An optimal board system: 

\textit{supervisory board vs. management board}\footnote{I gratefully acknowledge Prof. Yanagawa for his guidance, and Akifumi Ishihara and Naoki Takayama for very useful comments.}

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Abstract

We examine relative effectiveness of two kinds of boards, \textit{a supervisory board} and \textit{a management board}. The former is a board such that controls CEO’s opportunistic behavior directly by depriving the CEO of CEO’s control right, whereas the latter is a board such that does not control CEO’s opportunistic behavior directly but induces the CEO to make a desirable decision by board’s advice. We obtain three conclusions: a supervisory board is more desirable if CEO’s objective significantly deviates maximization of shareholder value, the CEO prefers being monitored if CEO’s control benefit is sufficiently small and CEO’s deviation from maximization of shareholder value is not significantly small or large, and a management board is effective even if board’s objective also deviates maximization of shareholder value while a supervisory board is not effective.

1 Introduction

In corporate governance research, functions of a board of directors are frequently discussed. Although boards have various functions, the one which is especially focused on is its monitoring function: boards try to obtain CEO’s private information - for example, his ability, and his effort and so on - to dissolve informational asymmetry between shareholders and the CEO, to decide proper compensation for the CEO and to decide whether to fire the CEO or not. There are many papers which analyze a board as “a group of monitors (auditors)” theoretically. Kofman and Lawarrée (1993) introduces two type auditors, an internal auditor and an external auditor in their contract model. An internal auditor is an auditor that needs no fixed cost but may collude with an agent, whereas an external auditor is an auditor that needs fixed cost but does not collude with agents. Kofman
and Lawarrée show that first best is attainable by appropriate contract including punishment for a management and an auditor with agents.

On the other hand, we can give Hermalin and Weisbach (1998) as a representative paper of corporate governance research itself. In their paper they regard a board as an agent who obtains information about CEO’s ability by monitoring or automatically and uses it to decide CEO’s compensation and his tenure, and describe the course in which independence of a board is endogenously determined by bargaining between a board and a CEO. And Osano (2002) analyzes almost the same case.

Raheja (2003) considers the interaction between inside directors and outside directors. In Raheja’s model, she regards inside directors as agents who provide private information for outside directors and outside directors as agents who verify inside directors’ information, and she considers balance of inside directors and outside directors and the size of boards. Warther (1998) focuses on directors’ behavior in a case that they may be dismissed by CEO.

In addition to these papers, there are many papers analyzing a board that focus on board’s monitoring function in business decisions or non-business decisions. Then why are only monitoring functions of boards noticed? i

As Berle and Means (1932) noted, modern corporations separate shareholders and management. Although corporate law and systems else offered various incentive schemes to dissolve agency problems generated by the separation of ownership and control, they could not prevent all problems, in particular opportunistic behavior of management. Many pointed out that it was due to the pooling of execution and monitoring. That is, because the monitor could be being monitored at the same time, monitoring functions were significantly lowered.

The U.S. type of corporate governance system clearly separates in the system executives, who are management that may conduct opportunistic behaviors, and directors, who are management that monitors executives. In Japan the Kansayaku-kai (audit committee) system leaves a part of monitoring function to Kansayaku (auditors), but it differs from the separation of execution and monitor because under the Kansayaku-kai system both Kansayaku (auditors) and directors have monitoring right.

However, are directors in the board system before the separation of execution and monitoring really inefficient? The answer is “No”. Such a board may have efficient aspects. It is because it is superior to boards in the separation of execution and monitor, in the point of CEO’s incentive to reveal his private information. When directors (including executives) control CEO, they frequently need the information such that only CEO can obtain. A board is a group of experts and therefore is superior to CEO in the point that a board can transform CEO’s information to more effective information for decision making.

Then, CEO may not want to reveal his private information while directors need it to make a more efficient decision, because the revelation of CEO’s information may bring loss to CEO.

In the case of boards whose monitoring function is intensified, for example, when the board obtains CEO’s information and CEO may conduct opportunistic behavior, the board may deprive CEO of his control right and may make deci-
sions by itself with CEO’s information, but it is loss for CEO such that want to satisfy his desire for his reputation and power or the other objective except maximization of shareholder value. Therefore CEO does not want to reveal his private information to the board.

On the other hand, in the case of boards under the pooling of execution and monitoring? In this case, a board and CEO are co-operative. CEO reveals his private information to the board and the board sends more efficient information in return to CEO (it is called “advice” in general). Then because the board never invades CEO’s control right, CEO has stronger incentives to reveal than in the case of a monitoring (supervisory) board.

When we consider this incentive problem, is a monitoring board always more efficient than a board such that advise CEO? Adams and Ferreira (2005) analyze boards taking this incentive into account.

Adams and Ferreira (2005) focus on these two roles of boards, analyze a board as an agent which has both two functions and furthermore consider a dual board system, which is a board system in which a firm has two boards; one board has only a supervisory role the other only an advising (management) role. Their model supposes that a board is a group of experts as well as a group of monitors, and supposes that cooperation of a CEO, an agent who has best access to information necessary to make decisions, and a board, an agent which is a group of directors that has higher specialty about management, law, accounting and so on, brings more efficient decision. Furthermore they take it into account that CEO’s incentive to reveal his private information to a board, and as a result they argue that a slightly biased board (in a sense that a board has preference close to a CEO) is more efficient than an unbiased board.

However, even if a board has two roles, does it play two roles evenly? The answer is “No”. As we state above, parts of Japanese boards cooperate with CEOs and directors advise CEOs on business decisions, rather than exercise their supervisory role. On the other hand, it is monitoring function of a board that is focused on in the process in which independence of directors are strengthened in many countries. That is, many boards which are observed in reality mainly play only one of two roles.

Here, we call a supervisory board a board that plays only a supervisory role and a management board a board that plays only a management (advising) role. Which of two boards, a supervisory board and a management board, is effective to protect shareholder value under what condition? This is the main issue that we consider in this paper.

In addition to this, we consider the following issues in this paper:

1. Which of two boards do a CEO and a board itself prefer?
2. Which of two boards are robust even if a board itself is biased?

Which of two boards do a CEO and a board itself prefer? In countries that have multiple board systems, determination of a board system that a firm adopts is a consequence of the interaction between stakeholders, especially, directors and a CEO, or, shareholders and the management. In either event a CEO has a great impact on determination of a board system that the firm adopts, and there-
fore if a CEO prefers a board system which is *not desirable* for shareholders, the firm may adopt an undesirable one. That is, if all CEOs prefer a board which is not desirable for shareholders and have a great impact on the determination process of a board system that the firm adopts, the claim can be affirmed that corporate law restricts a board type that a firm adopts. In Japanese corporate law, for example, large companies can choose a board system among *Kansayaku-kai* (audit committee) system, *Iinkai-secchi* (committee board) system and so on, but it may be favorable that corporate law unites these multiple board systems. As well as a CEO, a board has a significant impact on determination of a board that the firm adopts. Therefore it is important that we consider board’s preference for a board system that the firm adopts.

Which of two boards is more robust even if a board itself is biased? In this paper, at first we assume that a board maximizes shareholder’s benefit. However this assumption is obviously unrealistic. Kaplan and Reishus (1990) partially supports this assumption but does not totally support it. Hence we need to analyze efficiency of two boards in a situation that boards are biased given CEO’s bias.

This paper proceeds as following: Section 2 shows concrete sketches of our model. In Section 3, 3.1 shows equilibrium in a case of a *supervisory board*, and 3.2 shows equilibrium in a case of a *management board*. In these subsections, we assume that a board maximizes shareholder’s benefit (more precisely, a board minimizes the variance of shareholder value). In subsection 3.3 we consider conditions for which a *supervisory board* (or a *management board*) is more desirable for shareholders, the CEO and a board itself than a *management board* (or a *supervisory board*). In subsection 3.4 we examine robustness of two boards against change of a board’s bias. And, Section 4 is concluding remark.

