Purchasing Seats for High School Admission in China*

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Abstract

Many Chinese cities once gave students the option of paying higher tuition to attend their preferred schools. This seat-purchasing mechanism is neither strategy-proof nor stable. Our paper combines administrative and survey data to estimate students' preferences and conduct welfare analysis. We find that changing from a deferred acceptance mechanism to the so-called cadet-optimal stable mechanism reduces students' welfare but that adopting the observed seat-purchasing mechanism alleviates this welfare loss. Under the latter approach, upper-tier schools collect significantly more tuition—with a minimal change in student quality—whereas collecting more tuition results in middle-tier schools facing substantial uncertainty about student quality.

1 Introduction

The analysis of centralized school choice mechanisms has become a key focus of research in market design (Abdulkadiroglu and Sönmez 2003). In extant literature on the school choice problem, the effects of monetary transfers between students and schools is seldom considered because public schools are either free or have fixed (and low) tuition. Yet unlike school choice systems in most other counties, many Chinese cites have—starting in the 1990s—offered students the option of paying higher tuition and thereby gaining admission

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to public schools.[1] This procedure is referred to as the *Ze Xiao* (ZX) policy.[2] The ZX policy is a practical application of the *matching with contracts* model (Kelso and Crawford 1982; Hatfield and Milgrom 2005; Hatfield and Kojima 2008, 2010; Hatfield et al. 2017). Analyzing student responses to a "price menu" for an individual good in matching markets may shed light on broader applications, such as considering how financial aid affects the school choice problem.

However, this policy provoked controversy because it was viewed as unfair to students whose families cannot afford the higher costs (Shen and Wu 2006). The controversy lasted for more than a decade and was somewhat defused in 2012, when the Ministry of Education announced restrictions on the ZX policy and requested that public high schools cease using it within three years. Many cities did abandon the policy for high school admissions, including Shanghai (which ceased using it in 2012), Beijing (in 2014), and Shenzhen and Tianjin (in 2015). However, when the Chinese government decided to discontinue its ZX policy in the face of widespread objections, it had never been rigorously analyzed by policy makers or researchers. Yet the questions that naturally arose still merit examination. For instance: Does this policy reduce (as often claimed) or rather enhance student welfare? Does it have the same affect on all students? If the ZX policy led to welfare losses for students, did that loss stem from the option to purchase admissions or was the matching mechanism itself flawed?

This paper addresses these questions by exploiting a new data set covering high school admissions for the period 2012–2014 in a large Chinese city.[3] The admission records indicate that about a fifth of all students were admitted to high schools because they paid higher tuition. We combine these admission records with data from a 2014 survey to explore the strategic behaviors evident in student applications; we also estimate student preferences regarding schools and tuition. We then use those estimated preferences to evaluate how student welfare might have been affected by the ZX policy. In addition, we address the potential trade-off faced by schools—given that selling seats to students may increase school profits but reduce the quality of admitted students.

High school admission in our focal city is based on a centralized matching process that uses standardized tests (strict school priority). The City Education Bureau adopted a typical ZX policy for its admission procedure until that policy was contraindicated in 2014 (see Appendix J for details of the ZX policy in various Chinese cities). In particular: (i) students

[1] Zhu Kaixuan, then chairman of the state education commission, publicly addressed the seat-purchasing problem in public schools. In 1995 he argued, in the *People's Daily*, against paying higher tuition to purchase admission to compulsory education.

[2] "Ze Xiao" is Chinese for "school selection".

[3] Confidentiality restrictions prevent this city from being identified by name.

and schools were "price takers" in the admission procedure, while both the basic and the higher tuition levels were set by the local government; and (ii) the number of seats for sale in each school (i.e., the ZX *quota*) was also controlled by the government. When ranking the various schools, potential students' rank-ordered lists (ROLs) also indicated whether or not the student was willing to pay higher tuition in order to gain admission to each school on her preferred list—that is, if her admission would otherwise be denied. Admitted students who paid only the basic tuition are referred to as *normal* students; those whose admission depended on paying higher tuition are *ZX students*. From 2008 to 2013, the mechanism used to assign students to schools was the *Chinese parallel purchasing seats* (CPPS) mechanism, an indirect extension of the *Chinese parallel* (CP) mechanism (Chen and Kesten 2017). The difference between these two mechanisms is that the former incorporates an additional stage with each choice on the ROL to facilitate matching ZX students to their preferred school. In that stage, students who are willing to pay higher tuition have higher priority than others to receive seats from the ZX quota. Prior to 2008, the matching algorithm usually followed the so-called Boston mechanism by adding a similar additional stage; this procedure is referred to as the *Boston mechanism with purchasing seats* (BMPS) option.

Our theoretical analysis shows the CPPS mechanism has some undesirable features. It is not strategy-proof, which allows students to "game" the system by misreporting their true preferences. Moreover, equilibrium outcomes of this mechanism can be inefficient and unstable. One way to overcome these imperfections—while retaining the option to purchase admission—is to adopt the *cadet-optimal stable mechanism* (COSM). This mechanism is proposed by Sönmez and Switzer (2013), who study the cadet–branch matching in the US Army whereby cadets may choose a longer term of enlistment in exchange for being allowed to choose where they will be stationed. The COSM is an extension of the *student-proposing deferred acceptance* (DA) mechanism described by Gale and Shapley (1962) and shares its favorable properties, which include being both stable and strategy-proof.

The theoretical properties of these mechanism motivate us to investigate real-world student behavior and welfare consequences. One difficulty with any empirical analysis of the school choice problem is estimating student preferences when only the submitted applications can be observed. The reason is that, if the adopted mechanism is *not* strategy-proof, then students have an incentive to misreport their true preferences when submitting their rank-ordered lists. Our survey, which covered nearly half of those who graduated from middle school in 2014, aimed to uncover students' true preferences and thereby counter, to some extent, the problems associated with assessing those preferences in the presence of strategic behavior. A comparison of survey responses and the ROLs actually submitted indicates that only 1.4% of the students reported their true preferences on their respective ROLs. Students also sought to increase their chances of being admitted by strategically maintaining sufficient gaps between their ROL choices.

We estimate students' preferences in two steps. In the first step, survey results are used to estimate student preferences over schools without considering strategic behavior in ROLs. Given that the ZX policy ceased after 2013, the survey data cannot be used to identify any ZX policy–related parameters (e.g., tuition). So in the second step, the ROLs submitted in 2012 and 2013 are used to estimate other parameters. In this step, we assume that students have homogeneous beliefs about the likelihood of being admitted to each school and that they try to maximize their expected utilities in a rational manner. Our estimated results indicate that a 1-unit increase in school quality (see Section 4.2 for the definition) is associated with high-scoring students being willing to pay an additional 226 yuan—or about \$43.6 (US)—to attend that school; in contrast, medium- and low-scoring students are willing to pay only 93 yuan and 78 yuan (respectively) for that privilege.

Using the estimated student preferences, we conduct counterfactual experiments that enable assessment of how the different matching mechanisms perform. We use the simulated matching outcomes under the DA mechanism as the benchmark. When that mechanism is replaced with the COSM, student welfare is reduced (on average) by 71 yuan when 10% of the seats are reserved for ZX students.[4] This welfare loss increases to 380 yuan and 856 yuan when the quota for ZX students is increased to 30% and 50%, respectively. These results reflect the direct influence of the seat-purchasing option given that the two mechanisms in question are both strategy-proof. If the DA mechanism is instead replaced by the CPPS mechanism, the average welfare loss due to purchasing seats is only 4.5 yuan (resp., 620 yuan) when the ZX quota is 10% (resp., 50%); this follows because more students (especially the medium-scoring ones) can attend their preferred schools by gaming the system. Adopting the BMPS can further reduce the average welfare loss.

Our study allows for investigating the welfare of schools as well, a long-overlooked topic that is nonetheless important in studies of the school choice problem—that is, because competition among schools to recruit the best students is evident across all education levels and especially after the compulsory education levels. We measure the welfare of high schools in terms of (a) the quality of admitted students and (b) the profit derived from collecting student tuition. For upper-tier high schools, the tuition collected increases (once the DA mechanism is replaced) in proportion to the ZX quota across all mechanisms that have seat-purchasing options; meanwhile, the quality of their students (measured by percentage grade) declines by no more than 1.2%. Although an increase in collected tuition is observed with respect to (w.r.t.) middle-tier high schools, the seat-purchasing option leads to substantial

[4]On June 1, 2013, the exhchange rate was $\$1 : 6.1$ yuan.

variation in student quality for these schools: that quality may decline by more than 4%. For lower-tier schools, the ZX policy's effects on both collected tuition and student quality are uncertain.

This paper is closely related to the work of Sönmez (2013) and Sönmez and Switzer (2013), who investigate cadet–branch matching. We extend the theoretical results and complement these outcomes by offering an empirical analysis. Our work is also directly related to the extensive theoretical literature addressing the centralized school choice problem.[5] More specifically, there is a growing literature that discusses the role of multi-level financial aid in the school choice problem. Hassidim et al. (2016) study a large sample of college admissions with different levels of financial aid. Hassidim et al. (2017) discover that, in a matching procedure for Israeli psychology Master's programs, many applicants make the mistake of highly ranking programs that offer less financial aid.

The research undertaken here contributes to a growing body of empirical work on the school choice mechanism. One thread of that literature uses the preferences reported under non–strategy-proof mechanisms to estimate student preferences (Hwang 2015; He 2016; Calsamiglia et al. 2017; Agarwal and Somaini 2018). Other papers focus on strategy-proof mechanisms. Abdulkadiroğlu et al. (2015) treat preferences reported under the DA mechanism as students' true preferences and then use those preferences to analyze the demand for particular schools in New York City. Fack et al. (2018) propose an approach for estimating preferences that does not require truth telling to be the unique equilibrium under the DA mechanism. Several empirical papers (e.g., Burgess et al. 2014; Ajayi 2017; Akyol and Krishna 2017) bear similarities to our strict priority setting.

Reasearch has witnessed an increasing use of survey data by scholars exploring strategic behavior under matching mechanisms. Budish and Cantillon (2010) conduct a survey on student preferences for offered courses to study the course allocation mechanism at Harvard Business School, and Rees-Jones (2018) provide survey-based evidence of preference misrepresentation. Burgess et al. (2014) use survey data to assess directly the preferences of students over schools. Surveys are used also by De Haan et al. (2015) to analyze the Boston mechanism's deficiencies and by Kapor et al. (2017) to study heterogeneous beliefs in the school choice problem.

The rest of this paper proceeds as follows. In Section 2 we present school choice mechanisms that incorporate seat-purchasing options and develop the theoretical properties of those mechanisms. Section 3 provides details on the local ZX policy's background, after which Section 4 describes our data and analyzes students' strategic behavior in the applications. We present the empirical model and our estimates of student preferences in Section 5,

[5] See Pathak (2011) for a survey on the school choice problem from the perspective of market design.

and in Section 6 we conduct counterfactual experiments across mechanisms. Section 7 concludes with a summary of our findings.

2 School Choice with a Purchasing Seats Option

Before introducing the model, we first illustrate the ZX policy's potential influence on students and schools by way of a descriptive example. Suppose there are three schools—an upper-tier school, a middle-tier school, and a lower-tier school—and suppose that students always report their true preferences. Students are first assigned to schools by a centralized matching mechanism (ϕ) without the seat-purchasing option, and exam scores are the sole criterion used to admit students. Then we use a new mechanism(ψ) that features the option to purchase seats, where the proportion λ of a school's capacity is devoted to ZX seats.

For the upper-tier school, the top $1 - \lambda$ of students admitted under mechanism ϕ is unaffected by adoption of the new mechanism ψ. However, some of the next λ of students—who are unwilling to pay higher tuition to attend this school—are assigned to other schools, after which lower-scoring students take those seats and pay higher tuition. The rest of these λ students are still admitted by this school, but they pay higher tuition. As a result, the dispersion of quality students increases and the average quality of admitted students declines.

For the middle-tier school, some of the students assigned to it under the old mechanism will now attend the upper-tier school under the new mechanism because they are willing to pay higher tuition. Furthermore, this middle-tier school admits some students who—under the old mechanism—would have been assigned to the upper-tier school. Of the remaining students, those who were just barely admitted under the old mechanism must now pay higher tuition in order to keep their seats; otherwise, they will be assigned to the lower-tier school and their seats will be taken by lower-scoring students who do pay the higher tuition. Here the dispersion of student quality increases but the average quality of students is indeterminate. Finally, some of the lower-tier school's students will, under a ZX policy, gain admission to better-quality schools by paying the higher tuition. The lower-tier school then admits some students who were assigned to a better school under the old mechanism.

This example illustrates that some high-scoring students may suffer a welfare loss under the ZX policy. They must either pay higher tuition to attend the good schools or matriculate at a lower-quality school. Yet at the same time, low-scoring students can take advantage of this opportunity to gain admission into better schools by paying additional tuition. The effect of the ZX policy is determined by three factors: the quota of ZX seats, student preferences (demand for schools), and matching mechanisms.

Chinese Parallel Purchasing Seats (CPPS) Mechanism

Here we formally define a school choice problem when purchasing seats is an option. There is a finite set of students, $I = \{i_1, \ldots, i_n\}$, as well as a finite set of schools, $J = \{j_1, \ldots, j_m\} \cup \emptyset$; we use $\emptyset$ to denote the case where a student does not attend any school. Every school has the same tuition structure $C = \{c_0, c_1\}$ with $c_0 < c_1$, where c_0 is the basic tuition paid by normal stuents and c_1 is the higher tuition for ZX students.[6] Each school has two quotas, q_j^a and q_j^z, for (respectively) normal and ZX students. We assume that $\sum_{j \in J}(q_j^a + q_j^z) \geq n$, which guarantees that the number of students is less than the schools' total capacity. Each student has a strict preference π_i over schools and tuition (i.e., over the set $J \times C$), where $(j, c_0)\pi_i(j, c_1)$ means that student i strictly prefers paying basic tuition for a seat in school j to paying higher tuition for a seat in the same school. All schools share the same strict normal priority $\succ$ over students.

According to a *contract* $x = (i, j, c) \in I \times J \times C$, student i is assigned a seat in school j by paying tuition c; hence (j, c) is student i's *assignment*. A *matching* X is a set of contracts such that (a) each student appears in only one contract and (b) no school appears in more contracts than its total quota of students. Let $\mathcal{X}$ denote the set of all matching outcomes.

A *mechanism* is a strategy space A_i for each student i along with an outcome function, $\psi\colon (A_{i_1} \times A_{i_2} \times \cdots \times A_{i_n}) \to \mathcal{X}$, that selects a matching outcome for each strategy vector $\mathbf{a} = (a_{i_1} \times a_{i_2} \times \cdots \times a_{i_n}) \in (A_{i_1} \times A_{i_2} \times \cdots \times A_{i_n})$. A *direct* mechanism is one for which the strategy space of each student is simply the set of all preferences Π over $J \times C$. It follows that a direct mechanism is simply a function $\psi\colon \Pi \to \mathcal{X}$ that selects a matching outcome for each preference profile.

A matching is *stable* if: (i) there is no unselected contract (i, j, c) such that student i prefers assignment (j, c) to her current assignment and also that i's priority is high enough to be selected by j after paying cost c; (ii) no student prefers a pair (j, c) with an unfilled quota to his current assignment; and (iii)no school would rather reject one of the contracts that includes it. In turn, a mechanism ψ is stable if it always selects a stable matching. A mechanism ψ is *strategy-proof* if, for each student, it is at least a weakly dominant strategy to report her true preference.

Chen and Kesten (2017) study the Chinese parallel (CP) mechanism,[7] which reflects a *permanency-execution period* represented by a vector $\mathbf{e}$=$(e_1, e_2, \ldots)$ for $e \in \mathbb{N}$. Thus, within each matching round j, a total of e_j subchoices are considered; the algorithm implements a deferred acceptance procedure whereby applications are tentatively held until no new

[6]The model can easily be extended to accommodate multiple levels of tuition; see Sönmez and Switzer (2013).

[7]Their paper also gives additional details about the college admission reform in China.

proposals are made. Assignments are finalized after all e_j choices have been considered. The CPPS mechanism is an extension of the CP mechanism; but unlike the CP mechanism, the CPPS mechanism used to assign students is not a direct mechanism. In particular, each student is asked (i) to rank her school preferences and (ii) to indicate, for each ranked school, whether she wants the *ZX option* (i.e., would pay higher tuition) to attend that school if she is not assigned a seat there as a normal student.

Under the CPPS mechanism, each school j allocates its ZX seats based on the *ZX priority* $\succ^+$, which is constructed as follows. First, if student i chooses the ZX option w.r.t. j but student i' does not, then $i \succ^+ i'$; second, if both i and i' choose the ZX option w.r.t. j (or if neither does), then $i \succ^+ i'$ if and only if $i \succ i'$. Hence the relative priority of two students does not change under $\succ^+$ unless one of them chooses the ZX option w.r.t. school j and the other student does not.

The CPPS mechanism with $\mathbf{e}$=$(e_1, e_2, \ldots)$ selects the matching outcome as described next.

Round 1:

• Each student applies to her first choice. Each school j follows the normal priority while tentatively holding the top q_j^a applicants in the normal pool. Among remaining applicants, the school tentatively holds the top q_j^z applicants in its ZX pool based on the ZX priority. All other applicants are rejected.

In general:

• Each rejected student i who has not yet applied to her (e_1)th-choice school applies to her next-preferred school. A student who has been rejected by all her first e_1 choices does not apply to any other schools until the next round. Each school j reviews the new applicants, along with those currently held in the normal pool, and then tentatively holds the top q_j^a applicants in its normal pool based on the normal priority. School j next considers all remaining applicants, along with those currently held in its ZX pool, and tentatively holds the top q_j^z applicants based on the ZX priority. The other applicants are rejected.

• The round terminates whenever each student either is held in a school's pool or has been rejected by all her first e_1 choices. At this point, all tentative assignments become final. For each school, the remaining quotas are denoted $q_{j,2}^a$ and $q_{j,2}^z$ for normal and ZX students, respectively.

Round $k > 1$

• Each student applies to her $\left(\sum_{j=1}^{k-1} e_j + 1\right)$th-choice school. Then, as in Round 1, each school j tentatively holds the top $q_{j,k}^a$ applicants in the normal pool (again, based on the normal priority). Among the remaining applicants, the school tentatively holds the top $q_{j,k}^z$ applicants based on the ZX priority. All other applications are rejected.

In general:

• Each rejected student i who has not already applied to her $\left(\sum_{j=1}^{k} e_j\right)$th-choice school applies to her next-preferred school. A student who has been rejected by all her first $\sum_{j=1}^{k} e_j$ choices does not apply to any other schools until the next round. Each school j reviews the new applicants, along with those currently held in the normal pool, and tentatively holds the top $q^a_{j,k}$ applicants in its normal pool based on the normal priority. Then j considers all remaining applicants, along with those currently held in its ZX pool, and tentatively holds the top $q^z_{j,k}$ applicants based on the ZX priority. The other applicants are rejected.

• The algorithm terminates when each student is admitted to a school and all the tentative assignments are final. Each student who receives a normal seat pays tuition c_0. Each student who receives a ZX seat after choosing the ZX option for those schools pays the higher tuition c_1, while each student who receives a ZX seat but did *not* choose the ZX option pays only the basic tuition c_0.

