Storable good monopoly: the role of commitment

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PRELIMINARY DRAFT

Abstract

We study dynamic monopoly pricing of storable goods in an environment where demand changes over time.

The literature on durables has focused on incentives to delay purchases. Our analysis focuses on a different intertemporal demand incentive. The key force on the consumer side is advance purchases or stockpiling. In the case of storable goods the stockpiling motive has been documented in recent empirical literature. Advance purchases can also arise in the case of durables, although the literature has not focused on this case.

We show that if the monopolist cannot commit, then prices are higher in all periods, and social welfare is lower, than in the case in which the monopolist can commit. This is in contrast with the analysis in the literature on the Coase conjecture.

1 Introduction

According to most standard models in industrial organization a change in the price of a good at some future date has no effect on current incentives for producers because current consumer behavior is only affected by current prices. However, a large fraction of production involves goods for which intertemporal demand incentives may play a large role. A large literature considers one important consequence of durability. Durability can generate a special kind of intertemporal demand incentives, namely the incentive to postpone purchases in the

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expectation of better deals in the future. This emerges most starkly in the Coase conjecture (Coase 1972, Gul, Sonneschein, and Wilson 1986) where, under certain circumstances, durability and intertemporal demand incentives combine to generate a striking contrast with textbook monopoly analysis: if consumers are patient, or transactions can occur quickly, the power of the monopolist to extract surplus is completely undermined, and the monopoly distortion disappears. Although many papers in this literature deliver less extreme outcomes, (e.g., Sobel 1991), a consistent picture has emerged: when goods are durable monopoly power may not be as bad as in the textbook model because lack of commitment undermines monopoly power leading to lower prices and higher welfare.

We focus on an alternative intertemporal demand incentive which has been neglected in the durable goods literature: the incentive to anticipate purchase of to stockpile in the expectation of higher future prices. These incentives are present in numerous markets. Several recent empirical studies (in particular, Erdem, Keane, and Imai (2002) and Hendel and Nevo 2004, a and b) have documented that the timing and size of purchases by consumers responds to the timing of price changes in a quantitatively important way that can be attributed at least in part to stockpiling motives.[1] This evidence pertains mostly to groceries, but there are a large number of other goods for which it is at least plausible to think that stockpiling motives may be important (oil, and various intermediate goods, see Hall and Rust 2004 for a detailed study).

This paper provides an analysis of monopoly pricing in the presence of a stockpiling motive. In our model, goods are storable and demand varies deterministically over time. We characterize the optimal price sequence when the monopolist can commite and contrast this with the equilibrium when the monopolist does not commit. We show that, under certain conditions, lack of commitment leads to unifomrly higher prices (i.e., in all periods), to lower welfare is lower, and to higher wasteful storage. This result is in stark contrast with the results in the literature on the Coase conjecture: a monopoly analysis that takes into account another form of intertemporal demand incentives finds worse distortions than the standard textbook model.

To gain an initial intuition for our main result, consider a two period problem. Suppose that the marginal cost of storage is constant and is smaller than the difference between the static monopoly prices. If the monopolist charged the static monopoly price in each period, consumers would have an incentive to purchase more in the first period and stockpile for second period consumption. In the equilibrium with commitment the monopolist counters this consumer incentive by announcing current and future prices such that the difference between prices is small enough to ensure that consumers do not stockpile. The intuition is that the monopolist makes sure that consumers pay at the second period what they would

[1] Hendel and Nevo point out that accounting properly for purchases due to stockpiling is important for distinguishing between short-run and long-run elasticities.

See also Aguirregabiria (1999), Pesendorfer (2002), and Rust (2002).

otherwise spend for storage. Subject to the requirement that the difference in prices is equal to the marginal cost of storage, the monopolist chooses in the first period the price at which the sum of the marginal revenues is equal to zero.

Suppose now that the monopolist lacks commitment. In equilibrium the difference between the two prices is still equal to the marginal cost of storage. Yet, if consumers did not store in the first period the monopolist would charge the static monopoly price in the second period generating an incentive to stockpile. Equilibrium storage and the second period price are jointly determined: given the first period price, equilibrium storage has to be such that future demand is reduced to a point where the monopolist will have the incentive to charge a price that justifies that level of storage. In the first period the monopolist chooses the price that equates the sum of marginal revenue to the marginal loss in future profits due to an increase in stockpiling. The first period price (and consequently the second period price) is then higher than under commitment.

The key effect is that, absent commitment, in the second period, the monopolist does not take into account the fact that an increase in the second period price increases storage, thereby shifting sales to the first period when prices are lower. Thus, every increase in the second period price leads to a loss in the profit margin proportional to the difference between second and first period prices. When the monopolist can commit, this effect is taken into account, leading to overall lower prices, higher profits, lower storage, and higher welfare.

The analysis of this paper may lead to a more cautious evaluation of contracts that enhance a firm's commitment ability: the policy advice that emerges from the literature on the Coase conjecture is to be suspicious of any contractual arrangements, such as rental or leasing contracts, that enhance commitment since these may restore monopoly power, and lead to higher prices and lower welfare. In contrast, in our model, enhancing a monopolist's ability to commit may lead to lower prices and reduce wasteful storage.

We have focused most of our analysis on storable goods that have the feature that they are perishable in consumption but can be stored for future consumption (e.g., canned foods, laundry detergent, soft drinks, gasoline...). This provides a particularly stark scenario because in the case of storables, the *only* intertemporal demand incentive is stockpiling. Storability does not generate demand postponement incentives because, when prices are dropping, there is no incentive to store. In contrast, in the case of durables, stockpiling incentives and demand postponement incentives can be present at different times in the same market due to demand seasonality. In these circumstances the overall effect of commitment on prices and welfare is difficult to assess since it depends on the exact nature of the cycle. The previous literature has assumed away the possibility of stockpiling incentives in durables. However, we show that even in the case of durables there are plausible environments in which lack of commitment can lead to higher prices. More generally, our analysis identifies an effect which suggests that in environments with intertempor demand fluctuations, which are common in many markets, Coase's stark predictions can be significantly altered.

2 Related Literature

Anton and Das Varma (2003) study a two period duopoly model in which consumers can store first period purchases. They study the impact of storability on the intertemporal price path. Thet find that prices increase over time if consumers are patient and storage is affordable. The low initial prices are a consequence of the firms' incentive to capture future market share from their rival. In contrast to the duopoly case, the demand shifting incentives do not show up under monopoly or competition. Under these market structures there is no incentive to capture future market share, so the price dynamics are absent.

Several theoretical papers offer models of price dispersion (Varian (1980), Salop and Stiglitz (1982), Narasimhan (1988) and Rao (1991)), interpreted as sales, however, these are competitive models and they do not capture the dynamics of demand generated by sales. Hong, McAfee and Nayyar (2000) is a competitive industry model, where consumers are assumed to chose a store based on the price of a single item and can store up to one unit. Jeuland and Narasimhan (1985) present a model in which storability may allow a monopolist to price discriminate among consumers because of a negative correlation between demand and cost of storage.

There is a vast literature on durable goods.[2] The literature that is most related to our paper is the one on the Coase conjecture. This literature started with a paper by Coase (1972). Bulow (1982) and Gul, Sonnenschein, and Wilson (1986) are two of the early papers that provided a formal analysis of Coase's conjecture. These papers discuss models in which a good is perfectly durable, consumers have unit demands, and differ in their valuations for the good. The Coase conjecture states that the seller's ability to extract surplus from a buyer can be completely undermined by an inability to commit not to make more attractive offers in the future. Specifically, if buyers are very patient, or offers can be made very quickly, the seller will, in equilibrium, offer very low prices from the beginning of the game. In the limit, the initial price (and hence the profits of the seller) converge to the lower bound of the valuations of the buyer, and almost all consumers purchase almost immediately. This implies that the equilibrium is asymptotically efficient.

Sobel (1991) (see also Conlisk, Gerstner, and Sobel 1984, Sobel 1984, and Board 2004) describes a model of a market with a durable good monopolist in which, at every date a mass of new consumers enter. Consumers have unit demands and two possible valuations for the good. Sobel (1991) characterizes the set of equilibria under the assumption that the monopolist cannot commit. Board (2004) assumes that the monopolist commits and allows for a more general time path of entry of consumers. An important feature of the analysis in this strand of the literature is the possibility of price cycles, namely sales. There are several dimensions in which our analysis differs from this literature.In particular, we focus on a different effect: demand anticipation and we obtain different results on the effect of

[2] For a recent survey see Waldman (2003).

commitment.

3 Example

We first present a simple example to give an initial intuition for the basic forces at play in this environment. The next section considers the general environment.

Assume that there are two periods and that demand increases between periods one and two. Specifically, in the first period demand is $D_1(p) = 1 - p$. in the second period demand is: $D_2(p) = 2 - p$. Assume that a consumer can store between periods 1 and 2 at a cost of $c(S) = cS$ when he stores S units. Assume that costs of production are zero. Absent storage, the optimal solution is $p_1 = \frac{1}{2}$, $p_2 = 1$. If $c > \frac{1}{2}$ this solution is sustainable because at those prices no consumer would choose to store.

In the remainder of this section we assume that $c < \frac{1}{2}$.

