A Role of Common Morality in Social Choice*

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Abstract

We propose an extended framework of social choice, which incorporates the common morality of the society. The purpose of this paper is to investigate the role of common morality in social decision situations. Common morality includes non-welfaristic considerations such as rights, procedures, and historical considerations. Each individual has a comprehensive preference based on welfare and common morality. First, we consider the society, where there exists at least one common-sense moralist who always gives unequivocal priority to common morality. The existence of a common-sense moralist is indispensable to satisfy the desirable requirements in a social decision. A (comprehensive) social preference is amalgamation of individual comprehensive preferences of the members in the society. We impose three well-known Arrovian conditions, namely, the Pareto principle, independence of irrelevant alternatives, and non-dictatorship, on a social ordering function. Under such conditions, social preference ordering gives unequivocal priority to common morality. In other words, we characterize a social preference ordering which has a non-welfaristic feature. Our result implies that common-sense morality in the society plays a crucial role for the evaluation of social decision situations. Furthermore, we consider the general domain of common morality, which is the largest domain consistent with the existence of common morality, and impose modified versions of the Arrovian conditions. Under the general domain of common morality, there exists no common-sense moralist. We show that under these conditions, social preference ordering gives unequivocal priority to common morality.

Keywords common morality; social preference ordering; Arrovian framework; welfarism

JEL classification D63; D71

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1 Introduction

The standard economic theories, including the social choice theory, have judged the desirability of action based on the welfaristic assessment of consequential states.[1] This doctrine is called *welfarism*, and it is dominant in modern moral theories. One of the central features of utilitarianism is welfarism, and many critique for utilitarianism is for its welfarism.[2] Welfarism is "the principle that the goodness of a state of affairs depends ultimately on the set of individual utilities in that state" (Sen, 1979). The extremal form of welfarism is as follows: "individual utility is the only good or value which has basic moral significance." On the other hand, opponents of welfarism such as Rawls (1971), Dworkin (1977) and other political philosophers insist on the importance of non-welfaristic aspects of the human well-being. According to them, in order to evaluate human or social acts, there exists many important considerations such as rights, friendships, procedural justice, and freedom of choice, which cannot be considered by welfarism.[3]

Consider the following example. Suppose that a civil war breaks out in a neighboring country, leading to the influx of many refugees. Should our country accept them? Should we judge such a situation based only on welfaristic information as our ultimate aim? We should evaluate this choice situation depending on considerations such as human rights, duty, and reciprocal relationships based on historical consideration. Such non-welfaristic concepts have a closed relationship with morality of common sense (common morality) in the society.[4]

In this paper, we study an extended framework of the social choice theory, which incorporates not only welfare from social outcomes but also common morality of the society. The main purpose of this paper is the characterization of social preference that gives unequivocal priority to common morality. We term such preference as *common morality priori* (*CM-priori*) social ordering. *CM*-priori social ordering requires that common morality of the society, which is a non-welfaristic desirability, to play a crucial role for the evaluation of social decision situations. We give two characterizations of the CM-priori social ordering. Our first characterization has five features as follows:

(F.1) Each individual has an extended preference over the extended alternatives.

(F.2) The society consists of welfarists and common-sense moralists.

(F.3) There exists at least one common-sense moralist.

(F.4) A social (extended) preference over the extended alternatives is the image of social ordering function whose domain is the set of individual preferences.

(F.5) A social ordering function satisfies the conditions of the *Pareto principle*, *independence of irrelevant alternatives (IIA)*, and *non-dictatorship.*

Under (F.1)–(F.5), the social preference ordering gives unequivocal priority to common morality.

(F.1) is the fundamental feature of this paper. The standard economics theories consider only the individual's welfaristic evaluation over consequences. However, in general, a choice situation is described as follows: an agent (or society) chooses outcome x under circumstance θ. A circumstance includes a non-welfaristic information about the choice situation. I refer to a pair (x, θ) as

[1] Campbell and Kelly (2002) and Sen (1986) are excellent surveys on Arrovian social choice theory.

[2] Other important features of utilitarianism are consequentialism and sum-ranking.

[3] In Suzumura (1999), the importance of nonconsequential aspect is emphasized, and he gives persuasive examples.

[4] In his great book, Sidgwick (1874) gave valuable arguments on common morality. He distinguished the difference between common morality (common sense of morality) and common sense.

an extended alternative. Each individual has his welfaristic ordering over the set of social consequences. Welfaristic preferences over the consequences vary according to individuals. On the other hand, based on common morality, each individual has a common ordering over the set of circumstances. Furthermore, each individual has an ordering over the set of extended alternatives based on welfaristic preference and common morality. We call it an extended preference.

(F.2) and (F.3) state the types of extended preferences tha individuals can have.[5] As mentioned in (F.2), we focus on the society where every individual is either a welfarist or a common-sense moralist. A welfarist gives unequivocal priority to welfare over common morality. He/she considers common morality (over circumstance) if and only if welfares from two choice situations are indifferent. On the other hand, common-sense moralists give unequivocal priority to common morality. It is only when the circumstances are same between two choice situations that they are ranked by welfaristic ordering. Moreover, we assume that the ranking of common morality is homogeneous among the individuals in the society.[6]

At this point, we remark on the differences between Suzumura and Xu (2004) and our analysis in the framework structure and assumptions. An important difference is the assumption in the domain condition of a social ordering function (Suzumura and Xu (2004) call it an extended social welfare function). They focus on the freedom of choice in the choice situation, and non-welfaristic (or non-consequential) morality is the cardinality of the opportunity set. Thus, common-sense morality is a permanent feature in their study. On the other hand, in our framework, common-sense morality is not a permanent feature. As long as the orderings of common morality is common among all individuals in the society, all common morality is available: that is, the admissible set of orderings over the circumstances is all logically possible linear orderings.

The third feature (F.3) is the existence of a common-sense moralist. If there exists no common-sense moralist, then the CM-priori social preference and the Pareto principle are inconsistent (Lemma 1). Moreover, if there exists at least one common-sense moralist, then there exists a social welfare function that satisfies the Pareto principle, IIA, and non-dictatorship.[7] That is, if there exists at least one common-sense moralist, then we obtain a possibility result. Furthermore, if all individuals are welfarists, then there exists no social welfare function that satisfies all three Arrovian axioms. Proposition 1 states that the existence of a common-sense moralist is a necessary and sufficient condition for the existence of a social ordering function satisfying our conditions. These results imply that the existence of individuals with common morality priori preference has a critical role in the extended Arrovian framework.

By (F.4), an extended social preference is the amalgamation of individual preference ordering of the members in the society. In other words, a social ordering function is a mapping such that given the lists of individual extended preferences, the image is the social (extended) preference. (F.5) states three conditions that a social ordering function satisfies. We impose three well-known Arrovian axioms: the Pareto principle, IIA, and non-dictatorship.

