Oligopolistic and Monopolistic Competition Unified

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Abstract

We study an equilibrium for a product-differentiated market in which oligopolistic firms and monopolistically competitive firms coexist. We assume that the number of monopolistically competitive firms changes given the number of oligopolistic firms and the equilibrium size is determined when no firms earn positive profits. We show that there may exist multiple equilibria, and the size of monopolistically competitive firms decreases and social welfare level increases as the number of oligopolistic firms increases.

Keywords: *oligopoly, monopolistic competition, product differentiation*

1 Introduction

The theory of oligopoly is a classical method to investigate decision-makings of firms with market power. In an oligopolistic market, the behaviors of firms are strategically interdependent: the behavior of each firm affects on that of its rivals. There also exist barriers to enter into an oligopolistic industry and the number of firms is limited. It is easy to give examples of heterogeneous oligopolistic industries: i.e., airline, automobile, and prosports. The model of heterogeneous oligopoly has been applied for industrial organization and international trade (Dixit(1980, 1986), Bulow, Geanakoplos and Klemperer (1983), Krugman (1984), Eaton and Grossman (1986)).

On the other hand, the theory of monopolistic competition, pioneered by Chamberlim (1933), has been recently employed as the central method for analyzing some other markets: i.e., apparel, catering, publishers and information technology. Since there are no barriers to entry or exit, a number of firms which produce differentiated goods always compete. The model of monopolistic competition has been applied for industrial

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organization, international trade, and economic geography (Spence (1976), Dixit and Stiglitz (1977), Krugman (1979, 1991), Fujita, Krugman and Venables (1991), Ottaviano, Tabuchi and Thisse (2002)).

Both theories are no doubt useful to analyze the actual market mechanisms. However, we also recognize a fact that many industries consist a few large firms and many small firms. In these industries, a few large firms behave oligopolistically: they determine their strategies taking into account their rivals' reactions. On the contrary, small firms are monopolistically competitive and achieve the remaining market shares. There are no strategic interactions among the firms.

In this paper, we formulate a product-differentiated market in which oligopolistic and monopolistically competitive firms coexist. We use a utility function in Shimomura and Ishikawa (2004), which includes the models of Spence(1976) and Dixit-Stiglitz (1977). This utility function enables us to examine in each case that the income effect exists or not. We also assume that the number of monopolistically competitive firms is more quickly adjusted than that of oligopolistic firms. That is, the size of monopolistically competitive firms changes given the number of oligopolistic firms. We show that there may exist multiple equilibria, the size of monopolistically competitive firms decreases and social welfare level increases as the number of oligopolistic firms increases.

2 Consumer

We tentatively suppose that the economy produces a fixed set of differentiated products, each of which is supplied by oligopolistic firms and monopolistically competitive firms.

We also assume that there is a perfectly competitive market in which all the firms produce homogenous goods with identical linear cost functions whose values are measured in terms of numéraire. We thus always set the price of the good to be γ, the constant marginal cost of the competitive firms.

Let N be a positive integer which denotes the *number* of the oligopolistic firms. We also set M a positive real number which denotes the *size* of the monopolistically competitive firms. Then N represents the diversity of differentiated goods sold by oligopolistic firms and the interval $[0, M]$ represents the set of indices of differentiated products sold by monopolistically competitive firms.

We suppose that there exists a representative agent in this economy, whose behavior coincides with the aggregation over the whole group of the existing consumers. We assume the agent is endowed with L units of numéraire, holds ownership shares of all firms, and has a preference relation represented by the utility function:

$$\frac{\beta}{\eta}\left(\sum_{j=1}^{N} Q(j)^{\rho} + \int_{0}^{M} q(i)^{\rho} di\right)^{\eta/\rho} x^{\alpha} + z,$$

$$\alpha + \eta < 1, \quad 0 < \eta < 1, \; 0 < \alpha < 1, \; \beta > 0, \quad \text{and} \quad 0 < \rho < 1$$

where $Q(j)$ is an output level of oligopolistic firm j, $q(i)$ is an output level of monopolistically competitive firm i, x is a consumption of the competitive good and z is that of numéraire. The representative agent thus solves the following problem:

$$\begin{aligned} \text{Maximize} \quad & \frac{\beta}{\eta}\left(\sum_{j=1}^{N} Q(j)^{\rho} + \int_0^M q(i)^{\rho} di\right)^{\eta/\rho} x^{\alpha} + z, \\ \text{subject to} \quad & \sum_{j=1}^{N} P(j)Q(j) + \int_0^M p(i)q(i)di + \gamma x + z \leq Y \end{aligned}$$

where $P(j)$ is the price of oligopolistic product j, $p(i)$ is the price of monopolistically competitive product i, and Y is income.

We take two steps to compute the solution. The first step is to consider the following minimization problem:

$$\text{Minimize} \quad \int_0^M p(i)q(i)di, \qquad \text{subject to} \quad \left[\int_0^M q(i)^{\rho} di\right]^{1/\rho} = Q(0).$$

We then identify $Q(0)$ as the output index for monopolistically competitive goods. The first order condition for interior maximum is:

$$\begin{aligned} & p(i) = \mu\rho q(i)^{\rho-1}\left[\int_0^M q(i)^{\rho} di\right]^{(1-\rho)/\rho}, \quad \text{for all} \quad i \in [0, M] \quad \text{and} \\ & \int_0^M q(i)^{\rho} di = Q(0)^{\rho}, \end{aligned}$$

where μ is the Lagrangian multiplier. This gives the equality of marginal rate of substitution to price ratios, i.e., $p(i)/p(j) = q(i)^{\rho-1}/q(j)^{\rho-1}$ for any pair of $i, j \in [0, M]$. We thus set $q(i)/p(i)^{1/(\rho-1)} = q(j)/p(j)^{1/(\rho-1)} \equiv R$, then $q(i) = Rp(i)^{1/(\rho-1)}$. We introduce the *price index* for differentiated goods supplied by monopolistically competitive firms:

$$P(0) \equiv \left[\int_0^M p(i)^{\rho/(\rho-1)} di\right]^{(\rho-1)/\rho}. \tag{1}$$

Then we obtain $Q(0)$ and $q(i)$:

$$\begin{aligned} Q(0) =& R\left(\int_0^M p(i)^{\rho/(\rho-1)} di\right)^{1/\rho} = RP(0)^{1/(\rho-1)}, \quad \text{i.e., } R = \frac{Q(0)}{P(0)^{1/(\rho-1)}}, \quad \text{and} \\ q(i) =& Rp(i)^{1/(\rho-1)} = Q(0)\left(\frac{p(i)}{P(0)}\right)^{1/(\rho-1)} \quad \text{for all} \quad i \in [0, M]. \end{aligned} \tag{2}$$

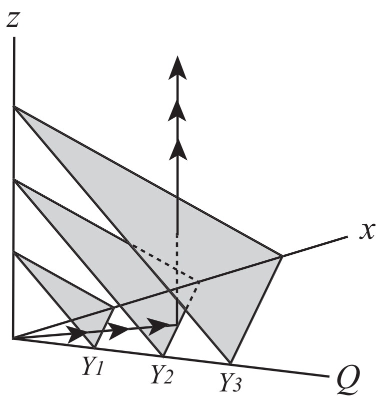


Figure 1: Income-Consumption Path when Utility Function is Quasi-Linear

The second step is to substitute (1) and (2) into the original maximization problem. Then we obtain the reduced maximization problem:

$$\text{Maximize} \quad \frac{\beta}{\eta}\left(\sum_{j=0}^{N} Q(j)^{\rho}\right)^{\eta/\rho} x^{\alpha} + z, \qquad \text{subject to} \quad \sum_{j=0}^{N} P(j)Q(j) + \gamma x + z \leq Y.$$

We discuss the case that both $Q(j)$ and x are positive, but take into account the possibility that $z = 0$ when Y is low (See Fig.1). By the Kuhn-Tucker theorem, the first order condition is given by:

$$\beta\left(\sum_{j=0}^{N} Q(j)^{\rho}\right)^{(\eta-\rho)/\rho} Q(j)^{\rho-1}x^{\alpha} = \lambda P(j), \tag{3}$$

$$\frac{\alpha\beta}{\eta}\left(\sum_{j=0}^{N} Q(j)^{\rho}\right)^{\eta/\rho} x^{\alpha-1} = \lambda\gamma, \tag{4}$$

$$z(1-\lambda) = 0, \quad z \geq 0, \quad 1-\lambda \leq 0; \quad \text{and} \tag{5}$$

$$Y - \sum_{j=0}^{N} P(j)Q(j) - \gamma x - z = 0, \tag{6}$$

where λ is the Lagrangian multiplier.

