The Concavity of Time Discount Function: An Experimental Study

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Abstract

This paper presents a new anomaly in the literature of time preference, which is future bias. The experiment with 37 subjects also provides the evidence in favor of an inverse S-shaped time discount function rather than a hyperbolic (and convex) time discount function. The time discount function seems to be locally concave. If time discount function is concave around t, then the decision-maker exhibits future bias.

Future bias has seldom been reported in the literature, since it is not easily identified. To detect future bias, this experiment lets subjects compare two future reward options whose interval is shorter than 4 weeks. The new elicitation method is also independent of the confounding factors that are common in the existing experiments. Specifically, the method does not put any assumption on the form of utility function and does not assume that time and reward are separable either. This is a great advantage that enable us to estimate time preference more precisely.

Keywords: intertemporal choice, time preference, future bias, inverse S-shaped discounting, reverse time inconsistency

JEL Classifications: C91, D81, D99

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1 Introduction

Time inconsistency in behavioral economics usually refers to present bias, and the hyperbolic discount function is the representative model to describe the time inconsistency.[1] The present bias is the tendency of a decision-maker in intertemporal choices that s/he overvalues and prefers an immediate-but-small reward to a delayed-but-large reward. The present bias is also consistent with *decreasing impatience.*, i.e., it would not really matter to her/him even if the length of the delay was more extended. Many experimental researches have done to detect this tendency in the literature.

In this paper, however, I examine another type of time inconsistency that is the *future bias*. In his famous "movie star kisses" experiment, Loewenstein (1987) characterizes the similar type of anomaly as *reverse time inconsistency*. With the reverse time inconsistency, subjects tend to postpone a reward until the near future. Rubinstein (2006) also introduces future bias as a possible but hypothetical time inconsistent behavior. Sayman and Öncüler (2009) have observed that subjects actually exhibiting reverse time inconsistent behavior in a longitudinal experiment. Takeuchi (2011) finds that two-thirds of the subjects in his non-parametric elicitation are *increasing impatience*. This finding implies that these subjects exhibit future bias as well.

The present bias and the future bias, both, cause a preference reversal illustrated in the example below.[2]

Q1. Which of the following reward options do you prefer?
  (1) $100 paid in 52 weeks
  (2) $110 paid in 53 weeks (∗)
Q2. Which of the following reward options do you prefer?
  (3) $100 paid today (∗)
  (4) $110 paid in a week

Many respondents, as indicated the asterisk, answer that they prefer (2) rather than (1), and they prefer (3) to (4). It is straightforward: they are willing to wait another week for the additional $10 in Q1, though the same one week delay becomes too long when it is compared against the immediate payment option (3).

Observe that those respondents exhibit present bias and preference reversal. In Q1, they prefer the latter ($110 in a week) option. Suppose that option (2) is granted to them in accordance with their choice, and suppose further that 52 weeks have past since then. Then, option (1) and (2) become option (3) and (4), respectively. At that moment, the respondents would choose ($100 today) over ($110 in a week), while they preferred the opposite 52 weeks ago. This preference reversal is referred as time inconsistent.

For future bias, just imagine the opposite. Some prefer ($100 in 52 weeks) to ($110 in 53 weeks), while they choose ($110 in a week) over ($100 today) in Q1. That is, the additional one-week delay does not really matter today, but it bothers if it is added to 52-week delay. Table 1 summarizes the four possible responding patterns to these 2 questions.

Future bias might be counter-intuitive. Note however that the future bias does not necessarily mean that one cannot exhibit present bias. Rather, one can exhibit *both* of present bias

[1] Frederick et al. (2002) provide a review of time preference experiments in economics and Green and Myerson (2004) provide an overview of studies in psychology. In the experiments on pigeons and rats, see Monterosso and Ainslie (1999) for a survey.

[2] This example is based on Keren and Roelofsma (1995).

Table 1: present bias, future bias and consistent behavior

		Q2	
		(3) \$100 in 52 weeks	(4) \$110 in 53 weeks
Q1	(1) \$100 today	consistent impatient	inconsistent present bias
	(2) \$110 in a week	inconsistent future bias	consistent patient

Note. For each of Q1 and Q2, a decision-maker chooses if s/he waits another week (longer) to take the additional \$10 or not. Present bias is detected if a decision-maker chooses (1) over (2) in Q1 though s/he prefers (4) to (3) in Q2.

and future bias. Even when her/his impatience is locally increasing around $t = 0$ and it causes future bias, s/he can be overall present biased. Sayman and Öncüler (2009) and Takeuchi (2011), independently, notice that an inverse S-curve time discount function captures the time inconsistent preference. Thus, I conduct an experiment to directly check the concavity of time discount function.

2 Model

In this section, I define present bias and future bias and then characterize the future bias using time discount function.

2.1 Definition

First, let (x, t) denote an option that will pay x at time t. Define the discounted present value of the option $V(x, t)$ as follows:

$$V(x, t) = D(x, t)u(x) \tag{1}$$

where $D(x, t)$ is the discount factor and $u(x)$ is the instantaneous utility of the reward. Notice that D depends not only on t but also on the reward magnitude x, though the literature always assume that x and t are separable.[3]

Until recently, the literature also commonly defined u as a linear function, implying risk neutrality.[4] This restrictive assumption systematically biases the estimation of the time preference, that is, subjects are estimated to be myopic and impatient. Andersen et al. (2008) and Takeuchi (2011) show that the estimated discounting factor $D(t)$ would be too small if the estimation ignored the risk averseness of subjects. In this paper, I do not need to assume linear utility function.

[3] In the literature, Takeuchi (2011) and Attema et al. (2009) are exceptions. Their non-parametric estimation do not assume hat x and t are separable and are still capable of eliciting time preference.

[4] See Table 1 of Takeuchi (2011) that summarizes 37 experimental studies since 1980. u is assumed to be linear in 22 papers out of 27 papers published in and before 2005. There are only 5 papers that allowed non-linearity of u. But, it already counts 7 papers, where u is non-linear, out of 10 papers published since 2006.

For a non-parametric definition of present and future bias, Takeuchi (2011) introduces an equivalent delay function T on $\{(x, x') \in \mathbb{R}^2_+ | x \leq x'.\}$. Suppose that a subject is indifferent between two options $(x, 0)$ and (x', T). Then, $T(x, x')$ is the delay that makes these two options the same to a subject.

Definition. $T(x, x')$ is an *equivalent delay* such that $(x, 0) \sim (x', T(x, x'))$.

Definition (Time consistency)**.** A subject is *time consistent* if T is modular.

Definition (Present bias)**.** A subject exhibits *present bias* if T is strictly submodular.

Let us show that this definition is corresponding to the present bias of Table 1. Suppose that you exhibit present bias, that is, you prefer ($100 today) to ($110 in 1 week) in Q1 and ($110 in 53 weeks) to ($100 in 52 week) in Q2.

($100 today) is better than ($110 in 1 week), so you are not willing to wait one week to get $110 instead of the immediate $100. The one week is too long. It follows that $T(\$100, \$110) < 1$. Next, let $\$z$ denote the present value of ($100 in 52 week) and observe that $T(\$z, \$100) = 52$ by definition. It follows that $T(\$z, \$100) + T(\$100, \$110) < 53$.

Similarly, according to your choice in Q2, there must be $\alpha > 0$ such that $T(\$z+\alpha, \$110) = 53$ where $\$z+\alpha$ is the present value of ($110 in 53 weeks). Consider $T(\$z, \$110)$, how long you are willing to wait to get $110 instead of receiving $z now. It should be longer than 53 weeks for which you are willing to wait to get $110 instead of $\$z + \alpha$ now. Thus, $T(\$z, \$110) > 53$. Altogether with the inequality above, it follows that

$$T(\$z, \$100) + T(\$100, \$110) < T(\$z, \$110) \tag{2}$$

which means that T is strictly submodular.

Recall that the formal definition of present bias, *decreasing impatience*, is previously given by Prelec (2004). My definition of present bias is equivalent to decreasing impatience.

Definition (Prelec (2004))**.** A subject is said to exhibit *decreasing impatience* if for any $\delta > 0$, $x_2 > x_1 > 0$, $(x_1, t_1) \sim (x_2, t_2)$ implies $(x_1, t_1 + \delta) \prec (x_2, t_2 + \delta)$.

Proposition 1 (Takeuchi (2011))**.** Decreasing impatience of Prelec (2004) is equivalent to present bias.

Accordingly, the future bias, or *increasing impatience*, can be defined as follows.

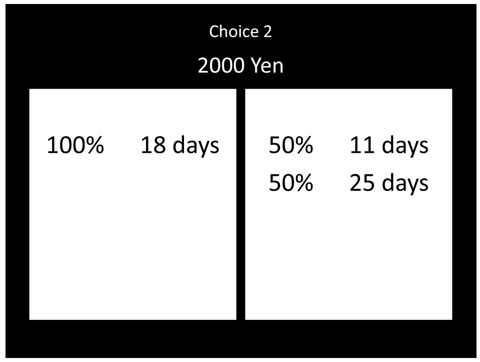
Definition (Future bias)**.** A subject exhibits *future bias* if T is strictly supermodular. Equivalently, a subject exhibits *increasing impatience* if for any $\delta > 0$, $x_2 > x_1 > 0$, $(x_1, t_1) \sim (x_2, t_2)$ implies $(x_2, t_2 + \delta) \prec (x_1, t_1 + \delta)$.

2.2 Characterization

Let us characterize future bias on time discount function $D(x, t)$. Define the hazard rate as follows.

$$h(x, t) = -\frac{\partial D(x, t)/\partial t}{D(x, t)}. \tag{3}$$

This represents the instantaneous impatience at time t. For a convenient interpretation, just assume that people discount a future reward only because they will die with a slight



Note. This is one of the simplest question. Respondents choose which option they prefer, the left one (receiving 2000 JPY in 18 days) or the right one (receiving 2000 JPY in y days) where y will be 11 with 50% chance or 25 with 50% chance. Notice that the expected delay of the right option $50\% \times 11$ days $+ 50\% \times 25$ days is equal to 18 days. If a respondent chooses the left, then her/his time discount function D is concave in t around $t \in [11, 25]$.

Figure 1: Questionnaire sample screen shot (translated from Japanese)

chance before receiving the reward (i.e., they do not have time preference, but it is just a risk preference). Observe that the hazard rate is the conditional probability of sudden death at time t given his/her survival up to t. If $h(x, t)$ is high, s/he behaves as if s/he becomes impatient at time t. Therefore, $h(x, t)$ is the instantaneous impatience at t.

Since the hazard rate is the instantaneous impatience, it characterizes the time preference.

Proposition 2 (Takeuchi (2011))**.** The hazard function $h(t)$ is decreasing (increasing) in t if and only if the time preference exhibits decreasing (increasing) impatience.

Corollary. The time preference exhibits increasing impatience (i.e., future bias), if $D(x, t)$ is concave in t.

The concavity of time discount function is the sufficient condition for future bias, which I directly test in experiments.

See Figure 1. It shows one of the simplest choices that respondents answer in the experiment. At that screen, respondents will choose the left option (receiving 2000 JPY in 18 days) or the right option (receiving 2000 JPY in y days) where y will be 11 with 50% chance or 25 with 50% chance. Notice that the expected delay of the right option $50\% \times 11\text{days} + 50\% \times 25\text{days}$ is equal to 18 days. Suppose a respondent chooses the left over the right, then

$$D(x, 18)u(x) > 0.5D(x, 11)u(x) + 0.5D(x, 25)u(x), \text{ where } x = 2000. \tag{4}$$

Neither risk averse nor risk neutrality plays any role in this comparison, since the reward level is constant and the payment of the reward (2,000 JPY) is guaranteed. For notational clarity, drop x from the inequality (yet, do not ignore x). Rather, this elicitation method is independent of the reward magnitude. It yields,

$$D(18) > \frac{D(11) + D(25)}{2} \text{ if and only if choose the left option in Figure 1.} \tag{5}$$

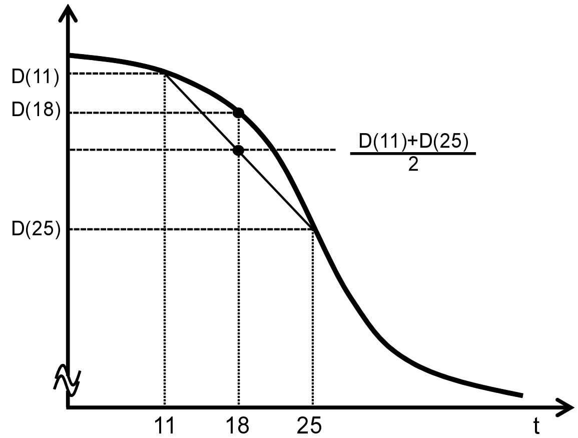


Figure 2: Concave time discount function

It immediately implies that D is concave as shown in Figure 2.

3 Method

37 subjects (college students) were recruited in two sessions on the same day in February 2011. The advertisement tells that the experiment will last 90 minutes at longest and subjects receive 2,000 Japanese yen for their participation,[5] and that we are likely to deposit additional money ranged from 2,000 to 6,000 yen to their bank account later by wire-transfer.[6]

At the beginning of each session, subjects are given printed instructions. After the instructions are read aloud, subjects are encouraged to ask questions. Then, the screen in front of the classroom displays 22 questions sequentially, one of which is shown in Figure 1. Each question is projected for 40 seconds, during which subjects mark left or right on an answer sheet. Then the question crawls toward the top of the screen (just like a credits end of a movie) and disappears, while the next question crawls in from the bottom. As this transition takes 10 seconds, the projection of all the questions lasts 18 minutes and 20 seconds (= 50 seconds $\times$ 22).

After subjects responded to all questions, for each of them, we randomly select one question to determine the outcome using a bingo machine. The choice on the question (left or right) will specify the payment day. If the chosen option stochastically specifies it, then we throw two 10-sided dice and pick an integer from 1 to 100.

All of these procedure is described in the instruction. The last section of the instruction reads as follows (translated from Japanese) :

> As this is an experiment in economics, we implement everything as specified by

[5]The monthly average exchange rate is \$1=82.33 JPY in February 2011. The 2000 yen show-up fee is equivalent to 24.29 dollars. A hourly wage of an average part-time job for students is around 850 to 1000 yen. In fact, for a temporary part-time job, the university administrative rules set 950 yen per hour for undergraduate students.

[6]In Japan, wire-transfer is the most common way to send and receive money through banking system. People do not use checks at all.

> this instruction. There is no trick or cheat at all. As for the bank deposit, please save the written pledge printed in the orange colored paper. Even if an unexpected event happens to the experimenter, we have an arrangement, an assistant will take care of the payment.

The written pledge reads:

> Kan Takeuchi will deposit ____ Yen to your bank account that you specified in an attached sheet ____ days after from today, on ____ year/ ____ month/ ____ day. If the bank is closed on this designated day, for observing a holiday or on weekend, the deposit will take place on the next business day.