2 The Model

2.1 Information and Preference

The firm exists, and shareholders, the CEO and the board face a project choice problem. A project set is denoted by a set of real numbers. Because a CEO has the best access to the firm specific information, only the CEO can observe his private signal, denoted by $\theta \in \mathbb{R}$, which specifies a relevant set. We assume that $\theta$ is hard information so that the CEO and the other players cannot distort $\theta$. What those that observe $\theta$ can do is to reveal or to conceal.

Both the CEO and the board believe *ex ante* that $\theta$ is an uniform, supported by a set of real numbers. A relevant set given by $\theta$ is the set of three alternatives that are considered to be the most favorable projects for both the CEO and the board. That is, the relevant set implies that there exists an optimal project for shareholders, denoted by $\epsilon + \theta$, in the relevant set. An optimal project is one such
that maximizes shareholder value:

\[ U_V = -(y - \epsilon)^2. \]  

(1)

Therefore shareholder value is maximized and is equal to 0 when \( y = \epsilon \). The CEO cannot find the optimal project by himself because he does not have his expertise and other resources (for example, his time) enough to find it. Both the CEO and the board believe *ex ante* that \( \epsilon \) is distributed as below:

\[
(p(\epsilon = \epsilon_0), p(\epsilon = \epsilon_1), p(\epsilon = \epsilon_2)) = (q_0, q_1, q_2).
\]

where \( q_i > 0 \) (\( i = 0, 1, 2 \)) and \( q_0 + q_1 + q_2 = 1 \). For simplicity, we suppose that \( \epsilon_0 = 0, \epsilon_1 = \frac{1}{2}, \epsilon_2 = 1 \) and \( q_i = \frac{1}{3} \) (\( i = 0, 1, 2 \)).

On the other hand, the board cannot observe the CEO’s private information \( \theta \) at all as long as the CEO does not reveal it, because the board has only the limited access to the firm specific information. However, the board can find an optimal project, which is its private information, by spending one unit of its time if the board receives his private information from the CEO, because directors of the board are experts on management, finance, and law, or have affluent knowledge and experience in various industries and they can obtain more useful information by an information resource.

A board is one of two type boards; a *supervisory board* and a *management board*. A *supervisory board* is one that chooses a project by itself if necessary, whereas a *management board* is one that leaves project choice to a CEO and advises him on project choice. A supervisory board has its preference below:

\[ U_S(y, \pi; \epsilon) = -(y - \epsilon)^2 - \frac{\pi^2}{2} \]  

(2)

where \( y \in \mathbb{R} \) is a project chosen by the CEO or the (supervisory) board and \( \pi \in [0, 1] \) is its monitoring intensity. \( \frac{\pi^2}{2} \) is its disutility generated by its monitoring, which is monotone increasing and convex with respect to \( \pi \). Therefore a supervisory board has the same preference as shareholders except its disutility generated by monitoring.

On the other hand, a management board has its preference below:

\[ U_M(a; \epsilon) = -(y - \epsilon)^2 \]  

(3)

where \( a \in [0, 1] \) is its advice to the CEO. That is, a management board has the same preference as shareholders.

That is, preferences of two boards are almost the same as shareholders, and this implies that we assume that a board has perfect integrity for shareholders. This assumption is modified in 3.4. And, to neglect team production problem of directors, we assume that a board acts as one agent.

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1The equation (1) means deviation from the optimal project and therefore it is not appropriate that we call this “shareholder value”. However, since this means loss for shareholders generated by irrelevant project choices, it does not matter to call this “shareholder value”.

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The CEO has his preference below:

\[ U_C(y, i; \epsilon, g) = -(y - \epsilon + g)^2 + \chi b \]  

where \( i \) is his decision about whether he reveals his private information or not, that is,

\[ i = \begin{cases} 
1 & \text{if the CEO reveals his information to the board} \\
0 & \text{otherwise} 
\end{cases} \]

\( g \geq 0 \) is a project choice bias, \( \chi \) is an indication function such that:

\[ \chi = \begin{cases} 
1 & \text{if the CEO retains his control right} \\
0 & \text{otherwise} 
\end{cases} \]

and \( b \geq 0 \) is his control benefit. \(^2\) The existence of \( b \) and \( g \) brings incentives and disincentives for the CEO to reveal his private information. In the case of a supervisory board, if the CEO reveals his private information, the board may make an efficient decision for the CEO but monitoring intensity increases (that is, the probability of losing his control benefit increases). On the other hand, in the case of a management board, if the CEO reveals his private information, a good advice by the board would be efficient for the CEO but a bad one inefficient. Therefore, the CEO may face trade-off between revealing and concealing his private information. After observing his private information, the CEO makes a decision about whether to reveal it or not.

Next, the board spends one unit of its time to observe to its private information if it receives the CEO’s private information and observes it after one unit time. From this time, the game proceeds in the different way between in the case of a supervisory board and of a management board.

In the case of a supervisory board, the board decides its monitoring intensity \( \pi \). The board can deprive the CEO of his control right with probability \( \pi \) and then the board chooses a project by itself, whereas the CEO retains his control right with probability \( 1 - \pi \) and then he chooses a project. Although Adams and Ferreira (2005) assume that when the CEO retains his control right the board advises him, in our model, for simplicity we assume that a supervisory board cannot transmit its private signal to the CEO. On the other hand in the case of a management board, the CEO must retain his control right. Therefore the CEO can choose a project regardless of quality of board’s advice.

2.2 Timing

We can summarize two games (ones in a case of a supervisory board and of a management board) as the following (Figure 2.1.1 and Figure 2.1.2 show the

\(^2\)Control benefit is one that the CEO gains by retaining his power, and CEO’s orientation toward his power in the firm and so on composes his control benefit. On the other hand, the existence of a project choice bias means that the CEO chooses projects that maximizes something except shareholder value, for example, firm’s revenue, well being of employees and so on.
game trees of the two games):

The case of a supervisory board

period 1 The CEO observes his private signal \( \theta \) with probability one and decides whether to reveal it or not.

period 2 The supervisory board costs one unit of its time to observe its private signal \( \epsilon \).

period 3 The supervisory board decides its monitoring intensity. After that, it observes \( \epsilon \) with probability one.

period 4 If monitoring is successful (with probability \( \pi \)), the supervisory board obtains CEO’s control right and chooses a project. If monitoring fails (with probability \( 1 - \pi \)), the CEO retains his control right and chooses a project.

period 5 The benefit is realized and distributed.

Note that monitoring intensity is determined before the supervisory board observes his private signal \( \epsilon \). In the case of a management board, the game is modified as to from period 2 to period 4 in the case of a supervisory board. The game is given as:

The case of a management board

period 1 The CEO observes his private signal \( \theta \) with probability one and decides whether to reveal it or not.

period 2 The management board costs one unit of its time to observe its private signal \( \epsilon \).

period 3 The management board observes \( \epsilon \) with probability one.

period 4 The management board decides its advice to the CEO and the CEO chooses a project based on its advice.

period 5 The benefit is realized and distributed.

We analyze this model next section.

