Abstract

This paper explores the outcome of the noncooperative environmental decision-making under transnational externalities when the national policies are chosen by the elected politicians. Specifically, we re-examine the extent of a voter's incentives for supporting politicians who are less green than she is, a phenomenon called “political race to the bottom”. The median voter (principal) strategically appoints his delegate (agent) who independently decides the level of environmental investments (as inputs for the global public good) which generate transnational benefits. The new feature of our model is the introduction of complementarity between public inputs, while previous studies supposed perfect substitution. Our analysis derives some new results. The extent of “political race to the bottom” diminishes as public inputs become more complementary, and if its degree exceeds a certain point, “political race to the top” emerges, without supposing effects of other factors including international trade. We further examine the case with perfect substitution. Equilibrium is in fact asymmetric. Although one of the elected politicians pays no attention to the environment, the other country results in self-representation.

Keywords: environmental policy; transnational externalities; strategic delegation; strategic complementarity

JEL classification: D62, D72, D78, Q58.

1 Introduction

The phenomenon of the strategic voting (strategic delegation) is well acknowledged in different contexts. Absent of commitments by the politicians, citizens, including the median voters, deliberately choose the delegate who has a different preference than themselves in order to pursue the strategic advantages. Examples include the capital-levy problem (Fischer (1980), Persson and
Tabellini (1994)), which Kehoe (1989) and Persson and Tabellini (1995) extended to the context of international policy coordination. They showed that, while uncoordinated policy making suffers from a conventional free-rider problem, policy coordination may in fact intensify the distortion due to the lack of commitments.

In the context of environmental policy, Buchholz et al. (2005) explored voters’ incentives and the outcome of an international environmental agreement when the governments of two countries are elected by their citizens through the strategic voting. In stage 1, a delegate (agent) is elected in each country who is most preferred by the median voter (principal); and in stage 2, two delegates choose environmental policy in each country. The delegate may be any citizen including the principal himself (referred to as self-representation by Segendorff (1998)), but there is an inherent advantage of choosing the type of the delegate different from the voter himself (called the strategic voting). Buchholz et al. (2005) showed (i) the strategic-delegation motive results in supporting a candidate who is less green than the median voter (called “political race to the bottom”); (ii) in the extreme case of global pollution, elected politicians pay no attention to the environment, and the resulting environmental policies are totally ineffective. Roelfsema (2007), on the other hand, developed another theoretical model with non-cooperative Pigouvian taxes and claimed that, if the median voter cares sufficiently for the environment, he has an incentive to delegate policy making to a politician that cares more for the environment than himself. However, Roelfsema (2007) mentions only a possibility of such cases.

One may argue that strategic delegation and the resultant “political race to the bottom” may be generated by the nature of environmental policies. The previous studies, including Segendorff (1998), Buchholz et al. (2005), and Roelfsema (2007), presupposed that the effects from countries’ environmental policies are perfect substitutes. Being analogous to a Cournot competition, each country’s reaction function, which is derived from the optimality condition equalizing the marginal benefit and the marginal cost from policy making, suggests that a country adapts to a less green policy abroad by choosing a greener policy. Anticipating the delegate’s ex post behavior, citizens may ex ante elect a less eco-friendly delegate “to commit their country to a more aggressive policy” (Buchholz et al. 2005, p. 188). A less eco-friendly politician will choose a less green (i.e., more aggressive) policy, and due to perfect substitution, other countries, in response to it, will reduce polluting economic activities, which will in turn improve the environment in their country. Thus, the citizens can shift the burden of policies for a cleaner environment to other countries by strategic delegation.

For some environmental problems, however, this extreme assumption of perfect substitution seems inappropriate. As an example which shows complementary effect of environmental action by a country on others, one may think of acute pollution of the common river/sea by chemical contaminants such as mercury. One country’s polluting behavior would ruin health, immunity, and preservation of species in neighboring countries. Sulfur emissions may be another example. Sulfur dioxide contained in emissions reacts with water vapor in the atmosphere to produce sulfuric acid. It falls to the earth as an acidic rain and damages soils, lake, and forests. In these examples, non-regulation by one country damages the well-being of all countries belonging to the group.
We can regard environmental action by an individual country as a public input for the quality of the environment, so that we can apply Hirshleifer’s (1983) social composition function in order to formalize the notion of complementarity. The cases of acute chemical contamination and sulfur emissions correspond to Hirshleifer’s (1983) “weakest-link” formula, where the level of the public good is determined by the minimum of the quantities individually provided. Variety of other cases can be covered with respect to the degree of complementarity, by parametrically treating the degree of complementarity that lies between the conventional perfect-substitution and the weakest-link.1

Our analysis first demonstrates that strategic delegation by the median voter (principal) towards a greener government can occur, when selecting a greener delegate (agent) causes the reaction by a foreign country towards a greener policy (Lemmas 1 and 2). In equilibrium, the extent of “political race to the bottom” by strategic delegation diminishes as public inputs individually provided for the global public good become more complementary. If its degree exceeds a certain point, “political race to the top” emerges, without supposing effects of other factors including international trade. The efficient outcome (in terms of the median voters) is obtained in the case of the weakest-link technology, while the inefficiency limit arises under perfect substitution (Lemma 3). An intriguing real-world example of international voluntary cooperation is found in the case of substantial sulfur reductions at the Helsinki Protocol (see Barrett (2003)). The parties to the Protocol successfully reached the reduction target and some of them achieved reductions over the required level. We can consider the reduction in sulfur emissions by an individual country as an input to the global public good whose benefits are measured by the weakest-link rule. The successful results of the Protocol as well as the strong incentives by the signatories are derived from the complementary structure.

