Japanese Package Auction (JPA):
Practical Design for 4G Spectrum Allocation in Japan

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(University of Tokyo)

Two Papers in Japanese Language:


松島斉 (2012b) 「4 G 周波数オークション・ジャパン設計案」「経済セミナー」 6, 7 月号掲載予定

### Beauty Contest in February 2012: Platinum Band (900MHz)

<table>
<thead>
<tr>
<th></th>
<th>eAccess</th>
<th>DoCoMo</th>
<th>KDDI</th>
<th>SB</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard A</strong></td>
<td>1 + 3</td>
<td>1 + 0</td>
<td>1 + 1</td>
<td>1 + 2</td>
</tr>
<tr>
<td><em>(subjective)</em></td>
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<tr>
<td><strong>Standard B</strong></td>
<td>1 + 1</td>
<td>1 + 1</td>
<td>1 + 0</td>
<td>1 + 1</td>
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<tr>
<td><em>(subjective)</em></td>
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<td></td>
</tr>
<tr>
<td><strong>Standard C</strong></td>
<td>2 + 0</td>
<td>0 + 2</td>
<td>0 + 2</td>
<td>2 + 2</td>
</tr>
<tr>
<td><strong>No platinum? Many Contractors?</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>(objective)</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Total Score</strong></td>
<td>8</td>
<td>5</td>
<td>5</td>
<td>9</td>
</tr>
</tbody>
</table>

**Win!**
The ‘First’ Spectrum Auction in Japan

March 2012: Spectrum Law Reform

‘Beauty Contest’ ⇒ ‘Auction’

cf. Multimedia Broadcasting
US and other countries initiated auction much earlier.

Japanese government auctions spectrum licenses in 2013, I suppose.
4G Technologies (1)

3.4GHz ~ 3.6GHz (200MHz Bandwidth)

High Speed, High Capacity

Competing Technologies:

- FDD (Frequency Division Duplex): LTE-Advanced:
  - SB, DoCoMo, E-Access

- TDD (Time Division Duplex): WiMAX2:
  - KDDI
  - cf. TD-LTE (SB)
4G Technologies (2)

Both TDD and FDD need lot size 20MHz

Divide 200MHz into 10 lots:

<table>
<thead>
<tr>
<th>20MHz</th>
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<th>20MHz</th>
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</thead>
</table>

Five Technological Constraints:
FDD need ‘Paired’ lots: ‘Uplink’ and ‘Downlink’
TDD works with ‘Unpaired’
‘Uplink Brock’ and ‘Downlink Brock’ need 40MHz separation
Centers of Uplink and downlink of each FDD need 120MHz separation
Each FDD run parallel with each other
## 10 Licenses Allocation

Spectrum License: Usage of Lot for a period (TDD, FDD up, or FDD down?)

- **8 Lots Maximal for FDD (2 Lots Minimal for TDD):**

<table>
<thead>
<tr>
<th>FDD1 Up (SB)</th>
<th>FDD2 Up (DoCoMo)</th>
<th>FDD3 Up (E)</th>
<th>FDD2 Uplink (New1)</th>
<th>TDD1 Up (KDDI)</th>
<th>TDD2 Down (New2)</th>
<th>FDD1 Down (SB)</th>
<th>FDD2 Down (DoCoMo)</th>
<th>FDD3 Down (E)</th>
<th>FDD4 Down (New1)</th>
</tr>
</thead>
</table>

- **Less than 8 for FDD:**

<table>
<thead>
<tr>
<th>FDD1 Up</th>
<th>FDD2 Up</th>
<th>FDD3 Up</th>
<th>TDD1</th>
<th>TDD2</th>
<th>TDD3</th>
<th>TDD4</th>
<th>FDD1 Down</th>
<th>FDD2 Down</th>
<th>FDD3 Down</th>
</tr>
</thead>
</table>

- **10 Lots Maximal for TDD:**

<table>
<thead>
<tr>
<th>TDD1</th>
<th>TDD2</th>
<th>TDD3</th>
<th>TDD4</th>
<th>TDD5</th>
<th>TDD6</th>
<th>TDD7</th>
<th>TDD8</th>
<th>TDD9</th>
<th>TDD10</th>
</tr>
</thead>
</table>

- **Multiple Brocks for TDD:**

<table>
<thead>
<tr>
<th>FDD1 Up</th>
<th>FDD2 Up</th>
<th>TDD1</th>
<th>TDD2</th>
<th>TDD3</th>
<th>TDD4</th>
<th>FDD1 Down</th>
<th>FDD2 Down</th>
<th>TDD5</th>
<th>TDD6</th>
</tr>
</thead>
</table>
Multiple Purposes for Spectrum Auction Design (1)