The subjects know me as a faculty member of the university, at least, through the recruitment process. I sign the pledge at the site of the experiment.

Table 2 presents all 22 questions. The last two columns show the number of respondents who choose the left option and the right option, respectively. The questions are clustered into 8 groups in the following manner. Group A $\{2, 12, 20\}$ for each of which the expected length of delay is 11 days. Similarly, Group B $\{1, 8, 17\}$: 11 days. Group C $\{5, 10, 18\}$: 18 days. Group D $\{7, 14, 21\}$: 49 days. Group E $\{4, 15, 22\}$: 49 days. For the other questions $\{3, 13, 6, 16, 9, 19, 11\}$, the expected length of delay is slightly different between the left and right options, since they are designed to test Allais paradox in this context.

4 Results

First, note that in every question either the left or the right option is indicating the concavity around the offered delay. In the first question, for example, subjects compare the following two options:

	Left		Right	
Q1	50%	4 days	80%	4 days
	50%	18 days	20%	39 days

In this question, choosing the left option is consistent with concavity. The left option is preferred, when

$$\frac{1}{2}D(x,4)u(x) + \frac{1}{2}D(x,18)u(x) > \frac{4}{5}D(x,4)u(x) + \frac{1}{5}D(x,39)u(x). \tag{6}$$

This equation is reduced to

$$D(x,18) > \frac{3}{5}D(x,4) + \frac{2}{5}D(x,39). \tag{7}$$

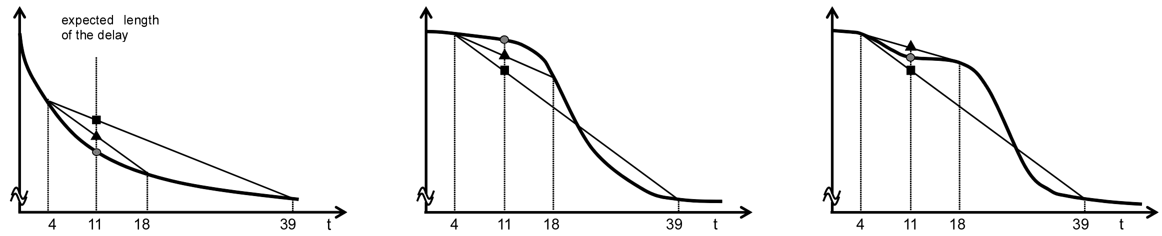
Since $18 = \frac{3}{5}4 + \frac{2}{5}39$, this is consistent with a concave time discount function. The first result presents a new anomaly in the literature of the time preference.

Observation 1. The subjects choose an option that is consistent with concave time discount functions, on average, at 9.14 questions out of the 15 questions of Group A–E. The standard deviation is 3.67, the minimum is 3 questions.

Table 2: Full list of the questions

	reward (yen)	left option		right option		choice left	choice right
1	2000	50%	4	80%	4	(26)	(11)
		50%	18	20%	39		
2	2000	99%	4	100%	7	(4)	(33)
		1%	304				
3	2000	100%	40	89%	40	(23)	(14)
				1%	700		
				10%	4		
4	6000	70%	4	85%	4	(14)	(23)
		30%	154	15%	304		
5	2000	100%	18	50%	11	(23)	(14)
				50%	25		
6	6000	89%	700	90%	700	(7)	(30)
		11%	40	10%	4		
7	2000	100%	49	70%	4	(17)	(20)
				30%	154		
8	2000	100%	11	50%	4	(20)	(17)
				50%	18		
9	4000	100%	40	89%	40	(31)	(6)
				1%	700		
				10%	4		
10	2000	75%	11	100%	18	(4)	(33)
		25%	39				
11	2000	50%	7	49%	4	(32)	(5)
		50%	none	1%	154		
				50%	none		
12	2000	100%	7	98%	4	(34)	(3)
				2%	154		
13	2000	89%	700	90%	700	(7)	(30)
		11%	40	10%	4		
14	2000	70%	4	85%	4	(12)	(25)
		30%	154	15%	304		
15	6000	85%	4	100%	49	(15)	(22)
		15%	304				
16	6000	100%	40	89%	40	(18)	(19)
				1%	700		
				10%	4		
17	2000	80%	4	100%	11	(8)	(29)
		20%	39				
18	2000	50%	11	75%	11	(17)	(20)
		50%	25	25%	39		
19	4000	89%	none	90%	none	(23)	(14)
		11%	40	10%	4		
20	2000	98%	4	99%	4	(18)	(19)
		2%	154	1%	304		
21	2000	85%	4	100%	49	(18)	(19)
		15%	304				
22	6000	100%	49	70%	4	(21)	(16)
				30%	154		

Note. For every question of Group A–E, the expected length of delay of the left option and the right option is identical. There are some options with "none" delay, which means the reward becomes unavailable with the designated probability. The number in parenthesis indicates how many subjects (out of 37) choose the left or right option.



(a) Convex (b) Concave (c) Winding

Note. For Group B questions $\{1, 8, 17\}$, there are $2^3 = 8$ variations of responses, each of which identifies the shape of time discount function. This figure illustrates three of them. The circle, triangle and square marks represent, $D(11)$, $\frac{1}{2}D(4) + \frac{1}{2}D(18)$, and $\frac{4D(4)+D(39)}{5}$, respectively.

Figure 3: Classifications of time discount function

Remark 1. Recall that a subject would never choose such an option, if her/his decision making was based on the standard model with a convex discount function. We observe, however, the opposite. Most of the subjects choose the options which are not consistent with the convex time discount curve. This anomaly has never been reported before.

I classify 37 subjects for each of the question Groups, according to their responses, in the following 4 categories: Convex, Concave, Winding, Cyclical. Figure 3 illustrates these concepts based on Group B questions $\{1, 8, 17\}$. In these questions, there are three options, whose expected lengths of delay are all 11 days.

The preference order of the subjects in Convex indicates

$$\text{Convex: } D(11) < \frac{D(4) + D(18)}{2} < \frac{4D(4) + D(39)}{5} \tag{8}$$

This corresponds to the time discount function of panel (a) of Figure 3. Note that almost all models in the literature assumes that time discount function is convex.

Panel (b) presents one of the following cases,

$$\text{Concave: } D(11) > \frac{D(4) + D(18)}{2}, \text{ and } \begin{cases} \dfrac{D(4) + D(18)}{2} < \dfrac{4D(4) + D(39)}{5}, \\ \text{or} \\ D(11) > \dfrac{4D(4) + D(39)}{5} \end{cases} \tag{9}$$

The time discount function in pane (b) is locally concave, while it may look overall inverse S-shaped. Subjects exhibit future bias where the time discount is concave, and they may exhibit present bias as well. I will discuss later why previous researches seldom detect future bias.

There are other two classifications that time discount function is not necessarily well-behaved. Winding is one of them, which is shown in panel (c).

$$\text{Winding: } \frac{D(4) + D(18)}{2} > \frac{4D(4) + D(39)}{5} \text{ and } D(11) < \frac{D(4) + D(18)}{2} \tag{10}$$

Finally, in the three questions some respondents do not satisfy transitivity in their choice. They belong to Cyclical.

Observation 2. For each of Group A–Group E, the 37 subjects are classified as follows:

Questions	Expected delay	Convex	Concave	Winding	Cyclical
Group A $\{2, 12, 20\}$	7 days	0	34	2	1
Group B $\{1, 8, 17\}$	11 days	3	19	11	4
Group C $\{5, 10, 18\}$	18 days	0	23	5	9
Group D $\{7, 14, 21\}$	49 days	13	16	4	4
Group E $\{4, 15, 22\}$	49 days	12	20	2	3

In Group A and C, there is no subject classified into Convex. In Group A, B and C, more than half subjects are classified into Concave. These contrast with Group D and E, where the expected length of the delay is 49 days. 13 and 12 subjects are Convex, respectively.

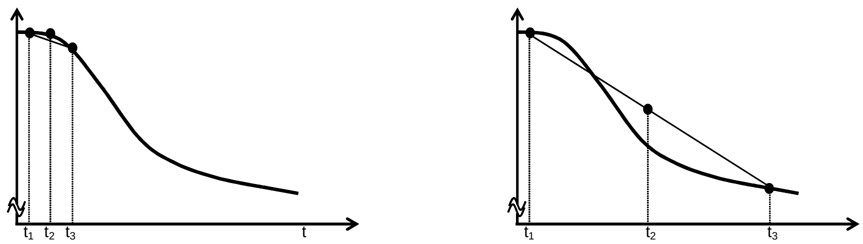
Remark 2. To explain this observation, the inverse S-shaped time discount function shown in Figure 2 serves better than the standard convex time discount function. Recall that if a subject has the inverse S-shaped discount curve, s/he will be classified as Concave around a short delay and also be classified as Convex around a longer delay. In this sense, the observation shown in the table above is consistent with the inverse S-shape curve. Few of the subjects are Convex at questions where the expected delays are 7, 11 and 18 days, although one third of them are Convex when the expected delay is 49 days.

5 Discussions

Future bias, although it is theoretically plausible, has rarely been reported in the literature. I suppose two major reasons. Firstly, the literature focuses on present bias simply because it is observed much more frequently in reality than future bias and because there are few situations associated with future bias. Moreover, the previous researches on time preference employ a hyperbolic discount function to show the present bias, which can never detect future bias. Some of the researchers perhaps noticed "misbehavior" that would imply future bias, but I suppose that the misbehavior was disregarded as just an error.

The other reason is more technical. In almost all previous experiments, the time intervals are too long to find the concavity of time discount function. See Figure 4. In both of the panels, assume t_2 is the median point between t_1 and t_3, though the intervals are different. In panel (a), the interval between t_1 and t_3 is short enough to find the concavity, while it is too long in panel (b). Takeuchi (2011) find that the subjects exhibit future bias and that their time discount function is concave around $t = 22$ days (around 3 weeks). Sayman and Öncüler (2009), the other research that finds future bias, set a time interval shorter than 4 weeks. Thus, to identify concavity and future bias, the time ranges or intervals in experiments ideally should be less than 2 weeks. Most of the previous studies, however, have much longer time ranges and therefore they do not recognize future bias.

To provide a psychological account for future bias, I stress again that future bias is not contracted with present bias and that the existing model can easily take incorporate future



(a) short interval (b) too long interval

Note. (a) A short interval in timings can detect the concavity of the curve and future bias. (b) A long interval cannot detect it and just imply the convexity.

Figure 4: The length of intervals and the detection of concavity

bias. Consider the quasi-hyperbolic discount function that discounts any **future** reward by $\beta < 1$;

$$\text{Quasi-hyperbolic:} \quad D(t) = \begin{cases} 1 & \text{if } t = 0, \\ \beta e^{-rt} & \text{otherwise.} \end{cases} \tag{11}$$

Ask "when does the **future** really start?" We are all standing at the present $t = 0$ which is the very frontier of the past time line. Thus, technically, we can call any $t > 0$ the future. Note however that the present $t = 0$ is floating into the future with ourselves. The point of $t = 0$ is always moving forward (even while you are reading this sentence), with the speed of 1 second per second. Thus, it is not intuitively easy to draw a clear line between the present and the future. Suppose that a subject perceives ($10 in 10 minutes) as a future reward, but suppose that s/he does not think ($10 in 3 minutes) is a future reward because 3 minutes are not long enough for her/him to feel any disutility of delay. In this sense, we may have to reconsider the notation of "present". The present is not a single point on the time line, instead, it can be extended to the near future. Then, we can re-define the quasi-hyperbolic discount as follows:

$$D(t) = \begin{cases} 1 & \text{if } 0 \le t \le \hat{t} \text{ for some } \hat{t} > 0, \\ \beta e^{-rt} & \text{otherwise.} \end{cases} \tag{12}$$

where $\hat{t} > 0$ is the long enough delay (3 minutes in the above example) at which a decision-maker starts feeling the pain of delay. Observe that this time discount function is an inverse S-shaped curve. It implies that the decision-maker exhibits future bias if time interval of future rewards is less than $\hat{t}$ and exhibits present bias otherwise. The future bias is not a strange phenomenon at all, but it can be characterize by a reasonable extension to the existing models.

6 Concluding remarks

This paper presents a new anomaly in the literature of time preference, which is future bias. The experiment with 37 subjects also provides the evidence in favor of an inverse S-shaped time discount function rather than a hyperbolic (and convex) time discount function. The result implies that the time discount function is locally concave.

The experimental design also has several advantages. Previous studies commonly assume that (i) t and x are separable, namely, $D(t, x)$ is invariant in x, and that (ii) $u(x)$ is linear or CRRA. These assumptions have been serious confounding factors in the estimation. Notice that my experimental method in this paper assumes neither of these, and it still elicits the concavity/convexity of time discount function.

If time discount function is concave around t, then the decision-maker exhibits future bias. Since the future bias is not easily identified, it has seldom been reported in the literature. To detect it, experiments should have let subjects compare two future reward options whose time interval is shorter than 2 weeks.

The inverse S-shaped time discount also imply that time perception is not monotone. Time runs slow around $t = 0$ and then it goes fast where the discount function changes its shape from concave into convex. Again, it seems that time runs slow in far future since it is too far to feel the disutility of any additional delay: who cares the difference between "1 year" and "1 year plus 3 minutes"?

It is important to identify the cognitive temporal distance of a given delay to construct models of intertemporal choice. Some people feel that 3-minute delay is long (thus they discount), while the others feel that it is not even a delay. In other words, each individual has her/his own time perception, how s/he recognizes a physical time line in the internal clock, which is not necessarily linear. Takahashi (2005), for example, shows that a logarithmic time perception may lead to a hyperbolic time discount. Moreover, Liberman and Trope (2008) argue that people tend to focus on the concrete and detailed features of an event if they are told that the event will happen in near future, though they tend to focus on more abstract aspects of the event if it will happen in some far future. This is based on so-called Construal Level Theory (CLT). We should re-examine economic models of intertemporal choices, if this CLT plays a role in the process of discounting. For example, assume that the amount of reward x is not an abstract but rather a concrete aspect of a future reward option. Then, we should consider which of the following is more plausible : (i) the utility function $u(x)$ is no longer time invariant, or (ii) the discount function $D(x, t)$ still captures the CLT.

Though these issues are not only interesting but also critically important in the literature, I leave them for *future* researches. At this present, this paper serves to show a new anomaly, future bias, and a new behavioral model, an inverse S-shaped time discount function.

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Non-parametric test of time consistency: Present bias and future bias

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Abstract

This paper reports the elicited time preference of human subjects in a laboratory setting. The model allows for non-linear utility functions, non-separability between delay and reward, and time inconsistency including future bias in addition to present bias. In particular, the experiment (1) runs a non-parametric test of time consistency and (2) estimates the form of time discount function independently of instantaneous utility functions, and then (3) the result suggests that many subjects exhibiting *future bias*, indicating an inverse S-curve time discount function. (*Games and Economic Behavior*, vol. 71, No. 2, pp.456–478, 2011.)