3.1 Commitment

Consider the following maximization problem for the seller:

$$\begin{aligned} \max_{p_1,p_2} \pi(p_1, p_2) &= (1 - p_1 + S)\, p_1 + (2 - p_2 - S)\, p_2 \\ s.t.\ p_2 - p_1 &= c. \end{aligned}$$

We will show that the solution to this problem, with $S = 0$ is the monopoly solution under commitment.

The solution of this problem is

$$\begin{aligned} p_1^c &= \frac{3}{4} - \frac{1}{2}c \\ p_2^c &= \frac{3}{4} + \frac{1}{2}c. \end{aligned}$$

At these prices consumers are indifferent between storing and not storing. However, in equilibrium, storage must be zero. To see this, observe first that profits are decreasing in S: the more the consumer stores, the more the monopolist sells second period consumption at a price of p_1 instead of at the price of p_2; the monopolist loses c for every additional unit of S. But the monopolist can induce zero storage by reducing p_2 slightly below $\frac{3}{4} + \frac{1}{2}c$ while leaving p_1 unchanged at $p_1 = \frac{3}{4} - \frac{1}{2}c$. These two facts imply that it must be the case that storage is zero in an equilibrium with commitment.

3.2 No commitment

Note first that the commitment solution cannot be an equilibrium without commitment. To see this, assume that $p_1 = \frac{3}{4} - \frac{1}{2}c$, and that $S = 0$. Then, in the second period the monopolist will charge the static monopoly price $p_2 = 1$. Furthermore, for $c < \frac{1}{2}$, we then have $p_2 - p_1 > c$ implying that S cannot be zero.

In order to construct an equilibrium without commitment, suppose that all consumers have stored $S \geq 0$ units in period 1. Then, in equilibrium, the monopolist will choose the second period price to maximize

$$V_2(p_2, S) = (2 - p_2 - S)\, p_2.$$

The solution is

$$p_2(S) = 1 - \frac{1}{2}S. \tag{1}$$

Given p_1, for S to be interior, it must be the case that

$$p_2(S) = p_1 + c \tag{2}$$

so that consumers are indifferent between storing and not storing.

This implies that

$$S(p_1) = 2 - 2p_1 - 2c \tag{3}$$

and that second period profits are $V_2(p_1) = (p_1 + c)^2$.

Thus, at the first period the present value of profits is:

$$V_1(p_1) = (1 - p_1 + 2 - 2p_1 - 2c)\, p_1 + (p_1 + c)^2.$$

which is maximized by

$$p_1^{nc} = \frac{3}{4}. \tag{4}$$

This allows us to obtain the equilibrium amount of storage from equation (3)

$$S^{nc} = \frac{1}{2} - 2c. \tag{5}$$

Since $S^{nc} \geq 0$ this equation implies that a necessary condition for this construction to characterize an equilibrium is $c \leq \frac{1}{4}$. Substituting S^{nc} into equation (1) we obtain the equilibrium second period price.

$$p_2^{nc} = \frac{3}{4} + c. \tag{6}$$

Equations (4),(6), and (3) characterize the equilibrium when $0 < c \leq \frac{1}{4}$.

In the region where $\frac{1}{4} < c < \frac{1}{2}$ the boundary condition $S \geq 0$ is binding, implying that $S_b^{nc} = 0$. Thus, the second period price must be the static monopoly price $p_{2b}^{nc} = 1$. The first period price is given by $p_{1b}^{nc} = 1 - c$. To see why this must be the case, note first that $p_2 - p_1 \leq c$ otherwise consumers will purchase all their consumption in the first period (which clearly cannot be optimal). Furthermore, for $c < \frac{1}{2}$, $1 - c > \frac{1}{2}$ implying that p_{1b}^{nc} is larger than the static monopoly price corresponding to first period demand. Thus, the monopolist clearly has no incentive to choose $p_1 > 1 - c$.

3.3 Comparison

Consider first the case in which $c \leq \frac{1}{4}$. In this case,

$$p_1^{nc} - p_1^c = p_2^{nc} - p_2^c = \frac{1}{2}c.$$

Thus, prices are uniformly higher without commitment. Furthermore, while under commitment storage is zero, absent commitment, when $c < \frac{1}{4}$, storage is positive. Note that storage here is unambiguously wasteful: consumers are indifferent between purchasing second period consumption in the first period at an effective price of $p_1^{nc} + c$ or purchasing consumption in the second period at a price of $p_2^{nc} = p_1^{nc} + c$. The monopolist however would obviously prefer it if consumers purchased in the second period.

Thus, profits, consumer surplus, and social welfare are all higher under commitment.

When $\frac{1}{4} < c < \frac{1}{2}$, we had a boundary solution for storage. In this case,

$$p_1^{nc} - p_1^c = p_2^{nc} - p_2^c = \frac{1}{4} - \frac{1}{2}c > 0.$$

Thus, prices are again lower under commitment, and, again, profits, consumer surplus, and social welfare are all higher under commitment. There is however no longer a difference in storage between the two scenarios.

It is interesting to note that the difference in prices between the two commitment scenarios is not monotonic in c. This can be understood by considering two extreme cases. If $c = 0$, then $p_1^c = p_2^c = p_1^{nc} = p_2^{nc} = \frac{3}{4}$. Because storage is costless prices cannot increase between the two periods: the monopolist is effectively selling in one period to the aggregate demand $3 - 2p$ and there is no difference between the two commitment scenarios. When $c > \frac{1}{2}$, then $p_1^c = p_1^{nc} = p_1^m = \frac{1}{2}$, $p_2^c = p_2^{nc} = p_2^m = 1$. When storage is very expensive, static monopoly prices prevail and, again, there is no difference between the two commitment scenarios. When $0 < c < \frac{1}{2}$, then there is a difference between the two scenarios. Thus, the difference in prices between the two commitment scenarios is not monotonic in c. This difference is maximal at $c = \frac{1}{4}$ which is the boundary value for positive storage under no commitment.

4 General Analysis

In this section we develop and analyze dynamic pricing incentives by a monopolist with changing demand for a storable good under two scenarios: commitment and lack of commitment.

4.1 The model

A monopolist faces a demand for a storable good in each one of T periods. For simplicity we assume that there is no cost of production and no discounting.

At each period t the monopolist can take two actions: post a price $p_t \geq 0$ and be willing to sell at that price, or post no price $(\emptyset_t)$ and shut down.

Demand in each period t comes from a continuum of identical consumers[3] whose utility is quasi-linear in the consumption of the good, x_t, and money

$$U_t(x_t, m_t) = u_t(x_t) + m_t.$$

We initially assume that the cost of storage is linear $c(S) = cS$.[4] At each date t, given any sequence of prices $p_t, ..., p_T$ and their current inventory, S_{t-1}, consumers choose purchases q_t (if the market is open), consumption levels c_t and storage levels S_t to maximize

$$\sum_{i=t}^{T} [U_i(x_i, m_i) - q_i p_i - cS_i]$$

subject to

$$q_i = x_i + S_i - S_{i-1}.$$

Let $D_t(p_t)$ be the static demand function associated with this environment, i.e., the maximizer of $u_t(q) - qp_t$. Preferences are assumed to be sufficiently regular that the resulting demand functions $D_t(p_t)$ are such that the revenue functions $TR_t(p_t) = D_t(p_t)p_t$ are twice continuously differentiable and the marginal revenue functions $MR_t(p_t)$ are strictly decreasing. Denote by p_t^m the static monopoly price at period t (the maximizer of $TR_t(p)$). We assume that $p_t^m < p_{t+1}^m$ for all $t = 1, ..., T-1$. Thus, demand is such that static monopoly prices are increasing over time. We consider the case of fluctuating demand in Section 5.1. We also assume that $D_t(p_T^m) > 0$ for every t and that $c < \min_t \{p_t^m - p_{t+1}^m\}$ for all $t = 1, ..., T-1$. The first assumption merely guarantees that the monopolist never shuts down any market. The second assumption ensures that the incentives of the monopolist are affected by the

[3] The case of heterogeneous consumers raises a number of complications because of the aggregation of storage decisions. We discuss this case briefly in Section 5.

[4] This assumption is relaxed in Section 5.2.

presence of storage. Dealing with the case in which this condition is not satisfied in every period is straightforward but tedious. Nothing of substance is affected by this assumption.

In equilibrium, it turns out that the storage decision of the consumer at period t only depends on the prices at periods t and $t+1$: the current and next period prices. However, in order to characterize the equilibrium, optimal storage decisions must be defined for all possible prices, in which case, period t storage decisions can depend on the sequence of all future prices. We now describe how consumers' optimal storage decisions are made given a sequence of prices. Most of this discussion is only used in the proofs of lemmas 6 and 8 in the appendix.

Given a sequence of equilibrium prices $\{p_t\}_{t=1}^{T}$ (we exclude mixed strategies here), at $t=1$ storage S_1 is determined as follows:

If $p_2 - p_1 < c$ then $S_1 = 0$.