We then carry out our analysis under the supposition of symmetric equilibrium, commonly dealt with in the previous studies. With a higher degree of complementarity, the elected delegate more concerns about the environment and hence provides a greater quantity of the public input (Proposition 1). Symmetric equilibrium actually exists in complementary as well as moderately substitutable cases. For sufficiently high degrees of substitution, on the other hand, the symmetric solution does not satisfy the second-order condition of optimization, so that equilibrium, if any, is asymmetric. This result is the modification of the previous study by Buchholz et al. (2005); by supposing the symmetric equilibrium, Buchholz et al. (2005) predicted the extreme case of the political race to the bottom, in which both of the elected politicians pay no attention to the environment. Indeed, under perfect substitution, one of the elected politicians pays no attention to the environment, but the other country results in self-representation (Proposition 2).

Section 2 defines the utility of citizens and the timing of the game. Section 3 derives the relation between the degree of substitution and the equilibrium public input level. It also discusses the citizen’s welfare.

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1The analysis on complementarity has been extended theoretically and empirically by Cornes (1993), Conybeare et al. (1994) and others (see Cornes and Sandler (1996)). Hirshleifer (1983) also introduced another aggregation technology called “the best-shot” rule, where the socially available amount is given by their maximum. We will not treat this case in our paper.
2 Citizens, Governments, and Transnational Policy Effects

2.1 Citizens’ Utility

We have two symmetric countries. Each country \( i \) \((i = 1, 2)\) faces a trade-off between environmental quality (as a public good) and the private cost to enhance it. Each country consists of a continuum of citizens. Citizens are identical with respect to the income and the attributes of the use of emissions-generating activities (such as transport and heating). Country \( i \)'s resource cost for the improvement of environment, denoted by \( g_i \), is equally shared by the citizens.\(^2\) The \( g_i \)'s from each country contributes to the global environmental quality, denoted by \( G \). Citizens within each country have heterogeneity with respect to the assessment of \( G \). It is represented by a constant \( \eta \), continuously distributed over the interval \((0, \infty)\). The utility of a citizen in country \( i \) with the taste parameter \( \eta \) is given by:

\[
- g_i + \eta \log G.
\]  
(1)

The \(- g_i\) in the first term represents country \( i \)'s resource cost for the improvement of environment.\(^3\) The second term captures the benefit from the environmental quality, \( G \). The \( g_i \)'s from each country are reflected to \( G \) by the following functional relationship:

\[
G = (g_i^q + g_j^q)^{\frac{1}{q}},
\]  
(2)

where \( q < 1 \), \( q \neq 0 \). The new feature of our model is the introduction of complementarity between benefits from public inputs provided by each country, while previous studies examining strategic delegation presupposed that they are perfect substitutes. With the specification in (2), we would like to capture various types of the global public good and the implications for the strategic voting. Consider, for example, the case of acute pollution of the common river/sea by chemical contaminants, where one country’s polluting behavior would ruin health, immunity, and preservation of species in neighboring countries. This limit case resembles that of the “weakest-link” originated by Hirshleifer (1983). Variety of other cases can be covered with respect to the degree of complementarity, where a higher \( q \) means lower complementarity. Representative cases are named as follows:

- \( q \to 1 \): perfect substitutes;
- \( q \to 0 \): Cobb-Douglas;
- \( q \to -\infty \): weakest-link.

\(^2\)One can think of, for example, the case of the regulation towards the use of the eco-friendly gasoline, which uniformly affects the citizen’s expenditures.

\(^3\)Namely, let \( \omega_i \) be the initial endowment (or GDP) of country \( i \) without an action for environment. Undertaking \( g_i \) reduces the endowment of country \( i \) to \( \omega_i - g_i \), which is equally distributed to the citizens who are identical except the valuation for environment (\( \eta \)). Dealing with the net payoffs from the environmental actions, we omit \( \omega_i \).
2.2 Timing

The taste parameter of a delegate in country \( i \), who was given authority of decision-making, is denoted by \( \eta_i \). Then the timing of the game is defined as

1. the decisive voter (principal) in each country \( i \) simultaneously and independently chooses type \( \eta_i \) as his delegate (agent);
2. each delegate \( i \) simultaneously and independently chooses \( g_i \).

The delegate may be any citizen including the principal himself. This case is referred to as self-representation, following Segendorff (1998). The concept we rely on to seek equilibrium of the game is subgame perfect Nash. We concentrate on pure-strategy equilibrium, and in order to explore it, examine the game backwards.

3 Equilibrium

3.1 Choice of the Quantity of a Public Input by a Delegate

In stage 2, each delegate \( i \), with his taste parameter \( \eta_i \) and that of another delegate \( \eta_j \) begin given, selects \( g_i \) that maximizes his utility. From (1), the utility of delegate \( i \) is given by

\[
V_i = -g_i + \eta_i \log G. \tag{3}
\]

The first-order conditions for delegates \( i \) and \( j \) are:

\[
1 = \eta_i \frac{g_i^{q-1}}{g_i^q + g_j^q} = \eta_i \frac{1}{1 + \left( \frac{g_j}{g_i} \right)^q} \frac{1}{g_i};
\]

\[
1 = \eta_j \frac{g_j^{q-1}}{g_i^q + g_j^q} = \eta_j \frac{1}{1 + \left( \frac{g_i}{g_j} \right)^q} \frac{1}{g_j}. \tag{4}
\]

Note that the second-order condition for delegate \( i \) globally holds for any \( q < 1 \):

\[
\frac{g_i^{q-2}(-g_i^q + (q-1)g_j^q)}{(g_i^q + g_j^q)^2} < 0.
\]

From (4), we have

\[
\frac{g_j}{g_i} = \left( \frac{\eta_j}{\eta_i} \right)^{\frac{1}{q-1}}. \tag{5}
\]

This relation shows that with a lower \( q \), the divergence between two delegates’ taste parameters \( \eta_i \) and \( \eta_j \) has a smaller impact on the divergence between equilibrium public input levels provided in
two countries. On the other hand, with higher \(q\), the possible divergence between \(\eta_i\) and \(\eta_j\) allows the divergence of \(g_i\) and \(g_j\) in stage 2, which gives a scope for the possibility of an asymmetric equilibrium which we deal with in Section 3.