Keywords: intertemporal choice, time preference, present bias, future bias, reverse time inconsistency, increasing impatience, decreasing impatience.

JEL Classifications: C91, D81, D99

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1 Introduction

People are typically averse to a delayed reward and prefer an option that pays a smaller reward immediately. This positive *time preference* has been observed in experiments on pigeons, rats, and humans.[1] It has been shown that when a reward is delayed into the future, its present value decreases. More interestingly, the time inconsistent preference referred to as *present bias* has been frequently reported: subjects put more weight on the value of a present reward than a future one. This phenomenon is also called *decreasing impatience*, as the subject's impatience decreases into the future (Prelec, 2004).[2]

There are many experimental studies on time preference, most of which commonly rely on several assumptions. Those assume, for instance, that the time discounting function is separable from the reward amount, the subjects believe that delayed rewards will be delivered for sure, the monetary rewards are non-fungible, and that the utility function is linear and time invariant. These often become the common confounding factors in experimental studies on time preferences.

Among these, this paper particularly addresses the separability assumption between discounting and utility and the linearity assumption on the utility function, presenting an experiment that elicits the time preference of subjects in a laboratory setting. Suppose that $V(x,t) = D(t,x)u(x)$ denotes the present discounted value of a reward x that will be paid at time t. The separability assumption forces us to ignore the second element of $D(t,x)$, which necessarily biases the result. As seen in this formula, any parametric assumption on the utility function u also naturally entails a bias in the estimation of time discount D. To incorporate these estimation biases, the present study develops a non-parametric test of time preference.

Specifically, this experiment runs a non-parametric test based on *equivalent delay function*. It also estimates the form of time discount function independently of that of the instantaneous utility function by converting delay into risk. Then I find the result suggesting that many subjects exhibit future bias.

First, I define the equivalent delay function on two reward options ($T : X \times Y \to \mathbb{R}_+$) and show that the modularity of the function T characterizes time inconsistent behavior. For a present reward x and a larger but delayed reward y, the equivalent delay $T(x,y)$ specifies the delay of y with which the two reward options are equally valuable. Then I show the sub(super)modularity of the function corresponds to the decreasing (increasing) impatience. Eliciting T for several pairs of (x,y) provides non-parametric characterization of time inconsistency. To the best of my knowledge, this method is new to the literature.

Second, the experiment converts delay into risk and attempts to estimate the form of the time discount function independently of the utility function. The advantage of this approach is to separate the time discount factor from the non-linearity of the utility function, both of which are often confounded in the literature.

[1] For humans, Frederick et al. (2002) provide a comprehensive review of time preference from an economic perspective, while Green and Myerson (2004) provide an overview of studies in psychology. In the experiments on pigeons and rats, the reward is food/water or the access to it. Time preference is referred to as impulsive behavior in the literature. See Monterosso and Ainslie (1999) for a survey.

[2] Halevy (2008) calls this diminishing impatience

Section 4 illustrates the potential biases in the estimation that might be caused if the utility function is assumed to be linear.

Finally, the result indicates that many subjects exhibit future bias and increasing impatience for few weeks from the present moment. The previous experimental studies assumed that subjects exhibited present bias and hyperbolic time discounting, but they seemingly overlooked the other time inconsistency, which I call *future bias*. While the present bias means that subjects tend to overvalue the immediate reward and result in a myopic preference reversal, the future bias is the opposite. Although there have been few studies of future bias, Loewenstein (1987) has already found the same type of anomaly as *reverse time inconsistency*. With reverse time inconsistency, subjects tend to postpone taking a reward until the near future. Rubinstein (2006) also introduces future bias as a possible (hypothetical) time inconsistent behavior. Very recently Sayman and Öncüler (2009) observed subjects actually exhibiting reverse time inconsistent behavior in their longitudinal experiment. My concurrent study finds similar evidence for future bias.

Notice that future bias and present bias are not conflicting, but they may coexist within a subject. Future bias, however, has not been frequently observed, since it can be elicited under certain conditions. Specifically, the interval between two future options has to be sufficiently short and the options have to be close to $t = 0$. In addition, to capture such a time preference, an inverse S-curve time discount function must be employed. Thus, I use the generalized Weibull function and show that some subjects have such time discount functions, i.e., they are concave for the first part and convex for the rest.

The rest of the paper is organized as follows. The next section explains the motivation and the model. Section 3 presents the experimental design. Section 4 presents the analysis and main results. In Section 5 I discuss the results, and Section 6 concludes the paper.

2 The model

This section describes a new experimental design. Unlike existing studies, this experiment estimates time preference without making any parametric assumption on the utility function. Before introducing the design, I highlight the importance of the approach.

Let $(x, t) \in \mathbb{R}^2_+$ represent an option that pays x at time t. The present value V of the option is

$$V(x, t) = D(t, x)u(x)$$

where D is a discount function and u is the instantaneous utility of the reward x. By observing preferences of subjects over several options, the experiments estimate the functional form of D. Although there is extensive research on this matter, the utility function $u(x)$ is, in most experimental studies, assumed to be linear (see Table 1).[3]

[3]There are a few exceptions. Kirby and Santiesteban (2003) compare $u(x) = x$ with $u(x) = \sqrt{x}$ and find no significant difference in goodness-of-fit. Andersen et al. (2008) , Fernández-Villaverde and Mukherji (2006) and Ida and Goto (2009)

Study	Utility Function	Adjustment Procedure	Elicitation Method
Thaler (1981)	linear	amount	matching
Benzion et al. (1989)	linear	amount	matching
Rachlin et al. (1991)	linear	amount/delay	choice
Holcomb and Nelson (1992)	n.a.	amount/delay	choice
Bohm (1994)	linear	amount	choice/matching
Keren and Roelofsma (1995)	linear	amount/delay	choice
Kirby and Maraković (1995)	linear	amount	matching
Wahlund and Gunarsson (1996)	linear	amount	matching
Ahlbrecht and Weber (1997)	linear	amount	choice/matching
Cairns and van der Pol (1997)	linear	amount	matching
Green et al. (1997)	linear	amount	choice
Kirby (1997)	linear	amount	matching
Chapman and Winquist (1998)	linear	amount	matching
Holden et al. (1998)	non-linear	amount	matching
Chapman et al. (1999)	linear	amount	matching
Coller and Williams (1999)	linear	amount	choice
Kirby et al. (1999)	linear	amount/delay	choice
Chesson and Viscusi (2000)	n.a.	delay	matching
Hesketh (2000)	linear	amount/delay	choice
Anderhub et al. (2001)	linear	amount	matching
Read (2001)	linear	amount	matching
van der Pol and Cairns (2001)	linear	amount/delay	choice
Warner and Pleeter (2001)	linear	amount	choice
Harrison et al. (2002)	linear	amount	choice
Kirby and Santiesteban (2003)	linear and $x^{0.5}$	amount	matching
Rubinstein (2003)	non-linear	amount/delay	choice
Harrison et al. (2005)	linear	amount	choice
Fernández-Villaverde and Mukherji (2006)	CRRA	—	choice
Andersen et al. (2008)	CRRA	amount	choice
Kinari et al. (2009)	linear	amount/delay	choice
Sayman and Öncüler (2009)	linear	amount	matching
Attema et al. (2009)	n.a.	delay	matching
Ida and Goto (2009)	CRRA	amount/delay	choice
Tanaka et al. (2010)	CRRA	amount	choice
Benhabib et al. (2010)	linear	amount	matching
Coller et al. (2011)	CRRA	amount	choice
This concurrent study	n.a.	delay	matching

Note. The instantaneous utility of reward x is usually assumed to be *linear*, i.e., $u(x) = x$. The adjustment procedure is either *amount* or *delay* or both. In the amount-adjustment procedure, subjects are asked to choose what amount of a present (future) reward x makes itself equally valuable to the other future (present) reward, and the delay of the future option is fixed. In the delay-adjustment procedure, subjects are asked to choose the delay of a future reward that makes it equally worth to a given present reward. Elicitation Method: In order to elicit those amounts or delays, subjects are given (a list of) two fixed options in the *choice method* or asked to specify the amount or delay in the *matching method*.

Table 1: Methodologies of Empirical Estimations of Time Preference

The linear utility assumption may become problematic, in particular, when we are interested in the functional form of D, e.g., whether it is exponential or hyperbolic discounting. Suppose that, for example, a subject is indifferent to any pair of options from $\{x_i, t_i\}_{i \in I}$, that is $V(x_i, t_i) = V(x_j, t_j)$ for any $i, j \in I$. Next suppose that one researcher assumes $u(x) = x$ and that she finds $D(t) = e^{-rt}$ fits the data perfectly. Given the same data, however, another researcher assumes that $u(x) = \log(x)$ and finds that $D(t) = 1/kt$ explains the behavior of the subject well. Thus, the former concludes that D is exponential, while the latter concludes D is hyperbolic. Notice that these two different conclusions do not necessarily contradict each other: they simply reflect the difference in the assumptions on the utility function.[4]

Furthermore, a parametric assumption on u can produce a *magnitude effect*, one of the anomalies commonly reported in the literature.[5] Thaler (1981) reports that subjects answered, on average, that they were indifferent between the two options in each of the following pairs respectively: ($15, now) vs. ($60, one year later) and ($3000, now) vs. ($4000, one year later). As long as the utility function is assumed to be linear, there is no discount factor that is consistent with these two choices ($15/60 \neq 3000/4000$). This anomaly is called the magnitude effect because the discount factor depends on the amount of the reward. It becomes easy, however, to find a constant discount factor once we allow general utility functions.[6] This observation does not necessarily contradict the discounted utility framework; it just reflects the restrictive assumption made on the utility function.

A new experimental design, therefore, should elicit time preference without invoking the linearity assumption on the utility function. The following subsections present two methods that estimate time preference independently of u. The first one elicits time preference or time inconsistent behavior without considering the utility function, and the second one estimates the time discount function without the linearity assumption on u.

I have to stress that the second method still relies on the separability between x and t. Almost all experimental research on time preference make the separability assumption and do not capture possible interactions.[7]

2.1 Modularity of equivalent delay function

The first part of this experiment elicits time preference for a set of rewards, $x_0 < x_1 < x_2$. I define the equivalent delay that makes the present value of a future large reward equal to the value of a present small

assume a constant relative risk-aversion (CRRA) utility function. Rubinstein (2003) does not impose any assumptions. The novel experimental design of Attema et al. (2009) does not require the functional form (utility-free). Tanaka et al. (2010) estimate parameters for CRRA utility functions incorporating with loss aversion and probability weighting function.

[4] A trivial example of such a data set is $\{(x_i, t_i) | x_i = e^{rt_i}.\}$.

[5] Frederick et al. (2002) refer to the magnitude effect as one of the six commonly observed anomalies. It is referred to as *amount-dependent discounting* in the psychology literature. See the extensive survey by Green and Myerson (2004).

[6] For example, $u(x) = x^{0.42} + 45.9$ can accommodate the anomaly above. That is, $u(15)/u(60) = u(3000)/u(4000) = 0.95$. Masatlioglu and Ok presented this numerical example in an earlier version of their paper.

[7] There is an exception, which is, the research by Benhabib et al. (2010) that allows for a fixed cost of present bias.

reward. Then, I show that the modularity of that equivalent delay function characterizes time inconsistency.

Suppose that a subject is indifferent between two options (x, t) and (x', t'). I set $t = 0$ for the following argument. For a given $(x, 0)$ and x', this experiment elicits the equivalent delay $T(x, x')$ that makes the two options the same to a subject.[8] Let T be such a function on $\left\{(x, x') \in \mathbb{R}^2_+ \middle| x \leq x'.\right\}$.

Definition. $T(x, x')$ is an *equivalent delay* such that $(x, 0) \sim (x', T(x, x'))$.

Present and future biases, if any, are detected in the properties of this function T. Notice that, by transitivity, $D(T(x_0, x_1), x_1) \times D(T(x_1, x_2), x_2) \equiv D(T(x_0, x_2), x_2)$ holds regardless of the form of D and T. First, the following definition is straightforward.

Definition. A subject is *time consistent* if T is modular.

If T is modular, a subject will not exhibit time inconsistent preference reversal. For example, the standard exponential discount function, $D(t, x) = e^{-rt}$, implies that

$$T(x_0, x_1) + T(x_1, x_2) = T(x_0, x_2)$$

Figure 1 illustrates this concept. Suppose that a subject responds as follows: (\$5 today)$\sim$(\$10 in 10 days), (\$5 today)$\sim$(\$15 in 16 days) and (\$10 today)$\sim$(\$15 in Z days). This means $T(5, 10) = 10$, $T(5, 15) = 16$ and $T(10, 15) = Z$. Assume that the subject is given the (\$15 in 16 days) option. Then, suppose the first 10 days have passed and there are still 6 days to go. Imagine that she is offered another option of (\$10 today) at that moment. She compares the two options (\$10 today) and (\$15 in 6 days). If she is time consistent, she is still indifferent and she is willing to wait 6 days to get \$15. Recall she has already answered (\$10 today)$\sim$(\$15 in Z days). Thus, $Z = 6$ corresponds to time consistency, implying $T(5, 10)+T(10, 15)=T(5, 15)$.

In the example above, the subject compares (\$10 today) and (\$15 in 6 days) at day $T(5, 10)$. But she would be willing to wait only Z days to get \$15. If she has present bias, then the immediate \$10 becomes more attractive than the future option of (\$15 in 6 days). Thus, $Z < 6$ or $T(5, 15)-T(5, 10) > T(10, 15)$. Formally, I define this present bias as follows.

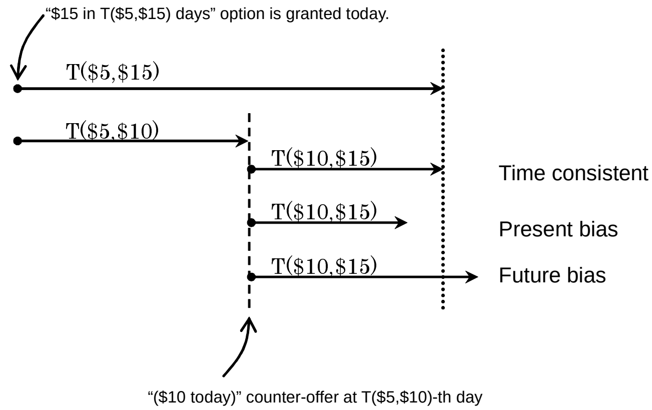
Definition. A subject exhibits *present bias* if T is strictly submodular.