If $p_2 - p_1 \geq c$, let τ^* be the lowest period t with $t \geq 3$ such that $p_{\tau^*} - p_1 < (\tau^* - 1)c$. Furthermore, define $D(2, \tau^*) \equiv \sum_{t=2}^{\tau^*-1} D_t(p_1 + (t-1)c)$. Then

$$S_1 = \begin{cases} [0, D(2,\tau^*)] & \text{if } p_2 - p_1 = c \\ D(2,\tau^*) & \text{if } p_2 - p_1 > c \end{cases}$$

In words, when profitable for the consumer to store (i.e., when $p_2 - p_1 \geq c$) we need to figure out the date of the next purchase, τ^*. Today's storage, $D(2,\tau^*)$, is the sum of planned consumption from $t=2$ to τ^*-1, given an effective price equal to p_1 plus the cost to store up to the respective date, $(t-1)c$.

Given S_{t-1}, we can then obtain the optimal S_t, for $t = 2, ..., T-1$, as follows. Let $V(S_{t-1}, \tau)$ be the value of the following maximization problem.

$$\max_{\{x_k\}_t^\tau} \sum_{k=t}^{\tau} (u_k(x_k) - c(k-t)x_k) \tag{7}$$

$$s.t. \quad \sum_{k=t}^{\tau} x_k \leq S_{t-1} \quad .$$

$V(S_{t-1}, \tau)$ is the highest utility the consumer can achieve consuming optimally out of her starting storage S_{t-1} between period t and τ (namely, without purchasing during that time).

Denote by τ^* the first $\tau \geq t$ such that $p_\tau \leq \frac{V(S_{t-1},\tau)}{\partial S_{t-1}} + (\tau - t)c$. Thus, τ^* is the first period after t in which the consumer purchases a positive quantity (note that $\tau^* = t$ is allowed).

At every period k such that $t \leq k \leq \tau^*$ storage is determined by the following condition

$$S_k = S_{k-1} - x_k$$

where x_k is obtained from the solution to the problem in equation 7. Next, denote by $\tau^{**} > \tau^*$ the first period such that $p_{\tau^{**}} - p_{\tau^*} < (\tau^{**} - \tau^*)c$. Namely, τ^{**} is the first date after τ^* in which the consumer would like to make a purchase. Define $D(\tau^{**}, \tau^*) \equiv \sum_{k=\tau^*+1}^{\tau^{**}-1} D_k(p_{\tau^*} + (k - \tau^*)\,c)$. $D(\tau^{**}, \tau^*)$ is the counterpart of $D(2, \tau^*)$, namely, the sum of planned consumption until the next purchase. Then

$$S_{\tau^*} = \begin{cases} [0, D(\tau^{**}, \tau^*)] & \text{if } p_{\tau^*+1} - p_{\tau^*} = c \\ D(\tau^{**}, \tau^*) & \text{if } p_{\tau^*+1} - p_{\tau^*} > c \end{cases}$$

Note that, for $p_{\tau^*+1} - p_{\tau^*} > c$, S_{τ^*} is continuous and differentiable in p.

Finally, for $\tau^{**} > j > \tau^*$, we have

$$S_j = \sum_{k=j+1}^{\tau^{**}-1} D_k(p_{\tau^*} + (k - \tau^*)\,c).$$

4.2 Commitment

Under commitment, the monopolist chooses a sequence of either prices p_t or shut down decisions $\emptyset_t$ to maximize total profits

$$V(\sigma) = \sum_{t=1}^{T} [D_t(p_t) - S_{t-1} + S_t] p_t$$

where $S_0 \equiv S_T \equiv 0$ and, for $t = 1, ..., T-1$, S_t is specified as in section 4.1.

Although the reasoning is more elaborate, the appendix shows that the essential properties of the commitment equilibrium generalize beyond the example presented in Section 3: prices must increase at rate c and storage is zero. The intuition is that in equilibrium consumers should not anticipate stockpile and pay for storage. If they did the monopolist could announce a slight reduction of future prices and induce consumers to postpone purchases and pay a higher price rather than the cost of storage.

Furthermore, given our assumption that $D_1(p_T^m) > 0$, it is never optimal to shut down any market.

These properties imply that in an equilibrium with commitment $p_t^c = p_1^c + (t-1)c$ for $t = 1, ..., T$ and $S_t = 0$ for $t = 1, ..., T-1$.

Thus, the entire strategy of the monopolist is identified by p_1^c. In particular, under commitment the problem of the monopolist is to choose price p_1 to maximize total profit

$$V_1(p_1) = \sum_{t=1}^{T} D_t(p_1 + (t-1)c)(p_1 + (t-1)c).$$

This leads to the following proposition.

Proposition 1 *Let p_1^c be the unique solution of*

$$\sum_{t=1}^{T} MR_t(p_1^c + (t-1)c) = 0. \tag{8}$$

Then, the sequence $\{p_t^c = p_1^c + (t-1)\,c\}_{t=1}^{T}$ is the unique equilibrium sequence of prices under commitment. In equilibrium storage is zero for all t.

An obvious consequence of this characterization is that $p_1^c > p_1^m$ and $p_T^c < p_T^m$.

Observe that in the subgame starting at each period t the sequence of prices $\{p_t^c\}$ is not optimal. Consider for instance the last period. Given that $S_{T-1} = 0$ the monopolist has an incentive to increase his profits by charging p_T^m instead of p_T^c. Thus, the optimal sequence of prices under commitment never constitutes an equilibrium of the game without commitment.

4.3 No Commitment

We will first construct the equilibrium in the case in which storage is interior. As in the example (Section 3) this holds in equilibrium if c is not too high. This construction is necessary to then extend the analysis to the case in which storage may be zero.

Consider the final period problem. Define

$$V_T(p, S) \equiv [D_T(p) - S_{T-1}]p.$$

Given S_{T-1}, in period T the monopolist chooses a price p_T to maximize $V_T(p, S_{T-1})$. The optimal price $p_T^{nc} = p_T(S_{T-1})$ must satisfy the following necessary conditions

$$\begin{aligned} \left.\frac{\partial V(p, S_{T-1})}{\partial p}\right|_{p_T^{nc}} &= 0 \\ \left.\frac{\partial^2 V(p, S_{T-1})}{\partial^2 p}\right|_{p_T^{nc}} &\leq 0. \end{aligned}$$

In particular the first order condition becomes

$$MR_T(p_T^{nc}) = S_{T-1}. \tag{9}$$

Let us consider the storage decision at time $T-1$ and define $S_{T-1}(p)$ the equilibrium amount of storage at period $T-1$ as a function of the price at period $T-1$. Since, whenever S_{T-1} is positive, $p_T - p_{T-1} = c$, in equilibrium of a subgame starting with p_{T-1}, we must have that $S_{T-1}(p_{T-1})$ solves

$$p_T\left(S_{T-1}\left(p_{T-1}\right)\right) = p_{T-1} + c.$$

Substituting into $V_T(p_T^{nc}, S)$ we obtain $V_T(p_{T-1})$: last period profits as a function of the previous period price.

Given S_{T-2}, the value of profits in period $T-1$ is given by

$$V_{T-1}(p_{T-1}, S_{T-2}) = (D_{T-1}(p_{T-1}) - S_{T-2} + S_{T-1}(p_{T-1}))\, p_{T-1} + V_T(p_{T-1}).$$

We can then obtain recursively the value of profits at period t:

$$V_t(p_t, S_{t-1}) = (D_t(p_t) - S_{t-1} + S_t(p_t))\, p_t + V_{t+1}(p_t)$$

where $S_t(p_t)$ must be such that, given $S_t(p_t)$, $p_{t+1}(S_t(p_t))$ satisfies $p_{t+1}(S_t(p_t)) = p_t + c$, i.e., the optimal price at period $t+1$ given S_t is exactly c higher than p_t.

The first order conditions for an optimum at period t are

$$MR_t(p_t^{nc}) - S_{t-1} + S_t(p_t^{nc}) - p_t \left.\frac{\partial S_t(p)}{\partial p}\right|_{p_t^{nc}} + \left.\frac{\partial V_{t+1}(p)}{\partial p}\right|_{p_t^{nc}} = 0.$$

Lemma 1 *In equilibrium,*
(i) $\frac{\partial V_{t+1}(p)}{\partial p} = -\frac{\partial S_t(p)}{\partial p} p_{t+1}^{nc}$,
(ii) $\frac{\partial S_t(p)}{\partial p} = \left.\frac{\partial^2 V_{t+1}(p, S_t)}{\partial^2 p}\right|_{p_{t+1}^{nc}}$

Proof. Property (i) follows immediately from the envelope theorem. To prove property (ii), note that, because of property (i) for any price p (not necessarily the optimal one), the first order condition at period $t+1$ implies that

$$S_t(p) \equiv MR_{t+1}(p_{t+1}^{nc}) + S_{t+1}(p_{t+1}^{nc}) - c \left.\frac{\partial S_{t+1}(p_{t+1})}{\partial p_{t+1}}\right|_{p_{t+1}^{nc}}.$$

As a consequence

$$\begin{aligned} \frac{\partial S_t(p)}{\partial p} &\equiv \frac{\partial}{\partial p}\left(MR_{t+1}(p_{t+1}^{nc}) + S_{t+1}(p_{t+1}^{nc}) - c \left.\frac{\partial S_{t+1}(p)}{\partial p}\right|_{p_{t+1}^{nc}} \right) \\ &\equiv \left.\frac{\partial}{\partial p_{t+1}}\left(MR_{t+1}(p_{t+1}) + S_{t+1}(p_{t+1}) - c\frac{\partial S_{t+1}(p_{t+1})}{\partial p_{t+1}} \right)\right|_{p_{t+1}^{nc}} \frac{\partial p_{t+1}^{nc}}{\partial p}. \end{aligned}$$

Recalling that for any price p the optimal price $p_{t+1}^{nc} = p + c$ we have that $\frac{\partial p_{t+1}^{nc}}{\partial p_t} = 1$. Moreover,

$$\frac{\partial}{\partial p}\left(MR_{t+1}(p) + S_{t+1}(p) - c \left.\frac{\partial S_{t+1}(x)}{\partial x}\right|_{x=p} \right) \equiv \frac{\partial^2 V_{t+1}(p, S_{t-1})}{\partial^2 p}.$$

■

Because of part (i) of Lemma 1, the first order conditions for period t can be written as

$$MR_t(p_t^{nc}) = S_{t-1} - S_t(p_t^{nc}) + c\,\frac{\partial S_t(p)}{\partial p}\bigg|_{p_t^{nc}} \quad t = 1, ..., T. \tag{10}$$

(For periods 1, and T, recall that $S_0 \equiv S_T \equiv 0$.)