First, we investigate the case z is positive. Then (5) reduces to $\lambda = 1$. Substituting $\lambda = 1$ into (3) and (4), we obtain:

$$\beta\left(\sum_{j=0}^{N} Q(j)^{\rho}\right)^{(\eta-\rho)/\rho} Q(j)^{\rho-1}x^{\alpha} = P(j); \quad \text{and} \tag{7}$$

$$\frac{\alpha\beta}{\eta}\left(\sum_{j=0}^{N} Q(j)^{\rho}\right)^{\eta/\rho} x^{\alpha-1} = \gamma, \tag{8}$$

From (7) and (8),

$$P(j)Q(j)^{1-\rho} = k^{1/(1-\alpha)} \left(\sum_{j=0}^{N} Q(j)^{\rho} \right)^{\frac{\rho(1-\alpha)-\eta}{\rho(\alpha-1)}}. \tag{9}$$

where $k = \beta \left(\frac{\alpha}{\eta\gamma} \right)^{\alpha}$. This gives $P(i)Q(i)^{1-\rho} = P(j)Q(j)^{1-\rho} \equiv \tau$. We thus obtain

$$\sum_{j=0}^{N} Q(j)^{\rho} = \tau^{\rho/(1-\rho)} \sum_{j=0}^{N} P(j)^{\rho/(\rho-1)} = P(j)^{\rho/(1-\rho)} Q(j)^{\rho} \sum_{j=0}^{N} P(j)^{\rho/(\rho-1)}. \tag{10}$$

Let P be the price index for all of the differentiated products. That is,

$$P \equiv \left[\sum_{j=0}^{N} P(i)^{\rho/(\rho-1)} \right]^{(\rho-1)/\rho}. \tag{11}$$

From (9), (10) and (11), we obtain

$$Q(i) = k^{1/(1-\eta-\alpha)} P(i)^{1/(\rho-1)} P^{\frac{\rho(1-\alpha)-\eta}{(1-\rho)(1-\eta-\alpha)}} \equiv d_o(P(i), P), \quad \text{and} \tag{12}$$

$$x = \left\{ \beta \left(\frac{\alpha}{\eta\gamma} \right)^{1-\eta} P^{-\eta} \right\}^{1/(1-\eta-\alpha)}. \tag{13}$$

where $d_o(P(i), P)$ is the demand function of oligopolistic product i when $z > 0$. From (12), $Q(0) = k^{1/(1-\eta-\alpha)} P(0)^{1/(\rho-1)} P^{\frac{\rho(1-\alpha)-\eta}{(1-\rho)(1-\eta-\alpha)}}$. Substituting this into (2), we have

$$q(i) = p(i)^{1/(\rho-1)} k^{1/(1-\eta-\alpha)} P^{\frac{\rho(1-\alpha)-\eta}{(1-\rho)(1-\eta-\alpha)}} \equiv d_m(p(i), P). \tag{14}$$

where $d_m(p(i), P)$ is the demand function of monopolistically competitive product i when $z > 0$.

Remark 1 *Let $z > 0$. The differentiated goods are substitutes (i.e., $\partial d_o(P(i), P)/\partial P > 0$ and $\partial d_m(p(i), P)/\partial P > 0$) if $\rho(1-\alpha)-\eta > 0$, and they are complements (i.e., $\partial d_o(P(i), P)/\partial P < 0$ and $\partial d_m(p(i), P)/\partial P < 0$) if $\rho(1-\alpha)-\eta < 0$. Notice that these relations are symmetric in our model.*

From (6), (12) and (13), we obtain z as a function of Y and P: $z = Y - \sum_{j=0}^{N} P(j)Q(j) - \gamma x = Y - (\eta+\alpha)/\eta \cdot k^{1/(1-\eta-\alpha)} P^{-\eta/(1-\eta-\alpha)}$. Thus, $z > 0$ is equivalent to

$$Y > \frac{\eta+\alpha}{\eta} k^{1/(1-\eta-\alpha)} P^{-\eta/(1-\eta-\alpha)}. \tag{15}$$

Next we consider the case of $z = 0$. The first order condition for utility maximization is

$$\beta \left(\sum_{j=0}^{N} Q(j)^{\rho} \right)^{(\eta-\rho)/\rho} Q(j)^{\rho-1} x^{\alpha} = \lambda P(j), \tag{16}$$

$$\frac{\alpha\beta}{\eta} \left(\sum_{j=0}^{N} Q(j)^{\rho} \right)^{\eta/\rho} x^{\alpha-1} = \lambda\gamma, \quad \text{and} \tag{17}$$

$$Y - \sum_{j=0}^{N} P(j)Q(j) - \gamma x = 0. \tag{18}$$

From (16), (17) and (18), $x = \alpha/(\eta\gamma) \cdot P(i)^{1/(1-\rho)} Q(i) P^{\rho/(\rho-1)}$ and $Y - P(i)^{1/(1-\rho)} Q(i) P^{\rho/(\rho-1)} - \gamma x = 0$. Then we have

$$Q(i) = \frac{\eta}{\eta + \alpha} Y P(i)^{1/(\rho-1)} P^{\rho/(1-\rho)} \equiv D_o(P(i), P, Y), \tag{19}$$

$$x = \frac{\alpha}{\gamma(\eta + \alpha)} Y, \quad \text{and} \tag{20}$$

$$q(i) = \frac{\eta}{\eta + \alpha} Y p(i)^{1/(\rho-1)} P^{\rho/(1-\rho)} \equiv D_m(p(i), P, Y). \tag{21}$$

where $D_o(P(i), P, Y)$ is the demand function of oligopolistic product i and $D_m(P(i), P, Y)$ is that of the monopolistically competitive products when $z = 0$.

Remark 2 *When $z = 0$, the differentiated goods are always substitutes (i.e., $\partial D_o(P(j), P, Y)/\partial P > 0$ for all $j \in \{1, \cdots, N\}$ and $\partial D_m(p(i), P, Y)/\partial P > 0$ for all $i \in [0, M]$).*

3 Oligopolistic Firm

We assume that each oligopolistic firm selects its output level given those of the other firms to maximize its profit. That is, the solution to the interactive profit maximization problem is given as a Cournot-Nash equilibrium of the following normal-form game:

- players: $i \in \{1, \cdots, N\}$;
- strategy for player i: $Q(i)$; and
- payoff function for player i:

  $$\Pi^i(Y, Q(0); Q(1), \cdots, Q(N)) = \Phi^i(Y, Q(0); Q(1), \cdots, Q(N))Q(i) - cQ(i) - F,$$

  for all $i \in \{1, \cdots, N\}$, where $\Phi^i(\cdot)$ is the inverse demand function for the products of oligopolistic firm i, Y is income, $Q(0)$ is the output level of monopolistically competitive firms, c is the marginal cost and F is the fixed cost.

3.1 Case without Income Effect

First we deal with the case that income effect exists. The profit-maximization problem for oligopolistic firm i is given by

$$\text{Maximize} \qquad \Phi^i(Y, Q(0); Q(1), \cdots, Q(N))Q(i) - cQ(i) - F.$$

From (12), the inverse demand function is

$$\Phi^i(Y, Q(0); Q(1), \cdots, Q(N)) = Q(i)^{\rho-1} k^{1/(1-\alpha)} \left[Q(i)^\rho + \sum_{j \neq i} Q(j)^\rho \right]^{\frac{\rho(1-\alpha)-\eta}{\rho(\alpha-1)}} \tag{22}$$

since

$$P = k^{1/(1-\alpha)} \left[Q(i)^\rho + \sum_{j \neq i} Q(j)^\rho \right]^{\frac{1-\eta-\alpha}{\rho(\alpha-1)}}. \tag{23}$$

Notice that Φ^i is actually independent of Y. Thus the profit function of oligopolistic firm i is

$$\Pi^i(Y, Q(0); Q(1), \cdots, Q(N)) = k^{1/(1-\alpha)} Q(i)^\rho \left[Q(i)^\rho + \sum_{j \neq i} Q(j)^\rho \right]^{\frac{\rho(1-\alpha)-\eta}{\rho(\alpha-1)}} - cQ(i) - F. \tag{24}$$

We have the following lemma.

Lemma 1 *Given any $Q(j)$, $j \neq i$, there exists a unique $Q(i)$ that maximizes Π^i.*

Proof. See Appendix.

From (24), the first order condition is:

$$k^{1/(1-\alpha)} \rho Q(i)^{\rho-1} \left[Q(i)^\rho + \sum_{j \neq i} Q(j)^\rho \right]^{\frac{2\rho(1-\alpha)-\eta}{\rho(\alpha-1)}} \left[\frac{\eta}{\rho(1-\alpha)} Q(i)^\rho + \sum_{j \neq i} Q(j)^\rho \right] = c.$$

Substituting (23) into this equation, we have

$$\frac{\rho(1-\alpha)-\eta}{\rho(1-\alpha)} k^{\frac{1-2\rho}{1-\eta-\alpha}} Q(i)^\rho P^{\frac{2\rho(1-\alpha)-\eta}{1-\eta-\alpha}} + \frac{1}{\rho} cQ(i)^{1-\rho} = k^{\frac{1-\rho}{1-\eta-\alpha}} P^{\frac{\rho(1-\alpha)-\eta}{1-\eta-\alpha}}. \tag{25}$$

We also obtain the proposition about the relation between $Q(i)$ and $Q(j)$ for all $i \neq j$.

Proposition 1 *Let $i \neq j$. If the differentiated goods are substitutes, $Q(i)$ and $Q(j)$ may be strategic substitutes or complements. If the differentiated goods are complements, $Q(i)$ and $Q(j)$ are always strategic complements.*

Proof. See Appendix.

3.2 Case with Income Effect

Next, we consider the case with income effect. The profit-maximization problem for oligopolistic firm i is given by

$$\text{Maximize} \qquad \Phi^i(Y, Q(0); Q(1), \cdots, Q(N))Q(i) - cQ(i) - F.$$

The inverse demand function is

$$\Phi^i(Y, Q(0); Q(1), \cdots, Q(N)) = \left(\frac{\eta}{\eta+\alpha}Y\right) Q(i)^{\rho-1} \left[Q(i)^\rho + \sum_{j \neq i} Q(j)^\rho\right]^{-1} \tag{26}$$

since

$$P = \left(\frac{\eta}{\eta+\alpha}Y\right) \left[Q(i)^\rho + \sum_{j \neq i} Q(j)^\rho\right]^{-1/\rho}. \tag{27}$$

Thus the profit function of oligopolistic firm i is

$$\Pi^i(Y, Q(0); Q(1), \cdots, Q(N)) = \left(\frac{\eta}{\eta+\alpha}Y\right) Q(i)^{\rho} \left[Q(i)^\rho + \sum_{j \neq i} Q(j)^\rho\right]^{-1} - cQ(i) - F. \tag{28}$$

We then have the following lemma.