By substituting (5) into (4), we derive a public input level in each country as a function of two delegates’ types (values of taste parameters):

\[
g_i = \frac{\eta_i}{1 + \left(\frac{\eta_i}{\eta_j}\right)^{\frac{1}{1-q}}} = \frac{\eta_i \eta_j^\frac{q}{q-1}}{\eta_i^{\frac{1}{q-1}} + \eta_j^\frac{1}{q-1}};
\]

\[
g_j = \frac{\eta_j}{1 + \left(\frac{\eta_i}{\eta_j}\right)^{\frac{1}{1-q}}} = \frac{\eta_j \eta_i^\frac{q}{q-1}}{\eta_i^{\frac{1}{q-1}} + \eta_j^\frac{1}{q-1}}.
\]

3.2 Election of a Delegate

We now examine stage 1. Here, the agent is selected by the election in each country. The structure of the preferences here guarantees the single-crossing property by Gans and Smart (1996), so that we regard the median voter in a country as being pivotal in the election in stage 1 and hence a principal. Represent the taste parameter of the median voter in each country as \(\eta_m\), relying on the assumption of symmetric countries. While \(V_i\) represents the utility of the delegate of country \(i\) as shown in (3), the utility of the median voter of that country is represented by \(V_m^i\):

\[
V_m^i = -g_i + \eta_m \log G.
\]

Anticipating a delegate’s subsequent behavior, the median voter, as a principal, selects his delegate \(\eta_i\) as an agent. His choice that maximizes (7) is given by the following first-order condition:

\[
\frac{\partial V_m^i}{\partial \eta_i} = \left[-1 + \eta_m \frac{\partial \log G}{\partial g_i}\right] \frac{\partial g_i}{\partial \eta_i} + \eta_m \frac{\partial \log G}{\partial g_j} \frac{\partial g_j}{\partial \eta_i} = 0.
\]

In order to understand the nature of the strategic voting, consider first the case where the median voter chooses himself as the delegate (called self-representation) in stage 1. The \(\eta_m\)’s preferred level of the public input in stage 2 is given by \(-1 + \eta_m \frac{\partial \log G}{\partial g_i} = 0\) from (4). Plugging this formula into (8) makes the first term of (8) be zero, so that, if the last term of (8) is non-zero, then the self-representation (\(\eta_i = \eta_m\)) is not consistent with (8).

In the present model, differentiating (6) with respect to \(\eta_i\), we have:

\[
\frac{\partial g_i}{\partial \eta_i} = \frac{\partial g_i}{\partial \log g_i} \frac{\partial \log g_i}{\partial \eta_i} = g_i \left(\frac{1}{\eta_i} - \frac{q \eta_i^\frac{2}{q-1} - 1}{\eta_i^{\frac{2}{q-1}} + \eta_j^\frac{2}{q-1}}\right)
\]

\[
= \frac{g_i}{\eta_i} \frac{1}{\eta_i^{\frac{2}{q-1}} + \eta_j^\frac{2}{q-1}} > 0,
\]
namely, a greener politician aims for a more eco-friendly domestic policy. We also have:

\[
\frac{\partial g_j}{\partial \eta_i} = \frac{\partial g_j}{\partial \log g_j} \frac{\partial \log g_j}{\partial \eta_i} = g_j \left( \frac{q}{q - 1} \eta_i \frac{\eta_i^{q-1}}{\eta_i^{q-1} + \eta_j^{q-1}} \right)
\]

(10)

\[
\frac{\partial \log G}{\partial g_j} = g_j \eta^{q-1} \frac{\eta_i^{q-1}}{\eta_i^{q-1} + \eta_j^{q-1}} > 0.
\]

(11)

We now turn to (8). The principal foresees the reaction by the foreign government to his choice of a greener politician \(\frac{\partial g_j}{\partial \eta_i}\). For instance, if \(\frac{\partial g_j}{\partial \eta_i} < 0\), then \(-1 + \eta^m \frac{\partial \log G}{\partial g_i} > 0\) in (8), so that the median voter should support the candidate who is less green than he is \((\eta_i < \eta^m)\). The strategic voting thus has a disincentive effect on the domestic environmental policy. Buchholz et al. (2005) showed this phenomenon in the case of the perfect substitute.

In our model, for \(q < 1\) and \(q \neq 0\), the sign of (10) is opposite to the sign of \(q\):

**Lemma 1** \(\frac{\partial g_j}{\partial \eta_i} < 0\) if \(q > 0\), and \(\frac{\partial g_j}{\partial \eta_i} > 0\) if \(q < 0\).

The intuition of this lemma is clear. As shown in (11), when \(q > 0\) (the regional policies are substitutable), a more eco-friendly politician in country \(i\) who brings higher \(g_i\) reduces the marginal product of \(g_j\). This causes a conventional free-riding effect for the provision of the public good in country \(j\). However, the opposite happens when \(q < 0\) (the regional policies are complementary). In this case, a greener policy in country \(i\) increases the marginal product of \(g_j\), and therefore induces higher \(g_j\).

Bring the consequence of Lemma 1 into (8). The sign of the third term, \(\eta^m \frac{\partial \log G}{\partial g_i} \frac{\partial g_j}{\partial \eta_i}\), is negative (resp. positive) when \(q > 0\) (resp. \(q < 0\)). Therefore, compared to the self-representation, political race to the bottom (resp. top) occurs when \(q > 0\) (resp. \(q < 0\)). An inspection of (8) induces the following conclusion:

**Lemma 2** When \(q > 0\), the choice of a politician with \(\eta_i > \eta^m\) is a dominated strategy by the principal. When \(q < 0\), the choice of a politician with \(\eta_i < \eta^m\) is a dominated strategy by the principal.