This allows us to state the following proposition

Proposition 2 *Assume that the monopolist cannot commit, and that storage is interior. In equilibrium, the first price p_1^{nc} must satisfy the following equation*

$$\sum_{t=1}^{T} MR_t(p_1^{nc} + (t-1)c) = c\sum_{t=1}^{T-1} \frac{\partial S_t(p)}{\partial p}\bigg|_{p_1^{nc}+(t-1)c} \tag{11}$$

and at all other $t = 2, ..., T$,

$$p_t^{nc} = p_1^{nc} + (t-1)c.$$

Furthermore, in all periods, prices under commitment p_t^c are lower than the corresponding prices without commitment p_t^{nc}.

Proof. To obtain equation (11), sum equations (10) over all t's and recall that $p_{t+1}^{nc} = p_t^{nc} + c$.

To compare with the commitment solution, characterized in Proposition 1 we need to sign $\frac{\partial S_t(p)}{\partial p}\Big|_{p_1^{nc}+(t-1)c}$. By part (ii) of Lemma 1,

$$\frac{\partial S_t(p)}{\partial p_t} = \frac{\partial^2 V_{t+1}(p, S_{t-1})}{\partial^2 p}\bigg|_{p_{t+1}^{nc}} \leq 0 \;\; \forall t = 1, T-1 \tag{12}$$

where the inequality holds because of optimality of p_{t+1}^{nc}. Furthermore, it is easy to show that for time T, the inequality is strict:

$$\frac{\partial S_{T-1}(p)}{\partial p_{T-1}} = MR'\left(p_T^{nc}\right) < 0.$$

Since the right-hand sides of equations (11) and (8) are the same decreasing functions of p_1, it must be the case that $p_1^{nc} > p_1^c$. Since in both scenarios prices increase at rate t, prices must be lower under commitment in all periods. ■

We now consider the case in which the non negativity constraint for storage is binding.

For any $\tau = 2, ...T$, consider an artificial problem in which the monopolist only faces consumers between periods τ and T. Denote by $\left\{p_{t,\tau}^{nc}\right\}_{t=\tau}^{T}$ the corresponding equilibrium price sequence. Specifically, $p_{\tau,\tau}^{nc}$ must satisfy the following equation

$$\sum_{t=\tau}^{T} MR_t(p_{\tau,\tau}^{nc} + (t-\tau)c) = c\sum_{t=\tau}^{T-1} \left.\frac{\partial S_t(p)}{\partial p}\right|_{p_{\tau,\tau}^{nc}+(t-\tau)c}. \tag{13}$$

Furthermore, set

$$p_{t,\tau}^{nc} = p_{\tau,\tau}^{nc} + (t-\tau)c \text{ for all } \ t = 1, ..., T. \tag{14}$$

We can now state a result for the case in which storage may be zero in some periods.

Proposition 3 *Assume that the monopolist cannot commit, and that $c \geq c^*$. Let $\tau - 1$ be the last period in which storage is zero. Then,*

(i) In equilibrium, prices $\{p_t^{nc}\}_{t=1}^{T}$ must satisfy equations (13) and (14).

(ii) In all periods, prices under commitment p_t^c are lower than the corresponding prices without commitment p_t^{nc}.

Proof. (i) If storage is zero at period $\tau - 1$, the monopolist enters period τ as if the prior periods did not exist. Thus, it is as if period τ is the same as period 1 in the equilibrium characterized in Proposition 2. Thus, for $t \geq \tau$ equations (13) and (14) must characterize equilibrium prices. For $t < \tau$, clearly $p_{t,\tau}^{nc} \geq p_{\tau,\tau}^{nc} + (t-\tau)c$ otherwise storage could not be zero. But it cannot be the case that $p_{t,\tau}^{nc} > p_{\tau,\tau}^{nc} + (t-\tau)c$. The proof of this case is analogous to a similar step in the proof of Lemma 8 in the appendix.

(ii) Note first that the solution under commitment is unchanged by the fact that we are considering the binding storage case for no commitment. Storage was binding in any event in the case of commitment. For any $\tau = 2, ...T$, consider the analogous artificial problem in which the monopolist only faces consumers between periods τ and T. Denote by $\left\{p_{t,\tau}^{c}\right\}_{t=\tau}^{T}$ the corresponding equilibrium price sequence. Specifically, $p_{\tau,\tau}^{c}$ solves the following equation

$$\sum_{t=\tau}^{T} MR_t(p_{\tau,\tau}^{c} + (t-\tau)c) = 0 \tag{15}$$

and $p_{t,\tau}^{c}$ is given by

$$p_{t,\tau}^{c} = p_{\tau,\tau}^{c} + (t-\tau)c \text{ for all } t = \tau, ..., T.$$

By Proposition 2, $p_{t,\tau}^{c} < p_{t,\tau}^{nc}$ for all $t = \tau, ..., T$. Furthermore, it is easy to see that $p_{\tau,\tau}^{c} > p_{\tau,1}^{c}$ since the latter is given by equation (8). Thus, prices under commitment are lower than prices without commitment. ∎

5 Extensions

We now consider a number of extensions of the analysis. We first discuss the case in which demand may also decrease. Then we discuss the case of convex storage costs.

5.1 T periods Cycles

Consider now a deterministic demand cycle $D_1(p)$,... $D_T(p)$, $D_{T+1}(p)$,... $D_{2T-1}(p)$. Assume that this cycle is symmetric in the sense that $D_2(p) = D_{2T-1}(p)$, $D_3(p) = D_{2T-2}(p)$,... For each demand call p_t^m the static monopoly price. Assume that $p_1^m < ... < p_T^m$ and assume also that the marginal cost of storage c is such that $c < \min\{p_{t+1}^m - p_t^m\}$. We want to study and compare prices along the equilibrium path of the game under commitment and absent commitment.

If the jumps in demand between periods T and $T+1$ and between periods $2T-1$ and $2T$ are sufficiently large, then the characterization for this problem is straightforward given the analysis in the previous section: when demand is increasing apply the previous characterization. When demand is decreasing, set prices to be static monopoly prices.

However, in general, the analysis can be complicated by the fact that when demand starts falling at period $T+1$ the static monopoly price p_{T+1}^m might be so high that consumers set positive storage at period T. Similarly it might happen that the static monopoly price p_{2T-1}^m is so low that the consumers store for period 1. In other words, the peak and bottom of the cycle of static monopoly prices may not coincide with that of the equilibrium prices.

In order to solve for the equilibrium we propose an algorithm that can be used to obtain equilibrium prices both in the case of commitment and when there is no commitment. For this reason in the description of the algorithm we will talk about "equilibrium" without specifying whether we will be referring to the equilibrium under commitment or to equilibrium absent commitment. Similarly, without further specification we will use the notation p_t^{eq} to indicate an equilibrium price at period t.

As we pointed out, the upward trend of equilibrium price does not necessarily coincide with the upward trend in static monopoly prices. Hence we use notation t_1 to indicate the period in the cycle at which equilibrium prices start rising. We will also call t_N the period in the cycle after which equilibrium prices start declining. Finally we will call t_n the period at which the n^{th} ascending price is observed in equilibrium. Finally we will call $\{t_n\}$ a generic sequence demands.

Solution algorithm:
Start the algorithm by considering the sequence $\{t_n\}$ where $n = 1, ..., T$ and $t_1 = 1$.

Step 1: compute the equilibrium for the sequence $\{t_n\}$ and then go to step 2.