Our second characterization of CM-priori social ordering has five features; they are as follows:

(F'.1) Each individual has an extended preference over the extended alternatives.

(F'.2) Every individual in the society has common morality in the weak sense.

[5] That is, these conditions state the restriction for domain of social ordering function. The issue of domain restriction is summarized in Gaertner (2002).

[6] In the context of extended sympathy, many authors require that each individual's sympathy for another individual is identical. See Hammond (1976).

[7] This result is essentially based on the study of Suzumura and Xu (2004). They show that if there exists at least one individual who has nonconsequential preference, then there exists a social welfare function that satisfies the Pareto principle, IIA, and non-dictatorship. As noted before, our formulation is different for their formulation. This implies that we obtain a similar result.

(F'.3) There exists no common-sense moralist and welfarist.

(F'.4) A social (extended) preference over the extended alternatives is the image of social ordering function whose domain is the set of individual preferences.

(F'.5) A social ordering function satisfies the *conditional Pareto principle*, *conditional IIA* and *non consequential-dictatorship.*

Under (F'.1)–(F'.5), the social preference ordering gives unequivocal priority to common morality.

(F'.1), (F'.2), and (F'.4) are the same as the first characterization result. On the other hand, (F'.3) and (F'.5) are different features; we explain these features below. In the previous characterization, the requirement of every individual to be a welfarist or common-sense moralist is strong. According to (F'.3), we do not require the existence of welfarists or common-sense moralists in the second characterization, and consider the general domain of common morality, which is the largest domain consistent with the existence of common morality.

(F'.5) states three conditions that a social ordering function satisfies. Under the general domain of common morality, we impose three modified versions of Arrovian axioms: conditional Pareto principle, conditional IIA, and non consequential-dictatorship. The conditional Pareto principle is a weak version of the Pareto principle, and requires that the principle of unanimity functions when consequences or circumstances of two alternatives are the same. The conditional IIA is a weak version of the IIA. It is noteworthy that under the general domain of common morality, there exists no non-dictatorial social ordering function, which satisfies the Pareto principle and IIA, but there exits a non-dictatorial social ordering function, which satisfies the conditional Pareto principle and conditional IIA. Non consequential dictatorship is stronger than the standard definition of non-dictatorship.

This paper is organized as follows. In Section 2, we present our extended Arrovian framework. Furthermore, we introduce W-priori ordering and CM-priori ordering. Section 3 considers the society where contains of welfarists and common-sense moralists. In Section 4, we extend the preference domain and find that a similar result holds. We conclude the paper in Section 5. The Appendix includes the proof of Proposition 1.

2 Notation and Definitions

Let X with $|X| \geq 3$ be a set of social consequences. Each member of X is a (social) option by (social) act. Let Θ with $|\Theta| \geq 3$ be a set of circumstances. An element of Θ describes the non-welfaristic aspect of choice situation, witch is related to common morality of the society. A set of social extended alternatives is $X \times \Theta$: $(x, \theta) \in X \times \Theta$ means that the social consequence x is chosen in the circumstance θ. Let $N := \{1, 2, \ldots, n\}$ with $n \geq 2$ be a finite set of individuals on the society.

R is *reflexive*, *complete*, and *transitive* binary relations on $X \times \Theta$. The symmetric and the asymmetric part of R are denoted by I and P, respectively. Let $\wp$ be the set of all logically possible ordering over $X \times \Theta$. Each individual $i \in N$ has a (extended) preference ordering $R_i \in \wp$ over the set of social alternatives $X \times \Theta$. Then a profile $\mathbf{R} = (R_1, R_2, \cdots, R_n) \in \wp^n$ is an $n-$tuple of individual preference orderings on $X \times \Theta$. The restriction of preference profile $\mathbf{R}$ on $X \times \Theta$ to the subset Γ of $X \times \Theta$ is denoted $\mathbf{R}|\Gamma$.

An *admissible preference domain* is a subset $\mathcal{D}$ of $\wp^n$. A *social ordering function (SOF)* is a mapping $f : \mathcal{D} \to \wp$ associating a unique preference $f(\mathbf{R}) \in \wp$ with each profile $\mathbf{R} \in \mathcal{D}$. For simplicity, $f(\mathbf{R})$ is denoted as R.

2.1 Restriction on Preference Domain: Common Morality

Let $\succsim_i$ be complete ordering over X. For all $x, y \in X$, $x \succsim_i y$ holds if and only if individual i weakly prefers consequence x to consequence y. Then $\succsim_i$ represents welfaristic aspect of individual i. Let J be the asymmetric part of complete, transitive and antisymmetric binary relation over Θ, *i.e.*, for all $\theta, \theta' \in \Theta$, $\theta J \theta'$ or $\theta' J \theta$ or $\theta = \theta'$. For all $\theta, \theta' \in \Theta$, $\theta J \theta'$ means that circumstance θ is more desirable in the view of common morality than circumstance θ'. We assume that the evaluation for circumstance is homogeneous among individuals in the society. Binaly relation J represents common morality in the society. We can also interpret that J represents social identity or social norms in community. The admissible set of orderings over circumstances Θ is all logically possible linear orderings. Hence, we allow the possibility of various morality.

Each individual i has extended preference R_i based on his welfaristic preference $\succsim_i$ and common morality J.[8] Let $\mathcal{D}_J \subset \wp^n$ be a preference domain which contains all logically possible profiles satisfying the following two conditions: for all $i \in N$, (a) $\forall x, y \in X, \forall \theta \in \Theta, (x, \theta) R_i (y, \theta) \Leftrightarrow x \succsim_i y$, and (b) $\forall (x, \theta), (y, \theta') \in X \times \Theta, x \sim_i y \Rightarrow [(x, \theta) R_i (x, \theta') \Leftrightarrow \theta J \theta']$. We call this the *general domain of common morality.* In the rest of this paper, we investigate a domain $\mathcal{D}$ which is subset of $\mathcal{D}_J$.

Now we define welfare-priori (*W-priori*) preference ordering and common morality-priori (*CM-priori*) preference ordering.

Definition 1. R is
(i) *W-priori* if and only if $x \succ y \Rightarrow (x, \theta) P (y, \theta')$ for all $(x, \theta), (y, \theta') \in X \times \Theta$.
(ii) *CM-priori* if and only if $\theta J \theta' \Rightarrow (x, \theta) P (y, \theta')$ for all $(x, \theta), (y, \theta') \in X \times \Theta$.

According to these definitions, W-priori preference is an ordering which gives unquavocal priority to welfaristic evaluation, and CM-priori preference is an ordering which gives unquavocal priority to common morality, which represents non-welfarictic evaluation in society.

