac{\eta^m(\lambda^m-1)}{1+\eta^m\lambda^m}} > 1$. When gain-loss utility is the same in both dimensions, this implies that

$$\beta_D(\theta) > \int_{\underline{\theta}}^{\theta} \frac{1+\eta^g\lambda^g}{F_1(\theta)\,(1+\eta^m\lambda^m)} x f_1(x)dx = \int_{\underline{\theta}}^{\theta} \frac{x f_1(x)}{F_1(\theta)} dx. \tag{16}$$

Hence, we have overbidding in the Dutch auction.

For the FPA, notice first that

$$\lim_{\theta \to \underline{\theta}} \beta_I(\theta) = \int_{\underline{\theta}}^{\theta} \frac{1+\eta}{F_1(\theta)\,(1+\eta\lambda)} x f_1(x)dx.$$

Hence, we have underbidding for the lowest type.

Next, we derive a condition on the slope of $\beta_I(\theta)$. For β_I to be an equilibrium, the first-order necessary condition of (1) for a local upward deviation yields

$$\begin{aligned} \left.\frac{\partial U(\tilde{\theta}|\theta)}{\partial \tilde{\theta}}\right|_{\tilde{\theta}=\theta} &= f_1(\theta)\left\{1+\eta^g\lambda^g F_1(\theta) + \eta^g\left[1-F_1(\theta)\right]\right\}\theta \\ &\quad - f_1(\theta)\left\{1+\eta^m F_1(\theta) + \eta^m\lambda^m\left[1-F_1(\theta)\right]\right\}\beta_I(\theta) \\ &\quad - \beta_I'(\theta)F_1(\theta)\left\{1+\eta^m\lambda^m\left[1-F_1(\theta)\right] + \eta^m\lambda^m F_1(\theta)\right\} \\ &\leq 0. \end{aligned}$$

This inequality can be re-arranged as follows:

$$\beta_I'(\theta) \leq -\frac{f_1(\theta)}{F_1(\theta)}\left\{\frac{1+\eta^m F_1(\theta)+\eta^m\lambda^m\left[1-F_1(\theta)\right]}{1+\eta^m\lambda^m}\right\}\beta_I(\theta) + \frac{f_1(\theta)}{F_1(\theta)}\left\{\frac{1+\eta^g\lambda^g F_1(\theta)+\eta^g\left[1-F_1(\theta)\right]}{1+\eta^m\lambda^m}\right\}\theta.$$

Balzer and Rosato (2021) show that the PPE in (2) is obtained as the solution to the differential equation when the above inequality binds. When gain-loss utility is the same in both dimensions, this equation becomes

$$\beta_I'(\theta) = -\frac{f_1(\theta)}{F_1(\theta)}\left\{\frac{1+\eta F_1(\theta)+\eta\lambda\left[1-F_1(\theta)\right]}{1+\eta\lambda}\right\}\beta_I(\theta) + \frac{f_1(\theta)}{F_1(\theta)}\left\{\frac{1+\eta\lambda F_1(\theta)+\eta\left[1-F_1(\theta)\right]}{1+\eta\lambda}\right\}\theta.$$

Call $\beta_I^{rn}(\theta)$ the equilibrium bidding function for the risk-neutral benchmark. If $\beta_I(\theta) < \beta_I^{rn}$ for all types, then the claim is trivially satisfied for $p = 1$. Otherwise, there exists a θ where β_I crosses β_I^{rn} from below; i. e., $\beta_I(\theta) = \beta_I^{rn}(\theta)$ and $\beta_I'(\theta) > (\beta_I^{rn})'(\theta)$. This implies

$$\frac{\eta(\lambda-1)\left\{F_1(\theta)\beta_I^{rn}(\theta) - \left[1-F_1(\theta)\right]\theta\right\}}{1+\eta\lambda} > 0. \tag{17}$$

Since $\beta_I^{rn}(\theta) < \theta$ it follows that $F_1(\theta) = F(\theta)^{n-1} > 0.5$. Hence, at least all types with $F(\theta) < (0.5)^{\frac{1}{N-1}}$ underbid. Moreover, since the left-hand side in (17) is increasing in θ, we have that $\beta_I'(\theta_0) > (\beta_I^{rn})'(\theta_0)$ for all $\theta_0 > \theta$. Hence, all types above θ_0 overbid. ■

Proof of Corollary 2: The inequality $Dutch > FPA$ follows from Corollary 1. The equality $FPA = SPA$ follows from the results in Balzer and Rosato (2021). Finally, the inequality $SPA > English$ follows from von Wangenheim (2020). ■

In the proof of Proposition 7, we use the following ancillary result.