Step 2: compare $p_{t_N+1}^m$ with $p_{t_N}^{eq}$ and compare p_{2T-1}^m with $p_{t_1}^{eq}$:

Step 2.1: if $p^m_{t_N+1} \leq p^{eq}_{t_N} + c$ and $p^m_{t_1-1} \geq p^{eq}_{t_1} - c$ stop[5]: the sequence of price is an equilibrium;

Step 2.2: if $p^m_{t_N+1} \leq p^{eq}_{t_N} + c$ and $p^m_{t_1-1} < p^{eq}_{t_1} - c$ create a new sequence $\{t'_n\}$ where $n = 1, ..., N+1$, $t'_1 = t_1 - 1$ and $t'_N = t_N$. Then start again from step 1;

Step 2.3: if $p^m_{t_N+1} > p^{eq}_{t_N} + c$ and $p^m_{t_1-1} \geq p^{eq}_{t_1} - c$ create a new sequence $\{t'_n\}$ where $n = 1, ..., N+1$, $t'_n = t_1$ and $t'_N = t_N + 1$. Then start again from step 1;

Step 2.4: if $p^m_{t_N+1} > p^{eq}_{t_N} + c$ and $p^m_{t_1-1} < p^{eq}_{t_1} - c$ create a new sequence $\{t'_n\}$ where $n = 1, ..., N+2$, $t'_n = t_1 - 1$ and $t'_n = t_N + 1$. Then start again from step 1.

Because the cycle is of finite length, the algorithm must eventually end.

Once the process has stopped set the equilibrium price for all the remaining periods not included in $\{t_n\}$ equal to p^m_t.

Lemma 2 *At all periods t except, possibly, t_1 and t_N, the increase in equilibrium prices is lower than the decrease in equilibrium prices.*

Proof. When prices increase they increase by c, when they drop they drop by $|p^m_{t+1} - p^m_t|$ which, by assumption, is greater than c for all t. ■

5.1.1 Commitment

In case of commitment this algorithm delivers the unique solution. Notice first that the conditions of optimality of the previous section do not depend on the fact that demand is increasing. Even within a cycle the optimality condition is that

$$\sum_{t=1}^{2T-1} MR_t(p_t) = 0 \tag{16}$$

with

$$\begin{aligned} p_{t_n} &= p_{t_1} + (n-1)c \text{ for } n = 1, ..., N \\ p_t &= p^m_t \text{ for all other periods } t. \end{aligned}$$

[5] When searching for a no commitment equilibrium this condition becomes

$$\begin{aligned} p^m_{t_N} &< p^{eq}_{t_N} + c \\ p^m_{t_1-1} &> p^{eq}_{t_1} - c. \end{aligned}$$

The algorithm above divide the cycle into two sequences. The first sequence is of length N. In this sequence prices are increasing and distanced by c one from the other. Moreover, when performing Step 1 of the algorithm we have that for all the t_n in this sequence

$$\sum_{n=1}^{N} MR_{t_n}(p_{t_1} + (n-1)c) = 0.$$

The other sequence is of length $2T - 1 - N$. In this sequence prices are decreasing and equal to the static monopoly prices so that $MR_t(p_t^m) = 0$ for all the periods in this sequence. Summing over all periods we have that condition (16) is satisfied. This show that the algorithm identifies the equilibrium.

5.1.2 No commitment

Within a cycle, when prices increase

$$p_{t+1} = p_t + c$$

whereas when prices decrease

$$\begin{aligned} p_t &= p_t^m \\ S_{t-1} &= 0 \\ \left.\frac{\partial S_{t-1}(p)}{\partial p}\right|_{p_{t-1}^{eq}} &= 0 \end{aligned}$$

The optimality condition that equilibrium prices must satisfy is

$$\sum_{t=1}^{2T-1} MR_t(p_t) = c \sum_{t=1}^{2T-1} \left.\frac{\partial S_{t-1}(p)}{\partial p}\right|_{p_{t_n-1}^{eq}}. \tag{17}$$

Upon convergence the proposed algorithm breaks the cycle into two sections. One is of length N. In this section prices are increasing and distanced by c one from the other. Moreover

$$\sum_{n=1}^{N} MR_{t_n}(p_{t_n}^{nc}) = c \sum_{n=1}^{N} \left.\frac{\partial S_{t-1}(p)}{\partial p}\right|_{p_{t_n}^{nc}}$$

with

$$\begin{aligned} p_{t_n} &= p_{t_1} + (n-1)c \text{ for } n = 1, ..., N \\ p_t &= p_t^m \text{ for all other periods } t. \end{aligned}$$

The other part is of length $2T - 1 - N$, prices $p_t = p_t^m$ are decreasing, $S_{t-1} = 0$ and $\left.\frac{\partial S_{t-1}(p)}{\partial p}\right|_{p_{t-1}^{eq}} = 0$. Combining these two we obtain condition (17) above.

5.1.3 Comparison

Before we compare prices along the equilibrium paths it is useful to state the following two lemmas

Lemma 3 *At any iteration, if at stage* 2 *only period* $t_N + 1$ *is added to a sequence* $\{t_n\}$ *then equilibrium prices* p_t *increase for all* $t = t_1, ..., t_N$*. At any iteration, if at stage* 2 *only period* $t_1 - 1$ *is added to a sequence* $\{t_n\}$ *then equilibrium prices* p_t *decrease for all* $t = t_1, ..., t_N$*.*

Proof. Consider first using the algorithm to find an equilibrium with commitment.

Suppose then that at step 2 only period $t_N + 1$ is added to a previous sequence $\{t_n\}$. Consider then the first order condition computed at the equilibrium price p_{t_1} and notice that

$$\sum_{n=1}^{N} MR_{t_n}(p_{t_1} + (n-1)c) + MR_{t_N+1}(p_{t_1} + Nc) > 0.$$

This means that the new equilibrium price $p'_{t_1} > p_{t_1}$. Because in equilibrium $p_{t_n} = p_{t_1} + (n-1)c$, $p'_t > p_t$ for all $t = t_1, ..., t_N$.

Similarly, suppose that at step 2 only period $t_1 - 1$ is added to the sequence $\{t_n\}$. Consider the first order condition computed at $p_{t_1-1} = p_{t_1} - c$ and notice that

$$MR_{t_1-1}(p_{t_1} - c) + \sum_{n=1}^{N} MR_{t_n}(p_{t_1} + (n-1)c) < 0.$$

Hence the equilibrium price $p_{t_1-1} > p_{t_1} - c$. As a consequence new equilibrium prices $p'_t > p_t$ for all $t = t_1, ..., t_N$

Consider now using the algorithm to find an equilibrium without commitment.

Suppose that at step 2 only period $t_N + 1$ is added to a previous sequence $\{t_n\}$. Because $p^{eq}_{t_N+1} = p_{t_1} + Nc$ and because $MR_{t_N+1}(p)$ is decreasing, if the new equilibrium price $p'_{t_1} < p_{t_1}$ then, by condition (9), $S'_{t_N} > S_{t_N}$. It is easy to verify that $V_{t_1}(p'_{t_1}, 0) < V_{t_1}(p_{t_1}, 0)$. This means that the new equilibrium price $p'_{t_1} > p_{t_1}$. Because in equilibrium $p_{t_n} = p_{t_1} + (n-1)c$, $p'_t > p_t$ for all $t = t_1, ..., t_N$.

Suppose now that at step 2 only period $t_1 - 1$ is added to the sequence. Notice that the previous equilibrium price p_{t_1} had to satisfy

$$MR_{t_1}(p_{t_1}) + S_{t_1} - c \left.\frac{\partial S_{t-1}(p)}{\partial p}\right|_{p_{t_1}} = 0.$$

After adding demand $t_1 - 1$ we have that $S_{t_1-1} \geq 0$. The first order condition at period t_1 computed at p_{t_1} implies that

$$MR_{t_1}(p_{t_1}) - S_{t_1-1} + S_{t_1} - c \left.\frac{\partial S_{t-1}(p)}{\partial p}\right|_{p_{t_1}} \leq 0.$$

As a consequence, when period $t_1 - 1$ is added price $p'_{t_1} < p_{t_1}$. Because in equilibrium $p_{t_n} = p_{t_1} + (n-1)c$, $p'_t > p_t$ for all $t = t_1, ..., t_N$. ■

We can then use this lemma to prove that:

Lemma 4 *In the equilibrium under commitment prices start to increase and start to decrease no sooner than in the equilibrium without commitment. Moreover, prices are higher when the monopolist lacks commitment.*

Proof. Call t_N^c and t_1^c the last and first period of ascending prices in equilibrium and when the monopolist can commit. Call t_N^{nc} and t_1^{nc} the corresponding periods in equilibrium when the monopolist lacks commitment.

Moreover at each iteration of the algorithm call $\{t_n\}^c$ and $\{t_n\}^{nc}$ the sequence related to the search for a commitment and a no commitment equilibrium respectively.

At *Step 1* of the very first iteration the two equilibria are computed over the same sequence $\{t_n\}$ and we find two sequences $\{p_{t_n}^c\}$ and $\{p_{t_n}^{nc}\}$.

By Proposition 2 prices under commitment are lower than prices under no commitment. This means that at *Step 2* if either $p_{t_N}^{nc} + c \leq p_{t_N+1}^m$ and/or $p_{t_1-1}^m < p_{t_1}^c - c$ then, at the next iteration, the sequences $\{t_n\}^c$ and $\{t_n\}^{nc}$ will be the same. If instead $p_{t_N}^c + c \leq p_{t_N+1}^m$ and $p_{t_N}^{nc} + c > p_{t_N+1}^m$ then period $t_N + 1$ will be added only to the sequence $\{t_n\}^{nc}$ and eventually, upon convergence, $t_N^c < t_N^{nc}$.

