International Currency and Public Goods Provision

(preliminary draft)

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June 2004

Abstract

This paper studies how government uses inflation tax to finance public goods affects the circulation of a currency in a two-country search theoretic model. Agents obtain utility from private good and the public good of his own country. Each government prints fiat money to purchase goods, taxes on money holdings, and provides public goods by purchasing private goods from its fellow citizens. While government purchases increase the demand for private goods, it also induces a crowding-out effect by reducing the matching rates among private agents. We find that, a higher inflation tax rate makes a currency less likely to circulate locally and internationally. A cost and a benefit accompany a policy of lower inflation tax. The cost is that a better equilibrium associated with a higher inflation tax is eliminated. A benefit is that it also eliminates the equilibria that entail lower welfare. The negative impact of a country's inflationary policy on the realm of circulation of its currency provides an inflation discipline.

1 Introduction

Historically, government policies have been designed to influence the value and circulation of a certain object as a medium of exchange. For example, governments may designate the legal tender status to a certain currency. In some of the medieval economies the sovereign debased coins to collect seigniorage in order to finance war. The widespread use of purely fiat money is a 20th-century development. One objective of government to issue fiat money is to finance public goods. Fiat money is an efficient means of payment for the government to purchase goods, and allows it to use inflation tax to finance spending. Although it is an efficient way for the government to finance public goods, inflation tax may deteriorate the value and realm of circulation of a currency, particularly in a world with multiple currencies. Obviously, inflation tax can not be used without limit: a government can generate seigniorage tax only if the currency issued remains in circulation. On the other hand, governments may have incentive to impose a higher inflation tax when its currency circulate abroad because the tax burden falls partially on foreigners.[1]

This paper studies how government uses inflation tax to finance public goods affects the circulation of a national currency in a two-country search theoretic model. Each country consists of infinitely-lived private agents and a government. A representative agent obtains utility from private good, and the public good of his own country. Each government prints fiat money to purchase goods, taxes on money holdings, and provides public goods by purchasing private goods from its fellow citizens. While government purchases increase the demand for the private goods, it also induces a crowding-out effect by reducing the matching rates among private agents. Agents interact with home and foreign agents in different frequencies, reflecting the relative country size and the degree of international economic integration. Government in each country

[1] For example, Canzoneri (1989) uses a two-country model with cash-in-advance constraints to show that a government will opt for an inflationary bias and too much public spending if their tax falls partially on foreigners. Canzoneri and Diba (1992) argue that currency substitution could provide an inflation discipline in two-country money-in-the-utility-function model. Cooper and Kempf (2003) use a two-country overlapping generations model to show the gains from monetary union arising from reduced transactions costs and lower inflation. All the above studies are silent on the issue why one currency emerges as an international medium of exchange while another does not. Hence, it's hard to answer the question as how inflation tax affects the realm of circulation of a national currency and how this effect works as a discipline on inflationary bias.

chooses a set of policies including the rate on taxing money holdings and the amount of public goods provided. Agents choose which money to hold to conduct trade. In so doing they must take into account which money yields higher frequency of trade, and the risk of confiscation (a proxy for inflation) that each currency is subject to.

We study how inflation tax affects whether one country's currency is more likely to circulate locally and internationally, and welfare. If the degree of economic integration is sufficiently low, national currency circulates only locally; there is no international currency and no international trade. We find that if a government raises the inflation tax, its citizens are more likely to use foreign currency. Hence, under inflationary policy, staying autarchy is not the best response unless the population size is big enough to compensate the negative impacts of confiscating currency and difficulties in trade caused by the crowding-out effect of higher inflation tax.

As the economic integration is higher, a national currency may start to circulate internationally. In particular, the country with a bigger size, representing larger economies of scale in trading, is more likely to make its currency to circulate internationally. If a country adopts a sufficiently higher inflation tax than the other country does, it may induce its citizen to abandon the use of national currency and also reduces the possibility that its currency circulate abroad. The reason is that, higher government's purchases increase the probability for domestic sellers to sell goods, but for all agents the meeting rates with fellow and foreign citizens are reduced due to the crowding-out effects. A higher home inflation tax not only deters home agents' incentive to accept national currency, but also reduces foreign agents' willingness to accept it. As the degree of economic integration is higher, the 'imported' impact due to inflation tax and crowding-out effect would be stronger.

However, a higher inflation tax does not necessarily preclude a currency from circulating abroad. We find from numerical experiments that, when supply of both currencies is moderate, if the smaller country adopts a higher inflation tax, the equilibrium where its currency circulates home and abroad is more likely to exist than the one in which the currency circulates only locally. Higher domestic inflation tax increases agents' incentive to abandon national currency; however, this effect will be lower if the currency circulates abroad also, because there are more home sellers and so the trade opportunity for the fellow citizens increases. On the other hand, if the currency circulates only locally, the tax burden falls completely on local citizens and so they may stop using the national currency. However, if currency supply is much lower, higher domestic inflation makes the national currency less likely to circulate internationally. Because of the currency shortage at home, agents of smaller country have higher incentive to accept other country's currency.

We then consider a policy game in which the two governments choose tax rates on their respective currencies, measuring the payoff of each government by the utility of its own representative agent. Numerical examples show, among others, that the equilibrium tax rate of a currency is higher when it becomes an international currency than otherwise. Welfare may be improved for the issuing country of the international currency when its currency supply is not too low, for that circulation of a currency abroad may create currency shortage at home. We also find that a government can implement a policy to interfere the existence of equilibrium so that its fellow citizens enjoy higher welfare (e.g., adopting a lower inflation tax to induce its currency circulate internationally). However, a cost and a benefit accompany such a policy. The cost of adopting a lower inflation tax is that a better equilibrium associated with a higher inflation tax (which generates higher seigniorage tax to provide public goods) is eliminated. A benefit is that it also eliminates the equilibria that entail lower welfare.[2]

2 The Basic Model

The Environment

Time is discrete and the horizon is infinite. There is a $[0, 1]$ continuum of infinitely-lived agents with unit mass. The agents are divided into two regions, Home and Foreign. Let $n \in (0, 1)$ be the size of Home population. There are k ($k \geq 3$) types of indivisible goods. Within each economy, there are equal proportions of k types of agents, who specialize in consumption, production and storage. A type i agent derives utility only from consuming type i good and can produce only good $i + 1$ (mod k). Agent i can only store his production good costlessly

[2]Previous studies on how trade frictions and government policy influence the circulation and value of a medium of exchange include the following. Aiyagari and Wallace (1997) and Li and Wright (1998) consider government transaction policy regarding which money and at which price they accept in a trade. Soller Curtis and Waller (2000) study how government punishment and enforcement policies affect the circulations and prices of legal and illegal fiat currencies. The most related paper is Li (1995) where buyers choose search intensity, and higher search intensity incurs higher disutility but increases trading frequency. Government taxing fiat money holding increases the risk (cost) of holding money and so induces higher level of search intensity, which may be welfare improving.

up to one unit; he can neither produce nor store other types of goods. Hence, there is no double coincidence of wants. Let $u > 0$ be the instantaneous utility from consuming an agent's consumption good and δ his discount rate.