Different Chinese cities adopted this mechanism with heterogeneous permanency-execution periods $\mathbf{e}$ (see Appendix J for details). A special case of the CPPS mechanism—when $e_j = 1$ for all j—is the "Boston mechanism with purchasing seats option", where the assignments made after each choice are final. When the Boston mechanism was being used by most Chinese provinces for college admissions, this BMPS mechanism was adopted as the secondary school admission procedure throughout the country.

The Chinese parallel mechanism is not strategy-proof, from which it follows that revealing one's true preferences w.r.t. schools under the CPPS (or BMPS) mechanism may not be a weakly dominant strategy (see Appendix A for an example). Because of this flaw, it is difficult to judge whether the seat-purchasing option is, per se, a "good" or "bad" choice for students. The reason is that students may suffer a welfare loss when they game a CPPS-based system.

Sönmez and Switzer (2013) study a similar system that matches cadets to military bases in the United States. These authors propose the "cadet-optimal stable mechanism", which is strategy-proof and allows players to retain the "purchasing" option. In our context, each student's strategy space is Π under the COSM, which makes it a direct mechanism. Here $\widetilde{\succ}$, the ZX priority of school j, is adjusted as follows. Suppose school j is considering two applicants, i and i', for ZX seats. Then: (i) if i's application is (j, c_1) and i''s application is (j, c_0), then the school prefers i to i' (i.e., $i \widetilde{\succ} i'$); (ii) if both applicants choose (j, c_0) or (j, c_1), then $i \widetilde{\succ} i'$ if and only if $i \succ i'$.

Given the submitted preference lists, the COSM selects the outcome as follows.

Round 1. Each student applies to her first choice. Each school j tentatively holds the top q^a_j students (with their contracts) whose first choices are (j, c_0) based on the normal priority ($\succ$) in its normal pool. Among the remaining applicants, the school tentatively holds the top q_j^z students (with their contracts) whose first choices are (j, c_1) or (j, c_0) based on the ZX priority ($\widetilde{\succ}$) in its ZX pool. The other applicants are rejected.

Round $k > 1$. Each rejected student applies to her next choice. Each school j considers the new applicants whose choices are (j, c_0) along with those who are held in the normal pool (with their contracts) from the previous round; then each j tentatively holds the top q_j^a applicants (with their contracts) in the normal pool based on the normal priority. Among the remaining applicants, j considers the new applicants whose choice is (j, c_1) or (j, c_0) along with those who are held in its ZX pool with their holding contracts from the previous round; it then holds the top q_j^z applicants based on the ZX priority. The other applicants are rejected.

This algorithm terminates when each student is tentatively held by a school, at which point the tentative assignments become final. A student i who is assigned a seat in j pays tuition c_0 if her assigned contract is (i, j, c_0) or pays c_1 if the assigned contract is (i, j, c_1).

Some properties of the COSM are similar to those of the DA mechanism. Moreover, the matching outcome under the COSM is weakly preferred by all students to any stable matching.[8] The COSM rules out the possibility of students gaming the system by misreporting their preferences. Under the CPPS (and BMPS) mechanism, however, students *can* achieve better outcomes by misreporting their preferences. These mechanisms share some deficiencies, as revealed in the following two propositions.

Proposition 1. *Nash equilibrium outcomes under the CPPS mechanism with* $e_1 > 1$ *can be unstable and also Pareto inferior to outcomes under the COSM.*

Proposition 1 states that, even in a Nash equilibrium, the matching outcome under the CPPS mechanism may exhibit undesired properties (e.g., instability) and may be Pareto-dominated by the COSM. Our next result indicates that, although the BMPS equilibrium outcome can be stable, like the Boston mechansim (Ergin and Sönmez (2006)), it is still Pareto inferior to outcomes under the COSM.

Proposition 2. (i) *The set of Nash equilibrium outcomes under the BMPS is equal to the set of stable matchings.*

(ii) *Every Nash equilibrium outcome of the BMPS is Pareto dominated by the corresponding outcome of the COSM.*

[8] Sönmez and Switzer (2013) prove additional theoretical properties of the COSM.

3 Background on High School Admissions

The schools in our focal city can be categorized into several types based on their educational goals after students graduate from middle school. There are general high schools that prepare students for colleges and universities in China, foreign language schools (or classes) for foreign colleges or universities, fine arts schools for the fine arts colleges in China, and vocational schools for the labor market. General high schools can also be categorized into public and private high schools.

The City Education Bureau requires that all schools, regardless of type or ownership, join the centralized admission system as it pertains to middle school graduates. In addition, each student who undergoes this admission procedure must register at the school to which she is assigned by the system. Hence no outside option is available for students who intend to continue their education in this city.[9]

At the end of March in each year, the Bureau presents an admission plan that includes the quota of students that can be allocated to each school.[10] The quota for each public high school j comprises three parts: quotas q_j^e for early admission students,[11] quotas q_j^a for normal admission students, and quotas q_j^z for ZX students. In mid-May, students submit their rank-ordered lists of schools. Thereafter, all students take the centralized high school entrance exam in early June. During 2012–2014, the full mark (i.e., the highest possible score) on this the exam was 665.[12] Once the exams are graded, students are assigned to the schools by a centralized matching mechanism. All schools adopt the same strict normal priority (exam scores) over students.

Each student can list at most three schools on her ROL; students also select (or not) the ZX option w.r.t. those schools. Finally, every student must indicate whether she will accept a random assignment in the event she is rejected by her three preferred schools.

Local public high schools play a dominant role in preparing students for college. Thus, gaining entry into a public high school is the only hope most students have for attending college in China. Yet high school education in China involves more than compulsory education, and local public high schools can accommodate fewer than half of all middle school

[9]To avoid an unacceptable assignment, a student may either forgo the admission procedure or leave the application blank. Another way to avoid an undesirable assignment is to register at—but not actually attend—the assigned school. By paying additional costs, such students can instead attend schools in other cities.

[10]The admission quotas for private and vocational schools are announced at the same time.

[11]Students who can receive early admission are determined by a separate procedure, and this procedure does not directly affect the normal admission procedure, so we exclude these students from the analysis.

[12]Prior to 2012, the highest possible score was 650; after 2014, it was 780.

graduates. After receiving the students' ROLs and exam scores, the Bureau determines and publishes a public high school admission threshold (hereafter simply "the threshold") based on the score distribution and total available seats. Only students whose scores are above that threshold will be considered for a seat in public high schools. The threshold is meant to guarantee that the number of qualified students does not exceed the total number of available seats in public high schools.

Because each student's rank-ordered list contains no more than three schools, the matching mechanism used by the Bureau differs slightly from the model described in Section 2; it uses what we refer to as a *constrained* mechanism (Haeringer and Klijn 2009) in that the matching algorithm terminates after each student's three choices have been considered. Unmatched students who have indicated acceptance of a random assignment are then randomly assigned to public high schools that still have available seats; the rest must find their own paths either to continue their schooling or to join the labor market. Before 2008, the (constrained) BMPS was employed to assign students. Since then, the (constrained) CPPS mechanism—with permanency-execution periods $(2, 1)$—has been used. This new mechanism's matching algorithm lasts two rounds. The first and second choices in students' ROLs are considered in the first round, and their third choices are considered in the second round. Without loss of generality, hereafter we shall reference the CPPS mechanism when describing the specific mechanism used in this city (i.e., without stipulating its permanency-execution periods).

The tuition structure of public high schools also differs from our baseline model. Given that the exam score is the unique admission criterion, each school establishes a cut-off for its normally admitted students. The annual basic tuition paid by normal students is 1,600 yuan (about $260 in 2013), so a public high school education is relatively inexpensive.[13]

Three levels of the higher tuition paid by ZX students are based on their exam scores. A ZX student pays a total of 3,333.3 yuan annually if her score is within 10 points of the school's cut-off, 5,000 yuan if it is within 11–20 points, and 6,000 yuan per year if it is within 21–30 points.[14] No school is allowed to admit a ZX student whose exam score is more than 30 points below its cut-off. Note that students submit their ROLs prior to taking the exam, and they can indicate only "yes" or "no" to the ZX option—that is, without making any stipulations about tuition levels. In accordance with instructions from the Ministry of Education, the local education bureau discontinued the ZX option after its 2013 admission

[13]In 2013, a local urban household's annual disposable income was 35,227 yuan; hence the basic tuition amounts to 4.5% of that income.

[14]Unlike normal students, who pay their tuition each year, ZX students must make a lump-sum payment for all three years of their high school education.

process was completed.

4 Data Description

4.1 Data Source and Sample Selection

Since we are analyzing the ZX policy, which is designed specifically for public high schools, we focus on the students qualified for admission to those schools.

The data set we use consists of two parts, administrative data and survey data. The former comprise admission records from 2012 through 2014. Those records include the three choices listed on students' ROLs, exam scores, final assignments, whether a student was admitted as a normal student or as a ZX student, and each student's middle school and home address. We also have some data on school characteristics: admission quotas, tuition, and dormitory accommodations.

In the administrative record, a total of 41,939 students were included in the 2012–2014 admission records. We first exclude students who were admitted by schools with special quotas, which did not affect the normal admission procedure (13.3%). Students excluded for this reason were those admitted early or by fine arts schools as well as those on sports or art scholarships.[15] Second, we exclude students whose exam scores were below the threshold (48.6%), since they were not qualified for admission to public high school. Finally, we exclude all students whose assignment outcomes were inconsistent with official rules or home addresses are missed (11.35%).[16] After these exclusions, our final sample size from the administrative data was 11,217.

In early May 2014, we conducted a survey of middle school graduates that asked each student to list five high schools she might attend and to rank them based on her preferences. The surveyed students were asked explicitly to report their genuine preferences, and there was no compelling reason for them not to honor this request. Because the survey was conducted just two weeks before students submitted their ROLs, it seems unlikely that their preferences would change within that short period (see Appendix H for details about the survey).

[15] An early admission decision is one that is made before students submit their ROLs. A student who is admitted early is still required to take the exam and to list the pre-admitting school as her first choice. Students admitted to fine arts schools must take an additional (art) exam; their admission process is handled separately from other students.

[16] For example, a few students were assigned to schools at which the cut-off was higher than their actual exam scores.

Unlike most surveys that seek to discover students' true preferences (Budish and Cantillon 2010; Kapor et al. 2017), we did not ask them to simply rank their favorite schools. Instead, respondents were asked to rank those schools they think that they might attend based on their true preferences. Recall that the exam score is the only admission criterion, and note that the highest admission cut-off may be more than 80 points higher than the lowest cut-off. Our survey design aims to avoid instances of a low-scoring student ranking schools at which she had no chance of being admitted—although such a student could list three schools with low cut-offs in her ROL. That possibility could lead to top schools being overreported in the survey, which would complicate attempts to compare the survey responses and reported ROLs of low-scoring students. The reliability of our survey is discussed further in Section 4.3.

We surveyed 6,980 students in 2014, or about half (49.17%) of the middle school graduates in that year's admission records. After we matched these students with the final administrative data sample just described—and deleted the invalid observations (e.g., students who ranked no school or only one school in the survey)—we were left with 1,447 survey observations for the subsequent analysis. Thus our survey covers 43.74% of the selected sample in 2014.

4.2 School Characteristics

In the administrative data, all nonpublic high schools were coded with a single number; we therefore treated all these schools as a whole without distinctions. Table 1 summarizes the characteristics of public high schools over the study period. A total of 13 public high schools were identified, with three special classes in 2012. Special classes are designed to admit gifted students and are independently operated; they also have their own admission quotas in the matching mechanism. In the table's last row, the changes in total number of public high schools reflect the addition of special classes in some years.

To assess the quality of public high schools, our proxy is the quality of students whom they admitted in previous years. More specifically: each school's quality is measured as the average high school entrance exam scores (percentage grade) of students (in the 10th to 90th percentile of those scores) admitted over the previous three years. Our school quality measure is highly correlated (0.96) with the schools' college admission rate, which is valued by most students and their parents. We did not use the college admission rate itself to measure school quality because (a) that information was missing for some of the schools and (b) the college admission rate is not publicly available to students.[17] However, both the high school admission cut-off and the score distribution on the high school entrance exam

[17] Also, the lowest-tier high schools refused to reveal their college admission rate.

are public information. (In the robustness check described in Section 5.5, we obtain similar results when using the available information on college admission rates to measure school quality.) We do not take a separate approach to estimating the schools' added-value when measuring school quality. The reason is that, when students and their families evaluate school quality in a school choice problem, they seldom consider value added; instead, they use such straightforward indexes as the school's rank, college admission record, or admission cut-off. Because we seek to mimic student strategies when estimating their preferences, there is little to be gained by considering a more complicated approach to estimate the "true quality" of schools.

We also conducted a supplemental survey of 44 middle school teachers in 2016 (see Appendix I for details). In that survey, we asked teachers to indicate the extent to which they agreed that a high school's education quality can be represented by the incoming students' test scores; most of the teachers (64%) answered "Strongly Agree" or "Agree" and a few of them (27%) answered "Neither Agree nor Disagree", but only 9% of them answered "Disagree". We also asked teachers to quantify each high school's quality by a number from 0 to 100. The survey result is also high correlated (0.93) with our quality measurement in the estimate.

The first row of Table 1 summarizes school quality for each school (as defined in Section 4.1). The average is approximately 80, with a standard deviation of 12.[18] School quality is stable across years, which reflects the stability not only of admission cut-offs but also of how students perceive the schools' relative ranks. There is considerable variation in the normal admission quotas. The largest school can admit 600 students; at the other extreme, a small, "special class" school admits but 40 students each year. The decrease in the average normal admission quota across years can be attributed to the newly established special classes and the increased number of early admissions. The average quota for ZX students ranges between 95 and 100 across years, with a standard deviation of about 35. Special classes and also four public high schools do not admit ZX students.[19] The table's fourth row indicates that the number of schools providing dormitories increased from nine in 2012 to thirteen in 2014.

[18]A school's quality is defined by its percentage grade (see the second paragraph of this section). To scale the measurement in the estimate, we multiply the percentage grade by 100. For example, if the school quality is 80% then we record it as 80 and not as 0.8.

[19]In fact, special classes and one public high school are not *allowed* to admit ZX students. The other three public high schools are the admission procedure's "leftover" schools. These schools admit students with scores above the threshold and then, if any unassigned seats remain, ZX students with scores below the threshold.

4.3 Student Characteristics and Behaviors

Exam score distributions are summarized in the first panel of Table 2. The first data column gives the percentile benchmarks, and the next three columns report the corresponding absolute scores across years. Exam scores are slightly lower in 2013 than in 2012 and 2014, but the variation in absolute scores of the same percentile level never exceeds 1.7% of the full mark. This finding confirms that exam scores were stable across years.

Our analysis focuses on students who were qualified to be assigned to public high schools. Approximately 94.3% of these students, whose scores were above the threshold, received seats in public high schools in 2012—as compared with 95.1% and 90.3% in 2013 and 2014, respectively. These values indicate that most students who qualified for admission to take seats in the public high schools end up going there rather than entering other types of schools.

The second panel of Table 2 reports the number of schools on students' submitted ROLs. More than 93% of the students submit full (three-school) lists, approximately 5% of them list two schools, and fewer than 1% of all students list only one school.[20]

The table's third panel shows the assignment results, which exhibited similar patterns in 2012 and 2013. About 30% (resp. 37%) of students were assigned to their first (resp. second) choice, and approximately 11%–13% of students were rejected by all three of their preferred schools. Some 13%–15% (resp. 5%–6%) of students were assigned to their first (resp. second) choice as ZX students. No ZX student was assigned to her third choice. After cancellation of the ZX policy in 2014, fewer students (26%) were assigned to their first choice and more students (17%) were rejected by all three choices.

Because the Chinese parallel mechanism is not strategy-proof, it is difficult to assess—while referring only to submitted ROLs—the extent to which students misrepresent their true preferences. Our survey data provide an opportunity for direct comparisons between each student's true ordinal preferences and her strategic behavior. More than 60% of the surveyed middle school graduates ranked five schools, 17% of them ranked four schools, and approximately 21% of them ranked fewer than four schools (see Table 8 in Appendix B).

The admission cut-offs of schools reflect their popularity among students. We define a *popular* school as one whose first-round cut-off is higher than the threshold; that is, the demand for admission to these schools is greater than the number of available seats.[21] At the same time, schools whose cut-offs are equal to the threshold are referred to as *leftover* schools.

[20]Schools that are listed twice in the same ROL are treated as a single school.

[21]Since no school's second-round cut-off was higher than the threshold when its first-round cut-off was equal to the threshold, no confusion can arise if we base popularity on only the first-round cut-offs.

Figure 1 shows the average admission cut-offs of schools chosen by students in the survey and the ROLs.[22] Students are grouped into four categories according to their score percentiles. In the survey, the top 10% students' exam score school cut-offs average 606.1 and 599.4 for (respectively) their first and second choices; the average cut-off for third choices (593.2) is another 6 points lower. The gaps between the third and fourth choices and the fourth and fifth choices in the survey are 5 and 9 points, respectively. The choices of students in the other three groups follow a similar pattern. Within a group, the average cut-off gap between consecutive choices is approximately 6 points and never more than 10 points. Between groups, the average cut-off for the first choice of the 80th–90th percentile students is 6 points lower than that for the highest decile of students, and this average cut-off decreases by another 9 points (to 591) for the 70th–80th percentile students. The average first-choice cut-off of students below the 70th percentile of exam scores is 585. For each additional choice, average cut-offs are similarly decreasing (at a rate of 4–10 points) in exam scores.

The decline in average cut-off of students' first choice when their scores decrease indicates that the surveyed students answered our questions truthfully by listing and ranking schools to which they might actually be admitted. The gaps between consecutive choices within groups in the survey indicate that student preferences w.r.t. schools were decreasing in the popularity of those schools; in 2014, the consecutive cut-off gaps for two popular schools were between 3 and 9 points. Also, the small cut-off gaps (4–10 points) between consecutive choices within each group implies that the preferences reported in the survey are reliable enough to be viewed as the students' true preferences.

In the rank-ordered lists, the average cut-offs for the first choices of students whose exam scores were above the 70th percentile nearly coincide with the corresponding parts in the survey, although the average cut-offs for the first choices of low-scoring students (i.e., with exam scores below the 70th percentile) are 6 points lower than in the survey. However, the gap between the first and second choices increases significantly with declining exam scores. The gap in the average cut-offs between the first and second choices for the top 10% students is almost the same as that in the survey, but this gap increases to 19 points for the 80th–90th percentile students and to about 25 points for the two groups of low-scoring students. Finally, the average cut-offs for third choices are consistently close to the threshold (of 535) for all groups in the ROLs.

When compared with the survey data, the large gaps between consecutive ROL choices reveal students' strategic behavior in their submitted preferences: maintaining a sufficiently large gap between choices toward the end of increasing their chances of being admitted to

[22] The corresponding table can be found in Appendix B.

some school.[23] The coincidence between the first choices in the survey and the ROLs indicates that students prefer applying to their favorite attainable schools. This coincidence, and the small cut-off gaps among choices reported in the survey, provide further evidence that the surveyed students accurately reported their five favorite attainable schools. Yet students, and especially those who were not in the top-scoring group, strategically manipulated their reported preferences in the ROLs so as to increase their overall likelihood of being admitted—that is, in the event of being rejected by their first choices. Thus the second choices in the ROLs of 80th–90th percentile (resp., 70th–80th percentile) students are close to their fourth (resp., fifth) choices in the survey. Moreover, most students (across all four groups) chose a leftover school as their third choice because the ROL is restricted to only three choices.