If instead $p_{t_1-1}^m < p_{t_1}^c - c$ and $p_{t_1-1}^m \geq p_{t_1}^c - c$ then period $t_1 - 1$ will be added only to the sequence $\{t_n\}^c$ and eventually, upon convergence, $t_1^{nc} < t_1^c$.

Moreover, by lemma 3 whenever only period $t_N + 1$ is added to a sequence the new equilibrium prices will decrease. Because in equilibrium $p_{t_N}^c < p_{t_N}^m$ if $p_{t_N}^c + c \leq p_{t_N+1}^m$ and $p_{t_N}^{nc} + c > p_{t_N+1}^m$ then even if prices p_t^{nc} are on a descending phase they are still higher than commitment prices.

Similarly, by lemma 3 whenever only period $t_1 - 1$ is added to a sequence the new equilibrium prices will increase. Because in equilibrium $p_{t_1}^{nc} \geq p_{t_1}^m$, if $p_{t_1-1}^m < p_{t_1}^c - c$ and $p_{t_1-1}^m \geq p_{t_1}^c - c$ then prices p_t^{nc} remains above commitment prices even if the latter are equal to the static monopoly prices. ■

5.2 Convex Cost of Storage

Assume that the cost of storage is given by a twice continuously differentiable function $c(S)$ with $c'(S) > 0, c''(S) > 0$, and $c(0) = c'(0) = 0$.

There are two main differences with the previous analysis: (1) storage is now positive under commitment as well. (2) it is no longer possible to characterize the equilibrium price sequence simply by obtaining the first price. As a result, we no longer obtain as crisp a result comparing commitment and no commitment. However, we show that prices cannot be uniformly higher under commitment, and we have computed examples with specific functional forms in which the result of the previous section generalizes.

Consider any fixed sequence of prices $\{p_t\}_{t=1}^T$. Suppose that the buyer begins date t with a stock S_{t-1} of the good. Let $S_t^*(p_1, ..., p_T)$ be the optimal storage choice by the consumer. The following Lemma provides a simple characterization of the solution of the buyer's problem.

Lemma 5 *Assume that $p_t < p_\tau$ for $t < \tau$, and that $p_t \leq p_T^m$ for all t. Then, the buyer always purchases a positive amount at every date and store a positive amount at every date except for date T. At date t the consumer stores quantity S_t that solves*

$$c'(S_t) = p_{t+1} - p_t \tag{18}$$

and consumes

$$x_t = D_t(p_t) + S_{t-1}.$$

Thus, at date t the consumer purchases $b_t = D_t(p_t) + S_t - S_{t-1}$ units.

By Lemma 5, we can write the consumer's optimal storage decision at period t as a function of period t and period $t+1$ prices only. Denote by $S_t(p_t, p_{t+1})$ the optimal storage decisions at period t as defined by equation (18).

5.2.1 Commitment

By Lemma 5, given a sequence of increasing prices $p_1, ..., p_T$, monopoly profits can be written as

$$\begin{aligned} \pi(p_1, ..., p_T) \;=\;& [D_1(p_1) + S_1(p_1, p_2)]p_1 \\ &+ \sum_{t=2}^{T-1}[D_t(p_t) - S_{t-1}(p_{t-1}, p_t) + S_t(p_t, p_{t+1})]p_t + [D_T(p_T) - S_{T-1}(p_{T-1}, p_T)]p_T \end{aligned} \tag{19}$$

Let us assume for the moment that prices form an increasing sequence. Then, under commitment, the monopolist chooses $(p_1, ..., p_T)$ at period 1 to maximize the right-hand side of equation (19).

Recall that $MR_t(p_t) = D_t(p_t) + D_t'(p_t)p_t$ and write the first order conditions as:

$$\begin{aligned} MR_1(p_1) + S_1(p_1, p_2) - \frac{\partial S_1(p_1, p_2)}{\partial p_1}(p_2 - p_1) &= 0 \\ MR_t(p_t) - S_{t-1}(p_{t-1}, p_t) + S_t(p_t, p_{t+1}) & \\ - \frac{\partial S_{t-1}(p_{t-1}, p_t)}{\partial p_t}(p_t - p_{t-1}) - \frac{\partial S_t(p_t, p_{t+1})}{\partial p_t}(p_{t+1} - p_t) &= 0 \quad t = 2, ..., T-1 \\ MR_T(p_T) - S_{T-1}(p_{T-1}, p_T) - \frac{\partial S_{T-1}(p_{T-1}, p_T)}{\partial p_T}(p_T - p_{T-1}) &= 0 \end{aligned}$$

Summing these rows we obtain

$$\sum_{t=1}^{T} MR_t\left(p_t^c\right) = 0. \tag{20}$$

This equation is the counterpart of equation (8) that we obtained in the case of linear costs of storage. Note however, that equation (20) is not as informative: because prices are now not necessarily rising at a constant rate c, we need T conditions to obtain each price.

5.2.2 No Commitment

The construction of the equilibrium absent commitment is quite similar to the analysis in Section 4. The main difference is that equilibrium storage $S_t\left(p_t\right)$ at date t must satisfy

$$c'\left(S_t\left(p_t\right)\right) = p_{t+1} - p_t.$$

Appropriately modifying the of the analysis of Section 4, we obtain that equilibrium is characterized by the system:

$$\begin{cases} MR_1(p_1^{nc}) = -S_1^{nc} + \left(p_2 - p_1\right) \frac{\partial S_1(p_1)}{\partial p_1} \\ \ldots \\ MR_t(p_t^{nc}) = S_{t-1}^{nc} - S_t^{nc} + (p_{t+1}^{nc}(p_t) - p_t) \frac{\partial S_t^{nc}(p_t)}{\partial p_t} \\ \ldots \\ MR_T(p_T^{nc}) = S_T^{nc} \end{cases}$$

Summing these rows we obtain

$$\sum_{t=1}^{T} MR_t(p_t^{nc}) = \sum_{t=1}^{T} \left((p_{t+1}^{nc} - p_t) \frac{\partial S_t^{nc}(p_t)}{\partial p_t} \right).$$

Going through similar steps as in the proof of Lemma 1 we can show that

$$\frac{\partial S_t^{nc}\left(p_t\right)}{\partial p_t} = \frac{\frac{\partial^2 V_{t+1}}{\partial p_{t+1}^2}}{1 - c''\left(S_t^{nc}\right) \frac{\partial^2 V_{t+1}}{\partial p_{t+1}^2}} \leq 0.$$

Because $p_{t+1}^{nc} > p_t^{nc}$, and because (as in the previous section) we can prove that $\frac{\partial S_{T-1}^{nc}(p_{T-1})}{\partial p_{T-1}} < 0$, we can conclude that $\sum_{t=1}^{T} MR_t(p_t^{nc}) < \sum_{t=1}^{T} MR_t(p_t^c)$. Because MR_t are decreasing functions for all t, we can conclude that prices under commitment cannot be uniformly higher. There is also a sense in which they have to be lower "on average." We have computed equilibria with several specific functional forms and we have always found that prices are uniformly lower under commitment.

5.3 Stockpiling with constant demand

We have focused on an environment in which stockpiling arises due to increasing prices generated by increasing demand. However, it is possible to show that, with heterogeneous consumers, prices may also vary because of price discrimination: the monopolist typically charges high prices serving high demand consumers, and periodic sales are conducted in which both high and low types stocks up for future consumption. Because high types run out of inventory faster, more surplus can be extracted from them. Thus, it is possible to obtain strockpiling motives even with constant demand. Similar results would then obtain in such environments.

6 Demand Anticipation in Durables: A Simple Model

In this section we show that the basic force behind our results is not exclusive to storable goods, but present in durables as well. We present a durable good set up, under quite reasonable assumptions, in which demand anticipation is present. The consequence of demand anticipation is that lack of commitment on the part of the monopolist leads to higher prices in every period.

We assume there is a two-periods demand cycle. We purposely concentrate on a simple cycle to avoid demand postponement (a la Coase). Allowing for demand postponement and anticipation in the same model may turn interesting, but quite difficult to solve, and beyond this paper.

We assume demand comes from overlapping generations of consumers. Each period a new geneartion starts enjoying the good. Each generation enjoys the durable for two periods. Thus they could purchase the durable when they are young, and enjoy it for two periods. Otherwise, they might wait to purchase when they are old, in which case they enjoy the product for a single period. We assume there are no active secondary markets.

The market is assumed to be open for T period. To model the demand cycle, we assume that the preferences of the consumers born in an odd period can be summarized by $D_1(p_1)$. $D_1(p_1)$ summarizes their willingness to pay for the durable when they are young. Namely, it incorporates their utility for two periods. When they consider purchasing the durable in their second period of life, their preferences can be summarized by $D_1(\frac{p_1}{2})$. Namely, for each quantity of the product they are only willing to pay half the price they would pay in the first period.

In even periods the generation of buyers born has the following preferences: $D_2(p_2)$. Lets call the consumers period-1 consumers and period-2 consumers, respectively.

We assume there are potential costs associated with advance purchases. The product deteriorates at rate $1 - \delta$ with $\delta \leq 1$. It costs $c \geq 0$ to store one unit of the product. For convenience, we assume the storage costs c is paid per undepreciated unit. We restric attention to those parameters of the model for which $p_2^m \geq \frac{p_1^m}{\delta} + c$, where p_t^m is the static monopoly price in period t.