Observe, for example, that hyperbolic discount functions imply $T(x_0, x_1)+T(x_1, x_2) < T(x_0, x_2)$. In fact, when u is continuous, this present bias is consistent with the *decreasing impatience* of Prelec (2004). See the appendix for proof.

Definition (Prelec (2004))**.** A subject is said to exhibit *decreasing impatience* if for any $\delta > 0$, $x_2 > x_1 > 0$, $(x_1, t_1)\sim(x_2, t_2)$ implies $(x_1, t_1 + \delta)\prec(x_2, t_2 + \delta)$.

Proposition 1. *Decreasing impatience is equivalent to present bias.*

[8] Noor (2010) similarly defines more general *time compensation function*, $\Psi_{s,l}(t)$. $T(x, x')$ is equivalent to $\Psi_{x,x'}(0)$.



Note. (\$5 today), (\$10 in 10 days) and (\$15 in 16 days) are equally valuable to a subject by transitivity. Assume that the subject is given the (\$15 in 16 days) option and then 10 days have passed. Note that she stands on the broken line in the middle of the figure and there are still 6 days to go for \$15. Then, imagine she is offered another option of (\$10 today) at that moment. She compares the two options (\$10 today) and (\$15 in 6 days). If she is time consistent, she is still indifferent and she is willing to wait 6 days to get \$15. Thus, $Z = 6$ or $T(5,10)+T(10,15)=T(5,15)$. If her preference is present biased, then she is not willing to wait 6 days. Instead, she chooses (\$10 today) option. It implies $Z < 6$ or $T(5,10)+T(10,15)<T(5,15)$. The future bias is defined for the opposite reversal.

Figure 1: Present and future bias

As this nonparametric test does not depend on the separability between x and t, it is compatible with the fixed cost representation of present bias proposed by Benhabib et al. (2010). Following their approach, let us suppose that any future option incurs a fixed cost of b and that the discount function is exponential (e.g., $V(x,t) = e^{-rt}u(x)-b$ or $V(x,t) = e^{-rt}u(x-b)$). These representations also imply that $T(x_0,x_1)+T(x_1,x_2) < T(x_0,x_2)$. See the appendix for the derivation.

Remark 1. The present bias due to the fixed cost of future reward is also characterized by the submodularity of T.

Accordingly, the opposite time inconsistent preference, future bias, implies that subjects become more impatient with delay as time goes into the future. *Future bias* or *increasing impatience*, can be defined in the following way.

Definition. A subject exhibits *future bias* if T is strictly supermodular. Equivalently, a subject exhibits *increasing impatience* if for any $\delta > 0$, $x_2 > x_1 > 0$, $(x_1,t_1)\sim(x_2,t_2)$ implies $(x_2,t_2+\delta)\prec(x_1,t_1+\delta)$.

This experiment runs a longitudinal analysis for $x_0 < x_1 < x_2$ and finds that most subjects exhibit future bias (see Result 1 in Section 4).

2.2 Converting delay into uncertainty

The second part of the experiment is to estimate the time discount function D independently of u. That is, the experiment does not estimate the form of the utility function. Note however this estimation still relies on the separability assumption. The discounted present value of (x, t) is denoted by $D(t)u(x)$ hereafter.

This method observes data points for the time discount function without using any parametric assumption on the utility function. To do so, it elicits both of the equivalent delay and the equivalent probability in within-subject design.[9] Thus, in this part, we need to estimate the ratio of the instantaneous utilities of two rewards.

Let $(x, p) \in \mathbb{R}_+^2$ represent a lottery that pays x with probability p and 0 otherwise. Suppose a subject is indifferent between a pair of lotteries, (x, p) and (x', p'). The separability assumption yields the following:

$$pu(x) = p'u(x'), \text{ if } (x, p) \sim (x', p') \tag{1}$$

For a given (x, p) and x', I elicit the probability p' that makes the subject indifferent between the two lotteries.[10] In this experiment, I set $p = 1$, so one option definitely pays the reward, and it follows that

$$p' = \frac{u(x)}{u(x')} \tag{2}$$

Recall that the same subject has reported the equivalent delay t' that makes herself indifferent between the immediate option that pays x and the delayed option that pays x' in t'. By the separability assumption, It follows that

$$D(t') = \frac{u(x)}{u(x')} \tag{3}$$

The two equations (2) and (3) yield one point for D in time-probability space; this point satisfies the following identity:

$$D(t') = p' \tag{4}$$

which holds without any parametric assumption on u.[11]

In the first part, $t = 0$ means that one option pays x immediately without any delay, while in the second part, the lottery with $p = 1$ pays x for sure. Thus, subjects compare the delayed reward with the immediate one, and they assess an uncertain option by comparing it with the certain reward.[12] By transitivity, $(x, t) \sim (x, p)$, so subjects use such comparisons to indirectly convert p into t (or vice versa).

[9] Notice there are two underlying assumptions. One is that subjects are expected utility maximizers and the other is that u is time invariant.

[10] I use only this probability equivalence (PE) method, not a certainty equivalence (CE) method, which elicits the certainty equivalent x for a given lottery (x', p'). Since this experiment intends to examine the correspondence between the time delay and the risk for a pair of fixed rewards, the CE method cannot be applicable. However, note that the systematic bias and the discrepancy between the PE and CE method are reported in Hershey and Schoemaker (1985).

[11] Recall that subjects are assumed to be EU maximizer. If the prospect theory applies here, that is, subjects transform p into subjective weighting $\pi(p)$, then the identity above should be $D(t') = \pi(p')$. Note that, however, the estimated time discount function represents the corresponding risk p' to the given delay t'. Thus, this experimental design still integrates the risk and time preferences.

[12] The front end delay (FED) design is used to control the transaction cost of the rewards and the immediacy effect in the recent experimental studies (Andersen et al., 2008; Benhabib et al., 2010; Coller and Williams, 1999). With the FED, the earlier

2.3 Discount function and hazard function

This subsection introduces the basic concept of survival analysis into the time preference framework. Because the experiment lets subjects convert delay into risk, the survival analysis framework is appropriate. To understand the underlying concept better, let us consider why humans, even animals, discount future rewards. There can be several explanations: they are mortal, there is a future uncertainty, (opportunity) cost of waiting, and so forth (Yaari, 1965). Alternatively, one can simply say that they have *pure* time preference, aside from risk. Yet, I compound all of those plausible explanations into one function of time, $D(t)$; this is consistent with assuming that time discounting is caused by the uncertain nature of the future (Green and Myerson, 1996; Stevenson, 1986). I similarly suppose there is an underlying single process of discounting any uncertainty or risk, including the future.[13] For further discussion and justification, see Bommier (2006), Dasgupta and Maskin (2005) and Rachlin et al. (1991).

Suppose a subject makes an intertemporal decision, at time 0, as if she presumes that the future reward is uncertain for some reason. I assume that she has consciously or subconsciously determined her subjective probability that the reward is no longer available to her at time t. Denote this by $F(t)$, which is called the *failure function* in this context. The *survival function* is defined as $D(t) = 1 - F(t)$. Note that it corresponds to the time discount function. Then, the *hazard function* is defined as follows:

$$h(t) = F'(t)/D(t)$$

which is the conditional probability that the reward becomes unavailable at time t given that it has been available up to time t. The hazard rate is also referred to as the instantaneous discount rate (e.g., Laibson (1997)), and it represents her impatience at a given moment t.

Prelec (2004) shows that $\ln D(t)$ is convex in t if and only if the time preference exhibits decreasing impatience. Then, a corollary immediately follows, since $-h(t) = d \ln D(t)/dt$:

Corollary 1. *The hazard function $h(t)$ is decreasing (increasing) in t if and only if the time preference exhibits decreasing (increasing) impatience.*

That is, when I characterize the time inconsistent behavior of subjects, it is sufficient to examine the hazard function. Compared to the hazard function, the time discount function is more familiar and intuitive. Therefore, I estimate both.

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option will not be paid immediately; instead, it will be paid with a little delay (see Harrison and Lau (2005) for a discussion). Although I was aware of the advantage, I did not adopt it for the following reason. Suppose two delayed options are offered, (x, t) and (x', t') where $0 < t < t'$. Note that, in theory, the time discount function depends on both timings (see Masatlioglu and Ok (2007)). That is, $D(t, t')$ is not necessarily equal to $D(0, t' - t)$ or $D(0, t')/D(0, t)$. Thus, we cannot use that observation to elicit $D(0, t')$. In addition, It is important to keep the symmetric structure between the time and risk preference tasks. As Keren and Roelofsma (1995) and Halevy (2008) argue, the immediacy effect and the certainty effect have several common properties. If that is the case, the immediate reward ($t = 0$) corresponds to the certain reward ($p = 1$). It is not certain, however, what p would correspond to a seven-day FED ($t = 7$).

[13] Prelec and Loewenstein (1991) review and contrast the anomalies in both expected utility theory and discount utility theory. For example, the decreasing impatience ("common difference effect") corresponds to the "common ratio effect (anomaly)" in

$$D(t,\theta,r,q) = \frac{1}{\left[1+\theta(rt)^q\right]^{\frac{1}{\theta}}} \quad \text{and} \quad h(t,\theta,r,q) = -D'(t)/D(t)$$

θ	q	$D(t)$	
0	q	$e^{-(rt)^q}$	Weibull
0	1	e^{-rt}	exponential discount function
1	q	$\left[1+(rt)^q\right]^{-1/q}$	log-logistic
1	1	$\left[1+rt\right]^{-1}$	hyperbolic discount function

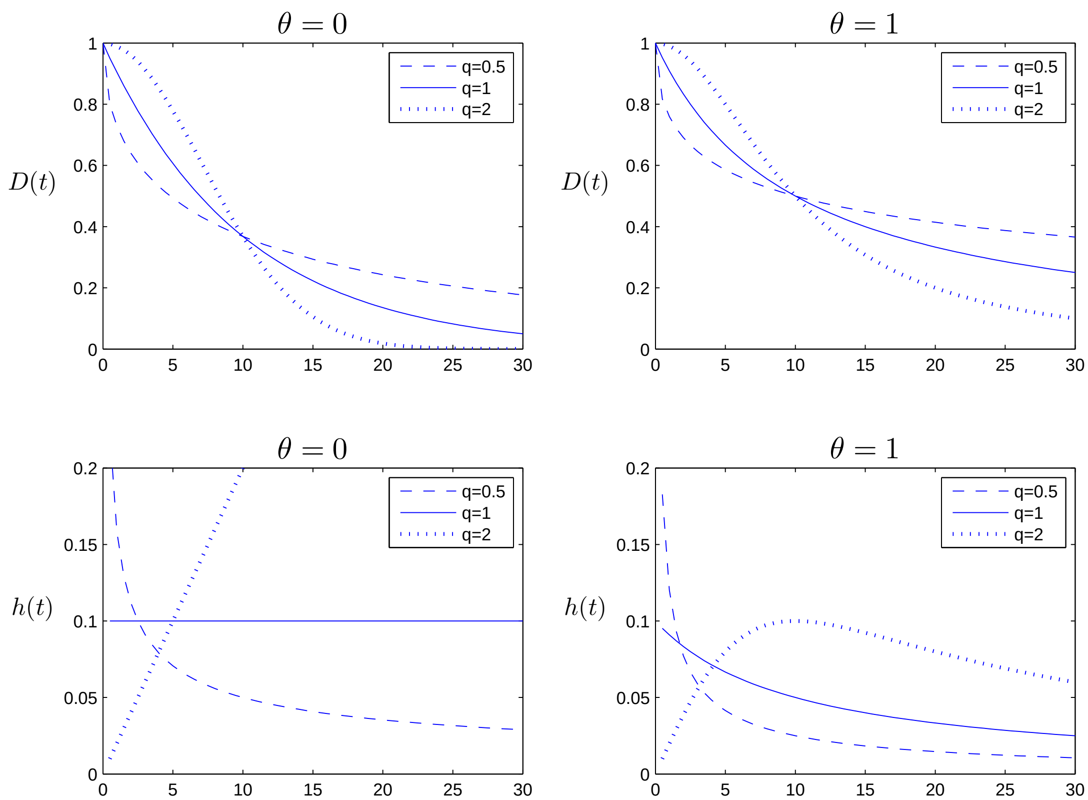


Figure 2: Example plots of $D(t)$ and hazard functions

As for the functional form of D, I assume the following:

$$D(t, \theta, r, q) = \frac{1}{[1 + \theta(rt)^q]^{\frac{1}{\theta}}} \tag{5}$$

where $\theta \in (0, 1], r \in [0, \infty)$ and $q \in [0, \infty)$. This D, called the generalized Weibull model, is a further-generalized version of the *generalized hyperbolic* of Loewenstein and Prelec (1992). They propose $D(t) = (1 + \alpha t)^{-\beta/\alpha}$ in their original notation, which is a special case of the above $D(t)$ when $q = 1$, $\alpha = \theta r$, and $\beta = r$. Note that, while the generalized hyperbolic form represents only decreasing impatience (present bias), the generalized Weibull function (5) can represent increasing impatience (future bias) as well.[14] This is the advantage of the generalized Weibull function.

Remark 2 (slope of hazard functions)**.**
1) If $q \leq 1$, then $h'(t) \leq 0$ (with strict inequality if $\theta > 0$), implying present bias and decreasing impatience.
2) If $q = 1$ and $\theta = 0$, then $h'(t) = 0$, implying time consistency and constant impatience.
3) If $q > 1$, then $h'(t) \geq 0$ for $t < \bar{t}$ and $h'(t) \leq 0$ for $t > \bar{t}$ where $\bar{t} = \left[(q-1)/r^q\theta\right]^{1/q}$, implying future bias and increasing impatience.

Remark 3 (inverse S-curve $D(t)$)**.** If $q > 1$, then $D(t)$ is an inverse S-curve function. That is, $D(t)$ is concave for $0 \leq t \leq \hat{t}$ and convex for $\hat{t} \leq t$, where $\hat{t} = \left[(q-1)/r^q(\theta + q)\right]^{1/q}$.

Figure 2 illustrates the functional form of D for several parameter values. When $\theta = 1$, D is called the Weibull model, and its hazard rate depends on q: constant for $q = 1$ and increasing (decreasing) when $q > 1$ (when $q < 1$). If $\theta = 0$, D is called the log-logistic model, which nests a hyperbolic discount function ($q = 1$).

Benhabib et al. (2010) use a novel approach to determine whether an exponential or hyperbolic discount function fits better. They parameterize time preferences by θ with fixed $q = 1$. Here, I would like to expand their approach by estimating both θ and q together. As Corollary 1 shows, it is sometimes more informative to estimate the slope of $h(t)$ rather than the functional form of D.