One drawback of a non–strategy-proof mechanism is that students who submit strategically modified ROLs may take advantage of the naïve students who reveal their true preferences (Pathak and Sönmez 2008).[24] We can estimate the proportion of naïve players in our data set by directly comparing the schools listed in the survey and in the ROLs. Only 20 students (1.38% of all observations) submitted ROLs that matched their lists in the survey. In fact, there may be even fewer naïve students because reporting true preferences could be a weakly dominant strategy for some students (e.g., those in the top-scoring groups). Our findings here accord with previous research in suggesting that few students submit an ROL without any strategic considerations—especially when a strict criterion is used to assign students.[25]

5 Empirical Model and Preference Estimate

To estimate student preferences, we simply adjust the structure of tuition fees—based on the local admission rule—in the school choice problem from Section 2. Recall that there is a set of tuition fees $C = \{c_0, c_1, c_2, c_3\}$, where c_0 is the basic tuition for normal students while c_1, c_2 and c_3 are the higher tuition amounts paid by ZX students; here $c_{t'} < c_t$ for $t < t'$.

[23]This finding is consistent with the literature that suggests students behave strategically under non–strategy-proof mechanisms (see e.g. Abdulkadiroğlu et al. 2005; Chen and Sönmez 2006; Abdulkadiroğlu et al. 2017).

[24]Calsamiglia et al. (2017) indicate that, in Barcelona's local school choice setting, the proportion of such naïve students is less than 4%.

[25]Unlike the assignment of students via coarser criteria (e.g., walking zones or siblings), high school admission in our context offers no safe choice for students before their exam scores are known; it follows that estimating this score is a student's first strategic move. Hence one must anticipate an extremely low percentage of naïve students among those subject to admission procedures like the ones described here.

Student i's (indirect) utility from being assigned to public high school j with tuition $c_{ij} \in C$ is

$$u_{i,j,c} = \sum_l \beta^l y_j^l + \sum_w \beta^w x_i^w y_j^w + \beta^D f(d_{ij}, X_i, Y_j) + \sum_k \alpha^k (c_{ij} - c_0) x_i^k + \varepsilon_{ij} \tag{1}$$

and that the utility from being assigned to nonpublic high school o is

$$u_{i,o} = F_o + \varepsilon_{io}. \tag{2}$$

Here $Y_j \equiv \{y_j\}$ is a vector of school j's observed characteristics; $X_i \equiv \{x_i\}$ is a vector of student i's observed characteristics; d_{ij} is the home–school distance;[26] F_o is the fixed effect of nonpublic high schools; and ε_{ij} and ε_{io} are i's idiosyncratic taste for (respectively) public high school j and nonpublic high schools. In the estimate, we assume that the home–school distance is additively separable and independent of unobserved student preferences; in addition, we normalize the coefficient d_{ij} for the home–school distance to be -1.[27]

We do not present the random coefficient model for estimating students' heterogeneous preferences for observed school characteristics (as in, e.g., Abdulkadiroğlu et al. 2015; Agarwal and Somaini 2018) owing to our data's limited variation. In China, a general high school's sole education goal is to prepare students for the college entrance exams. Except with regard to quality, schools' observed characteristics—for example, facilities—are fairly homogeneous. Even their teaching programs are fully controlled by the local education bureau. Furthermore, students who are qualified to gain seats in local public high schools exhibit similar preferences for schools (see Appendix H for details of students' survey responses). To avoid the mistake of choosing the wrong empirical model, we present an alternative random coefficient model in Appendix G and then compare the resulting estimates; the random coefficient model performs worse than does the nonrandom coefficient model on both the within-sample and the out-of-sample test.

We follow Abdulkadiroğlu et al. (2015) in not explicitly modeling an outside option. The reason for this choice is that, as mentioned in Section 3, no outside option can be observed in the current admission record. In addition, we make the following assumption.

Assumption 1. *The terms ε_{ij} and ε_{io} are independent of the explanatory variable, X_i, Y_j, d_{ij}, C, and F_o. Both ε_{ij} and ε_{io} are independent and identically distributed (i.i.d.) and exhibit a type I extreme value distribution with cumulative distribution function (CDF) $F(\varepsilon)$.*

[26] The road distance d_{ij} is calculated via Google Maps by inputting the focal school's address and the student's home address.

[27] Unlike admission to elementary and middle schools, the high school admission procedure does not consider the locations of school districts or homes. Hence we assume that, in this city, the school choice mechanism does not directly influence residential decisions or local housing prices.

We use both the administrative data and our survey data to estimate student preferences. The advantage of survey data is that our estimates can proceed without having to account for students' strategic behavior when they submit their ROLs. However, our survey data cannot reveal student preferences w.r.t. ZX options because the ZX policy was discontinued after 2013; thus, in 2014, all students paid the same basic tuition for all public high schools. As a result, α cannot be identified by the survey data. Hence we divide our estimation procedure into two steps. First, the survey data from 2014 are used to estimate the vector of parameters *unrelated* to the ZX option—that is, $\boldsymbol{\beta} = \{\beta\}$. Second, the vector $\boldsymbol{\alpha} = \{\alpha\}$ of parameters related to the ZX policy are estimated from the student ROLs submitted prior to 2014.

5.1 Step One: Estimating the Non-ZX-Related Parameters $\boldsymbol{\beta}$

In this step, we focus on the survey data without considering students' strategic behavior when submitting their ROLs. Each surveyed student ranked five schools that she believed herself capable of attending. This procedure implies that the student first selects the schools for which admission is a distinct possibility and then, after identifying those schools, ranks them. That process complicates our constructing a model of how these middle school graduates select schools in the first place. For example, if a school with a high admission cut-off does not make the surveyed student's list, then it is difficult to distinguish between (a) her preferring the listed schools to the focal school and (b) her thinking that admission to the high–cut-off school is not possible. From the evidence presented in Section 4.3, we conclude that the survey responses reflect students' true preferences—that is, conditional on their belief in the possibility of admission. To simplify the estimation process, we focus on the listed schools' ranks in the survey (i.e., without considering the unlisted schools). In other words, we do not attempt to infer the relative ranks of listed and unlisted schools.

While referring to the survey data, we use the rank-ordered logit model (Beggs et al. 1981) to estimate $\boldsymbol{\beta}$.[28] Given a surveyed student i's ranked school list $(j_1, \ldots, j_{l_i})_i$ of length $l_i \leq 5$, we conclude that j_1 is her favorite school among all the l_i schools on her survey list, that j_2 is her second-favorite school, and so on. The joint probability of these choices is

$$\Pr(u_{i,j_1} > u_{i,j_1} > \cdots > u_{i,j_{l_i}}) = \prod_{k=1}^{l_i-1} \frac{e^{\mu_{i,j_k}}}{e^{\mu_{i,j_k}} + e^{\mu_{i,j_{k+1}}} + \cdots + e^{\mu_{i,j_{l_i}}}}, \tag{3}$$

where $\mu_{i,j}$ is the deterministic component of $u_{i,j}$ or $u_{i,o}$.[29] Then the log-likelihood function

[28] Because $c_{ij} = c_0$ in this step, $\boldsymbol{\alpha}$ does not appear in the utility function.

[29] More precisely, $\mu_{i,j} = \sum_l \beta_l y_j^l + \sum_w \beta_w x_i^w y_j^w + \beta_D f(d_{ij}, Y_j)$ when j is a public high school and $\mu_{i,j} = F_o$ when j is not a public high school.

can be written as

$$\log L_1(\boldsymbol{\beta}) = \sum_{i=1}^{n}\sum_{k=1}^{l_j-1} \mu_{i,j_k} - \sum_{i=1}^{n}\sum_{k=1}^{l_i-1} \log\left(\sum_{s=k}^{l_i} e^{\mu_{i,j_s}}\right). \tag{4}$$

Now we can estimate $\boldsymbol{\beta}$ by using maximum likelihood estimation.[30]

5.2 Step Two: Estimating the ZX-Related Parameters $\boldsymbol{\alpha}$

In the second step, we estimate $\boldsymbol{\alpha}$ while considering students' strategic behavior in the admission procedure. After plugging the estimated $\hat{\boldsymbol{\beta}}$ into equations (1) and (2), we can rewrite student i's utility function as

$$u_{i,j,c} = \hat{u}_{i,j} + \sum_{k} \alpha^k (c_{ij} - c_0) x_i^k + \varepsilon_{ij}, \tag{5}$$

$$u_{i,o} = \hat{F}_o + \varepsilon_{io}, \tag{6}$$

where $\hat{u}_{i,j} = \sum_l \hat{\beta}^l y_j^l + \sum_w \hat{\beta}^w x_i^w y_j^w + \hat{\beta}^D f(d_{ij}, X_i, Y_j)$.

In light of the evidence (from Section 4.3) that few students report their true preferences when submitting ROLs, we model their strategic behavior by assuming that students submit ROLs that are optimal given a set of beliefs about their likelihood of being admitted. There is a growing literature indicating that students may form heterogeneous beliefs about such probabilities (Kapor et al. 2017) or make mistakes in ROLs (Hwang 2015; Hassidim et al. 2016; Artemov et al. 2017). However, there is no easy way to define "mistakes" in our administrative data: students submit their ROLs before taking the exams, and a student may have an accurate idea about admission cut-offs yet face an uncertainty about her exam performance. We therefore make the following assumption.

Assumption 2. *Students are fully informed about their own preferences, and they maximize their expected utility in a rational manner.*

Students' Decision Problem

At the start, each student submits an ROL $a_i = \{(j_i^1, v_i^1), (j_i^2, v_i^2), (j_i^3, v_i^3), t_i\}$; here $v_i^k \in \{0,1\}$ indicates whether student i selects the ZX option for her kth choice j_i^k, and $t_i \in \{0,1\}$ indicates whether i accepts a random assignment if she is rejected by all three of her chosen

[30] We assume that the utility function has an additively separable form; it is therefore easy to show that $\log L_1$ is globally concave in $\boldsymbol{\beta}$—from which it follows that there exists a unique maximum of the likelihood function.

schools. Next, each student takes the entrance exam and receives a score s_i. Student i's decision problem is to select the a_i that maximizes the expected payoff. Formally, we have

$$\max_{a_i \in A_i} \sum_{j \in \{j_i^k\}} \left[I_{hj} \left(\sum_{c \in C} P_{i,j,c} u_{i,j,c} \right) + I_{oj} P_{i,o} u_{i,o} \right] + \widetilde{P}_{a_i} \tilde{u}_i. \tag{7}$$

Here A_i is the set of all possible choices for student i; the terms I_{hj} and I_{oj} are indicators for whether school j is (respectively) a public or a nonpublic high school; $P_{i,j,c}$ represents the probability that student i is assigned to school j with tuition c; and $P_{i,o}$ is the probability that student i is assigned to a nonpublic high school. Finally, $\widetilde{P}_{a_i}$ and $\tilde{u}_i$ represent (respectively) the probability and payoff of student i being rejected by all three of her chosen schools.

Beliefs and Admission Probabilities

Students evaluate their likelihood of being admitted to each school before submitting their ROLs. Admission requires that the student's score be no less than the school's admission cut-off. Those cut-offs are announced to the public after the annual admission season. Compared with the previous year's admission cut-offs, none of the popular schools' cut-offs (with one exception) between 2011 and 2013 increased by more than 4% or decreased by more than 2% (see Figure 4 in Appendix B). Furthermore, the list of popular and leftover schools did not change across years. We therefore assume that students form the correct beliefs about admission cut-offs in the current year—that is, given the stability of those cut-offs and of the exam score distribution. An alternative assumption is that students use the previous year's admission cut-offs to form their beliefs (i.e., that students exhibit "adaptive expectations"). Estimated results based on this assumption are reported in Section 5.5.

Given student i's ROL a_i, she is assigned to her kth choice, school j, as a normal student if and only if her score s_i is no less than the school's cutoff as the kth choice, $\bar{S}_j^k$ and rejected by her $k-1$th choice j', formally, the probability can be written as

$$P_{i,j,c_0} = \Pr \left(\bar{S}_j^k \le s_i < (\bar{S}_{j'}^{k-1})^{(1-v_i^{k-1})} \left(\hat{S}_{j'}^{k-1} \right)^{v_i^{k-1}} \right), \tag{8}$$

where $\bar{S}_{j'}^{k-1}$ is school j''s cut-off as the $(k-1)$th choice for the normal students and $\hat{S}_{j'}^{k-1}$ is its cut-off for the ZX students.[31] The probability of being admitted by her kth choice as a ZX student with tuition c_t and $t \in \{1, 2, 3\}$ is

$$P_{i,j,c_t} = v_i^k \Pr \left(\max\{\hat{S}_j^k, \bar{S}_j^k - 10t\} \le s_i < \bar{S}_j^k - 10(t-1) \right). \tag{9}$$

[31] When $k = 1$, we put $\bar{S}_j^{k-1} = \infty$ and $v_i^{k-1} = 0$.

When j_i^k is a nonpublic school, the admission probability $P_{i,o}$ is a special case of equation (8) in which $\bar{S}_j^k$ equals the threshold S^*.

Taking the perspective of student i, we assume that she predicts her exam score will be $m_i + \eta_i$; here m_i represents either i's mock exam score or her true ability (by which she estimates her exam score) and η_i is the uncertainty. We assume that η_i is i.i.d. and distributed normally as $N(0, \delta)$. Note that m_i cannot be directly observed from the data. Instead, we use the student's actual exam score s_i as the estimate of m_i. We simplify our estimation process by setting $\delta = 20$, which is 3% of the full mark.[32] After we replace s_i with $s_i + \eta$ in equations (8) and (9), the admission probabilities can be expressed as the CDF of the standard normal distribution (see Appendix D for the functional forms).

Likelihood Function and Identification

A student's choice consists of the combination of her three choices and her ZX options. It follows that the size $|A_i|$ of student i's choice set is equal to $|(|J| \times 2)^3 \times 2|$; this amounts to more than 50,000 alternatives. To simplify calculations, we rule out a few weakly dominant strategies and thereby limit the choice set to $A_i' \subset A_i$, as described next.

First, if a student lists a leftover school as her first or second choice, then the rest of her choices should be blank. Hence no student will be admitted by a school that is listed after a leftover school. Second, if a student lists a popular school as her first or second choice, then her subsequent choice should *not* be blank. Third, no student's ROL can select a particular school more than once. Fourth, no student selects the ZX option for her third choice. According to the admission records, students are admitted by their third choice only when those choices are leftover schools, which admit all students as normal students.[33] Fifth, a student accepts the randomly assigned school if he is rejected by all of her listed schools. So if a student in those circumstances does not accept the randomly assigned school, then her only option is to attend a nonpublic high school. In the admission records, all nonpublic high schools have admission cut-offs that are below the threshold; in other words, their admission probability is equivalent to that of leftover high schools. The implication is that, if a student would rather attend a nonpublic high school than be randomly assigned to a leftover school, then he should list that nonpublic school as one of her three choices.[34]

[32]The estimated results when $\delta = 13.3$ (2% of the full mark), when $\delta = 26.6$ (4% of the full mark), and when $\delta = 33.35$ (5% of the full mark) are reported in Section 5.5.

[33]Selecting the ZX option for one's third choice does not affect the admission result. In the data set, we do not observe any student who was admitted as a ZX student for her third choice.

[34]In the admission records, 1.21% of the students did not accept the random assignment after being rejected by their preferred schools.

After excluding all these weakly dominated strategies, we can simplify our expression for student i's submitted ROL so that it reads as follows: $a_i = \{(j_i^1, v_i^1), (j_i^2, v_i^2), j_i^3\}$. Hence the choice set A_i' incorporates alternatives and so is significantly smaller than its parent set A_i.

Although we can observe the students' three choices in ROLs, their choices w.r.t. the ZX option cannot be observed from the admission records. All we know in that regard is whether a student is assigned to a school as a normal or a ZX student. Therefore, students' ZX options can be partially (or sometimes fully) inferred from their assignment results. Suppose, for example, that a student is assigned to her first choice as a ZX student; then we know she must have selected the ZX option for that choice. If she is assigned to the second choice but was qualified for admission by the first choice as a ZX student, then we can infer that she did not select the ZX option for the first choice. Hence observations can be categorized into three groups. The first group (G_1) includes only students whose ZX options in ROLs can be unambiguously inferred from the admission records data. The second group (G_2) comprises students whose decisions w.r.t. ZX options can be observed or inferred for either their first choice or second choice but not for both. We use $\tilde{a}_i$ to denote the partially inferred or observed choice of student i (i.e., when v_i^1 or v_i^2 is unknown).

For an observation in G_1, we write the probability of observing an ROL a_i as $\Pr(a_i \in A_i^*)$; here $A_i^* \subset A_i'$ is the optimal solution set of the student's problem in equation (7). For student i in G_2, we can observe (or infer) whether i selected the ZX option for her first choice but not for her second choice; however, we do know that he either: (a) selected the ZX option for her second choice, $a_i^+ = \{(j_i^1, v_i^1), (j_i^2, 1), (j_i^3)\}$; or (b) did not select that option, $a_i^- = \{(j_i^1, v_i^1), (j_i^2, 0), (j_i^3)\}$. Hence the probability of observing $\tilde{a}_i$ is $\Pr(a_i^+ \in A_i^*) + \Pr(a_i^- \in A_i^*)$. Similarly, if the ZX choice for the second choice is unknown then the probability of observing $\tilde{a}_i$ is $\Pr(a_i^+ \in A_i^*) + \Pr(a_i^- \in A_i^*)$, where $a_i^+ = \{(j_i^1, 1), (j_i^2, v^2), (j_i^3)\}$ and $a_i^- = \{(j_i^1, 0), (j_i^2, v^2), (j_i^3)\}$.

The total log-likelihood function for the entire sample can therefore be expressed as follows:

$$\log L_2(\boldsymbol{\alpha}) = \sum_{i \in G_1} \log(\Pr(a_i \in A_i^*)) + \sum_{i \in G_2} \log[\Pr(a_i^+ \in A_i^*) + \Pr(a_i^- \in A_i^*)]. \qquad (10)$$

Identification of the model's parameters is similar to that for a multinomial discrete choice model, which has been established based on general conditions (Matzkin 1993). Our model differs only in that each student considers the admission probabilities of schools and then chooses the option with the highest expected payoff (Calsamiglia et al. 2017). The identification power comes from the observed variation in choices by students who were rejected, as normal students, by either their first or second choices. When students submit their ROLs, they must decide whether or not to pay higher tuition in order to increase the probability of being admitted by their first or second choices. In a simplified example with only two schools (A and B), suppose that student i lists school A before school B. If she chooses the ZX option for school A, this implies that she would rather attend school A as a ZX student (and pay higher tuition) than attend school B as a normal student (assuming that school A rejects her as a normal student); otherwise, she should not choose the ZX option for school A (see Appendix C for a proof of the identification in this example). There is no closed-form solution to Equation (10).[35] We therefore estimate parameters using the maximal simulated likelihood estimate with the logit-smoothed accept–reject simulator (Train 2009, ch. 5); see our Appendix D for details.

5.3 Estimation Results

Table 3 presents estimated coefficients for the utility function. Columns 1 and 2 report the results when student–school interaction terms are not considered; column 4 gives results for the full model without the school fixed effect. We focus on column 3, which corresponds to the full model with the school fixed effect. Rows 2–4 of column 3 report student preferences regarding school quality. Students are classified into three groups based on their exam scores: high-scoring students, whose scores are above the 90th percentile; medium-scoring students, whose scores are between the 70th and 90th percentile; and low-scoring students, whose scores are below the 70th percentile yet above the threshold. The top students are much more sensitive to school quality than are students in the other two groups. For example, if school quality increases by 1 unit then high-scoring girls are willing to travel an additional distance of nearly 0.54 kilometers; the corresponding distances for medium- and low-score girls are 0.2 km and 0.18 km, respectively. In the same situation, high-scoring (resp., medium- and low-scoring) boys are willing to travel an additional 2.75 km (resp., 1.03 and 0.92 km).