We need ‘Compromise (Patchwork)’:

- **Efficiency and Incentives:**

  Package Auction
cf. Non-Package: Sequential Auction
SMRA (Porter and Smith (2006))

VCG Mechanism
cf. Core-Selecting Mechanism

- **Neutrality of Technology:**

  How many TDD or FDD? \(\Rightarrow\) Not government but auction answers.
cf. UK 2.6GHz Auction (Cramton (2009))
Multiple Purposes for Spectrum Auction Design (2)

- DM Complexity:
  Evaluation for too many packages is complicated

- Item Division:
  Allocate not lots but ‘rights’ to get TDD or FDD

- Value Discovery:
  Information revelation through Clock Auction

- Revenue: ‘Revenue Equivalence Theorem’
- Entrance (Competition) Promotion
- Deposit
- Reserve prices
- Consistency with Spectrum Law
Main Contribution of My Talk

Package Auction is generally difficult in practice:

Substitutes and Complements
Package Auction $\Rightarrow$ Non-Package Auction such as SMRA
UK 2.6GHz
Multi-Band Setting

Can we design practical package auction for 4G Japan?

Yes we can! because not many licenses and high homogeneity

Let me show four auction designs named ‘Japanese Package Auctions (JPA):

JPA1, JPA2, JPA3, JPA4
$n \geq 2$ Bidders: SB, DoCoMo, KDDI, E-Access, new comers

Bidder $i$ can purchase at most $l_i \equiv \min[L_i, l]$ number of licenses:

$L_i$ is exogenous, $l$ is endogenous

Bidder $i$ deposits $lD_i$ yen

$$\sum_{i=1}^{n} l_i > 10 \text{ must be satisfied}$$
Environment (2)

Reserve price for each license: $M$ yen

Preferential Treatments:

- Increase of bidder $i$'s valuation by $H_i$ yen per license
- Entrance (Competition) Promotion
Package Auction

Bidder makes ‘Package Bids’ cf. SMRA

DM Complexity:

Too many packages to evaluate: \[ \sum_{k=1}^{l_i} \binom{10}{k} \] packages:

- \( l_i = 5 \): 637 packages
- \( l_i = 10 \): 1023 packages

We need device to calm DM complexity \( \Rightarrow \) ‘Item Division’
Item Division

Item 1: Right to obtain unpaired lot for TDD (2 licenses)
Item 2: Right to obtain Paired lots for FDD (1 license)

Bidder makes bid for item vector \( a_i = (a_{i,1}, a_{i,2}) \in A_i \):

\[ 2a_{i,1} + a_{i,2} \leq l_i \text{ and } a_{i,1} \leq 4 \]

Item vector determines Business ‘scale’ (decisive factor)

Small number of bids are sufficient:

\[ l_i = 5: \quad 11 \text{ item vectors} \]
\[ l_i = 10: \quad 35 \text{ item vectors} \]

Item allocation \( a = (a_i)_{i=1}^{n} \in A : \quad \sum_{i=1}^{n} (2a_{i,1} + a_{i,2}) = 10 \text{ and } \sum_{i=1}^{n} a_{i,1} \leq 4 \)
Japanese Package Auction (JPA) (1)

4 Types:

**JPA1:**
- Item Division: DM Complexity
  - Two Stages: Item (right) allocation stage, License Allocation Stage
- VCG Mechanism: Efficiency and Incentives

**JPA2:**
- Value Discovery: DM Complexity
  - Three Stages: Value Discovery Stage, Item allocation with Assistance Stage, License Allocation Stage
- Private Values: Revealed Preference Activity Rule (RP)
Japanese Package Auction (JPA) (2)

**JPA3:**
Interdependent Values: Modified Revealed Preference Activity Rule

Three Stages:
- Modified Value Discovery Stage
- Modified Item allocation with Assistance Stage
- License Allocation Stage

**JPA4:**
Apply Auction even for License Allocation

Two Stages:
- Item allocation Stage
- Modified License Allocation Stage

**JPA5:** Include Everything!
JPA1 (1)

- **Item Allocation Stage**

  Bidder makes Bid $b_i(a_i)$ for item vector $a_i \in A_i$:
  
  $b_i(0) = 0$

  Free Disposal: $[a_i \geq a'_i] \Rightarrow [b_i(a_i) \geq b_i(a'_i)]$

Example: $l_i = 5$

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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</thead>
<tbody>
<tr>
<td>Item 1</td>
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<td>[ ]</td>
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<tr>
<td>Item 2</td>
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</table>
JPA1 (2)