3 Experimental design

The aim of this experiment is (i) to find any time inconsistent behavior (present bias or future bias), (ii) to specify the functional form of the discount function, $D(t)$, and (iii) to evaluate biases that would have been caused by the linearity assumption on $u(x)$. The experiment consists of four components: a delayed-payment task to elicit time preferences, a lottery choice task to elicit risk preferences, a psychological survey, and a demographic survey.

expected utility theory, and the present bias is equivalent to the certainty effect anomaly. The similar structures of those anomalies support my view.

[14] θ is introduced to capture any unobservable heterogeneity, or *frailty*. Assume that the frailty a is a multiplicative effect on the hazard function, $h(t|a) = ah(t)$ and that the unobserved a follows a Gamma distribution, $\mathrm{G}(1/\theta, \theta)$. This results in the D given above (Mudholkar et al., 1996).

3.1 Assessing Time and Risk Preferences

In the time preference part, subjects are asked a set of ten questions in the following format:

> To me, 'receiving $x today' is equally as good as 'receiving $y in __ days',

where $x < y$. Subjects are told that they must wait longer to get the larger amount of money, $y, and they are asked to identify the *longest acceptable delay* that makes the two options the same. In every case, subjects receive their earnings as money orders. The actual amounts of x and y are one of $5, $10, $15, $20 and $25; thus, the combinations of two different rewards make ten questions in total.

Similarly, in the risk preference part, subjects answer questions having the following format:

> To me, 'receiving $x for sure' is equally as good as 'receiving $y with __ % chance'.

I ask subjects to report the *lowest acceptable odds of winning* the lottery for $y. Again, subjects are told that they must play the lottery to get the larger amount, $y. The actual values of x and y are identical to those in the time preference part. Earnings were paid in cash for this part.

To induce subjects to report their true *delay* and *odds*, I employ the Becker-DeGroot-Marschak (BDM) mechanism.[15] In the time preference part, a computer program randomly chooses a proposed delay. If it is shorter than the longest acceptable delay that a subject reports, then the subject gets $y right after the proposed delay; otherwise, the subject gets $x at the end of the experiment. In the risk preference part, a number between 0 to 100 is randomly selected. If it is less than the lowest acceptable odds of winning, then the subject does not play the lottery and receives $x for sure; otherwise, she plays the lottery. Her chance of winning the lottery is the probability (%) that the computer generates. She gets $y if she wins the lottery and nothing if she loses.

Regarding the BDM mechanism, there are two possible issues: (i) it may be too complicated for subjects to understand and (ii) subjects may form decisions based on the underlying distribution of the possible valuation.[16] The instructions, therefore, explain the incentive compatibility property through the use of examples and the instructions go on to explain why any false report may make them worse off. Then the instructions explicitly tell subjects that their "best response is always to answer the questions truthfully."[17] As for the range of the potential valuation, in the risk preference part, subjects are told that $p \sim U[0, 100]$. This range is the most neutral to behavior because it suggests nothing particular about the odds of winning. In the time preference part, I set $t \sim U[1, 120]$. Since any range of t can be an anchor in the decision-making process, subjects are not told anything about the range of the possible proposed delay.[18]

[15] Becker et al. (1964).

[16] See Bohm et al. (1997) that find the sensitivity of BDM to the underlying distribution of valuation.

[17] Since the purpose of the BDM mechanism in this experiment is not to test the mechanism but to make subjects reveal their valuation, I believe that it is appropriate to teach the subjects about the incentive property. After they read the instructions for the time preference part, subjects answer two review questions on the mechanism. Out of 55 subjects, 35 answered both questions correctly and 12 answered one of the questions correctly.

[18] There were two subjects who asked about the possible range of the delay. I answered them by saying that there was a range of a proposed delay, from which the computer program would choose a number and I did not tell the range. Then, I repeated

The multiple choice list of Holt and Laury (2002) may be ideal for eliciting risk attitude; however, its grid size is too coarse to fit into equation (4). Alternatively, I could adopt the iterative multiple price list or the newly developed adaptive instrument (see Harrison et al. (2005) and Eckel et al. (2005), respectively), which would provide a finer grid and increment unit. However, both mechanisms require subjects to answer many questions to elicit a single t.[19] Thus, I run BDM.[20] After subjects complete the tasks, they fill out a survey.

3.2 Procedure

Each session involves ten to fifteen subjects; five sessions yield 56 subjects in total. In three of these sessions, subjects firstly complete the time preference part and then the risk preference part. The order of the two parts is reversed in the other two sessions.[21]

At the beginning of each session, subjects are given printed instructions. After the instructions for the first part are read aloud, subjects are encouraged to ask questions. Then, they answer the ten questions for the first part. The same procedure is repeated for the second part. The ten questions are separated into four groups depending on the amount of the smaller reward in the question.[22] The computer screen displays each group of questions individually and subjects report their *delay* (or *odds*) by answering questions. When the submit button is clicked, the computer proceeds to another screen. After she goes through those four screens, the computer shows her ten answers in a table format and offers a chance to revise the answers. On average, it takes 2 minutes and 25 seconds to complete ten questions.[23] To prevent any wealth effect and/or feedback, subjects are informed of the result at the end of the experiment session, not at the end of each part.

In the time preference part, the reward is paid with a US Postal Money Order.[24] If a subject earns the "$x today" option, she will get the money order when she leaves the session. Those who get the future option are asked to write their mailing address on a stamped envelope and the money order, and seal it into the envelope. Then I collect the envelopes and mail them after the proposed delay. All of these procedures are

their best response was still to answer questions truthfully regardless of the range.

[19] In addition, it seems to me that the mechanisms are not necessarily incentive compatible.

[20] Attema et al. (2009) independently develop another experimental design with the same spirit. For a given pair of rewards $x < x'$, they elicit the length of interval between the two rewards that makes the two options equally good. Suppose $(x, 0) \sim (x', t_1)$. In the next question, let subjects compare (x, t_1) and (x', t_2) and elicit t_2 that makes $(x, t_1) \sim (x', t_2)$. This sequence of adaptive questions yields the shape of the time discount function. See their paper for more detail. Note that, due to its adaptive nature, this method would not be incentive compatible if the reward were real money.

[21] I do not observe a significant order effect in the reported *delays*. However, the subjects in the last two sessions who completed the risk preference part first significantly reported the lower *lowest acceptable odds* than those in the first three sessions. The mean difference is 8.48 percent points and the p-value of t-test is 0.051.

[22] The first group consists of four questions, whose reward pairs are ($5, $10), ($5, $15), ($5, $20) and ($5, $25). The next group includes ($10, $15), ($10, $20) and ($10, $25).

[23] It took 145.1 seconds on average for a subject to complete the time preference task and 144.7 seconds for the risk preference task. Of 56 subjects, 21 revised their answers in the time preference part and 18 revised their answers in the risk preference part.

[24] There are several implementations of delayed payments. Harrison et al. (2005) have the Danish Ministry of Economic and Business Affairs transfer the delayed payment into the subjects' bank account. Anderhub et al. (2001) and Coller and Williams (1999) give a post-dated check to subjects. Benhabib et al. (2010) send a check to the subject's mailing address. Tanaka et al. (2010) assign to a village leader to deliver future rewards to participants in the village.

session	number of subjects	task 1	task 2	survey
1	10	time	risk	yes
2	10	time	risk	yes
3	10	risk	time	yes
4	11	risk	time	yes
5	15	risk	time	yes

Table 2: Session Summary

written in the instructions.

The 56 student subjects were recruited at the University of Michigan. No subject was used in more than one session. All sessions were conducted in the RCGD lab at the University of Michigan, and each session lasted approximately 70 minutes. The average earning (including money orders) was $22.77 plus $5 for a participation fee. I used a z-Tree program to run this experiment (Fischbacher, 2007). Table 2 summarizes the tasks and number of subjects for each of the five sessions I conducted.

4 Results

In this section, I present three main results. First, I characterize time inconsistent behavior and observe the future bias. Second, I estimate the time discount function using the inverse S-curve function. Finally, I illustrate the biases caused by the linearity assumption on $u(x)$.

Figure 3 gives an overview of the distribution of responses.[25] On average, subjects are willing to wait longer and take higher risk as the distance between the small and large rewards becomes larger. In what follows, I examine individual subject behavior in more detail.

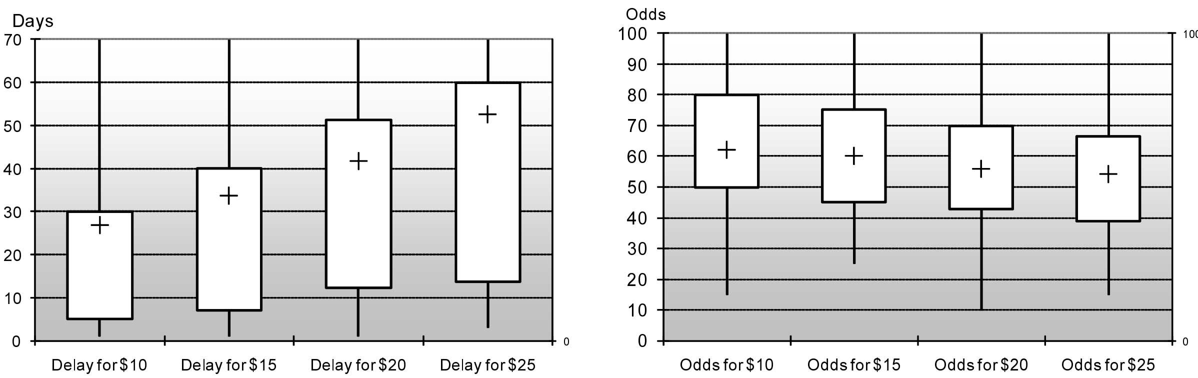
4.1 Future Bias

To run the longitudinal analysis, I select three different rewards from $\{5, 10, 15, 20, 25\}$ to classify subject responses according to the criteria above. For each subject, there are ten combinations, generating 550 observations in total.[26] Figure 4 summarizes the distribution of the difference, $T(x_0, x_1)+T(x_1, x_2)-T(x_0, x_2)$ for $x_0 < x_1 < x_2$. As seen, it is skewed to the right and the number and magnitudes of future biases are greater than those of present biases. The first main finding in this paper is that subjects exhibit significantly more future bias than present bias.

Result 1 (Future Bias)**.** Of the 550 observations, 362 are future biased preference reversals and 93 are present biased preference reversals. The number of future biases is significantly greater than that of present biases.

[25] There was a subject who answered 366 (days) to all ten delay questions. The value 366 was the longest delay that subjects could input. I refer this subject as ID56.

[26] In this analysis, I excluded the data of ID56, since his response always implies future bias no matter what his true time preference is.



Note. These boxplots summarize the responses of 56 subjects to the first four questions in the time and risk preference parts. In the questions for the time (risk) preference part, they are asked to report how many days of delay (what odds) make the larger reward option and the $5 today (for sure) option equal. The box covers the half of those responses in the middle and the cross symbol indicates the mean. As the amount of the larger reward increases, they are willing to wait longer and take higher risks, on average.

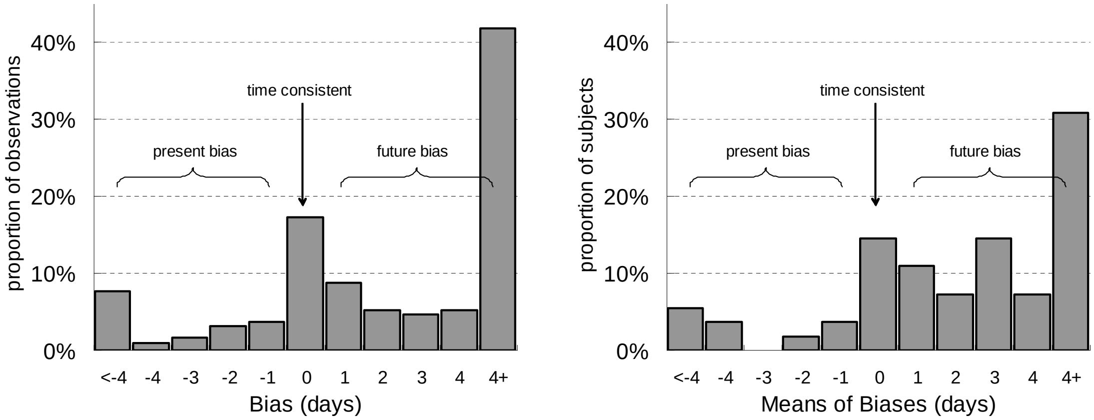
Figure 3: Reported delay and odds for the larger reward against $5 today (for sure) option

Support. The null hypothesis is that the median is zero for the difference between $T(x_0, x_1) + T(x_1, x_2)$ and $T(x_0, x_2)$. The sign test, assuming a binomial distribution of $B(n = 550, p = 0.5)$, indicates that the null hypothesis can be rejected at any level above 0.00%. Furthermore, for each subject, I take the mean of the ten observations of the difference between $T(x_0, x_1)+T(x_1, x_2)$ and $T(x_0, x_2)$ and run the sign test. Assuming $B(n = 55, p = 0.5)$, the p-value is 0.01%. □

This future bias result is surprising, since most analyses in the literature assume present bias. Only recently have Read (2001) and Attema et al. (2009) found increasing impatience in subject behaviors that indicates future bias preference. Sayman and Öncüler (2009) conducted experiments with longitudinal design and found *reverse* time-inconsistent choice behavior, which is also consistent with the future bias found in this experiment. Most of experiments in the literature, however, support present bias. I suppose there are two reasons for that: the time range and the estimation methods of those experiments.

First, in many of the prior experiments, subjects reveal their time preferences over a long time range which is usually longer than one month. Thus, those experiments do not capture future bias that seemingly occurs in the immediate future. According to Table 1 of Frederick et al. (2002) which summarizes the time range of 42 experiments, there are eight studies in which the time range is shorter than one month. The other 34 experiments elicit the time preference over future options that would pay a reward after one month.

As seen in Table 3 of the next subsection, however, the subjects exhibit future bias within a short period, which is on average 22.4 days from the current moment and then exhibit present bias thereafter. Sayman and Öncüler (2009) also report that the reverse time-inconsistency is more likely to be observed when the delay is shorter than four weeks. Therefore, it is indicated that future bias has been overlooked due to the



Note. This figure summarizes the distributions of time consistent behavior, the present and future biases. $T(x_i, x_j)$ denotes the delay that makes two options, (x_i today) and (x_j in $T(x_i, x_j)$ days), the same to a subject. The bias (days) is defined $T(x_0, x_1) + T(x_1, x_2) - T(x_0, x_2)$ for any three rewards, $x_0 < x_1 < x_2$. Thus, positive (negative) bias corresponds to future (present) bias. There are ten combinations of three rewards for each of 55 subjects. The left panel shows the distribution of the biases for all 550 combinations and the right panel shows the distribution of the mean value of bias of each subject. In the right panel, the range of k days includes any mean value in $[k - \frac{1}{2}, k + \frac{1}{2})$. As seen in both panels, the distributions are skewed to the right.

