

# Persuasion in Global Games with Application to Stress Testing

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# Motivation

- Coordination: central to many socio-economic environments
- Damages to society of mkt coordination on undesirable actions can be severe
  - Monte dei Paschi di Siena (MPS)
  - creditors + speculators with heterogenous beliefs about size of *nonperforming loans*
  - default by MPS: major crisis in Eurozone (and beyond)
- Government intervention
  - limited by legislation passed in 2015
- Persuasion (stress test design): **instrument of last resort**

# Questions

- Structure of optimal stress tests?
  - What information should be passed on to mkt?
- “Right” notion of transparency?
- Optimality of
  - pass/fail policies
  - monotone rules
- Benefits to discriminatory disclosures?
- Properties of persuasion in global games?

## Related literature

- **Persuasion and Information design:** Myerson (1986), Aumann and Maschler (1995), Calzolari and Pavan (2006,a,b), Glazer and Rubinstein (2004, 2012), Rayo and Segal (2010), Kamenica and Gentzkow (2011), Ely (2016), Bergemann and Morris (2017), Lipnowski and Mathevet (2017), Mathevet, Pearce, Stacchetti (2017), Dworzak and Martini (2019), Dworzak and Pavan (2019)...
- **Persuasion in Games:** Alonso and Camara (2013), Barhi and Guo (2016), Taneva (2016), Mathevet, Perego, Taneva (2019)...
- **Persuasion with ex-ante heterogeneous receivers:** Bergemann and Morris (2016), Kolotilin et al (2016), Laclau and Renou (2017), Chan et al (2016), Basak and Zhou (2017), Che and Horner (2017), Doval and Ely (2017), Guo and Shmaya (2017)...
- **Discrimination and “Divide and Conquer”:** Segal (2006), Wang (2015), Yamashita (2016)...
- **Financial Regulation and Stress Test Design:** Goldstein and Leitner (2015), Goldstein and Sapra (2014), Alvarez and Barlevy (2017), Bouvard et al. (2015), Goldstein and Huang (2016), Williams (2017), Inostroza (2019)...
- **Global Games w. Endogenous Info:** Angeletos, Hellwig and Pavan (2006, 2007), Angeletos and Pavan (2013), Edmond (2013), Iachan and Nenov (2015), Denti (2016), Yang (2016), Morris and Yang (2019), Li et al (2019), Morris, Oyama, Takahashi (2019)...

# Plan

- Basic Model
- Perfect Coordination Property
- Pass/Fail Policies
- (Non-)Monotone Policies
- General Model and Results
- Micro-foundations

# Stylized Global Game of Regime Change

- Policy maker (PM)
- Agents  $i \in [0, 1]$
- Actions

$$a_i = \begin{cases} 1 & (\textit{pledge}) \\ 0 & (\textit{not pledge}) \end{cases}$$

- $A \in [0, 1]$  : aggregate pledge
- Default outcome:  $r \in \{0, 1\}$ , with  $r = 0$  in case of default
- Default rule

$$r = \begin{cases} 0 & \textit{if } A < 1 - \theta \\ 1 & \textit{if } A \geq 1 - \theta \end{cases}$$

- “fundamentals”  $\theta$  parametrize liquidity, performing loans, etc.
- $\theta$  drawn from an abs. continuous cdf  $F$ , with smooth density  $f$  strictly positive over  $\mathbb{R}$

# Stylized Global Game of Regime Change

- PM's payoff

$$U^P(\theta, A) = \begin{cases} W & \text{if } r = 1 \\ L < W & \text{if } r = 0 \end{cases}$$

- Agents' payoff from not pledging (safe action) normalized to zero
- Agents' payoff from *pledging*

$$u = \begin{cases} g > 0 & \text{if } r = 1 \\ b < 0 & \text{if } r = 0 \end{cases}$$