3 Analysis

3.1 Supervisory Board

At first we consider a project choice problem by the supervisory board and the CEO. As the previous section says, the supervisory board can decide a project with probability \( \pi \). Then the supervisory board chooses a project such that minimizes \( (y_S(i) - \epsilon)^2 \); \( y^*_S(1) = \epsilon \) if it receives CEO’s private information, and \( y^*_S(0) = \frac{1}{2} \) because it minimizes an expectation \( E_\epsilon[(y^*_S(0) - \epsilon)^2] \) if it does not receive the CEO’s private information.
On the other hand, the CEO can decide a project with probability $1 - \pi$. Then he chooses a project such that minimizes $(y_C - \epsilon + g)^2$ but he cannot observe $\epsilon$. Therefore, $y_C^* = \frac{1}{2} - g$ holds.

Next, we consider board’s monitoring intensity decision problem. Note that we assume that the board decides monitoring intensity before the board observes his private signal. The optimal monitoring intensity, $\pi$, maximizes:

$$EU_C = -\pi E[\{(y^*_S)^i - \epsilon\}^2] - (1 - \pi)E[\{(y^*_C - \epsilon)^2\} - \frac{\pi^2}{2}$$

subject to $0 \leq \pi \leq 1$, and by the first order condition and $y^*_C = \frac{1}{2} - g$, we obtain the optimal monitoring intensity:

$$\pi(1) = \sigma^2_M + g^2$$

$$\pi(0) = g^2$$

where $\sigma^2_M = E[\{(\epsilon - \frac{1}{2})^2\}] = \frac{1}{6}$. Because $E[\{(y^*_S)^i - \epsilon\}^2] = 0$ and $E[\{(y^*_S)^0 - \epsilon\}^2] = E\{(\epsilon - \frac{1}{2})^2\} = \sigma^2_M$, we obtain the following lemma:

**Lemma 3.1.** If $g > 0$, the monitoring intensity is strictly positive.

**Lemma 3.2.** The monitoring intensity when the CEO reveals his private information is always strictly larger than the one when the CEO does not reveal it.

The former lemma argues that the board always monitors the CEO, regardless of whether to observe CEO’s private information or not, if CEO’s object deviates from maximization of shareholder value. This is because project choice by the board is always better than one by the CEO in the situation of the existence of project choice bias and the board decides monitoring intensity based on the difference of benefit between in a case that the CEO chooses and a case the board chooses. The latter lemma may seem to be counterintuitive in ordinary terminology. However, we use the word “monitoring” in Adams and Ferreira’s sense. That is, monitoring in this paper does not mean the board’s audit behavior of CEO’s private information but its effort to take CEO’s control right from him, and $E[\{(y^*_S)^i - \epsilon\}^2] > E[\{(y^*_S)^0 - \epsilon\}^2]$ implies that the project choice by the board brings more benefit to itself when the board receives the CEO’s private information than when the board does not. Hence the latter lemma is intuitive rather than counterintuitive.

Now which revealing or concealing does the CEO choose? The CEO enjoys board’s efficient decision but possibility of losing his control benefit increases if the CEO reveals, while the board does not make an efficient decision but possibility of obtaining his control benefit increases. Obviously, the necessary and sufficient condition for which the CEO reveals is:

$$E_e U_C(i = 1) \geq E_e U_C(i = 0),$$

3We assume that the CEO conceals when he is indifferent between revealing and concealing.
where $E_{\epsilon} U_C(i) = -\pi(i)E_{\epsilon}[y_0^*(i) - \epsilon + g) + (1 - \pi(i))(-\sigma^2_M + b)$. Therefore by calculation we obtain:

**Proposition 3.1.** In the case of a supervisory board,

1. the CEO always reveals his private information $\theta$ to the board when $0 \leq b < \frac{1}{6}$, and
2. the CEO never reveals his private information $\theta$ to the board when $b \geq \frac{1}{6}$.

**Proof.** See Appendix.

At the same time we obtain the following corollary:

**Collorary 3.1.** The CEO’s decision about whether to reveal to the supervisory board or not is independent of his project choice bias.

We explain interpretation of proposition 3.2 in the latter half of the next subsection.

### 3.2 Management Board

A *management board* is a board such that advises the CEO on a project choice and leaves a project choice to the CEO, rather than monitors him. In this case, the board has incentive to distort its information, $\epsilon$, strategically because its strategic distortion of information may be able to lead the CEO’s project choice to desirable direction for the board. On the other hand, the CEO knows that the board has incentive to distort its information to him strategically. Although the equilibrium in such a situation is given in Crawford and Sobel (1982) and Adams and Ferreira (2005) applies it to the relationship between the management board and the CEO, we consider the case of discrete type space $\{\epsilon_0, \epsilon_1, \epsilon_2\}$ and discrete message space $A$ (we assume that $|A| = 3$) to obtain analytical solutions. Figure 3.2.1 shows the game tree of the advising game.

First we define the equilibrium.

**Definition 3.1.** A Perfect Baysian Nash Equilibrium for the advising game consists of family of advising rules $q(a | \epsilon)$, such that $\sum_{i=0}^{2} q(a_i | \epsilon) = 1$ for all $\epsilon \in \{\epsilon_0, \epsilon_1, \epsilon_2\}$, and a project choice function for the CEO, denoted $y_C(a)$, such that:

(a) for each $\epsilon \in \{\epsilon_0, \epsilon_1, \epsilon_2\}$, if $q(a' | \epsilon) > 0$ then

$$a' \in \arg \max_{a \in A} -[y(a) - \epsilon]^2$$

(b) for each $a \in A$,

$$y_C(a) \in \arg \max_{y_C} \sum_{i=0}^{2} -(y - \epsilon_i + g)^2 p(\epsilon_i | a)$$

where $p(\epsilon_i | a) = \frac{q(a | \epsilon_i)}{\sum_{i=0}^{2} q(a | \epsilon_i)}$. That is, the CEO has the belief about the board’s advising rules based on Bayes rule.
Now we define
\[ \sigma^2_e = E_e [E(\epsilon | q'(a | \epsilon)) - \epsilon]^2 \]
where \( q' (a | \epsilon) \) is an advising rule in equilibrium given \( g \) and \( \sigma^2_e \in \{0, \frac{1}{24}, \frac{1}{6}\} \).

**Definition 3.2.** Given \( g' \), an equilibrium is called the most informative if the profile of advising rules, a project choice and belief minimizes \( \sigma^2_e \) given \( g' \).

We can interpret the most informative equilibrium as “the equilibrium in which the board gives the most effective advice to the CEO” and we focus on the kind of equilibrium. Next, we obtain the following proposition by the definition of the most informative equilibrium:

**Proposition 3.2.** There exist PBNEs such that are the most informative under given \( g \) and they have the following property:

1. **(The case of \( g \leq \frac{1}{4} \))**
   a) The board chooses \( q (a | \epsilon_0) = 1, q(a' | \epsilon_1) = 1 \) and \( q(a'' | \epsilon_2) = 1 \) where \( a \neq a' \neq a'' \).
   b) The CEO chooses \( y_C (a) = -g \), \( y_C (a') = \frac{1}{2} - g \) and \( y_C (a'') = 1 - g \).
   c) \( \sigma^2_e = 0 \).

2. **(The case of \( \frac{1}{4} < g \leq \frac{3}{4} \))**
   a) The board chooses \( q (a | \epsilon_0) = 1, q(a' | \epsilon_1) = q(a' | \epsilon_2) = \mu \) and \( q(a'' | \epsilon_2) = \mu \) where \( a \neq a' \) and \( \mu \) is an arbitrary number belonging to \([0, 1]\).
   b) The CEO chooses \( y_C (a) = -g \) and \( y_C (a') = y_C (a'') = \frac{3}{4} - g \).
   c) \( \sigma^2_e = \frac{1}{24} \).