Regarding the efficiency property of the equilibrium, the following is shown in an Appendix:

**Lemma 3** Limit (In)efficiency: At any equilibrium,

\[
\eta^m \frac{\partial \log G}{\partial g_i} + \eta^m \frac{\partial \log G}{\partial g_j} - 1 = \frac{1}{1 - q}.
\]

(12)
The left-hand side of (12) is a sort of $MB - MC$ measure (degree of inefficiency): if both countries marginally increased the level of public inputs,\(^4\) how much welfare gain (net of the unit cost) would the median voter receive? When this value is zero, the level of the public inputs is efficient. It follows from (12) that the inefficiency measure

- is increasing in $q$ (implying a more intense race to the bottom with a higher $q$);
- is equal to unity when $q \to 0$ (usual free-riding); in the present context, this means that no strategic delegation emerges and hence no extra welfare loss is generated by delegation since $\frac{\partial g_j}{\partial \eta_i} \to 0$ in (10);
- goes to $\infty$ when $q \to 1$; this indicates $\min\{\eta_i, \eta_j\} \to 0$ since from (4) and (11), $\frac{\partial \log G}{\partial g_i} = \frac{1}{\eta_i}$;
- goes to 0 when $q \to -\infty$ (efficiency limit under the weakest link).

The final point above should be explained in more detail. The condition (8) suggests that the degree of inefficiency measured by the left-hand side of (12) is equal to 0 when $\frac{\partial g_i}{\partial \eta_i} = \frac{\partial g_j}{\partial \eta_i}$, i.e., when the median voter’s selection of delegate $i$’s type effects equally on public input provision in both countries. Indeed, it is straightforwardly shown from (5), (9), and (10) that $\lim_{q \to -\infty} \frac{\partial g_i}{\partial \eta_i} = \lim_{q \to -\infty} \frac{\partial g_j}{\partial \eta_i}$.

This property partially comes from the fact that the stage-2 reaction function lies along the 45\(^\circ\) line on $(g_i, g_j)$-space (Hirshleifer (1983, Figure 2)), and the increase of $\eta_i$ enhances $g_i$ and $g_j$ for the same proportion. The novel point in the present analysis is that the non-cooperative behavior by two principals can unintentionally yield the efficient provision of public inputs.\(^5\)

It is worthy to note that the consequences of Lemmas 1-3 is valid without assuming symmetricity of the equilibrium. Therefore, the above analysis on the reaction functions and welfare well illustrates the general properties of the equilibrium.

\(^4\)The conventional model of perfect substitutes considers the case of a unit increase by either one of the agents. Here, taking account of imperfect substitution, we consider the increase of the public input by both countries. A unit increase of $g_i$ ($g_j$) brings the benefit of $\eta^m \frac{\partial \log G}{\partial g_i} \left( \eta^m \frac{\partial \log G}{\partial g_j} \right)$ to the two median voters. The aggregate benefit including the spillover, net of the unit cost, is $2\eta^m \frac{\partial \log G}{\partial g_i} + 2\eta^m \frac{\partial \log G}{\partial g_j} - 2$. The per-capita expression is the left hand side of (12).

\(^5\)Following the convention of the literature (Segendorff (1998) and Buchholz et al. (2005)), the efficiency here is from the principal’s (the median voter’s) point of view. The conventional free-riding at stage 2 fails to maximize the agent’s well-being at that stage even in the case of perfect complement, whereas the strategic delegation allows the maximization of the principal’s well-being.
3.3 Symmetric Equilibrium

In order to get clear intuitions about the relationship between complementarity and the quantities of provided public inputs, we assume symmetricity of equilibrium, i.e., $\eta_i = \eta_j = \eta_n$ and hence $g_i = g_j = g_n$. We can calculate (6) as

$$g_n = \frac{\eta_n}{2}. \quad (13)$$

The result (13) shows that the equilibrium quantity of the public input depends only on the delegate’s type.

Employing (11) and (13), we derive $\frac{\partial \log G}{\partial g_i} = \frac{\partial \log G}{\partial g_j} = \frac{1}{\eta_n}$. Therefore, from (12), we have the relation between the principal and the delegate

$$\eta_n = \frac{2 - 2q}{2 - q} \eta_m, \quad (14)$$

and the environmental policy from (13) and (14):

$$g_n = \frac{1 - q}{2 - q} \eta_m. \quad (15)$$

Given $\eta^m$, $\eta_n$ and $g_n$ are strictly decreasing in $q$. Therefore, with a lower $q$, i.e., with a higher degree of complementarity, the elected delegate more concerns about the environment and hence provides a greater quantity of the public input. Denote the quantity of the public input under self-representation by $g_m \left(= \frac{\eta^m}{2} \text{ from (13)} \right)$. Similarly, let $V_n^m$ and $V_m^m$ denote the payoff of the median voter in symmetric equilibrium and under self-representation, respectively. The results so far are interpreted in comparison with self-representation:

**Proposition 1** The symmetric equilibrium is characterized as follows:

1. $\eta_n$ and $g_n$ are strictly decreasing in $q$;

2. when $q < (>)0$, $\eta_n > (<)\eta^m$, meaning that the delegate is more (less) eco-friendly than the principal;

3. when $q < (>)0$, $g_n > (<)g_m$, meaning the equilibrium public input level is higher (lower) than under self-representation;

4. when $q \to 1$, $\eta_n \to 0$ and $g_n \to 0$;

5. when $q \to -\infty$, $\eta_n \to 2\eta^m$ and $g_n \to \eta^m = 2g_m$, which is the first-best level of the public input in terms of the utility of the principal;
6. when $q > 0$, $V^m_n < V^m_m$ and thus the principal prefers self-representation to delegation.