By using concepts of W-priori and CM-priori preference ordering, we define welfarist and common-sense moralist, respectively.

Definition 2. An individual $i \in N$ is
(i) *Welfarist* if for all $\mathbf{R} \in \mathcal{D}$, R_i is W-priori preference ordering.
(ii) *Common-sense Moralist* if for all $\mathbf{R} \in \mathcal{D}$, R_i is CM-priori preference ordering.

In other words, a *welfarist* is an individual who uniformly has W-priori preference ordering in admissible preference domain, and a *common-sense moralist* is an individual who uniformly has W-priori preference ordering in admissible preference domain.

2.2 Axioms

We now introduce three axioms on the social ordering function f, which are called as Arrovian conditions.

Axiom 1. *Pareto Principle (PP)*
For all $(x, \theta), (y, \theta') \in X \times \Theta$, and for all $\mathbf{R} \in \mathcal{D}$, if $(x, \theta) P_i (y, \theta')$ holds for all $i \in N$, then we have $(x, \theta) P (y, \theta')$.

Axiom 2. *Independence of Irrelevant Alternatives (IIA)*
For all $\mathbf{R} = (R_1, R_2, \cdots, R_n)$, $\mathbf{R}' = (R'_1, R'_2, \cdots, R'_n) \in \mathcal{D}$, and for all $(x, \theta), (y, \theta') \in X \times \Theta$, if

[8] Our approach on the formulation of extended preference is close to Gravel (1994) rather than Suzumura and Xu (2001). Gravel (1994)'s extended alternatives is a pair of outcome and opportunity set. He consider two binary relation, ordering over the set of outcomes and ordering over the set of extended alternatives.

$[(x,\theta)R_i(y,\theta') \Leftrightarrow (x,\theta)R'_i(y,\theta')]$ for all $i \in N$, then $[(x,\theta)R(y,\theta') \Leftrightarrow (x,\theta)R'(y,\theta')]$ holds, where $R = f(\mathbf{R})$ and $R' = f(\mathbf{R}')$.

Axiom 3. *Non-Dictatorship (ND)*
There exists no $i \in N$ such that $(x,\theta)P_i(y,\theta') \Rightarrow (x,\theta)P(y,\theta')$ for all $(x,\theta),(y,\theta') \in X \times \Theta$.

These conditions are well-known. Thus, we have nothing to explain them.

3 Welfarist and Common-sense Moralist

In this section, we consider the society where every individual is either a welfarist or a common-sense moralist. Obviously, such domain is propser subset of the general domain of common morality $\mathcal{D}_J$. For a characterization of *CM*-priori social ordering, we require that there exists at least one common-sense moralist in the society. The following result implies that a characterization of *CM*-priori social ordering is impossible if all individual are welfarists and a social ordering function satisfies *PP*.

Remark 1. Suppose that every individual is a welfarist. If a social preference ordering R where $R = f(\mathbf{R})$ is CM-priori, a social ordering function does not satisfy *PP*.

Proof. Suppose that a social ordering R where $R = f(\mathbf{R})$ is CM-priori and social ordering function satisfies *PP*. Consider the following situation: $\theta J \theta'$ and $x \succ_i y$ for all $i \in N$. CM-priori social preference ordering imposes that $\theta J \theta' \Rightarrow (y,\theta)P(x,\theta')$. Since all individuals are welfarists, $(x,\theta')P_i(y,\theta)$ for all $i \in N$. By *PP*, we have $(x,\theta')P(y,\theta)$. This would result a contradiction. Q.E.D.

In order to emphasize the importance of the existence of a common-sense moralist, we present the following result.

Proposition 1. *Suppose that every individual is either a welfarist or a common-sense moralist. There exists a social ordering function satisfying PP, IIA and ND if and only if there exists at least one common-sense moralist.*

Proof of this proposition is in the Appendix. This result is essentially based on the study of Suzumura and Xu (2004). However, note that the framework of ours is different on several points from one of Suzumura and Xu (2004). In Suzumura and Xu (2004), the ordering over opportunity sets have a similar role to common morality in our paper. However, the ordering over opportunity sets is fix in Suzumura and Xu (2004), while we allow various orderings over circumstances and all logically possible linear orderings are possible. Essentially, the preference domain of our framework is superdomain of Suzumura and Xu (2004)'s domain. This difference is essential for a characterization result.

In the rest of this paper, we focus the society which include at least one common-sense moralist. Now we begin our task of the characterization of *CM*-priori social ordering. First we consider the polar case such that all individual are common-sense moralists. As we show as follows, if all individual in society are common-sense moralists, we obtain *CM*-priori social ordering by Pareto Principle.

Lemma 1. Suppose that every individuals is a common-sense moralist. If a social ordering function satisfies *PP*, then a social preference ordering is *CM*-priori.

Proof. For each $i \in N$, for all $x, y \in X$ and all $\theta, \theta' \in \Theta$,

$$\theta J \theta' \Rightarrow (x,\theta) P_i (y,\theta') \text{ and } \theta = \theta' \Rightarrow [(x,\theta) R_i (y,\theta') \Leftrightarrow x \succsim_i y].$$

Therefore, $\theta J \theta'$ implies $(x,\theta)P_i(y,\theta')$ for all $i \in N$. Since a social ordering function satisfies *PP*, $(x,\theta)P_i(y,\theta')$ for all $i \in N$ implies $(x,\theta)P(y,\theta')$. Then, for all $(x,\theta),(y,\theta') \in X \times \Theta$, $\theta J \theta' \Rightarrow (x,\theta)P(y,\theta')$. Q.E.D.

In general, there may exist not only a common-sense moralist and but also a welfarist. For such a situation, even if a social ordering function satisfies *PP*, it is not necessary that social preference is *CM*-priori. In order to characterize *CM*-priori social ordering, we need to impose *IIA* and *ND* for social ordering function.

We now present our main result in this section.

Theorem 1. *Suppose that every individual is either a welfarist or a common-sense moralist. If there exists at least one common-sense moralist, then a social preference ordering is CM-priori if a social ordering function satisfies PP, IIA and ND.*

In order to show this theorem, we divide this proof into several steps. Let $\mathcal{W}$ denote the set of welfarists, and $\mathcal{CM}$ denote the set of common-sense moralists. By assumptions, $\mathcal{W} \cup \mathcal{CM} = N$ and $\mathcal{CM} \neq \emptyset$. For each $i \in \mathcal{W}$, for all $x, y \in X$ and all $\theta, \theta' \in \Theta$,

$$x \succ_i y \Rightarrow (x,\theta)P_i(y,\theta') \text{ and } x \sim_i y \Rightarrow [(x,\theta)P_i(y,\theta') \Leftrightarrow \theta J \theta' \text{ and } (x,\theta)I_i(y,\theta') \Leftrightarrow \theta = \theta'].$$

For each $i \in \mathcal{CM}$, for all $x, y \in X$ and all $\theta, \theta' \in \Theta$,

$$\theta J \theta' \Rightarrow (x,\theta) P_i (y,\theta') \text{ and } \theta = \theta' \Rightarrow [(x,\theta) R_i (y,\theta') \Leftrightarrow x \succsim_i y].$$

Since a social ordering function satisfies *PP*, if there exists no welfarist, lemma 2 implies that social preference ordering is *CM*-priori. Therefore, until the end of this proof, we assume that there exists a welfarist, i.e., $\mathcal{W} \neq \emptyset$.

