

# Excess Liquidity against Predation:

Product Market Competition and Financial Contract

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The firm keeps a portion of its capital in the form of liquid assets to provide a reserve for unforeseen contingencies. These liquid assets together with the credit lines readily available to the firm play a key role in the analysis of predatory pricing. (Telser 1966, pp.261–2)<sup>1</sup>

## Abstract

We first show that, under a Cournot duopoly, a financial-constraint entrant has excess liquidity for a barrier to a “long-purse” incumbent’s predation. The excess liquidity does not however matters to the equilibrium output levels, if loan charges no interest.

Otherwise, the loan for the excess liquidity shrinks the entrant’s output levels and expands the incumbent’s. In addition, if we assume fixed cost, there may be equilibria where the entrant with little endowment further reduces his output because of the threat of liquidation (adverse “limited-liability effect”).

Keywords: Predation, Excess liquidity, Non-predation condition, Maximum output level for profitable predation, Minimum liquidity level for Non-predation, Adverse limited-liability effect.

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<sup>1</sup>The emphasis is Telser’s.

# 1 Introduction

The plausibility of predation has been argued in antitrust policy and industrial economics: Is it irrational or off equilibrium? The remarkable critique is Telser's. Telser (1966) emphasizes the role of the entrant's liquidity as a barrier to predation. In his simple reasoning, the entrant's liquidity which enables him to endure the predation raises the incumbent's cost of predation, and invites the incumbent's cooperation (in the form of merger or collusion in his paper.) He concludes that predation hardly occurs if the firms are rational.

We formalizes deliberately Telser's argument in a Cournot duopoly with a simple financial contract. In the Cournot competition, the incumbent could prey on the entrant by excess supply (larger than his equilibrium output level), which cuts the price and drives out the entrant who does not have sufficient liquidity to pay the production cost. The entrant borrows excess liquidity (out of the production cost of his equilibrium output) in case of such a predation. This reduces the incumbent's benefit of predation, because he has to increase the excess supply so as to succeed the predation and the excess supply eventually reduces his own profit. The entrant therefore holds in equilibrium enough liquidity to let the incumbent give up the predation. Since our model involves no credit constraint, the entrant can always borrows enough loan and vanishes the predation. This is what our model first describes. The threat of predation does not affect an equilibrium.

If loan costs interest, the threat of predation affects equilibrium output levels. The excess liquidity against predation, which is just left in the entrant's safe and not spent in equilibrium, takes a capital cost. Since minimum liquidity level to avoid predation depends on the output levels, the equilibrium output levels are distorted compared to that in the economy where the incumbent has no means to prey. Moreover, with fixed cost, there may be an equilibrium where the financial-constrained entrant puts an exaggerated weight on the Bad-state loss by threat of liquidation, and shrinks his output level further.

In our model the entrant avoids predation without cooperation or (would-be) multi-period endurance to predation as Telser's. Moreover, we present that threat of predation may however distorts equilibrium outputs. It is worthwhile noticing that we do not rely on assymetric information in product market, while the recent papers on predation (e.g. Fudenberg and Tirole, 1986; Poitevin, 1989) emphasize signalling effect of predation.

The paper goes as follows. In the next section, we describe the economy. In Section 3, we show optimal financial contract and equilibrium of the economy. By slight observation, we find the output levels do not differ from a simple Cournot model in equilibrium. In Section 4, a linear demand and zero marginal costs of the incumbent and the Good-state entrant are assumed. We obtain numerically the parameter's region where the excess liquidity is needed, which covers most of the plausible values. In Section 5, we introduce fixed cost and interest charged on the entrant's loan into the model. Then, as we explained above, threat of predation and shortage of the entrant's endowment distort equilibrium output levels. In the last section, we summarize the implication for empirical study and competition policy, and compare our model with other theoretical papers, especially Bolton and Scharfstein (1990) and Holmström and Tirole (1998). In Sections 3 and 5, formal discussions are too complicate to capture at the first glance though I insert interpretations among equations. So I give intuitive descriptions with graphs before the formalizations.

## 2 The Economy

A liquidity-constrained entrant (firm 1) and a “long-purse” incumbent (firm 2) are rivals in a Cournot quantity competition. The incumbent could reduce the entrants’ expected operating profit by excess supply, which makes the entrant difficult to borrow sufficient liquidity to cover the production cost and forces him to abandon the production. The entrant avoids such a predation by holding precautionary liquidity on entering, which raises cost for the predation. This is a brief illustration of our model. In this section we describe the economy in detail.

We specify first the schedule of events as follows.

**Date 0.** The entrant borrows long-term loan. Simultaneously, the entrant and the incumbent decide their output levels, which are publicly observable and verifiable.<sup>2</sup>

**Date 1.** The entrant’s state is realized: high production cost in the Bad state and low in the Good state. Since the state is observable only to the entrant, he tells the (initial) lender his state and they decide whether to proceed or to abandon the production. The proceeding/quitting is publicly observable and verifiable.

After the decision of proceeding, the production cost is paid by the entrant’s current liquidity holdings and (if necessary) additional short-term loan. Then, the two firms begin the production at the committed output levels.

**Date 2.** The entrant who proceeds the production and the incumbent sell the product. The entrant repays the short-term loan.

**Date 3.** The entrant (including the case where he quit the production) repays the long-term loan to the initial lender.

**Date 4.** Following the Date-0 financial contract, the entrant goes bankrupt or continues his business. In the case of bankrupt, the initial lender liquidates the entrant’s business and gets all the liquidation value. In the case of continuation, the entrant still holds control over the business and gets the private benefit.

### 2.1 The Production Technologies

Each firm has constant-return-to-scale technology. The incumbent’s unit cost is  $c_2$ , while the entrant’s  $c_1$  takes randomly at Date 1;  $c_g$  (the Good state) with probability  $\theta \in (0, 1)$  or  $c_b(> c_g)$  (the Bad state) with probability  $1 - \theta$ . This distribution is common knowledge, though its realized value is only known to the entrant. Since the production takes time, the entrant must commit at Date 0 to produce  $q_1$ , as well as the incumbent  $q_2$ , before  $c_1$  is realized at Date 1. These output levels are observable and verifiable. When he cannot acquire sufficient liquidity to pay the realized cost, the entrant has to quit the production.<sup>3</sup> Then the market is monopolized by the incumbent. Both firms are risk-neutral and there is no time discount.

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<sup>2</sup>In our model, the incumbent preys on the entrant to monopolize the current market (forcing the entrant to quit the current production), not the future (forcing him to go bankrupt). The incumbent is therefore interested in the amount of the entrant’s total liquidity holding, not the amount of loans.

<sup>3</sup>We assume that the entrant cannot reduce the output level after the realization of his state. This is a typical assumption in papers on excess liquidity as Holmstrom and Tirole (1998), and the adjustment of output level is not consistent with a Cournot model.

Moreover we will impose the entrant in equilibrium to sufficient liquidity to make the incumbent’s (complete) predation unprofitable, where complete means that the entrant is forced to exit, not only reduce the output (partial predation). Since the complete predation yields partial predation by the monopolization in

The product market is characterized as a Cournot competition. When the entrant proceeds the production, each firm  $i(= 1, 2)$  gets the sales  $R^i(\mathbf{q})$ , where  $\mathbf{q} := (q_1, q_2)$  at Date 2. When the entrant quits, only the incumbent gets the monopoly sales  $R^2(0, q_2)$ . We assume  $R^i$ 's continuity, continuous differentiability and concavity  $R_{ii}^i < 0, R_{jj}^i \leq 0$ , as well as  $R_{ij}^i < 0, R_{ij}^j < 0$  for each  $i = 1, 2$  and  $j \neq i$ . (Here  $R_j^i := \partial R^i / \partial q_j$ .)

The entrant's business yields, out of the operating profit  $R^1(\mathbf{q}) - c_1 q_1$ , the private benefit or the liquidation value at Date 4. If the entrant still holds control on the business, he gets the private benefit  $\pi$ , which is not verifiable. If the initial lender gets the control, he earns the liquidation value  $L(< \pi)$ . The proceeding or quitting of the production does not affect the private benefit and the liquidation value.

## 2.2 The Financial Contract

The entrant has two ways to acquire liquidity for paying the production cost, out of his endowed liquidity  $w_0$ . One way is to borrow long-term loan at Date 0. Another is to raise short-term loan at Date 1.

The Date-0 long-term loan involves an informational problem, namely the truth-telling of the entrant's state. Thus we design an "optimal financial contract" in the following section. Anyway the entrant has the precautionary liquidity  $B$  at Date 0, which works as a barrier to predation.

The Date-0 financial contract determines the amount of loan  $B - w_0$ , the entrant's output level  $q_1$ , the policy on whether to proceed or to abandon the production at Date 1 (the **quitting policy**),<sup>4</sup> the repayment at Date 3, the policy whether the entrant remains holding control over the business at Date 4 or the lender gains the control (the **liquidation policy**). We assume that the liquidation policy can be written as a stochastic term. That is, the probability of continuation can take any value in  $[0, 1]$ , not only  $\{0, 1\}$ . So the entrant and the lender write the contract as follows:

**Date 0.** The entrant borrows  $B - w_0$  from the lender.

**Date 1.** The entrant announces his state  $a = G(\text{Good}), B(\text{Bad})$ . The production proceeds or quits following the contract. The entrant produces the contracted output level  $q_2$ .

**Date 3.** The entrant repays  $D_a$  to the lender.

**Date 4.** With probability  $\beta_a \in [0, 1]$ , the entrant remains holding control on the business. Otherwise the lender gains the control and liquidate the business.<sup>5</sup>

We assume that the entrant's Date-1 announcement about his state is publicly known in the economy. As we will impose truth telling on the optimal Date-0 financial contract, anybody gets to know the entrant's true state. To distinguish Date-1 short-term loan with Date-0 long-term one, we assume that the Date-1 lender do not commit at Date 0 whether he lends the additional loan at Date 1 or not. We can therefore interpret the Date-0 loan

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the entrant's exit, avoidance of complete predation implies that of partial one. Our assumption is thus not restrictive as we concern about equilibrium.

<sup>4</sup>For convenience, we presume that the quitting policy can vary only to the state, not to the output levels. Moreover, the entrant cannot borrow at Date 1 from the initial lender in the case where the short-term loan cannot cover the production cost. In this case, regardless to the quitting policy, the production is forced to quit.

<sup>5</sup>Here, we assume that no one but the entrant himself can gain the private benefit  $B$ .

as credit line committed before the realization of the state, and the Date-1 loan as non-committed loan borrowed after the realization. After the realization of  $c_1$ , the entrant can never borrow the Date-1 loan, if

$$R^1(\mathbf{q}) + B < c_1 q_1.$$

Since the sales  $R^1(\mathbf{q})$  and the Date-0 liquidity holding  $B$  do not suffice the production cost, the additional loan spent for the production cannot be wholly returned to the Date-1 lender.<sup>6</sup> That is, the entrant goes certainly default in repayment of the additional loan. Then he cannot pay the production cost and is forced to abandon the production.

We therefore impose on the entrant **the Liquidity constraint** to proceed the production:

$$R^1(\mathbf{q}) + B \geq c_1 q_1.$$

With this constraint satisfied, even if the Date-0 liquidity holding  $B$  is not enough to pay the production cost  $c_1 q_1$ , the entrant can borrow any amount of money as an additional short-term loan with no premium from a new lender, since it is surely repaid at Date 2.

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<sup>6</sup>The Date-0 lender might cover rest of the repayment for the additional Date-1 loan. As this is not committed at Date 0, the Date-0 lender will not if  $R^1(\mathbf{q}) + B < c_1 q_1$ .

### 3 Equilibria

An equilibrium with entry consists of the optimal financial contract and the firms' output levels, given the entrant's quitting policy.

#### 3.1 Optimal Financial Contracts

Date-0 financial contracts are classified according to quitting policy:

**Pooling contracts** The entrants always proceeds the production.

**Separating contracts** Only the Good entrants proceeds the production.

In contracting the Date-0 loan, all the bargaining power belongs to the entrants. According to the Revelation Principle, we have only to focus on truth-telling contracts. So, given the equilibrium output levels and the quitting policy, the entrant sets the repayment  $D_a$  and the liquidation policy  $\beta_a$  so as to maximize his expected profit, satisfying the conditions below.

1. The entrant always announces his true state (**the truth-telling condition**).
2. The entrant can pay up the contracted repayment (**the limited-liability constraint**).
3. The lender earns zero net expected profit. (In this section, we assume that the lender has no outside option.)

The entrant's ex ante profit needs to be represented explicitly to determine the optimal output levels. Thus, we induce it in this subsection. We assume below that the amount of Date-0 loan  $B$ , the output levels  $\mathbf{q}$ , and the quitting policy are on-path. Let  $y$  be the cash balance at the beginning of Date 3:

$$y_g = R^1(\mathbf{q}) - c_g q_1 + B, \quad y_b = R^1(\mathbf{q}) - c_b q_1 + B.$$

Before precise calculation, we see intuitively the characteristics of optimal financial contracts. Suppose that the Good entrant's operation profit  $R^1(\mathbf{q}) - c_g q_1$  is positive. If not, the production yields an ex ante loss (and of course ex post), and it is natural that the entrant will not entry (though entry for getting the continuation value is possible.)

If the entrant can always pay up just the amount of the loan (**No default**), the contract can be very simple: the entrant repays just the borrowed amount of loan and never loses the control on business by the lender. Since the lender gains zero ex ante profit, the entrant absorbs all the total profit, namely the operating profit and the private benefit of the continuation:  $R^1(\mathbf{q}) - (\theta c_g + (1 - \theta)c_b)q_1 + \pi$  as for a Pooling contract without default (Non-default Pooling contract), which requires that the Bad entrant's operating loss is (if exists) within his endowed liquidity  $R^1(\mathbf{q}) - c_g q_1 + w_0 > 0$  (Non-default condition).

A Separating contract, which obviously involves no default of the Bad entrant, needs a little modification of the repayment. Its Truth-telling condition requires the differences between the repayments of the Good and the Bad entrant  $D_g - D_b$  to be larger than the operating profit  $R^1(\mathbf{q}) - c_b q_1$  which the Bad entrant would earn by pretending the Good and proceeding the production. To make this difference, the Bad entrant's repayment is set to be smaller than the borrowed amount and the Good one's to be larger. The entrant always hold control, so the entrant's ex ante net profit is still the operating profit and the private benefit, i.e.  $\theta(R^1(\mathbf{q}) - c_g q_1) + \pi$ .

As for a Pooling contract, if the Bad entrant's production incurs the operating loss larger than his endowed liquidity, he must spend some of the Date-0 loan to cover the production cost and cannot pay up the loan (**default**). The truth-telling condition requires the repayment minus the expected continuation value  $D_a - \beta_a \pi$  not to vary with the entrant's announcement  $a = G, B$ . Consequently, the default raises both the Good entrant's repayment and the Bad entrant's liquidation probability, so as to hold the lender's ex ante net profit from the repayment and the liquidation to be zero. These sizes are determined from the size of the default, namely the operating loss and the initial loan. The Good's repayment and the Bad's liquidity probability increases with the Bad-state operating loss.