- Supermodular game w. dominance regions:  $(-\infty, 0)$  and  $[1, +\infty)$

- $\mathbf{x} \equiv (x_i)_{i \in [0,1]} \in \mathbf{X}$ : signal profile with each

$$x_i \sim p(\cdot | \theta)$$

i.i.d., given  $\theta$

- $\mathbf{X}(\theta) \subset \mathbb{R}^{[0,1]}$  : collection of signal profiles consistent with  $\theta$
- $x_i = \theta + \sigma \xi_i$  with  $\xi_i \sim N(0, 1)$

# Disclosure Policies (Stress Tests)

- *Disclosure policy*  $\Gamma = (S, \pi)$ 
  - $S$ : set of scores/grades/disclosures
  - $\pi(\theta)$  : score given to bank of type  $\theta$

# Timing

- 1 PM announces  $\Gamma = (S, \pi)$  and commits to it
- 2  $(\theta, \mathbf{x})$  realized
- 3  $\pi(\theta)$  publicly announced
- 4 Agents simultaneously choose whether or not to pledge
- 5 Default outcome and payoffs

# Solution Concept: MARP

- Robust/adversarial approach
- PM does not trust her ability to coordinate mkt on her favorite course of action
- **Most Aggressive Rationalizable Profile (MARP):**  
minimizes PM's payoff across all profiles surviving *iterated deletion of interim strictly dominated strategies* (IDISDS)
- $a^\Gamma \equiv (a_i^\Gamma)_{i \in [0,1]}$ : MARP consistent with  $\Gamma$

# Perfect Coordination Property [PCP]

## Definition 1

$\Gamma = \{\mathcal{S}, \pi\}$  satisfies **PCP** if, for any  $\theta, \mathbf{x} \in \mathbf{X}(\theta)$ ,  $i, j \in [0, 1]$ ,  
 $a_i^\Gamma(x_i, \pi(\theta)) = a_j^\Gamma(x_j, \pi(\theta))$ , where  $a^\Gamma \equiv (a_i^\Gamma)_{i \in [0,1]}$  is MARP consistent with  $\Gamma$

# Perfect Coordination Property [PCP]

## Theorem 1

*Given any (regular)  $\Gamma$ , there exists (regular)  $\Gamma^*$  satisfying **PCP** and s.t., for any  $\theta$ , default probability under  $\Gamma^*$  same as under  $\Gamma$ .*

- Regularity: MARP well defined

# Perfect Coordination Property [PCP]

- Policy  $\Gamma^* = (S^*, \pi^*)$  removes any **strategic uncertainty**
- It preserves **structural uncertainty**
- Under  $\Gamma^*$ , agents know actions all other agents take but not *what beliefs rationalize such actions*
- Inability to predict beliefs that rationalize other agents' actions essential to minimization of risk of default
- “Right” form of transparency
  - conformism in beliefs about mkt response
  - ...not in beliefs about “fundamentals”

# PCP: Proof sketch

- Let  $r^\Gamma(\theta) \in \{0, 1\}$  be default outcome at  $\theta$  when agents play according to  $a^\Gamma$
- Let  $\Gamma^* = \{S^*, \pi^*\}$  be s.t.  $S^* = S \times \{0, 1\}$  and  
 $\pi^*(\theta) = (\pi(\theta), r^\Gamma(\theta))$
- **Key step:** given  $s^* = (\pi(\theta), 1) \Rightarrow$  MARP under  $\Gamma^*$  less aggressive than MARP under  $\Gamma$  given  $s = \pi(\theta)$ 
  - At any round  $n$  of IDIDS

$$a_{i,(n)}^\Gamma(x_i, \pi(\theta)) = 1 \Rightarrow a_{i,(n)}^{\Gamma^*}(x_i, (\pi(\theta), 1)) = 1, \quad \forall i, \forall x_i$$

- Given  $s^* = (\pi(\theta), 1) \Rightarrow$  each agent pledges irrespective of  $x_i$
- Given  $s^* = (\pi(\theta), 0) \Rightarrow$  each agent refrains from pledging, irrespective of  $x_i$
- For all  $\theta$ , prob. of default under  $\Gamma^*$  same as under  $\Gamma$

(formal proof)

Optimal policy combines:

- public **Pass/Fail** announcement
  - eliminate strategic uncertainty
- additional disclosures necessary to guarantee that, when  $r = 1$  is announced (i.e., when bank passed the test), all agents pledge under MARP

# Pass/fail Policies

- Can signals other than  $r = 0, 1$  be dispensed with?