The fourth claim in Proposition 1 corresponds to Proposition 4 (ii) in Buchholz et al. (2005): when public inputs are perfectly substitutable ($q \to 1$), in symmetric equilibrium, a politician who assigns no weight to the environment should be elected. It will be shown in the following subsection, however, that symmetric equilibrium does not exist for sufficiently high degrees of substitution.

We also show in the proposition that strategic delegation generates different outcomes according to the different degrees of substitution. When $q > 0$, the extent of the “political race to the bottom” diminishes as public inputs become more complementary. When $q < 0$, the opposite – “political race to the top” – emerges. Eventually, with the limit $q \to -\infty$, the first-best level of the public input is attained. This claim is consistent with the result from (12) in Lemma 3, and actually the equal level of the public input is provided in each country in equilibrium; with the weakest-link formula, a party supplying a higher level of a public input than another party can improve his payoff by reducing it.

### 3.4 Asymmetric Equilibrium and the Perfect-substitution Case

In the previous subsection, our analysis has been carried out under the supposition that the equilibrium delegate types are symmetric. However, if the second-order condition associated with (8) does not hold globally, we may have asymmetric solutions from (8) and the corresponding first-order condition for the median voter in country $j$. Actually, the following results are gained:

**Result 1** If $q < 0$, then the second-order condition is satisfied globally, and the equilibrium is unique.

**Result 2** If $q \leq q_a$ such that $q_a^2 + q_a - 1 = 0$ and $q_a > 0$, then the unique candidate of the symmetric equilibrium in (14) satisfies the second-order condition globally.

Note that $\frac{1}{2} < q_a < \frac{2}{3}$. Results 1 and 2 show that the assumption of the symmetric equilibrium is valid for $q \leq q_a$. 6 However, as $q$ becomes higher and closer to 1, the concavity of the stage-1 reaction function would not be guaranteed. While we can numerically show that the symmetric solution (14) satisfies the global optimality ($\eta_l = \eta_n$ being the maximum of (7) given $\eta_j = \eta_n$) for $q = \frac{2}{3}$ and $q = \frac{3}{4}$, the following holds:

**Result 3** The unique candidate of the symmetric equilibrium satisfies the second-order condition locally, if and only if $q \leq q_b$ such that $q_b^2 - 6q_b + 4 = 0$.

6 Though Result 2 does not eliminate the possibility of the existence of an asymmetric equilibrium, as long as there exists a symmetric equilibrium, it is a natural candidate to focus.
Note that $\frac{3}{4} < q_b < 1$ and hence $q_b > q_a$. Therefore, a pure-strategy equilibrium, if there exists, has to be asymmetric for a sufficiently high $q$. The following can be shown:

**Result 4** If $q > \frac{2}{3}$, there exists an asymmetric solution $(\eta_i, \eta_j)$, $\eta_i \neq \eta_j$, that satisfies the first-order conditions (not sure about the second-order condition).

It thus follows from Results 3 and 4 that when $q > q_b \left( \frac{3}{4} \right)$, then either (i) an asymmetric equilibrium exists, or (ii) a pure-strategy equilibrium does not exist. Accordingly, we have:

**Lemma 4** Let $q \to 1$. Then equilibrium, if any, is asymmetric.

Lemma 4 is a modification of Proposition 4 (ii) in Buchholz et al. (2005). In order to see why the symmetric solution is invalid to be an equilibrium for the cases of high substitutability, we now examine the case of the perfect substitution $(q = 1)$ which is analytically tractable. Consider first stage 2 where $\eta_i \neq \eta_j$. Without loss of generality, let $\eta_i < \eta_j$. As in Buchholz et al. (2005, p. 180), the first-order condition corresponding to (4) yields the corner solution for country $i$ ($g_i = 0$), whereas the interior solution for country $j$ yields, from (4) and $g_i = 0$, $g_j = \eta_j$. Therefore we have:

$$\frac{\partial g_i}{\partial \eta_i}|_{q=1, \eta_j > \eta_i} = 0; \quad \frac{\partial g_i}{\partial \eta_j}|_{q=1, \eta_j > \eta_i} = 0; \quad \frac{\partial g_j}{\partial \eta_i}|_{q=1, \eta_j > \eta_i} = 0; \quad \frac{\partial g_j}{\partial \eta_j}|_{q=1, \eta_j > \eta_i} = 1. \quad (16)$$

From (8), the following first-order conditions should hold with regard to each principal $i$ and $j$’s selection of their delegates:

$$\frac{\partial V^m_i}{\partial \eta_i} = -1 + \eta^m \frac{\partial \log G}{\partial g_i} \bigg|_{q=1, \eta_j > \eta_i} + \eta^m \frac{\partial \log G}{\partial g_j} \bigg|_{q=1, \eta_j > \eta_i} = 0; \quad (17)$$

$$\frac{\partial V^m_j}{\partial \eta_j} = -1 + \eta^m \frac{\partial \log G}{\partial g_j} \bigg|_{q=1, \eta_i > \eta_j} + \eta^m \frac{\partial \log G}{\partial g_i} \bigg|_{q=1, \eta_i > \eta_j} = -1 + \eta^m \frac{\partial \log G}{\partial g_j}. \quad (18)$$

For country $j$, (18) becomes zero at $\eta_j = \eta^m$ since $\frac{\partial \log G}{\partial g_j} = \frac{1}{\eta_j}$ here, which is the global optimum given $g_i = 0$ and (16). For country $i$, $\frac{\partial V^m_i}{\partial \eta_j} = 0$ for all $\eta_i < \eta_j = \eta^m$; $\eta_i = \eta^m$ still yields $G = \eta^m$ (the same level as $\eta_i = 0$) with now $g_i \geq 0$; when $\eta_i > \eta_j$, $G > \eta^m$ at stage 2, so that (16) and (18) applied to country $i$ imply $\frac{\partial V^m_i}{\partial \eta_i}|_{q=1, \eta_j < \eta_i} < 0$. In summary, $\eta_i = 0$, $\eta_j = \eta^m$ constitutes a pure-strategy equilibrium.