Although the entrant still absorbs all the ex ante net profit, liquidation cuts it since the liquidation value is smaller than the private value of continuation. The entrant therefore puts heavier weight than  $1 - \theta$  on the operating loss in the Bad state, so as to reduce the liquidation probability  $1 - \beta_b$ . To speak simply, threat of liquidation in default makes the entrant a coward bear ("adverse limited liability effect"). We will find later the entrant's ex ante net profit  $\phi R^1(\mathbf{q}) - (\theta c_g + (\phi - \theta)c_b)q_1 + \pi + (\phi - 1)w_0$  ( $\phi > 1$ ). The difference from that without default,  $(\phi - 1)\{R^1(\mathbf{q}) - c_b q_1 + w_0\}$  represents the distortion caused by the threat of liquidation.

We formally investigate optimal financial contract below.

### 3.1.1 Pooling Contracts

The optimal Pooling contract  $(D_a, \beta_a)_{a=g,b}$  satisfies

$$\max_{(D, \beta)} \theta(y_g - D_g + \beta_g \pi) + (1 - \theta)(y_b - D_b + \beta_b \pi) \quad (1)$$

such that

$$y_g - D_g + \beta_g \pi \geq y_b - D_b + \beta_b \pi, \quad (2)$$

$$y_b - D_b + \beta_b \pi \geq y_b - D_b + \beta_b \pi, \quad (3)$$

$$\theta(D_g + (1 - \beta_g)L) + (1 - \theta)(D_b + (1 - \beta_b)L) = B - w_0, \quad (4)$$

$$D_g \leq y_g, \quad D_b \leq y_b. \quad (5)$$

The first and second inequalities are the truth-telling conditions. The third represents the lender's zero ex ante net profit. The fourth is the limited-liability constraints. We state here only the solution of this maximization problem, as we proof it in Appendix A.

**Lemma 1.** Suppose that  $y_g \geq B - w_0$ , i.e.  $R^1(\mathbf{q}) - c_g q_1 + w_0 \geq 0$ .

If  $y_b \geq B - w_0$ , the optimal Pooling contract is

$$D_g = D_b = B - w_0, \quad \beta_g = \beta_b = 1.$$

If  $y_b < B - w_0$ , the optimal Pooling contract is

$$D_g = \phi(B - w_0) + (1 - \phi)y_b, \quad D_b = y_b,$$

$$\beta_g = 1, \quad \beta_b = 1 - \phi(B - w_0 - y_b)/\pi,$$

where  $\phi = (\theta + (1 - \theta)L/\pi)^{-1} > 1$ .

We refer the former type of contracts as **Non-default Pooling contracts**, and the latter as **Default-involved Pooling contracts**. The condition  $y_b \geq B - w_0$  means that

the entrant can pay up just the borrowing loan even in the Bad state. Otherwise, he cannot. Hereafter, we call  $y_b \geq B - w_0$  **the Non-default condition** and  $y_b < B - w_0$  **the Default condition**.

Since the lender's ex ante net profit is zero, the entrant absorbs all the operating profit and the Date-4 value of the business, i.e. the private benefit of continuation or the liquidation value. As the private benefit is assumed to be higher than the liquidation value, the entrant prefers continuation to liquidation. If he has sufficiently large endowment and borrows little from the lender, he can repay just the face value of the loan even in the Bad state and no liquidation is consistent with the truth-telling condition. Otherwise, he has to borrow so much that he cannot repay in the Bad state, which makes the threat of liquidation is required for the entrant's truth telling.

The entrant's ex ante profit is, in the case of a Non-default Pooling contract,

$$\begin{aligned} & \theta\{y_g - (B - w_0) + \pi\} + (1 - \theta)\{y_b - (B - w_0) + \pi\} \\ & = R^1(\mathbf{q}) - (\theta c_g + (1 - \theta)c_b)q^1 + \pi + w_0; \end{aligned}$$

in the case of a Default-involved contract,

$$\begin{aligned} & \theta\{y_g - (\phi(B - w_0) + (1 - \phi)y_b) + \pi\} + (1 - \theta)\{y_b - y_b + 1 - \frac{\phi}{\pi}(B - w_0 - y_b) + \pi\} \\ & = \phi R^1(\mathbf{q}) - (\theta c_g + (\phi - \theta)c_b)q^1 + \pi + \phi w_0. \end{aligned}$$

As we see, the ex ante net profit is just the expected operating profit (subtracted the distortion caused by the threat of liquidation for a Default-involved) and the continuation value.

Notice that the weights on the Good-state and the Bad-state payoffs are different between these two types of contracts. In a Default-involved Pooling contract, the entrant puts heavier weight on the operating loss in the Bad state<sup>7</sup> than a Non-default one, because the size of default (shortage to the face value of the loan) raises the Good entrant's repayment  $D_g$  and the Bad's liquidation probability  $\beta_b$ . Such a distortion from the expected profit based on the objective probability, which you can call "adverse limited liability effect"<sup>8</sup> affect the optimal output levels.

### 3.1.2 Separating Contracts

The optimal Separating contract  $(D_a, \beta_a)_{a=g,b}$  satisfies

$$\max_{(D, \beta)} \theta(y_g - D_g + \beta_g \pi) + (1 - \theta)(B - D_b + \beta_b \pi) \quad (6)$$

such that

$$y_g - D_g + \beta_g \pi \geq B - D_b + \beta_b \pi, \quad (7)$$

$$B - D_b + \beta_b \pi \geq y_b - D_b + \beta_b \pi, \quad (8)$$

$$\theta(D_g + (1 - \beta_g)L) + (1 - \theta)(D_b + (1 - \beta_b)L) = B - w_0, \quad (9)$$

$$D_g \leq y_g, \quad D_b \leq B. \quad (10)$$

<sup>7</sup>Notice that the Default condition and  $w_0 \geq 0$  implies  $R^1(\mathbf{q}) - c_b^1 < -w_0 \leq 0$ .

<sup>8</sup>Brander and Lewis (1986) formalize the "limited liability effect" of debt financing: the borrower puts lighter weight on the loss in default. Since, in our optimization of financial contract, the size of default determines the repayment in the solvent state and the probability of liquidation in default, limited liability affects the borrower's decision in the opposite direction to their model.



Similarly to Pooling contracts, the first and the second inequalities are the truth-telling conditions, the third represents the lender's zero ex ante net profit, and the last is the limited-liability constraint. We state here only the solution again, as we proof it in Appendix B.

Lemma 2 Suppose that  $y_g \geq B - w_0/\theta$ , i.e.  $\theta(R^1(\mathbf{q}) - c_g q_1) + w_0 \geq 0$ .

The Date-0 Separating contract

$$D_g = B - w_0 + (1 - \theta)(y_g - B), \quad D_b = B - w_0 - \theta(y_g - B), \quad \beta_g = \beta_b = 1$$

is optimal.<sup>9</sup>

The entrant's expected payoff is

$$\begin{aligned} & \theta \{y_g - (B - w_0 + (1 - \theta)(y_g - B)) + \pi\} + (1 - \theta) \{B - (B - w_0 - \theta(y_g - B)) + \pi\} \\ &= \theta \{\theta(y_g - B) + \pi + w_0\} + (1 - \theta) \{\theta(y_g - B) + \pi + w_0\} \\ &= \theta(R^1(\mathbf{q}) - c_g q_1) + \pi + w_0. \end{aligned}$$

Here again, the entrant absorbs all the ex ante net profit, namely the expected operating profit  $\theta(R^1(\mathbf{q}) - c_g q_1)$  and the private benefit of continuation  $\pi$ .

## 3.2 Optimal Output Levels

Pure strategy<sup>10</sup> Nash equilibria with entry are classified according to their financial contracts:

**Non-default Pooling equilibria** where a Non-default Pooling contract is used.

**Default-involved Pooling equilibria** where a Default-involved Pooling contract is used.

**Separating equilibria** where a Separating contract is used.

In an equilibrium, the firm's output levels must satisfy the following conditions, given the entrant's quitting policy and the financial contract between the entrant and the initial lender.

1. Each firm's output level maximizes the firm's ex ante profit (for the entrant, within the Liquidity constraint), given the rival's output and quitting policy: (11), (12)/(18), (19).
2. The incumbent cannot gain more if he increases his production quantity and reduce the entrant's revenue so as to make the entrant's Liquidity constraint more tight, given the entrant's quantity and initial borrowing (**the Non-predation condition**): (13)/(20).

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<sup>9</sup>The optimal Separating contract is however not unique. It is necessary for optimality that the contract satisfies the limited-liability constraint,  $\beta_g = \beta_b = 1$  (come from the optimization w.r.t  $\beta$ ),  $\theta D_g + (1 - \theta) D_b = B - w_0$  (from the lender's PC) and

$$y_b - B \leq D_g - D_b \leq y_g - B.$$

While in a Pooling equilibrium the production always proceed regardless of the entrant's announcement, it depends, in a Separating one, on the announcement whether to proceed or to quit. So the liquidity holding at the beginning of Date 2 varies between announcement, given the entrant's state ( $y_g$  vs.  $B$  for the Good entrant;  $B$  and  $y_b$  for the Bad), which makes the freedom of  $D_g - D_b$ .

<sup>10</sup>We mean, by the term "Pure", that the output levels and the quitting policy is deterministic (pure strategies).

3. The entrant cannot gain more if he changes his quitting policy (**the Non-deviation condition**): (15)/(22).

We first describe intuitively the discussion below. We first determine the equilibrium output levels  $\mathbf{q}^*$  simply from the FOCs: at the point where each firm's marginal sales equals his marginal cost. The Liquidity constraint must always be slack. If not, the incumbent could prey on the entrant by marginal increase of his output, which raises the incumbent's profit by the monopolization in the entrant's exit. The Non-predation condition thus requires the Liquidity constraint to be slack.

**The minimum liquidity level  $\underline{B}^*$**  is given from the Non-predation condition. The incumbent would prey on the entrant with excess supply, only if the predatory profit (the monopoly profit when the entrants exits and the duopoly profit otherwise) is higher than the equilibrium profit. As the entrant holds larger pre-cautious liquidity  $B$ , predation requires larger excess supply, which in turn reduces the incumbent's predatory profit itself. (See Figs. 1–2.) So the incumbent will not prey, if predation requires excess supply larger than **the incumbent's maximum output level for profitable predation  $\bar{q}_2^*$**  at which the predatory profit equals the equilibrium profit. At this maximum output  $\bar{q}_2^*$ , the entrant's Liquidity constraint must be satisfied to avoid the predation. That is the entrant must hold more pre-cautious liquidity than the operating loss  $c_1 q_1^* - R^1(q_1^*, \bar{q}_2^*)$ , which is the minimum liquidity level  $\underline{B}^*$ .

The Non-deviation condition and the Non-default/Default condition of the financial contract restrict existence of equilibrium. In a Pooling equilibrium, the Bad entrant's operating profit must be positive (though it is not sufficient) for Non-deviation. If not, he would prefer quitting of the production to proceeding, and could get yet more profit by adjusting his output level so as to maximize just the Good-state operation profit. The Default condition however requires the Bad entrant to incur an operating loss, and consequently there is no Default-involved Pooling equilibrium. In contrast, a Non-default Pooling equilibrium exists whenever the Non-deviation condition is satisfied, since the Non-deviation condition implies the operating profit even in the Bad state and consequently the Non-default condition. A Separating equilibrium does also, because it contains neither Non-default nor Default condition.

We proceed to formalize the discussion above.

### 3.2.1 Non-default Pooling Equilibrium

We determine the output level  $\mathbf{q}^N$  and the minimum liquidity level  $\underline{B}^N$  in a Non-default Pooling equilibrium. Given the entrant's pre-cautious liquidity  $B^N$ , each firm sets his output level so as to maximize its profit (within the Liquidity constraint for the entrant):

$$q_1^N = \arg \max_{q_1}^{\textcircled{c}} R^1(q_1, q_2^N) - (\theta c_g + (1 - \theta) c_b) q_1 \quad \text{---} \quad c_b q_1 \leq R^1(q_1, q_2^N) + B^N, \quad (11)$$

$$q_2^N = \arg \max_{q_2}^{\textcircled{c}} R^2(q_1^N, q_2) - c_2 q_2 \quad \text{---} \quad c_b q_1^N \leq R^2(q_1^N, q_2) + B^N. \quad (12)$$

Besides, the incumbent do not prey the Bad-state entrant (the Non-predation condition):

$$R^2(\mathbf{q}^N) - c_2 q_2^N \geq \max_{q_2}^{\textcircled{c}} \theta R^2(q_1^N, q_2) + (1 - \theta) R^2(0, q_2) - c_2 q_2 \quad \text{---} \quad c_b q_1^N \leq R^1(q_1^N, q_2) + B^N, \quad (13)$$

The LHS of (13) is the incumbent's equilibrium profit and the RHS is the predatory profit. To succeed the predation, he must raise his output level so high that the Bad-state entrant's

Liquidity constraint is violated. This is the constraint on the RHS. Such an excess supply in turn reduces the predatory profit itself.<sup>11</sup> The Non-predation condition (13) requires the entrant to have the precautionary liquidity  $B^N$  enough to lead the incumbent into giving up the predation.

As the entrant's Liquidity constraint in (11) must be slack so that the incumbent gives up further predation (13), we find that both firms' output levels are simply determined by the FOCs.

**Lemma 3.** The output levels  $(q_1^N, q_2^N) =: \mathbf{q}^N$  in a Non-default Pooling equilibrium are determined such as

$$\begin{aligned} R_1^1(\mathbf{q}^N) &= \theta c_g + (1 - \theta)c_b && (\text{FOC of } \max_{q_1} \{R^1(q_1, q_2^N) - (\theta c_g + (1 - \theta)c_b)q_1\}), \\ R_2^2(\mathbf{q}^N) &= c_2 && (\text{FOC of } \max_{q_2} \{R^2(q_1^N, q_2) - c_2 q_2\}). \end{aligned}$$

**Proof.** Suppose that the entrant's Liquidity constraint binds in a Non-default Pooling equilibrium:

$$R^1(\mathbf{q}^N) + B^N = c^H q_1^N.$$

By  $R_1^2 < 0$  we have

$$R^2(\mathbf{q}^N) < \theta R^2(\mathbf{q}^N) + (1 - \theta)R^2(0, q_2^N).$$

$R^2$ 's continuity and  $R_2^1 < 0$  yield that, for a sufficiently small  $\epsilon > 0$ ,

$$\begin{aligned} R^2(\mathbf{q}^N) &< \theta R^2(\mathbf{q}^N) + (1 - \theta)R^2(0, q_2^N + \epsilon), \\ R^1(q_1^N, q_2^N + \epsilon) + B^N &< c^H q_1^N. \end{aligned}$$

That is, the incumbent preys on the high-cost entrant by raising his output level to  $q_2^N + \epsilon$ , which violates (13). Therefore, the Liquidity constraint must be slack at equilibrium output levels and the equilibrium output levels satisfy the FOCs.

(Q.E.D.)