There are two distinguishable fiat money, Home currency and Foreign currency. Each currency is indivisible. An agent can store only one unit of good or one unit of currency at a time. Let m_h (m_f) denote the fraction of Home agents holding the Home (Foreign) currency. The inventory distribution of Home agents can be summarized by $X = (1 - m_h - m_f, m_h, m_f)$. Likewise, let m_h^* (m_f^*) denote the fraction of Foreign agents holding the Home (Foreign) currency. The inventory distribution of Foreign agents can be summarized by $X^* = (1 - m_h^* - m_f^*, m_h^*, m_f^*)$. Let m and $m^* \in (0, 1)$ denote the supply of the Home currency per Home agent and that of Foreign currency per Foreign agent, respectively. Then,

$$nm = nm_h + (1 - n)m_h^*, \ (1 - n)m^* = nm_f + (1 - n)m_f^*.$$

Agents are matched randomly in pairs. Let $\beta \in (0, 1)$. Without government policy, a Home agent meets a Home agent with probability n, and meets a Foreign agent with probability $\beta(1 - n)$. A Foreign agent meets a Home and Foreign agent with probability βn and $(1 - n)$, respectively. Thus, agents who live in different countries meet less frequently than a pair of agents who live in the same country. Note that the above description implies the probability of meeting a trade partner also depends on the size of country. We can interpret β as the degree of economic integration or, a measure of the trading frictions in international trade, and the country size as a measure of the trading frictions in local trade. For example, an increase in β reduces international trade frictions, because higher β makes it easier to meet with Foreign agents. Similarly, an increase in n reduces the local trade frictions in Home country because higher n makes it easier for the Home agents to meet with their fellow citizens. However, an increase in n reduces the relative size of Foreign country and so increases the local trade frictions in Foreign country.

Trade entails a one-for-one swap of inventories, and takes place if and only if both agents agree to trade. The trade partner's type and inventory are observable, trade histories are not. Agents are unable to commit to future actions, and proposed transfers cannot be enforced. Thus, people trade when there is single coincidence of wants, and all trades involve the use of a tangible medium of exchange.

The role of government in the provision of public goods

In each country there is a government whose role is to print fiat money, tax money holdings and provide public goods from the private goods that it purchases. An agent who holds Home (Foreign) currency is subject to a probability τ_h (τ_f) that his money would be confiscated by the government of Home (Foreign) country. The rate τ_h (τ_f) can be interpreted as tax rate that a government imposes on money holdings in order to provide public goods. We can also interpret τ_h (τ_f) as inflationary tax.

When a government purchases goods from private agents, it prints one unit of fiat money to the producer. We assume that a government buys goods only from its fellow citizens. We assume that it is not feasible that the government pays for its purchase with private goods (i.e., no barter trade between government and private agents). Whether or not an agent accepts a particular currency is part of his strategy, and he will obtain neither extra reward nor punishment for his decision. Let $n\gamma_h$ (resp. $(1-n)\gamma_f$) denote the arrival rate to each agent by Home (resp. Foreign) government which proposes to purchase his commodity. Thus, with government policy the arrival rate to a Home agent of Home (resp. Foreign) agents is $n(1-\gamma_h)$ (resp. $\beta(1-n)(1-\gamma_f)$). A Foreign agent meets a Home (resp. Foreign) agent with probability $\beta n(1-\gamma_h)$ (resp. $(1-n)(1-\gamma_f)$).[3] This implies that government's involvement in the economy creates a crowding-out effect, in that government purchases reduce the meeting probability and trade among private agents.

A government transforms the private goods it purchases into public goods from which every private agent in the country enjoys the utility of $\phi(g)$ where g is the total quantity of private goods purchased by the government in a unit of time. We assume $\phi(0) = 0$, $\phi'(g) \to \infty$ as $g \to 0, \phi'(g) > 0$ and $\phi''(g) < 0$. Public goods are nonstorable (e.g., army service). We may assume that Home government and Foreign government have different efficiency in providing public goods. For example, assume that the quantity of public goods g is a fraction θ of total consumption goods purchased by the government, and both countries may have different θ's. We may also assume that Home and Foreign agents have different preferences for public goods.

Strategies and equilibrium

[3] Li and Wright (1998) assumes the size of government and private agents is γ and $1-\gamma$, respectively, and the arrival rate of each type of agents is proportional to the size.

An agent chooses trade strategies to maximize his expected discounted utility, taking as given others' strategies and the distribution of inventories. We restrict our attention to pure strategies which only depend on his nationality and the objects he and his trading partner have in inventory. Thus, the Home agent's trade strategy can be described as

$$s_{ab} = \begin{cases} 1 \text{ if he trades object } a \text{ for } b \\ 0 \text{ otherwise,} \end{cases}$$

where $a, b = g, h$, or f, and $a \neq b$. Similarly, the Foreign agent's trade strategy is given by $s^*_{ab} = 0$ or 1. We consider only time-independent strategies. Given that the physical environment is stationary and the planning horizon is infinite, we can therefore confine our attention to steady-state equilibrium.

Let V_g, V_h and V_f denote the expected discounted utility to a Home agent holding his production good, the Home currency, and Foreign currency, respectively. Let P_{ab} (P^*_{ab}) denote the transition probability with which a Home (Foreign) agent switches his inventory from object a to object b. Then, the Bellman's equations are

$$V_g = [(1 - P_{gh} - P_{gf})V_g + P_{gh}V_h + P_{gf}V_f]/(1 + \delta) \tag{1}$$

$$V_h = [\tau_h V_g + P_{hg}(u + V_g) + (1 - P_{hg} - P_{hf} - \tau_h)V_h + P_{hf}V_f]/(1 + \delta) \tag{2}$$

$$V_f = [\tau_f V_g + P_{fg}(u + V_g) + P_{fh}V_h + (1 - P_{fg} - P_{fh} - \tau_f)V_f]/(1 + \delta). \tag{3}$$

Note that the first terms in the RHS of equality in (2) and (3) imply that, if an agent's currency is confiscated by the issuing government (with probability τ_h and τ_f that his money is confiscated by Home and Foreign government, respectively), his value becomes that of holding production good. The value functions and strategies must satisfy the following incentive compatibility constraints:

$$s_{gb} = 1 \text{ iff } V_g < V_b \ (b = h \text{ or } f)$$

$$s_{ag} = 1 \text{ iff } V_a < u + V_g \ (a = h \text{ or } f)$$

$$s_{ab} = 1 \text{ iff } V_a < V_b \ (a, b = h \text{ or } f).$$

For example, $V_g > V_f$ is the sufficient and necessary condition for a Home agent not to trade his production good for Foreign currency.

We restrict our attention to the equilibrium where agents always accept their local currency; i.e., Home currency is accepted by the Home agents and Foreign currency is accepted by the Foreign agents. We are left four types of equilibria – no international currency, Foreign currency is the only international currency, Home currency is the only international currency and both currencies circulate in both countries. We characterize the existence conditions in terms of β and n, the extent of international and local trade frictions.