Lemma 2 *Given any $Q(j)$, $j \neq i$, there exists a unique $Q(i)$ that maximizes Π^i.*

Proof. See Appendix.

Then the first order condition is

$$\left(\frac{\eta}{\eta+\alpha}Y\right)^\rho - P^\rho Q(i)^\rho = \frac{1}{\rho} c P^{-\rho} Q(i)^{1-\rho} \left(\frac{\eta}{\eta+\alpha}Y\right)^{2\rho-1}. \tag{29}$$

We also obtain the proposition about the relation between $Q(i)$ and $Q(j)$ for all $i \neq j$.

Proposition 2 *The output levels $Q(i)$ and $Q(j)$ are always strategic substitutes for all $i \neq j$.*

Proof. See Appendix.

4 Monopolistically Competitive Firm

4.1 Case without Income Effect

Next, we consider monopolistically competitive firms. We assume that each firm i has the identical cost function $Cq(i) + f$ where C is the marginal cost and f is the fixed cost. A monopolistically competitive firm maximizes its profit subject to the demand function for its own product given the market price index P.

First, we investigate the case without income effect. Monopolistically competitive firm i chooses $q(i)$ and $p(i)$ given P:

$$\begin{aligned} \text{Maximize} \quad & p(i)q(i) - Cq(i) - f \\ \text{subject to} \quad & q(i) = d_m(p(i), P). \end{aligned}$$

From (14), the inverse demand function is given by $p(i) = q(i)^{\rho-1} k^{\frac{1-\rho}{1-\eta-\alpha}} P^{\frac{\rho(1-\alpha)-\eta}{1-\eta-\alpha}}$, then the associated profit is $q(i)^{\rho} k^{\frac{1-\rho}{1-\eta-\alpha}} P^{\frac{\rho(1-\alpha)-\eta}{1-\eta-\alpha}} - Cq(i) - f$. Since $\rho < 1$, this attains a single peak. Thus the first order condition of the profit maximization is

$$\rho q(i)^{\rho-1} k^{\frac{1-\rho}{1-\eta-\alpha}} P^{\frac{\rho(1-\alpha)-\eta}{1-\eta-\alpha}} = C.$$

We thus obtain the profit-maximizing values independently of i:

$$p(0) = \frac{C}{\rho}, \quad \text{and} \quad q(0) = \left(\frac{\rho}{C}\right)^{1/(1-\rho)} k^{1/(1-\eta-\alpha)} P^{\frac{\rho(1-\alpha)-\eta}{(1-\rho)(1-\eta-\alpha)}}. \tag{30}$$

4.2 Case with Income Effect

Next, we deal with the case with income effect. The profit maximization problem of monopolistically competitive firm i is given by

$$\begin{aligned} \text{Maximize} \quad & p(i)q(i) - (Cq(i) + f) \\ \text{subject to} \quad & q(i) = D(p(i), P, Y). \end{aligned}$$

From (21), the inverse demand function is given by $p(i) = (\eta/(\eta+\alpha) \cdot Y)^{1-\rho} q(i)^{\rho-1} P^{\rho}$. Then the profit function is $(\eta/(\eta+\alpha) \cdot Y)^{1-\rho} q(i)^{\rho} P^{\rho} - Cq(i) - f$. Since $\rho < 1$, this attains a single peak. The first order condition for profit maximization is

$$\rho \left(\frac{\eta}{\eta+\alpha} Y\right)^{1-\rho} q(i)^{\rho-1} P^{\rho} = C.$$

Hence we have $p(0)$ and $q(0)$ independently of i:

$$p(0) = \frac{C}{\rho}, \quad \text{and} \quad q(0) = \left(\frac{\rho}{C}\right)^{1/(1-\rho)} \left(\frac{\eta}{\eta+\alpha} Y\right) P^{\rho/(1-\rho)}. \tag{31}$$

5 Equilibrium

5.1 Case without Income Effect

We define an equilibrium as a state that the representative consumer maximizes his utility subject to the budget constraint, oligopolistic firms and monopolistically competitive firms maximize their own profits and the profits of monopolistically competitive firms are zero. We suppose that the size of monopolistically competitive firms is adjusted until their profits are zero given the number of oligopolistic firms.

We characterize the equilibrium by the following equations: the demand functions, the profit-maximization of monopolistically competitive firms, the profit-maximization of oligopolistic firms and the zero-profit condition of monopolistically compeitive firms. From (11) and (23), the demand conditions tell

$$P=\left[P(0)^{\rho/(\rho-1)}+NP(*)^{\rho/(\rho-1)}\right]^{(\rho-1)/\rho}=k^{1/(1-\alpha)}\left[Q(0)^{\rho}+NQ(*)^{\rho}\right]^{\frac{1-\eta-\alpha}{\rho(\alpha-1)}}. \tag{32}$$

From (14) and (30), we have the equilibrium price and quantity of monopolistically competitive products:

$$P(0)=\frac{C}{\rho}M^{(\rho-1)/\rho},\quad \text{and} \tag{33}$$

$$Q(0)=\left(\frac{C}{\rho}\right)^{1/(\rho-1)}k^{1/(1-\eta-\alpha)}M^{1/\rho}P^{\frac{\rho(1-\alpha)-\eta}{(1-\rho)(1-\eta-\alpha)}}. \tag{34}$$

since $p(0)=P(0)M^{(1-\rho)/\rho}$ and $q(0)=Q(0)M^{-1/\rho}$. Substituting (34) into the zero profit condition of monopolistically competitive firm, we have

$$P=\kappa_1^{\frac{(1-\rho)(1-\eta-\alpha)}{\eta-\rho(1-\alpha)}}k^{\frac{1-\rho}{\eta-\rho(1-\alpha)}}, \tag{35}$$

where $\kappa_1=((1-\rho)/f)(C/\rho)^{\rho/(\rho-1)}$. Note that P is independent of M and N. From (25), the individual profit-maximizer of oligopolistic firm $Q(*)$ satisfies the following formula:

$$\frac{\rho(1-\alpha)-\eta}{\rho(1-\alpha)}k^{\rho/\{\eta-\rho(1-\alpha)\}}\kappa_1^{\frac{\rho(1-\rho)(1-\alpha)}{\eta-\rho(1-\alpha)}}Q(*)^{\rho}=1-\frac{1}{\rho}cQ(*)^{1-\rho}\kappa_1^{1-\rho}. \tag{36}$$

We see that $Q(*)$ is also independent of M and N.

Remark 3 *If the differentiated goods are substitutes, there exists a unique equilibrium $Q(*)$. However, there may exist one, two, or no equilibrium if the differentiated goods are complements.*

Proof. See Appendix.

We also have $Q(0)$ as a function of M:

$$Q(0)=\frac{f}{1-\rho}\cdot\frac{\rho}{C}M^{1/\rho}. \tag{37}$$

From (32), (33) and (35),

$$P(*)=N^{(1-\rho)/\rho}\left[\kappa_1^{\frac{\rho(1-\eta-\alpha)}{\rho(1-\alpha)-\eta}}k^{\rho/\{\rho(1-\alpha)-\eta\}}-\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}M\right]^{(\rho-1)/\rho}. \tag{38}$$

From (32), (35) and (37),

$$M=-\left(\frac{f}{1-\rho}\cdot\frac{\rho}{C}\right)^{-\rho}Q(*)^{\rho}N+\left(\frac{f}{1-\rho}\cdot\frac{\rho}{C}\right)^{-\rho}\kappa_1^{\frac{\rho(1-\alpha)(1-\rho)}{\rho(1-\alpha)-\eta}}k^{\frac{\rho}{\rho(1-\alpha)-\eta}}. \tag{39}$$

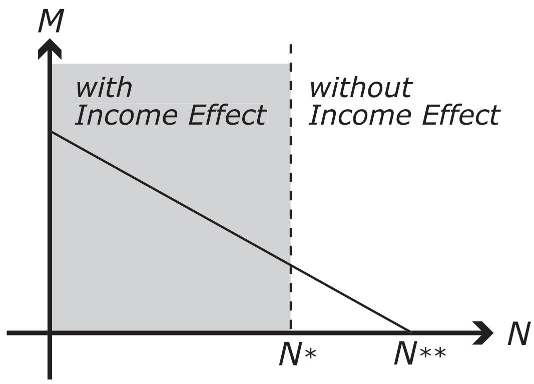


Figure 2: Diminishing the number of monopolistically competitive firms

Substituting (39) into (38),

$$P(*) = \kappa_1^{\rho-1} Q(*)^{\rho-1}, \tag{40}$$

which is independent of M and N. Thus we have some propositions for the case without income effect.

Proposition 3 *The price index of differentiated product market and both of the price index and the quantity of oligopolistic products are constant.*

Proof. From (35), (36) and (40), P, $P(*)$ and $Q(*)$ are constant. □

Proposition 4 *The size of monopolistically competitive firms decreases as that of oligopolistic firms increases.*

Proof. From (39), M is a decreasing linear function of N. □

Proposition 5 *The quantities of oligopilistic products are always strategic complements.*

Proof. See Appendix.