The Non-predation condition (13) yields the minimum liquidity level in a Non-default Pooling equilibrium  $\underline{B}^N$  such as

$$\underline{B}^N = c_b q_1^N - R^1(q_1^N, \bar{q}_2^N), \quad (14)$$

Here  $\bar{q}_2^N$  is the incumbent's maximum output for profitable predation in the equilibrium:

$$\begin{aligned} \theta R^2(q_1^N, \bar{q}_2^N) + (1 - \theta)R^2(0, \bar{q}_2^N) - c_2 \bar{q}_2^N &= R^2(\mathbf{q}^N) - c_2 q_2^N, \quad \text{and} \\ \theta R_2^2(q_1^N, \bar{q}_2^N) + (1 - \theta)R_2^2(0, \bar{q}_2^N) - c_2 &< 0. \end{aligned}$$

The entrant must hold precautionary liquidity larger than  $\underline{B}^N$  to satisfy the Non-predation condition.

[Fig. 1 enters here.]

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<sup>11</sup>Notice that the output levels must be committed before the entrant's exit.

In our model, the incumbent preys on the entrant by raising his output level  $q_2$ , as it decreases the entrant's operating sales  $R^1(q_1^N, q_2)$  and he faces in the Bad state difficulty in borrowing additional short-term loan to cover the production cost  $c_b q_1^N$ . To succeed the predation,  $q_2$  must be so large as to violate the entrant's Liquidity constraint

$$R^1(q_1^N, q_2) + B < c_b q_1^N.$$

The larger precautions liquidity  $B$  obviously forces the incumbent to raise  $q_2$  higher for success of predation.

If the incumbent succeeds the predation, he gets the monopoly profit when the entrant is the Bad state, so his predatory profit is

$$\theta R^2(q_1^N, q_2) + (1 - \theta)R^2(0, q_2) - c_2 q_2.$$

Since  $R_{22}^2(\cdot) < 0$ , this is also concave to  $q_2$ , and  $q_2$  higher than  $\bar{q}_2^N$  yields less profit than the equilibrium profit. So  $\bar{q}_2^N$  is indeed the maximum output level that the predation is profitable for the incumbent. Thus the reduced Non-predation condition (14) means that the incumbent cannot prey on the Bad-state entrant unless he ends up worse off than the equilibrium.

N.B. Since  $\bar{q}_2^N > q_2^N$  (by  $R_{22}^2(\cdot) < 0$ )<sup>12</sup> and  $R_{12}^1(\cdot) < 0$ , the minimum liquidity level  $\underline{B}^N$  and consequently the precautions liquidity  $B^N$  in any Non-default Pooling equilibrium let the Liquidity constraint slack:

$$R^1(\mathbf{q}^N) + B > c_b q_1^N.$$

As well as the Non-default condition  $R^1(\mathbf{q}^N) - c_b q_1^N + w_0 \geq 0$ , the Non-deviation condition restricts the existence of a Non-default Pooling equilibrium. If the entrant deviates to a Separating contract (quitting of the production in the Bad state), his optimal output level changes to  $\check{q}_1^N$  such as

$$R_1^1(\check{q}_1^N, q_2^N) = c_g \quad (\text{FOC of } \max_{q_1} \theta \{R^1(q_1, q_2^N) - c_g q_1\}).$$

Comparing the entrant's ex ante profit in equilibrium and in deviation, we have the Non-deviation condition:

$$R^1(\mathbf{q}^N) - (\theta c_g + (1 - \theta)c_b)q_1^N \geq \theta \{R^1(\check{q}_1^N, q_2^N) - c_g \check{q}_1^N\}, \quad (15)$$

which is rearranged to

$$(1 - \theta)(R^1(\mathbf{q}^N) - c_b q_1^N) \geq \theta \stackrel{\text{a}}{(R^1(\check{q}_1^N, q_2^N) - c_g \check{q}_1^N) - (R^1(\mathbf{q}^N) - c_b q_1^N)} \stackrel{\text{b}}{.}$$

Since the RHS is non-negative by the definition of  $\check{q}_1^N$ , the Non-deviation condition is stronger than the Non-default condition for all  $w_0 \geq 0$ . Intuitively, the Non-deviation condition requires the Bad-state operating profit to be positive, while the Non-default condition allows it to be negative within the entrant's endowment. It is worth notice that the entrant's endowment does not matter to the Non-deviation condition and consequently the existence of a Non-default Pooling equilibrium. So far we have clarified the Non-default Pooling equilibrium:

**Proposition 1.** If the Non-deviation condition (15) is satisfied, then a Non-default Pooling equilibrium exists. The output level  $\mathbf{q}^N$  is determined from Lemma 3. The Date-0 liquidity holding  $B^N$  can take any value larger than  $\underline{B}^N$  defined by (14).

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<sup>12</sup>Compare the FOC that  $q_2^N$  satisfies and the definition of  $\bar{q}_2^N$  and apply  $R_{22}^2(\cdot) < 0$ . In general, the incumbent's maximum output level for profitable predation is larger in equilibrium than his output level which satisfies the FOC.

### 3.2.2 Default-involved Pooling Equilibrium

In short, there is no Default-involved Pooling equilibrium, because of the inconsistency between the Default and the Non-deviation condition.

**Proposition 2.** There is no Default-Pooling equilibrium.

**Proof.** Suppose that  $\mathbf{q}^D$  is the output levels in a (would-be) Default Pooling equilibrium. Applying the argument in Lemma 3, we find that  $\mathbf{q}^D$  must satisfy

$$\begin{aligned} R_1^1(\mathbf{q}^D) &= \frac{\theta}{\phi}c_g + 1 - \frac{\theta}{\phi}c_b && \text{(FOC of } \max_{q_1}\{\phi R^1(q_1, q_2^N) - (\theta c_g + (\phi - \theta)c_b)q_1\}), \\ R_2^2(\mathbf{q}^D) &= c_2 && \text{(FOC of } \max_{q_2}\{R^2(q_1^N, q_2) - c_2q_2\}). \end{aligned}$$

Since  $\theta/\phi < \theta$  and  $R_{ij}^i < 0$ ,  $q_1^D < q_1^N$  and  $q_2^D > q_2^N$ .

If the entrant deviates to a Separating contract, his optimal output level changes to  $\check{q}_1^D$  such as

$$R_1^1(\check{q}_1^D, q_2^D) = c_g \quad \text{(FOC of } \max_{q_1}\theta\{R^1(q_1, q_2^D) - c_gq_1\}).$$

Hence, the Non-deviation condition is

$$\phi R^1(\mathbf{q}^D) - (\theta c_g + (\phi - \theta)c_b)q_1^D + \phi w_0 \geq \theta\{R^1(\check{q}_1^D, q_2^D) - c_g\check{q}_1^D\} + w_0,$$

which is rearranged to

$$(\phi - \theta)(R^1(\mathbf{q}^D) - c_bq_1^D) + (\phi - 1)w_0 \geq \theta^{\text{a}} (R^1(\check{q}_1^D, q_2^D) - c_g\check{q}_1^D) - (R^1(\mathbf{q}^D) - c_bq_1^D)^{\text{a}} \geq 0. \quad (16)$$

By the way, the Default condition and  $q_2^D > q_2^N$  yields

$$R^1(\mathbf{q}^D) - c_bq_1^D < R^1(\mathbf{q}^N) - c_gq_1^N \leq -w_0 \leq 0,$$

and then

$$(\phi - \theta)(R^1(\mathbf{q}^D) - c_bq_1^D) + (\phi - 1)w_0 < -(1 - \theta)w_0 \leq 0 \quad (17)$$

This contradicts (16).

(Q.E.D.)

### 3.2.3 Separating Equilibrium

We determine the output levels  $\mathbf{q}^S$  and the minimum liquidity level  $B^S$  in a Separating equilibrium similar to a Non-default Pooling one. First, given the entrant's precautionous liquidity  $B^S$ , the equilibrium output levels  $\mathbf{q}^S$  are defined as

$$q_1^S = \arg \max_{q_1}^{\text{f}} \theta(R^1(q_1, q_2^S) - c_gq_1) - c_gq_1 \leq R^1(q_1, q_2^S) + B^S, \quad (18)$$

$$q_2^S = \arg \max_{q_2}^{\text{a}} \theta R^2(q_1^S, q_2) + (1 - \theta)R^2(0, q_2) - c_2q_2. \quad (19)$$

Besides, the Non-predation condition is

$$\theta R^2(\mathbf{q}^S) + (1 - \theta)R^2(0, q_2^S) - c_2q_2^S \geq \max_{q_2}^{\text{a}} R^2(0, q_2) - c_2q_2 - c_gq_1^S \geq R^1(q_1^S, q_2) + B^S. \quad (20)$$

As Lemma 3, we can show that  $\mathbf{q}^S$  must be the interior solution of the maximization problems (18) and (19). (The proof is quite similar to Lemma 3, so we omit it.)

Lemma 4. The output levels  $\mathbf{q}^S$  in a Separating equilibrium are determined such as

$$R_1^1(\mathbf{q}^S) = c_g, \quad \theta R_2^2(\mathbf{q}^S) + (1 - \theta)R_2^2(0, q_2^S) = c_2.$$

The Non-predation condition yields the minimum liquidity level in a Separating equilibrium  $\underline{B}^S$  in the same way as a Non-default Pooling equilibrium:

$$\underline{B}^S = c_g q_1^S - R^1(q_1^S, \bar{q}_2^S), \quad (21)$$

Here  $\bar{q}_2^S$  is the incumbent's maximum output for profitable predation in the equilibrium:

$$\begin{aligned} R^2(0, \bar{q}_2^S) - c_2 \bar{q}_2^S &= \theta R^2(\mathbf{q}^S) + (1 - \theta)R^2(0, q_2^S) - c_2 q_2^N, \quad \text{and} \\ R^2(0, \bar{q}_2^S) - c_2 &< 0. \end{aligned}$$

[Fig. 2 enters here.]

The Non-deviation condition restricts the existence of a Separating equilibrium again. If the entrant deviates to a Pooling contract (proceeding the production in the Bad state), his optimal output changes to  $\check{q}_1^S$  such as

$$R_1^1(\check{q}_1^S, q_2^S) = \theta c_g + (1 - \theta)c_b \quad (\text{FOC of } \max_{q_1} \{R^1(q_1, q_2^S)\} - (\theta c_g + (1 - \theta)c_b)q_1).$$

Thus, the Non-deviation condition is

$$\theta R^1(\mathbf{q}^S) - c_g q_1^S \geq R^1(\check{q}_1^S, q_2^S) - (\theta c_g + (1 - \theta)c_b)\check{q}_1^S. \quad (22)$$

We therefore establish the existence and the characteristics of a Separating equilibrium.

**Proposition 3.** If the Non-deviation condition (22) is satisfied, then a Separating equilibrium exists. The output level  $\mathbf{q}^S$  is determined from Lemma 4. The Date-0 liquidity holding  $B^S$  is any value larger than  $\underline{B}^S$  defined by (21).

## 4 Example of Linear Demand

In this section we specify the demand and the cost structure so as to show clearly the existence of excess liquidity as a barrier to predation. We assume that the firms face a linear demand and for convenience that the Good entrant and the incumbent take no marginal cost:

$$R^i(q_1, q_2) = \{a - b(q_1 + q_2)\} q_i \quad (i = 1, 2), \quad c_g = c_2 = 0. \quad (23)$$

Applying Lemmas 3 and 4 to this case, we determine the output levels in a pooling and a separating equilibrium:

$$\mathbf{q}^N = \left( \frac{\mu}{3b} \frac{a - 2(1 - \theta)c_b}{3}, \frac{\mu}{3b} \frac{a + (1 - \theta)c_b}{3} \right), \quad \mathbf{q}^S = \left( \frac{\mu}{(4 - \theta)b}, \frac{\mu}{(4 - \theta)b} \frac{(2 - \theta)a}{3} \right),$$

The joint outputs when the entrant continues the production are

$$q_1^N + q_2^N = \frac{2a - (1 - \theta)c_b}{3b}, \quad q_1^S + q_2^S = \frac{(3 - \theta)a}{(4 - \theta)b}.$$

The expected operating profits are, in a Non-default Pooling equilibrium

$$R^1(\mathbf{q}^N) - (1 - \theta)c_b = \frac{1}{b} \frac{\mu}{3} \frac{a - 2(1 - \theta)c_b}{3} \mathbb{I}_2, \quad R^2(\mathbf{q}^N) = \frac{1}{b} \frac{\mu}{3} \frac{a + (1 - \theta)c_b}{3} \mathbb{I}_2;$$

in a Separating equilibrium

$$\theta(R^1(\mathbf{q}^S)) = \theta \cdot \frac{1}{b} \frac{\mu}{4 - \theta} \frac{a}{4 - \theta} \mathbb{I}_2, \quad \theta R^2(\mathbf{q}^S) + (1 - \theta)R^2(0, q_2^N) = \frac{1}{b} \frac{\mu}{4 - \theta} \frac{(2 - \theta)a}{4 - \theta} \mathbb{I}_2.$$

As noticed in Props. 1 and 3, a Non-default Pooling and a Separating Equilibria require the Non-deviation conditions for their existence. As for a Non-default Pooling one, the condition is reduces to

$$\begin{aligned} \frac{\mu}{3} \frac{a - 2(1 - \theta)c_b}{3} \mathbb{I}_2 &> \theta \frac{\mu}{3} \frac{a - (1 - \theta)c_b/2}{3} \mathbb{I}_2 \quad \text{by } \check{q}_1^N = \frac{a - (1 - \theta)c_b/2}{3b}, \\ \text{i.e. } \frac{c_b}{a} &< \frac{2}{(4 - \sqrt{\theta})(1 + \sqrt{\theta})}; \end{aligned} \quad (24)$$

as for a Separating one,

$$\begin{aligned} \theta \frac{\mu}{4 - \theta} \frac{a}{4 - \theta} \mathbb{I}_2 &> \frac{\mu}{4 - \theta} \frac{a}{4 - \theta} - \frac{1 - \theta}{2} c_b \mathbb{I}_2 \quad \text{by } \check{q}_1^S = \frac{a}{(4 - \theta)b} - \frac{1 - \theta}{2b} c_b, \\ \text{i.e. } \frac{c_b}{a} &> \frac{2}{(4 - \theta)(1 + \sqrt{\theta})}. \end{aligned} \quad (25)$$

We can easily show that the RHS of (24) is larger than that of (25) for all  $\theta \in (0, 1)$ . Thus either of the equilibria always exists.

N.B. The entrant prefers a Non-default Pooling equilibrium to a Separating one, if

$$\frac{\mu}{3} \frac{a - 2(1 - \theta)c_b}{3} \mathbb{I}_2 > \theta \frac{\mu}{4 - \theta} \frac{a}{4 - \theta} \mathbb{I}_2, \quad \text{i.e. } \frac{c_b}{a} < \frac{4 - \theta - 3\sqrt{\theta}}{2(1 - \theta)(4 - \theta)}. \quad (26)$$

By simple calculation, we find that the RHS of (26) is larger than  $1/2$ , while that of (24) is smaller than  $1/2$  for all  $\theta \in (0, 1)$ . That is, if the Bad-state marginal cost  $c_b$  is too much large compared to the level of demand  $a$ , the entrants is forced to give up Non-default Pooling equilibrium even though it is better than Separating one.

N.B. Non-default Pooling equilibrium would be trivial, in that  $q_1^N \leq 0$ , if

$$\frac{c_b}{a} \geq \frac{1}{2(1-\theta)}.$$

Since the RHS is however larger than  $1/2$ , there is no trivial Non-default Pooling equilibrium.