3. **(The case of \( g > \frac{3}{4} \))**
   a) The board chooses \( q (a | \epsilon_i) = \mu_1, q (a | \epsilon_i) = \mu_2 \) and \( q (a | \epsilon_i) = 1 - \mu_1 - \mu_2 \) for \( i = 0, 1, 2 \), where \( \mu_1 \) and \( \mu_2 \) are a profile of real numbers such that satisfies \( \mu_1 \geq 0, \mu_2 \geq 0 \) and \( \mu_1 + \mu_2 \leq 1 \).
   b) The CEO chooses \( y_C (a) = \frac{1}{2} - g \) for all \( a \in A \).
   c) \( \sigma^2_e = \frac{1}{6} \).

This proposition states that the more biased CEO’s choice is, the more inefficient the board’s advice is. Next, we consider the CEO’s decision problem about whether to reveal his private information or not. CEO’s expectational utility at the time when the CEO decides whether to reveal or not is given as:

\[ E_e U_C (i = 1) = -\sigma^2_e + b \]
and \( E_e U_C (i = 0) = -\sigma^2_M + b \leq -\sigma^2_e + b \) for all \( g \), we obtain the next proposition:

**Proposition 3.3.** In the case of a management board, the CEO always reveals his private information, \( \theta \), to the board regardless of values of \( b \) and \( g \).
This proposition argues that the CEO is always cooperative to the board in the case of a management board. There are two reasons for this: First, the board’s advising behavior brings no effect on the CEO’s control benefit \( b \) because the CEO can always choose a project regardless of whether the board advises or not. Second, the board’s advice is useful relative to the case of no information even if the CEO is too biased. Therefore, since the board’s advice always brings no undesirable effect on the CEO, he always reveals.

On the other hand, in the case of a supervisory board the CEO’s information revelation to the board brings positive effect and negative effect on the CEO. For explanation, we analyze the necessary and sufficient condition for revelation again.

\[
E_i U_C(i = 1) - E_i U_C(i = 0) = \pi(1) \sigma^2_M - (\pi(1) - \pi(0)) g^2 - (\pi(1) - \pi(0)) b
\]

The first term, \( \pi(1) \sigma^2_M \), is the benefit that the CEO gain when he reveals. The board intensifies its monitoring if the CEO reveals, and this gives him an incentive to reveal because the informed board can make an efficient decision when the CEO reveals. The second term and the third term are the CEO’s cost by revelation. The second term stems from the CEO’s project choice bias. Since the board chooses a project without considering the CEO’s benefit when the monitoring succeeds, given a project that the CEO chooses is biased by \( g \) from one that the board chooses. And this bias generates the second term. Furthermore, the third term, \( (\pi(1) - \pi(0)) b \), is the loss of the CEO’s control benefit by the increment of monitoring intensity.

However, since a part of the first term perfectly offsets the second term, only the relationship between \( b \) and \( \sigma^2_M \) is essential for the CEO’s decision as Proposition 3.1 states.

### 3.3 Which Board Is Better?

In the subsection 3.1 and 3.2, we analyzed actions of the CEO, the supervisory board and the management board. As a result, we found that a supervisory board intensifies monitoring when it receives CEO’s signal, and that a CEO may not reveal his private information to a supervisory board whereas he always reveals it to a management board. Next we consider which board each shareholder, the CEO and the board itself prefer.

Let \( EU_k(j; i) \) be player \( k \)’s (expected) utility \( (k \in \{V, C, S, M\}) \) in the case of \( j \)-type (\( j \) is a supervisory board (SV) or a management board (MG)). The CEO reveals in equilibrium in the case of a management board while by proposition 3.1 there are equilibria, ones in which the CEO reveals and the other in which the CEO does not reveal, in the case of a supervisory board.

First we consider shareholders. The necessary and sufficient condition for which shareholders prefer a supervisory board to a management board is:

\[
EU_S(SV; i) = -\pi(i) E_i [(\gamma_S(i) - \epsilon)^2] - (1 - \pi(i)) (\sigma^2_M + g^2) \\
g \geq -(\sigma^2_\epsilon + g^2) = EU_S(MG).
\]
By this condition, we obtain the following propositions:

**Proposition 3.4.** In each the equilibrium in which the CEO reveals or the one in which the CEO conceals, there exists \( \tilde{g}_S(i) \in (0, +\infty) \) such that:

1. shareholders always prefer a supervisory board if and only if \( g > \tilde{g}_S(i) \) for \( i = 0, 1 \),
2. shareholders always prefer a management board if and only if \( 0 \leq g \leq \tilde{g}_S(i) \) for \( i = 0, 1 \).

**Proof.** See Appendix.

This proposition argues that the intuition many people has that a supervisory board is better to protect shareholder’s interest is not necessarily correct (See Figure 3.3.1). In corporate governance literature in US and many European countries, the intensification of independence of directors, especially the increment of the ratio of independent directors, is often emphasized only because a non-independent board is considered to be unable to monitor the CEO effectively. However, this argument neglects the CEO’s incentive to reveal his private information which is useful for project choice and the fact that the board can advise the CEO even if it does not monitor him. For explanation of this point, first we analyze effect of the variation of \( g \) on shareholder value.

In the case of a supervisory board, increment of the project choice bias implies that of monitoring intensity, because increment of the project choice bias decreases shareholder value by direct bias of the CEO’s project choice when the board fails to monitor. Therefore, increment of the project choice bias brings the positive effect and the negative effect on shareholder value and net effect is positive for sufficiently large bias.

On the other hand, in the case of a management board increment of the project choice bias has only negative effects on shareholder value: the one is an effect that distorts the board’s advice, and the other is the increment of the direct bias of the CEO’s project choice.

Hence, for small bias a supervisory board is relatively less desirable for shareholders because the monitoring intensity of the supervisory board is low and then the CEO chooses an inefficient project with high probability, and for large bias a supervisory board is more desirable because increment of the monitoring intensity prevents the CEO’s inefficient project choice.

Next we consider the optimal board for CEO. One may think that the CEO always prefer a management board since a supervisory board may deprive the CEO of his control benefit. The following proposition argues that this intuition is not necessarily correct.

**Proposition 3.5.** In equilibrium, there exists \( \bar{b} \in (0, \frac{1}{6}) \) such that:

1. if \( b \leq \bar{b} \), there exists \( G(b) = (g_x(b), \tilde{g}_x(b)) \) such that the CEO always prefers a supervisory board to a management board if \( g \in G(b) \) and otherwise the
CEO always prefers a management board\(^4\), and

(2) if \(b > \bar{b}\), the CEO always prefers a management board to a supervisory board.

**Proof.** See Appendix.

This proposition is significantly important that it shows there exist cases in which the CEO itself prefers being monitored (See Figure 3.3.2). Then, why does the CEO prefer being monitored? The most important point for this is that the when the CEO’s control benefit is sufficiently small, a supervisory board can make the most efficient decision of the supervisory board, the management board and the CEO. When the project choice bias is sufficiently small, the monitoring intensity is low and the advice is not distorted much. However, increment of the project choice bias distorted the advice very rapidly. On the other hand, increment of the project choice bias does not intensify monitoring rapidly, and the project choice by the supervisory board is efficient because it observes its private information. But if the project choice bias is sufficiently large, the supervisory board decides a project with high probability and then the loss for the CEO that occurs by that the board chooses a project is significantly large. Hence there exist cases in which the CEO itself prefers being monitored. Figure 1, 2, 3 shows that the relationship between \(b\) and \(g\) and the CEO’s benefit.

Finally we consider the optimal board for the board itself. A board’s preference is the same as shareholders, except disutility accompanied with monitoring in the case of a supervisory board. One may think the board itself prefers a supervisory board because supervision accompanies with disutility. However the following proposition argues that this is not correct:

**Proposition 3.6.** In each the equilibrium in which the CEO reveals or the one in which the CEO conceals, there exists \(\bar{g}_{B}(i) \in (0, +\infty)\) such that:

1. the board always prefers a supervisory board if and only if \(g \geq \bar{g}_{B}(i)\) for \(i = 0, 1,\)
2. the board always prefers a management board if and only if \(0 \leq g < \bar{g}_{B}(i)\) for \(i = 0, 1,\)

**Proof.** See Appendix.

The preference for a board type of the board is the same as shareholders’ (See Figure 3.3.3).