To begin with, we show that there exists a local dictator over $\{(x,\theta)|x \in X\}$.

Step 1. For each $\theta \in \Theta$, there exists $d_\theta \in N$ such that $(x,\theta)P_i(y,\theta) \Rightarrow (x,\theta)P(y,\theta)$ for all $x, y \in X$.

Proof. Take any $\theta \in \Theta$. Consider preference profile $\mathbf{R} = (R_1, R_2, \cdots, R_n)$ restricted on $\{(x,\theta)|x \in X\}$. In assumption of preference domain, each individual preference ordering R_i corresponds to $\succsim_i$ under the restricted space $\{(x,\theta)|x \in X\}$. Since $\succsim_i$ is not restricted and $|\{(x,\theta)|x \in X\}| \geq 3$, we can apply Arrow's impossibility theorem to this subset of $X \times \Theta$: there exists $d_\theta \in N$ such that $(x,\theta)P_i(y,\theta) \Rightarrow (x,\theta)P(y,\theta)$ for all $x, y \in X$. Q.E.D.

We call individual d_θ the dictator under circumstance θ.

We want to show that a social preference ordering is *CM*-priori. According to Definition 1, formally, a social preference ordering is *CM*-priori if and only if

$$\text{for all } (x,\theta),(y,\theta') \in X \times \Theta,\ \theta J \theta' \Rightarrow (x,\theta)P(y,\theta').$$

Next, we show the following result.

Step 2. If for some profiles $\mathbf{R} \in \mathcal{D}$, there exists $(x,\theta),(y,\theta') \in X \times \Theta$ such that $\theta J \theta'$ and $(y,\theta')R(x,\theta)$ where $R = f(\mathbf{R})$, the dictator under circumstance θ is the same individual as the dictator under circumstance θ, *i.e.*, $d_\theta = d_{\theta'}$.

Proof. Suppose that $d_\theta \neq d_{\theta'}$. We consider the following preference profile:

$$\begin{aligned} \mathbf{R}|\{(x,\theta),(y,\theta')\} &= \mathbf{R}'|\{(x,\theta),(y,\theta')\} \\ (x,\theta)P'_{d_\theta}(z,\theta) &\text{ for a profile } \mathbf{R}' \\ (z,\theta')P'_{d_{\theta'}}(y,\theta') &\text{ for a profile } \mathbf{R}'. \end{aligned}$$

Note that since d_θ and $d_{\theta'}$ are distinct, the above profile is admissible. Since $\mathbf{R}|\{(x,\theta),(y,\theta')\} = \mathbf{R}'|\{(x,\theta),(y,\theta')\}$, by *IIA*, we have $(y,\theta')R'(x,\theta)$ where $R' = f(\mathbf{R}')$. Since d_θ is local dictator for the circumstance θ, $(x,\theta)P'_{d_\theta}(z,\theta)$ implies that $(x,\theta)P'(z,\theta)$. By the similar argument, we have that $(z,\theta')P'(y,\theta')$. By transitivity of R, combining with $(y,\theta')R(x,\theta)$, $(x,\theta)P'(z,\theta)$ and $(z,\theta')P'(y,\theta')$ imply that $(z,\theta')P'(z,\theta)$ where $R' = f(\mathbf{R}')$. However, since $\mathbf{R}' \in \mathcal{D}_J$, then we have that:

$$\forall i \in N : (z,\theta)P'_i(z,\theta'). \tag{1}$$

By *PP*, this implies $(z,\theta)P'(z,\theta')$. This would result a contradiction. Therefore $d_\theta = d_{\theta'}$. Q.E.D.

Step 3. If there exist distinct $\theta,\theta' \in \Theta$ such that $d_\theta \neq d_{\theta'}$, then a social preference ordering is *CM*-priori.

Proof. We use the reduction to absurdity to proof. Suppose that for $\mathbf{R} \in \mathcal{D}$, there exists $(x,\theta_1),(y,\theta_2) \in X \times \Theta$ such that $\theta_1 J \theta_2$ and $(y,\theta_2)R(x,\theta_1)$ where $R = f(\mathbf{R})$. To begin with, by the implication of Step.1, there exist a local dictator for each circumstance $\theta \in \Theta$. Furthermore, by Step. 2 and the assumption in Step. 3, $d_{\theta_1} = d_{\theta_2}$ holds.

Next we consider the following preference profile:

$$\begin{aligned} \mathbf{R}|\{(x,\theta_1),(y,\theta_2)\} &= \mathbf{R}''|\{(x,\theta_1),(y,\theta_2)\} \\ \theta_1 J''\theta^* &\text{ for a profile } \mathbf{R}'' \\ \theta^* J''\theta_2 &\text{ for a profile } \mathbf{R}'' \\ (x,\theta^*)P''_{d_{\theta^*}}(y,\theta^*) &\text{ for a profile } \mathbf{R}''. \end{aligned}$$

$J\theta_1 J''\theta^* \wedge \theta^* J''\theta_2$ is admissible. Moreover, by the implication of Step 2, since there exists $\theta^* \in \Theta$ such that $d_{\theta^*} \neq d_{\theta'}$, then the above profile is admissible.[9] If $d_{\theta_1} = d_{\theta_2} = d_{\theta^*}$, we can take a profile such that $(x,\theta^*)P''_{d_{\theta^*}}(y,\theta^*)$.[10]

Since $\mathbf{R}|\{(x,\theta_1),(y,\theta_2)\} = \mathbf{R}''|\{(x,\theta_1),(y,\theta_2)\}$, by *IIA*, we have

$$(y,\theta_2)R''(x,\theta_1) \tag{2}$$

where $R'' = f(\mathbf{R}'')$.[11] Since $R''_i \in \mathcal{D}_J$, then we have that; for all $i \in N$, $(x,\theta_1)P''_i(x,\theta^*)$. By *PP*, this implies $(x,\theta_1)P''(x,\theta^*)$. By the similar argument, we have $(y,\theta^*)P''(y,\theta_2)$. On the other hand, $d_{\theta^*} \in N$ is decisive over the pair $\{(x,\theta^*),(y,\theta^*)\}$, then $(x,\theta^*)P''_{d_{\theta^*}}(y,\theta^*) \Rightarrow (x,\theta^*)P''(y,\theta^*)$. Then we summarize as follows.

$$\begin{aligned} (x,\theta_1)P''(x,\theta^*) &\text{ where } R'' = f(\mathbf{R}'') \\ (x,\theta^*)P''(y,\theta^*) &\text{ where } R'' = f(\mathbf{R}'') \\ (y,\theta^*)P''(y,\theta_2) &\text{ where } R'' = f(\mathbf{R}''). \end{aligned}$$

By the transitivity of P, we have $(x,\theta_1)P''(y,\theta_2)$ where $R'' = f(\mathbf{R}'')$. This is incompatible with (2). This would result a contradiction. Thus, a social preference ordering is *CM*-priori. Q.E.D.