If we compare the trade-off between school quality and tuition cost, different groups exhibit various attitudes toward purchasing seats. If school quality increases by 1 unit, then high-scoring students would be willing to pay an additional 226 yuan whereas medium- and low-scoring students would be willing to pay only an extra 93 yuan and 78 yuan, respectively.

Students' valuation of school capacity, which we normalize to 100 seats, also varies across groups. All students prefer small schools when other variables are fixed, but medium-scoring students dislike large schools the most. When school capacity decreases by 100 seats, medium-scoring students are willing to travel an additional 1.54 km yet high-scoring (resp., low-

[35] No such solution exists because (a) the distribution of the summation of a type I extreme distribution does not itself follow a type I extreme distribution and (b) the ROLs are only partially observed.

scoring) students are willing to travel only 0.94 km (resp., 1.19 km) farther.

Table 3 also reports our estimates for other parameters. Rows 6–8 of column 3 show that high-scoring students have a somewhat unfavorable attitude toward special classes; in contrast, such classes are viewed positively by the other two student groups. Rows 9–10 reveal that a student's utility from attending a school increases when her exam score is close to (i.e., within 15% of) the average for other students admitted there. This outcome reflects peer pressure in schools. Rows 15–16 of column 3 indicate that a school's provision of dormitory accommodations can reduce students' negative concerns about travel distance, especially for girls.

5.4 Model Fit

Next we examine how well our preference estimates match the data. We conduct within-sample and out-of-sample tests to check the aggregate-level matching patterns. Table 4 compares the actual and predicted admission cut-offs of each high school.[36]

For the within-sample test, column 2 of the table reports schools' predicted cut-offs for year 2013. With only two exceptions, the gaps between the actual and predicted cut-off are less than 1% of the full mark (665). Column 5 reports the schools' predicted cut-offs for 2012. In this year, the gaps between predicted and actual cut-offs are less than 5.2 points (0.78% of the full mark) in 12 out of 13 schools and are about 2.9% of the full mark for the other school. The predicted results also correctly identify all the leftover schools, for which the cut-off is 530.

For the out-of-sample test, we estimate our parameters for preferences using the procedure described in Section 5 but while excluding the 2012 data. Then, using the newly estimated parameters, we simulate the behavior of students based on their 2012 preference profiles. The schools' predicted cut-offs are reported in column 8 of Table 4; note that the results are strongly similar to those for the within-sample test (column 5).

We also explore the aggregate-level matching patterns for students' first two school choices (see Table 10 in Appendix B). For our within-sample test, the data show that 30.7% (resp. 29.7%) of students were admitted by their first-choice school in 2013 (resp. 2012); our predictions are, respectively, 30.5% and 30.4%. More specifically, for 2013 we predicted that 18.8% of students would be admitted by their first choice as normal students and 11.7% would be admitted as ZX students; these predictions are close to the actual respective values

[36]Reported results are the admission cut-offs for the first round. The actual second-round cut-offs of all popular schools are infinity while those of all leftover schools are equal to the threshold. Given that our predicted results correctly identify all popular and leftover schools, we report results only for the first-round cut-offs.

of 15.6% and 15%. For 2012, we overpredicted (by 8%) the total number of students who were admitted by their second choices; however, all other differences between the predicted patterns and their true values are within 6%. In the out-of-sample test, we overpredicted the second-choice admission rate by 7%, but all other differences between the predicted and actual patterns are within 5.3% (and most are within 3.5%).

5.5 Other Robustness Check

We first use the college admission rates of schools in 2014 to measure school quality; the results are reported in the first panel of Table 5. High-scoring students remain the group most sensitive to school quality—that is, they are the students willing to pay the most for the privilege of attending good schools. This sensitivity decreases with rising student exam scores. Other estimated coefficients are close to the results reported in Section 5.3.

The estimated results in Section 5.3 are based on the rational expectation that students can predict the correct admission probabilities of schools. Because students must submit their ROLs before taking the entrance exam, the primary source of information for students is reasonably assumed to be information from previous years (adaptive expectations). Column 1 in the second panel of Table 5 reports the estimated ZX-related parameters when students use the prior year's admission cut-offs to estimate their likelihood of being admitted.[37] The reported results are similar to those derived under rational expectations and likely reflect the stability of admission cut-offs across years.

The strategic behavior of students may vary as a function of uncertainty about their exam scores. Columns 2–4 in the second panel of Table 5 report the estimated ZX-related parameters when the standard deviation δ of the exam score is, respectively, 13.3 (2% of the full mark), 26.6 (4% of the full mark), and 33.35 (5% of the full mark). These results are similar to the pattern evidenced in Table 1: high-scoring students are more willing (than are students in the other groups) to pay higher tuition for the purpose of securing a seat in their preferred school.

6 Counterfactual Analysis

The controversial ZX policy was canceled in 2014. We cannot evaluate this policy by directly comparing welfare variations before and after the policy change because, following the policy's cancellation, the City Education Bureau did not simply add the original ZX quo-

[37] Our estimate of the non–ZX-related parameters does not rely on any assumptions about student beliefs.

ta to the normal quota; instead, it substantially increased the quota for early admission.[38] Therefore, we conduct simulations to compare how different assignment mechanisms affect the welfare of students and schools—especially as regards discontinuance of the option to purchase seats.[39]

Using the estimated preferences, we simulate the students' application lists. In the simulation, we use the profiles of students and schools from the 2014 administrative data. Since there were no ZX students in that year, we treat the normal admission quota as the corresponding school's total capacity. To analyze the welfare effect of different ZX quotas, we run our experiments under three different setups: with the ZX quota accounting for 10%, 30%, and 50% of the total quota when the focal mechanism includes the option to purchase seats.[40]

We use the matching outcomes under the DA mechanism as our benchmark. When the COSM is adopted to replace the DA mechanism, the different outcome can be used to explain the ZX policy's effect because both mechanisms are strategy-proof and stable. To evaluate the mechanisms actually adopted, we also analyze the welfare changes when the CPPS and BMPS mechanisms replace the DA mechanism. Under the COSM and the DA mechanism, we assume that students' ROLs report their true preferences; under other mechanisms, we create ROLs that reflect each student's best response in equilibrium (see Appendix E for details). We use 1,000 simulations in which each student experiences a different vector of random utility shocks.

6.1 Students' Welfare

For each tested mechanism, we use the welfare-equalizing tuition adjustment Δyuan (Calsamiglia et al. 2017). This adjustment is defined as the amount of tuition that a student must pay (or be credited) under the DA mechanism to reach the utility level achieved under the replacement mechanism being tested.[41]

[38] In particular, the ratios of early admission quotas to normal quotas increased from 0.48 in 2013 to 0.64 in 2014.

[39] Pathak and Shi (2017) analyze the effectiveness of such counterfactual analyses of the school choice problem.

[40] The local government required that no school could admit ZX students totaling more than 20% of its capacity.

[41] All other parameters (except for tuition) remain fixed. Formally, let $u_{ij} = U(c_{ij})$ be i's utility derived from admittance to school j when paying tuition c_{ij} under the DA mechanism. If that mechanism is replaced by the focal new mechanism—in which case student i is assigned to school j' and achieves utility $u_{ij'}$—then the welfare-equalizing tuition adjustment (Δyuan) is the solution to $U(c_{ij} + \Delta\text{yuan}) = u_{ij'}$.

When the DA mechanism is replaced by the COSM, the average welfare of students falls as the ZX quota rises. Overall, students under the DA mechanism must pay additional 71 yuan (on average) to achieve the same utility level as under the COSM when the ZX quota is 10% of the total quota (see Figure 2a). This loss due to increased tuition becomes 380 yuan (resp., 856 yuan) when the ZX quota rises to 30% (resp., 50%). Different student groups experience a similar welfare loss that grows larger as the ZX quota increases. Namely, the medium-scoring student group suffers more than other groups: the tuition for the former students increases by 127 yuan when the ZX quota is 10% and by 1,209 yuan when the ZX quota is 50% of the total.

Table 6 identifies the percentage of "winners" (whose welfare increase) and "losers" (whose welfare decrease) when the DA mechanism is replaced. Under the COSM, the proportion of winners never exceeds 5.2% irrespective of the ZX quota and of which student group is considered. However, the proportion of losers is much greater than that of winners in all cases, especially when the ZX quota is increased to 50%; in that case, there are 20 times more losers than winners across all student groups. More precisely: for a 10% ZX quota, 3% of the high-scoring students attend their most-preferred schools by paying higher tuition (see Table 7); at the same time, nearly 4% of students from the same group are "priced out" and so can attend only their less-preferred schools. Among the medium-scoring students, 6% of them pay more to attend their preferred schools, 4% pay more to stay in their assigned schools, and 12% are priced out. For the low-scoring students, 1.6% are admitted by their preferred schools and 5.6% are priced out. When the ZX quota increases from 10% to 50%, only 5% (resp. 11%) of medium-scoring (resp. high-scoring) students get into their preferred schools and almost 60% (resp. 24%) of them are priced out.

These results constitute evidence for two main effects of the option to purchase seats. First, the number of the priced-out students and of those who must pay higher tuition to save their seats is much greater than the number of students who are accepted by their preferred schools; it is this effect that accounts for the welfare loss across all student groups. The second effect is that the seat-purchasing option prices out a large proportion of students—and especially of medium-scoring ones—when quotas are large.[42]

When the DA mechanism is replaced with the CPPS mechanism, the changes in student welfare follow much the same pattern as in the COSM case but with two differences (Figure 2b). One difference is that the welfare losses across all student groups are less than their counterparts under the COSM. Second, the medium-scoring student group experiences a slightly welfare gain equivalent to a 41-yuan reduction in tuition when the ZX quota is

[42]The amounts of the welfare gains and losses are given by (respectively) Table 11 and Table 12 in Appendix F.

10%. Table 6 shows that—across three different ZX quotas—both medium- and low-scoring student groups have more winners and fewer losers than do the same groups under the COSM. Thus more students overall attend their preferred schools by paying higher tuition. Table 7 shows, when the ZX quota is large (50%), slightly more high-scoring students are priced out but much fewer medium-scoring (37%) and low-scoring (9%) students are priced out. In addition, for a 10% ZX quota, 6% of medium-scoring students can attend their preferred schools without paying higher tuition, which explains this group's average welfare gain.

When the BMPS is adopted to replace the DA mechanism, the total average welfare loss decreases more than it does under the CPPS mechanism for all three evaluated ZX quotas (Figure 2c). Meanwhile, the low-scoring groups experience (on average) the same welfare gains for all ZX quota cases. Much as under the CPPS mechanism, both medium- and low-scoring student groups have—regardless of the ZX quota—more winners and fewer losers than do the same groups under the COSM (Table 6). So when the ZX quota is 10%, there are 16% more medium-scoring students and 13% more low-scoring students who are admitted to their preferred schools without paying higher tuition; when the ZX quota is 50%, there are 30% (resp. 3%) fewer medium-scoring (resp. low-scoring) students who are priced out of those schools.

The outcomes under the CPPS and BMPS mechanisms imply that, when the mechanism is not strategy-proof, students can game the system so that more relative low-scoring students can attend their preferred schools and fewer of them are priced out.

6.2 Schools' Welfare

There are several reasons why changes in schools' welfare have been ignored in the school choice literature. First, no mechanism can be optimal for both sides of a matching mechanism (Gale and Shapley 1962). Second, improved matching results for students (e.g., increasing their average welfare) is the primary goal of most studies addressing the school choice problem; this generalization holds in particular for public school systems. Third, the welfare of public schools—especially at the elementary and secondary level—is difficult to measure when their admission criteria (e.g., walking distance and/or whether siblings are in the same school) are "coarse".

Nevertheless, it is important to analyze schools' welfare. There is intense competition among schools with regard to admissions, and even more so following compulsory education. Although schools cannot make strategic moves in a centralized admission system, they still prefer to admit students of high quality. It follows that schools, which suffer a welfare loss when admission mechanisms undergo certain types of reform, have a strong incentive to block such reforms. When exam scores are used as the criterion for admission, it is easy to compare how different admission mechanisms affect schools' welfare.

The purpose of allowing schools to sell seats is to increase their profit. Hence the ZX policy offers two ways of analyzing school welfare: the quality of admitted students and the tuition collected by schools. As illustrated by the example in Section 2, the ZX policy may impose a trade-off. On the one hand, allowing students to buy seats will likely increase the income of schools; on the other hand, seat purchasing has the effect of dispersing high-quality students more widely among different schools.

Figure 3 plots the changes in quality and tuition for an upper-tier school (#183), a middle-tier school (#185), and a lower-tier school (#142). For the *upper-tier* school (Figure 3a), the collected fees increase in proportion to the ZX quota when the DA mechanism is replaced by the COSM. Across all mechanisms, if seats can be purchased then this school collects approximately 10% more tuition when the ZX quota is 10% and collects 30% more when the quota is increased to 30%. When the ZX quota is 50%, the tuition collected increases by more than 60% under the COSM and BMPS mechanisms but only by about 30% under CPPS. When the DA mechanism is replaced by the COSM in the case of a 10% ZX quota, student quality (as measured by the percentage grades) declines by only 0.06%, although it declines by an additional 0.51% (resp. 1.2%) when the ZX quota is 30% (resp. 50%); see Figure 3d. If either the CPPS or BMPS mechanism is adopted to replace the DA mechanism, then the decline in student quality does not exceed 1.05%. In view of our findings for other upper-tier schools (see Table 13 in Appendix F), the demand for elite schools is clearly such that they can profit significantly by selling seats yet without lowering the quality of admitted students.

Figures 3b and 3e illustrate the case of a *middle-tier* school. When the ZX quota is 10%, the collected tuition increases by about 10% under all three mechanisms that include the option to purchase seats. If the ZX quota is increased to 50% then, under the COSM and BMPS mechanisms, we observe a 43%–48% increase in collected tuition as compared with the baseline case and an increase of more than 60% under the CPPS mechanism. As for student quality, the change from the DA mechanism to the COSM does not alter the quality of admitted students by more than 1% when the ZX quota is 10% or 30%; when the ZX quota increases to 50%, however, student quality increases by 3.6%. If the DA mechanism is replaced by the CPPS mechanism, then student quality declines by more than 1% and decreases even further as the ZX quota increases. Under the BMPS mechanism, student quality is reduced by 4%–5%. When combined with our findings for the other middle-tier schools (see Table 13 in Appendix F), the results indicate that a seat-purchasing option can generate significant profits for most middle-tier schools. That said, these schools may then experience large variations—in both the positive and negative direction—in the quality of their admitted students.

Finally, Figures 3b and 3e plot our findings for a *lower-tier* school. When the ZX quota is 10%, the school collects approximately 13% higher tuition when the DA mechanism is replaced by the COSM; when the ZX quota increases to 30% (resp. 50%), this school collects 60% (resp. 237%) higher tuition. At the same time, student quality decreases slightly with increasing ZX quotas. The large change in tuition collection indicates that a number of students switch from middle-tier schools to low-tier schools to fill their empty seats when the ZX quota increases. When the CPPS mechanism is used to replace the DA mechanism, tuition collection experiences a similar trend as that under the COSM but at a reduced magnitude. In the CPPS case, student quality decreases slightly when the ZX quota increases. When the DA mechanism is replaced by the BMPS, this school collects 3.38% more tuition when the ZX quota is 10% but actually collects less tuition when the ZX quota increases to 30% or 50%. Meanwhile, student quality at this school declines somewhat. In short: for lower-tier schools, the ZX policy's effects on both collected tuition and student quality are uncertain.

7 Conclusion

Our paper investigates a controversial but long-ignored Chinese school choice policy, *Ze Xiao*. This policy allowed students to "purchase" seats at their desired schools by paying higher tuition. We find that the associated matching mechanisms are not strategy-proof and likely resulted in unstable matching outcomes. We combine high school admission records with survey data from China to estimate student preferences over schools and tuition. Our results indicate that high-scoring students are more willing than other students to pay an extra cost (higher tuition) to attend their preferred schools.

Using the estimated preferences, we conduct counterfactual experiments to evaluate the welfare consequences of the *Ze Xiao* policy. We find that, when the strategy-proof COSM replaces the deferred acceptance mechanism, students' welfare decreases across all student groups. However, replacing the DA mechanism with a non–strategy-proof mechanism (e.g., CPPS or BMPS) may alleviate those welfare losses—especially for medium-scoring students—because then more students can attend their preferred school by gaming the system. From the school's perspective, the seat-purchasing option helps upper-tier schools collect significantly more tuition and with only a limited decline (relative to the DA mechanism) in the quality of their admitted students. Yet for middle-tier schools, seat purchasing leads not only to a substantial increase in collected tuition but also to considerable uncertainty about the resulting quality of their admitted students. For lower-tier schools, the ZX policy's effects on both collected tuition and student quality are uncertain.

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Table 1: School Characteristics

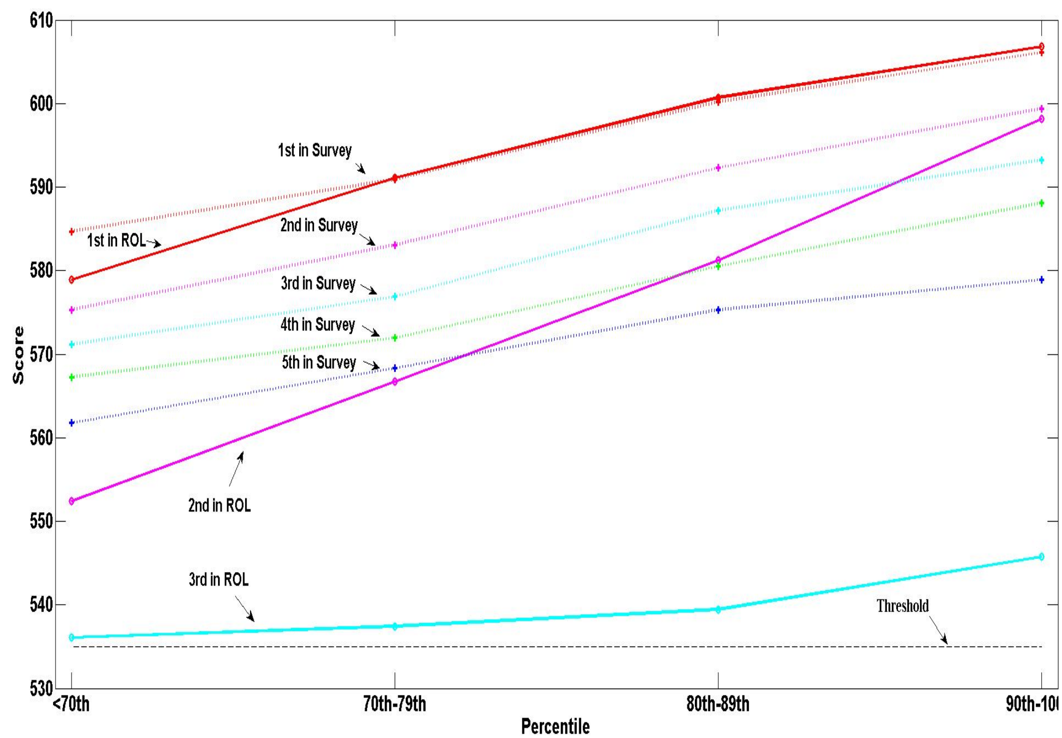
	2012			2013			2014		
	mean(s.d)	max	min	mean(s.d)	max	min	mean(s.d)	max	min
Quality	80(12)	97	64	81(12)	97	64	83 (11)	97	66
Normal Quota	215.9 (182.6)	600	40	197 (183.8)	600	40	186.4 (164.5)	600	40
ZX Quota	94.8(37.9)	146	22	101.3(33.1)	142	37			
♯ of schools with dorms	9			11			13		
♯ of schools	16			18			19		

Notes: Schools and special classes that did not admit ZX students are excluded when the ZX quota is calculated.