### 3.4 Robustness for Non-Independence of The Board

In the previous subsections, we analyzed the relationship between the optimal board and CEO’s integrity to shareholders. As a result, we found that a super-\(\bar{g}_{C}(b)\) is decreasing with respect to \(b\). Furthermore since for all \(b < b'(\leq \bar{b})\), \(G(b') \subset G(b)\), we can show the relationship between the optimal board for the CEO and \(b\) and \(g\) in figure 1.
visory board is desirable for shareholders if CEO’s project choice bias is sufficiently large and that there exist cases in which the CEO prefers being monitored. However, the assumption that the board has no project choice bias, that is, the board acts with perfect integrity for shareholders, is unrealistic.

In this subsection, we introduce a project choice bias into the board. Therefore the objective function of a supervisory board is modified as the following:

$$U_S = -(y - \epsilon + g_b)^2 - \frac{\pi^2}{2}$$

and the objective function of a management board is modified as the following:

$$U_M = -(y - \epsilon + g_b)^2$$

where $g_b \in [0, g]$ is the board’s project choice bias. The closer to 0 $g_b$ is the closer to shareholders the board’s preference is, and the closer to $g$ $g_b$ is the closer to the CEO the board’s preference is. Therefore we can interpret $g_b$ as independence of the board; small $g_b$ implies high independence.

A supervisory board maximizes:

$$-\pi E[(y_S^*(i) - \epsilon)^2] - (1 - \pi)E[(y_C^* - \epsilon + g_b)^2] - \frac{\pi^2}{2},$$

and $y_S^*(1) = \epsilon - g_b, y_S^*(0) = \frac{1}{2} - g_b$ and $y_C^* = \frac{1}{2} - g$ hold. These imply:

$$\pi(1) = \sigma_M^2 + (g - g_b)^2$$

$$\pi(0) = (g - g_b)^2.$$

**Proposition 3.7.** Monitoring intensity is decreasing with respect to board’s bias.

**Proof.** Obvious.

By substituting $\pi(i), y_S^*(i)$ and $y_C^*$ into $-\pi E[(y_S^*(i) - \epsilon)^2] - (1 - \pi)E[(y_C^* - \epsilon)^2]$, we obtain the maximized value.

**Proposition 3.8.** Given CEO’s bias, shareholder value is decreasing with respect to board’s bias.

**Proof.** Maximized shareholder values are given as:

$$E_{e}U_{V}(SV; i = 1) = -[\sigma_M^2 + (g - g_b)^2] g_b^2 - [1 - \{\sigma_M^2 + (g - g_b)^2]\] (\sigma_M^2 + g^2)$$

$$E_{e}U_{V}(SV; i = 0) = -(g - g_b)^2 (\sigma_M^2 + g_b^2) - \{1 - (g - g_b)^2\} (\sigma_M^2 + g_b^2).$$

Therefore we obtain:

$$\frac{\partial E_{e}U_{V}(SV; i = 1)}{\partial g_b} = -2g_b\sigma_M^2 - 2(g - g_b)(\sigma_M^2 + g^2) < 0$$

$$\frac{\partial E_{e}U_{V}(SV; i = 1)}{\partial g_b} = -4(g - g_b)(\sigma_M^2 + g^2) - 2g_b(g - g_b)^2 < 0$$

and the proof is completed. □

In a management board system, the advising game in a case that a management board is biased is defined as the following:
Definition 3.3. A Perfect Bayesian Nash Equilibrium for the advising game consists of family of advising rules $q(a | \epsilon)$, such that $\sum_{i=0}^2 q(a_i | \epsilon) = 1$ for all $\epsilon \in \{\epsilon_0, \epsilon_1, \epsilon_2\}$, and a project choice function for the CEO, denoted $y_C(a)$, such that:

(a) for each $\epsilon \in \{\epsilon_0, \epsilon_1, \epsilon_2\}$, if $q(a' | \epsilon) > 0$ then
   $$a' \in \arg \max_{a \in A} -[y(a) - \epsilon + g_b]^2$$
   (13)

(b) for each $a \in A$,
   $$y_C(a) \in \arg \max_{y_C} \sum_{i=0}^2 -(y - \epsilon_i + g)^2 p(\epsilon_i | a)$$
   (14)

where $p(\epsilon_i | a) = \frac{q(a | \epsilon_i)}{\sum_{i=0}^2 q(a | \epsilon_i)}$. That is, the CEO has the belief about the board’s advising rules based on Bayes rule.

According this definition of advising game, we can apply the case that the board is biased to propositions in subsection 3.2 by replacing $g$ with $g - g_b$.

Proposition 3.9. In the case of a management board, given $g$, shareholder value is non-decreasing with respect to $g_b$.

Proof. Given $g$, the increment of $g_b$ implies decreasing of $g - g_b$. Because we can apply proposition 3.3 to this case by replacing $g$ with $g - g_b$, decreasing of $g - g_b$ implies achievement of more informative equilibrium. And since $g$ is given, the project choice is never distorted more than in the case of small $g_b$. Therefore increment of $g_b$ brings that of shareholder value. $\square$

That is, since given constant CEO’s bias increment of board’s bias decreases the difference between CEO’s bias and board’s one, the quality of board’s advice improves.

What does effects given above bring comparative effectiveness of a supervisory board and a management board? Figure 3.4.1 shows the change of $EU_S(SV)$ and $EU_S(MG)$ in a case that $g_b$ changes from 0 to $g$ when $g = \frac{1}{2}$ and $b < \frac{1}{6}$ and Figure 3.4.2 shows the same on when $g = \frac{1}{4}$ and $b < \frac{1}{6}$. As Figure 3.4.1 shows, there exists a point such shareholders always prefer a management board if $g_b$ is larger than the point. And, as Figure 3.4.2 shows, if shareholders prefer the management board when board’s bias does not exist, the result does not change when board’s bias increases. This means that a management board is robust to a biased board. The reason for this is following: in a supervisory board system increment of board’s bias decreases monitoring intensity rapidly and deviates the board’s project choice when the board succeeds in taking CEO’s control right. Therefore given CEO’s bias the increment of board’s bias always brings a bad effect on shareholders. On the other hand in a management board system it improves the quality of advice, because the quality of advice is determined depending on only the difference between CEO’s bias and board’s one. Therefore
the closer to CEO’s bias board’s bias is, it can achieve more informative equilibrium. And, because board’s bias does not impact directly on CEO’s project choice, CEO’s decision does not deteriorate as long as CEO’s bias does not become larger. Therefore given CEO’s bias the increment of board’s bias always brings a good effect on shareholders.

As we explain above, increment of board’s bias brings opposite effects on a supervisory board and a management board. From this result we can state that in a situation that CEO’s bias is sufficiently high, a management board is more desirable for shareholders than a supervisory board if board’s bias is also sufficiently high. That is, what we can say from this result is that intensification of board’s monitoring function (for example, intensification of independence of directors) is rational if most of contemporary firms have a biased CEO and biased directors.

4 Concluding Remarks

In our paper, we answer two large questions about two-type boards.