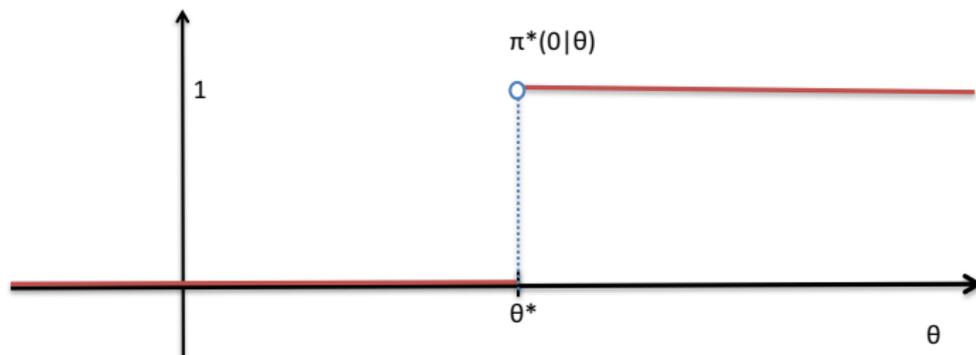
## Theorem 2

*Given any policy  $\Gamma$  satisfying PCP, there exists **binary policy**  $\Gamma^* = (\{0, 1\}, \pi^*)$  also satisfying PCP and s.t., for any  $\theta$ , prob of default under  $\Gamma^*$  same as under  $\Gamma$ .*

- MARP in threshold strategies: signals other than regime outcome can be dropped (averaging over  $s$ ) without affecting incentives
- Result hinges on Log-SM of  $p(x|\theta) \Rightarrow$  MLRP
  - co-movement between state  $\theta$  and beliefs

(Example)

# Optimality of Monotone Tests



# Sub-optimality of Monotone Tests

- Let  $\theta^{MS} \in (0, 1)$  be implicitly defined by

$$\int_0^1 u(\theta^{MS}, l) dl = 0 \quad (1)$$

- Let  $D^\Gamma \equiv \{(\underline{\theta}_i, \bar{\theta}_i) : i = 1, \dots, N\}$  be partition of  $[\underline{\theta}, \theta^{MS}]$  induced by  $\Gamma$  with

$$\Delta(\Gamma) \equiv \max_{i=1, \dots, N} |\bar{\theta}_i - \underline{\theta}_i|$$

denoting its *mesh*.

## Theorem 3

There exists  $\bar{\sigma} > 0$  and  $\mathcal{E} : (0, \bar{\sigma}] \rightarrow \mathbb{R}_+$ , with  $\lim_{\sigma \rightarrow 0^+} \mathcal{E}(\sigma) = 0$ , s.t. for any  $\sigma \in (0, \bar{\sigma}]$ , following is true: given any binary policy  $\Gamma$  satisfying PCP and s.t.  $\Delta(\Gamma) > \mathcal{E}(\sigma)$ , there exists another binary policy  $\Gamma^*$  with  $\Delta(\Gamma^*) < \mathcal{E}(\sigma)$  that also satisfies PCP and yields policy maker payoff strictly higher than  $\Gamma$ .

# Sub-optimality of Monotone Tests

- Small  $\sigma$ : PM cannot give pass to all  $\theta \in [\theta', \theta''] \subset [0, \theta^{MS}]$  with  $|\theta'' - \theta'|$  large
  - when  $\theta \in [\theta', \theta'']$ , most agents receive signals  $x_i \in [\theta', \theta'']$
  - if  $\pi(\theta) = 1$  all  $\theta \in [\theta', \theta'']$ , irrespective of shape of  $\pi$  outside  $[\theta', \theta'']$ , most agents with  $x_i \in [\theta', \theta'']$  assign high prob to  $\theta \in [\theta', \theta'']$ , to other agents assigning high prob to  $\theta \in [\theta', \theta'']$ , and so on
  - rationalizable for such agents to refrain from pledging