#### 4.1 Decomposing the Non-predation Condition in a Non-default Pooling Equilibrium

The Non-predation condition (13) is decomposed, in general, to these two conditions (see Fig. 1 again):

$$R^1(q_1^N, \check{q}_2^N) + B^N > c_b q_1^N, \quad (27)$$

$$R^2(\mathbf{q}^N) - c_2 q_2^N \geq \theta R^2(q_1^N, \tilde{q}_2^N(B^N)) + (1-\theta)R^2(0, \tilde{q}_2^N(B^N)) - c_2 \tilde{q}_2^N(B^N), \quad (28)$$

where

$$\begin{aligned} \check{q}_2^N &= \arg \max_{q_2}^{\textcircled{c}} \theta R^2(q_1^N, q_2) + (1-\theta)R^2(0, q_2) - c_2 q_2^{\text{a}}, \\ R^1(q_1^N, \tilde{q}_2^N(B)) + B &= c_b q_1^N. \end{aligned}$$

If the first condition (27) is violated, the incumbent gets the highest (would-be) predatory profit, since by the definition of  $\check{q}_2^N$  and  $R_1^1(\cdot) < 0$

$$\begin{aligned} \theta R^2(q_1^P, \check{q}_2^P) + (1-\theta)R^2(0, \check{q}_2^P) - c_2 \check{q}_2^P &\geq \theta R^2(\mathbf{q}^P) + (1-\theta)R^2(0, q_2^P) - c_2 q_2^P \\ &> R^2(\mathbf{q}^P) - c_2 q_2^P. \end{aligned}$$

Even though the entrant avoids the predation at  $\check{q}_2^N$ , the incumbent might set his output level so high  $\tilde{q}_2$  as to break the entrant's Liquidity constraint  $c_b q_1^N \leq R^1(q_1^N, \tilde{q}_2) + B$ .  $\tilde{q}_2^N(B)$  is the minimum level that the predation succeed, given the entrant's precautions liquidity  $B$ . The second condition (28) suggests that  $B^P$  must be enough large to raise  $\tilde{q}_2^P(B^P)$  so that the predation is not profitable for the incumbent.

We reduce these two conditions to the linear demand example specified in (23). The condition to exclude the predation at

$$\check{q}_2^N = \frac{a}{2b} - \frac{\theta}{2} q_1^N = \frac{(3-\theta)a + 2(1-\theta)\theta c_b}{6b},$$

i.e. (27) is reduced to

$$B^N > \frac{-(1+\theta)a + 2(1+3\theta-\theta^2)c_b}{6b} \frac{a - 2(1-\theta)c_b}{3b} \quad (29)$$

Besides, the condition to avoid the predation at

$$\tilde{q}_2(B^N) = \frac{2a - (1+2\theta)c_b}{3b} + \frac{3B^N}{a - 2(1-\theta)c_b},$$



i.e. (28) is reduced to

$$\begin{aligned}
& \theta R^2(q_1^N, \tilde{q}_2(B^N)) + (1 - \theta) R^2(0, \tilde{q}_2(B^N)) \\
&= \frac{(1 - \theta)a + (1 + 4\theta - 2\theta^2)c_b}{3} - \frac{3bB^N}{a - 2(1 - \theta)c_b} \frac{2a - (1 + 2\theta)c_b}{3b} + \frac{3B^N}{a - 2(1 - \theta)c_b} \\
&< \frac{1}{b} \frac{a + (1 - \theta)c_b}{3} = R^2(\mathbf{q}^N),
\end{aligned}$$

which is rearranged to

$$\begin{aligned}
& \frac{3bB^N}{a - 2(1 - \theta)c_b} + \frac{(1 + \theta)a - 2(1 + 3\theta - \theta^2)c_b}{3} \frac{3bB^N}{a - 2(1 - \theta)c_b} \\
& - \frac{(1 - \theta)a + (1 + 4\theta - 2\theta^2)c_b}{3} \frac{2a - (1 + 2\theta)c_b}{3} + \frac{a + (1 + \theta)c_b}{3} > 0. \quad (30)
\end{aligned}$$

## 4.2 Excess Liquidity in a Non-default Pooling Equilibrium

In the following cases the entrant needs excess liquidity as a barrier to predation, in that  $B^N$  must be positive and larger than  $c_b q_1^N - R^1(\mathbf{q}^N)$ . We see that  $B^N = 0$  violates either condition above, that is  $B^N = 0$  cannot constitute a Non-default Pooling equilibria.

[Fig. 3 enters here.]

Case 1: Consider the case where

$$\frac{1 + \theta}{2(1 + 3\theta - \theta^2)} \leq \frac{c_b}{a} < \frac{2}{(4 - \sqrt{\theta})(1 + \sqrt{\theta})}, \quad (31)$$

where  $B^N = 0$  violates the condition (29). Without precautionary liquidity  $B > 0$ , the incumbent succeeds in preying on the entrant at the optimal predatory output level  $\tilde{q}_2$ . That is, the incumbent enjoy the highest (would-be) predatory profit without “opportunity cost” to drive out the Bad entrant.

Case 2: Consider the case where

$$\begin{aligned}
& \frac{c_b}{a} < \frac{1 + \theta}{2(1 + 3\theta - \theta^2)}, \quad \text{and} \\
& (4\theta^3 - 7\theta^2 - 4\theta - 2) \frac{c_b}{a} + (-2\theta^2 + 9\theta - 1) \frac{c_b}{a} + (1 - 2\theta) \geq 0. \quad (32)
\end{aligned}$$

The former inequality of (32) assures that  $B^N = 0$  satisfies the condition to avoid the predation at  $\tilde{q}_2^N$  (29). But, under the latter of (32), the condition to avoid the predation at  $\tilde{q}_2^N$  (30) is not. That is, if the entrant had no precautionary liquidity  $B^N = 0$ , the incumbent is better off by the predation at  $\tilde{q}_2(0)$  than the equilibrium:

$$\begin{aligned}
R^2(q_1^N, \tilde{q}_2(0)) &= \frac{(1 - \theta)a + (1 + 4\theta - 2\theta^2)c_b}{3} \frac{2a - (1 + 2\theta)c_b}{3b} \\
&\geq \frac{a + (1 - \theta)c_b}{3} \frac{1}{b} = R^2(\mathbf{q}^N).
\end{aligned}$$

We therefore sum up the discussion above into the following proposition.

**Proposition 4.** Suppose that the Non-deviation condition (24) is satisfied. If either of eqs.(31)–(32) holds, a Non-default Pooling equilibrium exists and the precautionary liquidity  $B^N$  must be positive and larger than the required level to cover the production cost  $c_b q_1^N - R^1(\mathbf{q}^N)$ .

### 4.3 Excess Liquidity in a Separating Equilibrium

As in a Non-default Pooling equilibrium, we first decompose the Non-predation condition (20) into

$$R^1(q_1^S, \check{q}_2^S) + B^S > c_g q_1^S, \quad (33)$$

$$\theta R^2(\mathbf{q}^S) + (1 - \theta)R^2(0, q_2^S) - c_2 q_2^S \geq R^2(0, \check{q}_2^S(B^S)) - c_2 \check{q}_2^S(B^S), \quad (34)$$

where

$$\check{q}_2^S = \arg \max_{q_2} R^2(0, q_2) - c_2 q_2, \\ R^1(q_1^S, \check{q}_2^S(B)) + B = c_g q_1^S.$$

The former (27) is the condition to avoid the incumbent's predation at  $\check{q}_2^S$  where the would-be predatory profit is highest, and the latter is to avoid that at  $\check{q}_2^S(B)$  where the entrant's Liquidity constraint is binded. (See Fig. 2 again.)

These two conditions (33)–(34) are reduced in our specification (23) to respectively

$$\begin{aligned} B^S > -R^1(q_1^S, \check{q}_2^S) &= -\frac{2-\theta}{2(4-\theta)}a - \frac{6-\theta}{2(4-\theta)}a \frac{1}{b} \quad \text{by } \check{q}_2^S = \frac{a}{2b}, \\ \theta R^2(\mathbf{q}^S) + (1-\theta)R^2(0, q_2^S) &= \frac{(2-\theta)a}{4-\theta} \frac{1}{b} \\ &> R^2(0, \check{q}_2^S(B^S)) = \frac{a}{4-\theta} + \frac{4-\theta}{a} b B^S \frac{(3-\theta)a}{4-\theta} - \frac{4-\theta}{a} b B^S \frac{1}{b}, \\ \text{by } \check{q}_2^S(B) &= \frac{3-\theta}{4-\theta} \frac{a}{b} - \frac{4-\theta}{a} B \end{aligned}$$

Thus excess liquidity must exist if  $B^S = 0$  violates either of the inequalities above:

$$\begin{aligned} 0 &\geq \frac{2-\theta}{2(4-\theta)}a - \frac{6-\theta}{2(4-\theta)}a \frac{1}{b}, \\ \frac{(2-\theta)a}{4-\theta} \frac{1}{b} &\leq \frac{a}{4-\theta} - \frac{(3-\theta)a}{4-\theta} \frac{1}{b}, \end{aligned}$$

For all  $\theta \in (0, 1)$  the former inequality is never satisfied. That is, the incumbent never enjoys the highest would-be predatory profit, even though the entrant holds no pre-cautious liquidity. This is obvious in our specification because the Good entrant do not need the production cost. Thus, the excess liquidity  $B > 0$  must exist if the second inequality is satisfied:

$$(2-\theta)^2 \leq 3-\theta, \quad \text{i.e. } \theta \geq 0.5(3-\sqrt{5}) \approx 0.382.$$

**Proposition 5.** Suppose that the Non-deviation condition(25) is satisfied. If  $\theta \geq 0.5(3-\sqrt{5})$ , a Separating equilibrium exists and the pre-cautious liquidity  $B^N$  must be positive just as a barrier to predation.

[Fig. 4 enters here.]

## 5 Extension

Although we have clarified equilibrium condition and existence of excess liquidity as a barrier to predation, there is no interaction of financial contract, size of the minimum liquidity level for Non-predation and output levels. This comes from simplification of the model. In this section we modify the “basic” model described so far.

We first introduce the fixed cost assumption to the model. Then a Default-involved Pooling equilibrium arises. This implies that the entrant endowed small liquidity faces the threat of liquidation in the equilibrium and shrinks his output level (adverse limited liability effect).

We next give the initial lender an outside option with certain return rate. The long-term loan takes capital cost (the borrowing interest), even though the entrant will not spend it to buy inputs, just put it in his safe in case of predation. We see that the threat of predation matters to the decision of the output size.

### 5.1 With Fixed Cost

We assume that the entrant must pay fixed cost  $F$  on entry at Date 0. So he must borrow  $B + F - w_0$  at Date 0. This alters little of the propositions and their proofs above, except for the (Non-)Default condition and the Non-deviation conditions of Default-involved equilibrium, which affects its existence. (Remember that its non-existence in the basic model comes from the inconsistency between the Non-default condition and the Default condition.)

Intuitively speaking, the Non-deviation condition concerns the deviation after the fixed cost is paid, so it compares just the entrant’s operating profit in equilibrium and in deviation. In contrast, the Default/Non-default condition compares the entrant’s Date-2 liquidity holding  $R^1(\mathbf{q}) - c_1 q_1 + B$  with the total amount of borrowing  $B + F - w_0$ , namely the operating profit  $R^1(\mathbf{q}) - c_1 q_1$  with the fixed cost subtracted the endowed liquidity  $F - w_0$ . Hence, not only the operating profit but also the fixed cost matter to non-default.

Such a gap between the Non-deviation condition and the Default/Non-default one changes the existence condition of a Non-default and a Default-involved Pooling equilibrium. As for a Non-default Pooling equilibrium, the Non-deviation condition does not guarantee that the operating cost (plus the entrant’s endowment) is larger than the fixed cost, namely the Non-default condition. Then the equilibrium requires both two conditions for its existence. On the other hand, a Default-involved Pooling equilibrium may exist by the introduction of fixed cost, though it never exist in the basic model. If the fixed cost is sufficiently large, the operating profit may not compensate the fixed cost (minus the entrant’s endowed liquidity) and the default is inevitable, even if the operating profit is positive and satisfy the Non-deviation condition.

We formalize the discussion above. We just mention the optimal financial contracts which are proved similar to the basic model.

Lemma 1'. Suppose that  $y_g \geq B + F - w_0$ , i.e.

$$R^1(\mathbf{q}) - c_g q_1 + w_0 \geq F.$$

If  $y_b \geq B + F - w_0$  (the Non-Default condition), the optimal Pooling contract is a Non-default Pooling one:

$$D_g = D_b = B + F - w_0, \quad \beta_g = \beta_b = 1.$$

If  $y_b < B + F - w_0$  (the Default condition), the optimal Pooling contract is a Default-involved one:

$$D_g = \phi(B + F - w_0) + (1 - \phi)y_b, \quad D_b = y_b,$$

$$\beta_g = 1, \quad \beta_b = 1 - \phi(B + F - w_0 - y_b)/\pi,$$

where  $\phi = (\theta + (1 - \theta)L/\pi)^{-1} > 1$ .

Lemma 2' Suppose that  $y_g \geq B + (F - w_0)/\theta$ , i.e.

$$\theta(R^1(\mathbf{q}) - c_g q_1) + w_0 \geq F.$$

The Date-0 Separating contract

$$D_g = B + F - w_0 + (1 - \theta)(y_g - B), \quad D_b = B + F - w_0 - \theta(y_g - B), \quad \beta_g = \beta_b = 1$$

is optimal.

The entrant's ex ante profit is in the case of a Non-default Pooling contract

$$\begin{aligned} & \theta\{y_g - (B + F - w_0) + \pi\} + (1 - \theta)\{y_b - (B + F - w_0) + \pi\} \\ & = R^1(\mathbf{q}) - (\theta c_g + (1 - \theta)c_b)q_1 + \pi + w_0 - F; \end{aligned}$$

in the case of a Default-involved Pooling contract,

$$\begin{aligned} & \theta\{y_g - (\phi(B + F - w_0) + (1 - \phi)y_b) + \pi\} + (1 - \theta)\{y_b - y_b + 1 - \frac{\phi}{\pi}(B + F - w_0 - y_b) + \pi\} \\ & = \phi R^1(\mathbf{q}) - (\theta c_g + (\phi - \theta)c_b)q_1 + \pi + \phi(w_0 - F); \end{aligned}$$

in the case of a Separating contract

$$\begin{aligned} & \theta\{y_g - (B + F - w_0 + (1 - \theta)(y_g - B)) + \pi\} + (1 - \theta)\{B - (B + F - w_0 - \theta(y_g - B)) + \pi\} \\ & = \theta(R^1(\mathbf{q}) - c_g q_1) + \pi + w_0 - F. \end{aligned}$$

As the entrant's ex ante profit remains the same as the basic model, except to change the last terms “ $+w_0$ ” to “ $+w_0 - F$ ”, the output levels in a Non-default Pooling and a Separating equilibrium do not differ from Lemmas 3 and 5. The Non-deviation conditions of these two equilibria do not also change.<sup>13</sup> Therefore, Prop. 3 remains valid without any modification.