Which of two boards, a supervisory board and a management board, is more desirable for shareholders, a CEO, and a board itself? For sufficiently small CEO’s bias a management board is more desirable for shareholders and a board while for large CEO’s bias a supervisory board is more desirable. On the other hand, when CEO’s control benefit is sufficiently small and CEO’s bias is not significantly small or large, the CEO also prefers a supervisory board. And, the CEO also prefers a management board if shareholders prefer a management board, while in most of situations such that shareholders prefer a supervisory board the CEO prefers a management board.

We interpret these results as policy implications in corporate governance reform in corporate law. Although there are many countries such as Japan that have multiple alternatives about board type that firms adopt, this result gives significant implications to those countries’ governance reform. First, it may be inefficient governance reform that corporate law restricts board type that firms adopt to one board type because shareholders can prefer each board. In Japanese corporate law, we can regard that a management board corresponds to a board in the Kansayaku-kai system and a supervisory board a board in the linkai system. Then, this result implies that Japanese corporate law leaves both two systems and leaves choice of a board that each firm adopts to each firm if we neglect problems else.

Second, what we state above also has important problems. That is, if corporate law leaves choice of a board that each firm adopts to each firm but the CEO has the say, in law or in practice, about choice of a board that a firm adopts, in many cases a board which is undesirable for shareholders is chosen because in many cases CEO’s preference for a board is different from shareholders’ preference. However, if a board has its preference close to shareholders’ preference and a sufficient impact on board choice, a board which is desirable for shareholders can be chosen even if the general meeting of shareholders does not function.
adequately. Therefore the problem of board choice becomes that of incentives for a board to act for shareholders.

Which of two boards is more robust even if a board itself is biased? Given CEO’s project choice bias, a supervisory board becomes ineffective as board’s bias increases, while a management board becomes effective in reverse. This is because in a case of a supervisory board increment of board’s bias lowers monitoring intensity and brings a biased project choice, whereas in a case of a management board increment of board’s bias improves the quality of board’s advice for the CEO and brings no bad effect on shareholder value.

In many countries around the United States, independence of directors are gradually intensified because low independence is considered to lower the quality of monitoring through board’s collusion with the CEO and so on. This paper partially supports the course of this intensification of independence: if it is given that a board is a supervisory board, intensification of independence of a board implies improvement of board’s effectiveness. However, because we cannot learn easily which type a board is near to and intensification of board’s independence brings opposite effect on a supervisory board and a management board, thoughtless intensification of board’s independence may lower quality of board’s function. Therefore, this result suggests that intensification of board’s independence in a situation that we do not know board type sufficiently should not be done.

As we state above, the theme that we consider in this paper is closely related with endogenous determination of board structure. Hermalim and Weisbach (1998) and Hermalim and Weisbach (2003) analyze this point, and we will consider the model that determines board structure including board type endogenously.
5 Appendix

5.1 The Proofs of Propositions in Section 3

Proof of Proposition 3.1. By $y^*_S(1) = \epsilon$, $y^*_S(0) = \frac{1}{2}$ and $\pi(i) = (\sigma^2_M + g^2) - E_e[y^*_S(i) - e^2]$, we obtain:

\[ E_e U_C(i = 1) = (\sigma^2_M + g^2)(-g^2) + [1 - (\sigma^2_M + g^2)](-\sigma^2_M + b) \]  \hspace{1cm} (15)
\[ E_e U_C(i = 0) = g^2 [-(\sigma^2_M + g^2)] + (1 - g^2)(-\sigma^2_M + b). \]  \hspace{1cm} (16)

Since these equations imply that:

\[ E_e U_C(i = 1) - E_e U_C(i = 0) = -(\sigma^2_M - b)\sigma^2_M, \]  \hspace{1cm} (17)

where $\sigma^2_M = \frac{1}{12}$, the proof is completed. \(\square\)

Proof of Proposition 3.2. Since a number of types is three, there can be exist five kinds of PBNEs as the following:

(1) (pooling equilibrium)

(a) The board chooses $q(a \mid e_i) = \mu_1, q(a \mid e_i) = \mu_2$ and $q(a \mid e_i) = 1 - \mu_1 - \mu_2$ for $i = 0, 1, 2$, where $(\mu_1, \mu_2)$ is a profile of real numbers such that satisfies $\mu_1 \geq 0, \mu_2 \geq 0$ and $\mu_1 + \mu_2 \leq 1$.

(b) The CEO chooses $y_C(a) = \frac{1}{2} - g$ for all $a \in A$.

(c) $\sigma^2_r = \frac{1}{6}$

(2) (partially-pooling equilibrium 1)

(a) The board chooses $q(a \mid e_0) = 1, q(a' \mid e_1) = q(a' \mid e_2) = \mu$ and $q(a'' \mid e_1) = q(a'' \mid e_2) = \mu$ where $a \neq a'$ and $\mu$ is an arbitrary real number belonging to $[0, 1]$.

(b) The CEO chooses $y_C(a) = -g$ and $y_C(a') = y_C(a'') = \frac{3}{4} - g$.

(c) $\sigma^2_r = \frac{1}{24}$

(3) (partially-pooling equilibrium 2)

(a) The board chooses $q(a \mid e_0) = 1, q(a' \mid e_1) = q(a' \mid e_2) = \mu$ and $q(a'' \mid e_1) = q(a'' \mid e_2) = \mu$ where $a \neq a'$ and $\mu$ is an arbitrary real number belonging to $[0, 1]$.

(b) The CEO chooses $y_C(a) = \frac{1}{2} - g$ for all $a \in A$.

(c) $\sigma^2_r = \frac{1}{6}$

(4) (partially-pooling equilibrium 3)

(a) The board chooses $q(a \mid e_2) = 1, q(a' \mid e_0) = q(a' \mid e_1) = \mu$ and $q(a'' \mid e_1) = q(a'' \mid e_2) = \mu$ where $a \neq a'$ and $\mu$ is an arbitrary real number belonging to $[0, 1]$. 18
(b) The CEO chooses \( y_C(a) = 1 - g \) and \( y_C(a') = y_C(a'') = \frac{1}{4} - g \).

(c) \( \sigma_e^2 = \frac{1}{24} \).

(5) (separate equilibrium)

(a) The board chooses \( q(a \mid e_0) = 1 \), \( q(a' \mid e_1) = 1 \) and \( q(a'' \mid e_2) = 1 \) where \( a \neq a' \neq a'' \).

(b) The CEO chooses \( y_C(a) = -g \), \( y_C(a') = \frac{1}{2} \) and \( y_C(a'') = 1 - g \).

(c) \( \sigma_e^2 = 0 \).

First we show conditions for which there exists each equilibrium. Because the CEO maximizes \( \sum_{i=0}^{2} (y - e_i + g)^2 p(e_i \mid a) \) with respect to \( y_C \) and \( p(e_i \mid a) = \frac{q(a \mid e_i)}{\sum_{a=0}^{2} q(a \mid e_i)} \), we obtain:

\[
y_C(a) = \sum_{k=0}^{2} p(e_k \mid a) e_k - g.
\]

When the CEO chooses a project according to equation (12) and his belief is constituted by Bayes’ rule, what to do to check whether (1) ~ (5) constitute PBNEs under given \( g \) is to check whether the board has incentives to report messages except the one in equilibrium.

**The case of (1) (pooling equilibrium)**

Obviously, there exists this kind of equilibrium under all \( g \geq 0 \).

**The case of (2) (partially-pooling equilibrium 1)**

For the advising rules constituting a PBNE, it is necessary and sufficient that changing advising rules does not increase board’s payoff. Therefore necessary and sufficient conditions for which there exists this kind of PBNEs is that \( g \) satisfies :

\[
- g^2 \geq -\left( \frac{3}{4} - g \right)^2 = -\left( \frac{1}{2} + g \right)^2 \geq -\left( \frac{1}{4} - g \right)^2 = -\left( \frac{1}{4} + g \right)^2 \geq -(1 + g)^2.
\]

This implies that \( g \leq \frac{3}{8} \). (See Table 1 ~ 4 for payoffs when the board changes advising rules from ones in equilibrium to the other, for cases of (2), (4) and (5).)