Government selects Item Allocation \( a^* = (a_i^*)_{i=1}^n \in A \) by solving

\[
\max_{a \in A} \sum_{i=1}^{n} \{ b_i(a_i) + (2a_{i,1} + a_{i,2})H_i \}.
\]

Neutrality of Technology:

Government endogenously determines “Technology”:  

Total Number of FDD licenses:
\[
2 \sum_{i=1}^{n} a_{i,1}^*
\]

Total Number of TDD licenses:
\[
\sum_{i=1}^{n} a_{i,2}^*
\]

Number of FDD licenses each bidder purchases:
\[2a_{i,1}^*\]

Number of TDD licenses each bidder purchases:
\[a_{i,2}^*\]
• License Allocation Stage

Government randomly (or discretionally) selects license allocation $g$:

$$g(h) = (g_1(h), g_2(h))$$

Bidder $g_1(h)$ receives license $h$ for use of item $g_2(h)$

Five technological constraints required.
Bidder’s Payment $x_i^*$:

VCG Mechanism (modified):

\[ x_i \equiv \max \sum_{a \in A} \left\{ b_j(a_j) + (2a_{j,1} + a_{j,2})H_j \right\} - \sum_{j \neq i} \left\{ b_j(a_j^*) + (2a_{j,1}^* + a_{j,2}^*)H_j \right\}, \]

Define bidder’s payment as

\[ x_i^* \equiv \max[x_i - (2a_{i,1}^* + a_{i,2}^*)H_i, (2a_{i,1}^* + a_{i,2}^*)M]. \]

Strategy-Proofness: Quasi-Linearity, Private Values Reserve Prices $M = 0$ $H_i$ does not matter
We may need more device to calm DM complexity:

- **Value Discovery Stage**

  Ascending Clock:

  Information revelation a la Tâtonnement

  Discrete time horizon \( t = 1, 2, 3, ... \)

  Auctioneer offers and adjusts price vector \( p(t) = (p_1(t), p_2(t)) \)

  \( p(1) = (2M, M) \)

  Price grid per license \( \epsilon > 0 \)
Double Auction:
Bidder makes demand response as package \( d_i(t) = (d_{i,1}(t), d_{i,2}(t)) \)
\[
2d_{i,1}(t) + d_{i,2}(t) \leq l_i \quad \text{and} \quad d_{i,1}(t) \leq 4
\]
Government makes supply response to maximize revenue:
Case 1: \( 2p_1(t-1) = p_2(t-1) \)
   - Government is indifferent to supply response
Case 2: \( 2p_1(t-1) > p_2(t-1) \)
   - Government supplies 4 units of item 1 and 2 units of item 2
Case 3: \( 2p_1(t-1) < p_2(t-1) \)
   - Government supplies 0 unit of item 1 and 10 units of item 2

Ascending Prices:
   - Excess demand for an item increases its price
   - No excess demands end this stage.
Revealed Preference Activity Rule (RP): 

Bidder is required to be compatible with RP:

Valuation Function (with Free Disposal) \( v_i : A_i \rightarrow R_+ \cup \{0\}, \ v_i \in V_i \)

\( V_i(t) \subseteq V_i \): Set of valuation functions compatible with RP at time \( t \):

\[
v_i(d_i(t)) - \{2p_1(t)d_{i,1}(t) + p_2(t)d_{i,2}(t)\} \geq v_i(a_i) - \{2p_1(t)a_{i,1} + p_2(t)a_{i,2}\}
\]

\( A_i(t) \subseteq A_i \): Set of item vectors ‘feasible’ at time \( t \):

Given \((p(\tau), d_i(\tau))_{\tau=1}^{t-1}\) and \( p(t) : [a_i = d_i(t) \in A_i(t)] \iff [\bigcap_{\tau=1}^{t} V_i(\tau) \neq \emptyset] \)

Revealed Preference Activity Rule (RP): \( d_i(t) \in A_i(t) \) for all \( t \)
**JPA2 (4)**

Example: \( M = 1, \ v = 1, \ l_i = 3 \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>( p_1(t) )</th>
<th>( p_2(t) )</th>
<th>( d_{i1}(t) )</th>
<th>( d_{i2}(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>1</td>
<td>0</td>
<td>3</td>
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<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\nu_i(1, 0) &= 4 + Z_i \\
\nu_i(0, 1) &\leq 2 + Z_i \\
5 + Z_i &\leq \nu_i(0, 3) \leq 6 + Z_i \\
\nu_i(1, 1) &\leq \nu_i(0, 3)
\end{align*}
\]