We should however pay attention to the relation between the Non-deviation and the Non-default/Default condition. In a Non-default Pooling equilibrium, the Non-deviation condition (15) guarantees only

$$(1 - \theta)(R^1(\mathbf{q}^N) - c_b q_1^N) \geq \theta \stackrel{\text{a}}{(R^1(\check{q}_1^N, q_2^N) - c_g \check{q}_1^N) - (R^1(\mathbf{q}^N) - c_b q_1^N)} \geq 0,$$

while the Non-default condition is

$$R^1(\mathbf{q}^N) - c_b q_1^N \geq F - w_0. \quad (35)$$

For  $w_0 < F$ , even if the Non-deviate condition (15) is satisfied, the Non-default condition may be violated. So we further restrict the existence of a Non-default Pooling equilibrium.

<sup>13</sup>Notice that  $+w_0 - F$  in the equilibrium profit and the deviation profit cancel out in the Non-deviation conditions of these two equilibria, similarly to  $+w_0$  in the basic model.

**Proposition 1'.** If the Non-deviation condition (15) and the Non-default condition (35) is satisfied, then a Non-default Pooling equilibrium exists. The output level  $\mathbf{q}^N$  is determined from Lemma 3. The Date-0 liquidity holding  $B^N$  is any value larger than  $\underline{B}^N$  defined by (14).

The almost same argument goes in a Default-involved Pooling equilibrium. The Non-deviation condition is

$$\phi R^1(\mathbf{q}^D) - (\theta c_g + (\phi - \theta)c_b)q_1^D + \phi(w_0 - F) \geq \theta\{R^1(\check{q}_1^D, q_2^D) - c_g\check{q}_1^D\} + (w_0 - F), \quad (36)$$

which implies

$$(\phi - \theta)(R^1(\mathbf{q}^D) - c_b q_1^D) + (\phi - 1)(w_0 - F) \geq \theta \stackrel{\textcircled{a}}{(R^1(\check{q}_1^D, q_2^D) - c_g \check{q}_1^D) - (R^1(\mathbf{q}^D) - c_b q_1^D)} \stackrel{\text{a}}{.}$$

On the other hand, the Default condition

$$R^1(\mathbf{q}^N) - c_g q_1^N \leq F - w_0 \quad (37)$$

and  $q_2^D > q_2^N$ , which we will prove below, yield

$$R^1(\mathbf{q}^D) - c_b q_1^D < R^1(\mathbf{q}^N) - c_g q_1^N \leq F - w_0.$$

This implies only

$$(\phi - \theta)(R^1(\mathbf{q}^D) - c_b q_1^D) + (\phi - 1)(w_0 - F) < (1 - \theta)(F - w_0).$$

Thus, on the contrary to (17) in the basic model, there is a room where the Non-deviation and a Default condition are consistent. So we assert the existence of a Default-involved Pooling equilibrium.

As we have noticed in calculation of the entrant's expected payoff with a Default-involved Pooling contract, the output levels are distorted in a Default-involved Pooling equilibrium. As Lemma 3, we find that the Liquidity constraint must be slack and the output levels  $\mathbf{q}^D$  satisfy

$$R_1^1(\mathbf{q}^D) = \frac{\theta}{\phi}c_g + (1 - \frac{\theta}{\phi})c_b, \quad R_2^2(\mathbf{q}^N) = c_2. \quad (38)$$

Obviously, the entrant's reaction curve shifts downward from that in a Non-default Pooling equilibrium, leaving the incumbent's the same (Notice that  $\theta/\phi < \theta$ ). We induce therefore

$$q_1^D < q_1^N, \quad q_2^D > q_2^N.$$

That is, **the adverse limited liability effect** actually works by the introduction of fixed cost.

**Proposition 2'.** If the Non-deviation condition (36) and the Default condition (37) is satisfied, then a Default-involved Pooling equilibrium exists. The output levels  $\mathbf{q}^D$  are determined from the FOCs (38). Compared to a Non-default Pooling equilibrium, the entrant's output level shrinks while the incumbent's expands.

## 5.2 With Interest on Long-term Loan

In the previous subsection, we show the possibility that the financial-constrained entrant may shrink his output. This however comes from the entrant's (non-)deviation and threat of liquidation, not the incumbent's predation. In this subsection, we allow the lender to charge interest on the long-term loan. Then we find that the capital cost to borrow the excess liquidity for Non-predation affect the equilibrium output levels.

We assume that the initial lender has an outside option with certain return rate  $r > 0$ . Then, the lender's ex ante profit, namely the entrant's **capital cost** to borrow the long-term loan  $B - w_0$  is  $r(B - w_0)$ . we still assume fixed cost  $F \geq 0$  on entry.

We first illustrate informally how this capital cost distorts the equilibrium output levels. Since the entrant's output level  $q_1$  affects the minimum liquidity level  $\underline{B}$ , he optimizes  $q_1$  concerning not only the operating profit but also the capital cost  $r(\underline{B} - w_0)$ . Given the incumbent's output level  $q_2$ , the entrant's equilibrium output level  $q_1^*$  satisfies the FOC:

$$\begin{aligned} [\text{Marginal expected operating profit at } \mathbf{q}^*]^{14} &= [\text{Marginal capital cost}] \\ &= r(\partial \underline{B} / \partial q_1). \end{aligned}$$

As we know so far, the minimum liquidity level  $\underline{B}(\mathbf{q})$  is determined from the Non-predation condition such as

$$\underline{B}(\mathbf{q}) = [\text{Operating loss at } (q_1, \bar{q}_2)].^{15}$$

Remember that  $\bar{q}_2(\mathbf{q})$  is the incumbent's maximum output for profitable predation (see Figs. 5 and 6). Differentiation with regard to  $q_1$  yields

$$\partial \underline{B} / \partial q_1 = \underbrace{[\text{Marginal operating loss at } (q_1, \bar{q}_2)]}_{\text{Direct effect}} \underbrace{- R_2^1(q_1, \bar{q}_2) \times (\partial \bar{q}_2 / \partial q_1)}_{\text{Indirect effect}}.$$

We refer the former term of the RHS as the **direct effect** of marginal increase in the entrant's output  $q_1$  on the minimum liquidity level  $\underline{B}$ , and the latter as the **indirect effect**, which represents the marginal decrease (increase) in the entrant's sales caused by marginal increase (decrease) in the incumbent's maximum output  $\bar{q}_1$  corresponding to the increase in  $q_1$ . Consequently, the entrant's output level must satisfy

$$[\text{Marginal expected operating profit at } \mathbf{q}^*] - r[\text{Direct effect}] = r[\text{Indirect effect}].$$

Since the entrant's marginal sales  $R_1^1(\cdot)$  is assumed to be non-increasing to the incumbent's output and  $\bar{q}_2(\mathbf{q}^*)$  is larger than the incumbent's equilibrium output  $q_2^*$  (by  $R_{22}^2(\cdot) < 0$ ),<sup>16</sup>

$$\begin{aligned} [\text{Marginal expected operating profit at } \mathbf{q}^*] &\geq [\text{Marginal operating profit at } (q_1^*, \bar{q}_2)] \\ &= -r[\text{Direct effect}]. \end{aligned}$$

<sup>14</sup>This is  $R_1^1(q_1^N, q_2^N) - (\theta c_g + (1 - \theta)c_b)$  in a Non-default pooling equilibrium;  $R_1^1(q_1^D, q_2^D) - \{(\theta/\phi)c_g + (1 - (\theta/\phi))c_b\}$  in a Default-involved Pooling;  $\theta(R_1^1(q_1^S, q_2^S) - c_g)$ .

<sup>15</sup>This is  $R_1(q_1^P, \bar{q}_2^P) - c_g q_1^P$  in a Pooling equilibrium;  $R_1(q_1^S, \bar{q}_2^S) - c_b q_1^S$  in a Separating one.

<sup>16</sup>In addition, it is used to induce the following inequality that the expected marginal cost is not more than that the marginal cost in the Bad state that the entrant proceeds the production:  $\theta c_g + (1 - \theta)c_b < c_b$  for a Non-default Pooling equilibrium,  $(\theta/\phi)c_g + (1 - (\theta/\phi))c_b < c_b$  for a Default-involved Pooling; both are  $c_g$  for a Separating.

Combining these two (in-)equalities, we have

$$\begin{aligned} (1+r)[\text{Marginal expected operating profit at } \mathbf{q}^*] &\geq r[\text{Indirect effect}] \\ &= -rR_2^1(q_1^*, \bar{q}_2) \times (\partial \bar{q}_2 / \partial q_1) \end{aligned}$$

As  $R_2^1(\cdot) < 0$ , the marginal expected operating profit at  $\mathbf{q}^*$  is positive, and thus the entrant's equilibrium output is smaller than that in the basic model, if  $\partial \bar{q}_2 / \partial q_1$  is positive. That is, the marginal increase of the entrant's output lets the incumbent still better off by predation at higher output level.

This is true without any additional assumption in a Separating equilibrium. (See Fig. 5.) Increase of the entrant's output level  $q_1$  decreases the incumbent's profit without predation, i.e.  $\max_{q_2} \theta R^2(q_1, q_2) + (1-\theta)R^2(0, q_2) - c_2 q_2$ , while his (would-be) predatory profit  $R^2(0, q_2) - c_2 q_2$  remains the same (for all  $q_2$ ). This implies that the net benefit of the predation becomes large, and the maximum output level for profitable predation  $\bar{q}_2$  gets higher, i.e.  $\partial \bar{q}_2^S / \partial q_1 > 0$ .

In a Pooling equilibrium, the argument does not go as straightforward as above. (See Fig. 6.) The increase of  $q_1$  also decreases the incumbent's (would-be) predatory profit  $\theta R^2(q_1, q_2) + (1-\theta)R^2(0, q_2) - c_2 q_2$  (for all  $q_2$ ). This complicates the effect of  $q_1$  on the net benefit of predation and the maximum output for profitable predation  $\bar{q}_2$ . We however find that sufficiently low  $\theta$  guarantees increase of  $\bar{q}_2$ . With low  $\theta$ , the operating profit in predation is near the monopoly profit  $R^2(0, q_2) - c_2 q_2$  and hardly suffers from the increase of  $q_1$ , while that without predation  $\max_{q_2} R^2(q_1, q_2) - c_2 q_2$  decreases for all  $\theta \in (0, 1)$ .

Anyway, provided that marginal increase of the entrant's output level induces the incumbent's predation, the indirect effect on the minimum liquidity level is positive, and thus the entrant sets his output at the level where the entrant's marginal operating profit is still positive. That is, his response curve shift downward, while the incumbent's remains the same as the basic model. Therefore, the entrant's output level shrinks and the incumbent's expands in equilibrium by the capital cost.

Moreover, the capital cost incurs another problem: it drives out the entrant with little endowment from a Non-default to a Default-involved Pooling equilibrium and shrinks further his output level by the adverse limited liability effect.

We formally prove such an intuition below. We first list the optimal financial contract and the entrant's ex ante profit.

Lemma 1''. Suppose that  $y_g \geq (1+r)(B+F-w_0)$ , i.e.

$$R^1(\mathbf{q}) - c_g q_1 + (1+r)w_0 \geq rB + (1+r)F.$$

If  $y_b \geq (1+r)(B+F-w_0)$  (the Non-default condition), the optimal Pooling contract is a Non-default Pooling one:

$$D_g = D_b = (1+r)(B+F-w_0), \quad \beta_g = \beta_b = 1.$$

If  $y_b < (1+r)(B-w_0)$  (the Default condition), the optimal Pooling contract is a Default-involved one:

$$D_g = \phi(1+r)(B+F-w_0) + (1-\phi)y_b, \quad D_b = y_b,$$

$$\beta_g = 1, \quad \beta_b = 1 - \phi\{(1+r)(B+F-w_0) - y_b\}/\pi,$$

where  $\phi = (\theta + (1-\theta)L/\pi)^{-1} > 1$ .

Lemma 2'' Suppose that  $y_g \geq B + \{rB - (1+r)(w_0 - F)\}/\theta$ , i.e.

$$\theta(R^1(\mathbf{q}) - c_g q_1) + w_0 \geq r(B - w_0) + (1+r)F.$$

The Date-0 Separating contract

$$D_g = (1+r)(B+F-w_0) + (1-\theta)(y_g-B), \quad D_b = (1+r)(B+F-w_0) - \theta(y_g-B),$$

$$\beta_g = \beta_b = 1$$

is optimal.

The entrant's ex ante profit is in the case of a Non-default Pooling contract

$$\begin{aligned} & \theta\{y_g - (1+r)(B+F-w_0) + \pi\} + (1-\theta)\{y_b - (1+r)(B+F-w_0) + \pi\} \\ &= R^1(\mathbf{q}) - (\theta c_g + (1-\theta)c_b)q_1 + \pi + (w_0 - F) - r(B+F-w_0); \end{aligned}$$

in the case of a Default-involved Pooling contract,

$$\begin{aligned} & \theta\{y_g - (\phi(1+r)(B+F-w_0) + (1-\phi)y_b) + \pi\} \\ & + (1-\theta)\{y_b - y_b + 1 - \frac{\phi}{\pi}\{(1+r)(B+F-w_0) - y_b\} + \pi\} \\ &= \phi R^1(\mathbf{q}) - (\theta c_g + (\phi - \theta)c_b)q_1 + \pi + \phi(w_0 - F) - \phi r(B+F-w_0); \end{aligned}$$

in the case of a Separating contract

$$\begin{aligned} & \theta\{y_g - ((1+r)(B+F-w_0) + (1-\theta)(y_g-B)) + \pi\} \\ & + (1-\theta)\{B - (1+r)(B+F-w_0) - \theta(y_g-B) + \pi\} \\ &= \theta(R^1(\mathbf{q}) - c_g q_1) + \pi + (w_0 - F) - r(B+F-w_0). \end{aligned}$$

The entrant's ex ante profit is just that of the basic model subtracted the capital cost  $r(B-w_0)$  in a Non-default Pooling and a Separating equilibrium,  $\phi r(B-w_0)$  in a Default-involved Pooling one.<sup>17</sup>

### 5.2.1 Separating equilibrium

We first study a Separating equilibrium, which shows clearly that the capital cost affects the output levels.

Since the initial lender charges interest on long-term loan, the entrant sets the precautionary liquidity at the minimum liquidity level  $\underline{B}^S$ . We should first remember that  $\underline{B}^S$  is determined from the output levels. As the environment of the incumbent is nothing changed,  $\underline{B}^S$  is still defined by the Non-predation condition (21). We explicitly represent the minimum liquidity level  $\underline{B}^S$  as a function of the committed output levels  $\mathbf{q}$

$$\underline{B}^S(\mathbf{q}) = c_g q_1 - R^1(q_1, \bar{q}_2^S(\mathbf{q})),$$

and the incumbent's maximum output for profitable predation  $\bar{q}_2^S(\mathbf{q})$

$$\begin{aligned} & R^2(0, \bar{q}_2^S(\mathbf{q})) - c_2 \bar{q}_2^S(\mathbf{q}) = \theta R^2(\mathbf{q}) + (1-\theta)R^2(0, q_2) - c_2 q_2, \quad \text{and} \\ & R^2(0, \bar{q}_2^S(\mathbf{q})) - c_2 < 0. \end{aligned}$$

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<sup>17</sup>This is the same reason as the basic model: the capital cost affects the size of default as well as the operating loss in a Bad state, and the size of default is magnified in the ex ante profit, because of the threat of liquidation.