**The case of (3) (partially-pooling equilibrium 2)**

This kind of PBNEs gives the CEO the same project choice as the case of (1) and therefore gives the board the same payoff as the case of (1). Hence, necessary and sufficient condition is also the same: for all \( g \geq 0 \), there exists this kind of equilibrium.
The case of (4) (partially-pooling equilibrium 3)  Necessary and sufficient conditions for which there exists this kind of PBNEs is that \( g \) satisfies:

\[
\begin{align*}
-\left(\frac{1}{4} - g\right)^2 & \geq -(1 - g)^2 \\
-\left(\frac{1}{4} + g\right)^2 & \geq -\left(\frac{1}{2} - g\right)^2 \\
-g^2 & \geq -\left(\frac{3}{4} + g\right)^2.
\end{align*}
\]

This implies that \( g \leq \frac{3}{8} \).

The case of (5) (separate equilibrium 3)  Necessary and sufficient conditions for which there exists this kind of PBNEs is that \( g \) satisfies:

\[
\begin{align*}
-g^2 & \geq \left(\frac{1}{2} - g\right)^2 \\
-g^2 & \geq -(1 - g)^2 \\
-g^2 & \geq -\left(\frac{1}{2} + g\right)^2 \\
-g^2 & \geq -(1 + g)^2.
\end{align*}
\]

This implies that \( g \leq \frac{1}{4} \).

Therefore, there exist (1) ~ (5) types of PBNEs if \( g \leq \frac{1}{4} \), there exist (1), (2), (3) and (4) types of PBNEs if \( g \leq \frac{3}{8} \), and there exist (1) and (3) types of PBNEs if \( g > \frac{3}{8} \). This implies that the type (5) of equilibrium is the most informative when \( g \leq \frac{1}{4} \). (2) and (4) are the most informative when \( \frac{1}{4} < g \leq \frac{3}{8} \) and (1) and (3) are the most informative. □

<table>
<thead>
<tr>
<th>( \epsilon_0 )</th>
<th>( \alpha )</th>
<th>( \alpha' ) or ( \alpha'' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-g^2)</td>
<td>(-\left(\frac{1}{2} - g\right)^2)</td>
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<tr>
<td>(-\left(\frac{1}{2} + g\right)^2)</td>
<td>(-\left(\frac{3}{4} + g\right)^2)</td>
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Table 1: the case of (2)

<table>
<thead>
<tr>
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<th>( \alpha )</th>
<th>( \alpha' ) or ( \alpha'' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\left(\frac{1}{2} - g\right)^2)</td>
<td>(-\left(\frac{3}{4} - g\right)^2)</td>
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<tr>
<td>(-\left(\frac{1}{2} + g\right)^2)</td>
<td>(-\left(\frac{3}{4} + g\right)^2)</td>
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</tbody>
</table>

Table 2: the case of (4)

<table>
<thead>
<tr>
<th>( \epsilon_0 )</th>
<th>( \alpha )</th>
<th>( \alpha' ) or ( \alpha'' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-g^2)</td>
<td>(-\left(\frac{3}{4} - g\right)^2)</td>
<td>(-\left(\frac{1}{2} - g\right)^2)</td>
</tr>
<tr>
<td>(-\left(\frac{1}{2} + g\right)^2)</td>
<td>(-\left(\frac{3}{4} + g\right)^2)</td>
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</table>

Table 3: the case of (5)
Proof of Proposition 3.4. First we prove in the case that \( b < \frac{1}{6} \). When \( b < \frac{1}{6} \),

\[
EU_V(SV) - EU_V(MG) = \begin{cases} 
-\left(\frac{5}{6} - g^2\right)\left(\frac{1}{6} + g^2\right) - (-g^2) & \text{if } g \leq \frac{1}{4} \\
-\left(\frac{5}{6} - g^2\right)\left(\frac{1}{6} + g^2\right) - \left(-\frac{1}{24} - g^2\right) & \text{if } \frac{1}{4} < g \leq \frac{3}{8} \\
-\left(\frac{5}{6} - g^2\right)\left(\frac{1}{6} + g^2\right) - \left(-\frac{1}{6} - g^2\right) & \text{if } \frac{3}{8} < g
\end{cases}
\]

hold. If \( g \leq \frac{1}{4} \), \( EU_V(SV) - EU_V(MG) = -\frac{5}{36} + g^2 + g^4 < 0 \). If \( \frac{1}{4} < g \leq \frac{3}{8} \), \( EU_V(SV) - EU_V(MG) = -\frac{7}{72} + g^2 + g^4 < 0 \). If \( \frac{3}{8} < g \), \( EU_V(SV) - EU_V(MG) = \frac{1}{36} + g^2 + g^4 > 0 \). Therefore if \( b < \frac{1}{6} \), shareholders always prefer the supervisory board if and only if \( g > \frac{3}{8} \), and then we obtain \( \bar{g}_S(1) = \frac{3}{8} \).

Next we prove in the case that \( b \geq \frac{1}{6} \). When \( b \geq \frac{1}{6} \),

\[
EU_V(SV) - EU_V(MG) = \begin{cases} 
-g^2\left(\frac{1}{6}\right) - (1 - g^2)\left(\frac{1}{6} + g^2\right) - (-g^2) & \text{if } g \leq \frac{1}{4} \\
-g^2\left(\frac{1}{6}\right) - (1 - g^2)\left(\frac{1}{6} + g^2\right) - \left(-\frac{1}{24} - g^2\right) & \text{if } \frac{1}{4} < g \leq \frac{3}{8} \\
-g^2\left(\frac{1}{6}\right) - (1 - g^2)\left(\frac{1}{6} + g^2\right) - \left(-\frac{1}{6} - g^2\right) & \text{if } \frac{3}{8} < g
\end{cases}
\]

hold. If \( g \leq \frac{1}{4} \), \( EU_V(SV) - EU_V(MG) = -\frac{1}{6} + g^2 < 0 \). If \( \frac{1}{4} < g \leq \frac{3}{8} \), \( EU_V(SV) - EU_V(MG) = -\frac{1}{6} + g^2 < 0 \). Furthermore, if \( g > \frac{3}{8} \), \( EU_V(SV) - EU_V(MG) = g^4 > 0 \). Therefore if \( b \geq \frac{1}{6} \), shareholders always prefer the supervisory board if and only if \( g > \frac{3}{8} \), and then we obtain \( \bar{g}_S(0) = \frac{3}{8} \). Hence the proof is completed. □

Proof of Proposition 3.5. First we show that the CEO always prefers the management board if \( b \geq \frac{1}{6} \), that is, if the CEO reveals his private information only to the management board.

\( EU_C(SV) - EU_C(MG) \) is given the following:

\[
EU_C(SV) - EU_C(MG) = \begin{cases} 
-g^2\left(\frac{1}{6} + g^2\right) - (1 - g^2)\left(\frac{1}{6} - b\right) - b & \text{if } g \leq \frac{1}{4} \\
-g^2\left(\frac{1}{6} + g^2\right) - (1 - g^2)\left(\frac{1}{6} - b\right) - \left(-\frac{1}{24} + b\right) & \text{if } \frac{1}{4} < g \leq \frac{3}{8} \\
-g^2\left(\frac{1}{6} + g^2\right) - (1 - g^2)\left(\frac{1}{6} - b\right) - \left(-\frac{1}{6} + b\right) & \text{if } \frac{3}{8} < g
\end{cases}
\]

Because \( b \geq 0 \), in all cases \( EU_C(SV) - EU_C(MG) < 0 \) must hold for all \( g > 0 \). Hence the CEO always prefers the management board if \( b \geq \frac{1}{6} \).