We consider the boundary N^* with and without income effect. From (15), we have the range without income effect is given by

$$Y > \frac{\eta + \alpha}{\eta} \kappa_1^{\frac{\eta(1-\rho)}{\rho(1-\alpha)-\eta}} k^{\rho/\{\rho(1-\alpha)-\eta\}}. \tag{41}$$

where

$$Y = L + M\pi(0) + N\Pi(*) = L + N\{\kappa_1^{\rho-1} Q(*)^{\rho} - cQ(*) - F\}.$$

Thus we have

$$N^* = \left[\frac{\eta + \alpha}{\eta} \kappa_1^{\frac{\eta(1-\rho)}{\rho(1-\alpha)-\eta}} k^{\rho/\{\rho(1-\alpha)-\eta\}} - L\right] \{\kappa_1^{\rho-1} Q(*)^{\rho} - cQ(*) - F\}^{-1}. \tag{42}$$

Proposition 4 tells that there is a possibility that the monopolistically competitive firms disappear. From (39), we have the critical value N^{**} that satisfies $M = 0$:

$$N^{**} = Q(*)^{-\rho} \kappa_1^{\frac{\rho(1-\alpha)(1-\rho)}{\rho(1-\alpha)-\eta}} k^{\frac{\rho}{\rho(1-\alpha)-\eta}}. \tag{43}$$

From (42) and (43), the relation between N^* and N^{**} is

$$\begin{aligned}
N^* < N^{**} \text{ if } L - \frac{\alpha}{\eta}\kappa_1^{\frac{\eta(1-\rho)}{\rho(1-\alpha)-\eta}} k^{\rho/\{\rho(1-\alpha)-\eta\}} > Q(*)^{-\rho}\kappa_1^{\frac{\rho(1-\alpha)(1-\rho)}{\rho(1-\alpha)-\eta}} k^{\frac{\rho}{\rho(1-\alpha)-\eta}}(cQ(*)+F),\\
N^* = N^{**} \text{ if } L - \frac{\alpha}{\eta}\kappa_1^{\frac{\eta(1-\rho)}{\rho(1-\alpha)-\eta}} k^{\rho/\{\rho(1-\alpha)-\eta\}} = Q(*)^{-\rho}\kappa_1^{\frac{\rho(1-\alpha)(1-\rho)}{\rho(1-\alpha)-\eta}} k^{\frac{\rho}{\rho(1-\alpha)-\eta}}(cQ(*)+F),\\
N^* > N^{**} \text{ if } L - \frac{\alpha}{\eta}\kappa_1^{\frac{\eta(1-\rho)}{\rho(1-\alpha)-\eta}} k^{\rho/\{\rho(1-\alpha)-\eta\}} < Q(*)^{-\rho}\kappa_1^{\frac{\rho(1-\alpha)(1-\rho)}{\rho(1-\alpha)-\eta}} k^{\frac{\rho}{\rho(1-\alpha)-\eta}}(cQ(*)+F).
\end{aligned}$$

If $N \geq N^{**}$, i.e., $M = 0$, we obtain the following variables:

$$\begin{aligned}
P(*) =& cN\left\{\frac{\eta-\rho(1-\alpha)}{1-\alpha}+\rho N\right\}^{-1},\\
Q(*) =& c^{\frac{\alpha-1}{1-\eta-\alpha}} k^{1/(1-\eta-\alpha)} N^{\frac{\eta-2\rho(1-\alpha)}{\rho(1-\eta-\alpha)}}\left\{\frac{\eta-\rho(1-\alpha)}{1-\alpha}+\rho N\right\}^{\frac{1-\alpha}{1-\eta-\alpha}},\\
P =& cN^{(2\rho-1)/\rho}\left\{\frac{\eta-\rho(1-\alpha)}{1-\alpha}+\rho N\right\}^{-1} \quad \text{and}\\
Y =& L - NF + c^{\frac{-\eta}{1-\eta-\alpha}} k^{1/(1-\eta-\alpha)} N^{\frac{\eta(1-\rho)-\rho(1-\alpha)}{\rho(1-\eta-\alpha)}}\left\{\frac{\eta-\rho(1-\alpha)}{1-\alpha}+\rho N\right\}^{\frac{\eta}{1-\eta-\alpha}}\\
&\left\{(1-\rho)N - \frac{\eta-\rho(1-\alpha)}{1-\alpha}\right\}.
\end{aligned}$$

5.2 Case with Income Effect

Consider the simultaneous equations for the case with income effect. We also consider the formula for Y:

$$\begin{aligned}
Y =& L + N\Pi + M\pi = L + N(P(*)Q(*) - cQ(*) - F)\\
=& L + N\left\{\left(\frac{\eta}{\eta+\alpha}Y\right)^{1-\rho} Q(*)^{\rho}P^{\rho} - cQ(*) - F\right\}.
\end{aligned} \tag{44}$$

From (11) and (27),

$$P = \left[P(0)^{\rho/(\rho-1)} + NP(*)^{\rho/(\rho-1)}\right]^{(\rho-1)/\rho} = \left(\frac{\eta}{\eta+\alpha}Y\right)\left[Q(0)^{\rho} + NQ(*)^{\rho}\right]^{-1/\rho}. \tag{45}$$

From (21) and (31), we have the price and quantity of monopolistically competitive products at the equilibrium:

$$P(0) = \frac{C}{\rho}M^{(\rho-1)/\rho}, \quad \text{and} \quad Q(0) = \left(\frac{C}{\rho}\right)^{1/(\rho-1)}\left(\frac{\eta}{\eta+\alpha}Y\right)M^{1/\rho}P^{\rho/(1-\rho)}. \tag{46}$$

since $p(0) = P(0)M^{(1-\rho)/\rho}$ and $q(0) = Q(0)M^{-1/\rho}$. Substituting (46) into the condition $\pi(*) = 0$,

$$\frac{\eta}{\eta+\alpha}Y = \frac{f}{1-\rho}\left(\frac{C}{\rho}\right)^{\rho/(1-\rho)} P^{\rho/(\rho-1)}. \tag{47}$$

Thus we have

$$Q(0) = \frac{f}{1-\rho}\left(\frac{C}{\rho}\right)^{-1} M^{1/\rho} \tag{48}$$

From (29), (44), (45), (46) and (47), we obtain the following simultaneous equations:

$$P^{\rho/(\rho-1)} = M\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)} + N\kappa_1^{\rho}Q(*)^{\rho}, \tag{49}$$

$$P^{\rho/(\rho-1)} = \kappa_1^{\rho}Q(*)^{\rho}\left[1 - \frac{1}{\rho}c\kappa_1^{1-\rho}Q(*)^{1-\rho}\right]^{-1}, \quad \text{and} \tag{50}$$

$$P^{\rho/(\rho-1)} = \frac{\eta}{\eta+\alpha}\kappa_1\left\{L - NF + NQ(*)\left(\kappa_1^{\rho-1}Q(*)^{\rho-1} - c\right)\right\}. \tag{51}$$

From (49), (50) and (51),

$$\frac{\eta+\alpha}{\eta}\kappa_1^{\rho}Q(*)^{\rho} = \{(L-NF)\kappa_1 - cN\kappa_1 Q(*) + N\kappa_1^{\rho}Q(*)^{\rho}\}\left\{1 - \frac{1}{\rho}c\kappa_1^{1-\rho}Q(*)^{1-\rho}\right\} \tag{52}$$

$$M = \frac{\eta}{\eta+\alpha}\frac{1-\rho}{f}\left[L - NF - NQ(*)\left\{\frac{\alpha}{\eta}\kappa_1^{\rho-1}Q(*)^{\rho-1} + c\right\}\right]. \tag{53}$$

Remark 4 *There may exist one, two, or no equilibrium.*

Proof. See Appendix.

From (45) and (49),

$$P(*) = \kappa_1^{\rho-1}Q(*)^{\rho-1}. \tag{54}$$

Note that $Q(*)$ is independent of M. Consider the case where the profit oligopolistic firm is positive. We have some propositions about a differentiated product market with income effect.

Proposition 6 *The quantity of oligopolistic firms increases as that of oligopolistic firms increases.*

Proof. From (52),

$$\begin{aligned}\frac{dQ(*)}{dN} = &\left\{1 - \frac{1}{\rho}c\kappa_1^{1-\rho}Q(*)^{1-\rho}\right\}\{\kappa_1 F + c\kappa_1 Q(*) - \kappa_1^{\rho}Q(*)^{\rho}\} \\ &\left[-\frac{1}{\rho}c(1-\rho)\kappa_1^{1-\rho}Q(*)^{-\rho}\{(L-NF)\kappa_1 - cN\kappa_1 Q(*) + N\kappa_1^{\rho}Q(*)^{\rho}\}\right. \\ &\left.+Q(*)^{-1}\left\{-cN\kappa_1 Q(*) + \rho N\kappa_1^{\rho}Q(*)^{\rho} - \frac{\eta+\alpha}{\eta}\rho\kappa_1^{\rho}Q(*)^{\rho}\right\}\right]^{-1}.\end{aligned} \tag{55}$$