Figure 4: Distributions of time preference

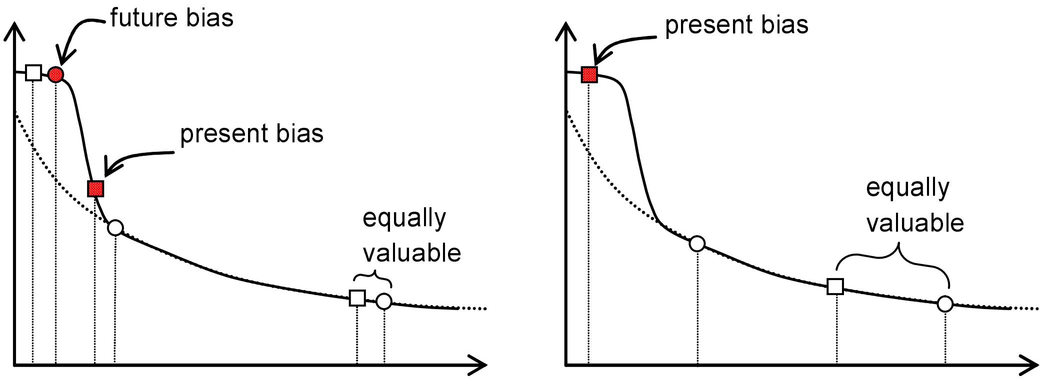
long time range of the experiments in prior research.

Secondly, although the previous studies found present bias behavior, they did not necessarily mean that the subjects did not have a future bias preference. The present bias and the future bias can coexist within a subject; while she exhibits future bias in the short period, the same person may also exhibit present bias over a long time range. Although she seems to have two inconsistent biases simultaneously, an inverse S-curve time discount function can consistently account for them together. Figure 5 illustrates the concept. The left panel shows that a subject who has the inverse S-curve time discounting exhibits both of the present and future bias. Notice, however, that the future bias can be elicited only when the interval between the two delayed options is sufficiently short. The right panel of the figure shows the case where the subject exhibits only present bias since the interval is too long to elicit the future bias.

To my knowledge, all previous studies that estimate time discount function focus on whether subjects have a present bias or not. Therefore, they did not need to employ an inverse S-curve function to estimate the subjects' time preference and they did not detect future bias. This is another reason why future bias has been rarely reported before.

I discuss other psychological reasoning for future bias and its implication in Section 5.

There is a remark on the result; decreasing discounting is not so prevalent in this experiment. I suppose that most of the subjects do not exhibit a present bias since the immediate reward is paid in a money



Note. The square and circle represent two delayed reward options. The dotted curve is a quasi-hyperbolic discount function and the solid curve is an inverse S-curve time discount function. Suppose that as long as the two options are on the dotted curve, they are equally valuable to a subject. Left panel: In the middle, the subject strictly prefers the square option and exhibits present bias, since the square is above the exponential discounting curve while the circle is still on the curve. However, when both are close to $t = 0$ and belong to the extended present in the subject's perception, the circle is further above the exponential curve than the square. Thus, she strictly prefers the circle option and exhibits future bias. Right panel: it shows the case where the interval between the two options is too long to elicit her future bias time preference.

Figure 5: The interval between options and elicitation of future bias

order, which incurs some cost to cash out. If we paid the subjects the immediate reward in cash and the future reward in a money order, they would probably have shown a present bias more frequently due to the transaction cost associated with the future option. There are other experiments that observe little decreasing discounting for the same reason. For example, Anderhub et al. (2001), who use a (post-dated) check for both immediate and future payments, observe little hyperbolic discounting (only 14 of their 61 subjects). The front-end delay (FED) also serves to equalize the transaction cost between immediate and future payments (see Harrison and Lau (2005) for discussion). In fact, Harrison et al. (2002) use a one-month FED and observe little hyperbolic discounting. This evidence implies that the hyperbolic discounting and decreasing discounting observed in other experiments might be attributed to, in part, the transaction cost of the future payment.

4.2 The inverse S-curve discounting

To characterize the future bias observations, I estimate parameters q_i, θ_i and r_i by a non-linear least-squares fit for each subject. Recall each subject generates ten paired-observations. Let (t_{ij}, p_{ij}) denote a data point obtained from subject i's response to question j, where t_{ij} is the reported delay and p_{ij} is the reported odds. For each subject, I solve a non-linear least-squares problem of the following form:

$$\min_{\theta,r,q} \sum_j \Big[p_{ij} - D(t_{ij}, \theta, r, q)\Big]^2 \tag{6}$$

where D is the generalized Weibull function defined in equation (5).

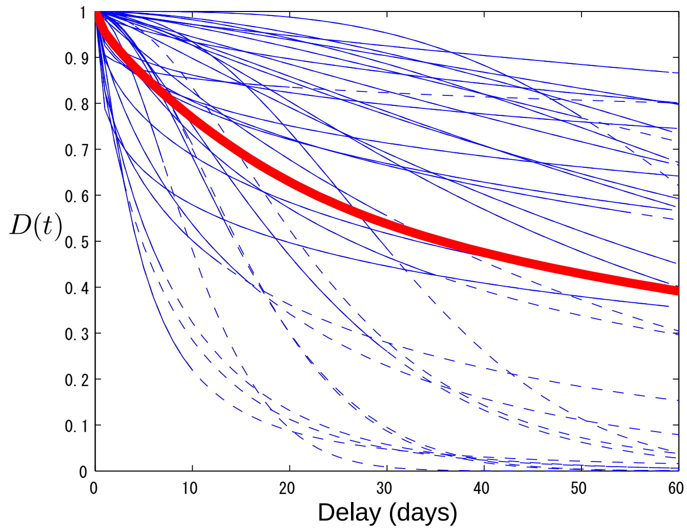


Figure 6: Estimated discount functions

Table 3 summarizes the estimated parameters for 33 subjects.[27] Figure 6 shows the estimated $D(t)$ for those 33 subjects. The solid portions of each curve depict $D_i(t)$ for $0 \leq t \leq \max_j \{t_{ij}\}$. The bold line in the middle represents the average discount factor for given t, that is $\bar{D}(t) = \sum D_i(t)/33$. Note that $\bar{D}(35.9) = 0.50$ means that a 36-day delay is equivalent to 50% of the risk, on average.

Result 2 (Inverse S-curve discounting)**.** Some subjects have an inverse S-curve $D_i(t)$. Overall, however, the average discount function $\bar{D}$ is hyperbolic.

Support. For 20 out of the 33 subjects in Table 3, the estimated q is greater than 1. This hump-shaped hazard function implies increasing impatience (future bias) at the beginning and decreasing impatience (present bias) later on. Non-linear least-squares for $t \in \{1, \ldots, 60\}$ finds $(\theta, r, q) = (0.915, 0.028, 0.890)$ fitting $\bar{D}$ with $R^2 = 0.9995$. Though not all $D_i(t)$ are hyperbolic, the average discount function $\bar{D}$ appears to be hyperbolic. □

Remark 4 (Extended present)**.** Result 2 suggests a concept of *extended present* (or extended immediacy). This means that the subjects recognize that the first few days following today all belong to the extended present and they discount rewards paid in those days moderately. Observe also that extended present is corresponding to future bias.

This concept is consistent with the experimental results of Coller et al. (2011) and Holcomb and Nelson (1992). Coller et al. (2011) show that a short (front end) delay attached to a small-soon reward eliminates the

[27]Note, however, that for the other 22 subjects the model has little explanatory power ($R^2 < 0.05$). I treat them separately and discuss this later.

ID	θ	r	q	R^2	D(1year)	$\hat{t}$
7	0.00	0.013	3.30	0.683	0.000	67.3
10	0.00	0.029	2.15	0.679	0.000	26.0
30	0.37	0.043	1.97	0.587	0.000	15.0
18	0.03	0.056	1.78	0.604	0.000	11.1
22	0.00	0.010	1.71	0.891	0.000	61.5
23	0.00	0.083	1.70	0.602	0.000	7.1
8	0.00	0.008	1.70	0.641	0.001	70.9
1	0.13	0.054	1.69	0.416	0.000	10.5
25	0.00	0.056	1.66	0.382	0.000	10.3
15	0.85	0.020	1.59	0.829	0.027	20.2
48	0.00	0.041	1.41	0.624	0.000	10.2
40	0.82	0.223	1.37	0.839	0.001	1.2
27	0.00	0.007	1.28	0.893	0.034	43.2
43	0.00	0.010	1.18	0.615	0.009	19.7
2	0.03	0.005	1.15	0.999	0.170	36.8
11	0.57	0.011	1.15	0.735	0.098	10.9
5	0.01	0.014	1.12	0.492	0.003	9.8
28	0.00	0.007	1.10	0.655	0.053	14.8
36	0.92	0.037	1.02	0.665	0.057	0.3
6	0.00	0.020	1.02	0.205	0.001	0.8
44	1.00	0.002	0.93	0.367	0.546	–
4	0.00	0.116	0.85	0.554	0.000	–
9	0.04	0.136	0.83	0.575	0.000	–
29	0.01	0.055	0.79	0.312	0.000	–
45	1.00	0.002	0.70	0.338	0.533	–
34	0.12	0.007	0.58	0.856	0.196	–
17	0.00	0.052	0.56	0.379	0.006	–
42	0.18	0.017	0.54	0.863	0.112	–
49	0.27	0.007	0.53	0.846	0.263	–
35	0.87	0.004	0.46	0.063	0.428	–
21	0.41	0.001	0.40	0.421	0.566	–
26	0.05	0.020	0.36	0.135	0.143	–
24	0.83	0.000	0.21	0.335	0.733	–
mean	0.26	0.04	1.18	0.578	0.121	22.4

Table 3: Estimates of Time Preference

Note. This table presents the estimates for 33 subjects for whom the model predicts well ($R^2 > 5\%$). See Table 4 for the other 23 subjects. The extrapolation of the function with the estimated parameters to $t = 365$ provides D(1year). The last column shows $\hat{t}=\left[(q-1)/r^q(\theta+q)\right]^{1/q}$ at which the form of the inverse S-curve D changes from concave to convex.

immediate effect or the present premium.[28] Their result shows that a seven-day front end delay eliminates the premium. On the other hand, Holcomb and Nelson (1992) do not find any effect of a one-day front end delay. Thus, the period between two and seven days from today constitutes the present in a sense that the present premium is still attached to those days. Similarly, in this experiment, the "present" seems to continue as the hazard rate remains low for the first few days of the future.

There are two caveats to the results. Observe that the discounting is unreasonably steep and that converting delay into risk does not work for some subjects.

First, as shown in Table 3, the annual discount factors estimated by extrapolation are very low, i.e., the discount function is very steep. The average time discount function also has a very short "half-life", which is only 36 days (i.e., $D(35.9\text{days}) = 0.50$). This result seems to be unreasonable, since the imputed annual discount rate from the average estimated $D(1\text{year}) = 0.121$ is 726%. Notice, however, that this incredibly high discount rate is not unique in the literature. For example, Table 1 of Frederick et al. (2002) summarizes 42 studies on time preference and includes 10 experiments that similarly observe unreasonably high discount rates. One may think that subjects were not certain whether they would really receive their future payment. If that was the case, the discount rates would be overestimated reflecting not only their time preference but also their suspicion about the plausibility of the future payment. However, half of those 10 experiments whose elicited discount rates are very high do not involve real future reward. Even though subjects do not need to, or simply cannot, be suspicious about the plausibility for the hypothetical rewards in the experiments, they still exhibit a very steep discounting function. Thus, the high discount rate itself does not indicate that the result is unacceptable.

The second caveat is that, for two-fifths of our subjects, the conversion of delay into risk does not work. The experimental method could not identify time preference by their risk-taking behavior or risk preference. While they are not willing to take higher risk for larger reward, they are still willing to wait longer for a larger reward.

Table 4 presents the estimated parameters with and without assuming $u(x) = x$ for those 23 subjects. The R^2 is almost zero, implying that the acceptable longest delay $T(x, y)$ for the larger reward y is not corresponding to the acceptable odds $p = u(x)/u(y)$ in our framework. The method intends to convert delay into risk (or vise versa) using BDM, but the result shows its limitation. In particular, the R^2 becomes much higher if $u(x) = x$ is assumed. It indicates that the elicited probability equivalence through BDM is not consistent with the subjects' risk preferences.

Remark 5 (Discrepancy between PE and delay)**.** For two-fifths of the subjects, there is a significant discrepancy between the probability equivalence (PE) and the acceptable delay elicited by BDM. It can be mostly

[28]The experiment has two treatments. In the first treatment, subjects are asked to choose one of two future rewards. In the second treatment, they are asked to choose from one immediate reward and another future reward. If there is an immediate effect (present bias) and a premium to accept any delayed reward instead of an immediate one, then the premium is present only in the second treatment.

	no assumption on $u(x)$				assumed $u(x) = x$				
ID	θ	r	q	R^2	θ	r	q	R^2	D(1year)
3	0.78	0.000	0.00	0.000	0.00	0.027	1.11	0.819	0.000
50	0.74	0.000	0.00	0.000	0.00	0.024	1.63	0.787	0.000
31	0.70	0.000	0.02	0.000	0.00	0.076	1.00	0.736	0.000
41	0.63	0.000	0.01	0.000	0.00	0.076	1.00	0.736	0.000
13	1.00	0.000	0.12	0.000	0.62	0.091	1.33	0.726	0.001
39	0.11	0.055	0.00	0.000	0.00	0.049	0.92	0.717	0.000
55	1.00	0.000	0.18	0.000	0.03	0.083	0.51	0.581	0.006
33	0.56	0.030	0.08	0.038	0.86	0.087	1.35	0.517	0.005
54	0.63	0.000	0.03	0.000	0.00	0.149	1.32	0.511	0.000
38	0.31	0.223	0.22	0.040	0.00	0.209	0.78	0.480	0.000
19	0.39	0.419	0.00	0.000	0.00	0.111	1.13	0.453	0.000
12	0.57	0.000	0.10	0.000	0.65	0.051	0.59	0.384	0.094
51	0.88	0.000	0.02	0.000	0.00	0.115	0.70	0.382	0.000
46	0.68	0.000	0.01	0.000	0.07	0.050	0.62	0.279	0.007
16	0.13	0.017	0.48	0.050	0.01	0.021	1.05	0.201	0.000
47	0.90	0.000	0.01	0.000	0.00	0.011	0.42	0.084	0.171
52	0.85	0.000	0.06	0.001	0.55	0.008	0.24	0.021	0.381
20	0.98	0.000	0.03	0.000	0.38	0.081	0.26	0.017	0.177
14	0.39	0.000	0.03	0.000	0.35	0.065	0.18	0.005	0.256
32	1.00	0.002	8.51	0.000	0.57	0.005	1.58	0.000	0.203
53	0.14	0.010	2.23	0.000	0.14	0.010	2.25	0.000	0.000
37	0.67	0.000	0.06	0.000	1.00	99.997	0.00	0.000	0.500
56	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	>0.800
mean	0.64	0.034	0.56	0.006	0.24	4.61	0.91	0.383	0.082

Table 4: Estimates of Time Preference

Note. The model, that converts delay into the equivalent risk, does not fit to responses for 23 subjects. As seen in this table, R^2 is almost zero for these subjects. The right half part shows the parameter estimation, imposing $u(x) = x$. In terms of R^2, the linearity assumption on $u(x)$ improves the fitting. The last row is corresponding to the subject, all of whose equivalent delays are binding at 366 days.

attributed to the inconsistency between PE and $u(x)/u(y)$.