Table 2: Student Characteristics

	2012		2013		2014	
Score Distributions						
Percentile	Abs. Scores		Abs. Scores		Abs. Scores	
90th	597		590.5		598	
80th	579.5		572		578	
70th	562		553		557.5	
60th	542		531		532.5	
Threshold	535		530		535	
	Freq.	Percent	Freq.	Percent	Freq.	Percent
Rank Ordered Lists						
3 Schools	3696	94.33%	3793	95.04%	3100	93.71%
2 Schools	191	4.87%	167	4.18%	189	5.71%
1 Schools	31	0.79%	31	0.78%	19	0.57%
Assignment Results						
1st Choice	1153	29.43%	1227	30.74%	875	26.45%
ZX students	542	13.83%	599	15.01%		
2nd Choice	1441	36.78%	1545	38.71%	1290	39%
ZX students	217	5.54%	262	6.56%		
3rd Choice	803	20.50%	751	18.82%	565	17.08%
ZX students	0	0	0	0		
Rejected by all 3	521	13.30%	460	11.53%	578	17.47%
Total observations	3918		3991		3308	

Figure 1: Average Admission Cutoffs of Schools: Survey versus ROLs



Notes: The y-axis represents absolute scores, and the x-axis represents the students' exam scores in percentile. The threshold for public high school admission is 535 (60.95 percentile) in 2014.

Table 3: Preference Parameters

	No student interactions		With student interactions	
	(1)	(2)	(3)	(4)
Quality	0.835***	0.296***		
	(0.011)	(0.020)		
Quality × H			0.539***	0.688***
			(0.155)	0.032
Quality × M			0.201***	0.376 ***
			(0.038)	(0.012)
Quality × L			0.181***	0.361***
			(0.030)	(0.014)
Special class	-1.006***	-2.121**		
	(0.325)	(1.015)		
Special class × H			-6.675***	-2.657***
			(1.972)	(0.560)
Special class × M			0.602	1.204***
			(1.592)	(0.342)
Special class × L			6.504	5.300***
			(5.591)	(1.193)
Score range			0.597	0.216
			(0.430)	(0.183)
Score range × Male			0.315	0.898***
			(0.550)	(0.220)
Same district			-1.896***	-2.401***
			(0.247)	(0.107)
Same district × Male			1.739***	2.586***
			(0.309)	(0.143)
Distance	-1	-1	-1	-1
Distance × Male			0.804***	0.933***
			(0.034)	(0.010)
Dorm	-3.924***	4.253***	4.445***	-0.907***
	(0.119)	(0.967)	(1.095)	(0.137)
Dorm × Male			0.684*	0.756***
			(0.307)	(0.164)
Capacity	-0.011	-1.969***		
	(0.055)	(0.136)		
Capacity × H			-0.941	0.217
			(0.835)	(0.318)
Capacity × M			-1.542***	-0.632***
			(0.291)	(0.081)
Capacity × L			-1.190***	-0.540***
			(0.237)	(0.062)
Cost	-2.878***	-2.370***		
	(0.001)	(0.002)		
Cost × H			-2.388***	-1.676***
			(0.008)	(0.004)
Cost × M			-2.156***	-1.910***
			(0.003)	(0.003)
Cost × L			-2.309***	-2.422***
			(0.007)	(0.004)
Non-public high school	43.909***	2.005 *	1.347*	13.364***
	(0.946)	(0.799)	(0.653)	(1.115)
School Fixed Effect		Y	Y	

Notes: Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Distance is measured by kilometer. Both normal and ZX quotas are normalized to 100 seats. Tuition is normalized to 1000 Yuan. H, M and L represent high-scoring, medium-scoring and low-scoring students respectively.

Table 4: Admission Cutoffs

	Within Sample						Out of Sample		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
School ID	True 2013	Predicted	Diff.	True 2012	Predicted	Diff.	True 2012	Predicted	Diff.
141	604.0	597.8	6.2	607.0	603.0	4.0	607.0	602.8	4.2
142*	530.0	530.0	0.0	535.0	535.0	0.0	535.0	535.0	0.0
147	552.5	562.6	-10.1	555.5	560.6	-5.1	555.5	559.8	-4.3
167	590.0	588.6	1.4	592.5	592.9	-0.4	592.5	592.1	0.4
173*	530.0	530.0	0.0	535.0	535.0	0.0	535.0	535.0	0.0
179	565.0	572.8	-7.8	571.5	570.9	0.6	571.5	570.7	0.79
181*	530.0	530.0	0.0	535.0	535.0	0.0	535.0	535.0	0.0
183	611.0	605.1	5.9	617.0	612.2	4.9	617.0	612.2	4.8
184*	530.0	530.0	0.0	535.0	535.0	0.0	535.0	535.0	0.0
185	580.0	576.1	3.9	583.0	580.6	2.4	583.0	580.5	2.5
186	578.0	574.0	4	583.0	577.3	5.7	583.0	577.2	5.8
187	594.5	595.1	-0.6	599.5	600.1	-0.6	599.5	599.7	-0.2
188	575.0	580.1	-5.1	571.5	591.1	-19.6	571.5	590.5	-19

Notes: This table indicates the within- and out-of-sample tests for the schools' cutoffs. The full mark is 665. The threshold is 535 in 2012 and 530 in 2013. * indicates the leftover schools with cutoff equal to the threshold.

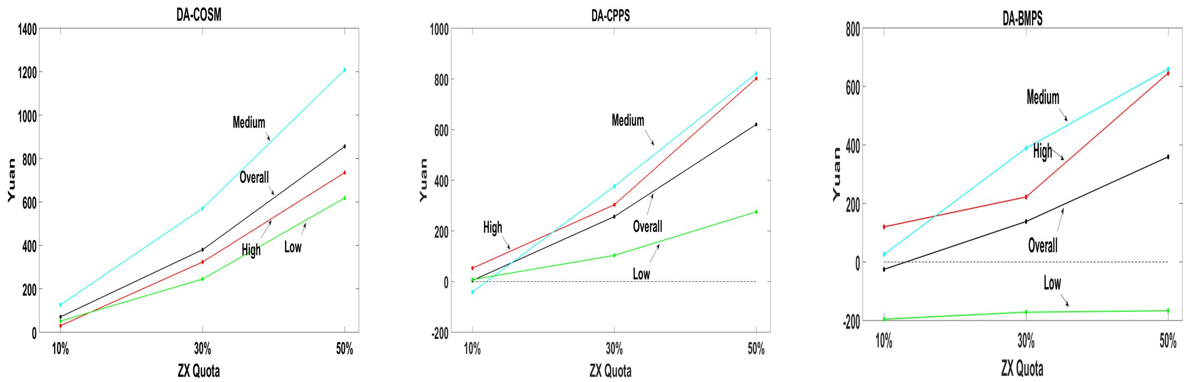
Table 5

Quality × H	0.245***	Capacity × H	0.467
	(0.057)		0.731
Quality × M	0.114***	Capacity × M	-0.544**
	(0.029)		(0.229)
Quality × L	0.074***	Capacity × L	-0.396*
	(0.029)		(0.194)
Special class × H	-6.617***	Cost × H	-2.899***
	(3.104)		(0.006)
Special class × M	1.324**	Cost × M	-2.682***
	(2.876)		(0.002)
Special class × L	6.698	Cost × L	-2.856***
	(12.089)		(0.015)
Score range	0.221	Distance	-1
	(0.416)		
Score range × Male	0.077	Distance × Male	0.787
	(0.497)		(0.035)
Same district	-1.873***	Dorm	5.364
	(0.241)		(1.347)
Same district × Male	1.698***	Dorm× Male	0.519
	(0.282)		(0.299)
Non-public high school	-6.602***	School Fixed Effect	Y
	(1.056)		

	(1)	(2)	(3)	(4)
Cost × H	-2.27***	-2.41***	-2.58***	-2.75***
	(0.007)	(0.005)	(0.018)	(0.011)
Cost × M	-2.01***	-2.03***	-2.27***	-2.50***
	(0.001)	(0.003)	(0.001)	(0.007)
Cost × L	-2.12***	-2.39***	-2.41***	-2.62***
	(0.007)	(0.004)	(0.009)	(0.01)
School Fixed Effect	Y	Y	Y	Y

Notes: The first panel is the estimated results based on the college admission rate as the school quality measure. The second panel are the estimated results for different assumptions about students' behaviors in ROLs. Col 1 represents the adaptive expectation assumption. Col 2-4 represents the s.d. of the uncertainty of exam score are 13.3, 26.6, and 33.35 respectively. Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Distance is measured by kilometer. H, M and L represent high-scoring, medium-scoring and low-scoring students respectively.

Figure 2: Welfare Change



(a) DA-COSM (b) DA-CPPS (c) DA-BMPS

Notes: These figures represents the welfare change when DA is replaced by another mechanism measured by the welfare-equalizing tuition. The y-axis represents the change of Yuan, and the x-axis represents the ZX quota.

Table 6: Winners and Losers (%)

	DA-COSM						DA-CPPS						DA-BMPS					
	10%		30%		50%		10%		30%		50%		10%		30%		50%	
	W	L	W	L	W	L	W	L	W	L	W	L	W	L	W	L	W	L
Overall	3.23	10.25	3.27	37.27	1.73	64.21	6.64	11.17	4.84	26.06	3.04	40.04	15.43	16.16	12.30	24.10	11.45	33.91
High	3.02	5.32	3.79	29.57	2.41	54.00	3.33	9.40	3.43	28.66	2.50	53.36	6.13	15.82	5.34	28.18	3.16	49.40
Medium	5.11	18.26	5.11	54.18	2.74	79.98	12.55	20.01	8.42	43.69	5.25	57.28	20.93	28.18	12.92	41.30	12.23	49.70
Low	1.59	6.68	1.07	27.48	0.18	57.60	3.71	4.16	2.59	7.01	1.36	12.45	17.90	4.93	17.51	4.23	17.61	5.87

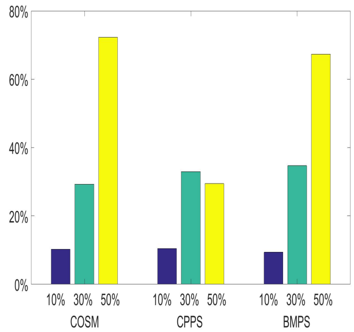
Notes: This table indicates the percentage change in the number of students whose utilities increase (winners) or decrease (losers) when the DA mechanism is replaced by the COSM, CPPS, and BMPS mechanisms. "W" represents winners, and "L" represents losers. For each mechanism change, utility changes are measured in three scenarios in which the ZX quotas are 10%, 30%, and 50% of the total quotas. "High" represents students whose scores are above the 90th percentile, 'Medium" represents students whose scores are between the 70th and 90th percentiles, and "Low" represents students whose scores are below the 70th percentile and above the threshold.

Table 7: Changes of Matching Assignments under the Purchasing Seats Option(%)

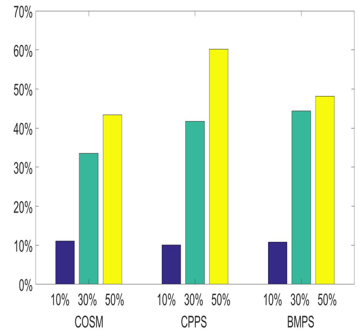
		ZX quota 10%				ZX quota 30%				ZX quota 50%			
		Same ZX	Better Normal	Better ZX	Worse Normal	Same ZX	Better Normal	Better ZX	Worse Normal	Same ZX	Better Normal	Better ZX	Worse Normal
COSM	H	1.43	0.00	3.02	3.90	10.52	0.00	5.58	17.27	20.36	0.00	11.16	23.93
	M	4.25	0.00	6.59	12.54	15.96	0.00	9.55	33.70	14.46	0.00	5.47	59.71
	L	0.98	0.00	1.63	5.66	4.06	0.00	1.10	23.38	4.13	0.00	0.30	53.36
CPPS	H	0.64	0.00	3.42	8.68	7.52	0.14	5.75	18.58	19.96	0.31	5.75	24.88
	M	0.49	6.07	8.03	17.97	8.18	1.98	11.04	30.61	13.38	2.03	8.05	37.29
	L	0.09	2.34	1.92	3.35	0.46	0.44	4.31	4.39	0.74	0.53	3.41	9.12
BMPS	H	0.00	3.63	4.06	14.25	8.37	0.01	9.42	15.66	27.40	0.01	9.53	15.26
	M	0.00	16.41	6.87	25.83	5.96	7.24	11.13	29.89	10.89	7.92	12.00	30.71
	L	0.04	13.38	5.22	4.19	0.00	12.94	6.67	2.13	0.19	14.70	5.44	3.16

Notes: This table indicates the percentage change in the number of students whose assignments are different under the purchasing seats option, when the DA mechanism is replaced by the COSM, CPPS, and BMPS mechanisms. When DA is replaced by another mechanism, "Same" means the student is assigned to the same school, "Better" represents the student is assigned to a more preferred school, and "Worse" represents the student is assigned to a less preferred school. "ZX" and "Normal"represents the student pays the basic and higher tuition respectively. "H", "M", and "L" represent high-scoring, medium-scoring and low-scoring students respectively.

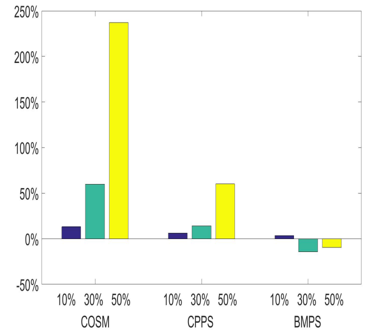
Figure 3: Student Quality and Tuition Collection



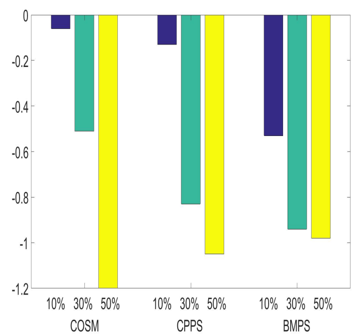
(a) 183 Tuition Collection



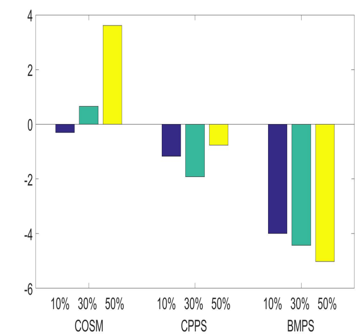
(b) 185 Tuition Collection



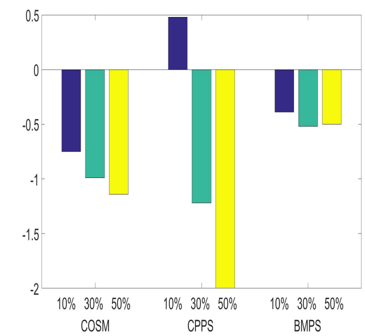
(c) 142 Tuition Collection



(d) 183 Student Quality



(e) 185 Student Quality



(f) 142 Student Quality

Notes: These figures represents the change of student quality and tuition collection of three schools when DA is replaced by another mechanism. The y-axis of Figure 3a, 3b and 3c are the percentage change of the tuition collected by schools. The y-axis of Figure 3d, 3e and 3f are the student quality measured by admitted students' percentage grades.

Appendices

Not for Publication

A Mathematical Proofs

Example to Indicate that the CPPS mechanism is not strategy-proof

For the CPPS mechanism with permanency-execution period $(e_1, e_2, ...)$ with $e_1 \geq 1$. There are three students i_1, i_2, i_3 and three high schools j_1, j_2, j_3 with one ZX seat each school and no normal seat. Students are ordered as $i_1 \succ i_2 \succ i_3$ by schools under the normal priority. Suppose that the true preference of student i_3 over schools and tuitions are as follows:

$$(j_1, c_0)\pi_{i_3}(j_2, c_0)\pi_{i_3}(j_1, c_1)\pi_{i_3}(j_2, c_1)\pi_{i_3}(j_3, c_0).$$

So student i_3's true preference over schools is $j_1\widetilde{\pi}_{i_3}j_2\widetilde{\pi}_{i_3}j_3$.

We need to show that no truthful strategy weakly dominates all other strategies.

Case 1: i_3 chooses the ZX option for j_1.

Given i_2 and i_3 choose the same strategy as $\{(j_1, 0), (j_3, 0), (j_2, 0)\}$, where 1 represents choosing the ZX option for the school and 0 otherwise.

If i_3 chooses the strategy as $\{(j_1, 1), (j_2, 0), (j_3, 0)\}$, then i_3 will receive the assignment (j_1, c_1). If i_3 switches to the strategy $\{(j_2, 0), (j_1, 1), (j_3, 0)\}$, she gets better off by receiving the assignment (j_2, c_0).

Case 2: i_3 does not choose the ZX option for j_1.

Given i_1's strategy as $\{(j_1, 0), (j_2, 0), (j_3, 0)\}$, and i_2's strategy as $\{(j_2, 0), (j_1, 0), (j_3, 0)\}$.

Subcase 2.1: $e_1 > 1$.

i_3 cannot receive an assignment better than (j_2, c_1) if she put j_1 as the first choice and does not choose the ZX option for it, because her normal priority is lower than i_1 and i_2. In this situation, if i_3 switches to the strategy $\{(j_2, 0), (j_1, 1), (j_3, 0)\}$, she gets better off by receiving the allocation (j_1, c_1).

Subcase 2.2: $e_1 = 1$.

In this mechanism, i_3 will be assigned to j_3 if she put j_1 as the first choice. If she switches to the strategy $\{(j_2, 1), (j_1, 0), (j_3, 0)\}$, then she gets better off by receiving the allocation (j_2, c_1).

Therefore, revealing the true preference over schools may not be a dominant strategy for i_3.

Proof of Proposition 1. There are four students i_1, i_2, i_3, i_4 and four schools j_1, j_2, j_3, j_4 with one ZX seat each and no normal seat. Schools order the students in the same way as $i_1 \succ i_2 \succ i_3 \succ i_4$. Students' preferences are as follows:

π_{i_1}: $(j_1, c_0)\pi_{i_1}(j_2, c_0)\pi_{i_1}(j_1, c_1)\pi_{i_1}(j_3, c_0)\cdots$

π_{i_2}: $(j_1, c_0)\pi_{i_2}(j_1, c_1)\pi_{i_2}(j_2, c_0)\pi_{i_2}(j_2, c_1)\pi_{i_2}(j_4, c_0)\pi_{i_2}(j_4, c_1)\pi_{i_2}(j_3, c_0)\pi_{i_2}(j_3, c_1)$.

π_{i_3}: $(j_1, c_0)\pi_{i_3}(j_3, c_0)\pi_{i_3}(j_1, c_1)\pi_{i_3}(j_2, c_0)\pi_{i_3}(j_3, c_1)\pi_{i_3}(j_2, c_1)\pi_{i_3}(j_4, c_0)\pi_{i_3}(j_4, c_1)$.