Second we show that the CEO always prefer the management board if both \( 0 \leq g \leq \frac{3}{8} \) and \( 0 \leq b < \frac{1}{6} \) hold. When \( 0 \leq g \leq \frac{1}{4} \),

\[
EU_C(SV) - EU_C(MG) = -\left(\frac{1}{6} + g^2\right)g^2 - \left[1 - (\frac{1}{6} + g^2)\right]\left(\frac{1}{6} - b\right) - b
= -g^4 - g^2b - \frac{b}{6} - \frac{5}{36}
\]

holds. Because \( EU_C(SV) - EU_C(MG) = -\frac{b}{6} - \frac{5}{36} < 0 \) if \( g = 0 \) and the RHS of the above equation is strictly decreasing with respect \( g \) when \( g \geq 0 \), for each \( g \in [0, \frac{1}{4}] \), \( EU_C(SV) - EU_C(MG) < 0 \), that is, the CEO always prefers a management board.
When $\frac{1}{4} < g \leq \frac{1}{5}$,

$$EU_C(SV) - EU_C(MG) = -\left(\frac{1}{6} + g^2\right)g^2 - \left[1 - \left(\frac{1}{6} + g^2\right)\right] \left(\frac{1}{6} - b\right) - \left(-\frac{1}{24} + b\right)$$

$$= -g^4 - g^2b - \frac{b}{6} - \frac{7}{72}$$

holds. Although $EU_C(SV) - EU_C(MG) = \frac{5b}{48} - \frac{233}{2304}$ when $g = \frac{1}{4}$, $\frac{5b}{48} - \frac{233}{2304} > 0$ implies $b > \frac{231}{256}$ and then it does not satisfy $b \leq \frac{1}{5}$; thus the CEO always prefers a management board also in this case.

Finally we show that there exists profiles of $b$ and $G(b)$ such that the CEO always prefers a supervisory board if and only if $g \in G(b)$ when $g > \frac{3}{8}$ and $b < \frac{1}{5}$.

When $g > \frac{3}{8}$ and $b < \frac{1}{5}$ hold, the most informative equilibrium is pooling equilibrium and the CEO reveals his private information. Therefore,

$$EU_C(SV) - EU_C(MG) = -\left(\frac{1}{6} + g^2\right)g^2 - \left[1 - \left(\frac{1}{6} + g^2\right)\right] \left(\frac{1}{6} - b\right) - \left(-\frac{1}{6} + b\right)$$

$$= -g^4 - g^2b - \frac{b}{6} + \frac{1}{36}$$

Solving the equation $-g^4 - g^2b - \frac{b}{6} + \frac{1}{36} = 0$ with respect to $g$, we obtain two real solutions:

$$g = \frac{\sqrt{1 - 6b}}{\sqrt{6}}, \quad -\frac{\sqrt{1 - 6b}}{\sqrt{6}},$$

and the LHS is strictly decreasing with respect to $g$ when $g \geq 0$. Therefore, if $EU_C(SV) - EU_C(MG) > 0$ when $g = \frac{3}{8}$ and $\frac{\sqrt{1 - 6b}}{\sqrt{6}} > \frac{1}{5}$ holds, there exists the area of $b$ such that there exists $G(b) = (g_C(b), \bar{g}_C(b)) = (\frac{3}{8}, \frac{\sqrt{1 - 6b}}{\sqrt{6}})$ such that $-g^4 - g^2b - \frac{b}{6} + \frac{1}{36} > 0$ holds if $g \in G(b)$. And, by calculation of $\frac{\sqrt{1 - 6b}}{\sqrt{6}} > \frac{3}{8}$ we obtain $b < \frac{5}{172} = \bar{b}$ and this satisfies $b < \frac{1}{5}$. Furthermore, if $b \geq \bar{b}$, $G(b) = \phi$. Hence the proof is completed. \(\square\)

**Proof of Proposition 3.6.** The way of proof is almost the same as proof of proposition 3.4. When $b < \frac{1}{5}$,

$$EU_B(SV) - EU_B(MG) = \begin{cases} 
[1 - \left(\frac{1}{6} + g^2\right)\left(\frac{1}{6} + g^2\right)] \left(\frac{1}{6} + g^2\right) - \frac{1}{2} \left(\frac{1}{6} + g^2\right)^2 - (-g^2) & \text{if } g \leq \frac{1}{4} \\
[1 - \left(\frac{1}{6} + g^2\right)\left(\frac{1}{6} + g^2\right)] \left(\frac{1}{6} + g^2\right) - \frac{1}{2} \left(\frac{1}{6} + g^2\right)^2 - \left(-\frac{1}{24} - g^2\right) & \text{if } \frac{1}{4} < g \leq \frac{3}{8} \\
[1 - \left(\frac{1}{6} + g^2\right)\left(\frac{1}{6} + g^2\right)] \left(\frac{1}{6} + g^2\right) - \frac{1}{2} \left(\frac{1}{6} + g^2\right)^2 - \left(-\frac{1}{6} - g^2\right) & \text{if } g > \frac{3}{8}
\end{cases}$$

hold. If $0 \leq g \leq \frac{1}{8}$, $EU_B(SV) - EU_B(MG) < 0$. If $g > \frac{1}{8}$, $EU_B(SV) - EU_B(MG) > 0$. Therefore if $b < \frac{1}{5}$, the board itself always prefers the supervisory board if and only if $g > \frac{3}{8}$, and then we obtain $g_B(1) = \frac{3}{8}$.

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5We neglect imaginary solutions.
When $b \geq \frac{1}{6}$,

\[
EU_B(SV) - EU_B(MG) = \begin{cases} 
-g^2 \left( \frac{1}{6} \right) - \left( 1 - g^2 \right) \left( \frac{1}{6} + g^2 \right) - \frac{1}{2} g^4 - (-g^2) & \text{if } g \leq \frac{1}{4} \\
-g^2 \left( \frac{1}{6} \right) - \left( 1 - g^2 \right) \left( \frac{1}{6} + g^2 \right) - \frac{1}{2} g^4 - \left( -\frac{1}{24} - g^2 \right) & \text{if } \frac{1}{4} < g \leq \frac{3}{8} \\
-g^2 \left( \frac{1}{6} \right) - \left( 1 - g^2 \right) \left( \frac{1}{6} + g^2 \right) - \frac{1}{2} g^4 - \left( -\frac{1}{6} - g^2 \right) & \text{if } g > \frac{3}{8}
\end{cases}
\]

hold. If $0 \leq g \leq \frac{3}{8}, EU_B(SV) - EU_B(MG) < 0$. If $g > \frac{3}{8}, EU_B(SV) - EU_B(MG) > 0$. Therefore if $b < \frac{1}{6}$, the board itself always prefers the supervisory board if and only if $g > \frac{3}{8}$, and then we obtain $\bar{g}_B(0) = \frac{3}{8}$. Hence the proof is completed. □
Figure 2.1.1: the game tree in a case of a supervisory board.
Figure 2.1.2: the game tree in a case of a management board.
Figure 3.2.1: the game tree of the advising game
Figure 3.3.1: which board do shareholders prefer?

Figure 3.3.2: which board does the CEO prefer?
Figure 3.3.3: which board does the board prefer?
Figure 3.4.1: the change of $EU_S(SV)$ and $EU_S(MG)$ w.r.t. $g_b$ when $g = \frac{1}{2}$

Figure 3.4.2: the change of $EU_S(SV)$ and $EU_S(MG)$ w.r.t. $g_b$ when $g = \frac{1}{3}$
References