These properties can be derived as extensions of equation (6) and Lemma 3 in a limit case of $q \to 1$. We therefore conclude:
Proposition 2  When public inputs in two countries are perfectly substitutable ($q \to 1$), equilibrium, if any, is such that

- $\eta_i \to 0$, $\eta_j \to \eta^m$;
- $g_i \to 0$, $g_j \to \eta^m$,

where $i, j \in \{1, 2\}$, $i \neq j$.

The result of Proposition 2 also applies to the framework of Buchholz (2005). First, the symmetric solution does not satisfy the second-order condition under the perfect substitution. Intuitively, high substitution allows the country’s best-response remote from the symmetric solution. Second, the extreme case of the political race to the bottom claimed by Buchholz et al. (2005) in fact does not happen. In (18), if country $i$ chooses $\eta_i = 0$, then, in the absence of the environmental action ($g_i = 0$) which in turn implies no reaction to the choice of $\eta_j$ ($\frac{\partial g_i}{\partial \eta_j} = 0$ in (16)), country $j$ is the sole provider of $G$, so that self-representation ($\eta_i = \eta^m$) is the best-response. (17) and the following derivation simply clarified that country $i$ cannot be better-off than $G = \eta^m$ and $g_i = 0$.

Appendix

Proof of Lemma 3

Substituting (6) into (9) and (10) shows that:

$$
\frac{\partial g_i}{\partial \eta_i} = \frac{g_i}{\eta_i} \left( -\frac{1}{q-1} \frac{g_j}{\eta_j} + \frac{g_i}{\eta_i} \right); \\
\frac{\partial g_j}{\partial \eta_i} = \frac{g_j}{\eta_i} \frac{q}{q-1} \frac{g_i}{\eta_i}. 
$$

(19)

From (4) and (11),

$$
\frac{\partial \log G}{\partial g_i} = \frac{1}{\eta_i}. 
$$

(20)

Substituting (19) and (20) to (8),

$$
\frac{\partial V_i^m}{\partial \eta_i} = (-1 + \frac{\eta^m}{\eta_i}) \left( -\frac{1}{q-1} \frac{g_j}{\eta_j} + \frac{g_i}{\eta_i} \right) \frac{g_i}{\eta_i} + \frac{\eta^m}{\eta_j} \frac{g_j}{\eta_j} \frac{q}{q-1} \frac{g_i}{\eta_i} = 0, 
$$

(21)

which is rearranged to yield:

$$
(-1 + \frac{\eta^m}{\eta_i}) \frac{g_i}{\eta_i} + \frac{g_j}{\eta_j} \frac{1}{q-1} + \frac{\eta^m g_j}{\eta_i \eta_j} = 0. 
$$

(22)
From (5), \( \frac{g_j}{\eta_j} = \left( \frac{\eta_i}{\eta_j} \right)^{\frac{q}{q-1}} \frac{g_i}{\eta_i} \) so that we yield:

\[
-1 + \frac{\eta^m}{\eta_i} = \left( \frac{1}{1 - q} - \frac{\eta^m}{\eta_i} \right) \left( \frac{\eta_i}{\eta_j} \right)^{\frac{q}{q-1}},
\]

(23)

and, symmetrically,

\[
\left( -1 + \frac{\eta^m}{\eta_j} \right) \left( \frac{\eta_i}{\eta_j} \right)^{\frac{q}{q-1}} = \left( \frac{1}{1 - q} - \frac{\eta^m}{\eta_j} \right).
\]

(24)

From (23) and (24),

\[
\left( -1 + \frac{\eta^m}{\eta_i} \right) \left( -1 + \frac{\eta^m}{\eta_j} \right) = \left( \frac{1}{1 - q} - \frac{\eta^m}{\eta_i} \right) \left( \frac{1}{1 - q} - \frac{\eta^m}{\eta_j} \right).
\]

(25)

After rearrangement, one can have:

\[
\eta^m \left( \frac{\eta_i}{\eta_j} \right) + \eta^m \left( \frac{\eta_j}{\eta_i} \right) = 1 + \frac{1}{1 - q}.
\]

(26)

Again from (20) and (26), we have derived (12). Q.E.D.

**Proof of Results 1-3**

From (5) and (22),

\[
\frac{\partial V^m}{\partial \eta_i} = \left( -1 + \frac{\eta^m}{\eta_i} + \frac{1}{q - 1} \left( \frac{\eta_i}{\eta_j} \right)^{\frac{q}{q-1}} + \eta^m \frac{\eta_i}{\eta_j} \right) \frac{g_i}{\eta_i} = 0.
\]

Let \( F(\eta_i) \equiv -1 + \frac{\eta^m}{\eta_i} + \frac{1}{q - 1} \left( \frac{\eta_i}{\eta_j} \right)^{\frac{q}{q-1}} + \eta^m \frac{\eta_i}{\eta_j} \) (treating \( \eta_j \) as a given parameter). Then:

\[
\frac{\partial^2 V^m}{(\partial \eta_i)^2} = F'(\eta_i) \frac{g_i}{\eta_i} + F(\eta_i) \frac{\partial (g_i/\eta_i)}{\partial \eta_i}.
\]

(27)