π_{i_4}: $(j_4, c_0)\pi_{i_4}(j_2, c_0)\pi_{i_4}(j_4, c_1)\pi_{i_4}(j_2, c_1)\cdots$

Consider the following strategy profile under the CPPS mechanism:

$a_{i_1} = \{(j_1, 1), (j_2, 0), (j_3, 0), (j_4, 0)\}$,

$a_{i_2} = \{(j_1, 0), (j_2, 0), (j_4, 1), (j_3, 0)\}$,

$a_{i_3} = \{(j_1, 1), (j_3, 0), (j_2, 0), (j_4, 0)\}$,

$a_{i_4} = \{(j_4, 0), (j_2, 1), (j_1, 0), (j_3, 0)\}$.

Then the matching outcome is

$\{(i_1, j_1, c_1), (i_2, j_2, c_0), (i_3, j_3, c_0), (i_4, j_4, c_0)\}$.

This strategy profile is a Nash equilibrium but not stable. Because i_1 prefer (j_2, c_0) to her assignment (j_1, c_1), and under the normal priority j_2 prefers i_1 to i_2. Furthermore, this outcome is Pareto dominated by the outcome of the COSM.

$\{(i_1, j_2, c_0), (i_2, j_1, c_1), (i_3, j_3, c_0), (i_4, j_4, c_0)\}$. □

Proof of Proposition 2. Part 1: For any Nash equilibrium strategy profile $(a_1, ..., a_n)$ and matching outcome τ of the BMPS mechanism, suppose τ is not stable under the true preference. Then there is an contract (i, j, c) such that student i prefers assignment (j, c) to her assignment in τ and either school j has an empty seat for tuition c or i has higher priority at school j than another student who receives a seat with tuition c. In the first case, the unstable matching implies i does not put j as the first choice if $c = c_0$, then i can move school j to the first choice and receives the assignment (j, c). In the second case, if $c = c_1$, the unstable matching implies either i does not choose j as the first choice and choose the ZX option for it. Then i can put j as the first choice and choose the ZX option for it, and i can receive the assignment (j, c). In either case, student i has the incentive to deviate, so the matching result is not an equilibrium.

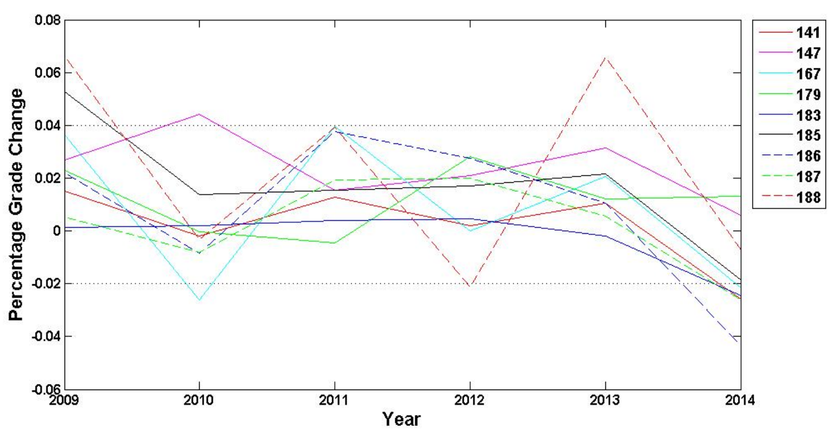
For a stable matching outcome τ, student i's assignment is (j, c). Then consider a strategy profile A as follow, if $c = c_0$, then student i put j as the first choice, if $c = c_1$, then student i put j as the first choice and choose the ZX option for it. Under this profile, every student receives the assignment in the first round and receives the same assignment as in τ. For student i, if she prefer an assignment (j', c') to the current assignment (j, c), since τ is stable, it implies the seats with tuition c' in school j' have assigned to other students who have higher priority to receive the seats. When $c' = c_1$, since the students who receive assignment (j', c') must put j' as the first choices and choose the ZX options for j', therefore even if student i put j as the first choice and choose the ZX option for j', she still cannot receive the assignment (j', c'). When $c' = c_0$, similarly, putting j' as the first choice cannot help i to receive (j', c_0). Therefore, student i has no way to deviate to get better assignment, and the strategy profile A is a Nash equilibrium.

Part 2 is straightforward, because Proposition 3 in Sönmez and Switzer (2013) have proven that students prefer the outcome under the COSM to any stable outcomes.

□

B More Results of Summary Statistics

Figure 4: Fluctuation of Admission Cutoffs



Notes: This figure indicates the fluctuation of admission cutoffs of schools as measured by percentage grade. The y-axis represents the change in school cutoff from the previous year, and the x-axis represents the year.

Table 8: Survey Length

	Freq.	Percent
5 schools	900	62.20%
4 schools	242	16.72%
3 schools	130	8.98%
2 schools	175	12.09%
Total	1447	100%

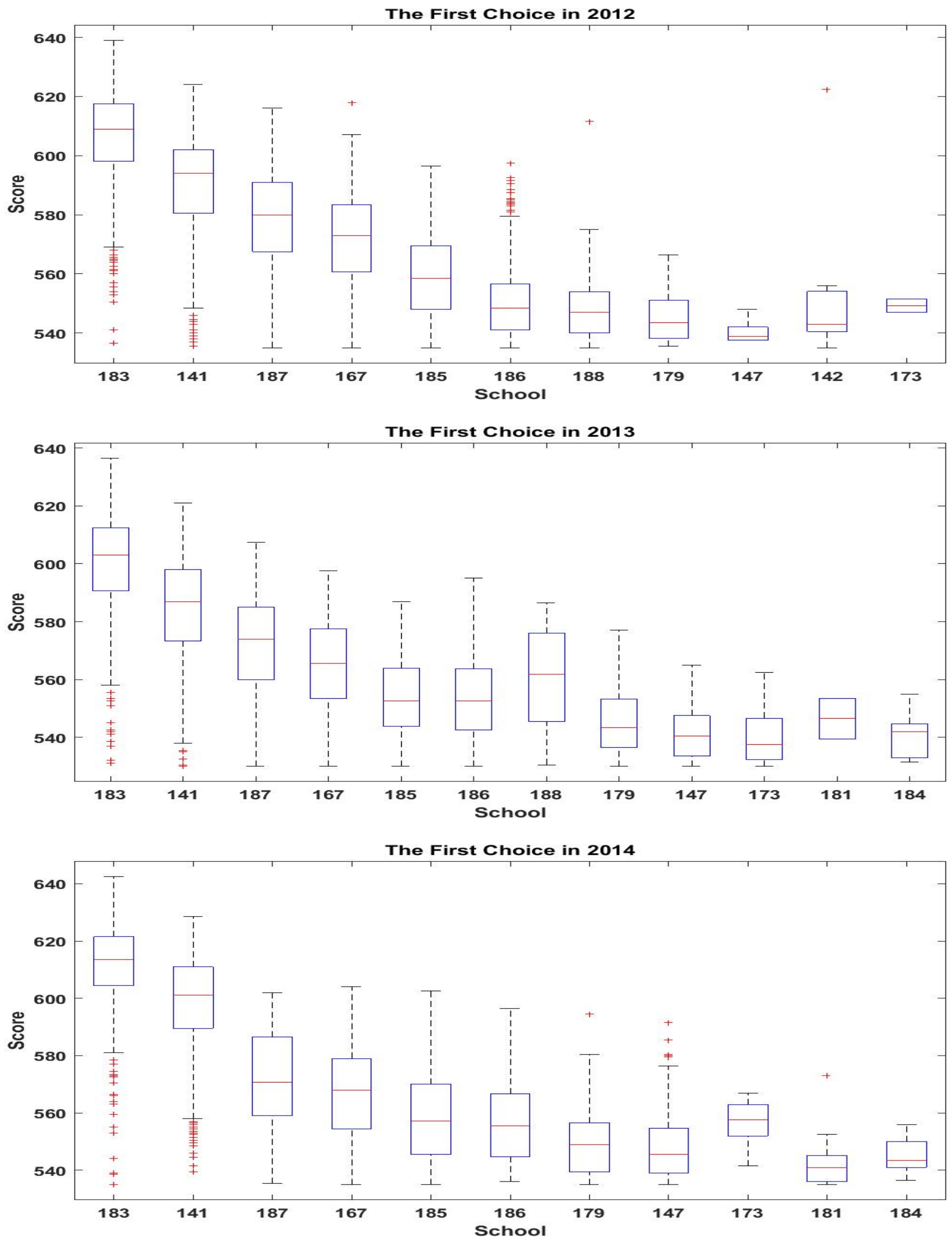
Notes: This table indicates how many schools surveyed students listed.

Table 9: Distance Distribution (km)

	Overall	2014	2013	2012
1st Choice	8.87	8.68	8.91	9.01
	(6.04)	(6.18)	(6.06)	(5.90)
2nd Choice	7.56	7.63	7.42	7.60
	(5.73)	(5.92)	(5.44)	(5.78)
3rd Choice	6.46	6.74	6.41	6.26
	(5.01)	(5.27)	(4.94)	(4.84)

Notes: This table indicates the distribution of the home-school distance in the ROLs. The standard deviations are reported in the parenthesis.

Figure 5: Score Distribution for the First Choice



Notes: These figures are the box plots of the first choice score distribution for each school that has been chosen by students as their first choices The x-axis represents the schools chosen as the first choices in the ROLs. The y-axis represents the exam score.

C Identification of Parameters $\boldsymbol{\alpha}$

In this section, we provide a simplified version for the identification of parameters Θ_2. The general case can be conducted from the simple version. Since Θ_2 contains the parameters related with the ZX policy, therefore, we consider only three schools, school 1, and 2. Students' ROLs contain only two schools, and the order of schools are fixed, school 1 must be in the first place and school 2 in the second position. The students can only decide whether choose the ZX option for the first schools in the ROLs, but not the second school. Also we assume there is only one ZX tuition level c_1 and the student's payoff of being rejected by both schools in the ROLs is zero. The students can be divided into two groups.

This simplified version emphasizes the identification of the ZX policy related parameters, so it only gives students a binary choice that whether choose the ZX option for their first listed schools.

If student i chooses the ZX option for school 1, then her expected payoff of action $a_i = \{(1,1),(2,0)\}$ is

$$P^{a_i}_{i,1,c_0}(\hat{u}_{i,1} + \gamma q^z_1 + \epsilon_{i1}) + P^{a_i}_{i,1,c_1}(\hat{u}_{i,1} + \gamma q^z_1 - \alpha(c_1 - c_0) + \epsilon_{i1}) + P^{a_i}_{i,2,c_0}(\hat{u}_{i,1} + \epsilon_{i2}),$$

and the payoff of not choosing ZX option for school 1 with action $a'_i = \{(1,0),(2,0)\}$ is

$$P^{a'_i}_{i,1,c_0}(\hat{u}_{i,1} + \gamma q^z_1 + \epsilon_{i1}) + P^{a'_i}_{i,2,c_0}(\hat{u}_{i,2} + \epsilon_{i2}).$$

Therefore, student i chooses the ZX option for school 1, i.e. $v^1_i = 1$ if and only if

$$U_{i,12} + \gamma Q - \alpha C > \tilde{\epsilon},$$

where $U_{i,12} = (P^{a_i}_{i,1,c_0} + P^{a_i}_{i,1,c_1} - P^{a'_i}_{i,1,c_0})\hat{u}_{i,1} + (P^{a_i}_{i,2,c_0} + P^{a'_i}_{i,2,c_0})\hat{u_{i,2}}$, $Q = (P^{a_i}_{i,1,c_0} + P^{a_i}_{i,1,c_1})q^z_1$, $C = P^{a_i}_{i,1,c_0}(c_1 - c_0)$, and $\tilde{\epsilon} = (P^{a'_i}_{i,2,c_0} + P^{a_i}_{i,2,c_0})\epsilon_{i2} - (P^{a_i}_{i,1,c_0} + P^{a_i}_{i,1,c_1} - P^{a'_i}_{i,1,c_0})\epsilon_{i1}$.

Therefore the probability of observing a decision $v^1_i = 1$ is $F(U_{i,12} + \gamma Q - \alpha C)$ where F is the cdf of $\tilde{\epsilon}$. Then the log-likelihood of an observation a is

$$L_1(\Theta_2) = v ln(F) + (1 - v) ln(1 - F).$$

The score function is

$$\frac{\partial L_1}{\partial \Theta_2} = \frac{y - F}{F(1-F)} \frac{\partial F}{\partial \Theta_2}.$$

Furthermore

$$\begin{aligned} E[\frac{\partial L_1}{\partial \Theta_2}\frac{\partial L_1}{\partial \Theta'_2}] &= \frac{1}{F(1-F)}\frac{\partial F}{\partial \Theta_2}\frac{\partial F}{\partial \Theta'_2} \\ &= \frac{f^2}{F(1-F)}\begin{pmatrix} Q^2 & -CQ \\ -CQ & C^2 \end{pmatrix}, \end{aligned}$$

where f is the pdf of $\tilde{\epsilon}$. Then it is easy to show that the information matrix $E\big[\frac{\partial L_1}{\partial \Theta_2}\frac{\partial L_1}{\partial \Theta_2'}\big]$ is positive definite. This result is equivalent to indicate that parameters γ and α are locally identified from the observed decisions.

D Maximum Simulated Likelihood Estimate

This appendix describes the algorithm used in the maximum simulated likelihood estimate to estimate the ZX related parameters with logit smoothed accept reject simulator. The procedure is implemented in the following steps, similar to the steps in Chapter 5 of Train (2009).

Step 1. Draw a value of J dimensional vector of errors, ϵ_i from type I extreme value distribution. Label the draw ϵ_i^r with $r = 1$ and the elements of the draw as $\epsilon_{i1}^r, ..., \epsilon_{iJ}^r$.

Step 2. Calculate the utility for each alternative. That is, $u_{i,j,c}^r = \tilde{u}_{i,j,c} + \epsilon_{ij}^r$, where $\tilde{u}_{i,j,c}$ is the deterministic part of the utility when student i enters school j and pay tuition c, and $u_{i,o}^r = \tilde{F}_o + \epsilon_{ij}^r$ that is denoted the utility when student i gets into a non-public high school.

Step 3. Given the beliefs and thus the admission probabilities, calculate the expected utility, $EU_i^r(a)$ of submitting a ROL $a = \{(j^1, v^1), (j^2, v^2), j^3\}$

In this step, the utility that the student i gets into one of her chosen school is $u_{i,j,c}^r$ obtained from step 2. The utility of being randomly assigned into a leftover school is $(\sum_{k=1,...,n_e}^{n_e} u_{i,j_{lo}^k,c_0}^r)/n_e$ where n_e is the number of leftover schools in year e, u_{i,j_{lo}^k,c_0}^r is the utility of i getting into the leftover school j_{lo}^k by paying the basic tuition c_0.

Given the student i's ROL $a = \{(j^1, v^1), (j^2, v^2), j^3\}$ and exam score s_i, the probability of i being admitted by school j^k as a normal student or by a non-public school can be calculated as follows:

$$P_{i,j_i^k,c_0} \text{or} P_{i,o} = \max\{0, P_i^{k-1} - \Phi((\bar{S}_{j_i^k}^k - s_i)/\eta)\},$$

where Φ is the cdf of the standard normal, $P_i^{k-1} = 1$ if $k = 1$, $P_i^{k-1} = \Phi((\bar{S}_{j_i^{k-1}}^{k-1} - s_i)/\eta)$ if $v^{k-1} = 0$, and $P_i^{k-1} = \Phi((\hat{S}_{j_i^{k-1}}^{k-1} - s_i)/\eta)$ if $v^{k-1} = 1$. The probability of being admitted by school j_i^k as a ZX student with tuition c is

$$P_{i,j_i^k,c} = \sum_{t=1}^{4} I(c_t = c)[\max\{0, \Phi((\bar{S}_{j_i^k}^k - 10(t-1) - s_i)/\eta) - \max\{\Phi((\bar{S}_{j_i^k}^k - 10t - s_i)/\eta), \Phi((\hat{S}_{j_i^k}^k - s_i)/\eta)\}\}],$$

Finally, the probability of being randomly assigned to a leftover school can be calculated as one minus the probability of being rejected by all three choices.

Step 4. For any student i in group 1, put these expected utilities into the logit formula, i.e.,

$$S_i^r = \frac{exp(Eu_i^r(a_i)/\lambda)}{\sum_{i'} exp(Eu_i^r(a_{i'})/\lambda)}, \tag{11}$$

where a_i is the student i's observed choice, $a_{i'}$ is her alternatives including a_i, and $\lambda > 0$ is a scale factor ($\lambda = 0.01$ in the reported results, the experimental results with other λs are available upon request).

For any student i in group 2, calculate $S_i^{r,2+}$ and $S_i^{r,2-}$ by using $a_i^{2+} = \{(j_i^1, v^1), (j_i^2, 1), j^3\}$ and $a_i^{2-} = \{(j_i^1, v^1), (j_i^2, 0), j^3\}$ to replace a_i in equation (11) respectively. Similarly, for any student i in group 3, calculate $S_i^{r,3+}$ and $S_i^{r,3-}$ by using $a_i^{3+} = \{(j_i^1, 1), (j_i^2, v^2), j^3\}$ and $a_i^{3-} = \{(j_i^1, 0), (j_i^2, v^2), j^3\}$ to replace a_i in equation (11) respectively.

Step 5. Repeat step 1-4 for R times, so that r takes the value from 1 to R.

Step 6. The simulated probability of student i in group 1 choosing the observed ROL a_i is the average of the values of the logit formula: $\hat{P}(a_i \in A_i^*) = \frac{1}{R}\sum_{r=1}^{R} S_i^r$. For the students in group 2, the simulated probability of observing a_i^2 is $\hat{P}(a_i^{2+} \in A_i^*) + \hat{P}(a_i^{2-} \in A_i^*) = \frac{1}{R}\sum_{r=1}^{R}(S_i^{r,2+} + S_i^{r,2-})$. Similarly, for the students in group 3, the the simulated probability of observing a_i^3 is $\hat{P}(a_i^{3+} \in A_i^*) + \hat{P}(a_i^{3-} \in A_i^*) = \frac{1}{R}\sum_{r=1}^{R}(S_i^{r,3+} + S_i^{r,3-})$.

Finally, the log-likelihood function can be calculated in the following equation.

$$\begin{aligned} LogL_2 &= \sum_{i \in G_1} log(P(a_i \in A_i^*)) \\ &\quad + \sum_{i \in G_2} log[P(a_i^{2+} \in A_i^*) + P(a_i^{2-} \in A_i^*)] + \sum_{i \in G_3} log[P(a_i^{3+} \in A_i^*) + P(a_i^{3-} \in A_i^*)]. \end{aligned}$$

E Simulations in Counterfactual Analysis

The section describes the simulation procedure used to analyze in the welfare comparison. We use the students' profiles from 2014. To simplified the calculation, the special classes and non-public schools are excluded. To calculate the equilibrium of the outcomes for different mechanism, the procedure is described as follows:

Step 1. For each student i, draw a value of J dimensional vector of errors, ϵ_i from type I extreme value distribution. Label the draw ϵ_i^r with $r = 1$ and the elements of the draw as $\epsilon_{i1}^r, ..., \epsilon_{iJ}^r$.