Suppose \( (p_1(4), p_2(4)) = (6, 2) \):

<table>
<thead>
<tr>
<th>Item 1</th>
<th>Item 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>(0,1)</td>
</tr>
<tr>
<td>(1,0)</td>
<td>(1,1)</td>
</tr>
</tbody>
</table>
- **Item Allocation with Assistance Stage**

  Example: Bidder decides absolute value $Z_i \geq 0$
  and then fills up blanks in [ ]

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</tbody>
</table>
Interdependent Values: RP requires too much: Weaken RP

- **Modified Value Discovery Stage**

  Apply UK Ofcom ‘License-Demand-Decreasing’ Rule (ad hoc, though):

  Bidder can select item vector whose size is at most previous demands

  \[ \hat{A}_i(t) \subseteq A_i: \text{Set of item vectors ‘quasi-feasible’ at time } t: \]

  Given \((p(\tau), d_i(\tau))_{\tau=1}^{t-1}\) and \(p(t)\):

  \[ [a_i = d_i(t) \in \hat{A}_i(t)] \Rightarrow [2d_{i,1}(t) + d_{i,2}(t) \leq \min_{\tau \in \{1,\ldots,t-1\}} \{2d_{i,1}(\tau) + d_{i,2}(\tau)\}] \]

  **Modified Revealed Preference Activity Rule:**

  \[ d_i(t) \in A_i(t) \cup \hat{A}_i(t) \text{ for all } t. \]
**JPA3 (2)**

Example: Bidder can select (1,0), simply because size is small enough

Bidder cannot select (1,1), because its size is too big and it is inconsistent with RP

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<td></td>
<td>(0,0)</td>
</tr>
<tr>
<td>(1,0)</td>
<td>(1,1)</td>
</tr>
</tbody>
</table>
Modified Item Allocation with Assistance Stage

For bidder incompatible with RP, we use only RP condition at the last time $t = t(a_i)$ that item vector $a_i$ is quasi-feasible, where:

$$t(a_i) \in \{1, \ldots, t^E\}$$

is the last time to satisfy

$$a_i \in \hat{A}_i(t)$$

Package bid $b_i(a_i) \neq b_i(d_i(t^E))$ must satisfy upper bound:

$$b_i(d_i(t(a_i))) - \{2p_1(t(a_i))d_{i,1}(t(a_i)) + p_2(t(a_i))d_{i,2}(t(a_i))\} \geq b_i(a_i) - \{2p_1(t(a_i))a_{i,1} + p_2(t(a_i))a_{i,2}\}$$

RP against smaller size $\Rightarrow$ Make consistent assistance!
**JPA3 (4)**

Example:

Suppose: bidder $i$ is inconsistent with RP:

\[ p(4) = (6,2), \quad d_i(4) = (1,0), \quad t^E = 4. \]

Notice:

\[ t(a_i) = 4 \quad \text{for} \quad a_i \not\in \{(1,1), (0,3)\} \]

\[ t(0,3) = t(1,1) = 3 \]

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<td>0</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(0 \sim 2 + Z_i)</td>
<td>(0 \sim 4 + Z_i)</td>
<td>(0 \sim 8 + Z_i)</td>
</tr>
<tr>
<td>1</td>
<td>[</td>
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<td>[</td>
<td>[</td>
</tr>
<tr>
<td></td>
<td>(6 + Z_i \sim 6 + Z_i)</td>
<td>(0 \sim 8 + Z_i)</td>
<td></td>
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</tbody>
</table>
Auction even for license allocation

- **Modified License Allocation Stage**

  Bidder submits $f_i(C)$ for any compatible subset of licenses with his/her item vector

  Government selects $g^*$ to solve

  $\begin{align*}
  (3) \quad & \max_{g} \sum_{i=1}^{n} f_i(C_i(g)) \quad \text{subject to technological constraints,} \\
  \text{where} & \quad C_i(g) \equiv \{h \mid g_i(h) = i\}
  \end{align*}$
JPA4 (2)

• VCG Mechanism (modified) \( x_i^* \):

\[
\gamma_i \equiv \max_{g \in G} \sum_{j \neq i}^{n} \beta_j(C_j(g)) - \sum_{j \neq i} \beta_j(C_j(g^*)),
\]

and define

\[
x_i^* \equiv \max[x_i - (2a_{j,1}^* + a_{j,2}^*)H_i, (2a_{j,1}^* + a_{j,2}^*)M] + r_i
\]