# Sub-optimality of Monotone Tests

- Next suppose  $\pi(\theta) = 0$  for all  $\theta \in [\theta', \theta''] \subset [0, \theta^{MS}]$  with  $|\theta'' - \theta'|$  large
  - suppose PM passes  $\theta \in \left[ \frac{\theta' + \theta''}{2}, \frac{\theta' + \theta''}{2} + \xi \right]$  and fails  $\theta \in \left[ \theta'' + \frac{\delta}{2}, \theta'' + \delta \right]$ , with  $\xi$  and  $\delta$  small chosen s.t ex-ante prob of passing same as under  $\Gamma$
  - agents with signals  $x \notin \left[ \frac{\theta' + \theta''}{2}, \frac{\theta' + \theta''}{2} + \xi \right] \cup \left[ \theta'' + \frac{\delta}{2}, \theta'' + \delta \right]$  have stronger incentives to pledge
  - incentives to pledge for agents with signals

$$x \in \left[ \frac{\theta' + \theta''}{2}, \frac{\theta' + \theta''}{2} + \xi \right] \cup \left[ \theta'' + \frac{\delta}{2}, \theta'' + \delta \right]$$

may be smaller; However, because for such individuals pledging was unique rationalizable action under  $\Gamma$ , provided  $\sigma, \xi, \delta$  are small, pledging continues to be unique rationalizable action under new policy

- PM can then pass also some types to the left of  $(\theta' + \theta'')/2$  while guaranteeing that all agents continue to pledge

# General Model

- General  $P(x|\theta)$
- Stochastic  $\Gamma$ :  $\pi : \Theta \rightarrow \Delta(S)$
- Default iff  $R(\theta, A, z) \leq 0$ 
  - $z$  drawn from  $Q_\theta$ : residual uncertainty
- PM's payoff

$$\hat{U}^P(\theta, A, z) = \begin{cases} \hat{W}(\theta, A, z) & \text{if } r = 1 \\ \hat{L}(\theta, A, z) & \text{if } r = 0 \end{cases}$$

- Agents' payoffs

$$\hat{u}(\theta, A, z) = \begin{cases} \hat{g}(\theta, A, z) & \text{if } r = 1 \\ \hat{b}(\theta, A, z) & \text{if } r = 0 \end{cases}$$

- Expected payoff differential:  $u(\theta, A)$

# General Model

- For any common posterior  $G \in \Delta(\Theta)$ , let  $\bar{U}^G(x)$  be expected payoff differential of agent with signal  $x$  who expects all other agents to pledge iff their signal exceeds  $x$
- Let  $\xi^G$  be the largest solution to  $\bar{U}^G(x) = 0$ 
  - $\xi^G = +\infty$  if  $\bar{U}^G(x) < 0$  for all  $x$
  - $\xi^G = -\infty$  if  $\bar{U}^G(x) > 0$  for all  $x$

# General Model

- Finally, let

$$\theta^G \equiv \inf \{ \theta : u(\theta, 1 - P(\xi^G | \theta)) \geq 0 \} .$$

- **Condition PC.** For any  $\Lambda \in \Delta(\Delta(\Theta))$  such that  $\int G d\Lambda(G) = F$ ,

$$\begin{aligned} \int (\int (U^P(\theta, 0)I_{\theta \leq \theta^G} + U^P(\theta, 1)I_{\theta > \theta^G}) dG(\theta)) d\Lambda(G) \\ \geq \\ \int (\int U^P(\theta, 1 - P(\xi^G | \theta)) dG(\theta)) d\Lambda(G) \end{aligned}$$

- Trivially satisfied when  $L(\theta, A, z)$  is invariant in  $A$  and there is no aggregate uncertainty (e.g.,  $z = 0$  a.s.)

## Theorem 4

(a) Given any  $\Gamma$ , there exists  $\Gamma^*$  satisfying PCP and s.t., for any  $\theta$ , agents' expected payoff under  $a^{\Gamma^*}$  is at least as high as under  $a^{\Gamma}$ .