[9] The reason is that since $d_1 = d_2$, there exist another individual $d_{\theta^*} \neq d_1$.

[10] These cases are considered in Step.4 and Step.5.

[11] "Suppose that for $\mathbf{R} \in \mathcal{D}$, there exists $(x,\theta_1),(y,\theta_2) \in X \times \Theta$ such that $\theta_1 J \theta_2$ and $(y,\theta_2)R(x,\theta_1)$ where $R = f(\mathbf{R})$"

Suppose that $d := d_\theta = d_{\theta'}$ for all $\theta, \theta' \in \Theta$. In other words, there exists an individual $d \in N$ such that for all $\theta \in \Theta$, $(x,\theta)P_d(y,\theta) \Rightarrow (x,\theta)P(y,\theta)$ for all $x, y \in X$. There exists the two case: (i) $d \in \mathcal{C}$, and (ii) $d \in \mathcal{NC}$. We first consider case (i).

Step 4. If there exists an individual $d \in \mathcal{W}$ such that for all $\theta \in \Theta$, $(x,\theta)P_d(y,\theta) \Rightarrow (x,\theta)P(y,\theta)$ for all $x, y \in X$, then a social preference ordering is *CM*-priori.

Proof. We now assume that for some profiles $\mathbf{R} \in \mathcal{D}$, there exists $(x,\theta),(y,\theta') \in X \times \Theta$ such that $\theta J \theta'$ and $(y,\theta')R(x,\theta)$ where $R = f(\mathbf{R})$. Take any $z \in X$. We want to show that $(z,\theta')P_d(x,\theta) \Rightarrow (z,\theta')P(x,\theta)$. Now, we consider a profile $\mathbf{R}' \in \mathcal{D}$ such that:

$$\mathbf{R}|\{(x,\theta),(y,\theta')\} = \mathbf{R}'|\{(x,\theta),(y,\theta')\}$$
$$(z,\theta')P'_d(x,\theta) \text{ and } (z,\theta')P'_d(y,\theta') \text{ for a profile } \mathbf{R}'.$$

Note that since individual d is a welfarist, this profile $\mathbf{R}'$ is possible. By the implication of *IIA*, we have that $(y,\theta')R'(x,\theta)$ where $R' = f(\mathbf{R}')$. Since individual d is the dictator under circumstance θ', we have that $(z,\theta')P'_d(y,\theta') \Rightarrow (z,\theta')P'(y,\theta')$. By the transitivity, $(z,\theta')P'(y,\theta') \wedge (y,\theta')R'(x,\theta)$ implies that $(z,\theta')P'(x,\theta)$. And this with only $(z,\theta')P'_d(x,\theta)$, without anything being specified about the preference of the other individuals between (x,θ) and (z,θ'). Hence, by *IIA*, individual d is decisive for (z,θ') against (x,θ).

Take any $w \in X$. Next, we want to show that $(z,\theta')P_d(w,\theta) \Rightarrow (z,\theta')P(w,\theta)$. We consider a profile $\mathbf{R}'' \in \mathcal{D}$ such that:

$$(z,\theta')P''_d(x,\theta), (x,\theta)P''_d(w,\theta), \text{ and } (z,\theta')P''_d(w,\theta) \text{ for a profile } \mathbf{R}''$$

By above argument, $(z,\theta')P''_d(x,\theta) \Rightarrow (z,\theta')P''(x,\theta)$. Furthermore, since individual d is the dictator under circumstance θ, $(x,\theta)P''_d(w,\theta) \Rightarrow (x,\theta)P''(w,\theta)$. By the transitivity of R, $(z,\theta')P''(x,\theta) \wedge (x,\theta)P''(w,\theta)$ implies $(z,\theta')P''(w,\theta)$. And this with only $(z,\theta')P''_d(w,\theta)$, without anything being specified about the preference of the other individuals between (z,θ') and (w,θ). Hence, by *IIA*, individual d is decisive for (z,θ') against (w,θ).

By this argument, if for some profiles $\mathbf{R} \in \mathcal{D}$, there exists $(x,\theta),(y,\theta') \in X \times \Theta$ such that $\theta J \theta'$ and $(y,\theta')R(x,\theta)$ where $R = f(\mathbf{R})$, then for θ and θ', $(z,\theta')P_d(w,\theta) \Rightarrow (z,\theta')P(w,\theta)$ for all $z, w \in X$.

Now, we show that welfarist d is the global dictator, *i.e.*, $(a,\bar{\theta})P_d(b,\hat{\theta}) \Rightarrow (a,\bar{\theta})P(b,\hat{\theta})$ for all $(a,\bar{\theta}),(b,\hat{\theta}) \in X \times \Theta$. We consider a profile $\mathbf{R}''' \in \mathcal{D}$ such that:

$$(a,\bar{\theta})P'''_d(b,\hat{\theta}), \theta J''' \hat{\theta}, \text{ and } \bar{\theta} J''' \theta' \text{ for a profile } \mathbf{R}'''.$$

Since $R'''_i \in \mathcal{D}_J$ for all $i \in N$, we have that $(a,\bar{\theta})P'''_i(a,\theta')$ and $(b,\theta)P'''_i(b,\hat{\theta})$ for all $i \in N$. *PP* implies that $(a,\bar{\theta})P'''(a,\theta')$ and $(b,\theta)P'''(b,\hat{\theta})$. Furthermore, since then for θ and θ', $(x,\theta')P_d(y,\theta) \Rightarrow (x,\theta')P(y,\theta)$ for all $x, y \in X$, $(a,\theta')P'''_d(b,\theta) \Rightarrow (a,\theta')P'''(b,\theta)$. By the transitivity, we have that $(a,\bar{\theta})P'''(b,\hat{\theta})$. Then $d \in \mathcal{C}$ is the global dictator. This would result a contradiction. Therefore, in this case, for all $(x,\theta),(y,\theta') \in X \times \Theta$, $\theta J \theta' \Rightarrow (x,\theta)P(y,\theta')$ where $R = f(\mathbf{R})$. Q.E.D.