Recall that $1 - 1/\rho \cdot c\kappa_1^{1-\rho}Q(*)^{1-\rho}$ and $(L - NF)\kappa_1 - cN\kappa_1 Q(*) + N\kappa_1^\rho Q(*)^\rho$ are always positive. Next we consider the sign of $-cN\kappa_1 Q(*) + \rho N\kappa_1^\rho Q(*)^\rho - \frac{\eta+\alpha}{\eta}\rho\kappa_1^\rho Q(*)^\rho$. Rearranging (52),

$$-\frac{1}{\rho}c\frac{\eta+\alpha}{\eta}\kappa_1 Q(*) = \left\{(L - NF)\kappa_1 - cN\kappa_1 Q(*) + N\kappa_1^\rho Q(*)^\rho - \frac{\eta+\alpha}{\eta}\kappa_1^\rho Q(*)^\rho\right\} \times \left\{1 - \frac{1}{\rho}c\kappa_1^{1-\rho}Q(*)^{1-\rho}\right\}.$$

Thus $(L-NF)\kappa_1 - cN\kappa_1 Q(*) + N\kappa_1^\rho Q(*)^\rho - (\eta+\alpha)/\eta \cdot \kappa_1^\rho Q(*)^\rho$ is always negative. We then have $cN\kappa_1 Q(*) > N\kappa_1 Q(*)^\rho - (\eta+\alpha)/\eta \cdot \kappa_1^\rho Q(*)^\rho > \rho\{N\kappa_1^\rho Q(*)^\rho - (\eta+\alpha)/\eta \cdot \kappa_1^\rho Q(*)^\rho\}$. Hence $-cN\kappa_1 Q(*) + \rho N\kappa_1^\rho Q(*)^\rho - ((\eta+\alpha)/\eta)\rho\kappa_1^\rho Q(*)^\rho$ is always negative. Consider the sign of $\kappa_1 F + c\kappa_1 Q(*) - \kappa_1^\rho Q(*)^\rho$. The profit of oligopolistic firm is

$$\Pi(*) = \kappa_1^{\rho-1}Q(*)^\rho - cQ(*) - F.$$

Thus $\kappa_1 F + c\kappa_1 Q(*) - \kappa_1^\rho Q(*)^\rho$ is positive if $\Pi(*)$ is negative. Then $dQ(*)/dN > 0$ if $\Pi(*) > 0$ and $dQ(*)/dN < 0$ if $\Pi(*) < 0$. □

Proposition 7 *The size of monopolistically competitive firms decreases as that of oligopolistic firms increases.*

Proof. From (53),

$$\frac{dM}{dN} = -\frac{\eta}{\eta+\alpha}\frac{1-\rho}{f}\left[F + Q(*)\left\{\frac{\alpha}{\eta}\kappa_1^{\rho-1}Q(*)^{\rho-1} + c\right\} + \frac{dQ(*)}{dN}N\left\{\frac{\alpha}{\eta}\rho\kappa_1^{\rho-1}Q(*)^{\rho-1} + c\right\}\right].$$

From Proposition 6, M is a decreasing function for N if $\Pi(*)$ is positive. □

Proposition 8 *The price index of the differentiated product market decreases as the number of oligopolistic firms increases.*

Proof. From (49),

$$\frac{dP}{dN} = \frac{\partial P}{\partial M}\cdot\frac{dM}{dN} + \frac{\partial P}{\partial N},$$

where $$\frac{\partial P}{\partial M} = \frac{\rho-1}{\rho}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\left[M\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)} + N\kappa_1^\rho Q(*)^\rho\right]^{-1/\rho},$$ and

$$\frac{\partial P}{\partial N} = \frac{\rho-1}{\rho}\left[M\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)} + N\kappa_1^\rho Q(*)^\rho\right]^{-1/\rho}\kappa_1^\rho\left\{Q(*)^\rho + \rho Q(*)^{\rho-1}N\frac{\partial Q(*)}{\partial N}\right\},$$

since $Q(*)$ is independent of $M(N)$ and N. Then

$$\frac{dP}{dN} = \frac{1-\rho}{\rho}\frac{\eta}{\eta+\alpha}\left[M\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)} + N\kappa_1^\rho Q(*)^\rho\right]^{-1/\rho} \times \left[\{\kappa_1 F + \kappa_1^\rho Q(*)^\rho + c\kappa_1 Q(*)\} - N\frac{\partial Q(*)}{\partial N}\rho\kappa_1^\rho Q(*)^{\rho-1}\left\{1 - \frac{1}{\rho}c\kappa_1^{1-\rho}Q(*)^{1-\rho}\right\}\right].$$

From (55),

$$\frac{dP}{dN} = \frac{1-\rho}{\rho}\frac{\eta}{\eta+\alpha}\left[M\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)} + N\kappa_1^\rho Q(*)^\rho\right]^{-1/\rho}\{\kappa_1 F + c\kappa_1 Q(*) - \kappa_1^\rho Q(*)^\rho\}$$
$$\times\left[1 - N\rho\kappa_1^\rho Q(*)^{\rho-1}A_2\left\{1-\frac{1}{\rho}c\kappa_1^{1-\rho}Q(*)^{1-\rho}\right\}^2\right]$$

where

$$A_2 = \left[-\frac{1}{\rho}c(1-\rho)\kappa_1^{1-\rho}Q(*)^{-\rho}\{(L-NF)\kappa_1 - cN\kappa_1 Q(*) + N\kappa_1^\rho Q(*)^\rho\}\right.$$
$$\left.+Q(*)^{-1}\left\{-cN\kappa_1 Q(*) + \rho N\kappa_1^\rho Q(*)^\rho - \frac{\eta+\alpha}{\eta}\rho\kappa_1^\rho Q(*)^\rho\right\}\right]^{-1}$$

Since $A_2 < 0$, the sign of $\partial P(M(N), N)/\partial N$ is determined by that of $\kappa_1 F + c\kappa_1 Q(*) - \kappa_1^\rho Q(*)^\rho$. Thus $\partial P(M(N), N)/\partial N$ is negative if $\Pi(*)$ is positive. $\square$

From Proposition 7, there is a possibility that the monopolistically competitive firms disappear. From (53), we have the critical value N^{**} that satisfies $M = 0$:

$$N^{**} = L\left\{\frac{\alpha}{\eta}\kappa_1^{\rho-1}Q(*)^\rho + cQ(*) + F\right\}^{-1}.$$

Note that $Q(*)$ is a function of N^{**}.

If $N > N^{**}$, i.e., $M = 0$, we obtain the following variables:

$$P(*) = \frac{c}{\rho}\frac{N}{N-1}, \qquad Q(*) = \frac{\rho\eta(N-1)}{cN\{\alpha\eta + \rho\eta(N-1)\}}(L-NF),$$
$$P = \frac{c}{\rho}(N-1)^{-1}N^{(2\rho-1)/\rho} \quad \text{and} \quad Y = \frac{(\eta+\alpha)N}{\alpha\eta+\rho\eta(N-1)}(L-NF).$$

6 Social Welfare

6.1 Case without Income Effect

From (13) and (15), the social welfare is calculated as

$$W = \frac{\beta}{\eta}\left(\sum_{j=1}^N Q(j)^\rho + \int_0^M q(i)^\rho di\right)^{\eta/\rho} x^\alpha + z = Y + \frac{1-\eta-\alpha}{\eta}k^{1/(1-\eta-\alpha)}P^{-\eta/(1-\eta-\alpha)}. \tag{56}$$

Then $dW/dN = dY/dN$ since P is constant in the case that income effect does not exist. Since $P(*)$ and $Q(*)$ are constant, and

$$\Pi(*) = \frac{C}{\rho}\left\{\frac{1-\rho}{f}\frac{C}{\rho}\right\}^{\rho-1} Q(*)^\rho - cQ(*) - F,$$

$\Pi(*)$ is also constant. From Proposition 3, $dY/dN = \Pi(*)$. Thus we have the following proposition.

Proposition 9 *If the profit of oligopolistic firm is positive (resp. negative), the social welfare increases (resp. decreases) as the number of oligopolistic firms increases.*

If monopolistically competitive firms disappear, i.e., $N \geq N^{**}$, the social welfare is given by

$$W = L - NF + c^{-\eta/(1-\eta-\alpha)} k^{1/(1-\eta-\alpha)} \left\{ \frac{\eta - \rho(1-\alpha)}{1-\alpha} + \rho N \right\}^{\eta/(1-\eta-\alpha)} N^{\frac{\eta(1-\rho)-\rho(1-\alpha)}{\rho(1-\eta-\alpha)}}$$
$$\left\{ \frac{1-\alpha-\eta\rho}{\eta} N - \frac{\eta-\rho(1-\alpha)}{1-\alpha} \right\}.$$

Note that $(1-\alpha-\eta\rho)/\eta \cdot N - \{\eta - \rho(1-\alpha)\}/(1-\alpha)$ is always positive when $\Pi(*) > 0$. Then we have

$$\frac{\partial W}{\partial N} = -F + c^{-\eta/(1-\eta-\alpha)} k^{1/(1-\eta-\alpha)} \left\{ \frac{\eta - \rho(1-\alpha)}{1-\alpha} + \rho N \right\}^{\frac{2\eta+\alpha-1}{1-\eta-\alpha}} N^{\frac{\eta-2\rho(1-\alpha)}{\rho(1-\eta-\alpha)}}$$
$$\left[\frac{(1-\rho)(1-\alpha-\eta\rho)}{1-\eta-\alpha} N^2 + \frac{\{\eta-\rho(1-\alpha)\}[(1-\alpha-\eta\rho)(1-2\rho) - \rho\{\eta-\rho(1-\alpha)\}]}{\rho(1-\alpha)(1-\eta-\alpha)} N \right.$$
$$\left. - \frac{\{\eta(1-\rho)-\rho(1-\alpha)\}\{\eta-\rho(1-\alpha)\}^2}{\rho(1-\alpha)^2(1-\eta-\alpha)} \right].$$

If F is relatively high, the social welfare decreases as N increases when $N \geq N^{**}$ (See Fig.3).