Differentiating these two equations with regard to  $q_1$ , we have

$$\frac{\partial \underline{B}^S}{\partial q_1} = \{c_g - R_1^1(q_1, \bar{q}_2^S(\mathbf{q}))\} - R_2^1(q_1, \bar{q}_2^S(\mathbf{q})) \frac{\partial \bar{q}_2^S}{\partial q_1}, \quad \{R_2^2(0, \bar{q}_2^S(\mathbf{q})) - c_2\} \frac{\partial \bar{q}_2^S}{\partial q_1} = \theta R_1^2(\mathbf{q}),$$

which yields

$$\frac{\partial \bar{q}_2^S}{\partial q_1} = \frac{\theta R_1^2(\mathbf{q})}{R_2^2(0, \bar{q}_2^S(\mathbf{q})) - c_2} > 0$$

by  $R_1^2(\cdot) < 0$  and the definition of  $\bar{q}_2^S$ .

[Fig. 5 enters here.]

The equilibrium output levels are still determined from the FOCs because of the Non-predation condition as well as the basic model.<sup>18</sup> Nevertheless, the entrant's FOC includes the marginal cost of raising  $\underline{B}^S$ :

$$\begin{aligned} \theta(R_1^1(\mathbf{q}^S) - c_g) &= r \frac{\partial \underline{B}^S}{\partial q_1}(\mathbf{q}^S), \\ \theta R_2^2(\mathbf{q}^S) + (1 - \theta)R_2^2(0, q_2^S) &= c_2. \end{aligned} \tag{39}$$

Substituting  $\partial \underline{B}^S / \partial q_1$  into the entrant's FOC yields

$$\theta(R_1^1(\mathbf{q}^S) - c_g) = r \{c_g - R_1^1(q_1, \bar{q}_2^S(\mathbf{q}^S))\} - R_2^1(q_1, \bar{q}_2^S(\mathbf{q}^S)) \frac{\partial \bar{q}_2^S}{\partial q_1}$$

Since  $R_1^1(\cdot) \leq 0$  and  $q_2^S < \bar{q}_2^S(\mathbf{q}^S)$ , we have

$$R_1^1(\mathbf{q}^S) - c_g \geq R_1^1(q_1, \bar{q}_2^S(\mathbf{q}^S)) - c_g.$$

Thus we obtain

$$(\theta + r)(R_1^1(\mathbf{q}^S) - c_g) \geq -r R_2^1(q_1, \bar{q}_2^S(\mathbf{q}^S)) \frac{\partial \bar{q}_2^S}{\partial q_1}$$

As we have verified that  $\partial \bar{q}_2^S / \partial q_1 > 0$ , this implies

$$R_1^1(\mathbf{q}^S) - c_g > 0.$$

That is, the entrant's response curve shifts downward (near the equilibrium) while that of  $q_2$  obviously remains the same.<sup>19</sup>

<sup>18</sup>To determine the entrant's best response uniquely, it is sufficient that the expected marginal sales subtracted the marginal capital cost decreases to the entrant's output level  $q_1$ , given the incumbent's  $q_2$ :

$$\begin{aligned} \frac{\partial}{\partial q_1} \theta R_1^1(\mathbf{q}) - r \{c_g - R_1^1(q_1, \bar{q}_2^S)\} - R_2^1(q_1, \bar{q}_2^S) \frac{\partial \bar{q}_2^S}{\partial q_1} \\ = \theta R_{11}^1(q_1, \bar{q}_2^S) + r \{R_{11}^1(q_1, \bar{q}_2^S) + 2R_{12}^1(q_1, \bar{q}_2^S) \frac{\partial \bar{q}_2^S}{\partial q_1} + R_{22}^1(q_1, \bar{q}_2^S) \frac{\partial^2 \bar{q}_2^S}{\partial q_1^2}\} < 0 \end{aligned}$$

This is guaranteed with a linear demand function ( $R_{22}^1(\cdot) = 0$ ).

<sup>19</sup>In fact  $q_2^S$  does not affect the size of  $\bar{q}_2^S$  as  $q_2^S$  satisfies the FOC above: differentiation yields in general

$$\frac{\partial \underline{B}^S}{\partial q_2} = -R_2^1(q_1, \bar{q}_2^S(\mathbf{q})) \frac{\partial \bar{q}_2^S}{\partial q_2}, \quad \{R_2^2(0, \bar{q}_2^S(\mathbf{q})) - c_2\} \frac{\partial \bar{q}_2^S}{\partial q_2} = \theta R_2^2(\mathbf{q}^S) + (1 - \theta)R_2^2(0, q_2) - c_2,$$

which is reduced in equilibrium to

$$\frac{\partial \bar{q}_2^S}{\partial q_2} = 0, \quad \text{and then} \quad \frac{\partial \underline{B}^S}{\partial q_2} = 0.$$

We therefore find that, with capital cost on the long-term loan, the entrant's output shrinks, and consequently the incumbent's expands. It is worth mentioning that this effect does not occur if the entrant is endowed with more liquidity than  $\underline{B}^S + F$  of the basic model, and otherwise it works regardless to the size of his endowment.

**Proposition 3''.** Suppose that the entrant's endowment is smaller than  $\underline{B}^S + F$  defined by (21) in the basic model and the Non-deviation condition<sup>20</sup>

$$\theta(R^1(\mathbf{q}^S) - c_g q_1^S) - r\underline{B}^S \geq R^1(R^1(\check{q}_1^S, q_2^S) - (\theta c_g + (1 - \theta)c_b)\check{q}_1^S),$$

where  $R^1(\check{q}_1^S, q_2^S) = \theta c_g + (1 - \theta)c_b$ , is satisfied. Then, a Separating equilibrium exists. The output levels  $\mathbf{q}^S$  are determined from the FOCs (39). Compared to the basic model and the fixed-cost model, the entrant's output level shrinks while the incumbent's expands.

### 5.2.2 Pooling equilibria

Fixed cost arises possibility of a Default-involved Pooling equilibrium, again. Its Non-deviation condition is

$$\phi R^1(\mathbf{q}^D) - (\theta c_g + (\phi - \theta)c_b)q_1^D + \phi(w_0 - F) - \phi r(\underline{B}^D - w_0) > \theta(R^1(\check{q}_1^D, q_2^D) - c_g \check{q}_1^D) + (w_0 - F), \quad (40)$$

where

$$R^1(\check{q}_1^D, q_2^D) = c_g.$$

Notice that, as the incumbent's output level fixed in Nash equilibrium, the entrant has no need to hold excess liquidity to avoid the predation in the deviation. Rearrangement of the inequality yields

$$(\phi - \theta)(R^1(\mathbf{q}^D) - c_b q_1^D) + (\phi - 1)(w_0 - F) - \phi r(\underline{B}^D - w_0) > \theta^{\textcircled{c}} (R^1(\check{q}_1^D, q_2^D) - c_g \check{q}_1^D) - (R^1(\mathbf{q}^D) - c_b q_1^D)^a.$$

By the way, the Default condition is<sup>21</sup>

$$R^1(\mathbf{q}^D) - c_b q_1^D < r(\underline{B}^D + F - w_0) - (w_0 - F), \text{ and} \\ R^1(\mathbf{q}^N) - c_b q_1^N < r(\underline{B}^N + F - w_0) - (w_0 - F). \quad (41)$$

The former yields<sup>22</sup>

$$\begin{aligned} & (\phi - \theta)(R^1(\mathbf{q}^D) - c_b q_1^D) + (\phi - 1)(w_0 - F) - \phi r(\underline{B}^D + F - w_0) \\ & < (\phi - \theta)\{r(\underline{B}^D + F - w_0) - (w_0 - F)\} + (\phi - 1)(w_0 - F) - \phi r(\underline{B}^D + F - w_0) \\ & = \phi - \theta r(\underline{B}^N + F - w_0) - (1 - \theta)(w_0 - F). \end{aligned}$$

Thus, if  $F$  is sufficiently larger than  $w_0$ , a Default-involved Pooling equilibrium may exist.

We confirm that, in a Non-default Pooling equilibrium, the Non-default condition restricts its existence as well as the Non-deviation condition. The former is

$$R^1(\mathbf{q}^N) - (\theta c_g + (1 - \theta)c_b)q_1^N + w_0 - \phi r(\underline{B}^N - w_0) > \theta(R^1(\check{q}_1^N, q_2^N) - c_g \check{q}_1^N) + w_0. \quad (42)$$

<sup>20</sup>In deviation, the incumbent's output level is fixed, so there is no need of excess liquidity.

<sup>21</sup>Notice that the latter does not necessarily imply the former. in contrast to the basic model.

<sup>22</sup>We focus to the case where  $q_1^D < q_1^N$  and  $q_2^D > q_2^N$  as before, so  $R^1(\mathbf{q}^D) - c_b q_1^D < R^1(\mathbf{q}^N) - c_b q_1^N$  holds in the model with interest on the Date-0 loan.

This is equivalent to

$$(1 - \theta)(R^1(\mathbf{q}^N - c_b q_1^N) > r(\underline{B}^N - w_0) + \theta^\circ (R^1(\tilde{q}_1^N, q_2^N) - c_g \tilde{q}_1^N) - (R^1(\mathbf{q}^N) - c_b q_1^N)^a,$$

while the Non-default condition:

$$R^1(\mathbf{q}^N) - c_b q_1^N > r(\underline{B}^N - w_0) - w_0. \quad (43)$$

Given the committed output levels  $\mathbf{q}$ , the minimum liquidity level in a Pooling equilibrium  $\underline{B}^P(\mathbf{q})$  is determined, in both a Non-default and a Default-involved Pooling equilibrium, such as

$$\underline{B}^P(\mathbf{q}) = c_b q_1 - R^1(q_1, \bar{q}_2^P(\mathbf{q})). \quad (44)$$

Here,  $\bar{q}_2^P(\mathbf{q})$  is the maximum output level for profitable predation:

$$\begin{aligned} \theta R^2(q_1, \bar{q}_2^P(\mathbf{q})) + (1 - \theta)R^2(0, \bar{q}_2^P(\mathbf{q})) - c_2 \bar{q}_2^P(\mathbf{q}) &= R^2(\mathbf{q}) - c_2 q_2, \text{ and} \\ \theta R_2^2(q_1, \bar{q}_2^P(\mathbf{q})) + (1 - \theta)R_2^2(0, \bar{q}_2^P(\mathbf{q})) - c_2 &< 0. \end{aligned}$$

Differentiation with regard to  $q_1$  yields

$$\begin{aligned} \frac{\partial \underline{B}^P}{\partial q_1} &= c_b - R_1^1(q_1, \bar{q}_2^P(\mathbf{q})) - R_2^1(q_1, \bar{q}_2^P(\mathbf{q})) \frac{\partial \bar{q}_2^P(\mathbf{q})}{\partial q_1}, \\ \frac{\partial \bar{q}_2^P}{\partial q_1} &= \frac{R_1^2(\mathbf{q}) - \theta R_1^2(q_1, \bar{q}_2(\mathbf{q}))}{\theta R_2^2(q_1, \bar{q}_2^P(\mathbf{q})) + (1 - \theta)R_2^2(0, \bar{q}_2^P(\mathbf{q})) - c_2}. \end{aligned}$$

As  $\theta$  approaches 0,

$$\frac{\partial \bar{q}_2^P}{\partial q_1} \longrightarrow \frac{R_1^2(\mathbf{q})}{R_2^2(0, \bar{q}_2^P(\mathbf{q})) - c_2} > 0.$$

[Fig. 6 enters here.]

The entrant's equilibrium output levels  $q_1^N, q_2^D$  are determined by the FOCs including the marginal capital cost:<sup>23</sup> in a Non-default Pooling equilibrium,

$$R_1^1(\mathbf{q}^N) - \{\theta c_g + (1 - \theta)c_b\} = r \frac{\partial \underline{B}^P}{\partial q_1}(\mathbf{q}^N),$$

$$\text{i.e. } R_1^1(\mathbf{q}^N) - \{\theta c_g + (1 - \theta)c_b\} + r\{R_1^1(q_1^N, \bar{q}_2^P(\mathbf{q}^N)) - c_b\} = r - R_2^1(q_1^N, \bar{q}_2^P(\mathbf{q}^N)) \frac{\partial \bar{q}_2^P}{\partial q_1}(\mathbf{q}^N); \quad (45)$$

in a Default-involved Pooling one,

$$\begin{aligned} R_1^1(\mathbf{q}^D) - \frac{1}{2} \frac{\theta}{\phi} c_g + \frac{\mu}{1 - \frac{\theta}{\phi} c_b} &= r \frac{\partial \underline{B}^P}{\partial q_1}(\mathbf{q}^D), \\ \text{i.e. } R_1^1(\mathbf{q}^D) - \frac{1}{2} \frac{\theta}{\phi} c_g + \frac{\mu}{1 - \frac{\theta}{\phi} c_b} + r\{R_1^1(q_1^D, \bar{q}_2^P(\mathbf{q}^D)) - c_b\} &= r - R_2^1(q_1^D, \bar{q}_2^P(\mathbf{q}^D)) \frac{\partial \bar{q}_2^P}{\partial q_1}(\mathbf{q}^D); \end{aligned} \quad (46)$$

In contrast, the incumbent's  $q_1^N, q_2^D$  is similar to the basic model: in either equilibrium,

$$R_2^2(\mathbf{q}^i) = c_2 \quad (i = N, D). \quad (47)$$

It follows from  $R_{12}^1(\cdot) \leq 0$  and  $q_2^i < \bar{q}_2^P(\mathbf{q}^i)$  ( $i = N, D$ ) that, in either Pooling equilibrium

$$\begin{aligned} R_1^1(\mathbf{q}^N) - \{\theta c_g + (1 - \theta)c_b\} &\geq R_1^1(q_1^N, \bar{q}_2^S(\mathbf{q}^N)) - c_b, \\ R_1^1(\mathbf{q}^D) - \frac{\theta}{\phi}c_g + 1 - \frac{\theta}{\phi}c_b &\geq R_1^1(q_1^D, \bar{q}_2^S(\mathbf{q}^D)) - c_b. \end{aligned}$$

Substituting the entrants FOCs into these two inequalities, we have

$$\begin{aligned} (1 + r) R_1^1(\mathbf{q}^N) - \{\theta c_g + (1 - \theta)c_b\} &\geq r - R_2^1(q_1, \bar{q}_2^P(\mathbf{q})) \frac{\partial \bar{q}_2^P}{\partial q_1}(\mathbf{q}), \\ (1 + r) R_1^1(\mathbf{q}^D) - \frac{\theta}{\phi}c_g + 1 - \frac{\theta}{\phi}c_b &\geq r - R_2^1(q_1^D, \bar{q}_2^P(\mathbf{q})) \frac{\partial \bar{q}_2^P}{\partial q_1}(\mathbf{q}^D). \end{aligned}$$

Assuming sufficiently small  $\theta$ , we obtain by  $\partial \bar{q}_2^P / \partial q_1 > 0$

$$R_1^1(\mathbf{q}^N) - \{\theta c_g + (1 - \theta)c_b\} > 0, \quad R_1^1(\mathbf{q}^D) - \frac{\theta}{\phi}c_g + 1 - \frac{\theta}{\phi}c_b > 0.$$

As a Separating equilibrium, in each Pooling equilibrium, the entrant's response curve shifts downward (near the equilibrium) by the capital cost.