Subsequent to this, we consider case (ii).

Step 5. If there exists an individual $d \in \mathcal{CM}$ such that for all $\theta \in \Theta$, $(x,\theta)P_d(y,\theta) \Rightarrow (x,\theta)P(y,\theta)$ for all $x, y \in X$, then there exists no social ordering function satisfying *PP*, *IIA* and *ND*.

Proof. If $\theta J\theta' \Rightarrow (x,\theta)R(y,\theta')$ for all $(x,\theta),(y,\theta') \in X \times \Theta$, individual $d \in \mathcal{CM}$ is the global dictator. This is inconsistent with *ND*. Then, for some profiles $\mathbf{R} \in \mathcal{D}$, there exists $(x,\theta),(y,\theta') \in X \times \Theta$ such that $\theta J\theta'$ and $(y,\theta')R(x,\theta)$ where $R = f(\mathbf{R})$. We consider a profile $\mathbf{R}' \in \mathcal{D}$ such that:

$$\mathbf{R}|\{(x,\theta),(y,\theta')\} = \mathbf{R}'|\{(x,\theta),(y,\theta')\}$$
$$(x,\theta)P_d'(y,\theta),(x,\theta')P_d'(y,\theta'), \text{ and } \theta J'\theta' \text{ for a profile } \mathbf{R}'.$$

Note that since individual d is a common-sense moralist, this profile $\mathbf{R}'$ is possible. By the implication of *IIA*, we have $(y,\theta')R'(x,\theta)$ where $R' = f(\mathbf{R}')$. Moreover, since individual d is the dictator over circumstance θ', $(x,\theta')P_d'(y,\theta') \Rightarrow (x,\theta')P'(y,\theta')$. By the transitivity, $(x,\theta')P'(y,\theta')\wedge (y,\theta')R'(x,\theta)$ implies that $(x,\theta')P'(x,\theta)$. Since $(x,\theta)P_i'(x,\theta')$ for all $i \in N$, by *PP*, $(x,\theta)P'(x,\theta')$. This would result a contradiction. Q.E.D.

4 Genaral Common Morality Domain

In the analysis of previous section, individuals in the society is either a welfarist or a common-sense moralist. In this section, we assume that the preference domain of social odering function is the general domain of common morality. Under this domain, individuals in the society esteeme common morality in some sense.

Before we introduce axioms, we discuss the implication of the standard Arrovian axioms in this domain. Under the general domain of common morality, *PP* and *IIA* implies dictatorial social ordering function.

Remark 2. Suppose that preference domain is common morality domain, *i.e.,* $\mathcal{D} = D_J$. There exists no social ordering function that satisfies *PP*, *IIA* and *ND*.

One possible interplitation of this fact is that in the general domain of common morality, *IIA* and *PP* is too strong. Hnece, we introduce the modified virsion of Arrovian axioms. The following axiom is a weaker virsion of *PP*.

Axiom 4. *Conditoinal Pareto Principle (CP)*
(i) For all $(x,\theta),(x,\theta') \in X \times \Theta$, and for all $\mathbf{R} \in \mathcal{D}$, if $(x,\theta)P_i(x,\theta')$ holds for all $i \in N$, then we have $(x,\theta)P(x,\theta')$.
(ii) For all $(x,\theta),(y,\theta) \in X \times \Theta$, and for all $\mathbf{R} \in \mathcal{D}$, if $(x,\theta)P_i(y,\theta)$ holds for all $i \in N$, then we have $(x,\theta)P(y,\theta)$.

Accoding to *CP*, (i) when consequences of two alternatves are same, circomestance θ is socially moe desireble than circumstance θ' if every individual prefer circumestance θ to circumestance θ', and (ii) when circumstances of two alternatves are same, consequence x is socially moe desireble than consequence y if every individual prefer consequence x to consequence y.

The following axiom is a weaker virsion of *IIA*.

Axiom 5. *Conditoinal Independence of Irrelevant Alternatives (CIIA)*
For all $\mathbf{R} = (R_1,R_2,\cdots,R_n)$, $\mathbf{R}' = (R_1',R_2',\cdots,R_n') \in \mathcal{D}$, and for all $(x,\theta),(y,\theta') \in X \times \Theta$, if $[(x,\theta)R_i(y,\theta') \Leftrightarrow (x,\theta)R_i'(y,\theta')] \wedge [(x,\theta)R_i(x,\theta') \Leftrightarrow (x,\theta)R_i'(x,\theta')] \wedge [(x,\theta)R_i(y,\theta) \Leftrightarrow (x,\theta)R_i'(y,\theta)]$ for all $i \in N$, then $[(x,\theta)R(y,\theta') \Leftrightarrow (x,\theta)R'(y,\theta')]$ holds, where $R = f(\mathbf{R})$ and $R' = f(\mathbf{R}')$.

Consider the following two profile $\mathbf{R}, \mathbf{R}'$:

$$\mathbf{R}|\{(x,\theta_1),(y,\theta_2)\} = \mathbf{R}'|\{(x,\theta_1),(y,\theta_2)\}$$
$$\mathbf{R}|\{(x,\theta_1),(x,\theta_2)\} = \mathbf{R}'|\{(x,\theta_1),(x,\theta_2)\}$$
$$\mathbf{R}|\{(x,\theta_1),(y,\theta_1)\} \neq \mathbf{R}'|\{(x,\theta_1),(y,\theta_1)\}$$

For this two profiles, *IIA* requires that $(x,\theta_1)R(y,\theta_2) \Leftrightarrow (x,\theta_1)R(y,\theta_2)$ where $R = f(\mathbf{R})$ and $R' = f(\mathbf{R}')$. On the other hand, *CIIA* does not require this.

The following axiom require that there exists no individual who has decisive power over consequences under all circumstance. The spirit of this axiom is same as *ND*, but the requirement is stronger than *ND*. The formal statement is as follows.

Axiom 6. *Non Consequential Dictatorship (NCD)*
There exists no $i \in N$ such that for all θ, $(x,\theta)P_i(y,\theta) \Rightarrow (x,\theta)P(y,\theta)$ for all $(x,\theta),(y,\theta) \in X \times \Theta$.

The main result in this section is as follows.

Theorem 2. *Suppose that preference domain is common morality domain, i.e., $\mathcal{D} = D_J$. A social preference ordering is CM-priori if a social ordering function satisfies CP, CIIA and NCD.*

Step 1. For each $\theta \in \Theta$, there exists $d_\theta \in N$ such that $(x,\theta)P_i(y,\theta) \Rightarrow (x,\theta)P(y,\theta)$ for all $x, y \in X$.