First of all, in any of these equilibria, we have $P_{fh} = P_{hf} = P^*_{fh} = P^*_{hf} = 0$. Given the tie-breaking rule [assume somewhere!], no two agents in the same country exchange Home currency and Foreign currency; indeed, for currency exchange to occur between two, say, Home agents, we need $s_{hf} = s_{fh} = 1$, which implies $V_f > V_h$ and $V_h > V_f$, a contradiction. Therefore, the only possibility for currency exchange is between agents from different countries. Due to the nature of equilibrium, this may happen only when both currencies circulate worldwide. In this case, we need to have, say, $V_h > V_f$ and $V^*_f > V^*_h$ (the opposite case has a similar consequence). If $\tau_h = \tau_f$ holds, then the two currencies are perfect substitutes, and therefore, $V_h = V_f$ and $V^*_f = V^*_h$, which is a contradiction. But, if, say, τ_h becomes smaller (resp. greater) than τ_f, then Home currency is more (resp. less) attractive for both Home and Foreign agents than Foreign currency. Thus, both Home and Foreign agents have the same incentive concerning the acceptance of currency, and therefore, there is no room for currency exchange.

Before conducting equilibrium analysis, let us calculate the value functions from (1), (2) and (3):

$$V_g = [(\delta + P_{fg} + \tau_f)P_{gh}P_{hg} + (\delta + P_{hg} + \tau_h)P_{gf}P_{fg}]\, u/P, \quad (4)$$

$$V_h = [P_{hg}(\delta + P_{gh})(\delta + P_{fg} + \tau_f) + P_{hg}P_{gf}(\delta + P_{fg}) + P_{gf}P_{fg}\tau_h)]\, u/P, \quad (5)$$

$$V_f = [P_{fg}(\delta + P_{gf})(\delta + P_{hg} + \tau_h) + P_{fg}P_{gh}(\delta + P_{hg}) + P_{gh}P_{hg}\tau_f)]\, u/P, \quad (6)$$

where

$$P = \delta\, [(\delta + P_{gh} + P_{hg} + \tau_h)(\delta + P_{fg} + \tau_f) + P_{gf}(\delta + P_{hg} + \tau_h)]\, .$$

Using the above value functions, we are able to state some general results.

Proposition 2.1. *1.* $u + V_g > V_g, V_h, V_f$.

2. $\max\{V_h, V_f\} > V_g$.

3. $V_h > (<) V_g$ *iff* $P_{hg}(\delta + P_{fg} + P_{gf} + \tau_f) > (<) P_{gf}P_{fg}$.

4. $V_f > (<) V_g$ *iff* $P_{fg}(\delta + P_{hg} + P_{gh} + \tau_h) > (<) P_{gh}P_{hg}$.

3 Equilibrium

3.1 Equilibrium with two local currencies: Equilibrium A

In this equilibrium a Home agent trades his production good for the Home currency, the Home currency for his consumption good, but does not accept Foreign currency ($u + V_g > V_h > V_g \geq V_f$). A Foreign agent trades his production good for the Foreign currency, the Foreign currency for his consumption good, but does not accept the Home currency ($u + V_g^* > V_f^* > V_g^* \geq V_h^*$). There is no international currency and no international trade in this equilibrium. The inventory distributions are given by $X = (1-m, m, 0)$ and $X^* = (1-m^*, m^*, 0)$. The transition probabilities in this equilibrium for a Home agent are:

$$\begin{aligned} P_{gh} &= n(1-\gamma_h)m/k + n\gamma_h, & P_{hg} &= n(1-\gamma_h)(1-m)/k \\ P_{fg} &= \beta(1-n)(1-\gamma_f)(1-m^*)/k, & P_{gf} &= P_{hf} = P_{fh} = 0. \end{aligned} \tag{7}$$

Note that P_{gh} incorporates the opportunity to sell goods to acquire money from private agents and government with probability $n(1-\gamma_h)$ and $n\gamma_h$, respectively. If a Home agent ever holds Foreign currency, then given others' strategies the chance that he can acquire consumption goods is from trading with Foreign sellers, of which the probability is $\beta(1-n)(1-\gamma_f)(1-m^*)/k$. Similarly, the transition probabilities for a Foreign agent are:

$$\begin{aligned} P_{gh}^* &= (1-n)(1-\gamma_f)m^*/k + (1-n)\gamma_f, & P_{fg}^* &= (1-n)(1-\gamma_f)(1-m^*)/k \\ P_{hg}^* &= \beta n(1-\gamma_h)(1-m)/k, & P_{gh}^* &= P_{fh}^* = P_{hf}^* = 0. \end{aligned} \tag{8}$$

Given the inventory distributions the balanced budget constraints for Home and Foreign country satisfy

$$\begin{aligned} n\gamma_h(1-m) &= \tau_h m, \\ (1-n)\gamma_f(1-m^*) &= \tau_f m^*, \end{aligned} \tag{9}$$

respectively. Thus, given the tax rate (τ_h, τ_f), the purchase of private goods from governments, (γ_h, γ_f), must satisfy (9). This also implies that the supply of Home and Foreign currency per capita does not change over time.

To find the existence conditions for Equilibrium A, we verify the incentive constraints $u + V_g > V_h > V_g > V_f$ and $u + V_g^* > V_f^* > V_g^* > V_h^*$.

From Proposition 2.1, $V_g \geq V_f$ and $V_g^* \geq V_h^*$ imply other inequalities. We have $V_g \geq V_f$ (Home agents do not accept Foreign currency) iff $\beta \leq \beta_A$, where

$$\beta_A = \frac{m[n(1-m) - m\tau_h][n(1-m) - m\tau_h + k\tau_h]}{[(1-n)(1-m^*) - m^*\tau_f][(1-m)(n+\delta k) - m\tau_h + k\tau_h]}.$$

Likewise, Foreign agents do not accept Home currency, or $V_g^* \geq V_h^*$, iff $\beta \leq \beta_A^*$, where

$$\beta_A^* = \frac{m^*[(1-n)(1-m^*) - m^*\tau_f][(1-n)(1-m^*) - m^*\tau_f + k\tau_f]}{[n(1-m) - m\tau_h][(1-m^*)(1-n+\delta k) - m^*\tau_f + k\tau_f)]}.$$

The above expressions are cumbersome, and the effect of an increase in, say, τ_f is ambiguous. Instead of analyzing the case for which intuition does not work well, we focus on the case where agents are sufficiently patient relative to matching frequency, i.e., we study the limiting situation where δ goes to zero. Taking the limit, we obtain

$$\lim_{\delta \to 0} \beta_A = \frac{m[n(1-m) - m\tau_h]}{(1-n)(1-m^*) - m^*\tau_f} \tag{10}$$

$$\lim_{\delta \to 0} \beta_A^* = \frac{m^*[(1-n)(1-m^*) - m^*\tau_f]}{n(1-m) - m\tau_h} \tag{11}$$

Given parameter values of m, m^*, k, τ_h, and τ_f, $\beta \leq \beta_A$, $\beta \leq \beta_A^*$ give the existence conditions of equilibrium A on (n, β) space, shown in Figures 1-a, 1-b.[4] To see how a policy pair (τ_h, τ_f) affects the existence region of equilibrium, note that given other parameters, an increase in τ_h leads to a decrease in β_A, while an increase in τ_f leads to an increase in β_A. Likewise, an increase in τ_h leads to an increase in β_A^*, while an increase in τ_f leads to a decrease in β_A^*. If we interpret (τ_h, τ_f) as a proxy for the rate of inflation, then this change is intuitive, i.e., the higher (resp. lower) the rate of inflation of Home (resp. Foreign) currency is, the more likely Home agents are to use Foreign currency.