Lemma 2. *Let $\alpha \in [0,1]$ be the weight on a bidder's most recent beliefs in her reference point as the Dutch auction unfolds. Then, for $\theta' > \theta$ the expected utility at price $\beta(\theta')$ of a type-θ bidder from an upward deviation from $\beta_D(\theta)$ to $\beta_D(\tilde{\theta})$ is given by*

$$U_\alpha(\tilde{\theta}|\theta,\theta') = \alpha U(\tilde{\theta}|\theta,\theta') + (1-\alpha)U_{Sticky}(\tilde{\theta}|\theta,\theta'), \tag{18}$$

where

$$\begin{aligned} U_{Sticky}(\tilde{\theta}|\theta,\theta') \quad = \quad & F_1(\tilde{\theta}|\theta')\left[\theta - \beta_D(\tilde{\theta})\right] - \eta^g\lambda^g\left[1-F_1(\tilde{\theta}|\theta')\right]F_1(\theta)\theta + \eta^g F_1(\tilde{\theta}|\theta')\left[1-F_1(\theta)\right]\theta \\ & +\eta^m\left[1-F_1(\tilde{\theta}|\theta')\right]F_1(\theta)\beta_D(\theta) - \eta^m\lambda^m F_1(\tilde{\theta}|\theta')\left[1-F_1(\theta)\right]\beta_D(\tilde{\theta}) \\ & -\eta^m\lambda^m F_1(\tilde{\theta}|\theta')F_1(\theta)\left[\beta_D(\tilde{\theta}) - \beta_D(\theta)\right]. \end{aligned} \tag{19}$$

Notice also that U_{Sticky} coincides with the expected utility in (1) for an upward deviation in the FPA after replacing the ex-ante probability $F_1(\tilde{\theta})$ to win with a bid of $\beta(\tilde{\theta})$ with the updated probability to win $F_1(\tilde{\theta}|\theta')$ at a given price $\beta(\theta')$.

Proof of Lemma 2: We separately analyze the three sources of gain-loss utility:

(i) Under strategy $\beta_D(\theta)$, at price $\beta_D(\theta')$ the winning probability is $F_1(\theta|\theta')$; hence the reference point puts a weight of $(1-\alpha)F_1(\theta) + \alpha F_1(\theta|\theta')$ on winning. A deviation to $\beta_D(\tilde{\theta})$ updates the reference point to $(1-\alpha)F_1(\theta) + \alpha F_1(\tilde{\theta}|\theta')$ and hence induces gain-loss utility

$$\alpha \left\{ \eta^g \theta \left[F_1(\tilde{\theta}|\theta') - F_1(\theta|\theta') \right] - \eta^m \lambda^m [F_1\left(\tilde{\theta}|\theta'\right) \beta_D(\tilde{\theta}) - F_1(\theta|\theta') \beta_D(\theta)] \right\}. \tag{20}$$

(ii) Next, we look at expected news utility $\mathbb{E}[N_\alpha(\tilde{\theta}|\theta,\theta')]$ from updates to the reference point that take place during the auction. As in the proof of Lemma 1, consider a price drop from $\beta_D(x)$ to $\beta_D(x-\Delta)$. With probability of $1 - F_1(x-\Delta|x)$ an opponent stops the clock; in this case, the bidder loses the auction and experiences gain-loss utility equal to

$$(1-\alpha)F_1(\theta) \left[\eta^m \beta_D(\theta) - \eta^g \lambda^g \theta\right] + \alpha F_1(\tilde{\theta}|x) \left[\eta^m \beta_D(\tilde{\theta}) - \eta^g \lambda^g \theta\right]. \tag{21}$$

With probability $F_1(x-\Delta|x)$, no opponent stops the clock, and the bidder updates her belief about winning to $F_1(\tilde{\theta}|x-\Delta)$ and experiences gain-loss utility equal to

$$\alpha \left[F_1(\tilde{\theta}|x-\Delta) - F_1(\tilde{\theta}|x)\right] \left[\eta^g \theta - \eta^m \lambda^m \beta_D(\tilde{\theta})\right]. \tag{22}$$