Proof. Take any $\theta \in \Theta$. To begin with, note that *CIIA* implies the following: for all $\mathbf{R} = (R_1, R_2, \cdots, R_n)$, $\mathbf{R}' = (R_1', R_2', \cdots, R_n') \in \mathcal{D}_{\mathcal{J}}$, and for all $(x,\theta),(y,\theta) \in X \times \Theta$, if $[(x,\theta)R_i(y,\theta) \Leftrightarrow (x,\theta)R_i'(y,\theta)]$ for all $i \in N$, then $[(x,\theta)R(y,\theta) \Leftrightarrow (x,\theta)R'(y,\theta)]$ holds, where $R = f(\mathbf{R})$ and $R' = f(\mathbf{R}')$. This is the standard definition of *IIA*. That is, *IIA* holds over a restricted alternatives $\{(x,\theta) \in X \times \Theta | \theta \in \Theta\}$.

Therefore, similar to Step 1 of the proof of Theorem 1, we can apply the Arrow's impposibility theorem to this subset $\{(x,\theta) \in X \times \Theta | \theta \in \Theta\}$ of $X \times \Theta$. Hence, we prove above statement. Q.E.D.

Step 2. If for some profiles $\mathbf{R} \in \mathcal{D}_J$, there exists $(x,\theta),(y,\theta') \in X \times \Theta$ such that $\theta J \theta'$ and $(y,\theta')R(x,\theta)$ where $R = f(\mathbf{R})$, the dictator under circumstance θ is the same individual as the dictator under circumstance θ, *i.e.*, $d_\theta = d_{\theta'}$.

Proof. Suppose that $d_\theta \neq d_{\theta'}$. We consider the following preference profile:

$$\mathbf{R}|\{(x,\theta),(y,\theta')\} = \mathbf{R}'|\{(x,\theta),(y,\theta')\}$$
$$\mathbf{R}|\{(x,\theta),(x,\theta')\} = \mathbf{R}'|\{(x,\theta),(x,\theta')\}$$
$$\mathbf{R}|\{(x,\theta),(y,\theta)\} = \mathbf{R}'|\{(x,\theta),(y,\theta)\}$$
$$(x,\theta)P'_{d_\theta}(z,\theta) \text{ for a profile } \mathbf{R}'$$
$$(z,\theta')P'_{d_{\theta'}}(y,\theta') \text{ for a profile } \mathbf{R}'.$$

where x, y, z are distinct. Note that since d_θ and $d_{\theta'}$ are distinct, the above profile is admissible. Since $\mathbf{R}|\{(x,\theta),(y,\theta')\} = \mathbf{R}'|\{(x,\theta),(y,\theta')\}, \mathbf{R}|\{(x,\theta),(x,\theta')\} = \mathbf{R}'|\{(x,\theta),(x,\theta')\}, \mathbf{R}|\{(x,\theta),(y,\theta)\} = \mathbf{R}'|\{(x,\theta),(y,\theta)\}$, we have $(y,\theta')R'(x,\theta)$ where $R' = f(\mathbf{R}')$.

By similar argument of Step 2 in the proof of Theorem 1, we have a contradiction, and $d_\theta = d_{\theta'}$. Q.E.D.

Proof of Theorem. Suppose that for $\mathbf{R} \in \mathcal{D}$, there exists $(x, \theta_1), (y, \theta_2) \in X \times \Theta$ such that $\theta_1 J \theta_2$ and $(y, \theta_2) R (x, \theta_1)$ where $R = f(\mathbf{R})$. To begin with, by the implication of Step.1, there exist a local dictator for each circumstance $\theta \in \Theta$. Furthermore, by Step. 2 and the assumption in Step. 3, $d_{\theta_1} = d_{\theta_2}$ holds. By *NCD*, there exists distinct $\theta, \theta' \in \Theta$ where $d_\theta \neq d_{\theta'}$. Now, we consider the following preference profile:

$$\begin{aligned}
\mathbf{R}|\{(x,\theta_1),(y,\theta_2)\} &= \mathbf{R}''|\{(x,\theta_1),(y,\theta_2)\} \\
\mathbf{R}|\{(x,\theta_1),(x,\theta_2)\} &= \mathbf{R}''|\{(x,\theta_1),(x,\theta_2)\} \\
\mathbf{R}|\{(x,\theta_1),(y,\theta_1)\} &= \mathbf{R}''|\{(x,\theta_1),(y,\theta_1)\} \\
&\theta_1 J'' \theta^* \text{ for a profile } \mathbf{R}'' \\
&\theta^* J'' \theta_2 \text{ for a profile } \mathbf{R}'' \\
(x,\theta^*) P''_{d_{\theta^*}} &(y,\theta^*) \text{ for a profile } \mathbf{R}''.
\end{aligned}$$

$J\theta_1 J'' \theta^* \wedge \theta^* J'' \theta_2$ is admissible. Since $\mathbf{R}|\{(x,\theta),(y,\theta')\} = \mathbf{R}'|\{(x,\theta),(y,\theta')\}, \mathbf{R}|\{(x,\theta_1),(x,\theta_2)\} = \mathbf{R}'|\{(x,\theta_1),(x,\theta_2)\}, \mathbf{R}|\{(x,\theta_1),(y,\theta_1)\} = \mathbf{R}'|\{(x,\theta_1),(y,\theta_1)\}$, we have $(y,\theta_2) R' (x,\theta_1)$ where $R' = f(\mathbf{R}')$.

By similar argument of Step 3 in the proof of Theorem 1, we have a contradiction, and this complete the proof. Q.E.D.

5 Concluding Remarks

In this paper, we attempt to found non-welfaristic social ordering based on an extended Arrovian framework. Non-welfaristic social preference ordering gives unequivocal priority to non-welfaristic information, such as procedure, freedom of choice, historical consideration. Important features of our foundation are; the existence of a common-sense moralist, who is non-welfarist, and the three Arrovian axioms, *i.e.*, Pareto Principle, Independence of Irrelevant Alternatives, and Non-Dictatorship. Similar to the study of Suzumura and Xu (2004), the existence of a common-sense moralist has the important role of the "existence" of a social ordering function. Our analysis is different from the framework of Suzumura and Xu (2004) in a important point. In our framework, the admissible set of orderings over the set of circumstances is all logically possible linear orderings. On the other hand, in Suzumura and Xu (2004), the admissible set of orderings over opportunity sets is unique: the cardinality of opportunity set. This difference of formulation is crucial to obtain our characterization result. In our framework, the three Arrovian conditions deliver *CM*-priori social preference ordering.

Appendix

In this section, we give the proof of proposition 1. As noted before, this proposition is essentially based on Suzumura and Xu (2004).

Suppose that all individual preference domains are W-priori or CM-priori.