Given a policy pair (τ_h, τ_f), if the degree of economic integration is sufficiently small, national currency circulates only locally; there is no international currency and no international trade. Other things being equal, this equilibrium does not survive if the country size is uneven. If n is sufficiently large, the trade with Home agents is so easy that Foreign agents would have incentives

[4] The parameters are $m = m^* = .2, k = 10$, $\tau_h = \tau_f = .1$.

to use Home currency. For a given n, Equilibrium A does not exist when β is sufficiently high, either. The higher the degree of economic integration becomes, the easier trade with foreigners, and the higher the incentive to accept foreign currency becomes.

The change in the existence region of equilibrium compared to the case with $(\tau_h, \tau_f) = (0, 0)$ depends on the relative magnitudes of τ_h and τ_f.[5] If $\tau_h > \tau_f > 0$, β_A curve shifts downward while β_A^* rotates clockwise (see Figure 1-b).[6] Similarly, if $\tau_f > \tau_h > 0$, β_A^* curve shifts downward while β_A rotates anti-clockwise. A higher Home inflation rate induces a higher incentive for Home agents to accept Foreign currency. The downward shift of β_A implies that under inflationary policy, staying autarchy is not the best response unless the population size of the country is big enough to compensate the negative impacts of confiscating currency (a proxy for inflation). Thus, for a given pair of (n, β), if a country adopts too high an inflation tax rate, it may destroy the equilibrium with two currency areas. To see the move of β_A^* when $\tau_h > \tau_f > 0$, notice that, though government's purchases increase the probability for private agents to sell goods, it also reduces the meeting rates among private agents. This crowding-out effect works like trade frictions. As the degree of economic integration is higher, the 'imported' impact due to inflation tax and crowding-out effect would be stronger. Given a high β, the crowding-out effect is high so not accepting Home currency is best response for Foreign agents even when n is larger than without policy. However, when β is low, the effect of τ_f dominates the 'imported' effect by τ_h and so the strategy of staying autarky survives at smaller n than without policy.

3.2 Equilibrium with one local currency and one international currency: Equilibrium F and H

We discuss the existence conditions for Equilibrium F, where Home currency is accepted only in Home country, while Foreign currency circulates in both Home and Foreign country as an international medium of exchange. Equilibrium H is the mirror image of Equilibrium F and can be characterized in a similar manner.

Equilibrium F requires $u + V_g > V_h, V_f > V_g$ and $u + V_g^* > V_f^* > V_g^* \geq V_h^*$. When agents follow these strategies, $m_h = m$ and so $X = (1 - m - m_f, m, m_f)$ and $X^* = (1 - m_f^*, 0, m_f^*)$.

[5]Notice that here we are considering changes in τ_h and τ_f at a time, which is different from the above comparative statics analysis.

[6]If the difference between both tax rates is larger, β_A^* rotates at a lower β.

The steady state requires that the ratios of commodity holders to the Foreign currency holders in the two countries be equalized, i.e.,

$$\frac{m_f^*}{1 - m_f^*} = \frac{m_f}{1 - m - m_f}.$$

From the steady state condition, $m_f = (1 - m)m_f^*$. Therefore we can rewrite the inventory distributions in terms of m_f^* as $X = ((1-m)(1-m_f^*), m, (1-m)m_f^*)$ and $X^* = (1-m_f^*, 0, m_f^*)$. The total supply of Foreign currency must equal the total amount circulates in both countries

$$(1 - n)m^* = n(1 - m)m_f^* + (1 - n)m_f^* = (1 - nm)m_f^* \tag{12}$$

The transition probabilities for a Home agent are

$$\begin{aligned} P_{gh} &= n(1 - \gamma_h)m/k + n\gamma_h \\ P_{gf} &= [n(1 - \gamma_h)(1 - m) + \beta(1 - n)(1 - \gamma_f)]m_f^*/k \\ P_{hg} &= n(1 - \gamma_h)(1 - m)(1 - m_f^*)/k \\ P_{fg} &= [n(1 - \gamma_h)(1 - m) + \beta(1 - n)(1 - \gamma_f)](1 - m_f^*)/k \\ P_{hf} &= P_{fh} = 0 \end{aligned} \tag{13}$$

and for a Foreign agent

$$\begin{aligned} P_{gf}^* &= [\beta n(1 - \gamma_h)(1 - m) + (1 - n)(1 - \gamma_f)]m_f^*/k + (1 - n)\gamma_f \\ P_{hg}^* &= \beta n(1 - \gamma_h)(1 - m)(1 - m_f^*)/k \\ P_{fg}^* &= [\beta n(1 - \gamma_h)(1 - m) + (1 - n)(1 - \gamma_f)](1 - m_f^*)/k \\ P_{gh}^* &= P_{fh}^* = P_{hf}^* = 0 \end{aligned} \tag{14}$$

where m_f^* satisfies (12). Given the inventory distributions the balanced budget constraints for Home and Foreign country are

$$\begin{aligned} n\gamma_h(1 - m)(1 - m_f^*) &= \tau_h m, \\ (1 - n)\gamma_f(1 - m_f^*) &= \tau_f m^*, \end{aligned} \tag{15}$$

respectively.

In the sequel, we take the limit of δ going to zero in making comparative statics and other characterizations, which implies that matching frequencies are sufficiently high relative to time preference. From Proposition 2.1, it suffices to check that Home agents accept Home currency ($V_g < V_h$), and that Foreign agents do not accept Home currency ($V_g^* \geq V_h^*$).

First, substituting (13) into the third and forth claims of Proposition 2.1, and taking the limit of δ going to zero, we have $V_h > V_g$ iff

$$f(n,\beta) = \beta_F > 0 \tag{16}$$

and $V_g^* \geq V_h^*$ iff

$$f^*(n,\beta) = \beta_F^* \leq 0 \tag{17}$$

where $f(n,\beta)$ and $f^*(n,\beta)$ are defined in the Appendix. Equilibrium F exists if and only if the two incentive constraints hold, given (13), (14) and (15). We depict the equilibrium region defined by (16) and (17) on the space of (n,β) in Figures 2-a, 2-b.