(b) Suppose  $p(x|\theta)$  is log-supermodular; then  $\Gamma^*$  is binary.

(c) In addition to  $p(x|\theta)$  being log-supermodular, suppose Condition PC holds. Then PM's payoff under  $\Gamma^*$  at least as high as under  $\Gamma$ .

- PCP: announcement of sign of agents' expected payoff under MARP

# Foundation for Monotone Tests

- Let

$$D^P(\theta) \equiv U^P(\theta, 1) - U^P(\theta, 0).$$

**Condition M:** Following properties hold:

- 1 The function  $U(\theta; x) \equiv u(\theta, 1 - P(x|\theta))$  is log-supermodular;
- 2 For any  $x$ , and any  $\theta_0, \theta_1 \in [\underline{\theta}, \bar{\theta}]$ , with  $\theta_0 < \theta_1$ ,  $\frac{D^P(\theta_1)}{D^P(\theta_0)} > \frac{p(x|\theta_1)U(\theta_1;x)}{p(x|\theta_0)U(\theta_0;x)}$

## Theorem 5

*Suppose  $p(x|\theta)$  log-supermodular, Condition PC holds, and Condition M holds. Given any  $\Gamma$ , there exists deterministic binary monotone  $\Gamma^* = (\{0, 1\}, \pi^*)$  satisfying PCP and yielding a payoff weakly higher than  $\Gamma$ .*

# Micro-foundations

- Former liabilities:  $D$
- Bank's legacy asset delivers
  - $v(\theta) \in \mathbb{R}$  end of period 1
  - $V(\theta)$  end of period 2
- Bank can issue
  - shares
  - new short-term debt
- Potential investors
  - endowed with 1 unit of capital
  - market orders

# Micro-foundations

- $Y(p, \theta, z)$ : exogenous demand for shares (alternatively, debt)
- Market clearing price  $p^*(\theta, A, z)$  solves

$$q + 1 - A = A + Y(p^*, \theta, z).$$

- Default:

$$R(\theta, A, z) = v(\theta) + \rho_S q p^*(\theta, A, z) - D \leq 0$$

- Analysis can be used to study
  - effect of different recapitalization policies
    - $(q_E, q_D)$
  - role of uncertainty for toughness of optimal stress tests
    - uncertainty about bank's profitability:  $\sigma$
    - uncertainty about macro variables:  $z$

# Conclusions

- Information design in coordination games with heterogeneously informed agents
- Application: Stress Test Design
  - Perfect coordination property (“right” notion of transparency)
  - Pass/Fail tests
  - Monotone rules
- Extension 1: PM uncertain about mkt prior beliefs
  - robust-undominated design (w. Piot Dworczak)
- Extension 2: timing of optimal disclosures

**THANKS!**

- Here allow for stochastic policies  $\pi : \Theta \rightarrow \Delta(S)$
- Let  $r(\omega; a^\Gamma) \in \{0, 1\}$  be default outcome at  $\omega \equiv (\theta, \mathbf{x}, s)$  when agents play according to  $a^\Gamma$
- Let  $\Gamma^* = \{S^*, \pi^*\}$  be s.t.  $S^* = S \times \{0, 1\}$  and  $\pi^*((s, r(\omega; a^\Gamma)) | \theta) = \pi(s | \theta)$ , all  $(\theta, s)$  s.t.  $\pi(s | \theta) > 0$
- After receiving  $s^* \equiv (s, 1)$ , agents use Bayes' rule to update beliefs about  $\omega \equiv (\theta, \mathbf{x}, s)$ :

$$\partial \Lambda_i^{\Gamma^+}(\omega | x_i, (s, 1)) = \frac{\mathbf{1}\{r(\omega; a^\Gamma) = 1\}}{\Lambda_i^\Gamma(1 | x_i, s)} \partial \Lambda_i^\Gamma(\omega | x_i, s)$$

where

$$\Lambda_i^\Gamma(1 | x_i, s) \equiv \int_{\{\omega: r(\omega; a^\Gamma)=1\}} d\Lambda_i^\Gamma(\omega | x_i, s)$$