We therefore conclude that the entrant's equilibrium output level shrinks, while the incumbent's expands, compared to the economy without interest, if the probability of the Good state is sufficiently low. Moreover, the entrant endowed with little liquidity is possibly driven out to a Default-involved equilibrium and shrinks further his output level<sup>24</sup> by the "adverse limited liability" effect.

**Proposition 1''.** Suppose that the entrant's endowment is smaller than  $\underline{B}^N + F$  defined by (14) in the basic model. If the Non-deviation condition (42) and the Non-default condition (43) are satisfied, then a Non-default Pooling equilibrium exists. If the Non-deviation condition (42) and the Default condition (43) are satisfied, a Default-involved Pooling equilibrium exists. The output levels  $\mathbf{q}^N$  are determined from the FOCs, (45) and (47). Compared to the basic model and the fixed-cost model, the entrant's output level shrinks while the incumbent's expands, under sufficiently small  $\theta$ .

**Proposition 2''.** Suppose that the entrant's endowment is smaller than  $\underline{B}^N + F$  defined by (14) in the basic model. If the Non-deviation condition (42) and the Default condition (43) are satisfied, a Default-involved Pooling equilibrium exists. The output levels  $\mathbf{q}^D$  are determined from the FOCs, (46) and (47). Compared to the fixed-cost model, the entrant's output level shrinks while the incumbent's expands, under sufficiently small  $\theta$ .

<sup>23</sup>As is in a Separating equilibrium, the entrant's best response is determined uniquely if the marginal sales minus the marginal capital cost decreases to the entrant's output level  $q_1$ , given the incumbent's  $q_2$ :

$$\begin{aligned} \frac{\partial}{\partial q_1} [R_1^1(\mathbf{q}) - r(c_b - R_1^1(q_1, \bar{q}_2^P))] - R_2^1(q_1, \bar{q}_2^P) \frac{\partial \bar{q}_2^P}{\partial q_1} \\ = R_{11}^1(\mathbf{q}) + r[R_{11}^1(q_1, \bar{q}_2^P) + 2R_{12}^1(q_1, \bar{q}_2^P) \frac{\partial \bar{q}_2^P}{\partial q_1}(\mathbf{q}) + R_{22}^1(q_1, \bar{q}_2^P) \frac{\partial^2 \bar{q}_2^P}{\partial q_1^2}(\mathbf{q})] < 0 \end{aligned}$$

This is guaranteed with a linear demand function ( $R_{22}^1(\cdot) = 0$ ) and a sufficiently low  $\theta$  ( $\theta \rightarrow 0$ ).

<sup>24</sup>This requires that the same condition as the entrant's unique best response.

## 6 Concluding Remarks

### Implication for Empirical Study and Competition Policy

We verify that even the entrant can avoid the “long-purse” incumbent’s predation by raising his liquidity holding by borrowing. The entrant’s output level however shrinks with interest charged on the loan. The entrant’s higher output invites more the incumbent’s predation, and consequently increases the minimum required level of liquidity. Thus the capital cost increases to the entrant’s output level. The threat of predation, namely the minimum liquidity level for non-predation, therefore reduces the entrant’s production.

Lerner (1995) studies 1980-88’s disk drive industry. After Hedonic regressions of products’ price and segmentation of product markets according to the drives’ attributes (mainly, diameters, densities, access time, products’ ages, and years of observation), he investigates whether price wars are triggered by entries of financially weak rivals.<sup>25</sup> The results are quite consistent with our predictions, though his observations are prices while ours are quantities (output levels). In 1980–83, which Lerner calls the era of “capital market myopia” quoted from Sahlman and Stevenson (1986), prices are wholly determined by the products’ attributes mentioned above; that is, there is no predatory pricing by large-purse incumbents. On contrary, in 1984–88, when entrepreneurs suddenly face difficulty in equity financing, predatory pricing is executed against these financially weak rivals.

The former result fits well with the prediction of our “basic” model that the entrant can avoid predation by raising enough liquidity. The latter corresponds that of our extension with interest that entrants facing capital cost (which I think represents the difficulty in external financing) shrinks his production by the threat of predation even if he actually avoids to exit the market. Although replication especially taking output level as the dependent variable and test in other product markets are required, Lerner’s empirical results confirms theoretical predictions of our model.

Our model has bold implication for competition policy. If entrants can easily access to capital market, especially venture capital who can arrange the form of financial contract with borrower’s circumstance, there is no need of Government’s aid to them or regulation of incumbents’ behavior: *laissez faire* solves the distortion induced by the incumbent’s predation through supplying sufficient excess liquidity. Otherwise, the Government has to remove threat of predation; for instance, a legal ban on incumbents’ price changes corresponding to entrants’ exit prevents the incumbents to prey with expectation of enjoying the monopoly profit.

### Further Extension Compared with Preceding Papers

Bolton and Scharfstein (1990) is the most distinguishable paper that correlates predation with financial contracting. They induce the optimal financial contract under threat of predation. Their model has two production periods, and the financial contract takes the advantage of threat of liquidation at the end of the first period to draw the borrower’s truth-telling on his state. This threat of liquidation in turn invites the rival’s predatory behavior. Thus the lender (which has all the bargaining power) increases the probability of liquidation to avoid predation, since the increased possibility of liquidation reduces the net benefit of predatory behavior.

Their prediction is apparently opposite to ours which the entrant borrows more under the threat of predation. This comes from difference in model building. The differences in

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<sup>25</sup>Lerner characterizes financial weakness of a firm in two aspects. First, the firm specializes in disk drive manufacturing, which means the absent of internal financing from other business. Second, the firm’s capital is below the median of the samples measured in its equity capital.

the meaning of predation is especially critical. Predation in their model is the entrant's exit at the second period, while in ours is the exit before the completion of the production at the "first period" in their term. So it is interesting to endogenize the "second period" continuation value, namely the Date-4 private benefit in our model.

Notice that in many theoretical models and empirical studies "predation" is thought as "predatory pricing" (Fudenberg and Tirole, 1986)<sup>26</sup> or not specified activity (Bolton and Scharfstein, 1990). Although we specify "predation" as excess supply in Cournot competition, the incumbent who supplies excessively intends to decline the price so that the entrant cannot pay the production cost and is forced to exit the market. Thus I believe that our model is not far from "predatory pricing",<sup>27</sup> and what the assumption of Cournot competition means in our model is only that firms must commit their output levels while price is flexible.

By the way, we have presented one of the reason why excess liquidity is required. Holmström and Tirole (1999) study a partial equilibrium of liquidity supply and demand between price-taking entrepreneurs and credit suppliers. Moral hazard causes a financial imperfection, in that an entrepreneur with positive NPV may be short in the short-term loan to cover the production cost and abandon the production. Appropriate credit line or borrowing liquidity in advance with covenant on the liquidity's usage (which is our Date-0 loan) solves partially the financial imperfection, while the Good-state firm holds excess liquidity which is not used for production. Holmström and Tirole argue on the redistribution of such an excess liquidity from the Good-state entrepreneurs to the Bad-state ones. If the aggregate supply of liquidity is exogenous or the firms' uncertainties are independent each other, financial intermediary serves the role of redistribution. Otherwise, the difficulty in redistribution of excess liquidity is problematic in macro economy.

While our model brings to monetary economics another *raison d'être* of excess liquidity, Holmström and Tirole's argument about the liquidity redistribution will give us another perspective of predation. Inefficient redistribution of excess liquidity caused by threat of predation, not by financial imperfection as Holmström and Tirole, possibly results in distortion of equilibrium outputs, without the assumption of interest on long-term loan. Multi-sector Cournot model with a financial intermediary is therefore another direction of our model's extension.

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<sup>26</sup>Although Telser (1966) also refers predation as "predatory pricing", the product market is not specified and the way to reduce the entrant's revenue is obscure.

<sup>27</sup>Poitevin (1989) construct a Cournot model of predation, which is also on the same line with ours.

## Appendix

### A Proof of Lemma 1

In the beginning, we reduce the truth-telling conditions (2), (3) and the lender's PC (4), substituting them into the entrant's expected profit (1). The truth-telling conditions (2), (3) are reduced to

$$-D_g + \beta_g \pi = -D_b + \beta_b \pi.$$

Substituting this into the lender's PC (4), we have

$$-D_b = (1 - \beta_b)L + \theta(\beta_g - \beta_b)(\pi - L) - (B - w_0).$$

Substituting these two equation into the entrant's expected profit (1) again, we obtain

$$\max \theta y_g + (1 - \theta)y_b + L + (\theta\beta_g + (1 - \theta)\beta_b)(\pi - L) - (B - w_0). \quad (48)$$

Since higher  $\beta_g, \beta_b \in [0, 1]$  yield obviously higher value of the objective function,  $\beta_g = \beta_b = 1$  is the would-be solution. This brings  $D_g = D_b = B - w_0$ . If  $y_b \geq B - w_0$ , this solution satisfies the limited-liability constraint (5), and is the feasible solution of (48).

Otherwise, the limited-liability constraint (5) restricts feasible values of  $\beta_g, \beta_b$ :

$$D_b = \beta_b(\theta\pi + (1 - \theta)L) - \theta\beta_g(\pi - L) + B - w_0 - L \leq y_b.$$

Since the solution of (48) must bind this, we have

$$D_b = y_b, \quad \beta_b = \frac{\phi}{\pi} \{y_b - (B - w_0) + L + \theta\beta_g(\pi - L)\},$$

which implies that higher  $\beta_b$  corresponds to higher  $\beta_g$ . Thus, we find that the solution of (48) with (5) is  $\beta_g = 1$  and

$$\beta_b = 1 - \frac{\phi}{\pi}(B - w_0 - y_b),$$

which yields

$$D_g = \phi(B - w_0) + (1 - \phi)y_b.$$

(Q.E.D.)

### B Proof of Lemma 2

The truth-telling conditions (7) and (8) allows some freedom of the solution, in contrast to those of a Pooling contract:

$$y_b - B \leq (D_g - D_b) - (\beta_g - \beta_b)\pi \leq y_g - B \quad (49)$$

The lender's PC (9) is equivalent to

$$\theta D_g + (1 - \theta)D_b = B - w_0 + (\theta\beta_g + (1 - \theta)\beta_b - 1)L.$$

Substituting this into the entrant's expected profit (6), we have

$$\max \theta y_g + (1 - \theta)B + L + (\theta\beta_g + (1 - \theta)\beta_b)(\pi - L) - (B - w_0). \quad (50)$$

$\beta_g = \beta_b = 1$  is a would-be solution. Under the assumption that  $y_g \geq B - w_0/\theta$ ,

$$D_g = \theta B + (1 - \theta)y_g - w_0 \leq y_g, \quad D_b = B - w_0 - \theta(y_g - B) \leq B$$

with  $\beta_g = \beta_b = 1$  satisfies the reduced truth-telling condition (49), the lender's PC (9), and the limited-liability constraints (10). This is therefore one of the feasible solution of (50).

(Q.E.D.)



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Fig. 1. The Non-predation condition w.r.t  $B$  for a Pooling eqm.

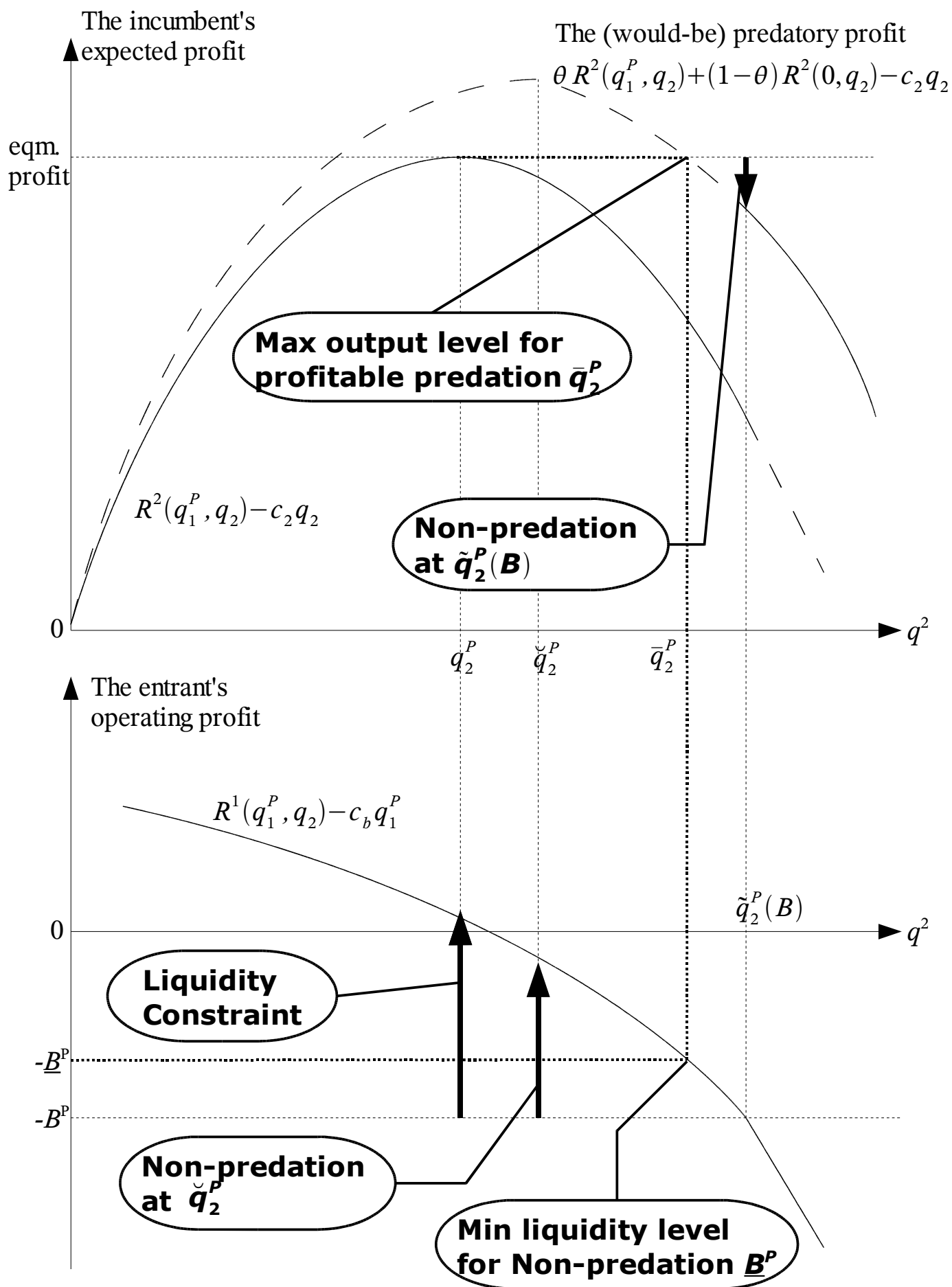
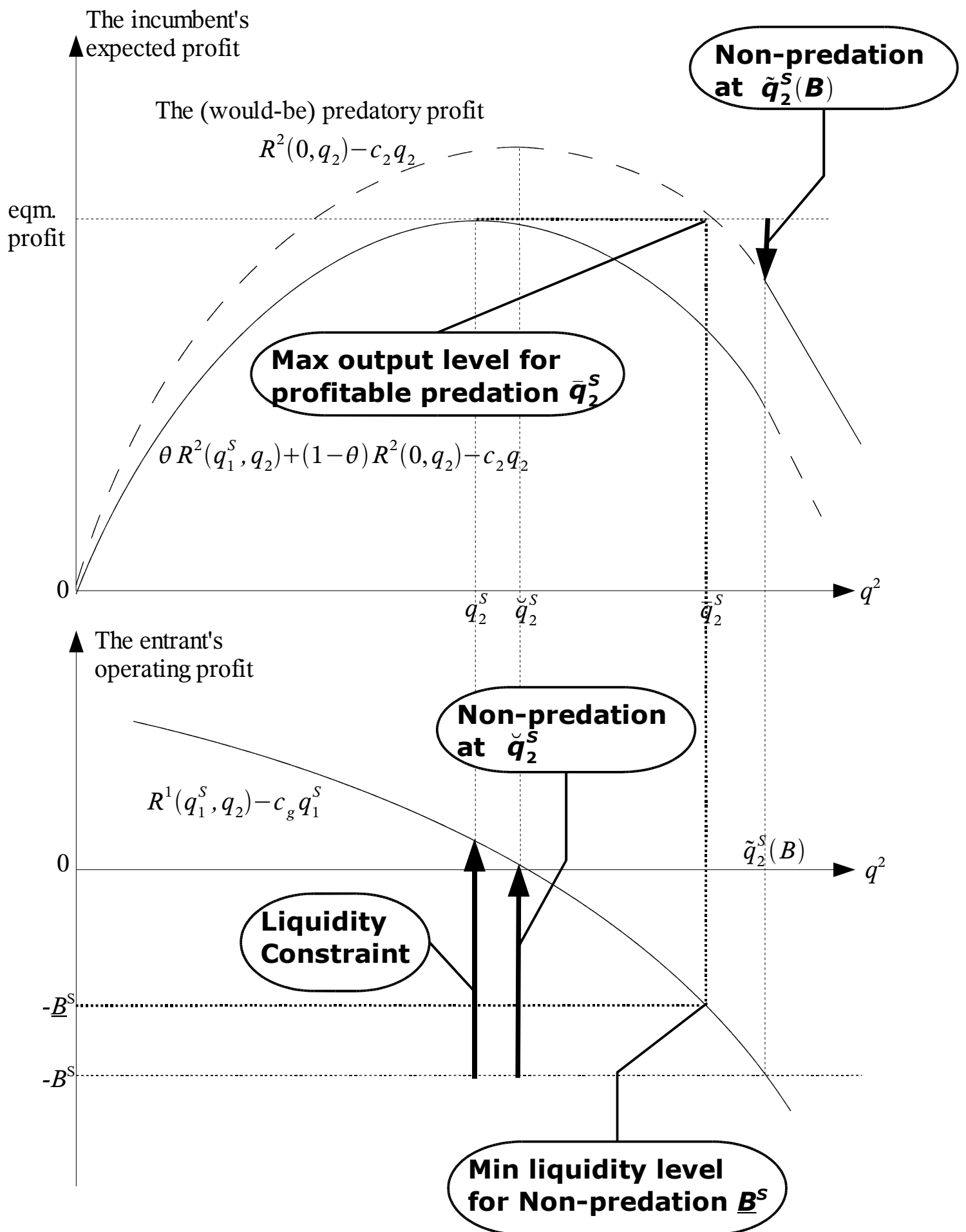