4.3 The effect of the linear utility assumption

In this subsection, I examine the effect and biases caused by the linear utility assumption. Thus, I explicitly assume $u(x) = x$, replacing p_{ij} by the ratio of rewards. For example, suppose that a subject is indifferent between (\$15 today) and (\$20 in 7 days), and between (\$15 for sure) and (\$20 with 90% chance). A paired observation should be (7 days, 90%) and infer $D(7) = 0.9$. In this subsection, assuming $u(x) = x$, I ignore the second part of the responses and thus I observe (7 days, 75%) where 75% = \$15/\$20. This infers $D(7) = 0.75$ instead.

Downward bias

In general, the ratio of two rewards is lower than the acceptable odds; \$15/\$20 $<$ 90% in the example above. Subjects are not willing to take the high risk (25%) of losing \$15 for an additional \$5. Thus, the linearity assumption causes downward bias on $D(t)$.

Result 3 (downward bias)**.** The linearity assumption biases estimates of $D(t)$ downward.

Support. The acceptable risks that subjects report is 66.26% on average, while the ratio of rewards is 0.50 overall. The difference is statistically significant ($p < 0.001$). Figure 7 compares the average discount factors, with and without the $u(x) = x$ assumption. As seen in Figure 7, the $u(x) = x$ assumption causes an overestimation of the discount factor. Fitting $D(t) = e^{-rt}$ to those $\bar{D}(t)$, I find that $\hat{r} = 0.0282$ for $\bar{D}(t)$ without the linearity assumption and $\hat{r} = 0.0560$ for the other. That is, the linearity assumption increases the discount rate (hazard rate) from 2.82% to 5.60%. □

Figure 8 illustrates such a downward bias for two subjects. For subject ID=34, the assumption shifts $D(t)$ so low that the discount rate becomes almost double. By comparing the equivalent delay and the equivalent uncertainty, the experiment estimates $D(t)$ independently of the functional form of $u(x)$. For example, for subject ID=10, it successfully estimates the increasing impatience or the concavity of $D(t)$ (see the lower-left panel of Figure 8). This would not be observable if I had assumed $u(x) = x$.

Variance bias

Our experimental method also reduces the variance in the data. For some subjects, it may be that $\text{var}(x_k/x_j) > \text{var}(u(x_k)/u(x_j))$, since u can be concave. Then, eliciting $u(x_k)/u(x_j)$, this experiment can reduce the variance of the data.

Result 4 (Variance bias)**.** The linearity assumption increases variance.

Support. The variance of the reported odds $\sigma_i^2 = \text{var}(p_{ij})$ is significantly less than the variance of the ratios of the rewards, 383.5. For 50 subjects, $\sigma_i^2 < 383.5$. □

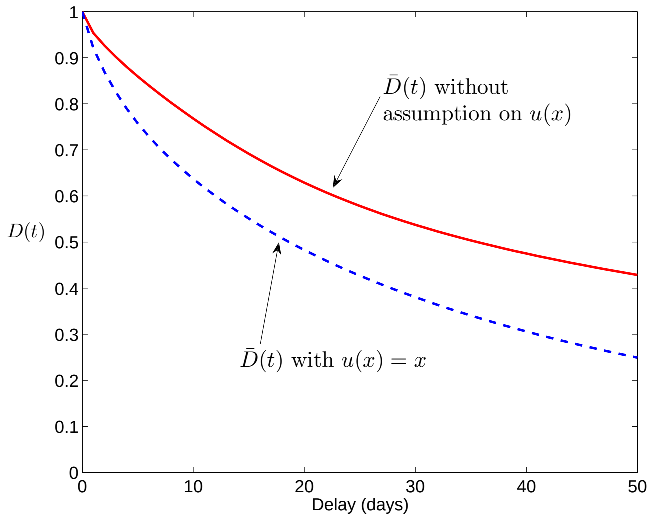


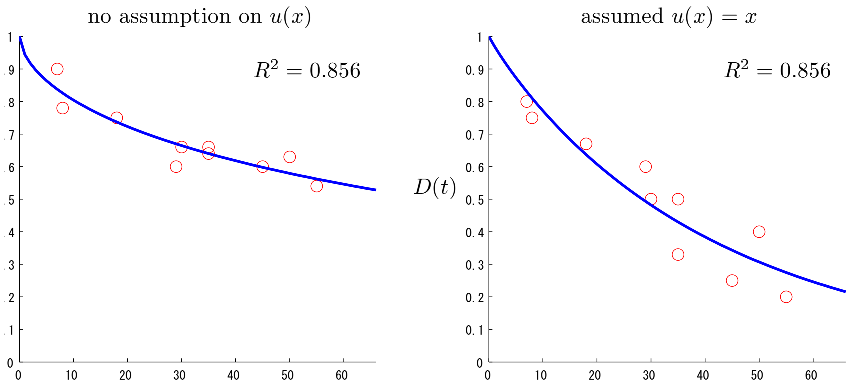
Figure 7: Overall downward bias

Figure 9 illustrates that the linearity assumption increases the variance of the data for subject ID=42. For this subject, 14 days of delay discount the value of rewards down to 65% on average (see the left panel). In fact, in this subject's responses, there are four paired observations: (14 days, 60%), (14 days, 65%), (14 days, 65%) and (14 days, 70%). However, the linearity assumption biases the reported odds into 0.5, 0.67, 0.75 and 0.8, respectively. As a result, R^2 also decreases from 0.863 to 0.671.

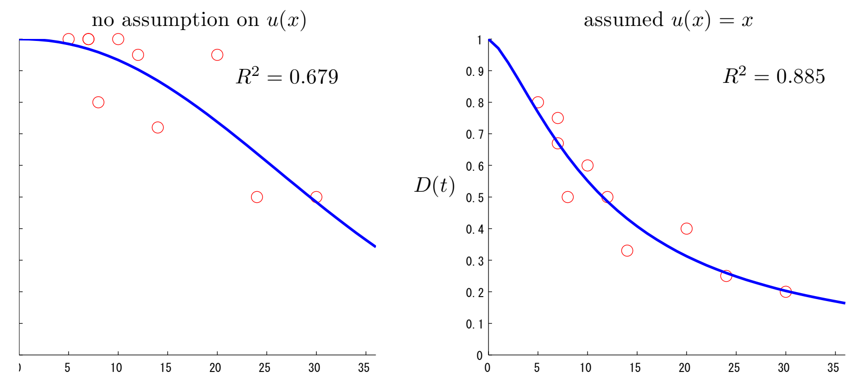
5 Discussion

This section discusses the following: the existence of pure time preference, the issues in the experimental method including plausibility of delay and fungibility of rewards, and psychological accounts for future bias, including the extended notion of present.

First, although this experimental design has some advantages, the interpretation needs to be done carefully. In this study, some subjects seem to have a pure time preference that cannot be attributed to a risk preference. For those 19 subjects, the R^2 of the non-linear least-squares of (6) is less than 0.01, meaning that $D(t)$ has no explanatory power. However, if I impose $u(x) = x$, then R^2 will be 0.38 on average, indicating that their time preferences are not identified by their elicited probability equivalences. It also suggests that there may be some other factor than risk causing time discounting. Detecting those factors is left for future research.



Subject ID=34



Subject ID=10

Note. These figures compare two estimations of $D(t)$: one without an assumption on $u(x)$ and the other with a $u(x) = x$ assumption. The linearity assumption causes downward bias. For ID=10, the experiment successfully elicits the concavity of $D(t)$ (the left panel). This would not be observed if I had assumed the linearity assumption.

Figure 8: Downward bias

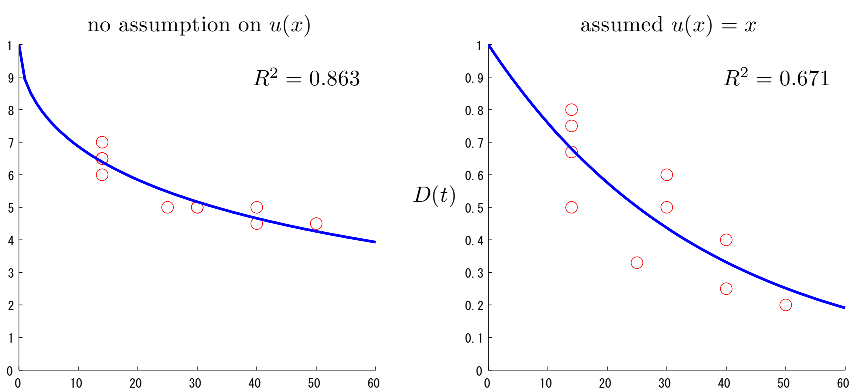


Figure 9: Variance bias (Subject ID=42)

5.1 Confounds in the experimental method

The potential confounds in the experimental method should be addressed.[29] In particular, the plausibility of future rewards and the fungibility of monetary rewards are the confounding factors that many experimental studies, including this paper, do not control.

When estimating the time preference of a subject, it is assumed that she is not skeptical about the plausibility of a future reward. However, the subject may avoid a future reward just because she does not think she will actually receive it. For example, the subject feels that an experimenter is unreliable, or she anticipates she may be moving far in the near future.[30] Then, her revealed time preference does not correctly reflect her true time preference. This type of subjective uncertainty cannot be easily separated from time preference and might result in an overestimation of time discounting.

The other issue is that a money order is a fungible reward in principle. Thus, most of the experiments including this paper do not necessarily elicit the psychological time preference for consumption of subjects. Note, however, that Reuben et al. (2010) recently showed that the time discounting for non-fungible chocolate and fungible money correlates with each other within a subject, indicating that monetary rewards are still useful for time preference experiments.

5.2 Psychological accounts of future bias

As for future bias, I consider other interpretations and psychological accounts, which include the unreliability of own future memory and the notion of extended present.[31]

First, a subject may anticipate that she is going to forget about a delayed reward. Suppose that she thinks her short-term memory is most likely to fade after several weeks, i.e., the hazard rate of the memory loss is increasing in time during those weeks and decreasing thereafter. Assume she is a little skeptical about the plausibility of the future payment but she still thinks the future payment will be delivered as long as she remembers it, by reclaiming it for example. Then, the revealed time preference results in the inverse S-shaped time discount function. It is left for further research to control this psychological factor.

Secondly, but most importantly, future bias observations suggest that the present is not a single point on the time line but, rather, that it extends into the immediate future. Observe also that the inverse S-shaped time discount function fits to this concept of the extended present. Then, a question arises: When does the *future* really start?

[29] I thank an anonymous referee for his/her detailed comments pointing out these issues.

[30] In our study, to minimize the skepticism, subjects were given a postal money order and wrote their addresses and names on the money order and an envelope, in which they sealed their money order.

[31] I am aware that the future bias can be explained by the same psychological process that causes a subadditive discounting (Read, 2001; Scholten and Read, 2006). The subadditive discount function means $D(0,t) > D(0,s) \times D(s,t)$ and implies present biased behavior. Read explains that "when an object or event is subdivided, each part is paid more attention than if it is part of a larger whole (p. 10)." Notice that a similar subadditivity of attention can lead to the opposite future bias in this framework: namely, if a subject interprets the difference between x_0 and x_1 as an object, then the acceptable delay is a function of the difference, i.e. $\tilde{T}(x_1 - x_0) = T(x_0, x_1)$. When there is subadditivity in $\tilde{T}$, it results in future bias observations.

One can ask when does the future really start and how many seconds, minutes, hours, days, or weeks separate the present from the future. If the next ten minutes do not belong to the future, then one would not discount any reward paid within those ten minutes. This suggests the notion that the present extends into the future and that, immediately after the current moment, a time discount function will not necessarily decrease. This concept of the extended present is also important for the application of quasi-hyperbolic discounting models.

Suppose that a subject perceives the period $[0, \bar{t}]$ as the "present" and discounts any reward arriving thereafter. Her time discount function can be simply characterized as follows:

$$D(t) = \begin{cases} 1 & \text{if } t \in [0, \bar{t}] \\ e^{-rt} & \text{if } t > \bar{t} \ . \end{cases}$$

This still captures the nature of quasi-hyperbolic discounting. But, notice also that it is consistent with both present bias and future bias. The concept of the extended present naturally adds another dimension to the quasi-hyperbolic discount function, and it will deepen our understanding of the time preference.

6 Concluding remarks

Time preference is one of the fundamental factors in any decision-making process. Understanding the nature of this time preference provides us with deep insight into human behaviors and economic decisions in both microeconomics and macroeconomics. In fact, there are many applications of this line of research: savings and investments, credit card markets, retirement, clinical decisions[32], procrastination, and addiction.[33]

In this paper, I elicit the time preference of subjects using a new experiment design. This experimental design is unique in the sense that it runs a non-parametric test of time consistency and it does not impose any parametric assumption on the utility function. The non-parametric test can be done by introducing equivalent delay function, and it is shown that the modularity of the function is corresponding to the standard definition of time inconsistency. This test is also independent of the separability assumption between x and t, which is the very first one of this kind in the literature.

In the parametric estimation of time discount function, I employ the generalized Weibull model to accommodate an inverse S-curve time discount function.

The experimental results suggest that some subjects exhibit both of increasing and decreasing impatience (i.e., future bias and present bias). These behavior patterns, future bias in particular, were rarely observed in previous experimental studies, as the standard experimental designs could estimate only present bias. The finding of future bias implies that the immediate future constitutes an extended present for subjects. That is, the future does not really start right away, but it starts after some delay. This time preference is

[32]Time-related aspects and delay discounting play important roles in clinical decisions. See Bos et al. (2005) and Ortendahl and Fries (2006) for reviews and discussions.