Lemma 2. If there exists at least one common-sense moralist, then there exists a social ordering function satisfying *PP*, *IIA* and *ND*.

Proof. In order to show the existence of an SOF which satisfies three conditions, we divide this proof into two steps. In step 1, we construct an SOF f. Further, in step 2, we show that the SOF f satisfies the requirements.

Step.1 By assumption, there exists $i \in N$ who is a common-sense moralist. Consider the following SOF: For all $(x,\theta),(y,\theta') \in X \times \Theta$,

$$\theta J \theta' \Rightarrow (x,\theta)P(y,\theta')$$
$$\theta = \theta' = \theta^* \Rightarrow [(x,\theta)R_l(y,\theta') \Leftrightarrow (x,\theta)R(y,\theta')]$$
$$\theta = \theta' \neq \theta^* \Rightarrow [(x,\theta)R_k(y,\theta') \Leftrightarrow (x,\theta)R(y,\theta')]$$

where $k,l \in N$ and $k \neq l$.

Step.2 By constriction, this SOF satisfies *PP*, *IIA*, and *ND*. A social preference R by this SOF is clearly reflexive and complete. Thus, we have only to show that R is transitive, *i.e.*, $\forall (x,\theta),(y,\theta'),(z,\theta'') \in X \times \Theta, (x,\theta)R(y,\theta') \wedge (y,\theta')R(z,\theta'') \Rightarrow (x,\theta)R(z,\theta'')$. This SWF imposes that $xRy \Leftrightarrow$ (a)$x_S \sim_\ell^S y_S \wedge xR_iy$ or (b)$x_S \succ_\ell^S y_S$. Then, to check transitivity, we consider the four possibilities that are mutually exclusive and exhaustive.

(1) $\theta J \theta'$ and $\theta' J \theta''$.
By transitivity of J, we have $\theta J \theta''$. This imply $(x,\theta)R(z,\theta'')$.

(2) $\theta J \theta'$ and $\theta' = \theta''$.
By transitivity of J, we have $\theta J \theta''$. This imply $(x,\theta)R(z,\theta'')$.

(3) $\theta = \theta'$ and $\theta' J \theta''$.
By transitivity of J, we have $\theta J \theta''$. This imply $(x,\theta)R(z,\theta'')$.

(4) $\theta = \theta'$ and $\theta' = \theta''$.

(4-a) $\theta = \theta' = \theta'' = \theta^*$.
In this case, $(x,\theta)R(y,\theta') \wedge (y,\theta')R(z,\theta'')$ implies that $(x,\theta)R_l(y,\theta') \wedge (y,\theta')R_l(z,\theta'')$. By transitivity of R, we have $(x,\theta)R_l(z,\theta'')$. By the feature of the SOF, $(x,\theta)R(z,\theta'')$.

(4-b) $\theta = \theta' = \theta'' \neq \theta^*$.
In this case, $(x,\theta)R(y,\theta') \wedge (y,\theta')R(z,\theta'')$ implies that $(x,\theta)R_k(y,\theta') \wedge (y,\theta')R_k(z,\theta'')$. By transitivity of R, we have $(x,\theta)R_k(z,\theta'')$.

Therefore, R is transitive.

Q.E.D.

Lemma 3. If there exists no common-sense moralist, then there exists no social ordering function satisfying *PP*, *IIA* and *ND*.

Proof. Since every individual in the society is welfarist, for all $i \in N$,

$$\forall (x,\theta),(y,\theta') \in X \times \Theta : x \succ_i y \Rightarrow (x,\theta)P_i(y,\theta') \text{ and } x \sim_i y \Rightarrow [(x,\theta)R_i(y,\theta') \Leftrightarrow \theta J \theta'] \quad (3)$$

First we prove the following claim:

(Claim) There exists $d \in N$ such that $(x,\theta)P_i(y,\theta') \Rightarrow (x,\theta)P(y,\theta')$
for all $(x,\theta),(y,\theta') \in X \times \Theta$ such as x and y are distinct.

Consider a triple $(x,\theta),(y,\theta'),(z,\theta'') \in X \times \Theta$ such as x,y,z are all distinct. Since every individual is a , then each individual's extended preference is not restricted over this triple. Since $|\{(x,\theta),(y,\theta'),(z,\theta'')\}| = 3$ and $\succsim_i$ is not restricted, Arrow's impossibility theorem is applied this triple: there exists a local dictator d over $\{(x,\theta),(y,\theta'),(z,\theta'')\}$. Now, take any extended alternatives (a,γ) and (b,γ') such as $a \neq b$ and $a,b \in X \setminus \{x,y,z\}$. Now we show that a local dictator over $\{(x,\theta),(y,\theta'),(z,\theta'')\}$ is also a local dictator over $\{(a,\gamma),(b,\gamma')\}$. Consider a triple $(a,\gamma),(y,\theta'),(z,\theta'') \in X \times \Theta$. For this triple, we can apply the Arrow's impossibility theorem: there exists a local dictator over $\{(a,\gamma),(y,\theta'),(z,\theta'')\}$. Obviously, the local dictator over

$\{(a,\gamma),(y,\theta'),(z,\theta'')\}$ is individual d. Moreover, consider a triple $(a,\gamma),(b,\gamma'),(z,\theta'') \in X \times \Theta$. Similar arguments show that individual d is a local dictator over $\{(a,\gamma),(b,\gamma'),(z,\theta'')\}$. Therefore, individual d is a dictator over pair $\{(a,\gamma),(b,\gamma')\}$. Thus, our claim is proved.

Next we show that for all $(x,\theta),(y,\theta') \in X \times \Theta$, if $x = y$, then $(x,\theta)P_d(y,\theta') \Rightarrow (x,\theta)P(y,\theta')$. Take any $(x,\theta),(y,\theta') \in X \times \Theta$ such that $x = y$. Since $D_d \in \mathcal{D}_J$, $(x,\theta)P_d(y,\theta')$ holds if and only if $\theta J \theta'$. Since all individual are consequentialist, then $x = y$ and $\theta J \theta'$ implies $(x,\theta)P_i(y,\theta')$ for all $i \in N$. By *PP*, we have $(x,\theta)P(y,\theta')$. Therefore, $(x,\theta)P_d(y,\theta') \Rightarrow (x,\theta)P(y,\theta')$.

Combining with our first claim, this implies that there exists a universal dictator over $X \times \Theta$. Then since for all $\theta \in \Theta$, $(x,\theta)P_d(y,\theta) \Rightarrow (x,\theta)P(y,\theta)$ for all $x, y \in X$, a social ordering function satisfies *NDC*. Q.E.D.

Proof of Proposition 1 The sufficiency is proved by lemma 4. The Necessity follows from lemma 5. Hence the proof is complete.

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