Since \( \frac{g_i}{\eta_i} > 0 \) from (6), at the point where the first-order condition is satisfied, \( F(\eta_i) = 0 \). We now show that, when \( q \leq q_\alpha \), for any \( \eta_i \) at which \( F(\eta_i) = 0 \), \( F'(\eta_i) < 0 \). If this is true, then: (i) the second-order condition is satisfied locally, and (ii) there is no \( \eta_i \) such that \( F(\eta_i) = 0 \) and \( F'(\eta_i) \geq 0 \), so that the first-order condition implies global optimality.
Differentiating $F(\eta)$ with respect to $\eta$,

$$F'(\eta) = -\frac{\eta^m}{\eta^2} + \frac{q}{(q-1)^2} \frac{1}{\eta_i} \left( \frac{\eta}{\eta_j} \right)^{\frac{q-1}{q}} + \frac{1}{q-1} \frac{\eta^m}{\eta_j^{\frac{q-1}{q}}} \left( \frac{\eta}{\eta_j} \right)^{\frac{q-1}{q}} + \frac{1}{q-1} \frac{\eta^m}{\eta_j^{\frac{q-1}{q}}} \left( \frac{\eta}{\eta_j} \right)^{\frac{q-1}{q}}$$

$$= \frac{1}{\eta_i \eta_j^{\frac{q-1}{q}}} \left( -\frac{\eta^m}{\eta_i} \left( \frac{\eta}{\eta_j} \right)^{\frac{q-1}{q}} + \frac{q}{(q-1)^2} \frac{1}{\eta_i} \left( \frac{\eta}{\eta_j} \right)^{\frac{q-1}{q}} + \frac{1}{q-1} \frac{\eta^m}{\eta_i} \left( \frac{\eta}{\eta_j} \right)^{\frac{q-1}{q}} + \frac{1}{q-1} \frac{\eta^m}{\eta_i} \left( \frac{\eta}{\eta_j} \right)^{\frac{q-1}{q}} \right). \tag{28}$$

The expression of (28) is negative for $q < 0$. In the proof of Result 4, we will show that there is no asymmetric solution when $q < 0$. This verifies Result 1.

Now, consider the case of $q \in (0, 1)$. Omitting $\eta_i \eta_j^{\frac{q-1}{q}} > 0$, we have:

$$\frac{-\eta^m}{\eta_i} \eta_j^{\frac{q-1}{q}} + \frac{q}{(q-1)^2} \frac{1}{\eta_i} \left( \frac{\eta}{\eta_j} \right)^{\frac{q-1}{q}} + \frac{1}{q-1} \eta^m \eta_i^{\frac{q-1}{q}}$$

$$= -\frac{-\eta^m}{\eta_i} \eta_j^{\frac{q-1}{q}} + \frac{1}{(q-1)^2} \frac{1}{\eta_i} \left( \frac{\eta}{\eta_j} \right)^{\frac{q-1}{q}} - \frac{1}{q-1} \eta^m \eta_i^{\frac{q-1}{q}} \left( \frac{1}{\eta_i} \right)^{\frac{q-1}{q}} - \frac{1}{q-1} \eta^m \eta_i^{\frac{q-1}{q}} \left( \frac{1}{\eta_i} \right)^{\frac{q-1}{q}}$$

$$= -\eta^m \eta_j^{\frac{q-1}{q}} + \frac{1}{q-1} \eta^m \eta_i^{\frac{q-1}{q}} - \frac{1}{q-1} \eta^m \eta_i^{\frac{q-1}{q}}$$

$$= \frac{q}{1-q} \eta^m \eta_i^{\frac{q-1}{q}} - \frac{1}{1+q} \eta^m \eta_i^{\frac{q-1}{q}} - \frac{1}{1+q} \eta^m \eta_i^{\frac{q-1}{q}}$$

$$= \frac{1}{1+q} \left( \frac{1}{1-q} \eta^m \eta_i^{\frac{q-1}{q}} - \frac{1}{1-q} \eta^m \eta_i^{\frac{q-1}{q}} - \frac{1}{1-q} \eta^m \eta_i^{\frac{q-1}{q}} \right)$$

$$= \frac{1}{1-q} \left( \frac{1}{1-q} \eta^m \eta_i^{\frac{q-1}{q}} - \frac{1}{1-q} \eta^m \eta_i^{\frac{q-1}{q}} - \frac{1}{1-q} \eta^m \eta_i^{\frac{q-1}{q}} \right)$$

which verifies Result 2.

Evaluating (29) at the symmetric solution $\eta_m$,

$$\frac{q}{1-q} \eta_m \eta_j^{\frac{q-1}{q}} - \frac{1}{1+q} \eta^m \eta_i^{\frac{q-1}{q}} - \frac{1}{1+q} \eta^m \eta_j^{\frac{q-1}{q}}$$

$$= \eta_m \left( \frac{q \eta^m - 2 \eta_m}{1-q} \right)$$

$$= \eta_m \left( \frac{q \eta^m}{1-q} \right) - \frac{4 \eta^m}{2-q}$$

$$= -\eta_m \left( \frac{q \eta^m}{1-q} \right) (q^2 - 6q + 4), \tag{31}$$

$$= -\eta_m \left( \frac{q \eta^m}{1-q} \right) (q^2 - 6q + 4),$$
which verifies Result 3. \textit{Q.E.D.}

**Proof of Result 4**

From the first-order condition (26), we obtain:

\[ \frac{\eta_i}{\eta_j} = \frac{\eta_i}{\eta^m} \frac{2 - q}{1 - q} - 1. \]  \hspace{1cm} (32)

Substituting this equation into (23) and re-arranging, we obtain:

\[ -1 + \frac{q \eta^m}{\eta_i - (1 - q) \eta^m} = \frac{1}{1 - q} \left( \frac{\eta_i}{\eta^m} \frac{2 - q}{1 - q} - 1 \right)^{\frac{q}{q - 1}}. \]  \hspace{1cm} (33)

Let \( L(\eta_i) \equiv -1 + \frac{q \eta^m}{\eta_i - (1 - q) \eta^m} \) and \( R(\eta_i) \equiv \frac{1}{1 - q} \left( \frac{\eta_i}{\eta^m} \frac{2 - q}{1 - q} - 1 \right)^{\frac{q}{q - 1}} \). (33) is equivalent to \( L(\eta_i) = R(\eta_i) \), which characterizes \( \eta_i \) that satisfies the first-order condition.