6.2 Case with Income Effect

In the case with income effect, the social welfare is given by

$$W = \frac{\beta}{\eta} \left(\sum_{j=1}^{N} Q(j)^\rho + \int_0^M q(i)^\rho di \right)^{\eta/\rho} x^\alpha + z \tag{57}$$

$$= \frac{1}{\eta} k \left(\frac{\eta}{\eta+\alpha} Y \right)^{\eta+\alpha} P^{-\eta} = \frac{1}{\eta} k \kappa_1^{\eta+\alpha} P^{\frac{\rho\alpha+\eta}{\rho-1}}. \tag{58}$$

Thus we have the following proposition.

Proposition 10 *In the case with income effect, the social welfare is increases (resp. decreases) if the profit of oligopolistic firm is positive (resp. negative) as the number of oligopolistic firms increases.*

Proof. From (58), we have

$$\frac{dW}{dN} = \frac{\rho\alpha+\eta}{\eta(\rho-1)} k \kappa_1^{\eta+\alpha} \frac{dP}{dN} P^{\frac{\rho\alpha+\eta+(1-\rho)}{\rho-1}}.$$

It means that dW/dN has the opposite sign of dP/dN. From Proposition 8, dW/dN is positive if $\Pi(*)$ is positive. □

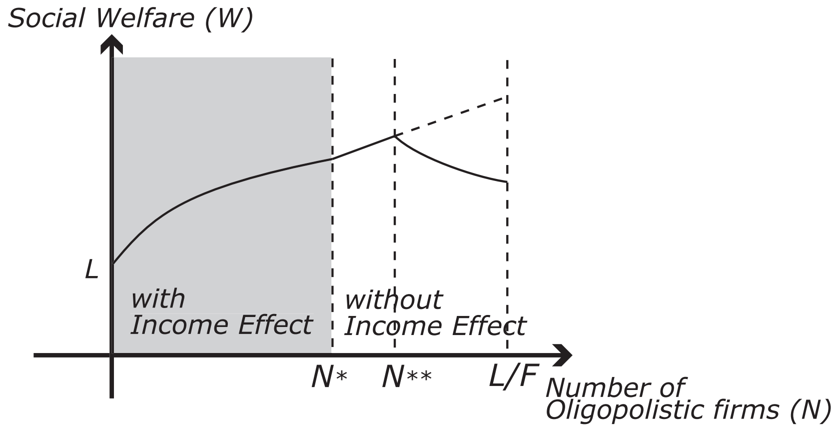


Figure 3: Social Welfare

We have to consider the boundary whether there exists income effect or not. From (15) and (35), the condition without income effect is

$$Y = N\Pi(*) > \frac{\eta+\alpha}{\eta} k^{\rho/\{\rho(1-\alpha)-\eta\}} \kappa_1^{\frac{\eta(1-\rho)}{\rho(1-\alpha)-\eta}}.$$

Recall that $\Pi(*)$ is constant and suppose that $\Pi(*) > 0$. Thus we have N^*, the boundary between the intervals with and without income effect:

$$N^* = \left[\frac{\eta+\alpha}{\eta} k^{\rho/\{\rho(1-\alpha)-\eta\}} \kappa_1^{\frac{\eta(1-\rho)}{\rho(1-\alpha)-\eta}} - L\right] \Pi(*)^{-1}.$$

There exists no income effect if $N > N^*$. On the other hand, there exists income effect if $N < N^*$. From Propositions 9 and 10, we have the following.

Proposition 11 *The social welfare increases as the number of oligopolistic firms increases.*

Appendix

We give proofs of the lemmas and propositions stated in the text.

Lemma 1 *Given any $Q(j)$, $j \neq i$, there exists a unique $Q(i)$ that maximizes Π^i.*

Proof. Fix $Q(j)$, $j \neq i$. Let $g_1(Q(i))$ be the function given by

$$g_1(Q(i)) \equiv Q(i)^\rho \left[Q(i)^\rho + \sum_{j \neq i} Q(j)^\rho \right]^{\frac{\rho(1-\alpha)-\eta}{\rho(\alpha-1)}}.$$

First, we prove the existence of $Q(i)$. We have

$$\frac{\partial g_1(Q(i))}{\partial Q(i)} = \rho Q(i)^{\rho-1} \left[Q(i)^\rho + \sum_{j \neq i} Q(j)^\rho \right]^{\frac{2\rho(1-\alpha)-\eta}{\rho(\alpha-1)}} \left[\frac{\eta}{\rho(1-\alpha)} Q(i)^\rho + \sum_{j \neq i} Q(j)^\rho \right].$$

Then we have $\lim_{Q(i)\to 0} \partial g_1(Q(i))/\partial Q(i) = \infty$. Rearranging this equation, we have

$$\frac{\partial g_1(Q(i))}{\partial Q(i)} = \rho Q(i)^{\frac{1-\eta-\alpha}{\alpha-1}} \left[1 + \frac{\sum_{j \neq i} Q(j)^\rho}{Q(i)^\rho} \right]^{\frac{2\rho(1-\alpha)-\eta}{\rho(\alpha-1)}} \left[\frac{\eta}{\rho(1-\alpha)} + \frac{\sum_{j \neq i} Q(j)^\rho}{Q(i)^\rho} \right].$$

Then we have $\lim_{Q(i)\to\infty} \partial g_1(Q(i))/\partial Q(i) = 0$. Note that $g_1(0) = 0$. Thus there exists at least one $Q(i)$.

If $g_1(Q(i))$ is concave, Π^i attains a single peak. Thus we examine $\partial g_1(Q(i))/\partial Q(i) > 0$ and $\partial^2 g_1(Q(i))/\partial Q(i)^2 < 0$. We have

$$\begin{aligned}
\frac{\partial g_1(Q(i))}{\partial Q(i)} =& \rho Q(i)^{\rho-1} \left[Q(i)^\rho + \sum_{j \neq i} Q(j)^\rho \right]^{\frac{2\rho(1-\alpha)-\eta}{\rho(\alpha-1)}} \left[\frac{\eta}{\rho(1-\alpha)} Q(i)^\rho + \sum_{j \neq i} Q(j)^\rho \right] > 0 \quad \text{and} \\
\frac{\partial^2 g_1(Q(i))}{\partial Q(i)^2} =& \left[Q(i)^\rho + \sum_{j \neq i} Q(j)^\rho \right]^{\frac{3\rho(1-\alpha)-\eta}{\rho(\alpha-1)}} \\
& \times \left[Q(i)^{2(\rho-1)} \left\{ \frac{-\eta(1-\eta-\alpha)}{(1-\alpha)^2} Q(i)^\rho + \frac{-(1+\rho)(1-\eta-\alpha) - \eta(1-\rho)}{1-\alpha} \sum_{j \neq i} Q(j)^\rho \right\} \right. \\
& \left. -(1-\rho) \left(\sum_{j \neq i} Q(j)^\rho \right)^2 Q(i)^{\rho-2} \right] < 0.
\end{aligned}$$

Thus Π^i has a unique maximizer given $Q(j)$ $j \neq i$. □

Proposition 1 *Let $i \neq j$. If the differentiated goods are substitutes, $Q(i)$ and $Q(j)$ may be strategic substitutes or complements. If the differentiated goods are complements, $Q(i)$ and $Q(j)$ are always strategic complements.*

Proof. The first order condition gives

$$\Pi_1^1 \equiv \frac{\partial \Pi^1}{\partial Q(1)} = \frac{\partial \Pi(Q(0); Q(1)(Q(1), \cdots, Q(2), \cdots, Q(N)), Q(2), \cdots, Q(N))}{\partial Q(1)} = 0.$$

Consider $\partial Q(1)/\partial Q(2)$. From the above equation, we have $\partial Q(1)/\partial Q(2) = -\Pi_{12}^1/\Pi_{11}^1$ where $\Pi_{11}^1 = \partial^2 \Pi(1)/\partial Q(1)^2$ and $\Pi_{12}^1 = \partial^2 \Pi^1/\partial Q(1)\partial Q(2)$. Since $\partial^2 g_1(Q(1))/\partial Q(1)^2 < 0$, $\Pi_{11}^1 = \partial^2 \Pi^1/\partial Q(1)^2$ is always negative. Thus the sign of $\partial Q(1)/\partial Q(2)$ coincides with that of Π_{12}^1. We have

$$\begin{aligned} \Pi_{12}^1 \equiv \frac{\partial^2 \Pi^1}{\partial Q(1)\partial Q(2)} =& \frac{\rho\{\eta - \rho(1-\alpha)\}}{1-\alpha} k^{1/(1-\alpha)} Q(1)^{\rho-1} Q(2)^{\rho-1} \left[Q(1)^\rho + \sum_{j\neq 1} Q(j)^\rho\right]^{\frac{3\rho(1-\alpha)-\eta}{\rho(\alpha-1)}} \\ &\times \left[\frac{\eta - \rho(1-\alpha)}{1-\alpha} Q(1)^\rho + \sum_{j\neq 1} Q(j)^\rho\right]. \end{aligned}$$

Thus,

$$\begin{aligned} \Pi_{ij}^i > 0 \quad &\text{if} \quad \rho(1-\alpha) < \eta \quad \text{or} \\ &\text{if} \quad \rho(1-\alpha) > \eta \quad \text{and} \quad \frac{\eta-\rho(1-\alpha)}{1-\alpha} Q(i)^\rho + \sum_{j\neq i} Q(j)^\rho < 0, \\ \Pi_{ij}^i < 0 \quad &\text{if} \quad \rho(1-\alpha) > \eta \quad \text{and} \quad \frac{\eta-\rho(1-\alpha)}{1-\alpha} Q(i)^\rho + \sum_{j\neq i} Q(j)^\rho > 0, \quad \text{for all } i \neq j. \end{aligned}$$