Equations (16) and (17) are too complicated for us to sign the comparative statics in general. However, we are able to do it if we evaluate the derivatives near $(\tau_h,\tau_f) = (0,0)$. We prove that $\frac{\partial\beta_F}{\partial\tau_h}|_{\tau_h=0,\tau_f=0} < 0$, $\frac{\partial\beta_F}{\partial\tau_f}|_{\tau_h=0,\tau_f=0} > 0$, $\frac{\partial\beta_F^*}{\partial\tau_h}|_{\tau_h=0,\tau_f=0} > 0$ and $\frac{\partial\beta_F^*}{\partial\tau_f}|_{\tau_h=0,\tau_f=0} > 0$ if k is not too small.[7] Therefore, as τ_h (resp. τ_f) increases from zero, $\beta = \beta_F$ shifts downward (resp. upward), while $\beta = \beta_F^*$ shifts upward (resp. upward). As τ_h increases from zero it requires a larger n (or higher β) for Home agents to accept Home currency. Similarly, Foreign agents' not-accepting-Home-currency strategy survives even at a larger n (or higher β). An increase in τ_f from the policy pair $(0,0)$ makes Home agents less likely to abandon its national currency, while Foreign agents less likely to accept Home currency. The latter result may be somewhat surprising; however, recall that higher inflation tax also increases government's purchases, which increases the probability for domestic sellers to sell goods. As the benefit compensates the cost of confiscation of currency, an increase in τ_f could make the equilibrium strategy survive at a larger n.

We now consider changes in τ_h and τ_f simultaneously at one time. Note first, when $\tau_h > 0$ and $\tau_f > 0$, β_F^* rotates clockwise, similar to the case in equilibrium A. However, the shift of β_F depends on the relative magnitudes of τ_h and τ_f. If τ_h is sufficiently larger than τ_f, β_F shifts downward and so for accepting Home currency to remain the best response it requires a higher Home country size than without policy. For other cases, β_F rotates anti-clockwise, implying the 'imported' effect from τ_f has different impact at different degree of economic integration. That is, when β is high (resp. low) accepting Home currency is the best response at a smaller (resp.

[7] $\frac{\partial\beta_F^*}{\partial\tau_f}|_{\tau_h=0,\tau_f=0} > 0$ iff $k > \overline{k} = \frac{m^*(1-n)(1-nm)}{n(1-m)(1-m^*-nm+nm^*)}$, and $\frac{\partial\overline{k}}{\partial n} < 0$, $\frac{\partial\overline{k}}{\partial m} > 0$, $\frac{\partial\overline{k}}{\partial m^*} > 0$.

larger) n than without policy. When β is high, the 'imported' crowding-out effect is so strong to offset the trading frictions caused by inflation tax τ_h that Home agents would have incentive to use Home currency even at a smaller n than without policy.

3.3 Equilibrium with the unified currency

In this equilibrium, two currencies are unified and become perfect substitutes. That is, both Home and Foreign currency circulate in both countries, $u + V_g > V_h, V_f > V_g$, and $u + V_g^* > V_f^*, V_h^* > V_g^*$. When agents follow these strategies, $X = X^*$, and $m_h = m_h^* = nm$, and the steady state that the ratios of commodity holders to the Home currency holders in the two countries are equalized gives $m_h^* = (1-m^*)m_h$. The inventory distributions are given by $X = (1-m_h, m_h, 0)$ and $m_f = m_f^* = (1-n)m^*$. The transition probabilities are

$$\begin{aligned}
P_{gh} &= nm[n(1-\gamma_h) + \beta(1-n)(1-\gamma_f)]/k + n\gamma_h \\
P_{gf} &= [n(1-\gamma_h) + \beta(1-n)(1-\gamma_f)](1-n)m^*/k \\
P_{hg} &= P_{fg} = [n(1-\gamma_h) + \beta(1-n)(1-\gamma_f)][1-nm-(1-n)m^*]/k \\
P_{gh}^* &= nm[\beta n(1-\gamma_h) + (1-n)(1-\gamma_f)]/k + (1-n)\gamma_f \\
P_{gf}^* &= [\beta n(1-\gamma_h) + (1-n)(1-\gamma_f)](1-n)m^*/k \\
P_{hg}^* &= P_{fg}^* = [\beta n(1-\gamma_h) + (1-n)(1-\gamma_f)][1-nm-(1-n)m^*]/k \\
P_{hf} &= P_{fh} = P_{fh}^* = P_{hf}^* = 0.
\end{aligned} \tag{18}$$

We find that, given any $\tau_h > 0$ and $\tau_f > 0$, the above conditions hold for all parameters, and so we conclude that this equilibrium exists for any $(n, \beta) \in (0,1)^2$.

4 Inflationary policy and coexistence of equilibria

We depict the coexistence of equilibria without policy and with policy on the space of (n, β) in Figures 3-a and 3-b, respectively. For the following discussions, we ignore equilibrium U for a moment and focus on equilibria A, F and H. When $\tau_h = \tau_f = 0$, the types of multiple equilibria include the coexistence of equilibria A, F and H, equilibria A and H, and equilibria A and F. Under policy (τ_h, τ_f), the feature of coexistence of equilibria is changed. For a large number of numerical examples we tried, the type of multiple equilibria include only the coexistence of equilibria A, F and H, and equilibria F and H. The inflationary policy suppresses the coexistence of equilibrium A with other types of equilibria, and further creates new type of coexistence (e.g., only equilibria F and H coexist).

We observe from Figure 3-b that the new feature of coexistence is due to the shift of β_F^* (resp. β_H^*) curve in equilibrium F (resp. H), which represents the incentive constraints that the citizens of the issuing country of international currency do not accept the other country's currency. Take equilibrium F as an example. The 'imported' impact of inflation tax and crowding-out effect is stronger in an equilibrium with international interaction than in one without. To have β_F^* curve lies above β_A^* curve, we need $V_g^* \geq V_h^*$ to hold for higher β in equilibrium F, but is violated in equilibrium A. That is, we need to show that inflationary policy creates higher trade friction in equilibrium F than in equilibrium A. In equilibrium F, Foreign sellers meet Home buyers holding Foreign currency with arrival rate less than β, due to the crowding-out effect. If a Foreign agent ever held Home currency, he would be subject to confiscation. These two effect is stronger when both countries are more integrated. Thus $V_g^* \geq V_h^*$ may still hold for higher β under which it does not in equilibrium A.

To have more intuition on the effect of inflationary policy on the coexistence of equilibria, we depict multiple equilibria on the space of (τ_h, τ_f) in Figures 4-a and 4-b, for a given pair of (n, β).[8] In the example of Figure 4-a, Home country is the bigger country and the existence of equilibria depends on the magnitude of τ_f. For a given τ_h, equilibria A, F and H coexist when τ_f is low. As τ_f increases, Foreign agents are more likely to accept Home currency and destroy equilibrium A. If τ_h is low, an increase in τ_f may induce Foreign agents to accept Home currency and eliminates equilibrium F. If τ_h is high, a further increase in τ_f makes Foreign agents stop using Foreign currency and eliminates equilibrium H instead, leaving equilibrium F as the only equilibrium.