Notice that combining expression (22) together with the second term in (21) yields exactly α times the expected gain-loss utility from the incremental update in the baseline model for the Dutch auction as calculated in the proof of Lemma 1. Hence, as ε approaches zero, the total expected gain-loss utility from all incremental updates from $\beta_D(\theta')$ to $\beta_D(\tilde{\theta})$ approaches

$$\left[1 - F_1(\tilde{\theta}|\theta')\right] (1-\alpha)F_1(\theta) \left[\eta^m \beta_D(\theta) - \eta^g \lambda^g \theta\right] + \alpha \mathbb{E}\left[N(\tilde{\theta}|\theta,\theta')\right]. \tag{23}$$

(iii) With probability $F_1(\tilde{\theta}|\theta')$, the bidder wins the auction at price $\beta_D(\tilde{\theta})$. While her beliefs have updated to winning with certainty at price $\beta_D(\tilde{\theta})$ when she does so, her reference point only fully adjusts when it "sinks in". Comparing the reference point of belief of $F_1(\theta)$ to win and pay $\beta_D(\theta)$ with the outcome to win and pay $\beta_D(\tilde{\theta})$ hence induces, from the perspective at price $\beta_D(\theta')$, an expected gain-loss utility of

$$F_1(\tilde{\theta}|\theta')(1-\alpha) \left\{ [1 - F_1(\theta)]\, \eta^g \theta - [1 - F_1(\theta)]\, \eta^m \lambda^m \beta_D(\tilde{\theta}) - F_1(\theta) \eta^m \lambda^m \left[\beta_D(\tilde{\theta}) - \beta_D(\theta)\right] \right\}. \tag{24}$$

Finally, by putting all three sources of gain-loss utility together with classical material utility $(1-\alpha)F_1(\tilde{\theta}|\theta') \left[\theta - \beta_D(\tilde{\theta})\right] + \alpha F_1(\tilde{\theta}|\theta') \left[\theta - \beta_D(\tilde{\theta})\right]$, we obtain the formula for $U(\tilde{\theta}|\theta,\theta')$ as in (18). ■

Equipped with the above result, we are now ready to prove Proposition 7.

Proof of Proposition 7: As in the proof of Proposition 2, we define

$$\Delta U_\alpha(\tilde{\theta}|\theta,\theta') := F_1(\theta')\left[U_\alpha(\tilde{\theta}|\theta,\theta') - U_\alpha(\theta|\theta,\theta')\right].$$

In a symmetric equilibrium, a type-θ bidder prefers executing her plan of buying at price $\beta_D(\theta)$ over buying at price $\beta_D(\tilde{\theta})$ at any clock price $\beta_D(\theta') > \beta_D(\theta)$ if and only if $\Delta U_\alpha(\tilde{\theta}|\theta,\theta') \leq 0$ for all $\theta' \geq \theta$ and all credible deviations $\tilde{\theta} \leq \theta'$. By equations (18) and (19), for any such upward deviations, we have

$$\begin{aligned}\Delta U_\alpha(\tilde{\theta}|\theta,\theta') &= \alpha \Delta U(\tilde{\theta}|\theta,\theta') \\ &\quad + (1-\alpha)\{(1+\eta^g)\,[F_1(\tilde{\theta}) - F_1(\theta)]\theta + \eta^g(\lambda^g-1)\theta F_1(\theta)[F_1(\tilde{\theta}) - F_1(\theta)] \qquad (25)\\ &\quad - (1+\eta^m\lambda^m[1-F_1(\theta)])\,[F_1(\tilde{\theta})\beta_D(\tilde{\theta}) - F_1(\theta)\beta_D(\theta)] \\ &\quad - \eta^m F_1(\theta)\beta_D(\theta)[F_1(\tilde{\theta}) - F_1(\theta)] - \eta^m\lambda^m F_1(\tilde{\theta})F_1(\theta)[\beta_D(\tilde{\theta}) - \beta_D(\theta)]\}. \qquad (26)\end{aligned}$$