[33]For example, nicotine-dependent (Reynolds et al., 2004) and alcoholic (Petry, 2001) individuals have more myopic time preferences than individuals without any addiction.

characterized by an inverse S-curve discount function that is concave for the first 22 days, on average, and convex thereafter. My method also corrects biases caused by the linearity assumption on utility functions, i.e., $u(x) = x$. The result shows that the estimated discount rate with the linearity assumption could be roughly twice as high as that without the assumption.[34]

This study considers delay as a risk factor and integrates risk and time preference. Although there is no definitive answer about how delay relates to risk, there are some clues. Many studies in psychopharmacology, for example, show that substance dependent (addicted) individuals tend to make impulsive intertemporal choices.[35] If drug abuse is high-risk behavior, there must be a common impulsive nature in both myopic time preference and risk-taking behavior. It is also known that the perception of a short time interval is influenced by dopamine.[36] More recently, McClure et al. (2007) find, by observing the fMRI images of subjects' brain, a brain region that seems to be responsible for the present bias.[37] These clues from various fields will reveal the relationship between risk and time preferences in a more systematic manner. At present, this paper serves to show the need for a systematic approach to the integration of time and risk, as well as the boundary between the present and the future.

A Proof

Proof of Proposition 1. Assume a subject exhibits decreasing impatience. Choose arbitrary $w < z \leq x_1 < x_2$. Let $t_1 = T(z, x_1)$, $t_2 = T(z, x_2)$ and $t_1 + \delta = T(w, x_1)$. By transitivity, it follows that $(z, 0) \sim (x_1, t_1) \sim (x_2, t_2)$. Decreasing impatience implies $(x_1, t_1 + \delta) \prec (x_2, t_2 + \delta)$, that is, $(x_1, T(w, x_1)) \prec (x_2, T(z, x_2) + T(w, x_1) - T(z, x_1))$. Substituting $(x_1, T(w, x_1)) \sim (x_2, T(w, x_2))$, it yields $(x_2, T(w, x_2)) \prec (x_2, T(z, x_2) + T(w, x_1) - T(z, x_1))$. Comparing these two options with the same reward x_2, observe $T(w, x_2) > T(z, x_2) + T(w, x_1) - T(z, x_1)$, which means submodularity.

Assume the present bias and T is submodular. Choose arbitrary $t_1 \geq 0$, $\delta \geq 0$ and $x_2 > x_1 > 0$. Suppose $(x_1, t_1) \sim (x_2, t_2)$. We want to show $(x_1, t_1 + \delta) \prec (x_2, t_2 + \delta)$. Find $y < z \leq x_1$ such that $t_1 = T(z, x_1)$ and $\delta = T(y, z)$. By submodularity there exists $w < y$ such as $t_1 + \delta = T(w, x_1)$. Notice that $T(w, x_2) > T(w, y) + T(y, z) + T(z, x_2) > T(y, z) + T(z, x_2) = \delta + t_2$. Therefore,

$$(x_1, t_1 + \delta) \sim (x_1, T(w, x_1)) \sim (x_2, T(w, x_2)) \prec (x_2, t_2 + \delta)$$

$\square$

Derivation of Remark 1. Let b denote the fixed cost of future rewards. b is zero if the reward is paid immediately.

[34] This result supports one of the main findings of Andersen et al. (2008).

[35] In psychopharmacology, there is extensive research on the relationship between addictive behavior and discounting. Reynolds (2006) and Bickel et al. (2007) provide comprehensive reviews of the literature.

[36] See the extensive survey by Cardinal (2006) for other examples.

[37] See also Kable and Glimcher (2007) for other arguments.

First, it is straightforward that the representation of $V(x,t) = e^{-rt}u(x) - b$ result in $T(x_0, x_1) + T(x_1, x_2) < T(x_0, x_2)$ for any three rewards $x_0 < x_1 < x_2$. Notice that $u(x) = e^{-rT(x,y)}u(y) - b$ for a pair of rewards $x < y$ and apply this equation for the three combinations of x_0, x_1 and x_2. Eliminate $u(x_0)$ and $u(x_1)$ from those equations and observe $u(x_2)\left[e^{-r[T(x_0,x_1)+T(x_1,x_2)]} - e^{-rT(x_0,x_2)}\right] = b$. For any positive fixed cost ($b > 0$), this means $T(x_0, x_1) + T(x_1, x_2) < T(x_0, x_2)$ and present bias.

Next, let us show that another representation of the fixed cost, $V(x,t) = e^{-rt}u(x - b)$, may also lead the present bias result. Consider $u(x_0) = V(x_1, T(x_0, x_1)) = V(x_2, T(x_0, x_2))$ and $u(x_1 - b) = V(x_2, T(x_1 - b, x_2))$. Altogether, they yield $T(x_0, x_1) + T(x_1 - b, x_2) = T(x_0, x_2)$. Notice that this equation implies $T(x_0, x_1) + T(x_1, x_2) < T(x_0, x_2)$, since $T(x_1 - b, x_2) > T(x_1, x_2)$ for $b > 0$. $\square$

B Instruction

Experimental Instruction — T/R

Instruction

You are about to participate in an economics experiment in which you will earn dollars as well as money orders based on the decisions you make. All earnings you make in the experiment are yours to keep. Please do not talk to each other during the experiment. If you have a question, please raise your hand and the experimenter will come and help you.

Overview

1. This experiment consists of 2 different parts and 2 parts of follow-up survey.
2. In the first part, you will be asked several questions about your timing preferences and will earn a money order. The amount of the money order depends on your answers.
3. In the second part, you will be offered several lotteries to choose from.
   If you win any, the cash reward will be paid to you at the end of this experiment.
4. Note that the two parts are completely independent of one another. That is, your choices and the earnings in one part do not affect those in the other part.
5. We will read the instruction for each part separately. First, we will read the instruction for the first part and you will complete the first task. Then, we will read the instruction for the second part and you will complete the second task. Finally, we will ask you to fill out some survey questions.
6. At the end of the experiment, each of you will be informed individually of your earnings for both parts, and you will then get paid.

Part 1: Delayed Payment Decision

In this part, we will pay you with a money order. The money order is issued by the US Postal Service and redeemable for the face value cash at any postal office. It may be also deposited to your bank account.

Task
You will answer a set of 10 questions assuming the following situation:

> A money order of \$A will be given to you at the end of experiment.
> Alternatively, if you are willing to wait, then instead of \$A, we will mail you a money order for \$B which is greater than \$A, i.e., \$B > \$A. Consider the acceptable longest delay for which you would be willing to wait to receive the larger amount.

Then, the question asks you to fill out the blank below:

Q: 'To me, "receiving \$A today" is equally as good as "receiving \$B in ____ days." '

You must wait to get the larger amount. Decide what length of delay makes the two options the same to you, and fill in that amount.

Note that "Receiving \$B in **T** days", it means you expect to receive the money order of \$B by mail in **T** days. The actual amounts of \$A and \$B vary from question to question.

If you get \$B money order, you will write your mailing address on a stamped envelope, sign the money order and seal it into the envelope. We will then mail the envelope later.

After each one of you answers all 10 questions, the computer will randomly select one of the questions. Your actual payment will be based on your answer to the selected question.

Procedure
To determine which of \$A or \$B you get, the computer will randomly choose a number. It will be generated independently of your answers to the questions. This number will become the actual delay for \$B, if you get \$B. Call that the *proposed delay*.

If the proposed delay is longer than your longest acceptable delay, you will not get \$B. Instead, you will get \$A at the end of the experiment.

If the proposed delay is shorter than or equal to your longest acceptable delay, you will get \$B. The proposed delay will be the actual delay. Thus, the \$B money order will arrive at your mailing address right after the proposed delay.

Example: (For purposes of illustration, we replace days with weeks.)

Suppose that you were asked the following question.

Q: 'To me, "receiving \$70 today" is equally as good as "receiving \$100 in __ weeks." '

If your answer was 10 weeks, i.e.,

'To me, "receiving \$70 today" is equally as good as "receiving \$100 in <u>10</u> weeks." '

Then, the computer randomly generates a number. If the number is greater than 10, e.g., if it is 14, then you do not get \$100. Instead, you will get \$70 today.

If the number is less than or equal to 10, then you will get \$100. For example, suppose that the number generated is 8. In this case, you will get \$100 in 8 weeks.

Any question?

Strategy:
Note that this procedure is such that your best response is to write down the longest delay for which you are willing to wait to get the larger amount, \$B.

We now show that truthful reporting is your best strategy. We will illustrate why you will never be better off sending a false report. Let us work through one example. Say that we offer you two amounts \$70 and \$100 and ask you to choose a time, **T** that is such that you would be indifferent to waiting **T** weeks and receiving

\$100 as opposed to receiving \$70 today. Let us just assume, for the sake of argument, that you would be indifferent between receiving \$70 today and receiving \$100 in 10 weeks. The question is should you tell us **T**=10 when we ask you?

To see why the answer is yes, let us say that you are thinking of not telling us the truth. There are two possible cases, under-reporting or over-reporting. We will show that in either case you might be worse off compared to telling the truth.

1) Under-reporting can make you worse off.
By reporting any shorter delay than your actual acceptable delay, **T**, you can never be better off, and sometimes be worse off.

Suppose that you falsely answered by saying that your acceptable delay was only 6 weeks, even though your true acceptable delay was 10 weeks, i.e.,

'To me, "receiving \$70 today" is equally as good as "receiving \$100 in 6 weeks." '

The computer randomly chooses a number to propose a delay. Suppose that the number generated is between 6 and 10, say, it is 9. Since this proposed delay is longer than that you reported, i.e., $9 > 6$, you receive \$70 today. But, the proposed delay is still shorter than your acceptable delay, and thus you would be willing to wait 9 weeks for \$100. Receiving \$70 today is worse than receiving \$100 in 9 weeks. You lose the opportunity to get the better outcome by falsely reporting shorter delay.

Thus, under-reporting will never make you better off.

What about stating **T** greater than 10 weeks?

2) Over-reporting can make you worse off as well.
By reporting any longer delay than your actual acceptable delay, **T**, you may end up waiting too long.

Suppose that you falsely answered by saying that your acceptable delay was 14 weeks, even though your true acceptable delay was 10 weeks. That is,

'To me, "receiving \$70 today" is equally as good as "receiving \$100 in 14 weeks." '

The computer randomly chooses a number to propose a delay. Suppose that the number generated is between 10 and 14, say, it is 13. Since the proposed delay is shorter than that you reported, i.e., $13 < 14$, you will get \$100. But, the actual delay, 13 weeks, is longer than your acceptable delay. You end up waiting too long. Thus, you lose the opportunity to get the better outcome by falsely reporting a longer delay.

Thus, over-reporting will never make you better off.

In sum, your best strategy is always to answer the questions truthfully.

Any question?

[the next part starts in a new page in the original format]

Part 2: Lottery Choice
Your earnings in this part will be paid in cash at the end of this experiment.

Task
You will answer a set of 10 questions assuming the following situation:

You are given two options:
1) receive \$Y for sure; or
2) play a lottery for \$Z, where \$Z > \$Y, and your odds of winning the lottery are P%.
Consider the lowest acceptable odds of winning with which you would be willing to play the lottery.

In a series of questions, you will be asked to fill out the blank below:

Q: 'To me, "receiving \$Y for sure" is equally as good as "receiving \$Z with __ % chance." '

You need to play a lottery to get the larger amount. Decide what odds of winning make the two options the same to you, and fill in that amount.

The actual amounts of \$Y and \$Z vary from question to question.

After each one of you answers all 10 questions, the computer will select one of the questions at random. Your actual payment will be based on your answer to the selected question.

Payment
To determine your chance of winning the lottery, the computer will randomly choose a number between 0% and 100%. Each of those numbers will be equally likely to be drawn, and the selected number will be the chance of winning.

If the chance of winning the lottery is less than your lowest acceptable odds of winning, you will not play the lottery. Instead, you will receive \$Y for sure.

If the chance of winning the lottery is greater than or equal to your lowest acceptable odds of winning, you will play the lottery. If you win the lottery, you will get \$Z; and if you lose, you will get nothing.

Example: (For purposes of illustration, we use different amounts than those actually given to you in the experiment.)

Suppose that you were asked the following question.

Q: 'To me, "receiving \$70 for sure" is equally as good as "receiving \$120 with __ % chance." '

Suppose your answer is 58%, i.e.,

'To me, "receiving \$70 for sure" is equally as good as "receiving \$120 with _58_ % chance." '

Then, the computer randomly generates a number between 0 and 100.

If the number is less than 58, e.g., if it is 23, then you do not get to play the lottery. Thus, you get \$70 for sure.

If the number is greater than or equal to 58, then you will play the lottery. For example, suppose that the number generated is 84. In this case, you will play a lottery for \$120 and your chance of winning is 84%. If you win the lottery, you will get \$120; and if you lose the lottery you will get nothing.

Any question?

Strategy:

Note that this procedure is such that your best response is to write down the minimum odds with which you are willing to play a lottery for \$Z.

We now show that truthful reporting is your best strategy. We will illustrate why you will never be better off sending a false report. Let us work through one example. Say that we offer you two amounts \$70 and \$120 and ask you to choose odds of a lottery, **P%**. Let us just assume, for the sake of argument, that you would be indifferent between receiving \$70 for sure and receiving \$120 with 58% chance. The question is should you tell us **P**=58 when we ask you?

To see why the answer is yes, let us say that you are thinking of not telling us the truth. There are two possible cases, under-reporting or over-reporting. We will show that in either case you might be worse off compared to telling the truth.

1) Under-reporting can make you worse off. By reporting any lower odds than your actual acceptable odds, you can never be better off, and sometimes be worse off.

Suppose that you falsely answered by saying that your acceptable odds were 43%, even though your true acceptable odds were 58%, i.e.,

'To me, "receiving \$70 for sure" is equally as good as "receiving \$120 with _43_ % chance." '

The computer randomly chooses a number between 0% and 100% to determine the chance of winning the lottery. Suppose that the number generated is between 43 and 58, say, it is 51. Since the number generated is greater than that you reported, i.e., $51 > 43$, you play the lottery and your odds of winning the lottery are 51%. But, it is lower than your acceptable odds, and thus playing the lottery is worse than receiving \$70 for sure. You end up playing a lottery with unacceptably low odds by falsely reporting lower odds.

Thus, under-reporting will never make you better off.

What about stating **P** greater than 58%?

2) Over-reporting can make you worse off as well. By reporting any higher odds than your acceptable odds, you may lose the opportunity to play a lottery even if it is preferred to receiving \$70 for sure.

Suppose that you falsely answered by saying that your acceptable odds were 77%, even though your true acceptable odds were 58%.

'To me, "receiving \$70 for sure" is equally as good as "receiving \$120 with _77_ % chance." '

The computer randomly chooses a number between 0% and 100% to determine the chance of winning the lottery. Suppose that the number generated is between 58 and 77, say, it is 66. Since the number generated is smaller than that you reported, i.e., $66 < 77$, you do not play the lottery and you receive \$70 for sure. But, the chance of winning the lottery, 66%, is greater than your acceptable odds. It means you still prefer playing the lottery to receiving \$70 for sure. Thus, you lost the opportunity to get the better outcome by falsely reporting higher odds.

Thus, over-reporting will never make you better off.

In sum, your best strategy is always to answer the questions truthfully.

Any question?

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