One may puzzle why equilibrium F where Foreign currency is the international currency, can survive at a higher inflation tax τ_f than equilibrium H? The reason lies in different steady state inventory distributions cross equilibria, which affect the frequency of trade and thus the incentive to accept a particular currency as a medium of exchange. A switch from equilibrium H to F increases the fraction of commodity holders of Foreign agents, but reduces that of Home

[8] The parameter values in the example of Figure 4-a are $m = m^* = .3$, $\beta = .1, \delta = .01, k = 10, n = .7$, for Figure 4-b are $m = m^* = .2$. The examples we present presume Home country as the bigger country. Similar results hold if Foreign country is the bigger country.

agents. Under equilibrium F, if Home agents wish to trade with Foreign sellers, they must hold Foreign currency. Given that the proportion of Foreign sellers is large, which implies trade is very easy, all agents are willing to bear the cost of high inflation to use Foreign currency. On the other hand, in equilibrium H, the fraction of Foreign sellers is smaller than at in equilibrium F. Given that holding Foreign currency can only trade with Foreign sellers, the incentive to give up using Foreign currency may be strong if Foreign inflation tax is high, and this eliminates the existence of equilibrium H.

However, if the quantity of currency m and $m^*$are smaller, we may find that equilibria A and F exist when τ_h is relatively big. In the example of Figure 4-b, Home country is the bigger country and equilibrium H exists for all values of τ_h and τ_f. For a given τ_f, if τ_h is too low, Foreign agents may have incentive to accept Home currency. Given τ_h, if τ_f is too high, Foreign agents may start to abandon Foreign currency. Compare Figures 4-a and 4-b we find that, lower currency supply suppresses the existence region of equilibria A and F, and enlarges the region for equilibrium H. That is, for a given policy (τ_h, τ_f), if currency supply is lower, it's less likely for the agents of smaller country not to accept other country's currency. In this example, equilibria A and F are more likely to exist when τ_h is high. Higher Home inflation makes it more likely the existence of equilibria where Home currency is not the international money.[9]

5 Policy and Welfare

We use the long-run expected utility of a representative agent as the welfare criterion. Let W (W^*) denote the welfare criterion for Home (Foreign) country, where

$$
\begin{aligned}
W &= (1 - m_h - m_f)V_g + m_h V_h + m_f V_f + [\theta n \gamma_h (1 - m_h - m_f)]^{1/\alpha} \\
W^* &= (1 - m_h^* - m_f^*)V_g^* + m_h^* V_h^* + m_f^* V_f^* + [\theta^*(1-n)\gamma_f(1 - m_h^* - m_f^*)]^{1/\alpha^*}.
\end{aligned}
$$

Let $\overline{W}$ denote total welfare, the weighted average of welfare of both countries by using the population size as the weight. Notice that the welfare criteria incorporate the expected utility of consuming public goods. The total quantity of public goods provided in Home (Foreign) country

[9] A switch from equilibrium H to equilibrium F increases the fraction of Foreign commodity holders and reduces that of Home commodity holders. When currency supply is low, the effect of the changes is smaller than when currency supply is high.

is equal to the transformation parameter θ (θ^*) multiplied by the total private goods purchased. The Home and Foreign agents' preference for public goods is determined by the parameter α and α^*, respectively. We first assume $\theta = \theta^* = 1$ and $\alpha = \alpha^* = 2$ in the following examples.

We discuss welfare issues from the observations of numerical experiments. In the following discussions we consider both countries adopt the tax rate that each government chooses independently to maximize welfare W and W^*, respectively. We find that, given (m, m^*), W and W^* are highest in equilibrium U. All agents prefer a fully integrated equilibrium because it results in highest frequency of trade. For most examples Home agents prefer equilibrium H to A to F, Foreign agents prefer equilibrium F to A to H, but the total welfare $\overline{W}$ is lowest in equilibrium A.[10] We do, however, find exceptional cases. For example, if Home country is the bigger country and issues less currency than Foreign country ($m < m^*$), the total welfare $\overline{W}$ is lowest in equilibrium H. This implies that, though a switch from equilibrium A to an equilibrium with international currency improves trade opportunity, the welfare gain of the issuing country may not be big enough to compensate the loss of the other country. When the Home currency supply is much lower, Home agents even prefer equilibrium F to H; i.e., agents from Home country enjoy higher welfare in the equilibrium where the circulation of its currency remains locally. The currency shortage at home makes the inflow of foreign currency a bless rather than a curse.

Given the above results, one may think whether a government can implement a policy to interfere the existence of equilibria so that its fellow citizens enjoy higher welfare. From Figure 4-b we observe that if τ_h is low enough, the only equilibria are equilibrium H and U. We also know that W is higher in equilibrium H than in equilibrium A and F when those equilibria coexist. Hence, Home government would have incentive to lower tax rate τ_h to eliminate other equilibria where Home welfare is lower. The magnitude of τ_h that can make Foreign agents start to accept Home currency depends on τ_f. However, one can find a threshold of τ_h, $\overline{\tau}$, below which Foreign agents start to accept Home currency regardless of τ_f. This policy ensures higher welfare to Home agents.[11] Notice that, if $\tau_h > \overline{\tau}$ and τ_f is not too high, there coexist equilibria A, F and

[10] Matsuyama et al. (1993) find a case in which a switch from equilibrium A to H can reduce welfare of Home agents when the supply of Home currency and β are relatively low. For the examples we tried, we do not find such a case. Our conjecture is that, in Matsuyama et al. (1993) the experiments are undertaken given fixed (m, m^*) an there are no other policy tools for improving welfare. However, although we fix (m, m^*), we do allow governments to choose tax rates to attain the highest welfare as possible.

[11] Of course, there coexist equilibrium U and H so it's a question of which equilibrium will be selected under

H. If Home government reduces inflation tax below $\overline{\tau}$ and we are in equilibrium H, welfare W is *lower* than that in equilibrium H with $\tau_h > \overline{\tau}$ but *higher* than that in equilibria A and F. Thus, the cost of adopting a lower inflation tax for a country is that a better equilibrium associated with a higher tax rate is eliminated. The benefit is that it also eliminates the equilibria that entail lower welfare.

6 Conclusion

Some economists have argued that inflationary bias is necessary in an environment where sovereigns have the ability to tax foreigners. Yet they are puzzled by the observation that the United States, the country that would seem best able to impose a seignorage tax on foreigners, have relatively stable monetary policy.[12] This paper may offer an answer to the above issue. We have shown that, a big country, by adopting a lower inflation tax, can insure the existence of equilibrium where its national currency circulate abroad. This would entail higher welfare to its fellow citizen, provided it would not cause currency shortage at home, because the tax burden partially falls on foreigners. A higher inflation tax may lead to an inferior outcome where its currency circulates only locally, under which the tax burden falls completely on its citizens. Thus, the negative impact of a country's inflationary policy on the realm of circulation of its currency provides an inflation discipline, an issue that cannot be answered in a framework without considering the endogenous emergence of an international currency.