The main difference between our model and previous models of durables (e.g. Sobel (1984)) is that buyers (in particular, type-2 consumers) are allowed to purchase before their need for the durable arises. In the previous literature, new buyers were only allowed to purchase as they enter the market. In our model, their need is born at a point in time, but they can purchase in advance.

In order to find the equilibrium of the T period model, we first characterize a single cycle. Namely, we find the equilirbium prices in the $T = 2$ case. We then show conditions under which the repetition of the 2-period prices forms an equilibrium of the T period model.

6.1 The Two-Period Cycle

WIn this section we consider a cycle of periods 1 and 2 in isolation. We studey both commitment and lack of commitment for this short horizon.

6.1.1 Commitment

Period-2 consumers are willing to buy in the first period if $p_2 \geq \frac{p_1}{\delta} + c$. Otherwise the static monopoly prices arise. Namely, if p_2 is sufficiently attractive period-2 consumers are not tempted to buy in advance and pay the costs associated with storage.

In the complementary case (namely, when the gap in static prices is large or the cost of storage low) period 2 consumers buy in the first period. In equilibrium, when period-2 consumers purchase in both periods, prices must be linked in the following way: $p_2 = \frac{p_1}{\delta} + c$. To enjoy a unit in the second period, period-2 consumers have to either pay p_2 or purchase $\frac{1}{\delta}$ units in the first period at price p_1 and pay the storage cost of the surviving unit, c. Equilibrium prices make period 2 consumers indifferent between buying in the first or the second period.

Assuming no advance purchases, the monopolist sets p_1 to maximize:

$$\pi\left(p_1, p_2\right) = D_1(p_1)p_1 + D_2\left(\frac{p_1}{\delta} + c\right)\left(\frac{p_1}{\delta} + c\right)$$

Thus

$$MR_1(p_1) + \frac{MR_2(\frac{p_1}{\delta} + c)}{\delta} = 0 \tag{21}$$

6.1.2 No Commitment

Under lack of commitment when $p_2^m \geq \frac{p_1^m}{\delta} + c$ we know there must be storage in equilibrium. Absent storage, the monopolist would face the whole demand by type 2 consumers in the second period, and consequently set the static monopoly price, which by assumption would make storing for period-2 consumers profitable.

Let us analyze the subgames in which period-2 consumers have stored q_1 units. If they bought q_1 in first period, the residual demand in the second period is $D_2(p_2) - \delta q_1$ so, second period prices maximize

$$\pi_2(p_2|q_1) = (D_2(p_2) - \delta q_1) p_2$$

The optimal $p_2^*(q_1)$ is dictated by:

$$MR_2(p_2(q_1)) - \delta q_1 = 0$$

Given p_1, first period purchases by pereiod-2 consumers must be such that the optimal price $p_2(q_1) = \frac{p_1}{\delta} + c$ (otherwise period-2 buyers are not indifferent between buying in the first or second period).

Moreover, notice that since the advance purchases, q_1, as well as p_2, are functions of p_1. Thus, we can represent $\pi_2(p_2|q_1)$ as a function of p_1 only. Call that function $\pi_2^*(p_1)$.

$$\pi_2^*(p_1) = \left(D_2\left(\frac{p_1}{\delta} + c\right) - \delta q_1(p_1)\right)\left(\frac{p_1}{\delta} + c\right)$$

Thus, in the first period, the monopolist maximizes the sum of first and second period profits:

$$(D_1(p_1) + q_1(p_1))p_1 + \pi_2^*(p_1)$$

Profits are maximized at a p_1 such:

$$MR_1(p_1) + q_1(p_1) + \frac{\partial q_1(p_1)}{\partial p_1}p_1 + \frac{\partial \pi_2^*(p_1)}{\partial p_1} = 0$$

By the envelope theorem $\frac{\partial \pi_2^*(p_1)}{\partial p_1} = -\delta \frac{\partial q_1(p_1)}{\partial p_1} p_2$. Substituting above, we get:

$$MR_1(p_1) + q_1(p_1) - \frac{\partial q_1(p_1)}{\partial p_1} c\delta = 0$$

Adding both f.o.c.s we get a condition readily comparable to the one from the commitment problem:

$$MR_1(p_1) + \frac{MR_2(\frac{p_1}{\delta} + c)}{\delta} = \frac{\partial q_1(p_1)}{\partial p_1} c\delta \tag{22}$$

6.2 The T-Period Model

Having characterized the short, two-period, cycle, we can show the conditions under which the price we found are an equilibrium of the longer horizon game, where generations of consumers keep coming for T periods.

We start conjecturing that prices p_1 and p_2 that we found above arise every odd and even period respectively. For them to form an equilibrium of the T-period game it has to be the case that no consumer prefers to either delay or advance purchases.

It is immediate to see that no consumers wants to wait or advance purchases for two periods or more, because every two periods the same prices prevail. Furthermore, notice that the prices we found already guarantee that the period-1 buyers does not want to delay purchase for a single period, nor period-2 buyers want to purchase one period in advance. It remains to be checked that period 1 consumers do not want to purchase in advance, nor period 2 consumers would like to wait one period, to enjoy a lower price. The former is immediate, since $p_1 < p_2$ there is no reason for period 1 buyers to buy in advance.

Period-2 buyers may want to wait for a lower price, since $p_1 = \delta(\ p_2 - c)$. The cost of waiting is the foregone utility on their first period of life. Thus, when considering withholding the purchase of the last unit they consume, the saving is $p_2 - p_1 = (1-\delta)p_2 + \delta c$, while the cost is half of the marginal utility of that unit, or $\frac{p_2}{2}$. Thus, as long as $p_2 > c(1+\frac{1}{2\delta-1})$ period-2 consumers have no incentive delay purchases and the prices we characterized above form an equilibrium of the $T-$period model.

6.3 Comparison of Prices

Remember that first and second period prices were linked by the relation $p_2 = \frac{p_1}{\delta} + c$ in both cases, thus, comparing first period price suffices to show whether prices are uniformly higher in one case or the other. Comparing 21 and 22 is imemdiate if we can sign $\frac{\partial q_1(p_1)}{\partial p_1}$.

Recall that determined as the advance purchase such: $MR_2(\frac{p_1}{\delta}+c) - \delta q_1(p_1) = 0$. Thus $\frac{\partial q_1(p_1)}{\partial p_1} < 0$ as long as the second order condition of the static problem holds. A negative $\frac{\partial q_1(p_1)}{\partial p_1}$ implies that $p_1^c < p_1^{nc}$ if either $\delta < 1$ or $c > 0$.

Prices under lack of commitment are higher. The price differential vanishes only when it becomes free to store.

When would our analysis fail? In most durable goods models in the literature prices decline over time so demand anticipation plays no role. Several common assumptions lead to declining price. First, non-increasing demand is sufficient to lead to non-increasing prices. This is a natural assumption to relax, either new customers or cyclical demand by incumbent customers is likely to make demand increases in a foreseeable way at different periods of time.

Second, perfectly functioning secondary markets make today's price incorporate the discounted resale value of the product. Hence, equilibrium prices under smooth secondary markets cannot increase in time.

Papers like Sobel (1984) have deviated from the previous two assumption, leading to cycling (sometimes increasing) prices. However, they did not allow the entering customers to purchase in advance. Thus demand anticipation was not a relevant force in the literature.

7 Appendix

Lemma 6 *Consider a period t, with $t = 1, ..., N$. Suppose that, in the commitment equilibrium, $S_i = 0$ for all $i = 1, ..., t-1$. Suppose also that markets t and $t+k$ are open. If $p_{t+k} - p_t = kc$ then $S_{t+k-1} = 0$.*

Proof. Assume by way of contradiction that there exists an equilibrium in which $S_i^c = 0$ for all $i = 1, ..., t$, market t and market $t+k$ are open at prices p_t and p_{t+k} respectively, $p_{t+k} - p_t = kc$ but $S_{t+k-1} > 0$.

Call τ the first period after $t+k$ at which either market $\tau+1$ is closed or, if market $\tau+1$ is open, $p_{\tau+1} - p_\tau > c$. If such τ does not exist because all markets after period $t+k$ are open and prices are distanced by no more than c, then set $\tau = T$. Consider the sequence σ^* where, with abuse of notation

$$\begin{array}{rl} p_i^* = p_i & \text{for } i = 1, ..., t+k-1 \\ p_i^* = p_i - (i-t)\varepsilon & \text{for } i = t+k, ..., \tau \\ p_i^* = p_i & \text{for } i = \tau+1, ..., T \end{array}$$

(by definition of k, under σ markets $i = t+k, ..., \tau$ are all open at price p_i). We will show that $\pi(\sigma^*) > \pi(\sigma)$.