If the differentiated goods are complements, i.e., $\rho(1-\alpha) < \eta$, $Q(i)$ and $Q(j)$ are always complements for all $i \neq j$. On the other hands, if the differentiated goods are substitutes, i.e., $\rho(1-\alpha) > \eta$, $Q(i)$ and $Q(j)$ may be strategically substitutes or complements for all $i \neq j$. □

Lemma 2 *Given any $Q(j)$, $j \neq i$, there exists a unique $Q(i)$ that maximizes Π^i.*

Proof. Let $g_2(Q(i))$ be the function given by

$$g_2(Q(i)) \equiv \left(\frac{\eta}{\eta+\alpha} Y\right) Q(i)^\rho \left[Q(i)^\rho + \sum_{j\neq i} Q(j)^\rho\right]^{-1}.$$

We examine the concavity of $g_2(Q(i))$. We obtain

$$\begin{aligned} \frac{\partial g_2(Q(i))}{\partial Q(i)} =& \rho \left(\frac{\eta}{\eta+\alpha} Y\right) Q(i)^{\rho-1} \sum_{j\neq i} Q(j)^\rho \left[Q(i)^\rho + \sum_{j\neq i} Q(j)^\rho\right]^{-2} > 0 \quad \text{and} \\ \frac{\partial^2 g_2(Q(i))}{\partial Q(i)^2} =& \rho \left(\frac{\eta}{\eta+\alpha} Y\right) Q(i)^{\rho-2} \sum_{j\neq i} Q(j)^\rho \left[Q(i)^\rho + \sum_{j\neq i} Q(j)^\rho\right]^{-3} \\ &\times \left\{(\rho-1)\left[Q(i)^\rho + \sum_{j\neq i} Q(j)^\rho\right] - 2\rho Q(i)^{2(\rho-1)}\right\} < 0. \end{aligned}$$

Thus Π^i has a unique maximizer given $Q(j)$ $j \neq i$. □

Proposition 2 *The output levels $Q(i)$ and $Q(j)$ are always strategic substitutes for all $i \neq j$.*

Proof. Consider $\partial Q(1)/\partial Q(2)$. Since $\partial^2 g_2(Q(1))/\partial Q(1)^2 < 0$, we examine the sign of Π_{12}. We have

$$\Pi_{12} = \rho^2 \left(\frac{\eta}{\eta+\alpha}\right) Q(1)^{\rho-1} Q(2)^{\rho-1} \left[Q(1)^\rho + \sum_{j\neq 1} Q(j)^\rho\right]^{-3} \left[Q(1)^\rho - \sum_{j\neq 1} Q(j)^\rho\right].$$

Note that $Q(i)^\rho < \sum_{j\neq i} Q(j)^\rho$. Thus $\Pi_{ij} < 0$, and $\partial Q(i)/\partial Q(j) < 0$ for all $i \neq j$. □

Remark 3 *If the differentiated goods are substitutes, there exists a unique equilibrium $Q(*)$. However, there may exist one, two, or no equilibrium if the differentiated goods are complements.*

Proof. We consider the following function:

$$\begin{aligned} h_1(Q(*)) = \frac{\rho(1-\alpha)-\eta}{\rho(1-\alpha)} k^{\rho/\{\eta-\rho(1-\alpha)\}} \left[\frac{1-\rho}{f}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\right]^{\frac{\rho(1-\rho)(1-\alpha)}{\eta-\rho(1-\alpha)}} Q(*)^\rho \\ + \frac{1}{\rho} c Q(*)^{1-\rho} \left[\frac{1-\rho}{f}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\right]^{1-\rho}. \end{aligned}$$

Thus we have

$$\begin{aligned} \frac{\partial h_1(Q(*))}{\partial Q(*)} = \frac{\rho(1-\alpha)-\eta}{1-\alpha} k^{\rho/\{\eta-\rho(1-\alpha)\}} \left[\frac{1-\rho}{f}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\right]^{\frac{\rho(1-\rho)(1-\alpha)}{\eta-\rho(1-\alpha)}} Q(*)^{\rho-1} \\ + \frac{1-\rho}{\rho} c Q(*)^{-\rho} \left[\frac{1-\rho}{f}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\right]^{1-\rho}. \end{aligned}$$

If $\rho(1-\alpha) > \eta$, $h_1(Q(*))$ is an increasing function of $Q(*)$. Since $h_1(0) = 0$, (36) has a unique solution.

If $\rho(1-\alpha) > \eta$, there exists $\bar{Q}(*)$ that satisfies $\partial h_1(Q(*))/\partial Q(*)$ equals to zero:

$$\bar{Q}(*) = \left[\frac{\rho\{\eta-\rho(1-\alpha)\}}{(1-\alpha)(1-\rho)}\right]^{1/(1-2\rho)} c^{1/(2\rho-1)} k^{\frac{\rho}{(1-2\rho)\{\eta-\rho(1-\alpha)\}}} \left[\frac{1-\rho}{f}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\right]^{\frac{(1-\rho)\{2\rho(1-\alpha)-\eta\}}{(1-2\rho)\{\eta-\rho(1-\alpha)\}}}.$$

If $\rho < 1/2$, $h_1(Q(*))$ takes the minimum value at $\bar{Q}(*)$. If $\rho > 1/2$, $h_1(Q(*))$ takes the maximum value at $\bar{Q}(*)$. Note that if $\rho = 1/2$, the sign of $\partial h_1(Q(*))/\partial Q(*)$ coincide with that of

$$\frac{\rho(1-\alpha)-\eta}{1-\alpha} k^{\rho/\{\eta-\rho(1-\alpha)\}} \left[\frac{1-\rho}{f}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\right]^{\frac{\rho(1-\rho)(1-\alpha)}{\eta-\rho(1-\alpha)}} + \frac{1-\rho}{\rho} c \left[\frac{1-\rho}{f}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\right]^{1-\rho}.$$

Thus (36) has a unique solution if $\rho < 1/2$ and $\rho(1-\alpha) > \eta$ or $\rho = 1/2$. If $\rho > 1/2$ and $\rho(1-\alpha) > \eta$, we have the following condition:

There exist two equilbria if

$$\frac{1}{\rho(1-\rho)}\left[\frac{\eta-\rho(1-\alpha)}{1-\alpha}\right]^{\frac{1-\rho}{1-2\rho}}\left(\frac{\rho}{1-\rho}\right)^{\rho/(1-2\rho)}\left[k^{1/(1-\eta-\alpha)}\frac{1-\rho}{f}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\right]^{\frac{\rho(1-\rho)(1-\eta-\alpha)}{(1-2\rho)\{\eta-\rho(1-\alpha)\}}} > 1,$$

and exists a unique equilbria if

$$\frac{1}{\rho(1-\rho)}\left[\frac{\eta-\rho(1-\alpha)}{1-\alpha}\right]^{\frac{1-\rho}{1-2\rho}}\left(\frac{\rho}{1-\rho}\right)^{\rho/(1-2\rho)}\left[k^{1/(1-\eta-\alpha)}\frac{1-\rho}{f}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\right]^{\frac{\rho(1-\rho)(1-\eta-\alpha)}{(1-2\rho)\{\eta-\rho(1-\alpha)\}}} = 1,$$

and exists no equilbrium if

$$\frac{1}{\rho(1-\rho)}\left[\frac{\eta-\rho(1-\alpha)}{1-\alpha}\right]^{\frac{1-\rho}{1-2\rho}}\left(\frac{\rho}{1-\rho}\right)^{\rho/(1-2\rho)}\left[k^{1/(1-\eta-\alpha)}\frac{1-\rho}{f}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\right]^{\frac{\rho(1-\rho)(1-\eta-\alpha)}{(1-2\rho)\{\eta-\rho(1-\alpha)\}}} < 1.$$

$\square$

Proposition 5 *The quantities of oligopilistic products are always strategic complements.*

Proof. From Proposition 1, the quantities of oligopolisitc products are strategic complements in the case of $\rho(1-\alpha) < \eta$. We consider the case $\rho(1-\alpha) > \eta$. From (36),

$$\begin{aligned} Q(*)^{\rho} =& \frac{\rho(1-\alpha)}{\rho(1-\alpha)-\eta}k^{\rho/\{\rho(1-\alpha)-\eta\}}\left[1-\frac{1}{\rho}cQ(*)^{1-\rho}\left[\frac{1-\rho}{f}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\right]^{1-\rho}\right] \\ &\times\left[\frac{1-\rho}{f}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\right]^{\frac{\rho(\rho-1)(1-\alpha)}{\eta-\rho(1-\alpha)}}. \end{aligned}$$

Thus we have

$$\begin{aligned} &\frac{\eta-\rho(1-\alpha)}{1-\alpha}Q(i)^{\rho}+\sum_{j\neq i}Q(j)^{\rho} = \frac{\rho(1-\alpha)(\rho-1)-\eta}{\rho\{\rho(1-\alpha)-\eta\}}k^{\rho/\{\rho(1-\alpha)-\eta\}} \\ &\times\left[1-\frac{1}{\rho}cQ(*)^{1-\rho}\left[\frac{1-\rho}{f}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\right]^{1-\rho}\right]\left[\frac{1-\rho}{f}\left(\frac{C}{\rho}\right)^{\rho/(\rho-1)}\right]^{\frac{\rho(\rho-1)(1-\alpha)}{\eta-\rho(1-\alpha)}} < 0. \end{aligned}$$