Previous studies using models with explicit trade frictions on the issue of international currency include the following. Matsuyama, Kiyotaki and Matsui (1993), of which the model is adopted in this paper, show that as the degree of economic integration is big enough there arises international currency. Zhou (1997) considers preference shocks to induce currency exchange in a framework similar to Matsuyama et al. (1993). Wright and Trejos (2001) considers a search model with divisible goods to study the determination of exchange rate. Soller Curtis and Waller (2003) show how currency restrictions (such as restrictions on the internal use of foreign currency) and government transactions policy affect the values of fiat currencies in a two-country

the low τ. Since equilibrium U yields even higher welfare to Home agents, this is not a concern.

[12] According to Fisher (1982), in the United States seigniorage averaged about .5 percent of GNP and only 2 or 3 percent of total revenue collected between 1960 and 1978. Seigniorage Tax accounted for about 15 percent of total revenue between 1973 and 1978 in Italy.

model. Ravikumar and Wallace (2002) shows that a uniform currency can eliminate inferior equilibria associated with distinct currencies where output and quantities of trade are less than the optimal quantities. The major differences of this paper from previous studies is that we explicitly consider an objective for government to issue fiat money, formulate how government's collection of seigniorage interacts with public goods provision, and study in a two-country world how the policies affect which currency is accepted as a medium of exchange and its welfare implications.

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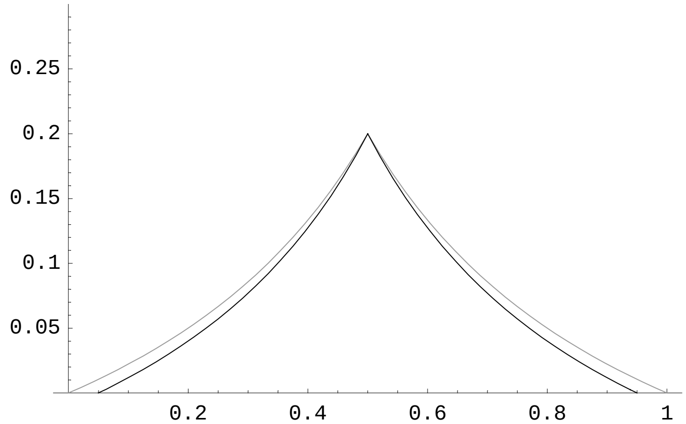
Equilibrium A

m = .2 m = .2 k = 10 u = 2 c = .1 delta → 0

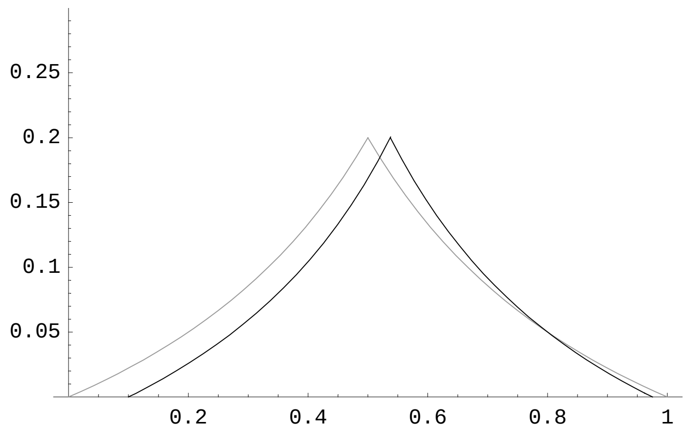
the region under the dark lines : $\tau_h = \tau_f = .2$

the region under the grey lines : $\tau_h = \tau_f = 0$



the region under the dark lines : $\tau_h = .4$ $\tau_f = .1$

the region under the grey lines : $\tau_h = \tau_f = 0$

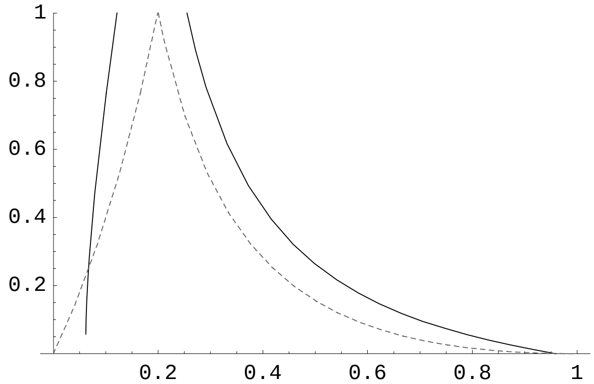


Equilibrium F

m = .2 m = .2 k = 10 u = 2 c = .1 delta → 0

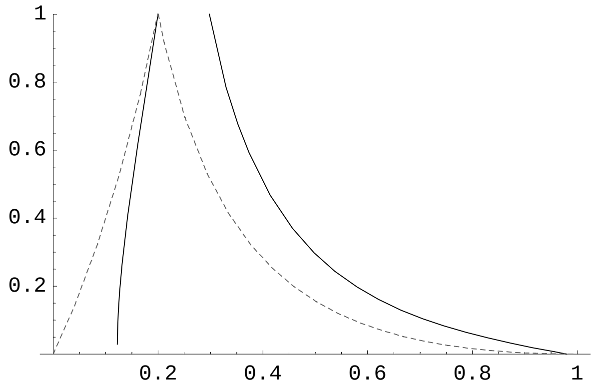
the region under the dark lines : $\tau_h = \tau_f = .2$

the region under the grey lines : $\tau_h = \tau_f = 0$



the region under the dark lines : $\tau_h = .4$ $\tau_f = .1$

the region under the grey lines : $\tau_h = \tau_f = 0$



Coexistence of equilibria

$m^{*} = 0.2 \quad m = 0.2 \quad \delta = 0.01 \quad k = 10 \quad \tau = \tau^{*} = 0$