Step 2. Calculate the utility function as, $u_{i,j,c}^r = \tilde{u}_{i,j,c} + \epsilon_{ij}^r$, where $\tilde{u}_{i,j,c}$ is the deterministic part of the utility. The parameters used to calculate it come from table 3.

Step 3. The DA mechanism and COSM are strategy-proof. For the DA mechanism, we treat students' true preferences across all schools as their reported ROLs. For the COSM, there are three tuition levels for each school. We treat students' true preferences across school-tuition pairs as their ROLs. Then we run the use serial dictatorship algorithm to match students and schools.

Step 4. The CP, CPPS, BM, BMPS mechanisms are not strategy-proof. We describe the calculation of the equilibrium outcomes as follows:

Step 4.1. For each of these non-strategy-proof mechanism, use the admission cutoffs generated by the DA mechanism as the first prior beliefs for all students.

Step 4.2. Use the prior beliefs to calculate the optimal choice for each student. When there are more than one choice as the optimal choices, then randomly choose one of them. Then each student reports the calculated optimal choice as the ROL.

Step 4.3. Given the submitted ROL, run the matching algorithm base on the definition of the mechanism to match students to schools. Then rank all N students by exam scores.

Step 4.4. The matching outcome from the last step generates new admission cutoffs for schools. Then use the these cutoffs as the new prior beliefs.

Start from the first student and let $k = 1$.

Step 4.5. Calculate the k-th student's best response to the prior beliefs. If there exists at least one choice of this student making him/her strictly better off, then jump to step 4.6. If there does not exist any choice of this student making him/her strictly better off, then let $k = k + 1$. If $k = N$, then jump to step 5. If $k < N$, then repeat step 4.5.

Step 4.6. Choose the k-th student's best response to replace his/her old choice in the submitted ROL. When there is more than one best response, then randomly choose one of them. Thereafter, repeat step 4.3.

Step 5. The current ROLs are the equilibrium strategies of the students.

After calculate one equilibrium outcome for each mechanism, repeat the step 1 to 5 for R times ($R = 100$ in the reported results).

F More results for the estimate and welfare comparison

Table 10: Admission Patterns (%)

	Within Sample						Out of Sample		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Data 2013	Predeted	Diff.	Data 2012	Predeted	Diff.	Data 2012	Predeted	Diff.
Total 1st Choice	30.7	30.5	0.3	29.4	30.8	-1.3	29.4	30.4	-1.0
Normal 1st	15.6	18.8	-3.2	15.5	19.6	-4.1	15.5	19.1	-3.6
Top	10.6	12.4	-1.7	10.0	11.8	-1.9	10.0	11.8	-1.9
Middle	3.7	6.3	-2.7	3.9	6.6	-2.7	3.9	6.3	-2.4
Low	1.3	0.1	1.2	1.7	1.2	0.5	1.7	1.0	0.6
ZX 1st	15.0	11.7	3.4	13.8	11.2	2.7	13.8	11.4	2.4
Top	5.0	4.4	0.6	4.9	5.1	-0.2	4.9	5.5	-0.7
Middle	8.2	6.7	1.5	7.5	5.7	1.8	7.5	5.5	2.0
Low	1.9	0.6	1.3	1.5	0.4	1.1	1.5	0.4	1.1
Total 2nd choice	38.9	32.4	6.4	36.7	28.5	8.2	36.7	29.7	7.0
Normal 2nd	32.0	28.0	4.0	31.0	25.0	6.0	31.0	25.7	5.3
Top	6.2	4.1	2.1	6.3	3.5	2.8	6.3	3.1	3.2
Middle	18.1	21.6	-3.4	14.7	17.5	-2.8	14.7	18.3	-3.6
Low	7.6	2.3	5.3	10.0	3.9	6.0	10.0	4.4	5.6
ZX 2nd	6.6	4.4	2.2	5.5	3.6	2.0	5.5	4.0	1.6
Top	0.2	0.5	-0.3	0.2	0.1	0.1	0.2	0.1	0.1
Middle	4.6	3.3	1.3	4.0	3.2	0.9	4.0	3.5	0.5
Low	1.8	0.6	1.3	1.3	0.3	1.0	1.3	0.3	1.0

Notes: This table indicates the within- and out-of-sample test of the matching patterns for the 1st and 2nd choices in the ROLs.

Table 11: Welfare Gain (%)

	DA-COSM			DA-CPPS			DA-BMPS		
	10%	30%	50%	10%	30%	50%	10%	30%	50%
Overall	939.4	1079.7	1118.5	1494.8	1318.6	1185.6	1541.9	1281.0	1253.4
	(904.9)	(990.9)	(946.3)	(1164.9)	(1017.3)	(943.4)	(1024.0)	(839.2)	(725.0)
Hihg	934.4	977.4	967.5	1208.0	1140.5	1095.3	1605.2	954.8	918.3
	(775.0)	(813.0)	(830.0)	(802.8)	(799.2)	(804.6)	(1025.6)	(862.6)	(787.2)
Medium	1070.7	1270.6	1280.7	1636.7	1532.4	1265.0	1627.6	1341.6	1218.6
	(1025.8)	(1114.8)	(1013.1)	(1325.0)	(1109.8)	(967.4)	(1223.2)	(1007.2)	(779.5)
Low	544.0	508.3	432.6	1249.3	849.6	1030.2	1427.8	1320.9	1326.6
	(486.5)	(460.1)	(421.9)	(641.3)	(680.0)	(1015.9)	(723.8)	(666.0)	(658.8)

Notes: This table indicates the average gain of welfare of the winners when the DA mechanism is replace by the COSM, CPPS and BMPS mechanisms. For each mechanism change, the welfare changes are measured in three scenarios in which the ZX quotas are 10%, 30%, and 50% of the total quotas. Top represents the students are the students whose score is above 90%, Middle represents the students whose scores are between 90% and 70%, and Low represents the students whose scores are below 70% and above the threshold. The standard deviations are in the parenthesis.

Table 12: Welfare Loss (%)

	DA-COSM			DA-CPPS			DA-BMPS		
	10%	30%	50%	10%	30%	50%	10%	30%	50%
Overall	-988.4	-1114.7	-1362.8	-928.7	-1229.4	-1638.3	-1315.7	-1227.8	-1483.4
	(577.4)	(584.2)	(779.4)	(904.0)	(917.0)	(1145.8)	(1509.6)	(1055.6)	(1010.8)
High	-1099.7	-1219.3	-1404.3	-991.8	-1194.1	-1554.3	-1380.7	-969.6	-1365.5
	(558.6)	(539.4)	(673.8)	(869.8)	(623.6)	(759.1)	(2235.9)	(1042.1)	(901.4)
Medium	-994.1	-1172.6	-1555.1	-820.9	-1155.5	-1550.7	-1301.5	-1361.0	-1627.1
	(588.1)	(596.5)	(810.4)	(757.6)	(772.1)	(1080.4)	(1031.6)	(920.9)	(999.2)
Low	-899.7	-911.5	-1074.7	-1306.9	-1791.0	-2324.6	-1219.8	-1413.8	-1143.3
	(546.7)	(549.1)	(726.8)	(1359.7)	(1868.2)	(2013.4)	(1281.1)	(1786.0)	(1515.3)

Notes: This table indicates the average loss of welfare of the winners when the DA mechanism is replace by the COSM, CPPS and BMPS mechanisms. For each mechanism change, the welfare changes are measured in three scenarios in which the ZX quotas are 10%, 30%, and 50% of the total quotas. Top represents the students are the students whose score is above 90%, Middle represents the students whose scores are between 90% and 70%, and Low represents the students whose scores are below 70% and above the threshold. The standard deviations are in the parenthesis.

Table 13: Tuition Collection vs Student Quality (%)

School		DA-COSM			DA-CPPS			DA-BMPS		
		10%	30%	50%	10%	30%	50%	10%	30%	50%
183	Δ Tuition	10.27	29.28	72.32	10.43	32.95	29.44	9.37	34.71	67.34
	Δ Qual.	-0.06	-0.51	-1.20	-0.13	-0.83	-1.05	-0.53	-0.94	-0.98
141	Δ Tuition	11.35	39.77	43.33	12.26	43.91	71.26	10.83	38.54	73.40
	Δ Qual.	-0.07	0.38	0.92	-0.76	-0.57	0.49	-0.46	-0.06	0.00
187	Δ Tuition	-1.04	-16.30	-25.45	1.76	-11.32	-14.85	5.66	18.35	46.71
	Δ Qual.	0.35	1.67	2.32	-0.46	1.17	1.20	-0.42	-0.03	0.32
167	Δ Tuition	10.61	34.92	3.75	11.69	44.30	74.62	9.77	38.50	73.21
	Δ Qual.	0.59	2.12	5.48	1.04	2.77	4.81	0.42	1.59	1.55
185	Δ Tuition	11.05	33.56	43.41	10.09	41.76	60.23	10.77	44.35	48.14
	Δ Qual.	-0.31	0.66	3.61	-1.17	-1.92	-0.77	-4.00	-4.43	-5.02
186	Δ Tuition	10.01	5.65	-33.15	6.51	16.27	33.77	9.00	8.45	16.27
	Δ Qual.	0.74	3.90	12.06	1.25	3.95	7.00	-2.64	-1.87	-1.78
179	Δ Tuition	12.10	18.55	-3.87	9.38	15.36	22.87	10.29	11.15	9.12
	Δ Qual.	0.40	3.50	9.23	0.99	2.92	4.08	-3.56	-2.71	-3.20
184	Δ Tuition	-3.87	-0.79	11.23	-3.61	4.25	38.77	-4.25	-3.04	0.51
	Δ Qual.	-0.51	-0.59	-0.17	-1.13	-2.34	-3.20	-0.19	-0.49	0.05
147	Δ Tuition	-4.86	-15.49	-34.29	1.13	3.84	4.84	2.55	1.59	-1.26
	Δ Qual.	0.81	4.04	10.25	0.55	1.51	2.89	4.45	3.60	3.71
181	Δ Tuition	22.18	102.13	340.05	0.28	6.93	23.46	-5.39	-1.81	0.27
	Δ Qual.	-0.91	-1.33	-1.11	0.24	0.28	0.38	0.69	1.80	1.57
173	Δ Tuition	17.54	82.14	153.26	4.85	15.39	26.69	-0.85	5.92	12.91
	Δ Qual.	-0.80	-1.03	2.45	0.85	1.43	2.20	3.26	3.76	4.10
142	Δ Tuition	13.12	59.95	237.27	6.11	14.13	60.03	3.38	-14.48	-9.60
	Δ Qual.	-0.75	-0.99	-1.14	0.48	-1.22	-2.00	-0.39	-0.52	-0.50

Notes: This table indicates the percentage change in the tuition collection and the student quality when the DA mechanism is replaced by the COSM, CPPS, and BMPS mechanisms. For each mechanism change, utility changes are measured in three cases in which the ZX quotas are 10%, 30%, and 50% of the total quotas.

Table 14: Standard Deviation of Student Quality

School ID	DA	COSM			CPPS			BMPS		
		10%	20%	30%	10%	20%	30%	10%	20%	30%
183	0.015	0.016	0.027	0.031	0.017	0.033	0.041	0.020	0.025	0.025
141	0.026	0.028	0.032	0.035	0.037	0.044	0.035	0.040	0.040	0.039
187	0.035	0.035	0.025	0.023	0.050	0.037	0.048	0.050	0.045	0.039
167	0.044	0.043	0.048	0.038	0.052	0.049	0.039	0.055	0.053	0.053
185	0.042	0.042	0.050	0.056	0.051	0.067	0.069	0.056	0.068	0.066
186	0.049	0.053	0.057	0.047	0.062	0.064	0.069	0.069	0.070	0.069
179	0.040	0.040	0.047	0.043	0.052	0.057	0.057	0.055	0.056	0.056
184	0.035	0.032	0.033	0.037	0.030	0.035	0.035	0.046	0.045	0.048
147	0.037	0.037	0.038	0.039	0.042	0.041	0.046	0.077	0.067	0.067
181	0.032	0.035	0.035	0.039	0.033	0.037	0.041	0.048	0.053	0.055
173	0.030	0.032	0.033	0.039	0.036	0.040	0.047	0.058	0.059	0.061
142	0.028	0.029	0.028	0.033	0.028	0.031	0.030	0.033	0.037	0.039

Notes: This table indicates the standard deviation (s.d.) of the admitted students quality under different mechanisms. Except the DA mechanism, the standard deviations are measured in three scenarios in which the ZX quotas are 10%, 30%, and 50% of the total quotas.

G Alternative Random Coefficient Model

The section presents an alternative model, random coefficient model, to estimate the coefficients. When student i attends school high school j and pays tuition c, her indirect utility function can be rewritten as:

$$u_{i,j,c} = \sum_{l} \beta^{l} y_j^{l} + \sum_{v} \gamma_i^{v} y_j^{v} + \sum_{w} \beta^{w} x_i^{w} y_j^{w} + \beta_D f(d_{ij}, X_i, Y_j) + \sum_{k} \alpha^{k} (c_{ij} - c_0) x_i^{k} + \varepsilon_{ij} \quad (12)$$

and that the utility from being assigned to nonpublic high school o is

$$u_{i,o} = F_o + \varepsilon_{io}. \quad (13)$$

We further assume that $\gamma_i \sim N(0, \sum_{\gamma})$. Here $Y_j \equiv \{y_j\}$ is a vector of school j's observed characteristics; $X_i \equiv \{x_i\}$ is a vector of student i's observed characteristics; d_{ij} is the home–school distance; F_o is the fixed effect of nonpublic high schools; and ε_{ij} and ε_{io} are i's idiosyncratic taste for (respectively) public high school j and nonpublic high schools. Both ε_{ij} and ε_{io} are i.i.d and follow type I extreme value distribution. In the estimate, we assume that the home–school distance is additively separable and independent of unobserved student preferences; in addition, we normalize the coefficient d_{ij} for girls to be -1.

The estimate results are reported in Table 15. Next we also examine how well these estimates match the data. We conduct within-sample and out-of-sample tests to check the aggregate-level matching patterns. For the out-of-sample test, we estimate our parameters for preferences using the procedure described in Section 5 but while excluding the 2012 data. Then, using the newly estimated parameters, we simulate the behavior of students based on their 2012 preference profiles. Table 16 compares the actual and predicted admission cut-offs of each high school.[43]

For the within-sample test, column 2 of the table reports schools' predicted cut-offs for year 2013. The predicted results correctly identify all left schools, but three schools' gaps between the actual and predicted cut-offs are more than 10 points, the largest gap are more than 3% of the full mark (28 points). Column 5 reports the schools' predicted cut-offs for 2012. The predicted results fail to identify one left over school and five schools' gaps are more than 10 points. The out-of-sample test indicates the similar results.

[43] Reported results are the admission cut-offs for the first round. The actual second-round cut-offs of all popular schools are infinity while those of all leftover schools are equal to the threshold. Given that our predicted results correctly identify all popular and leftover schools, we report results only for the first-round cut-offs.

We also explore the aggregate-level matching patterns for students' first two school choices (see Table 15). The predicted results indicate more than 60% of students are assigned to their first choices, however the data show that around 30% of students got seats in their first choice. The random coefficient model underpredited the second-choice admission by more than 10% for both 2012 and 2013.

Table 15

Coefficients			
Quality baseline	2.151***	Dorm baseline	1.292
	(0.399)		(0.833)
Quality × H	0.312	Dorm × Male	1.764***
	(0.473)		(0.557)
Quality × L	–0.631	Capacity	-1.031
	(0.417)		(0.694)
Special class baseline	1.347***	Capacity × H	2.149
	(0.290)		(0.406)***
Special class × H	0.215	Capacity × L	-2.132***
	(0.291)		(0.573)
Special class × L	1.525***	Same district	-1.420***
	(0.199)		(0.057)
Score range Baseline	-0.560	Same district × Male	0.970**
	(0.483)		(0.486)
Score range × Male	-0.839	Cost × H	-3.029***
	(0.526)		(0.035)
Distance	-1	Cost × M	-6.409***
			(0.013)
Distance × Male	0.504***	Cost × L	-5.610***
	(0.114)		(0.018)
Non-Public School	1.047***	School Fixed Effect	Y
	(0.069)		
Variance-Covariance			
Quality	1.600***	Score range	3.971***
	(0.325)		(0.991)
Quality × Special class	-0.655	Score range × Same district	0.140
	(0.445)		(0.336)
Quality × Score range	0.935 ***	Score range × Dorm	0.402
	(0.276)		(0.460)
Quality × Same district	1.147**	Score range × Capacity	3.129***
	(0.455)		(0.707)
Quality × Dorm	0.068	Same district	3.937***
	(0.344)		(1.142)
Quality × Capacity	2.808***	Same district × Dorm	-1.065*
	(0.688)		(0.631)
Special class	-0.437	Same district × Capacity	-0.318
	(0.938)		(0.606)
Special class × Score range	-0.128	Dorm	1.935**
	(0.431)		(0.716)
Special class × Same district	-0.422	Dorm × Capacity	2.298***
	(0.397)		(0.683)
Special class × Dorm	10.416***	Capacity	9.250***
	(2.243)		(1.285)
Special class × Capacity	-2.700**		
	(1.067)		

Notes: Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Distance is measured by kilometer. Both normal and ZX quotas are normalized to 100 seats. Tuition is normalized to 1000 Yuan.

Table 16: Admission Cutoffs

	Within Sample						Out of Sample		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	True 2013	Predicted	Diff.	True 2012	Predicted	Diff.	True 2012	Predicted	Diff.
141	604	588.1	15.9	607	592.3	14.7	607	592.3	14.7
142	530	530.0	0.0	535	535.0	0.0	535	535.0	0.0
147	552.5	543.8	8.7	555.5	551.1	4.4	555.5	551.1	4.4
167	590	582.4	7.6	592.5	585.5	7.0	592.5	585.5	7.0
173	530	530.0	0.0	535	535.0	0.0	535	535.0	0.0
179	565	561.7	3.3	571.5	564.4	7.1	571.5	564.4	7.1
181	530	530.0	0.0	535	535.0	0.0	535	535.0	0.0
183	611	583.0	28.0	617	589.2	27.8	617	589.0	28.0
184	530	530.0	0.0	535	535.0	0.0	535	535.0	0.0
185	580	575.7	4.3	583	580.8	2.2	583	580.7	2.3
186	578	569.1	8.9	583	568.9	14.1	583	568.7	14.3
187	594.5	580.2	14.3	599.5	588.5	11.0	599.5	588.5	11.0
188	575	584.5	-9.5	571.5	598.5	-27.0	571.5	598.5	-27.0

Notes: This table indicates the within- and out-of-sample tests for the schools' cutoffs using random coefficient model. The full mark is 665. The threshold is 535 in 2012 and 530 in 2013. * indicates the leftover schools with cutoff equal to the threshold.