- Let  $a_{(n)}^\Gamma$ ,  $a_{(n)}^{\Gamma^+}$  be most aggressive profile surviving  $n$  round of IDISDS under  $\Gamma$  and  $\Gamma^*$ , respectively

## Definition 2

Strategy profile  $a_{(n)}^{\Gamma^*}$  less aggressive than  $a_{(n)}^\Gamma$  iff, for any  $i \in [0, 1]$ ,

$$a_{(n),i}^\Gamma(x_i, s) = 1 \Rightarrow a_{(n),i}^{\Gamma^*}(x_i, (s, 1)) = 1$$

## Lemma 1

For any  $n$ ,  $a_{(n)}^{\Gamma^*}$  less aggressive than  $a_{(n)}^\Gamma$

- Proof by induction
- Let  $a_0^\Gamma = a_0^{\Gamma^*}$  be strategy profile where all agents refrain from pledging, regardless of their (endogenous and exogenous) information
- Suppose that  $a_{(n-1)}^{\Gamma^*}$  less aggressive than  $a_{(n-1)}^\Gamma$
- Note that  $r(\omega|a^\Gamma) = 0 \Rightarrow r(\omega|a_{(n-1)}^\Gamma) = 0$   
( $a_{(n-1)}^\Gamma$  more aggressive than  $a^\Gamma = a_\infty^\Gamma$ )
- Hence,  $r(\omega; a^\Gamma) = 1$  “removes” from support of agents’ beliefs states  $(\theta, \mathbf{x}, s)$  for which default occurs under  $a_{(n-1)}^\Gamma$

Because

- payoffs from pledging in case of default are negative
- payoff from **pledging** under  $\Gamma^*$  when agents follow  $a_{(n-1)}^\Gamma$

$$\begin{aligned}
 U_i^{\Gamma^*}(x_i, (s, 1); a_{(n-1)}^\Gamma) &= \frac{\int_{\omega} u(\theta, A(\omega; a_{(n-1)}^\Gamma)) 1_{\{r(\omega; a^\Gamma)=1\}} d\Lambda_i^\Gamma(\omega|x_i, s)}{\Lambda_i^\Gamma(1|x_i, s)} \\
 &> \frac{\int_{\omega} u(\theta, A(\omega; a_{(n-1)}^\Gamma)) d\Lambda_i^\Gamma(\omega|x_i, s)}{\Lambda_i^\Gamma(1|x_i, s)} \\
 &= \frac{U_i^\Gamma(x_i, s; a_{(n-1)}^\Gamma)}{\Lambda_i^\Gamma(1|x_i, s)}
 \end{aligned}$$

- Hence,  $U_i^\Gamma(x_i, s; a_{(n-1)}^\Gamma) > 0 \Rightarrow U_i^{\Gamma^*}(x_i, (s, 1); a_{(n-1)}^\Gamma) > 0$

- That  $a_{(n-1)}^{\Gamma^*}$  less aggressive than  $a_{(n-1)}^{\Gamma}$  along with supermodularity of game implies that

$$U_i^{\Gamma^*}(x_i, (s, 1); a_{(n-1)}^{\Gamma}) > 0 \Rightarrow U_i^{\Gamma^*}(x_i, (s, 1); a_{(n-1)}^{\Gamma^*}) > 0$$