Differentiation with respect to $\tilde{\theta}$ yields

$$\begin{aligned}&\frac{\partial \Delta U_\alpha(\tilde{\theta}|\theta,\theta')}{\partial\tilde{\theta}} = \alpha\frac{\partial \Delta U(\tilde{\theta}|\theta,\theta')}{\partial\tilde{\theta}} \\ &+(1-\alpha)\{[1+\eta^g+\eta^g(\lambda^g-1)F_1(\theta)]\theta f_1(\tilde{\theta}) - (1+\eta^m\lambda^m)f_1(\tilde{\theta})\beta_D(\tilde{\theta}) \\ &-(1+\eta^m\lambda^m)F_1(\tilde{\theta})\beta'_D(\tilde{\theta}) + \eta^m(\lambda^m-1)F_1(\theta)\beta_D(\theta)f_1(\tilde{\theta})\}. \qquad (27)\end{aligned}$$

Exploiting that the integrals in equation (11) vanish for $\theta' = \theta$, we obtain the following necessary condition for equilibrium:

$$\begin{aligned}0 &\geq \lim_{\theta'\searrow\theta} \frac{\partial \Delta U_\alpha(\tilde{\theta}|\theta,\theta')}{\partial\tilde{\theta}}\Big|_{\tilde{\theta}=\theta'} \\ &= [1+\eta^g+\eta^g(\lambda^g-1)F_1(\theta)]\theta f_1(\theta) \\ &- (1+\eta^m\lambda^m)\beta'_D(\theta)\,F_1(\theta) - [1+\eta^m\lambda^m - \eta^m(\lambda^m-1)F_1(\theta)]\,\beta_D(\theta)f_1(\theta) \\ &+ \alpha\,[1-F_1(\theta)]\,[\eta^g(\lambda^g-1)\theta + \eta^m(\lambda^m-1)\beta_D(\theta)]\,f_1(\theta). \qquad (28)\end{aligned}$$

An argument analogous to the one in the proof of Proposition 2 shows that the solution to the differential equation obtained by making condition (28) bind is the PPE for the Dutch auction. It is easy to verify that the slope of this differential equation

$$\beta_D'(\theta) = \frac{[1+\eta^g+\eta^g(\lambda^g-1)F_1(\theta)]\theta f_1(\theta) - [1+\eta^m\lambda^m - \eta^m(\lambda^m-1)F_1(\theta)]\,\beta_D(\theta)f_1(\theta)}{(1+\eta^m\lambda^m)\,F_1(\theta)} + \alpha\frac{[1-F_1(\theta)]\,[\eta^g(\lambda^g-1)\theta+\eta^m(\lambda^m-1)\beta_D(\theta)]\,f_1(\theta)}{(1+\eta^m\lambda^m)\,F_1(\theta)}$$

is increasing in α. Hence, given the same initial condition $\beta_D(\underline{\theta})F_1(\underline{\theta})=0$ it follows that $\beta_D(\theta)$ is increasing in α for all $\theta > \underline{\theta}$. ■

Proof of Proposition 8: First, we re-write Equation (8) as

$$U_\theta(\tilde{\theta},\theta') = F_1(\tilde{\theta}|\theta')[\theta-\beta_D(\tilde{\theta})] - \Lambda^g\theta F_1(\tilde{\theta}|\theta')\left[1-F_1(\tilde{\theta}|\theta')\right] - \Lambda^m\beta_D(\tilde{\theta})F_1(\tilde{\theta}|\theta')[1-F_1(\tilde{\theta}|\theta')].$$

Notice that this utility function is continuously differentiable in $\tilde{\theta}$ for any $\theta' > \tilde{\theta}$. A necessary condition for a CPE in the FPA is that for a type-θ bidder, $\theta=\tilde{\theta}$ maximizes $U_\theta(\tilde{\theta},\theta')$ at the beginning of the auction, i.e. for $\theta'=\overline{\theta}$. Differentiating $U_\theta(\tilde{\theta},\overline{\theta})$ with respect to $\tilde{\theta}$ and evaluating the resulting first-order condition at $\tilde{\theta}=\theta$ yields a differential equation whose solution provides us with the equilibrium bidding strategy. The symmetric CPE bidding strategy in the FPA, borrowed from Lange and Ratan (2010), is given by

$$\beta_I(\theta) = \int_{\underline{\theta}}^{\theta} \frac{1+\Lambda^g[2F_1(x)-1]}{F_1(\theta)\left\{1+\Lambda^m\left[1-F_1(\theta)\right]\right\}} x f_1(x)dx. \tag{29}$$