Suppose first that \( q < 0 \). Then \( L(\eta_i) \) is an increasing and convex function with respect to \( \eta_i \), with \( L(\eta^m) = 0 \). On the other hand, \( R(\eta_i) \) is an increasing and concave function with respect to \( \eta_i \), with \( R(\eta^m) > 0 \). From Lemma 2, we consider the best-response in the range of \( \eta_i \geq \eta^m \). For \( \eta_i \geq \eta^m \), \( L(\eta_i) \) and \( R(\eta_i) \) have the unique intersection at \( \eta_i = \frac{2 - 2q}{2 - q} \eta^m \equiv \eta_n \) represented at (14). Therefore, \( (\eta_i, \eta_j) = (\eta_n, \eta_n) \) is the unique equilibrium.

Next, suppose that \( 0 < q < 1 \). Then, both \( L(\eta_i) \) and \( R(\eta_i) \) are decreasing and convex functions with respect to \( \eta_i \). Notice first that \( L(\eta_i) < 0 \) for \( \eta_i < (1 - q) \eta^m \) but \( R(\eta_i) \), which represents \( \frac{1}{1 - q} \left( \frac{\eta_i}{\eta^m} \right)^{\frac{q}{q - 1}} \) in an equilibrium, has to be non-negative. Therefore, we restrict our attention to \( \eta_i \geq (1 - q) \eta^m \). Combining Lemma 2, we have \( \eta_i \in [(1 - q) \eta^m, \eta^m] \). Evaluating the derivatives at \( \eta_n = \frac{2 - 2q}{2 - q} \eta^m \), we have:

\[ \frac{L'(\eta_n)}{R'(\eta_n)} = -\frac{q}{\eta^m (1 - q) \eta^m} \frac{(2 - q)^2}{q^2} = \frac{q^2 - 3q + 2}{q^2 - q - 1 \eta^m}. \]  \hspace{1cm} (34)

If \( q > \frac{2}{3} \), then, since \( L'(\eta_n) < 0 \) and \( R'(\eta_n) < 0 \), (34) implies that \( L'(\eta_n) > R'(\eta_n) \). Then, combined with \( \lim_{\eta_i \rightarrow (1 - q) \eta^m \eta_n} L(\eta_i) = +\infty > R((1 - q) \eta^m) = (1 - q) \eta^{\frac{1}{q - 1}} \), \( (1 - q) \eta^m < \eta_n \) and \( L(\eta_n) = R(\eta_n) \), there exists \( \tilde{\eta}_i \in ((1 - q) \eta^m, \eta_n) \) such that \( L(\tilde{\eta}_i) = R(\tilde{\eta}_i) \). Also, \( L(\eta^m) = 0 = R(\eta^m) = (1 - q) \frac{2q}{q - 1} \), \( \eta_n < \eta^m \) and \( L(\eta_n) = R(\eta_n) \) imply the existence of \( \tilde{\eta}_i \in (\eta_n, \eta_m) \) such that \( L(\tilde{\eta}_i) = R(\tilde{\eta}_i) \). Notice
that, for \( \hat{\eta}_i \), there exists \( \hat{\eta}_j \) that satisfies (32) for \((\eta_i, \eta_j) = (\hat{\eta}_i, \hat{\eta}_j)\). Since \( \hat{\eta}_j \in (\eta_n, \eta_m) \) from (26), and \( L(\hat{\eta}_j) = R(\hat{\eta}_j) \) (since expressions are symmetric between \( i \) and \( j \)), \( \hat{\eta}_j = \hat{\eta}_i \), and for \( \hat{\eta}_j \) similarly defined, \( \tilde{\eta}_j = \tilde{\eta}_i \) can hold. This verifies Result 4. Q.E.D.

**Proof of Proposition 2**

Consider \( \tilde{\eta}_i \) defined in the proof of Result 4. If we regard \( \tilde{\eta}_i \) as a function of \( q \), it is a continuous function.\(^7\) Notice also that \( \lim_{q \to 1} \log R(\eta_i) = -\infty \) for all \( \eta_i > \eta_n \), so that \( \lim_{q \to 1} R(\eta^m) = 0 = L(\eta^m) \) and \( \lim_{q \to 1} R(\eta_i) < L(\eta_i) \) for \( q \to 1 \) and for all \( \eta_i \in (0, \eta^m) \). We conclude that \( \lim_{q \to 1} \tilde{\eta}_i(q) = \eta^m \): the sequence of the asymmetric solution converges to \( \tilde{\eta}_i = \eta^m \). From (26), \( \lim_{q \to 1} \tilde{\eta}_j(q) = 0 \). Now, \( (\tilde{\eta}_i, \tilde{\eta}_j) = (\tilde{\eta}_j, \tilde{\eta}_i) \), and since \( (\eta_i, \eta_j) = (0, \eta^m) \) satisfies the global optimality at \( q = 1 \) as we verified in the text, we have proved Proposition 2.

**References**


\(^7\)If there are multiple \( \tilde{\eta}_i \) such that \( \tilde{\eta}_i \in (\eta_n, \eta_m) \) and \( L(\tilde{\eta}_i) = R(\tilde{\eta}_i) \), pick any of \( \tilde{\eta}_i \) so that the sequence of \( \tilde{\eta}_i \)'s with different \( q \) conforms a continuous function \( \tilde{\eta}_i(q) \). Since the system of \( L(\tilde{\eta}_i) = R(\tilde{\eta}_i) = 0 \) is differentiable with respect to \( q \), it is possible to do that.