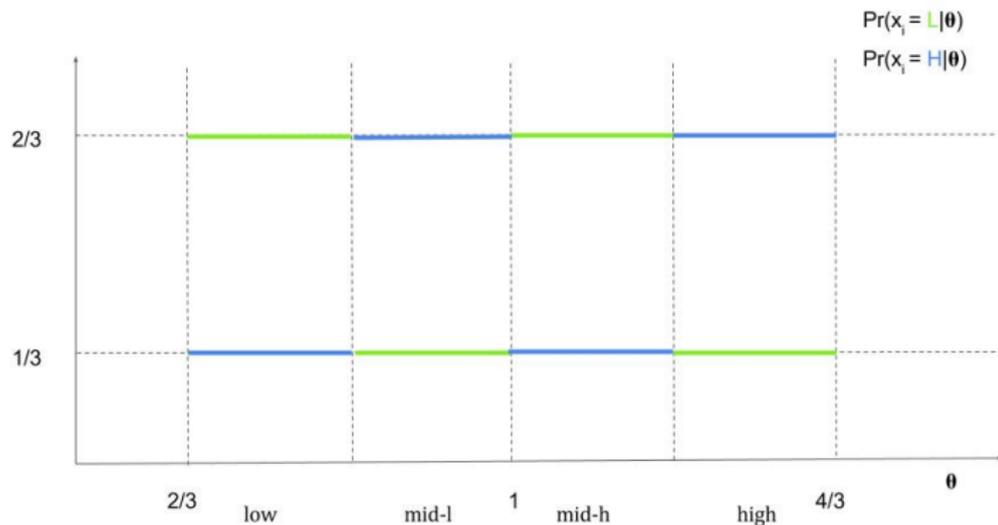
- As a consequence,

$$a_{(n),i}^{\Gamma}(x_i, s) = 1 \Rightarrow a_{(n),i}^{\Gamma^*}(x_i, (s, 1)) = 1$$

- This means that  $a_{(n)}^{\Gamma^*}$  less aggressive than  $a_{(n)}^{\Gamma}$ .

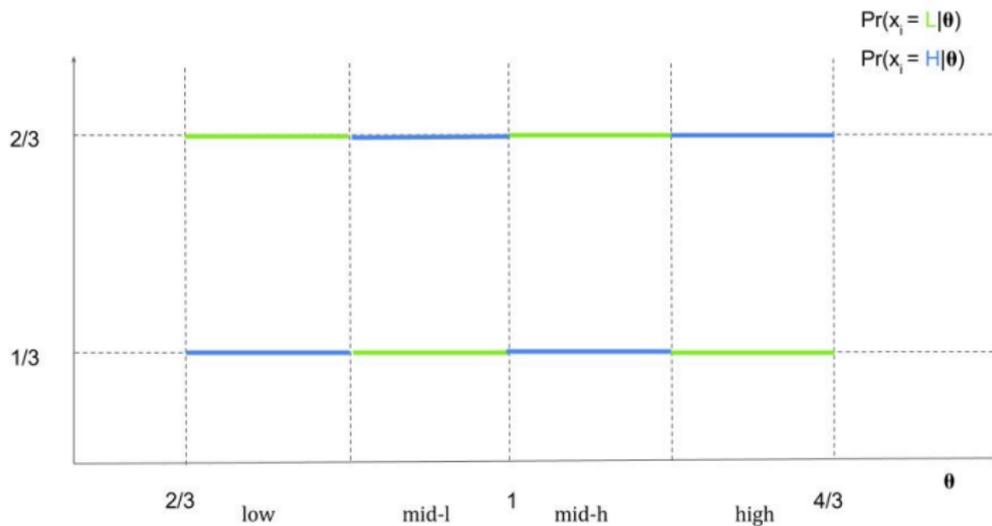
- Above lemma implies MARP under  $\Gamma^*$ ,  $a^{\Gamma^*} \equiv a_{(\infty)}^{\Gamma^*}$ , less aggressive than MARP under  $\Gamma$ ,  $a^{\Gamma} \equiv a_{(\infty)}^{\Gamma}$
- In turn, this implies that  $r(\omega; a^{\Gamma}) = 1$  makes it common certainty that  $r(\omega; a^{\Gamma^*}) = 1$
- Hence, all agents pledge after hearing that  $r(\omega; a^{\Gamma}) = 1$
- Similarly,  $r(\omega; a^{\Gamma}) = 0$  makes it common certainty that  $\theta < 1$ . Under MARP, all agents refrain from pledging when hearing that  $r(\omega; a^{\Gamma}) = 0$