Fig. 2. The Non-predation condition w.r.t  $B$  for a Separating eqm.



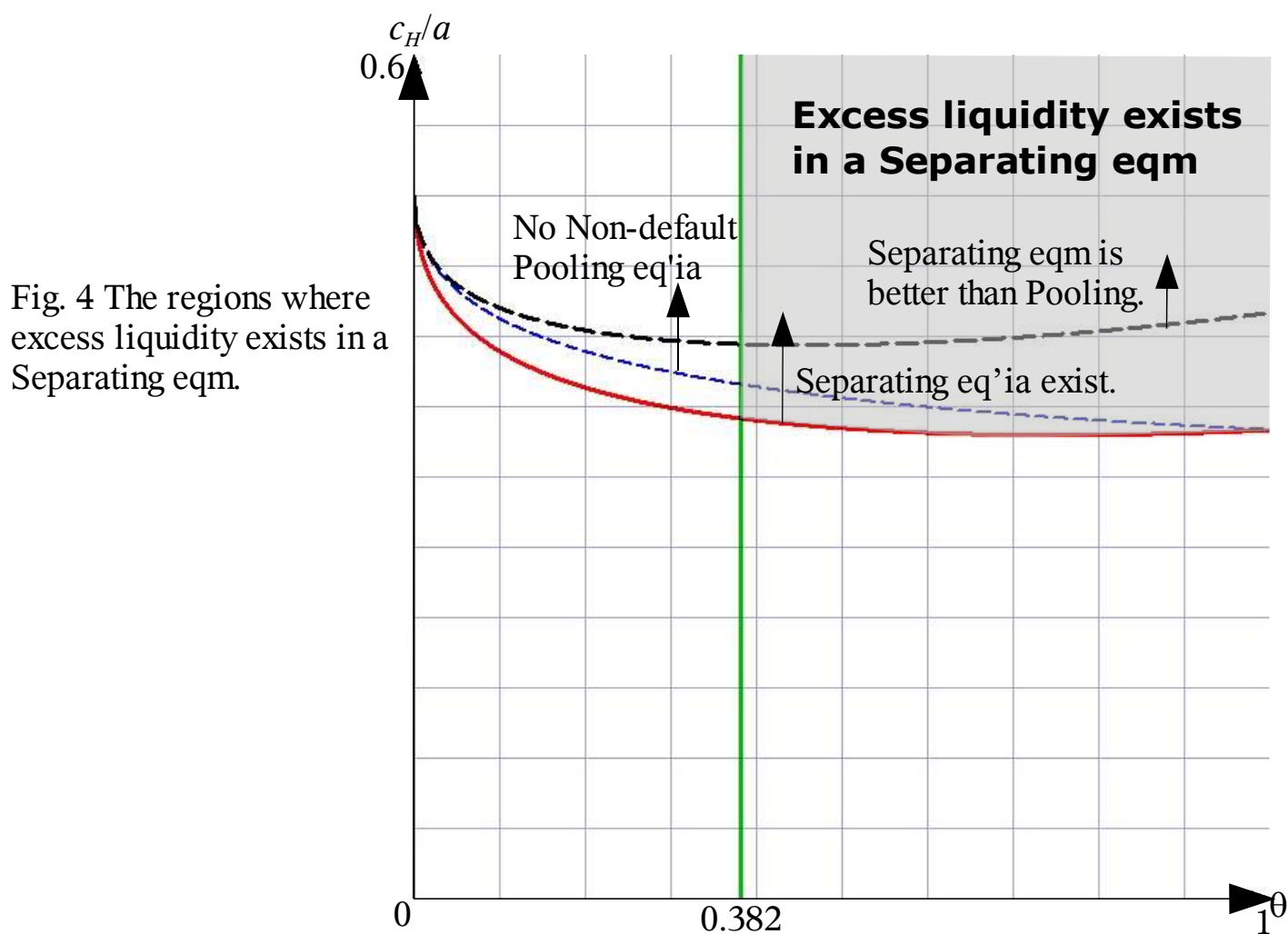
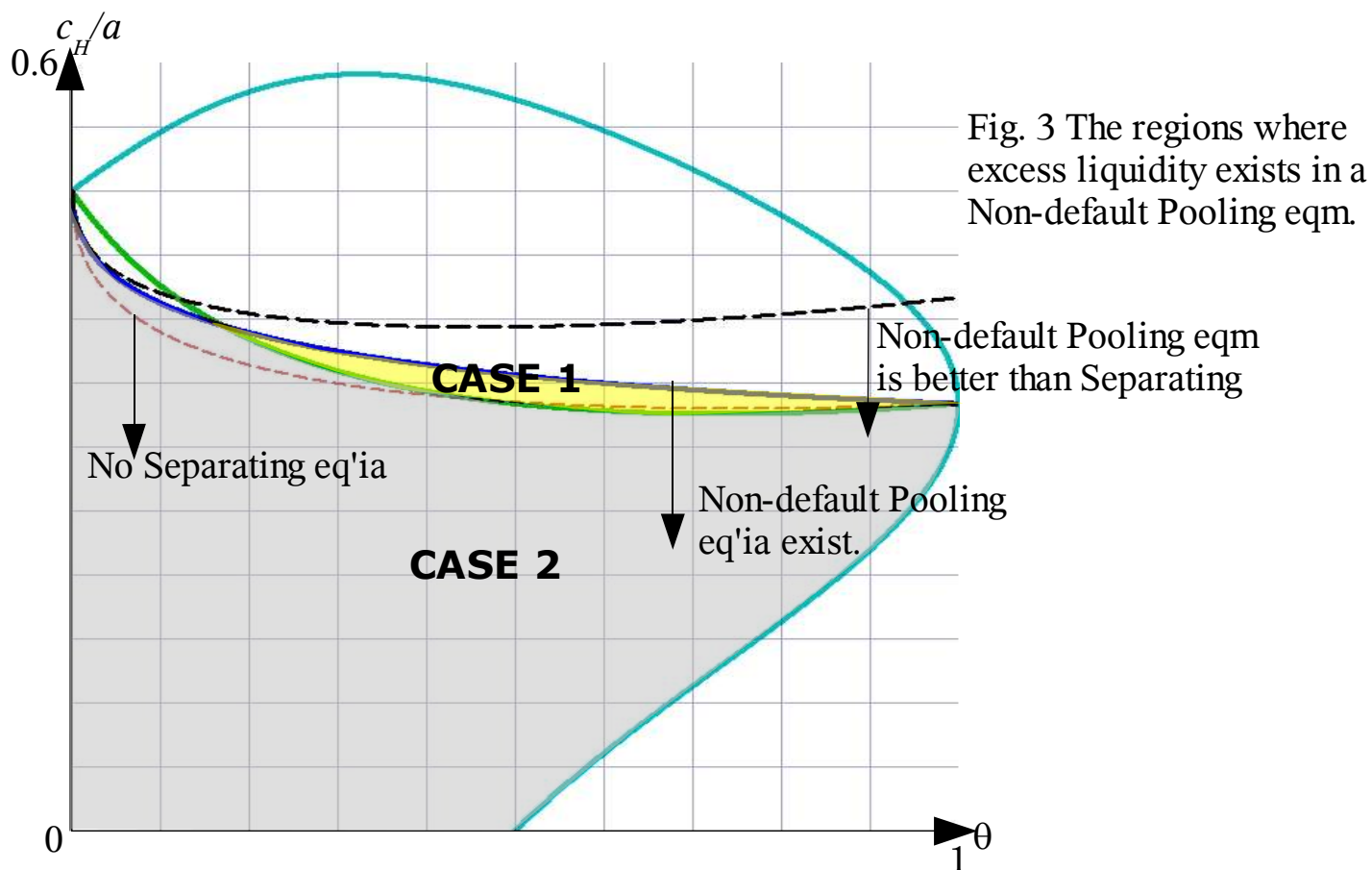


Fig. 5. Marginal operating profit and capital cost in a Separating eqm.

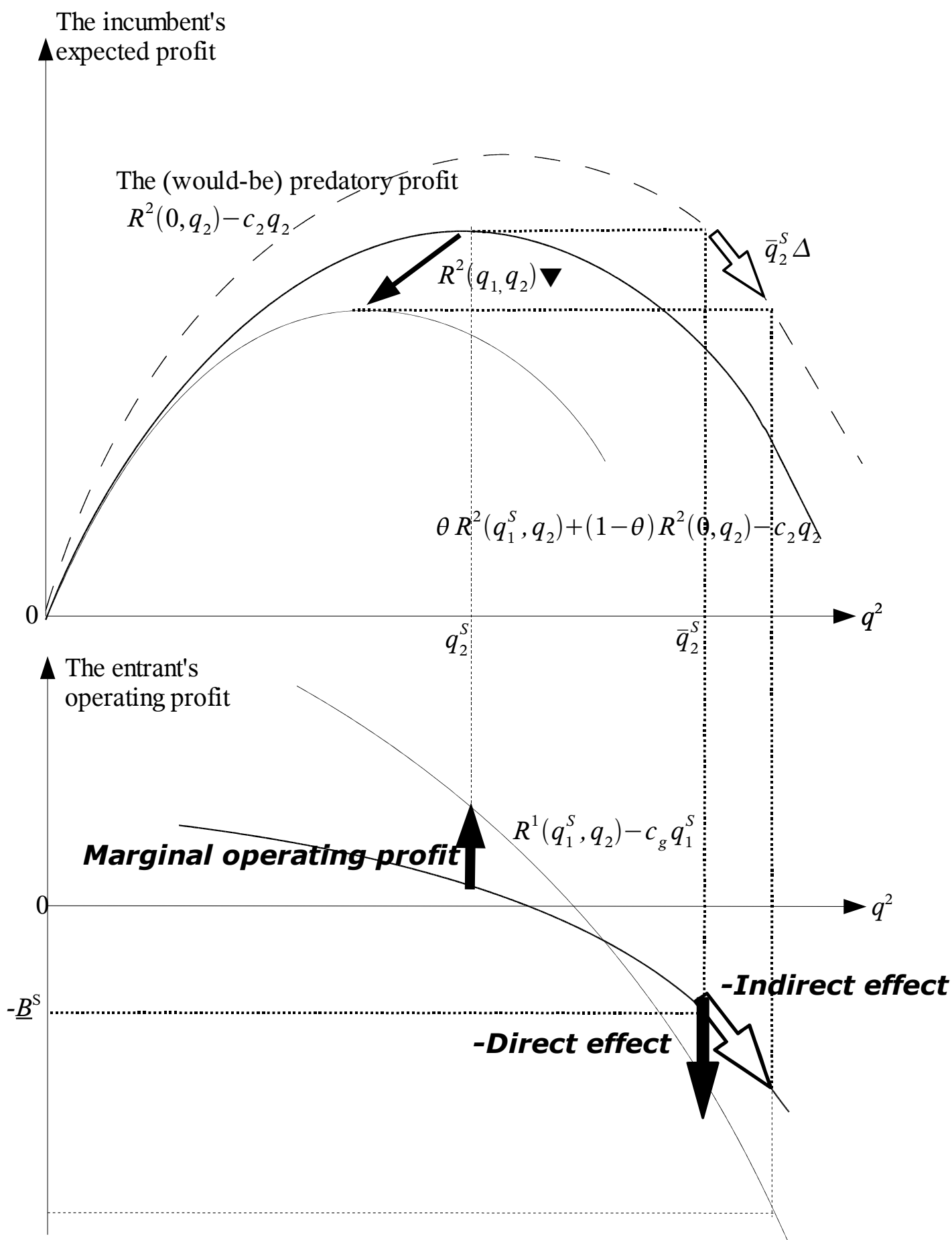


Fig. 6. Marginal operating profit and capital cost in a Pooling eqm.