For the Dutch auction, strategy $\beta_D(\theta)$ for type-θ is credible if and only if $\theta=\tilde{\theta}$ maximizes $U_\theta(\tilde{\theta},\theta')$ for $\theta'\to\theta$. Since

$$\frac{\partial U_\theta(\tilde{\theta},\theta')}{\partial\tilde{\theta}} = -F_1(\tilde{\theta}|\theta')\beta_D'(\tilde{\theta})\left\{1-\Lambda^m\left[1-F_1(\tilde{\theta}|\theta')\right]\right\} + f_1(\tilde{\theta}|\theta')\left\{\theta-\beta_D(\tilde{\theta})-\Lambda^g\theta\left[1-2F_1(\tilde{\theta}|\theta')\right]-\Lambda^m\beta_D(\tilde{\theta})\left[1-2F_1(\tilde{\theta}|\theta')\right]\right\},$$

evaluating the necessary condition at $\tilde{\theta}=\theta'=\theta$ yields the differential equation

$$-\beta_D'(\theta)+\frac{f_1(\theta)}{F_1(\theta)}\left[\theta-\beta_D(\theta)+\Lambda^g\theta+\Lambda^m\beta_D(\theta)\right]=0.$$

Using the initial condition $\beta_D(\underline{\theta})F_1(\underline{\theta})=0$, the solution to the above differential equation provides us with the unique equilibrium candidate:

$$\beta_D(\theta) = \frac{1}{F_1(\theta)^{1-\Lambda^m}}\int_{\underline{\theta}}^{\theta}\frac{1+\Lambda^g}{F_1(x)^{\Lambda^m}}xf_1(x)dx. \tag{30}$$

Since the equilibrium candidate is the only time-consistent candidate, it only remains to show sufficiency (i.e., global deviations to $\tilde{\theta} < \theta$ at $\theta = \theta'$). Suppose that when the clock price is $\beta_D(\theta)$, a type-θ bidder deviates to the plan of buying at price $\beta_D(\tilde{\theta}) < \beta_D(\theta)$. Such a deviation is only a concern if it is a credible plan; that is, if the bidder actually carries it through. This, however, is not the case. Indeed, since for $\theta = \tilde{\theta}$ we have $\frac{\partial U_\theta(\tilde{\theta},\tilde{\theta})}{\partial \tilde{\theta}} = 0$, a type-$\tilde{\theta}$ bidder would be indifferent towards a local upward deviation around price $\beta_D(\tilde{\theta})$. Since $\Lambda^g \leq 1$, $\frac{\partial U_\theta(\tilde{\theta},\theta')}{\partial \tilde{\theta} \partial \theta} = f_1(\tilde{\theta}|\theta') \left[(1-\Lambda^g) + \Lambda^g 2F_1(\tilde{\theta}|\theta')\right] > 0$ for any θ' and in particular for $\theta' = \tilde{\theta}$; hence, a type-θ bidder strictly benefits from such a local upward deviation at $\beta_D(\tilde{\theta})$. This establishes (30) as the unique SCPE in the Dutch auction.

Finally, in order to establish the revenue ranking, we need to show that

$$\beta_D(\theta) = F_1(\theta)^{\Lambda^m - 1} \int_{\underline{\theta}}^{\theta} \frac{(1+\Lambda^g)\, x f_1(x)}{F_1(x)^{\Lambda^m}} dx \geq \int_{\underline{\theta}}^{\theta} \frac{\left[(1-\Lambda^g) + 2\Lambda^g F_1(x)\right] x f_1(x)}{F_1(\theta)\left\{1 + \Lambda^m \left[1 - F_1(\theta)\right]\right\}} dx = \beta_I(\theta). \quad (31)$$

To establish (31), it is sufficient to show that

$$F_1(\theta)^{\Lambda^m} \int_{\underline{\theta}}^{\theta} \frac{1}{F_1(x)^{\Lambda^m}} f_1(x) x dx \geq \int_{\underline{\theta}}^{\theta} x f_1(x) dx,$$

which is equivalent to

$$\Leftrightarrow \int_{\underline{\theta}}^{\theta} \frac{x f_1(x)}{F_1(x)^{\Lambda^m}} dx \geq \int_{\underline{\theta}}^{\theta} \frac{x f_1(x)}{F_1(\theta)^{\Lambda^m}} dx.$$

The result then follows since $F_1(x)^{\Lambda^m} < F_1(\theta)^{\Lambda^m}$ for $\theta > x$. ■

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