# Example



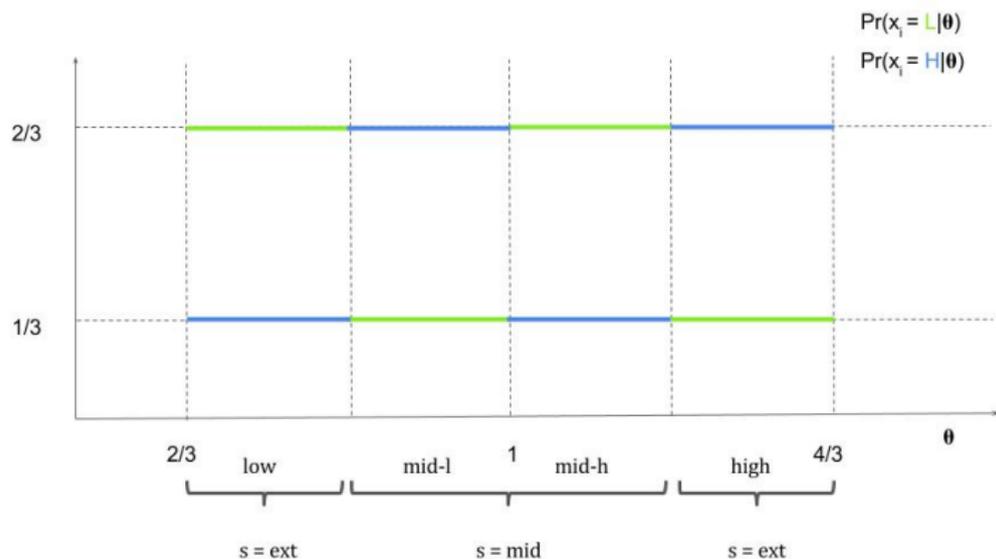
- Assume  $b = -g$
- Pledging rationalizable iff  $\Pr(r = 1) \geq 1/2$

# Example



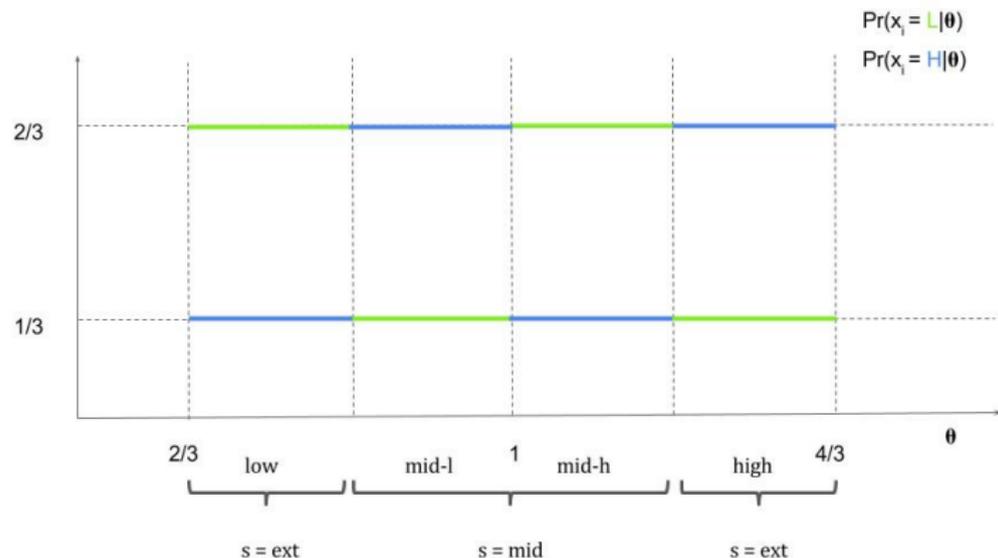
- No disclosure: under MARP,  $a_i^\Gamma(x_i) = 0$ , all  $x_i$

# Example



- Suppose PM informs agents of whether  $\theta$  is extreme or intermediate
- $a_i^\Gamma(x_i, s) = 1$ , all  $(x_i, s)$

# Example



- If, instead, PM only recommends to pledge (equivalently,  $\Gamma$  is pass/fail):  
 $a_i^\Gamma(x_i, 1) = 0$  for all  $x_i$
- **Suboptimality of P/F policies (+ failure of RP)**