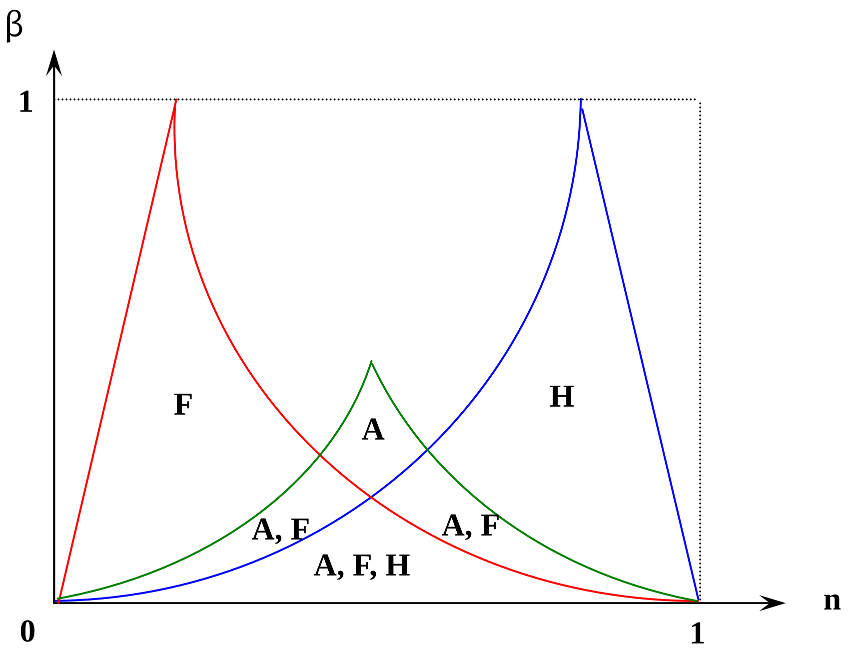


Figure 3 - *a*

$\tau = 0.05 \quad \tau^{*} = 0.15$

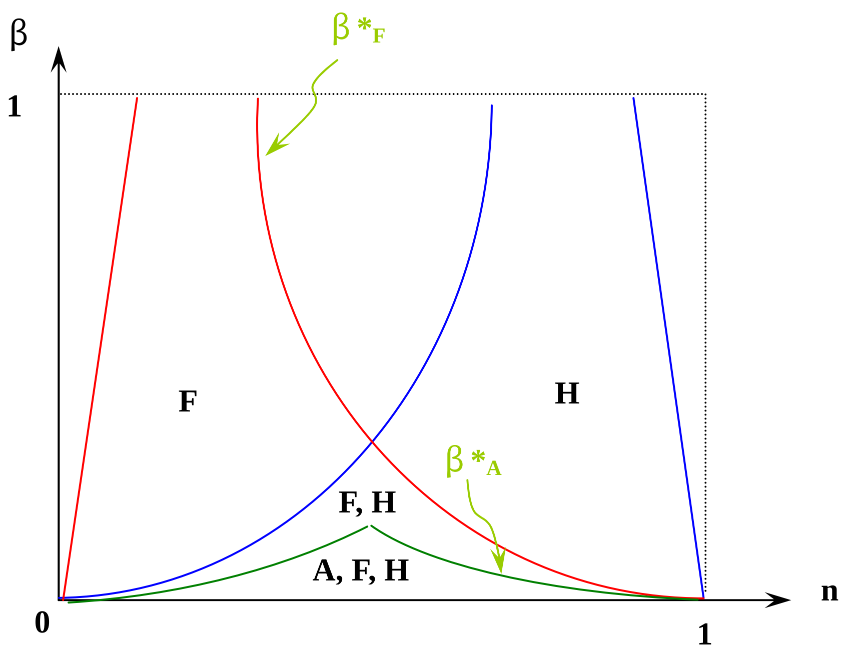


Figure 3 - *b*

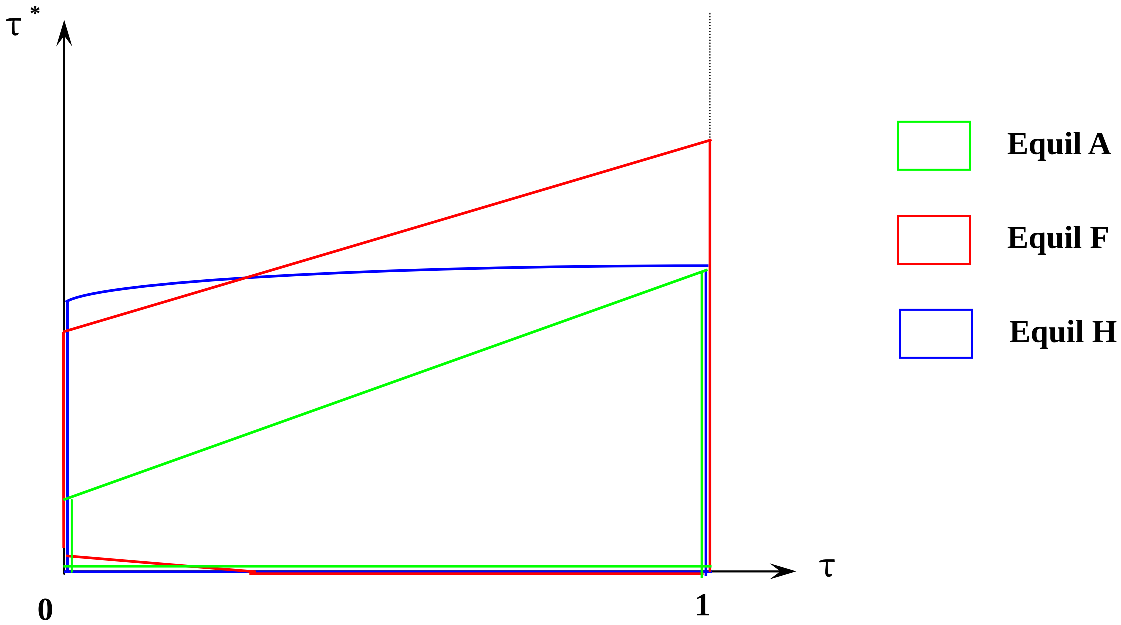


Figure 4 - a

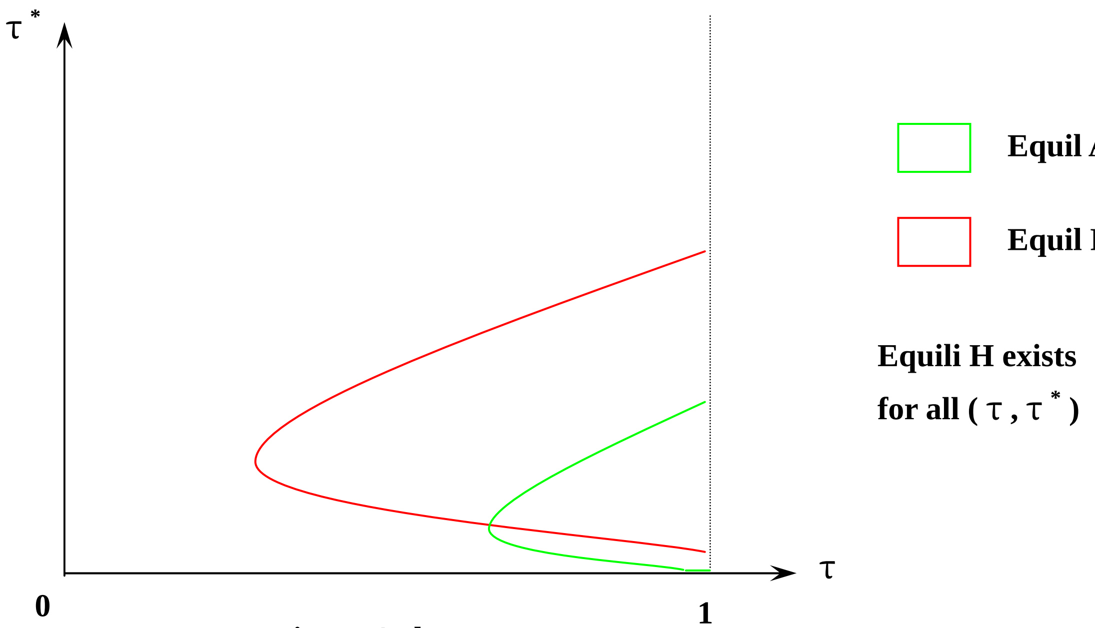


Figure 4 - b