Notice that under sequence σ^* the storage decision at periods $i = 1, ..., t+k-1$ and at periods $i = \tau+1, ..., T$ is the same as under sequence σ. On the contrary, under sequence σ^* price $p_{i+1}^* - p_i^* < c$ at all the periods $i = t+k, ..., \tau-1$. Hence the optimal storage decision of consumers will be

$$\begin{array}{rl} S_i^* = 0 & \text{for } i = 1, ..., \tau-1 \\ S_\tau^* = S_\tau & \text{for } i = \tau+1, ..., T. \end{array}$$

and for period τ

$$S_\tau^* = S_\tau - (\tau - t)\varepsilon \left. \frac{\partial S_\tau(p)}{\partial p} \right|_{p_\tau}$$

The difference between $\pi(\sigma)$ and $\pi(\sigma^*)$ accrues only from period t and from periods $i = t+k, ..., \tau+1$. Specifically

$$\begin{aligned} \pi(\sigma^*) - \pi(\sigma) &= -\varepsilon \sum_{i=t+k}^{\tau} (i-t) MR_i(p_i) + kcS_{t+k-1} \\ &\quad + \sum_{i=t+k}^{\tau} S_i(p_{i+1} - p_i) + [S_\tau^* p_\tau^* - S_\tau p_\tau - S_\tau^* p_{\tau+1} + S_\tau p_{\tau+1}] \\ &= -\varepsilon \sum_{i=t+k}^{\tau} (i-t) MR_i(p_i) + kcS_{t+k-1} \\ &\quad + \sum_{i=t+k}^{\tau} S_i(p_{i+1} - p_i) - (\tau - t)\varepsilon S_\tau^* + (\tau - t)\varepsilon \left. \frac{\partial S_\tau^c(p)}{\partial p} \right|_{p_\tau} (p_{\tau+1} - p_\tau) \end{aligned}$$

Notice that if ε is small enough all terms containing ε in the right hand side of the above equality become negligible, so that $\pi(\sigma^*) - \pi(\sigma) > 0$. ■

Lemma 7 *In a commitment equilibrium the monopolist opens all markets.*

Proof. Suppose that all markets $i = 1,..,t-1$ are closed whereas market t is open at price p_t. Suppose also that $D_i(p_t) > 0$ for all $i = 1,...,t-1$. If the monopolist opens all markets $i = 1,..,t$ at price $p_i = p_{t+1} - (t-i)c$ then, by Lemma 6, $S_i = 0$ for all $i = 1,...,t$ and profits would increase by

$$\sum_{i=1}^{t} D_i(p_i)p_i.$$

Because the monopolist will open at least market T at a price $p_t \le p_T^m$ and because $D_1(p_T^m) > 0$, it follows that all markets are open. ■

Lemma 8 *In a commitment equilibrium $p_{t+1}^c = p_t^c + c$ for all t.*

Proof. The proof proceeds by induction on t. Starting from $t = 1$ we will first show that in equilibrium it must be that $p_2^c - p_1^c \ge c$. Then, we will show that it cannot be that $p_2^c - p_1^c > c$. We will then use a similar argument to prove the statement for a generic t.

Consider an equilibrium price sequence $\sigma = \{p_t\}_{t=1}^T$ and assume by way of contradiction that $p_2 - p_1 < c$. If $p_1 > p_1^m$ then $MR_1(p_1) < 0$. The monopolist could then decrease p_1 and, by doing so, increase first period revenues and hence total profits. If instead $p_1 \le p_1^m$, call τ the earliest period at which $S_\tau > 0$. If $S_t = 0$ for $t = 2,...,T$ set $\tau = T$. By lemma 6 if $S_\tau > 0$ and $S_i = 0$ for all $i = 1,...,\tau-1$ it must be the case that $p_{\tau+1} - p_\tau > c$.

Consider now $\varepsilon < \min\limits_{1\le t\le\tau}\{|p_{t+1} - p_t - c|\}$ and consider the price sequence $\sigma^* = \{p_t^*\}_{t=1}^T$ such that

$$\begin{array}{rl} p_1^* = p_1 & \\ p_t^* = p_t + \frac{\varepsilon}{t-1} & \text{for } 2 \le t \le \tau \\ p_t^* = p_t & \text{for } \tau+1 \le t \end{array}$$

Then

$$\begin{array}{rl} S_t^* = 0 & \text{for } 2 \le t \le \tau-1 \\ S_t^* = S_t + \frac{\varepsilon}{t-1} \left.\frac{\partial S_t(p)}{\partial p}\right|_{p_t} & \text{for } t = \tau \\ S_t^* = S_t & \text{for } \tau+1 \le t \end{array}$$

Hence

$$\begin{aligned} \pi(\sigma^*) - \pi(\sigma) &= \varepsilon \sum_{t=\tau+1}^{\tau} \frac{MR_t(p_t)}{t-1} + [S_\tau^* p_\tau^* - S_\tau p_\tau - S_\tau^* p_{\tau+1} + S_\tau p_{\tau+1}] \\ &= \varepsilon \sum_{t=\tau+1}^{\tau} \frac{MR_t(p_t)}{t-1} + \frac{\varepsilon}{t-1} S_{\tau'}^* - \frac{\varepsilon}{t-1} \left.\frac{\partial S_t(p)}{\partial p}\right|_{p_t} (p_{\tau+1} - p_\tau) \end{aligned}$$

Since $p_1 \leq p_1^m$ and because $c < p_{t+1}^m - p_t^m$ for all t, then $MR_t(p_t) > 0$ for all t. Moreover, $\left.\frac{\partial S_t(p)}{\partial p}\right|_{p_t} \leq 0$. Hence $\pi(\sigma^*) > \pi(\sigma)$. This concludes the proof that $p_2 - p_1 \geq c$.

Now, assume by way of contradiction that $p_2 - p_1 > c$. Consider τ the lowest period at which $p_{\tau+1} - p_1 \leq (\tau+1)c$. Consider the sequence $\sigma^* = \{p_t^*\}_{t=1}^T$ such that

$$\begin{array}{cc} p_t^* = p_1 + tc - c & \text{for } 1 \leq t \leq \tau \\ p_t^* = p_t & \text{for } \tau + 1 \leq t. \end{array}$$

By lemma 6,

$$\begin{array}{cc} S_t^* = 0 & \text{for } t \leq \tau \\ S_t^* = S_t & \text{for } \tau + 1 \leq t \end{array}.$$

Hence,

$$\begin{aligned} \pi(\sigma^*) - \pi(\sigma) &= \sum_{t=1}^{\tau} D_t(p_1 + tc - c)(p_1 + tc - c) - \left[\sum_{t=1}^{\tau} D_t(p_1 + tc - c)\right] p_1 \\ &= c\sum_{t=1}^{\tau} D_t(p_1 + tc - c)(t-1) > 0 \end{aligned}$$

and the fact that $\pi(\sigma^*) > \pi(\sigma)$ concludes the proof.

Assume now that the statement holds for any period $t-1$ and consider period t. We will first prove that $p_{t+1} - p_t \geq c$ and then that $p_{t+1} - p_t \leq c$.

Assume by way of contradiction that $p_{t+1} - p_t < c$ and suppose that $MR_t(p_t) \leq 0$. Since $p_{\tau+1} - p_\tau = c$ for all $\tau < t$ and because $c < \min\{p_{t+1}^m - p_t^m\}$ this implies that $MR_i(p_i) < 0$ for all $i = 1, ..., t-1$. Hence, the monopolist could increase all revenues by decreasing all price p_i with $i = 1, ..., t-1$ by ε.

Suppose now that $MR_t(p_t) > 0$ and call τ the lowest period at which $S_\tau > 0$. By the lemma 6 it must be the case that $p_{\tau+1} - p_\tau > c$. Moreover, consider $\varepsilon < \min_{t \leq i \leq \tau}\{|p_{i+1}^c - p_i^c - c|\}$ and consider the price sequence $\sigma^* = \{p_t^*\}_{t=1}^T$ such that

$$\begin{array}{cc} p_i^* = p_i & \text{for } i = 1, ..., t \\ p_i^* = p_i + \frac{\varepsilon}{i-t} & \text{for } i = t+1, ..., \tau \\ p_i^* = p_i & \text{for } i = \tau+1, ..., T \end{array}$$

Then

$$\begin{array}{cc} S_i^* = 0 & \text{for } i = 1, ..., \tau - 1 \\ S_t^* = S_t + \frac{\varepsilon}{t-1} \left.\frac{\partial S_t(p)}{\partial p}\right|_{p_t} & \text{for } \tau = \tau' \\ S_i^* = S_i & \text{for } i = \tau+1, ..., T \end{array}$$

Hence, similarly to the argument used above

$$\pi(\sigma^*) > \pi(\sigma)$$

and this proves that $p_{t+1} - p_t \geq c$.

Now, assume by way of contradiction that $p_{t+1} - p_t > c$. Consider τ the lowest period for which $p_{\tau+1} - p_1 \leq (\tau + 1)c$. Consider the sequence $\sigma^* = \{p_t^*\}_{t=1}^T$ such that

$$\begin{array}{cc} p_i^* = p_i & \text{for } i = 1, ..., t-1 \\ p_i^* = p_t + (i-1)c & \text{for } i = t, ..., \tau \\ p_i^* = p_i & \text{for } i = \tau + 1, ..., T \end{array}$$

so that, by lemma 6,

$$\begin{array}{cc} S_i^* = 0 & \text{for } i = 1, ..., \tau \\ S_i^* = S_i & \text{for } i = \tau + 1, ..., T \end{array}.$$

Hence, similarly to what seen above,

$$\pi(\sigma^*) > \pi(\sigma).$$

This means that $p_{t+1}^c - p_t^c \leq c$ and this concludes the proof. ■

8 References

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