Table 17: Admission Patterns (%)

	Within Sample						Out of Sample		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Data 2013	Predeted	Diff.	Data 2012	Predeted	Diff.	Data 2012	Predeted	Diff.
Total 1st Choice	30.7	66.5	-35.8	29.4	64.5	-35.1	29.4	64.5	-35.1
Normal 1st	15.6	47.5	-31.9	15.5	47.3	-31.8	15.5	47.3	-31.8
Top	10.6	21.2	-10.6	10	20.4	-10.4	10	20.4	-10.4
Middle	3.7	19.8	-16.1	3.9	19.8	-15.9	3.9	19.9	-16.0
Low	1.3	6.4	-5.1	1.7	7.0	-5.3	1.7	7.0	-5.3
ZX 1st	15	19.0	-4.0	13.8	17.2	-3.4	13.8	17.2	-3.4
Top	5	0.0	5.0	4.9	0.1	4.8	4.9	0.1	4.8
Middle	8.2	17.8	-9.6	7.5	15.4	-7.9	7.5	15.5	-8.0
Low	1.9	1.2	0.7	1.5	1.7	-0.2	1.5	1.6	-0.1
Total 2nd Choice	38.9	26.2	12.7	36.7	25.4	11.3	36.7	24.9	11.8
Normal 2nd	32	19.3	12.7	31	19.9	11.1	31	19.4	11.6
Top	6.2	2.7	3.5	6.3	2.7	3.6	6.3	2.6	3.7
Middle	18.1	7.2	10.9	14.7	7.2	7.5	14.7	7.0	7.7
Low	7.6	9.3	-1.7	10	10.0	0.0	10	9.8	0.2
ZX 2nd	6.6	6.9	-0.3	5.5	5.5	0.0	5.5	5.5	0.0
Top	0.2	0.0	0.2	0.2	0.0	0.2	0.2	0.0	0.2
Middle	4.6	4.7	-0.1	4	3.4	0.6	4	3.5	0.5
Low	1.8	2.2	-0.4	1.3	2.1	-0.8	1.3	2.0	-0.7

Notes: Using the random coefficient model, this table indicates the within- and out-of-sample test of the matching patterns for the 1st and 2nd choices in the ROLs.

H Student Survey in 2014

Survey Overview

We cooperated with the local education bureau to conduct the student survey in mid-May 2014. 27 out of 42 of these schools agreed to cooperate with our research and let us survey their 9th grade students. The total number of participants was 8434 and the total number of high school exam takers was 14,194 in that year.

It takes about 10 minutes for one to finish answering all the questions on the survey at most. Two weeks before running the survey, we ran a pilot study of the survey to 60 students one week prior to our fieldwork.

The team of surveyors was led by a retired professor in educational psychology. The members of the team consisted of 20 college students. They were instructed in detail the survey process and their accountability to supervise the survey.

The survey asked the 9th grade students about:

- What aspects of a high school do they think as important when selecting schools.
- Students' true preferences over high schools based their study ability.
- For how many years' cutoff lines do the students look at before submitting their rank order lists?

Survey Process

Each day, starting from 7:00am, the survey team started to travel together to the targeted schools. They arrived at the first school at about 7:45am, then started the survey immediately after their arrival. Each member of our surveyor team supervised the survey for one classroom. The responsibility of our surveyor team members were distributing the paper form surveys and watching the students to make sure they are answering the questions and also to prevent them from looking at others' answers or communicating with each other.

After finishing collecting the answered surveys, the surveyor team would start traveling to the next middle school. In each survey day, the surveyor team surveyed 5 to 10 middle schools, depending on the distance between one school and another. During the survey dates, the surveys were all conducted before morning classes started, during class breaks, at noon before afternoon classes started, and after afternoon classes ended. The starting times were about 7:45am, 9:45am, 1:15pm, 2:45pm and 4:15pm. Each member of the surveyor team were paid by 300 yuan (approximately 45 USD) per survey day.

Outreach

At the beginning of the survey, we stated clearly that this survey was not related to students' high school admission and was for research only. Also, it would be kept completely confidential. Every time before starting the survey, the surveyors announced these points to the students and requested them answer according to the truth.

Questionnaire

Dear students: We are researchers of Educational Science Research Department. Please take a few minutes to complete this questionnaire. This questionnaire is only for research it has no relationship with the results of high school entrance exam, neither does it have any relationship with high school admission. Any personal information in this questionnaire will be treated as highly confidential. Please answer the questions carefully. Thank you!

School: Class: Name: Gender: Student ID:

Q1. Are you Arts or Sports Specialty Student? A. Yes B. No

Q2. Are you directly upgrading student? A. Yes B. No.

Q3. Are you quota student? A. Yes B. No.

Q4. Are you a student who graduated in previous years? A. Yes B. No.

Q5. Please choose the level of importance of the following factors that you consider when you choose general high schools or vocational schools:

Q5A. (1) The academic quality (e.g. college entrance exam scores):

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

(2) The employment condition after graduation and the professional training (Please answer this question if you are possible to choose vocational schools; do not answer if you do not consider vocational schools):

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5B. The infrastructure condition of schools (e.g. equipment, computers, sports fields):

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5C. Whether the school provides scholarship or tuition waive:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5D. The distance from school to home:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5E. Low pressure at school:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5F. Good study atmosphere of the school:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5G. The schools especially good performance at arts or sports:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5H. The strict management in students study and life:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5I. Schools environment (e.g. beautiful and clean campus, good safety condition around the campus):

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5J. Schools living condition for students (e.g. the quality of food, school bus condition, accommodation condition):

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5K. The outside-class life condition

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5L. Good classmates:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5M. Whether the school has special classes:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Please list other factors not listed above that you think important:

Q6. When you are considering the choice of high schools, how important are your opinion and other peoples opinion:

Q6A. Your own opinion:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q6B. Parents opinion:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q6C. You teachers suggestion:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q7. With your current study ability and scores, please list 7 schools - ordinary high schools or vocational schools you may consider as your choice (do not consider order):

Q8. Please pick up 5 schools that you want to get in most from the above 7, and list them in the order of intensity of your willingness to get in:

Q9. When filling your rank order list, how many previous years admission lines for the schools will you refer to:

A. Do not refer to any previous year admission lines

B. Refer to the admission lines for only last year

C. Refer to the admission lines for the past two years

D. Refer to the admission liners for the past three years

E. Refer to the admission lines for more than past three years

I Supplemental Middle School Teacher Survey in 2016

Survey Overview

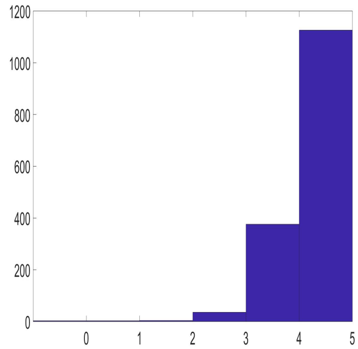
The 2016 survey was designed as a supplement of the survey conducted in 2014. This survey was designed to acquire middle school teachers opinions on high school quality and students' school choices. This survey was targeted at a middle school. The leader of the surveyor of the 2014 survey ran a teacher training program at this middle school and therefore had the chance to run a survey to its teachers. The survey covered all 44 teachers in that school.

The survey was conducted early February of 2016 right after the teacher training program ended. All the teachers were gathered together. The teachers were instructed that the survey was for research use only and would be kept confidential. Teachers were not required to write their names on the surveys. It took about 5 minutes to answer all the questions and the answered surveys were collected immediately. Later, the answer to the survey questions were inputted into an excel form by a research assistant who was a college student.

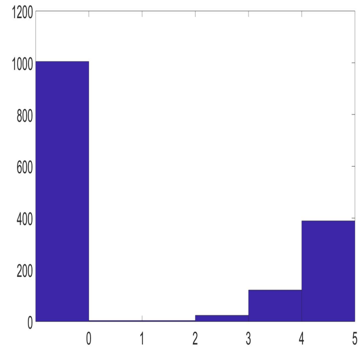
The main questions we asked the teachers are trying to acquire the knowledge of their opinions on

- High schools' education quality;

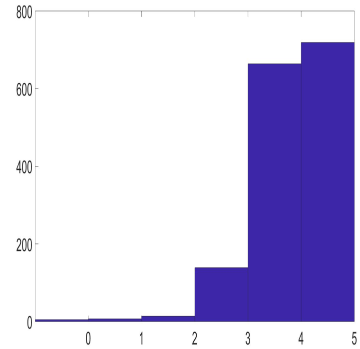
Figure 6: Student Survey Answers I



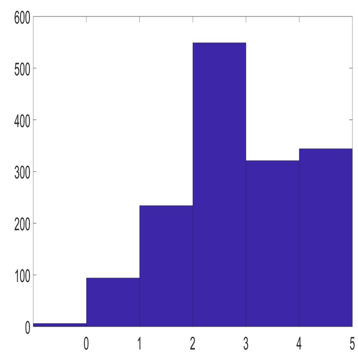
(a) Q5A1



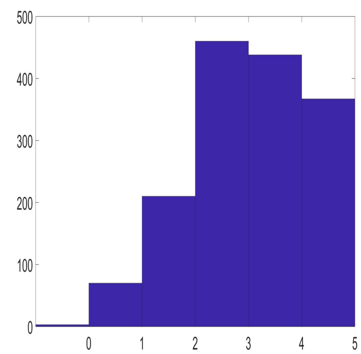
(b) Q5A2



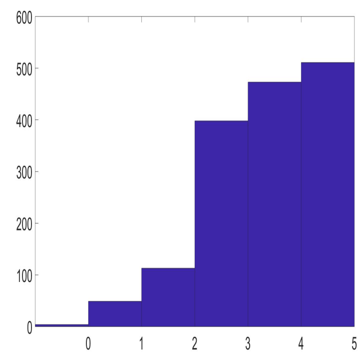
(c) Q5B



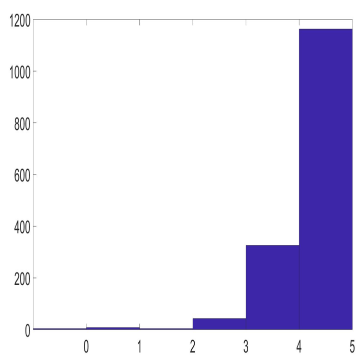
(d) Q5C



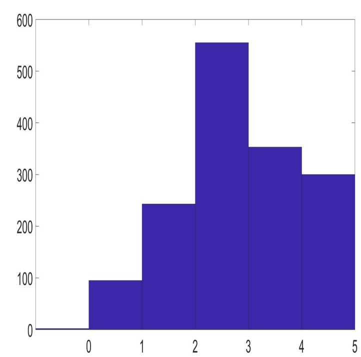
(e) Q5D



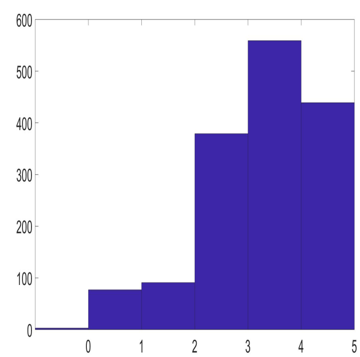
(f) Q5E



(g) Q5F

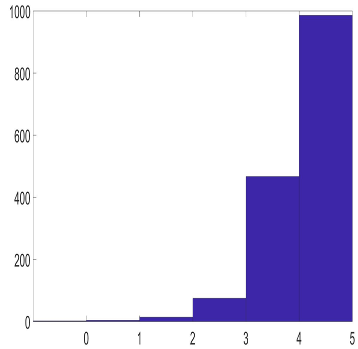


(h) Q5G

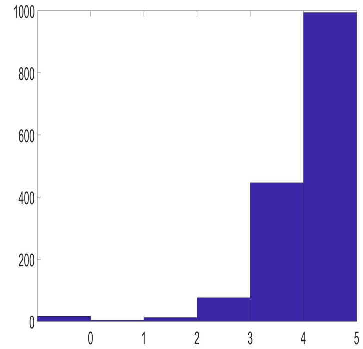


(i) Q5H

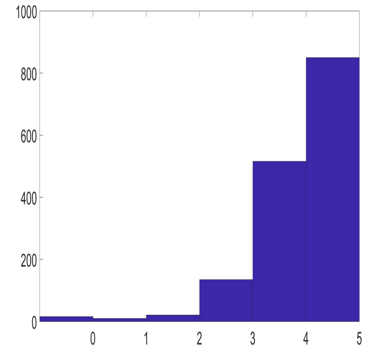
Figure 7: Student Survey Answers II



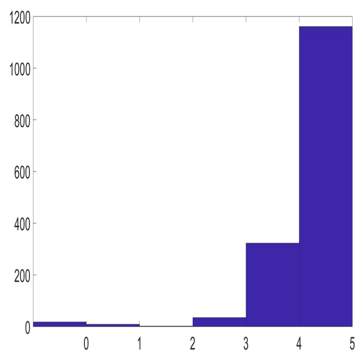
(a) Q5I



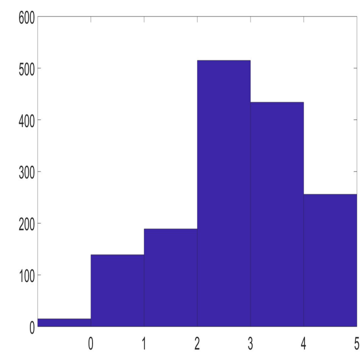
(b) Q5J



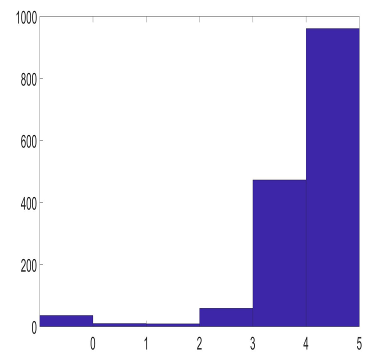
(c) Q5K



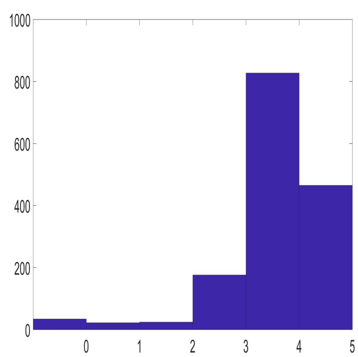
(d) Q5L



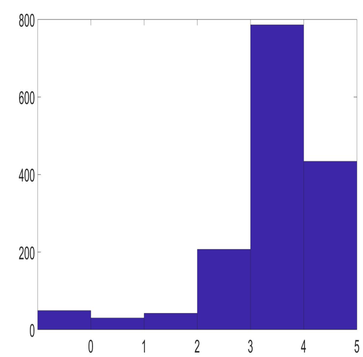
(e) Q5M



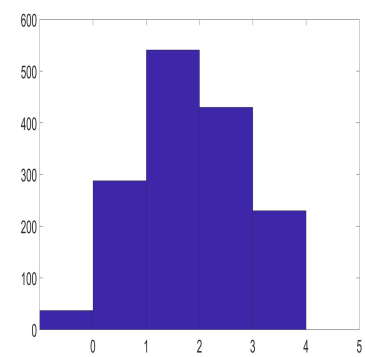
(f) Q6A



(g) Q6B



(h) Q6C



(i) Q9

- Whether incoming students' study ability can represent a high school's education quality;
- Whether students' preferences over high schools do not change across years;

Questionnaire:

1. From the perspective of education quality, please give marks to these schools (full point is 100 points)

List of the Public Ordinary High School Names Are Provided Here

2. From the perspective of education quality, please give marks to these special classes (full point is 100 points)

List of the Special Programs Are Provided Here

3. What fractions of the middle school graduate students preferences over the schools can your answer to the above two questions represent? (Ignore students test scores)

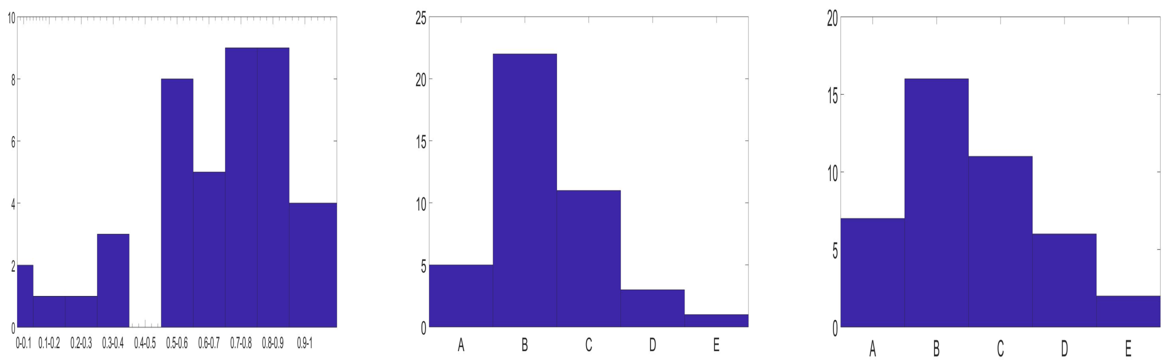
4. To what degree do you agree with the following statement: Students preferences over the public ordinary high schools did not change in the past 10 years

A. Strongly agree B. Agree C. Neutral D. Disagree E. Strongly disagree

5. To what degree do you agree with the following statement: The higher the cutoff line of a school (or special class) is, the better education quality it has

A. Strongly agree B. Agree C. Neutral D. Disagree E. Strongly disagree

Figure 8: Teacher Survey Answers



(a) (b) (c)

Notes: In Figure 8a, 30 (76%) teachers think their evaluations of the school quality can represent more than 50% of students' preferences. Almost half of teachers think their evaluations represent more than 70% of students' preference. In Figure 8b, 27 (64%) teachers chose "agree" or "strong agree" that students preferences over the public high schools did not change in the past 10 years. Only 4(9%) chose "disagree" or "strong disagree". In Figure 8c, 23 (54%) teacher chose "agree" or "strong agree" thatthe higher the cutoff line of a school is, the better education quality it has. Only 8(19%) chose "disagree" or "strong disagree".

J ZX policy in other cities in China

In this appendix, we describe the implementation of ZX policy in three direct-controlled municipalities of China, i.e., Beijing, Shanghai, and Tianjin.

Beijing integrated the ZX policy into its centralized high school admission system in 2005. After the Ministry of Education announced the cancellation of the ZX policy in 2012, the percentage of ZX students of each school decreased from 18% to 15% and further to 10% in 2013. The ZX policy was fully terminated in 2014. The basic tuition of public high schools was 1,600 Yuan/year for a normal student in 2011, whereas that for a ZX student cannot exceed 10,000 Yuan/year.

The admission mechanism of the ZX policy applied in Beijing was an adjusted constrained DA mechanism with purchasing seat options. In this process, no more than eight schools can be selected in the ROL. Each student can select no more than two options from each specific school choice. The options of a school include normal, ZX, special class, and dorm. This mechanism is a special case of CPPS mechanism, wherein the matching algorithm follows the CPPS mechanism with permanency-execution period (8, 0, 0,...).

Shanghai is one of the cities that discontinued the ZX policy immediately after the announcement from the Ministry of Education in 2012. The total percentage of ZX students was restricted within 15% for each school in 2011, which is the percentage for ZX policy in the previous year. The ZX tuition in Shanghai was charged according to the type of school. In district-level key high schools, the basic tuition for students was 2,400 Yuan/year, whereas the ZX tuition was 6,000 Yuan/year before 2011 and 4,266 Yuan/year in 2011. For the city-level key high schools, the basic tuition was 3,000 Yuan/year, whereas the ZX tuition was 10,000 Yuan/year before 2011 and 7,000 Yuan/year in 2011. For the boarding schools, the basic tuition was 4,000 Yuan/year, whereas the ZX tuition was 13,333 Yuan/year before 2011 and 9,333 Yuan/year in 2011. The admission mechanism adopted in Shanghai was the constrained COSM where no more than 15 schools can be selected from the ROL.

Tianjin cancelled its ZX policy in 2015. Before 2015, the ZX tuition was standardized across all general high schools at 8,000 Yuan/year, which was a fourfold increase in the basic tuition (2,000 Yuan/year). The matching algorithm used in Tianjin was a constrained CPPS mechanism with permanency-execution period (2, 8). The students can select two key high schools in the first round and eight ordinary high schools in the second round.