Thus, the quantities of oligopolisitc products are always complements. $\square$

Remark 4 *There may exist one, two, or no equilibrium.*

Proof. Let $h_2(Q(*)) \equiv \{(L-NF)\kappa_1 - cN\kappa_1 Q(*) + N\kappa_1^{\rho}Q(*)^{\rho}\}\left\{1-(1/\rho)c\kappa_1^{1-\rho}Q(*)^{1-\rho}\right\} - ((\eta+\alpha)/\eta)\kappa_1^{\rho}Q(*)^{\rho}$. Let $y = \kappa_1^{1-\rho}Q(*)^{1-\rho}$. Then we have

$$\begin{aligned} h_2(y) &= \left\{\kappa_1(L-NF)+Ny^{\rho/(1-\rho)}(1-cy)\right\}\left(1-\frac{1}{\rho}cy\right)-\frac{\eta+\alpha}{\eta}y^{\rho/(1-\rho)}, \\ \frac{\partial h_2(y)}{\partial y} &= -\frac{c}{\rho}\kappa_1(L-NF)+\frac{1}{\rho(1-\rho)}y^{\frac{2\rho-1}{1-\rho}}\left[N\left\{(\rho-cy)^2-cy(1-cy)(1-\rho)\right\}-\frac{\eta+\alpha}{\eta}\rho^2\right]. \end{aligned}$$

Let $g_2 \equiv N\left\{(\rho - cy)^2 - cy(1-cy)(1-\rho)\right\} - ((\eta+\alpha)/\eta)\rho^2$. Rearranging g_2, we have

$$g_2 = Nc^2(2-\rho)\left\{y - \frac{1+\rho}{2c(2-\rho)}\right\}^2 - \frac{4(2-\rho)\rho^2 - (1+\rho)^2}{4(2-\rho)}N - \frac{\eta+\alpha}{\eta}\rho^2.$$

Note that $4(2-\rho)\rho^2 - (1+\rho)^2 < 0$ when $0 < \rho < 1$. Thus we have the following condition:

$$\begin{array}{lll} \text{When} \quad N > \dfrac{\eta+\alpha}{\eta}, & g_2 > 0 & \text{on the domain that} \quad 0 < y < y_1, \ \text{ and } \ y_2^* < y, \quad \text{and} \\ & g_2 < 0 & \text{on the domain that} \quad y_1 < y < y_2. \\ \text{When} \quad N < \dfrac{\eta+\alpha}{\eta}, & g_2 > 0 & \text{on the domain that} \quad 0 < y < y_2, \quad \text{and} \\ & g_2 < 0 & \text{on the domain that} \quad y_2^* < y. \end{array}$$

where

$$\begin{aligned} y_1 =& \frac{N(1+\rho) - \{N^2(1+\rho)^2 - 4\rho^2(2-\rho)N(N-(\eta+\alpha)/\eta)\}^{1/2}}{2c(2-\rho)N}, \quad \text{and} \\ y_2 =& \frac{N(1+\rho) + \{N^2(1+\rho)^2 - 4\rho^2(2-\rho)N(N-(\eta+\alpha)/\eta)\}^{1/2}}{2c(2-\rho)N}. \end{aligned}$$

Thus we plot the graph of $g_2(y)$, $\partial h_2(y)/\partial y$ and $h_2(y)$ (See Fig.4), and have the following condition:

There exist two equilbria (Case1, See Fig.4(c)) if

$$\left\{\kappa_1(L-NF) + N\bar{y}^{\rho/(1-\rho)}(1-c\bar{y})\right\}\left(1 - \frac{1}{\rho}c\bar{y}\right) - \frac{\eta+\alpha}{\eta}\bar{y}^{\rho/(1-\rho)} < 0,$$

and exists a unique equilbria (Case2, See Fig.4(c)) if

$$\left\{\kappa_1(L-NF) + N\bar{y}^{\rho/(1-\rho)}(1-c\bar{y})\right\}\left(1 - \frac{1}{\rho}c\bar{y}\right) - \frac{\eta+\alpha}{\eta}\bar{y}^{\rho/(1-\rho)} = 0,$$

and exists no equilibrium (Case3, See Fig.4(c)) if

$$\left\{\kappa_1(L-NF) + N\bar{y}^{\rho/(1-\rho)}(1-c\bar{y})\right\}\left(1 - \frac{1}{\rho}c\bar{y}\right) - \frac{\eta+\alpha}{\eta}\bar{y}^{\rho/(1-\rho)} > 0,$$

where $\bar{y}$ satisfies the following equation:

$$c(1-\rho)\kappa_1(L-NF)\bar{y}^{\frac{1-2\rho}{1-\rho}} = N\left\{(\rho - c\bar{y})^2 - c\bar{y}(1-c\bar{y})(1-\rho)\right\} - \frac{\eta+\alpha}{\eta}\rho^2.$$

In terms of $Q(*)$, we have the following condition.

There exist two equilbria if

$$\left\{\kappa_1(L-NF) + N\kappa_1^{\rho}\bar{Q(*)}^{\rho}\left(1 - c\kappa_1^{1-\rho}\bar{Q(*)}^{1-\rho}\right)\right\}\left(1 - \frac{1}{\rho}c\kappa_1^{1-\rho}\bar{Q(*)}^{1-\rho}\right) < \frac{\eta+\alpha}{\eta}\kappa_1^{\rho}\bar{Q(*)}^{\rho},$$

and exists a unique equilbria if

$$\left\{\kappa_1(L-NF)+N\kappa_1^{\rho}Q\bar{(}*)^{\rho}\left(1-c\kappa_1^{1-\rho}Q\bar{(}*)^{1-\rho}\right)\right\}\left(1-\frac{1}{\rho}c\kappa_1^{1-\rho}Q\bar{(}*)^{1-\rho}\right)=\frac{\eta+\alpha}{\eta}\kappa_1^{\rho}Q\bar{(}*)^{\rho},$$

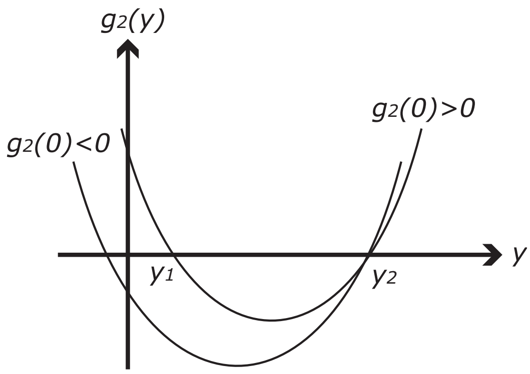
and exists no equilibrium if

$$\left\{\kappa_1(L-NF)+N\kappa_1^{\rho}Q\bar{(}*)^{\rho}\left(1-c\kappa_1^{1-\rho}Q\bar{(}*)^{1-\rho}\right)\right\}\left(1-\frac{1}{\rho}c\kappa_1^{1-\rho}Q\bar{(}*)^{1-\rho}\right)>\frac{\eta+\alpha}{\eta}\kappa_1^{\rho}Q\bar{(}*)^{\rho},$$

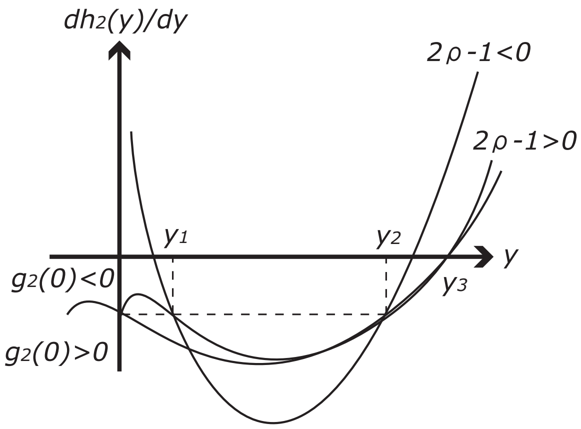
where $Q\bar{(}*)$ satisfies the following equation:

$$\begin{aligned}&c(1-\rho)(L-NF)\kappa_1^{2(1-\rho)}Q\bar{(}*)^{1-2\rho}\\&\quad=N\left\{\left(\rho-c\kappa_1^{1-\rho}Q\bar{(}*)^{1-\rho}\right)^2-c\kappa_1^{1-\rho}Q\bar{(}*)^{1-\rho}\left(1-c\kappa_1^{1-\rho}Q\bar{(}*)^{1-\rho}\right)\right\}-\frac{\eta+\alpha}{\eta}\rho^2.\end{aligned}$$

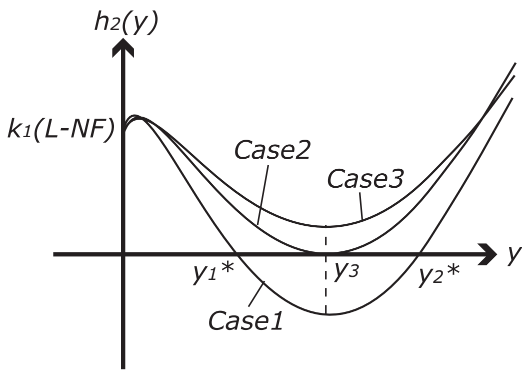
□



(a) $g_2(y)$



(b) $\partial h_2(y)/\partial y$



(c) $h_2(y)$

Figure 4: Multiple Equilibria $Q(*)$

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